Physics is all about the dynamics, on and off the lattice

Shoji Hashimoto (KEK, SOKENDAI)

Hiroshima, 1990~

- Tetsuya joined the particle theory lab of Hiroshima University, as the youngest faculty member.
- The head of the lab was Taizo Muta, who had an intention to study non-perturbative dynamics of the EW symmetry breaking: techni-color, top-condensation, NJL model, …
- Tetsuya was willing to learn new subjects (for him) with us students. In fact, it was an ideal learning experiences… Memories with (always) lost mugs.

Lattice gauge theory

- Muta gave me a theme: simulations of Lattice Gauge Theory (LGT). For me, LGT was a kind of toy to play with using numerical simulations. I learned how to program.

- Tetsuya was again willing to study together, and gave me a homework: plaquette → cube. I didn't even know what the model was representing. Memory remains with Tetsuya's herpes.

Physics Letters B 266 (1991) 107–111
North-Holland

PHYSICS LETTERS B

Gauge theory of antisymmetric tensor fields

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Received 8 April 1991

The compact lattice gauge theory of the second-rank antisymmetric tensor field in four dimensions is studied. We calculate the hypersurface tension by Monte Carlo simulation on a 10^4 lattice. It is found that the tension does not go to zero for a finite value of the coupling constant. This suggests that the theory has the confinement phase only. This result is consistent with the result obtained from the dilute gas approximation of the instanton. Some phenomenological implications on the string model are discussed also.

$$S_{\mathrm{g}} = \sum_{\mathrm{c}} \beta \left[1 - \cos\left(\sum_{\mathrm{p} \in \mathrm{c}} \theta(p) \right) \right].$$

Strong coupling constant

- Tetsuya joined a group of LQCD, later called JLQCD. The first subject was a determination of α_s. It was the first attempt to do it including the effect of dynamical fermion.

VOLUME 74, NUMBER 1 PHYSICAL REVIEW LETTERS 2 JANUARY 1995

Manifestation of Sea Quark Effects in the Strong Coupling Constant in Lattice QCD

