Physics
OSAKA UNIVERSITY

重イオン加速器を用いた高エネルギー光渦の生成

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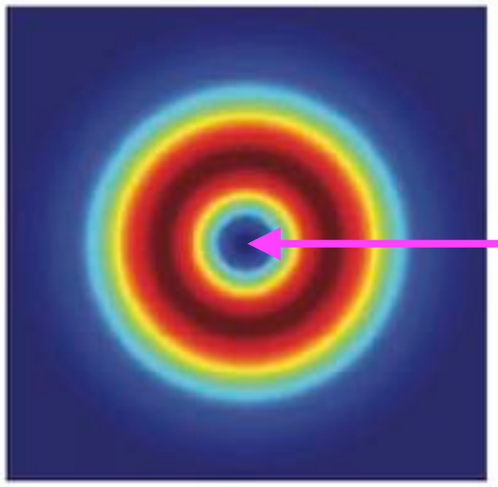
Twisted photons

Orbital angular momentum (OAM) of light

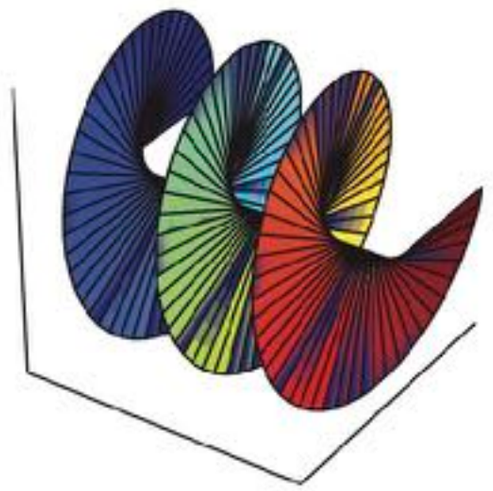
winding field phase ~ $e^{im\varphi}$

G. Molina-Terriza et al.
Nat. Phys. 3, 305 (2007)



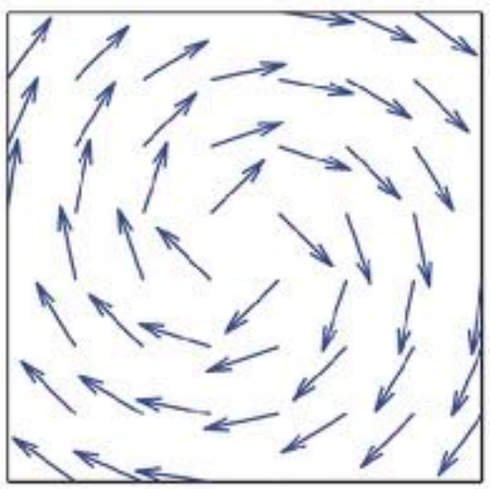
field intensity

phase singularity

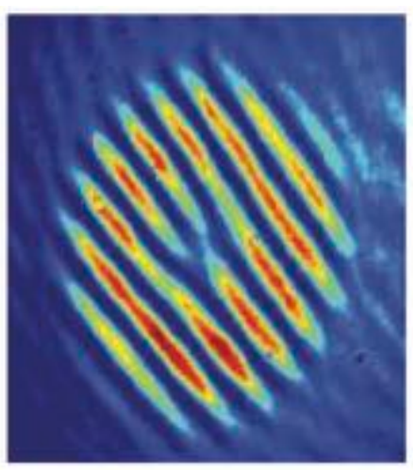


wave front

helicoid



transvers Poynting vector



interference pattern with PW

computer-generated hologram

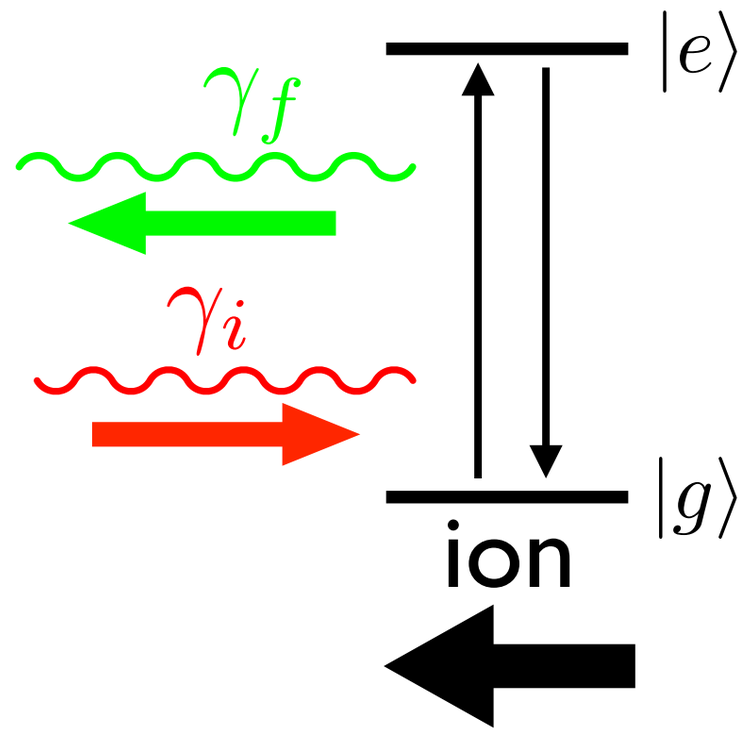
Energy up-conversion with boosted ions

Rayleigh scattering:

$$\gamma_i + |g\rangle \to |e\rangle \to |g\rangle + \gamma_f$$

Boosted ions: $E = \gamma M$

e.g. $\gamma \sim 10^3$ @LHC



Level splitting: E_{eg}

binding energy of H-like ion $= (Z^2/n^2)13.6$ eV

Resonance condition: $2\gamma\omega_i \simeq E_{eg}$

optical laser $\omega_i \sim 1$ eV $\longrightarrow$ $Z^2/2\gamma \sim 0.1$

heavy ion

Up-conversion:

$$\omega_f^{\max} \simeq 2\gamma E_{eg} \simeq 4\gamma^2\omega_i \sim 100 \text{ MeV}(2\gamma/10^4)^2$$

Bessel beam: gauge field

Jentschura, Serbo,
PRL106, 013001(2011)

Twisted photon as a superposition of plane waves

