原子・分子過程による ニュートリノ物理

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Refs. : A.Fukumi et al. PTEP (2012) 04D002, arXiv:1211.4904
D.N. Dinh, S.T. Petcov, N. Sasao, M.T., M.Yoshimura
PLB719(2013)154, arXiv:1209.4808

益川塾セミナー, 2014/01/22 @ 京産大

INTRODUCTION

What we know about neutrino mass and mixing

Masses:

$$\Delta m^2_{21} = 7.54 \times 10^{-5}\ \mathrm{eV}^2\,, \quad |\Delta m^2_{31(32)}| = 2.47\ (2.46) \times 10^{-3}\ \mathrm{eV}^2$$

Fogli et al. (2012)

$\sum m_\nu \leqslant 0.58\ \mathrm{eV}$ Jarosik et al. (2011)

Mixing: $U = V_{\mathrm{PMNS}}\, P$

$$V_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$P = \mathrm{diag.}(1, e^{i\alpha}, e^{i\beta})$ Majorana phases

Bilenky, Hosek, Petcov; Doi, Kotani, Nishiura, Okuda, Takasugi; Schechter, Valle

$$s^2_{12} \simeq 0.31,\ s^2_{23} \simeq 0.39,\ s^2_{13} \simeq 0.024$$

Fogli et al. (2012)

Undetermined properties of neutrinos

Absolute mass

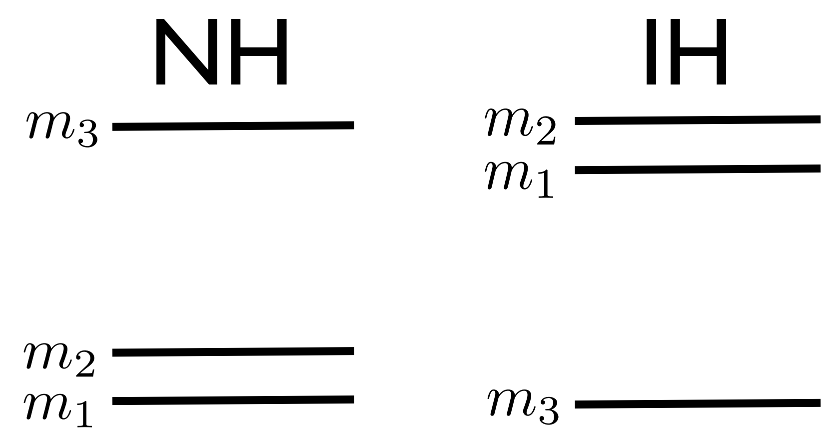
$$m_{1(3)} < 0.19\,\mathrm{eV}\,,\ \ 0.050\,\mathrm{eV} < m_{3(2)} < 0.58\,\mathrm{eV}$$

Mass type

Dirac or Majorana

Hierarchy pattern

normal or inverted



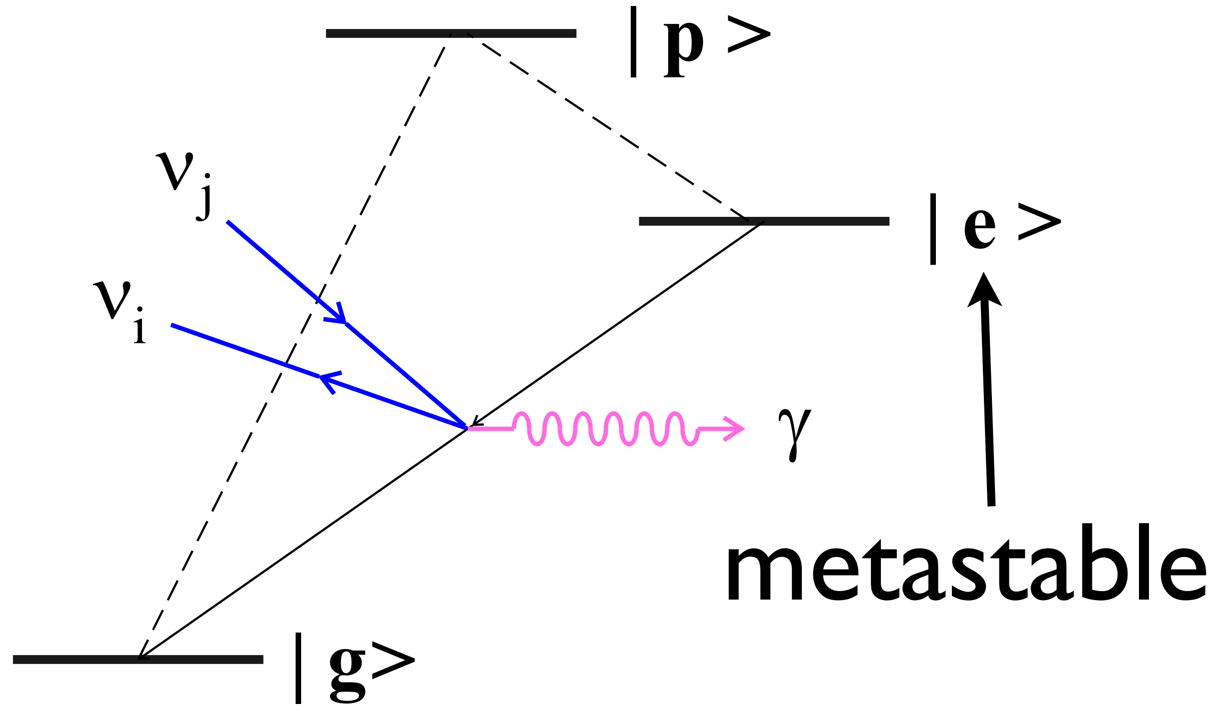
CP violation

one Dirac phase, two Majorana phases

Atomic/molecular processes may help.

RENP

Radiative Emission of Neutrino Pair (RENP)



Λ-type level structure

Ba, Xe, Ca+, Yb,...

H_2, O_2, I_2, ...

Atomic/molecular energy scale ~ eV or less

close to the neutrino mass scale

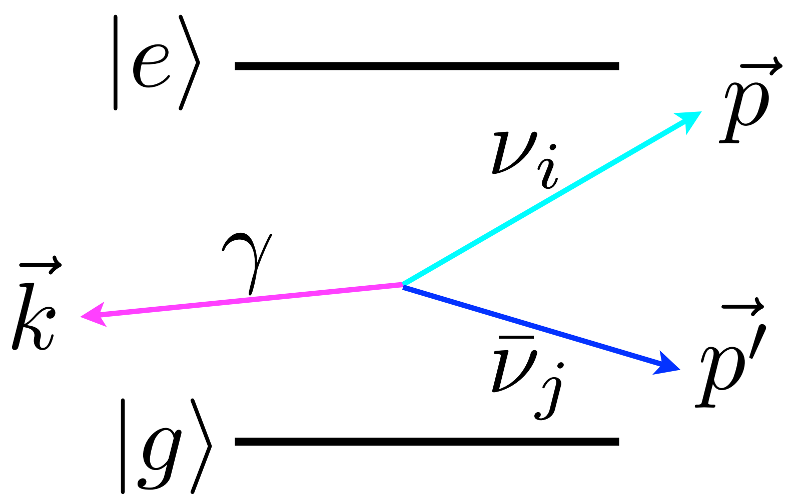
cf. nuclear processes ~ MeV

Rate $\sim \alpha G_F^2 E^5 \sim 1/(10^{33}\,\mathrm{s})$

Enhancement mechanism?

Macro-coherence

Yoshimura et al. (2008)



position of atom

$$\propto e^{-i(\vec{k}+\vec{p}+\vec{p}')\cdot\vec{x}_a}(2\pi)\delta(\epsilon_{eg}-\omega-E_p-E_{p'})$$

$$(\epsilon_{eg}=\epsilon_e-\epsilon_g,\ \omega=|\vec{k}|)$$

Macroscopic target of N atoms, volume V (n=N/V)

total amp. $\propto \sum_a e^{-i(\vec{k}+\vec{p}+\vec{p}')\cdot\vec{x}_a} \simeq \frac{N}{V}(2\pi)^3\delta^3(\vec{k}+\vec{p}+\vec{p}')$

$$d\Gamma \propto n^2V(2\pi)^4\delta^4(q-p-p') \qquad q^\mu=(\epsilon_{eg}-\omega,-\vec{k})$$

macro-coherent amplification

RENP spectrum

Energy-momentum conservation
due to the macro-coherence
→ familiar 3-body decay kinematics

Six thresholds of the photon energy

$$\omega_{ij} = \frac{\epsilon_{eg}}{2} - \frac{(m_i + m_j)^2}{2\epsilon_{eg}} \qquad i, j = 1, 2, 3$$

$\epsilon_{eg} = \epsilon_e - \epsilon_g$ atomic energy diff.

Required energy resolution $\sim O(10^{-6})\,\mathrm{eV}$

typical laser linewidth

$$\Delta\omega_{\mathrm{trig.}} \lesssim 1\,\mathrm{GHz} \sim O(10^{-6})\,\mathrm{eV}$$

RENP rate formula

dynamical factor

$$\Gamma_{\gamma 2\nu}(\omega, t) = \Gamma_0 I(\omega)\eta_\omega(t)$$

spectral function

overall rate

Overall rate (Xe-type target)

macro-coherence

~ field energy density

$$\Gamma_0 = \frac{3n^2 V G_F^2 \gamma_{pg}\epsilon_{eg} n}{2\epsilon_{pg}^3}(2J_p+1)C_{ep} \sim 1\,\mathrm{Hz}\,(n/10^{22}\mathrm{cm}^{-3})^3(V/10^2\,\mathrm{cm}^3)$$

$\gamma_{pg} : |p\rangle \to |g\rangle$ rate

$(2J_p+1)C_{ep}$: atomic spin factor

Spectral function

$$I(\omega) = F(\omega)/(\epsilon_{pg} - \omega)^2$$

$$F(\omega) = \sum_{ij} \Delta_{ij}(B_{ij} I_{ij}(\omega) - \delta_M B_{ij}^M m_i m_j)\theta(\omega_{ij} - \omega)$$

$$\Delta_{ij}^2 = 1 - 2\frac{m_i^2 + m_j^2}{q^2} + \frac{(m_i^2 - m_j^2)^2}{q^4} \qquad q^2 = (p_i + p_j)^2$$

$$I_{ij}(\omega) = \frac{q^2}{6}\left[2 - \frac{m_i^2 + m_j^2}{q^2} - \frac{(m_i^2 - m_j^2)^2}{q^4}\right] + \frac{\omega^2}{9}\left[1 + \frac{m_i^2 + m_j^2}{q^2} - 2\frac{(m_i^2 - m_j^2)^2}{q^4}\right]$$

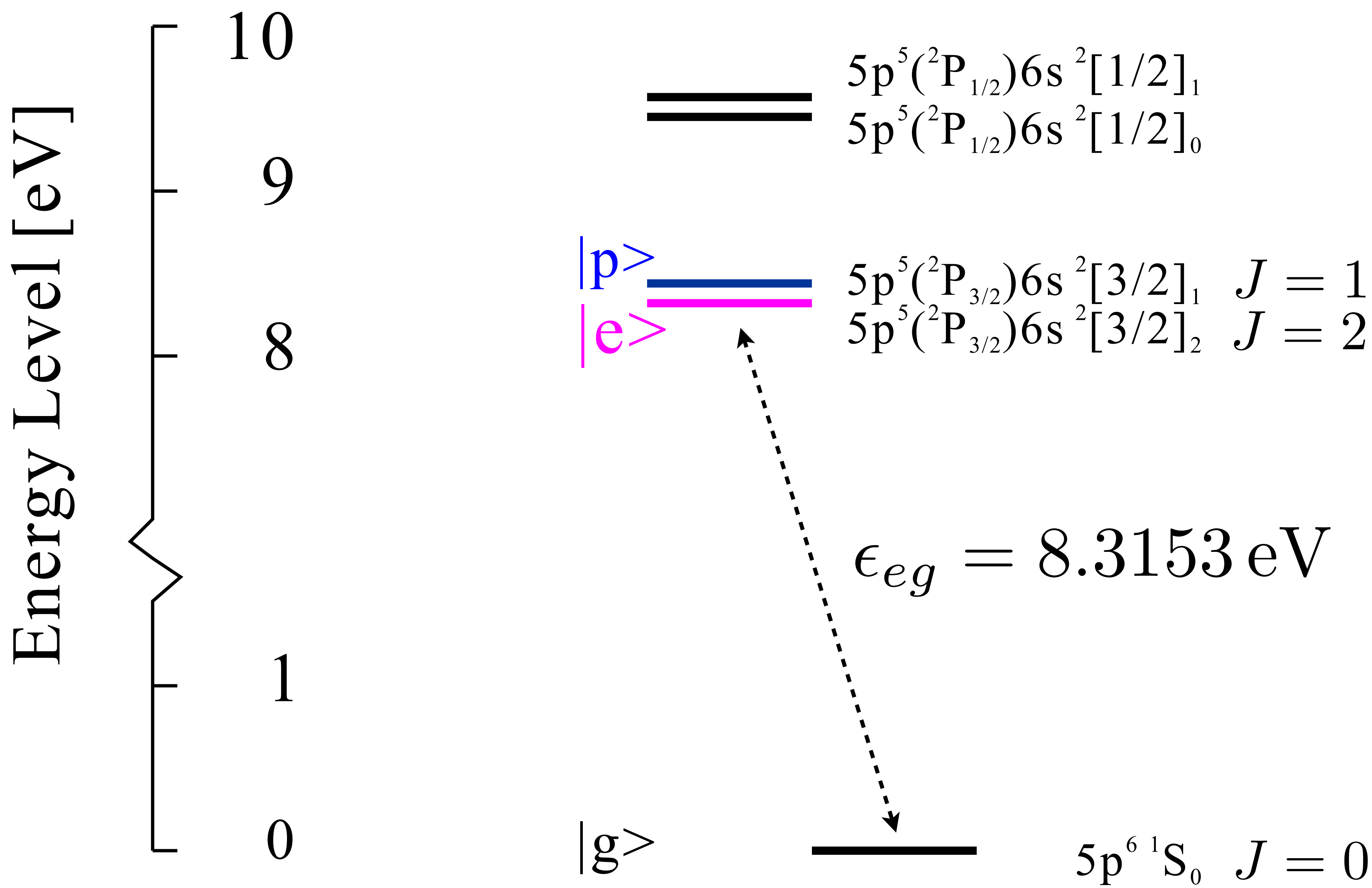
$\delta_M = 0(1)$ for Dirac(Majorana)

$$B_{ij} = |U_{ei}^* U_{ej} - \delta_{ij}/2|^2, B_{ij}^M = \Re[(U_{ei}^* U_{ej} - \delta_{ij}/2)^2]$$

Dynamical factor

$$\sim |\text{coherence} \times \text{field}|^2$$

Xe (gas target)

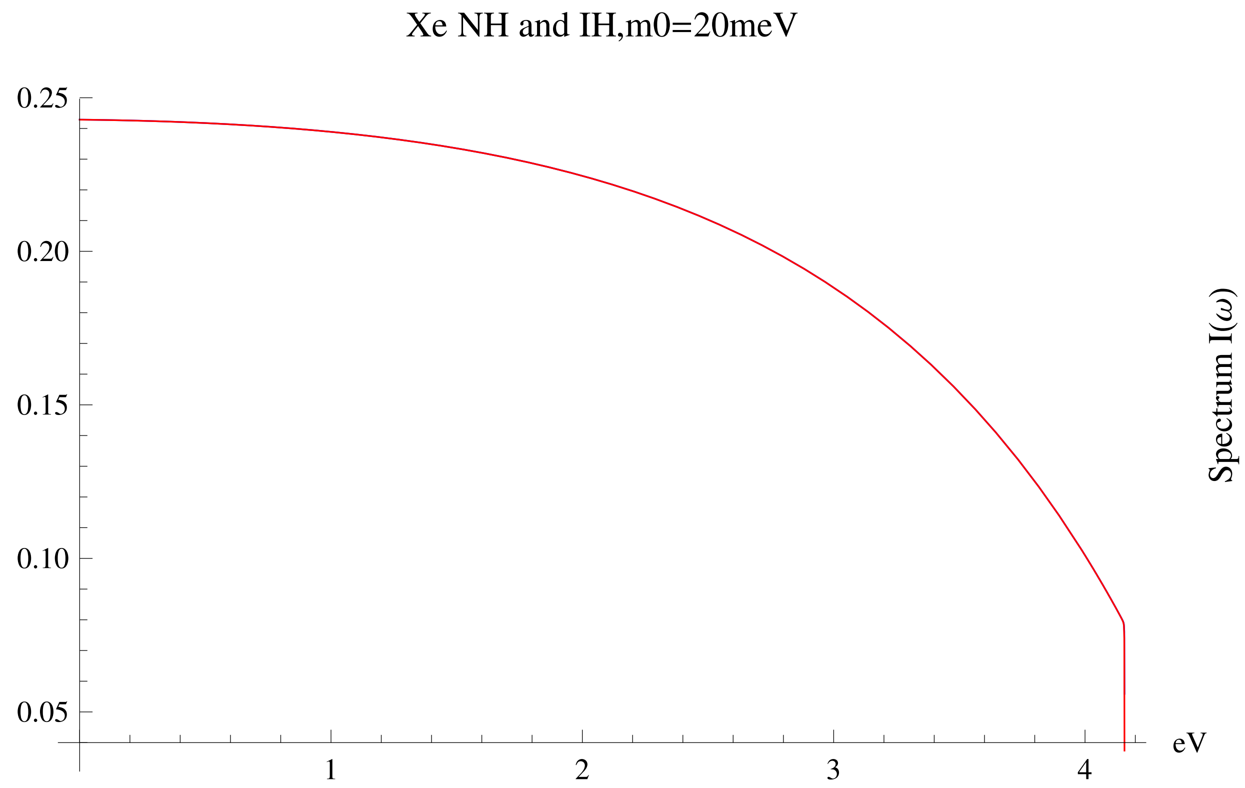


$$|e\rangle \leftrightarrow |p\rangle \quad \mathrm{M1}$$

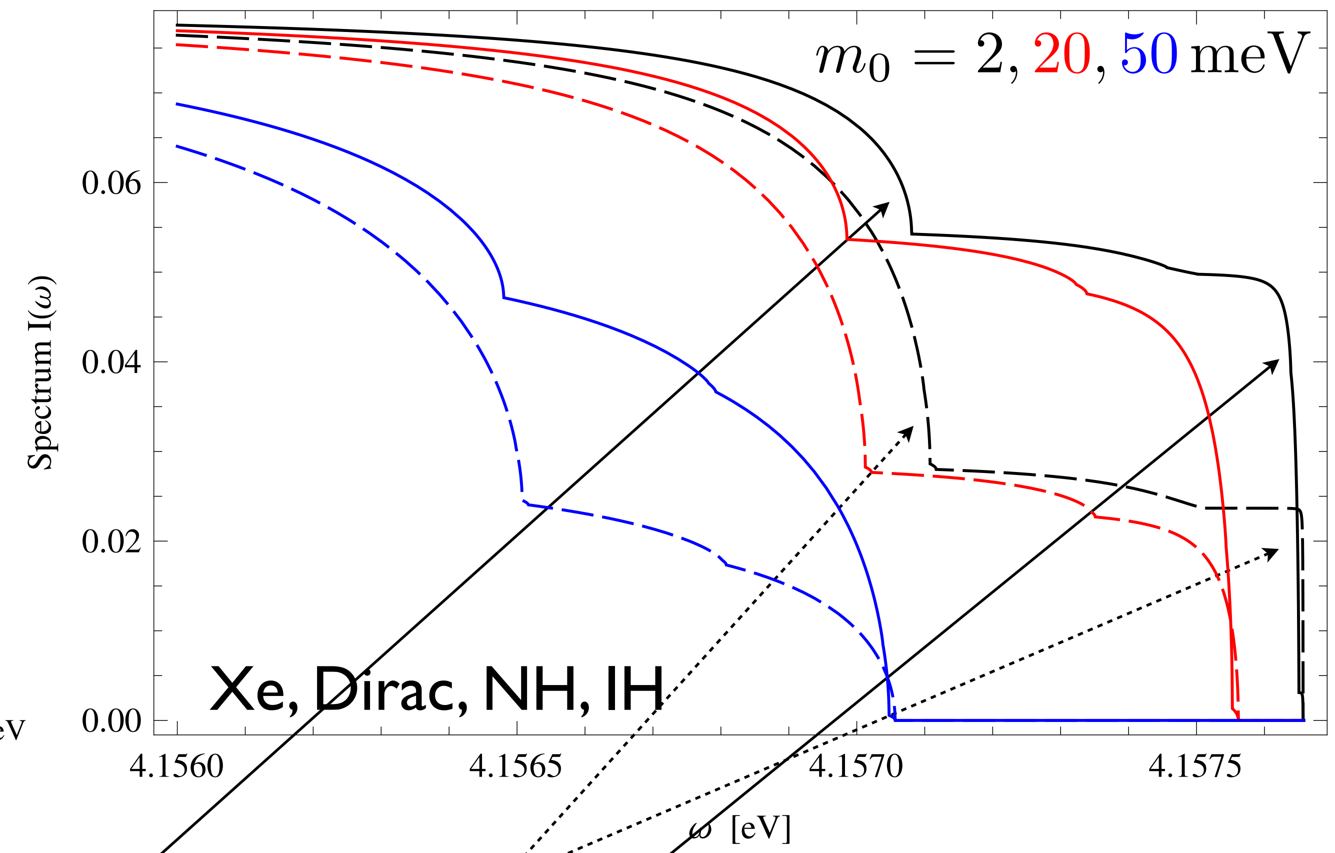
$$|p\rangle \leftrightarrow |g\rangle \quad \mathrm{E1}$$