PW: $A^{\mu}_{\boldsymbol{k}\lambda}(t,\boldsymbol{x}) = \varepsilon^{\mu}_{\lambda}(\boldsymbol{k}) e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}/\sqrt{2\omega}$

Twisted photon (Bessel beam):

$$A^{\mu}_{m k_T k_z \lambda}(t,\boldsymbol{x}) := \int a_{m k_T}(\boldsymbol{k}_T) A^{\mu}_{\boldsymbol{k}\lambda}(t,\boldsymbol{x}) dk_T^2/(2\pi)^2$$

$$a_{m k_T}(\boldsymbol{k}_T) := (-i)^m e^{im\varphi_k} \sqrt{2\pi/k_T}\, \delta(|\boldsymbol{k}_T| - k_T)$$

$$A^{\mu}_{m k_T k_z \lambda}(t,\boldsymbol{x}) = -i\lambda \sqrt{k_T/4\pi\omega}\, e^{-i(\omega t - k_z z)} \Big[e^{i(m-\lambda)\varphi} \cos^2\frac{\theta_k}{2} J_{m-\lambda}(k_T\rho)\eta^{\mu}_{\lambda}$$

$$- e^{i(m+\lambda)\varphi} \sin^2\frac{\theta_k}{2} J_{m+\lambda}(k_T\rho)\eta^{\mu}_{-\lambda} + \frac{i}{\sqrt{2}} e^{im\varphi} \sin\theta_k J_m(k_T\rho)\eta^{\mu}_0 \Big]$$

$$\eta^{\mu}_{\lambda} := (0, -\lambda, -i, 0)/\sqrt{2}\,,\ \eta^{\mu}_0 := (0,0,0,1)$$

$$\sin\theta_k := |\boldsymbol{k}_T|/|\boldsymbol{k}|$$ pitch angle

Orthogonality:

$$\int d^3x (A_{m'k'_Tk'_z\lambda'})^*_\mu \, A^\mu_{mk_Tk_z\lambda} = -(\pi/\omega)\delta(k_z - k'_z)\delta(k_T - k'_T)\delta_{mm'}\delta_{\lambda\lambda'}$$

Transversality: $\boldsymbol{\nabla} \cdot \boldsymbol{A}_{mk_Tk_z\lambda} = A^0_{mk_Tk_z\lambda} = 0$