Photon spectrum

Global shape



Threshold region



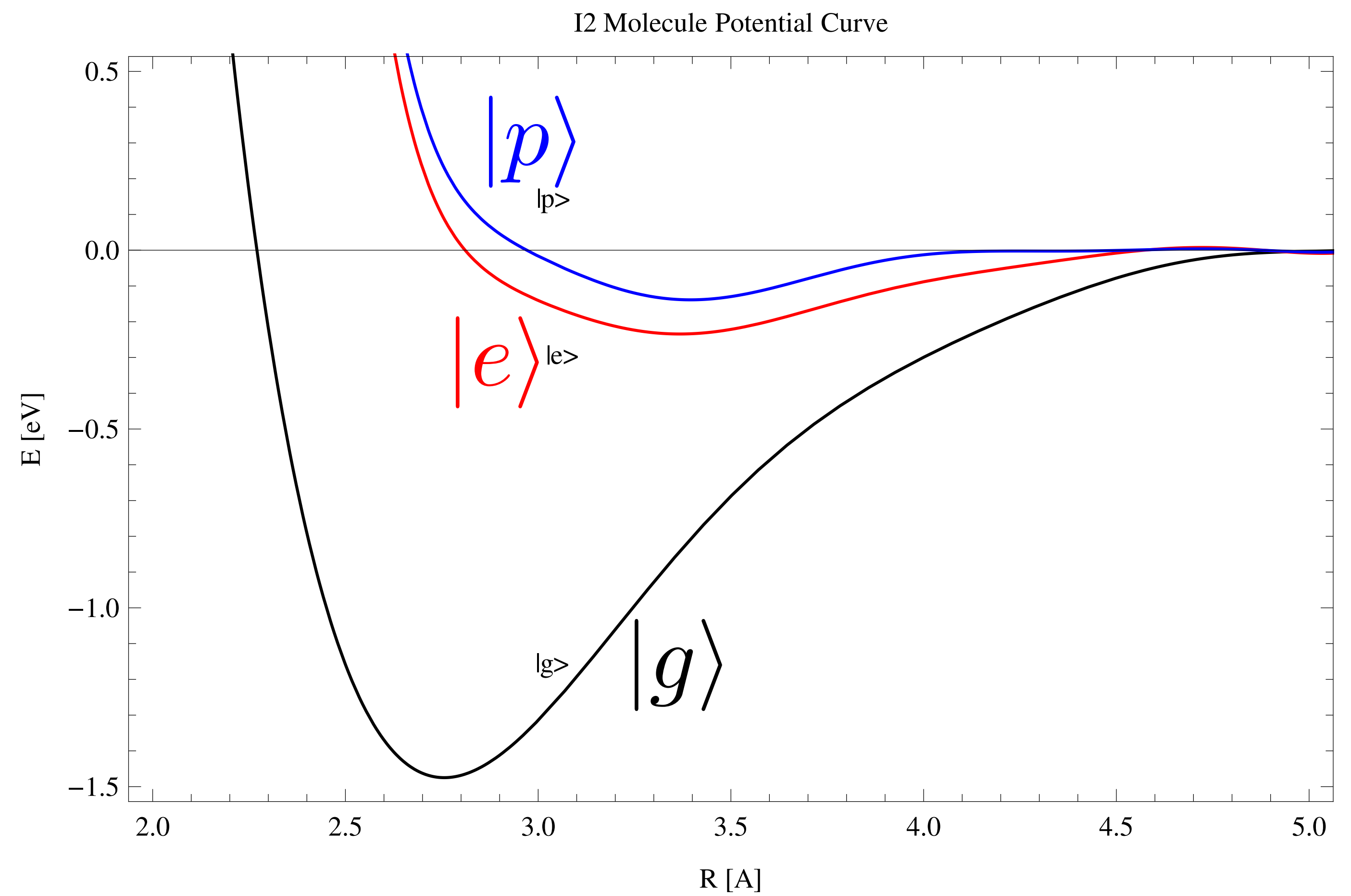
The threshold weight factors

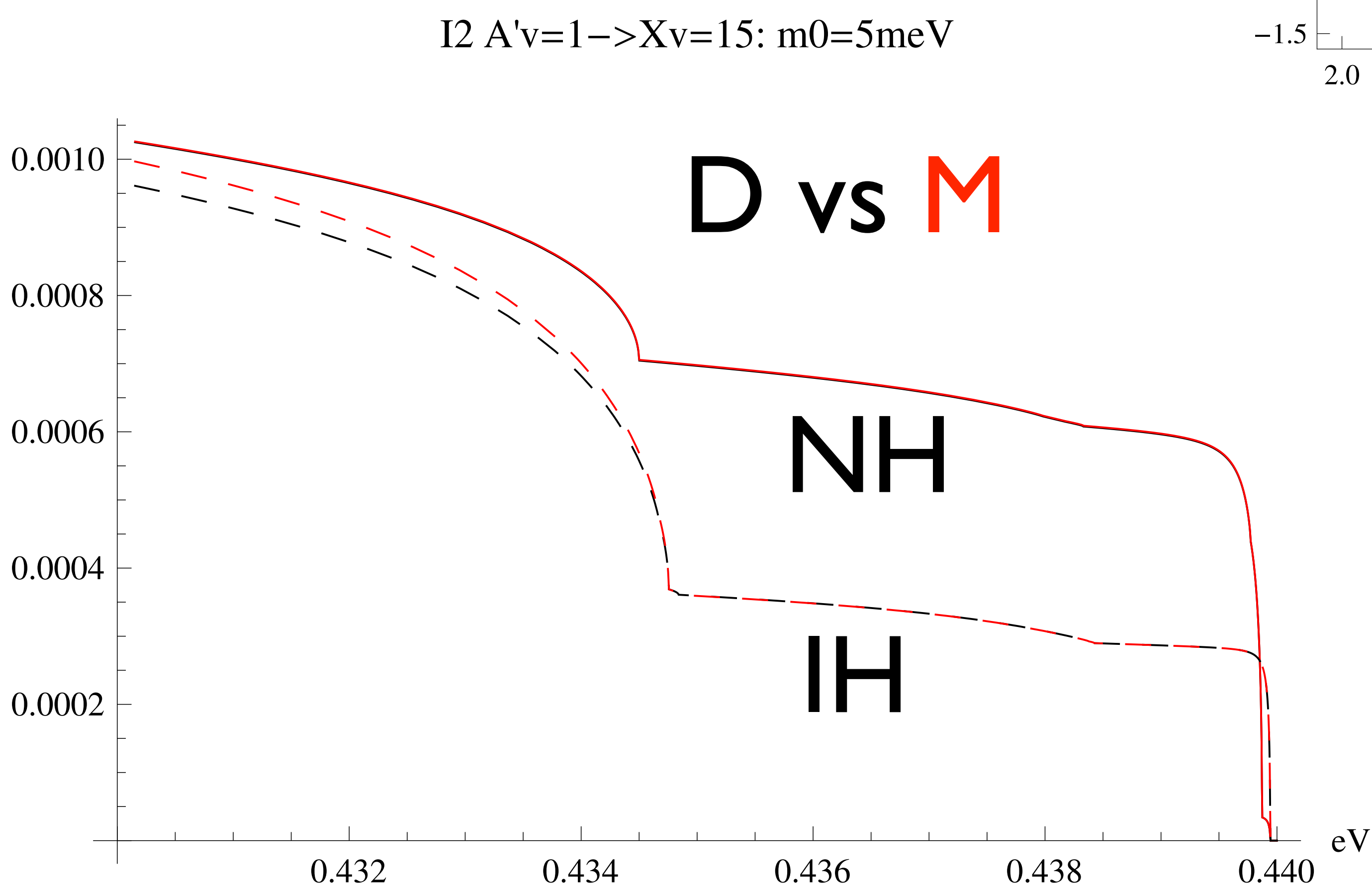
B_{11}	B_{22}	B_{33}	$B_{12}+B_{21}$	$B_{23}+B_{32}$	$B_{31}+B_{13}$
$(c_{12}^2 c_{13}^2 - 1/2)^2$	$(s_{12}^2 c_{13}^2 - 1/2)^2$	$(s_{13}^2 - 1/2)^2$	$2c_{12}^2 s_{12}^2 c_{13}^4$	$2s_{12}^2 c_{13}^2 s_{13}^2$	$2c_{12}^2 c_{13}^2 s_{13}^2$
0.0311	0.0401	0.227	0.405	0.0144	0.0325

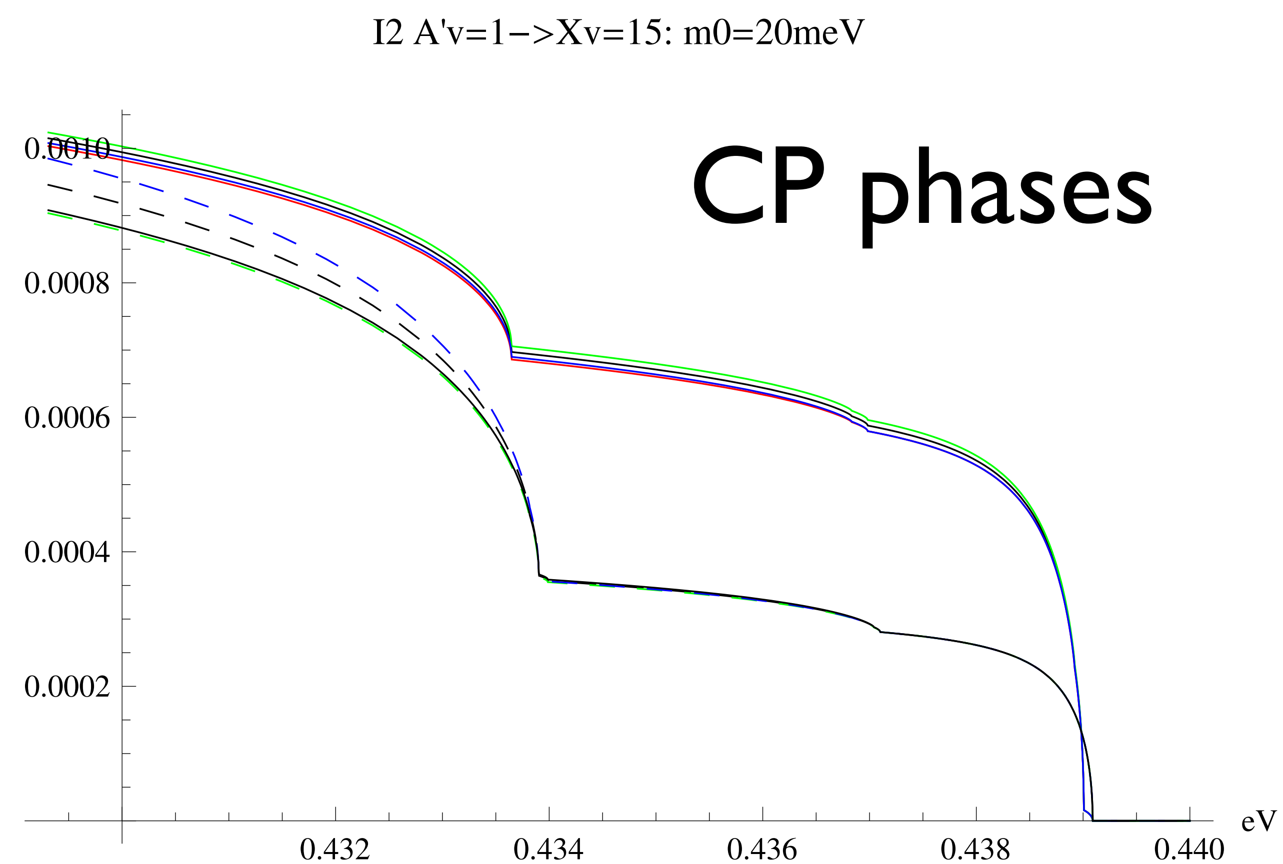
I2 molecule

potential curves

$$\epsilon_{eg} \sim 1\,\mathrm{eV}$$





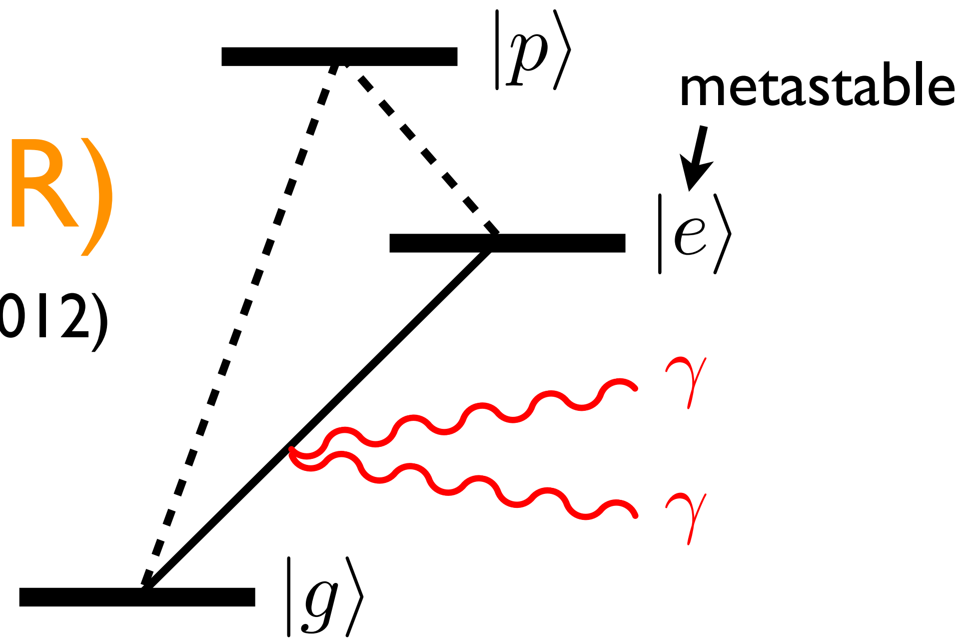


D-M diff. < 10%

PSR

Paired Super-Radiance (PSR)

M. Yoshimura, N. Sasao, MT, PRA86, 013812 (2012)



$$|e\rangle \to |g\rangle + \gamma + \gamma$$

prototype for RENP

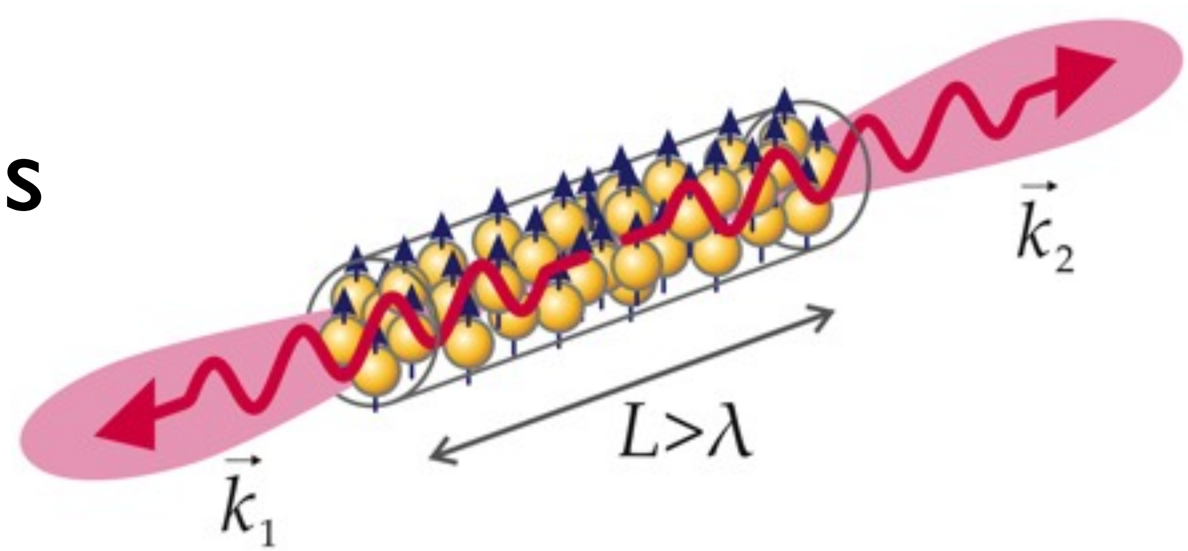
proof-of-concept for the macro-coherence

preparation of initial state for RENP

dynamical factor $\eta_\omega(t)$

background for RENP

A novel coherent process with two propagating fields/triggers



PSR Equation

Effective two-level interaction Hamiltonian

$|g\rangle,\ |e\rangle,$ ~~$|p\rangle$~~

$$\mathcal{H}_I = \begin{pmatrix} \alpha_{ee} & \alpha_{ge} e^{i\varepsilon_{eg} t} \\ * & \alpha_{gg} \end{pmatrix} E^2$$

$$\alpha_{ge} = \frac{2 d_{pe} d_{pg}}{\epsilon_{pg} + \epsilon_{pe}}\,, \qquad \alpha_{aa} = \frac{2 d_{pa}^2 \epsilon_{pa}}{\epsilon_{pa}^2 - \omega^2}\,, \qquad (a = g\,, e)$$

d_{pa} : dipole matrix element

Field (1+1 dim.)

$$\omega = \epsilon_{eg}/2$$

$$E = E_R\, e^{-i(\omega t - kx)} + E_L\, e^{-i(\omega t + kx)} + \text{c.c.}$$

$$k = \omega$$

$|e\rangle$

L-mover $e^{i\omega(t+x)}$

R-mover $e^{i\omega(t-x)}$

$$\sim e^{2i\omega t} = e^{i\epsilon_{eg} t}$$

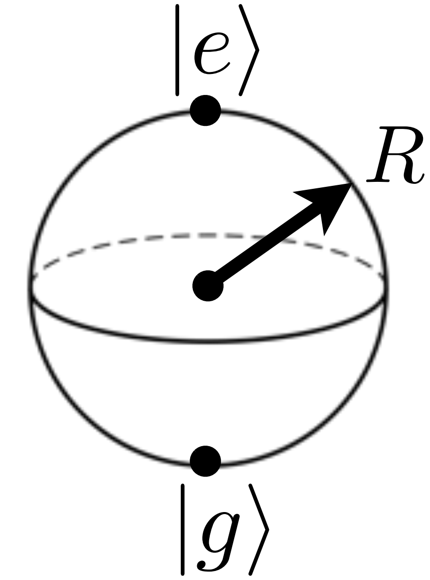
macro-coherence

$|g\rangle$

Bloch equation: $\partial_t \rho = i[\rho, \mathcal{H}_I]$

Maxwell equation: $\partial_t^2 \boldsymbol{E} = -[H, [H, \boldsymbol{E}]]$

$$H = \int d^3x \; [\mathcal{H}_{em} + \mathrm{tr}(\rho \mathcal{H}_I)]$$



Wikimedia Commons

Bloch vector: $R_i(x,t) = \mathrm{tr}(\rho\sigma_i)$

spatial grating

$$R_i = R_i^{(0)} + R_i^{(+)} e^{2ikx} + R_i^{(-)} e^{-2ikx}$$

Rotating wave approximation (RWA)

omitting fast oscillation terms

Slowly varying envelope approximation (SVEA)

$$|\partial_{x,t} E_{R,L}| \ll \omega |E_{R,L}|, \; |\partial_{x,t} R_i^{(0,\pm)}| \ll \omega |R_i^{(0,\pm)}|$$

Rescaling: $t_* = 2/\alpha_{ge}\epsilon_{eg}n$

$$x = t_*\xi,\ t = t_*\tau,\ E_{R,L}^2 = \omega n e_{R,L}^2,\ R_i^{(0,\pm)} = n r_i^{(0,\pm)}$$

$$r_T^{(0,\pm)} = r_1^{(0,\pm)} + i r_2^{(0,\pm)}$$

The master equation

$$\gamma_\pm = \frac{\alpha_{ee} \pm \alpha_{gg}}{2\alpha_{ge}}$$

$$\begin{aligned}
\partial_\tau r_T^{(0)} &= -4i\left[\gamma_-\left\{r_T^{(0)}(|e_R|^2+|e_L|^2) + r_T^{(+)}e_R^* e_L + r_T^{(-)} e_R e_L^*\right\}\right. \\
&\quad \left. - \left\{2r_3^{(0)}e_R^*e_L^* + r_3^{(+)}e_R^{*2} + r_3^{(-)}e_L^{*2}\right\}\right] - r_T^{(0)}/\tau_2\,, \\
\partial_\tau r_T^{(+)} &= -4i\left[\gamma_-\left\{r_T^{(+)}(|e_R|^2+|e_L|^2) + r_T^{(0)}e_R e_L^*\right\} - \left\{2r_3^{(+)}e_R^*e_L^* + r_3^{(0)}e_L^{*2}\right\}\right] - r_T^{(+)}/\tau_2\,, \\
\partial_\tau r_T^{(-)} &= -4i\left[\gamma_-\left\{r_T^{(-)}(|e_R|^2+|e_L|^2) + r_T^{(0)}e_R^* e_L\right\} - \left\{2r_3^{(-)}e_R^*e_L^* + r_3^{(0)}e_R^{*2}\right\}\right] - r_T^{(-)}/\tau_2\,, \\
\partial_\tau r_3^{(0)} &= 2i\left[\left(2r_T^{(0)}e_R e_L + r_T^{(+)}e_L^2 + r_T^{(-)}e_R^2\right) - (\text{c.c.})\right] - (r_3^{(0)}+1)/\tau_1\,, \\
\partial_\tau r_3^{(+)} &= 2i\left[2r_T^{(+)}e_R e_L + r_T^{(0)}e_R^2 - \left(2r_T^{(-)*}e_R^*e_L^* + r_T^{(0)*}e_L^{*2}\right)\right] - r_3^{(+)}/\tau_1\,.
\end{aligned}$$

$$\begin{aligned}
(\partial_\tau + \partial_\xi)e_R &= \frac{i}{2}\left[\left(\gamma_+ + \gamma_- r_3^{(0)}\right)e_R + \gamma_- r_3^{(+)}e_L + r_T^{(0)*}e_L^* + r_T^{(-)*}e_R^*\right] \\
(\partial_\tau - \partial_\xi)e_L &= \frac{i}{2}\left[\left(\gamma_+ + \gamma_- r_3^{(0)}\right)e_L + \gamma_- r_3^{(-)}e_R + r_T^{(0)*}e_R^* + r_T^{(+)*}e_L^*\right]
\end{aligned}$$