Poynting vector: $S_\rho = 0\,,\ S_\varphi \propto \sin\theta_k\,,\ S_z \neq 0$

Angular momentum:

$$\boldsymbol{J} = \int d^3x\, \boldsymbol{x} \times \boldsymbol{S} = -\mathrm{Re}\int d^3x \left[\dot{A}^*_j(\boldsymbol{x} \times \boldsymbol{\nabla})A_j + \dot{\boldsymbol{A}}^* \times \boldsymbol{A}\right]$$

$$=: \boldsymbol{L} + \boldsymbol{s}$$ OAM + spin

$$L_z = m - \lambda\cos\theta_k\,,\ \ s_z = \lambda\cos\theta_k \Rightarrow J_z = m$$

Afanasev, Carlson, Mukherjee, IJMP25, 1460048 (2014)

Heavy ion excitation by twisted photons

Relativistic effects ~ $O(Z\alpha)$

Dirac theory of Hydrogen-like ion:

interaction hamiltonian $H_I = e\,\boldsymbol{\alpha}\cdot\boldsymbol{A}$

wave function

$$\psi(\boldsymbol{x}) = \begin{pmatrix} \dfrac{G(r)}{r}\mathcal{Y}^{j_3}_{j\ell_A}(\theta,\varphi) \\ i\dfrac{F(r)}{r}\mathcal{Y}^{j_3}_{j\ell_B}(\theta,\varphi) \end{pmatrix}$$

transition matrix element: $|i\rangle \to |f\rangle$

$$\mathcal{M}_{fi} = e\int d^3x\,\psi^\dagger_f(\boldsymbol{x})\boldsymbol{\alpha}\psi_i(\boldsymbol{x})\cdot\boldsymbol{A}(t,\boldsymbol{x})e^{i\omega t}$$