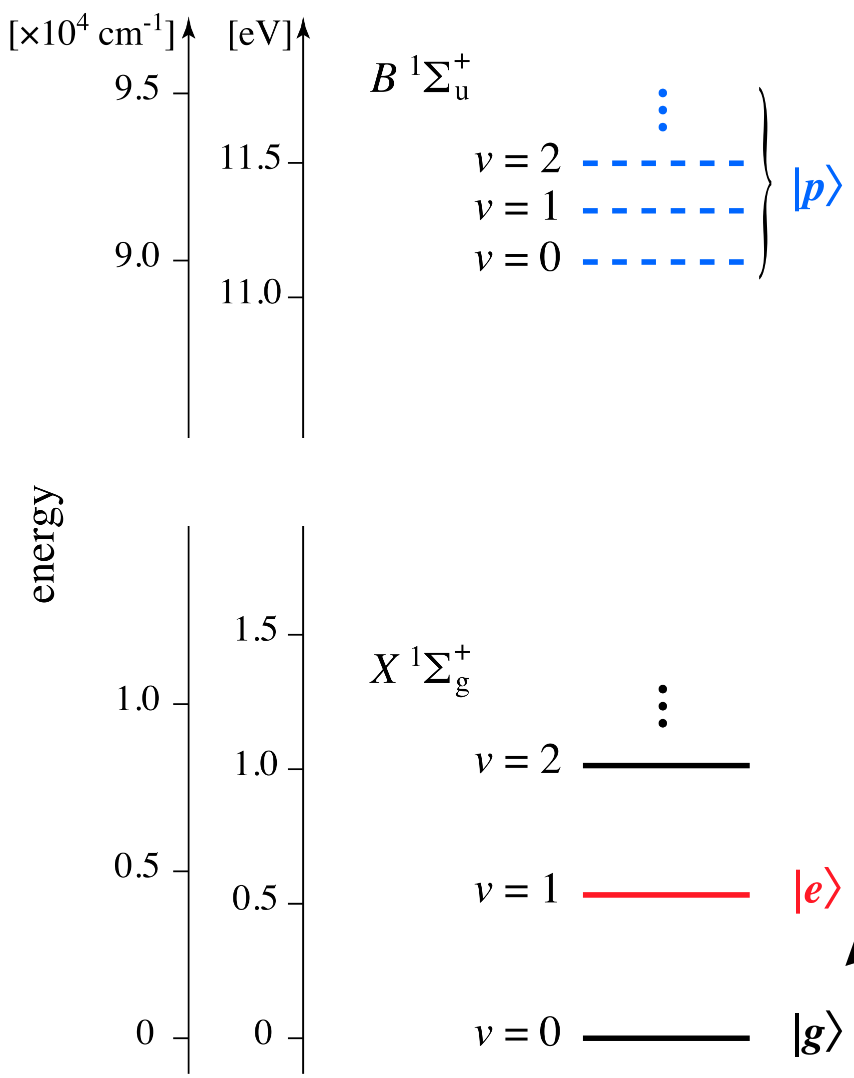
$\tau_i = T_i/t_*$ dimensionless relaxation times

PSR pH2 Numerical Results

Target system: para-hydrogen molecule

gas or solid

vibrational transition (electronic ground state)



$$|e\rangle = |Xv=1\rangle \longrightarrow |g\rangle = |Xv=0\rangle$$

no E1 transition

two-photon lifetime ~10^{16} sec.

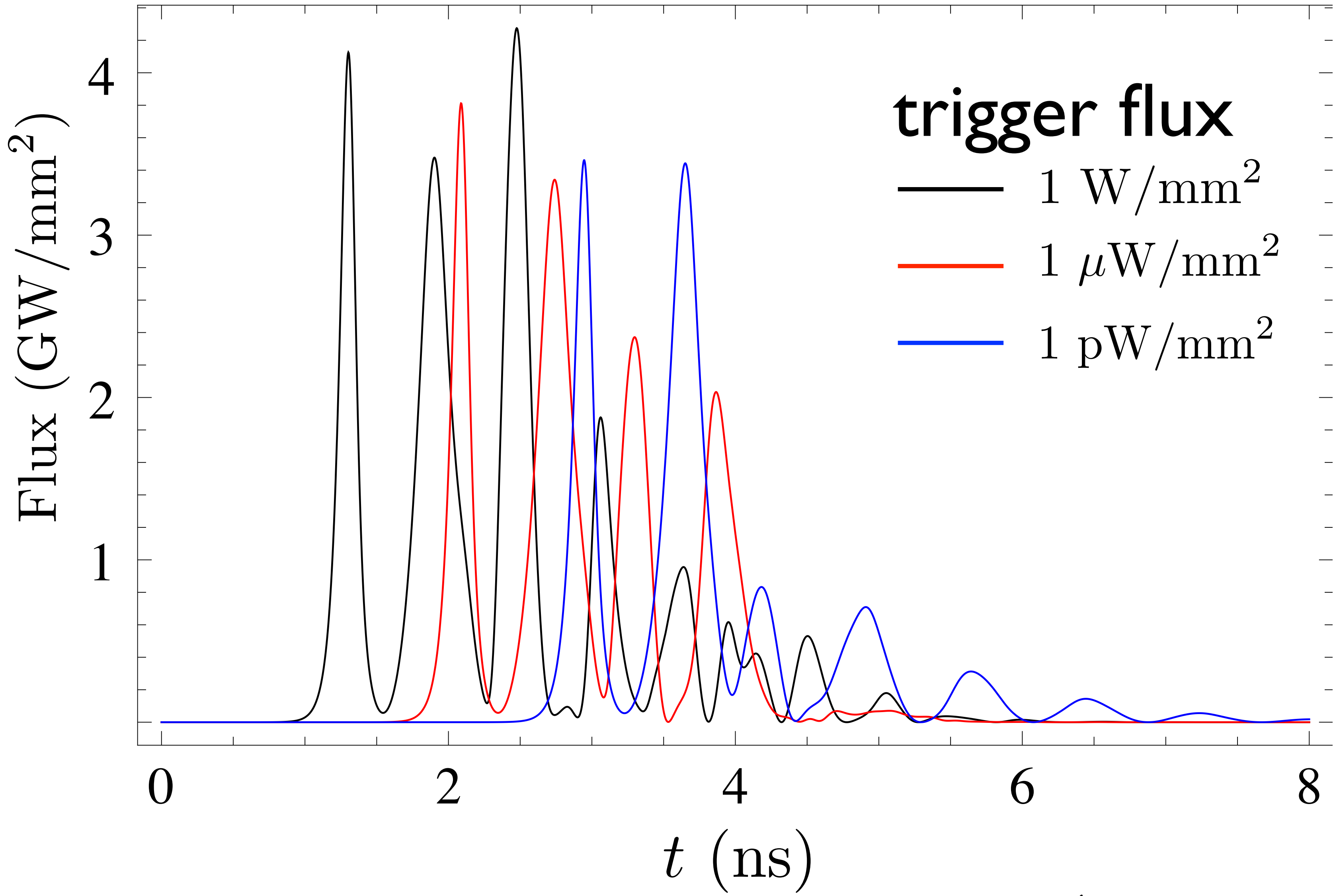
$$\epsilon_{eg} = 0.52\,\mathrm{eV},\ \gamma_{\pm} = 15.3,\ 0.64$$

$$t_* \sim 50\,\mathrm{ps}\frac{10^{21}\,\mathrm{cm}^{-3}}{n}$$

Explosive PSR

$$n = 1 \times 10^{21}/\mathrm{cm}^3 \quad T_1 = 1\,\mu\mathrm{s},\ T_2 = 10\,\mathrm{ns}$$

target length $L = 30\,\mathrm{cm} \quad r_T^{(0)} = 1 (= 2\rho_{eg}^{(0)*})$



burst time $\sim \dfrac{1}{\sqrt{|r_T|^2 - \gamma_-^2 r_3^2}} \log \dfrac{\epsilon_{eg} n}{|E_{\mathrm{trig.}}|^2}$

The dynamical factor

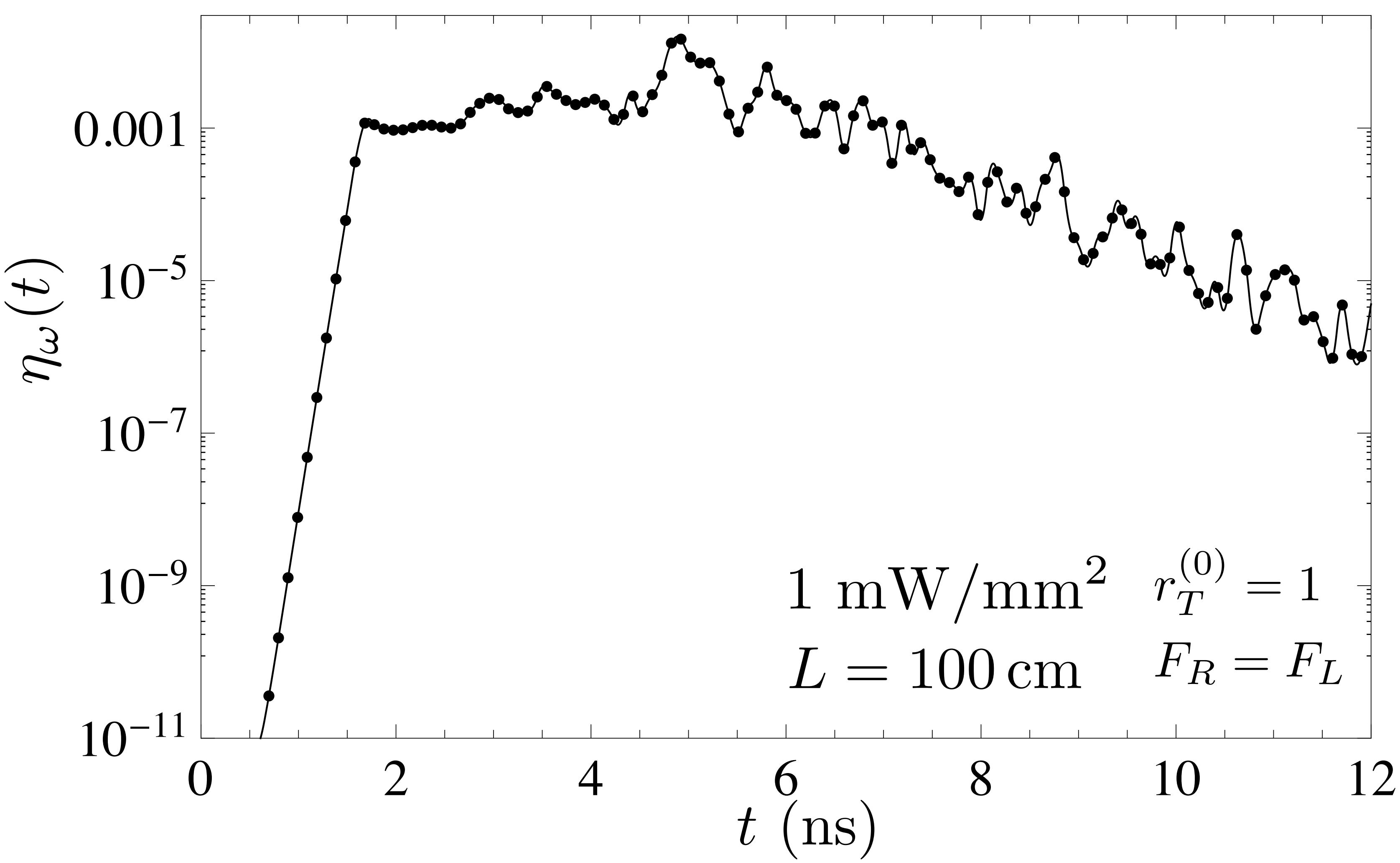
local field-medium activity

$$\eta_\omega(\xi,\tau) = \frac{1}{\epsilon_{eg} n^3} \left| \vec{E}^{-} \frac{R_1 - iR_2}{2} \right|^2 = \left| \left(e_R^* e^{-i\kappa\xi} + e_L^* e^{i\kappa\xi} \right) \frac{r_1 - ir_2}{2} \right|^2$$

$$= \frac{1}{4} \left[(|e_R|^2 + |e_L|^2)(|r_T^{(0)}|^2 + |r_T^{(+)}|^2 + |r_T^{(-)}|^2) + 2\Re\{ e_R^* e_L (r_T^{(0)*} r_T^{(+)} + r_T^{(0)} r_T^{(-)*}) \} \right]$$

average over the target length

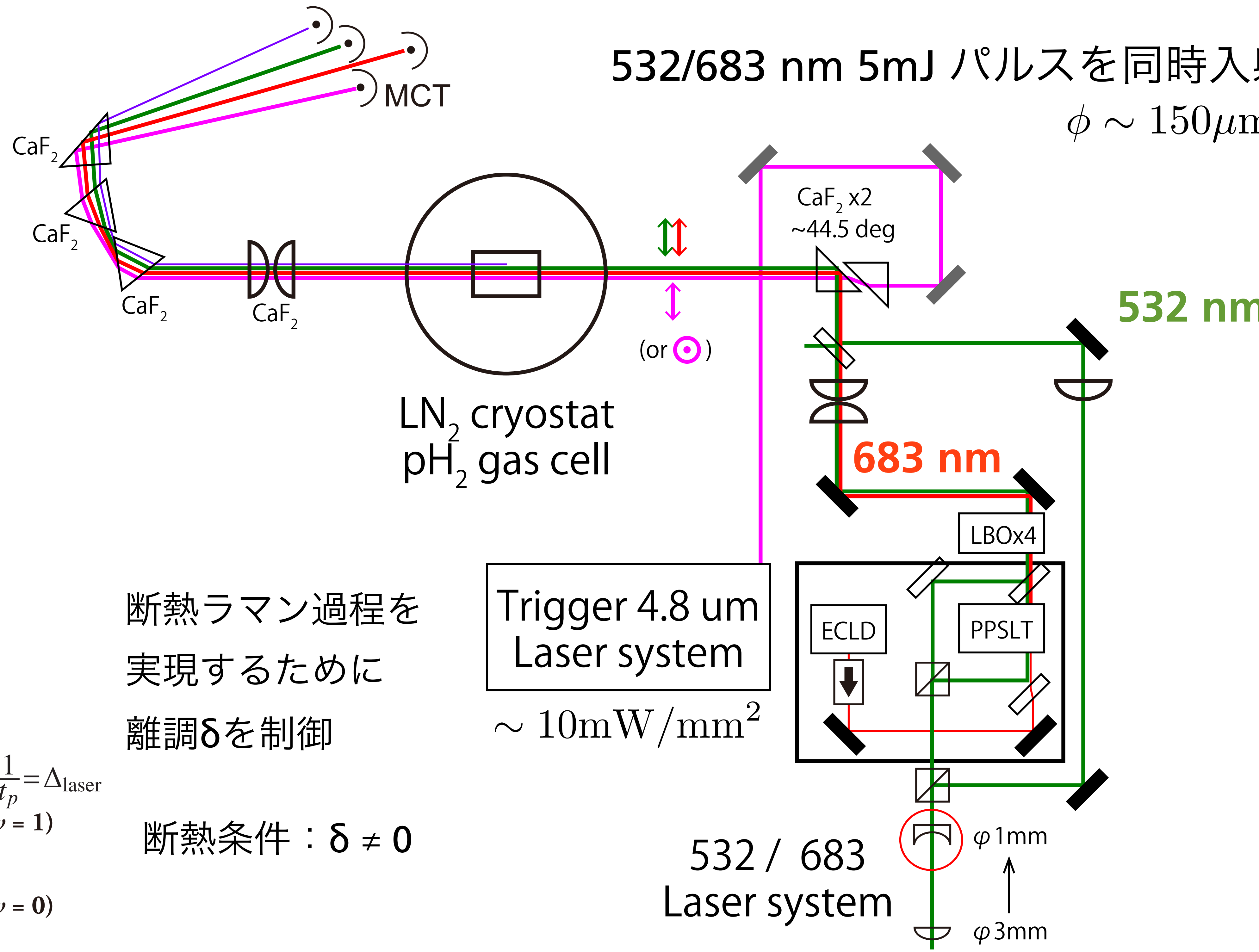
$$\eta_\omega(t) = \langle \eta_\omega(\xi,\tau) \rangle_\xi$$