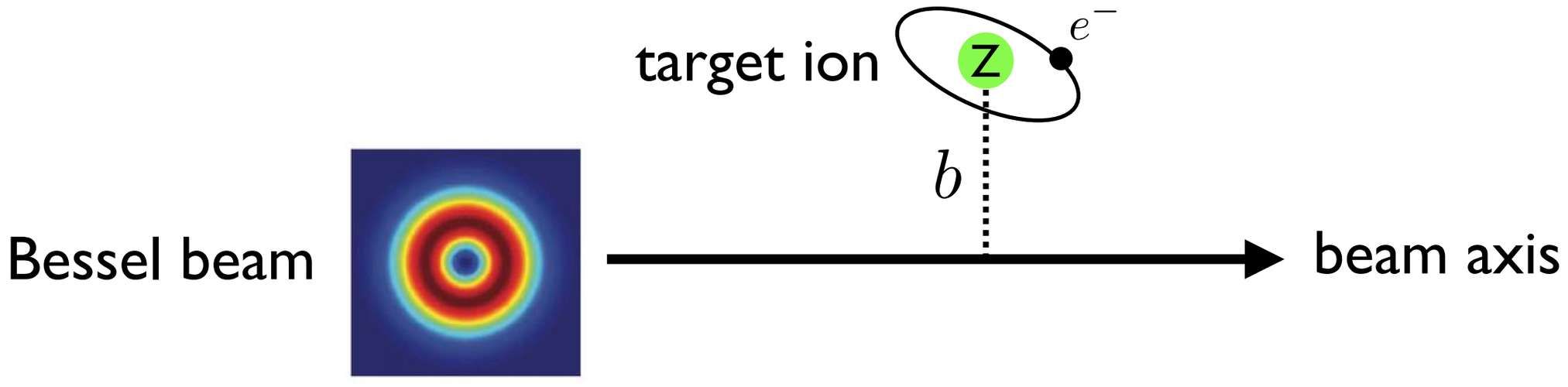
Twisted photon amplitude

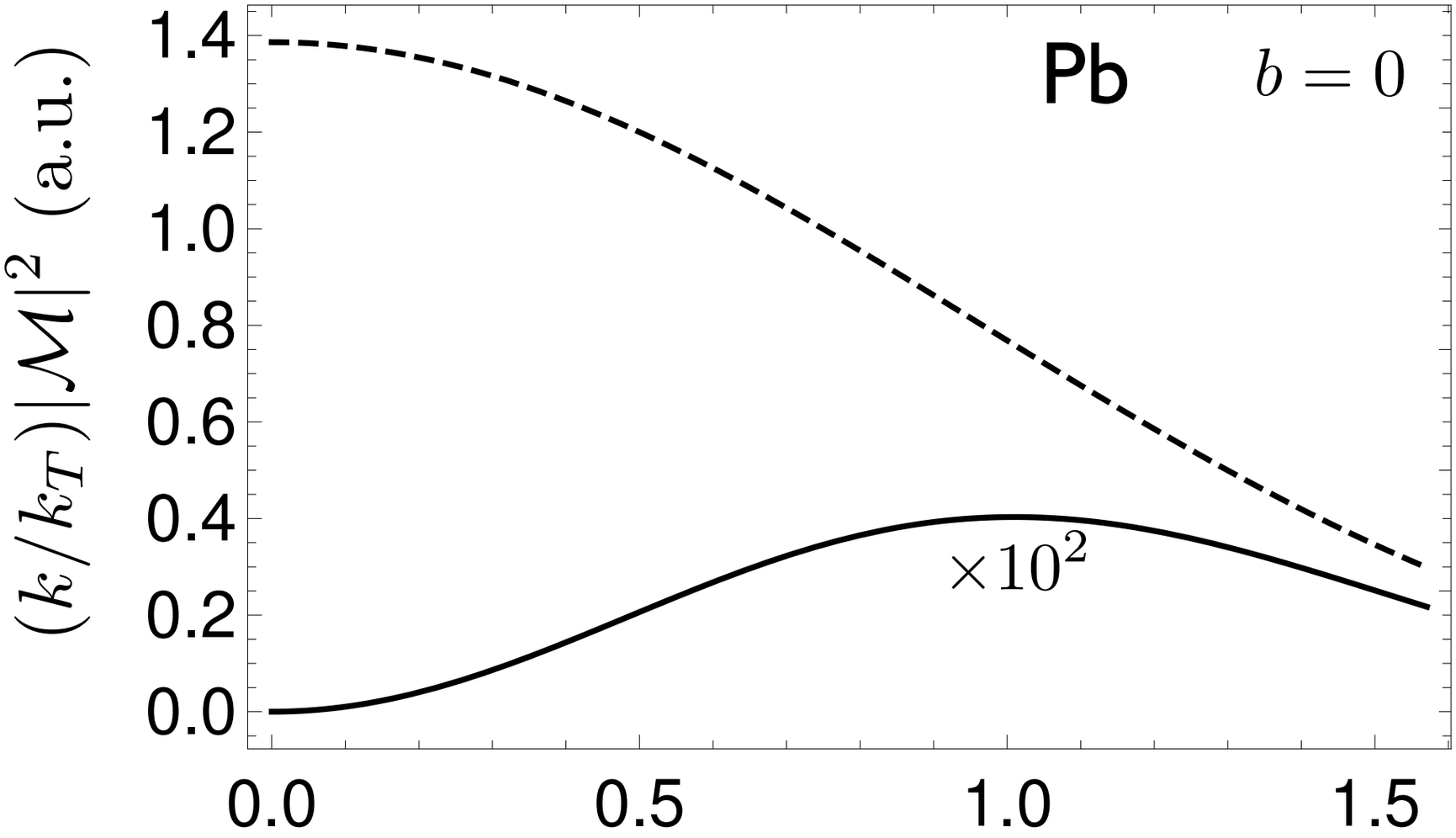
Scholz-Marggraf et al.
PRA90, 013425(2014)

a superposition of plane wave amplitudes

$$\mathcal{M}_{fi}^{(\mathrm{tw})} = (-i)^{2m+m_i-m_f}\sqrt{\frac{k_T}{2\pi}}\, e^{i(m+m_i-m_f)\phi_b}\, J_{m+m_f-m_i}(k_T b)$$
$$\times \sum_{m'_f, m'_i} d^{j_f}_{m_f m'_f}(\theta_k) d^{j_i}_{m_i m'_i}(\theta_k) \mathcal{M}^{(\mathrm{pl})}_{m'_f m'_i}$$

impact parameter: $\boldsymbol{b} = b(\cos\phi_b, \sin\phi_b, 0)$





——— $1\mathrm{s}_{1/2}(m_i = 1/2) \to 3\mathrm{d}_{5/2}(m_f = 5/2)\,,\ \lambda = 1\,,\ m = 2$

plane-wave forbidden

--------- $1\mathrm{s}_{1/2}(m_i = 1/2) \to 2\mathrm{p}_{3/2}(m_f = 3/2)\,,\ \lambda = 1\,,\ m = 1$

plane-wave allowed

Summary

- ✸ Twisted photons: OAM of light
- ✸ Energy up-conversion with boosted ions

$$\omega_f^{\max} \simeq 4\gamma^2 \omega_i \,, \;\; 2\gamma \sim 10^4$$ optical → gamma ray

- ✸ Twisted photons: forbidden → allowed
- ✸ Absorption & emission rates

Relativistic calculation for heavy ions

ONGOING