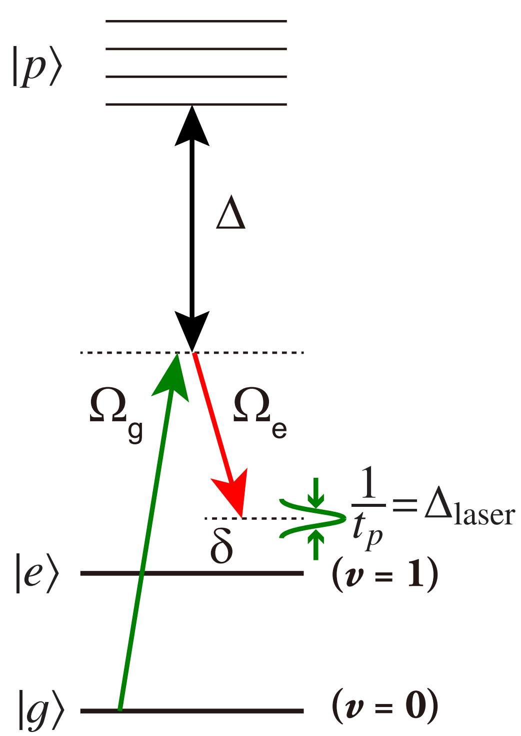


Experimental Setup

532/683 nm 5mJ パルスを同時入射

$\phi \sim 150\mu\mathrm{m}$





断熱ラマン過程を
実現するために
離調δを制御

断熱条件：$\delta \neq 0$

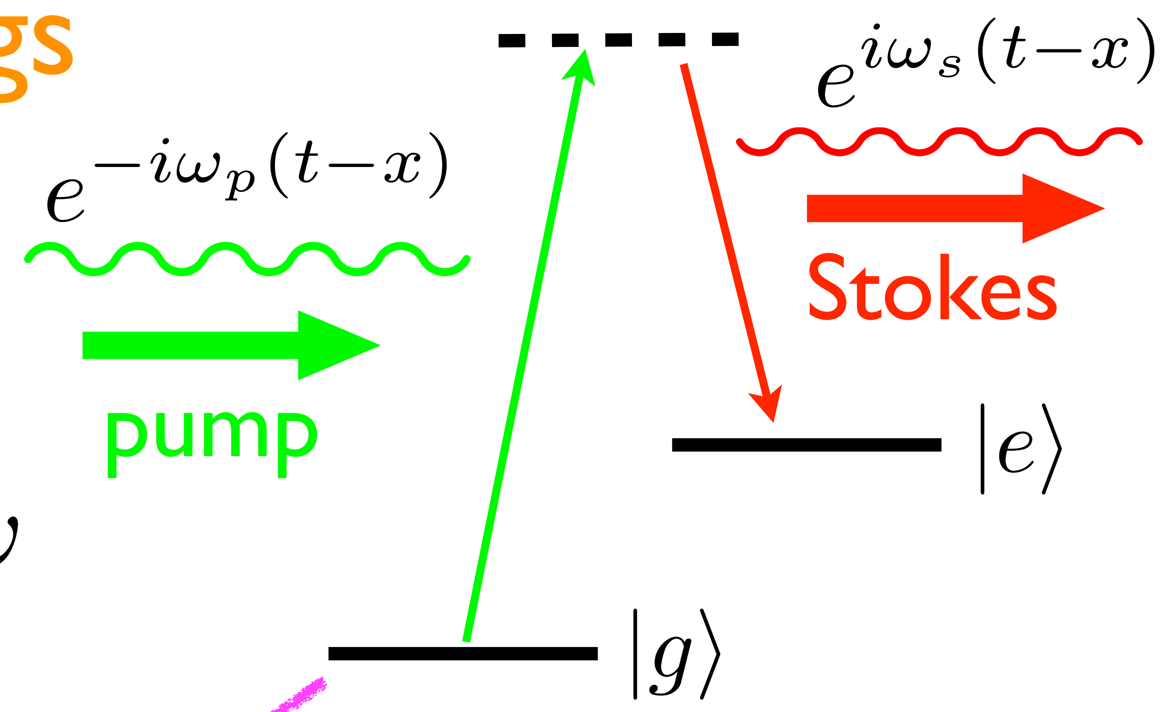
S. Kuma X00 2013/12/19

PSR with Spatial Gratings

How to populate $|e\rangle$

Raman scattering

$$\omega_p - \omega_s = \epsilon_{eg} = 2\omega$$

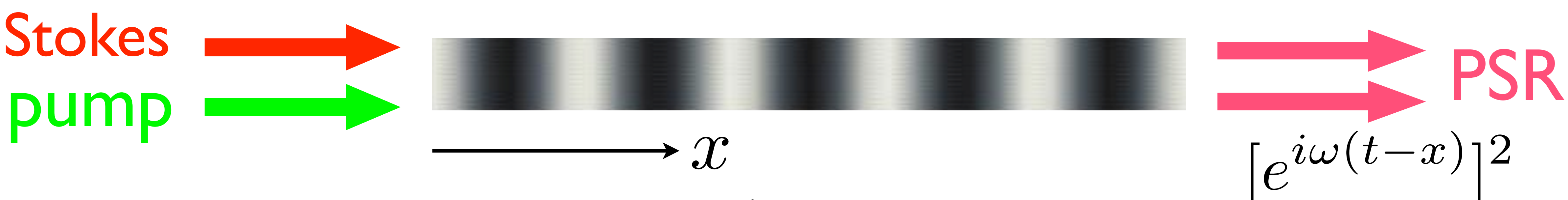


Generated coherence

spatial grating

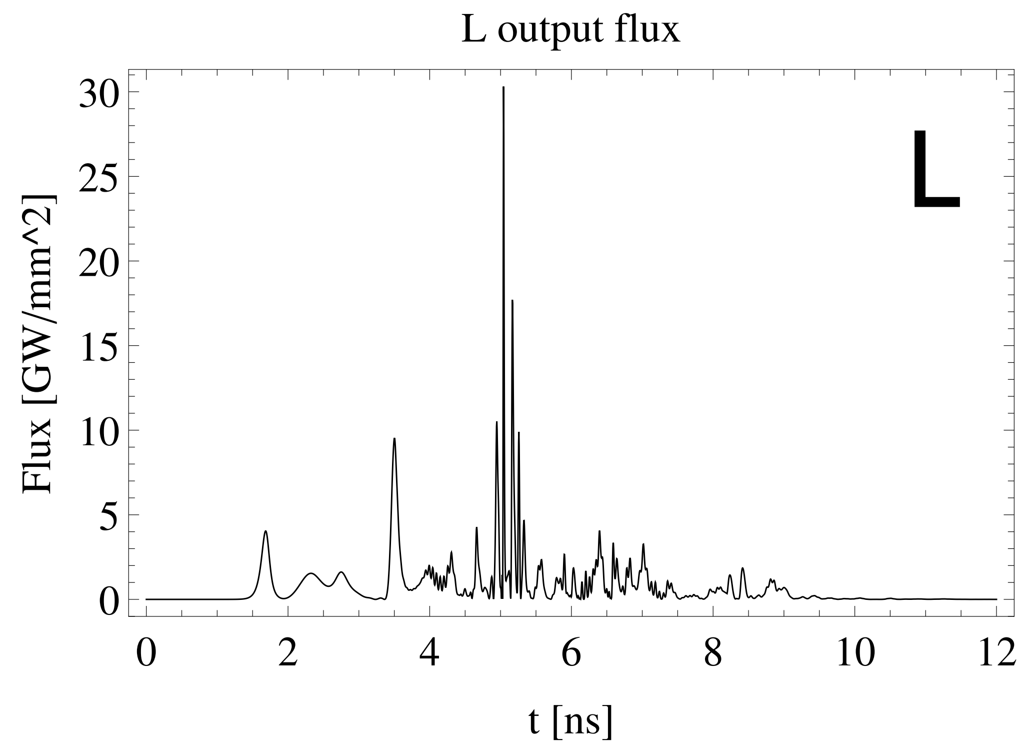
$$\rho_{eg} = \rho_{eg}^{(0)} + \rho_{eg}^{(+)} e^{2i\omega x} + \rho_{eg}^{(-)} e^{-2i\omega x}$$

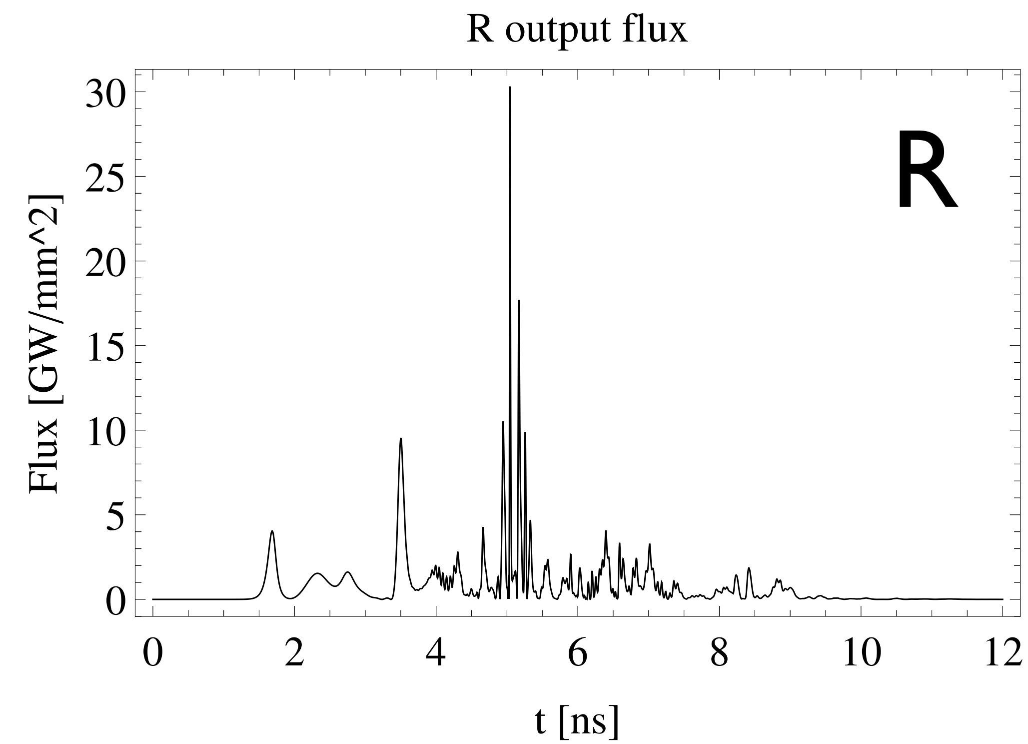
momentum memory in macroscopic medium



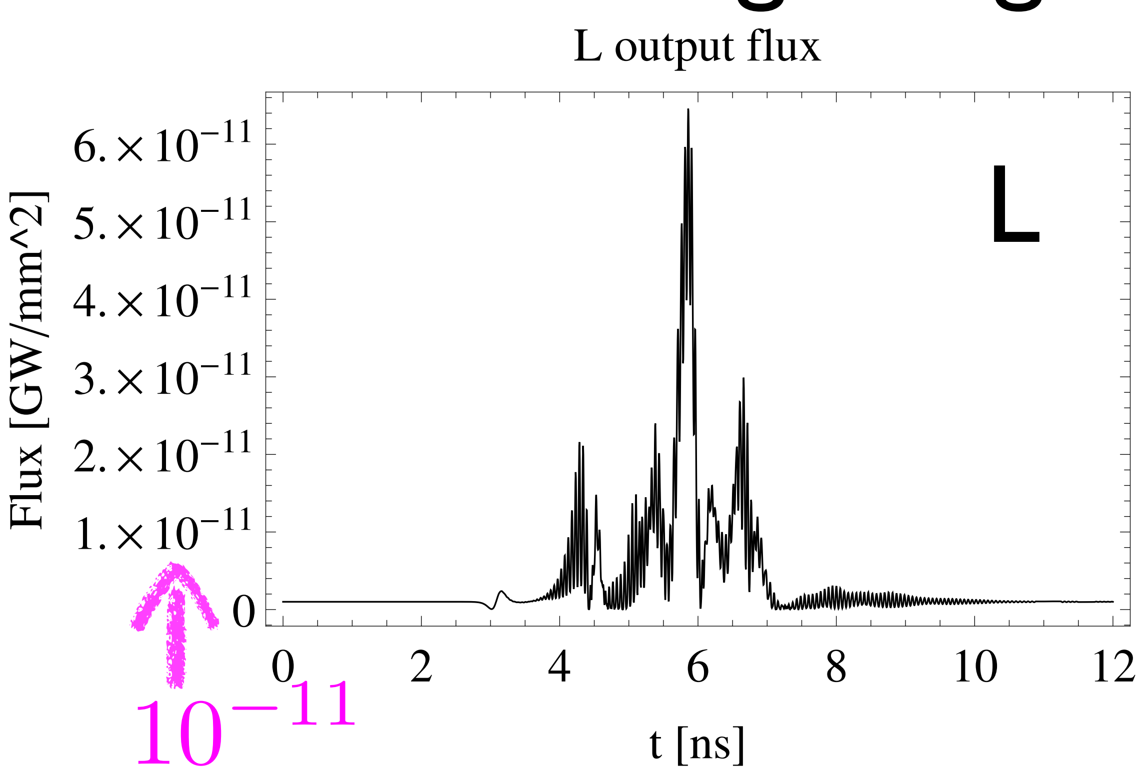
momentum conservation in macro-coherence → Unidirectional PSR

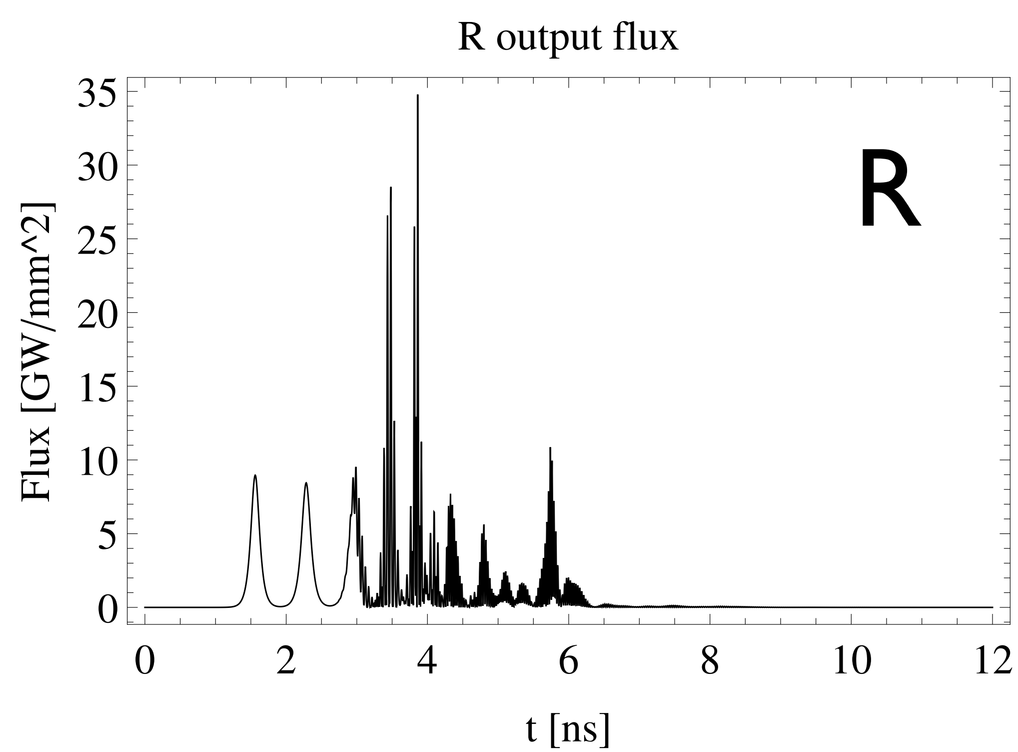
w/o the initial grating $(\rho_{eg}^{(0)}, \rho_{eg}^{(+)}, \rho_{eg}^{(-)}) = (1/2, 0, 0)$





w/ the initial grating $(\rho_{eg}^{(0)}, \rho_{eg}^{(+)}, \rho_{eg}^{(-)}) = (0, 1/2, 0)$





SUMMARY

Neutrino Physics with Atoms/Molecules

- ✸ RENP spectra are sensitive to unknown neutrino parameters.

  Absolute mass, Dirac or Majorana,
  NH or IH, CP

- ✸ The macro-coherence is essential.

  Proof by a companion QED process, paired super-radiance (PSR).

  The experiment at Okayama is ongoing.

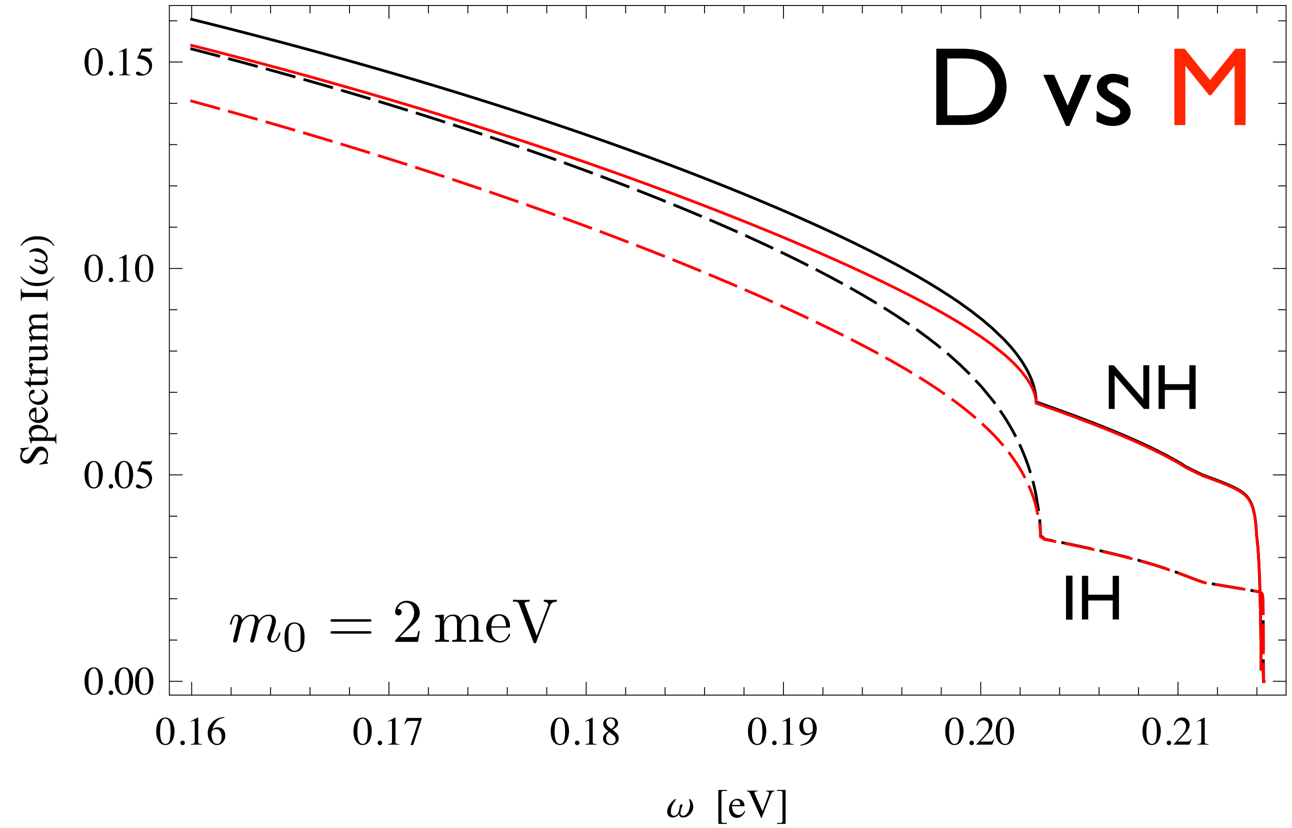
Atomic/molecular processes may help.

Backup Slides

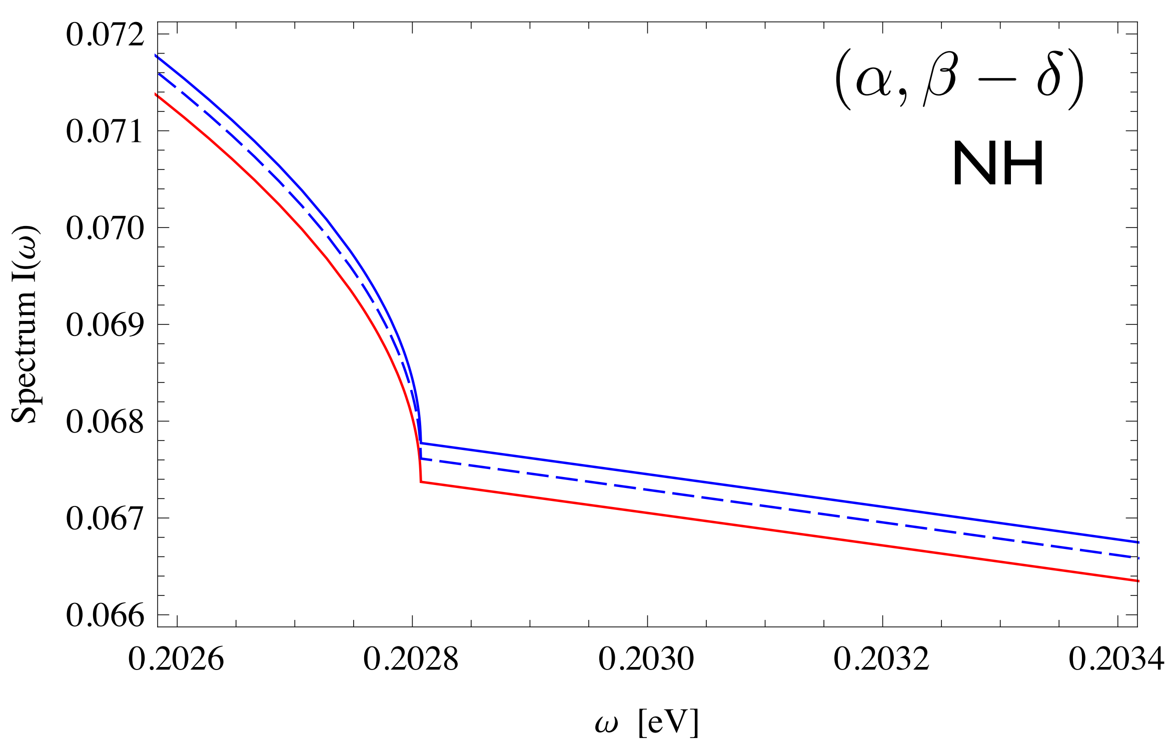
More on Dirac vs Majorana and CP phases

hypothetical atom

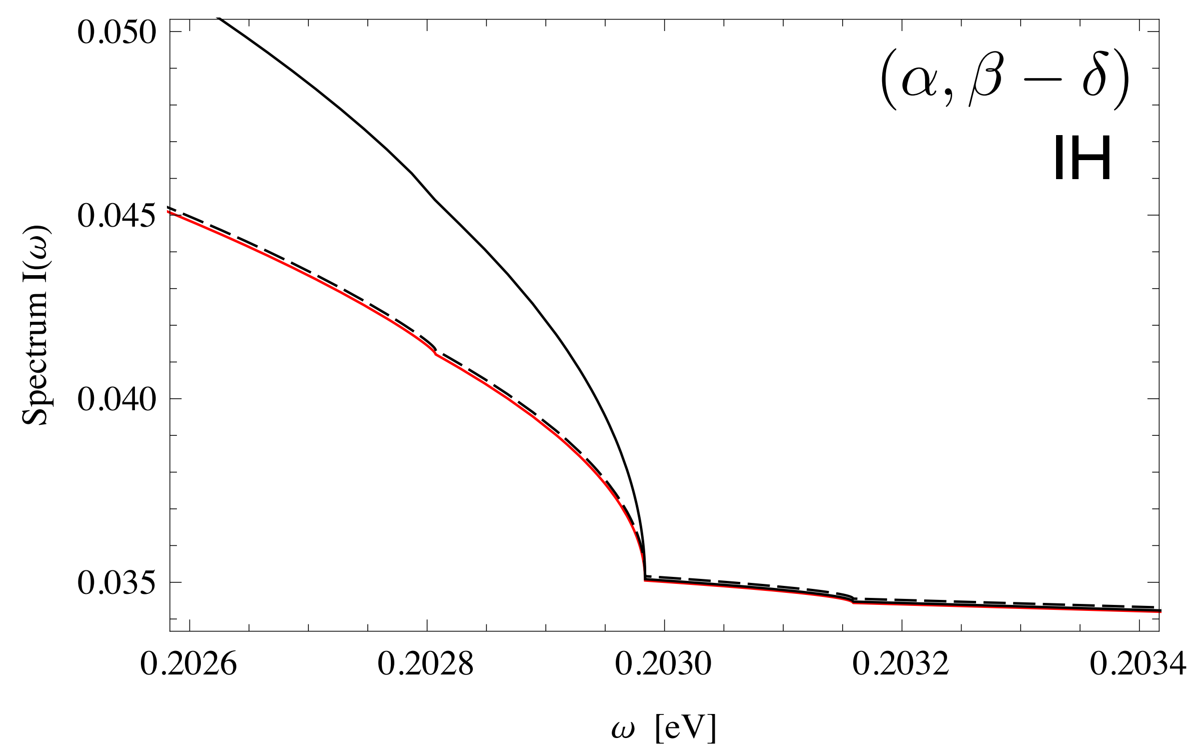
$$\epsilon_{eg} = 0.43\,\mathrm{eV}$$



CP phases (NH); Red=(0,0), Blue=(π/2,0), Blue Dashed=(0,π/2)



CP phases (IH); Red=(0,0), Black=(π/2,0), Black Dashed=(0,π/2)



Coherences in RENP

Atomic coherence $(|g\rangle + |e\rangle)/\sqrt{2}\,,\ \rho_{eg} = 1/2$

Target coherence $\left[\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)\right]^N$

$$\xrightarrow[J_-]{} \frac{1}{\sqrt{2^N}}\Big[|g\rangle(|g\rangle + |e\rangle)\cdots(|g\rangle + |e\rangle) + (|g\rangle + |e\rangle)|g\rangle\cdots(|g\rangle + |e\rangle) + \cdots\cdots\Big]$$

$$\Gamma \propto N^2$$

Macro-coherence

$$\Gamma \propto N^2/V = n^2 V$$