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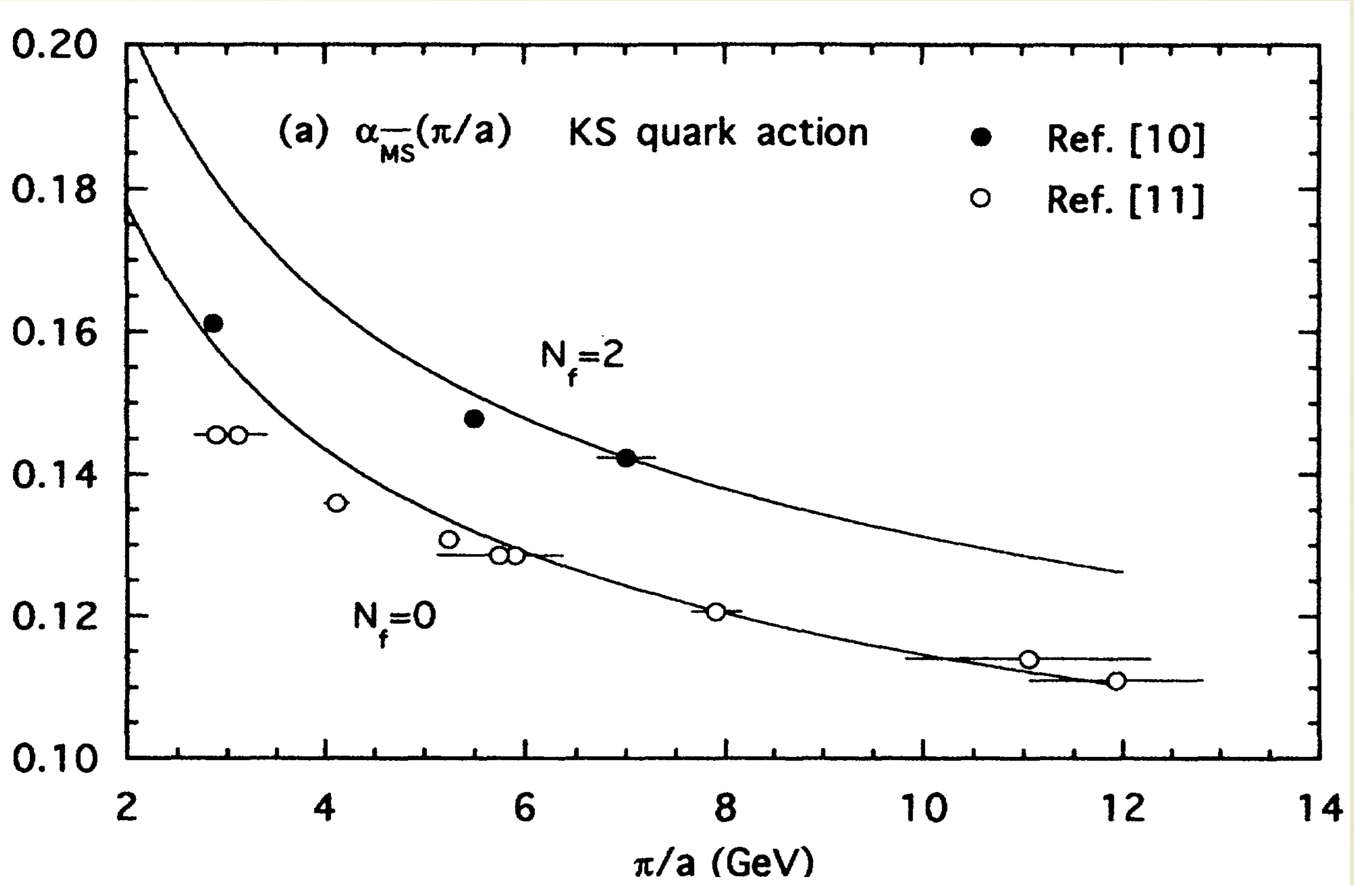
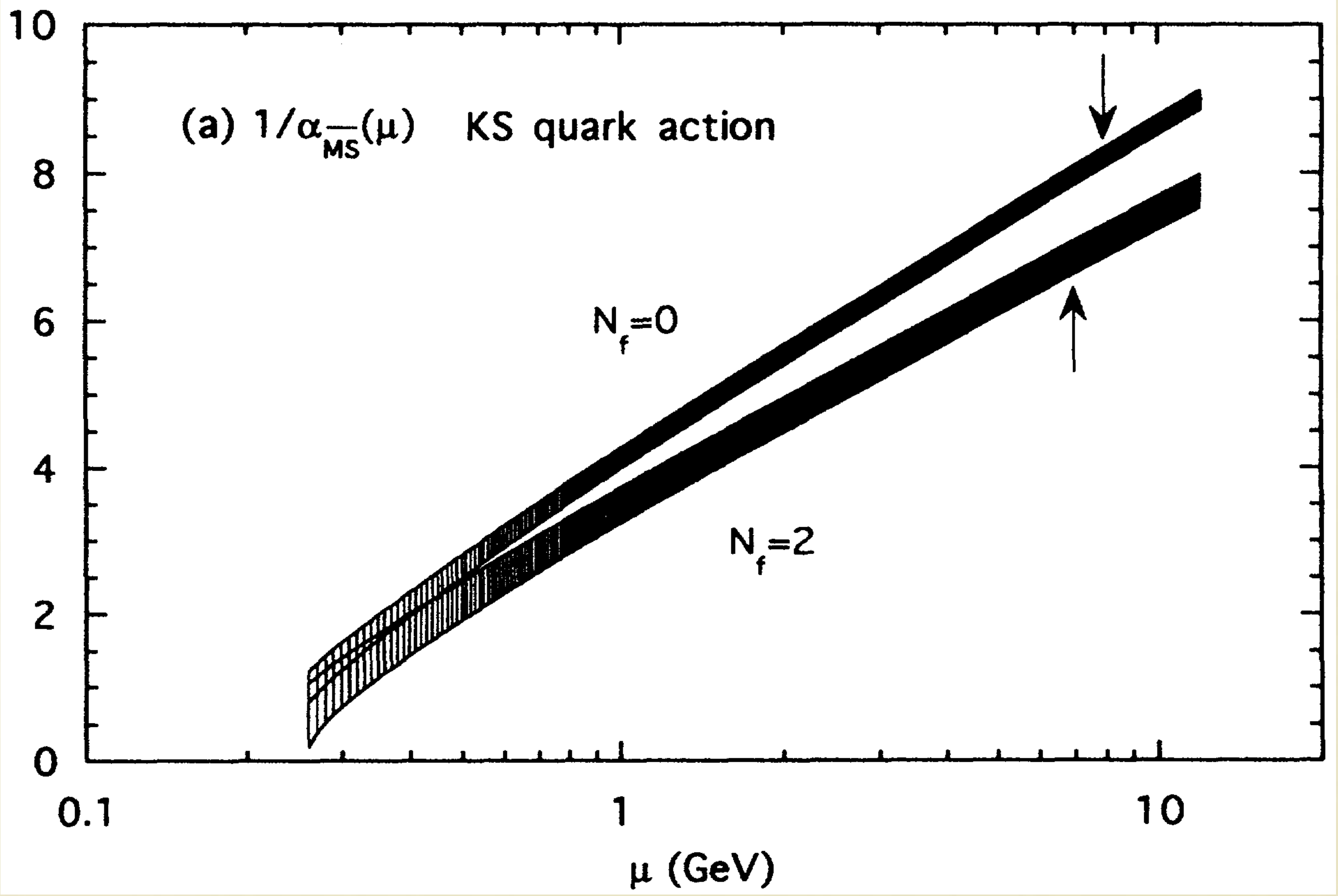
(Received 25 July 1994)

We demonstrate that sea quark effects of a magnitude expected from renormalization group considerations are clearly visible in the strong coupling constant measured in current full QCD simulations. Building on this result an estimate of $\alpha^{(5)}_{\overline{\rm MS}}(M_Z)$ ($\overline{\rm MS}$ denotes the modified minimal subtraction scheme) is made employing the charmonium $1S$-$1P$ mass splitting calculated on full QCD configurations generated with two flavors of dynamical Kogut-Susskind quarks to fix the scale.





α_s runs differently, indeed!

HEAVY QUARKS ON THE LATTICE

- Tetsuya left us for a visit at Fermilab in 1993-94 (?). His study there was about applications of LQCD to pheno.

- My Ph.D thesis was also about heavy quark effective theory (or called NRQCD) on the lattice.

- A lot of "chat" on UNIX between US and Japan about perturbative matching calculation. Through such discussions, I learned that "lattice" is indeed a kind of (lovely!) QFT.

Semileptonic!

- After a bunch of works with younger students in Hiroshima (I was already at KEK), Tetsuya joined the JLQCD collaboration for a calculation of B→πlν form factors.

PHYSICAL REVIEW D, VOLUME 64, 114505

Differential decay rate of $B\to\pi l\nu$ semileptonic decay with lattice nonrelativistic QCD

S. Aoki,[1] M. Fukugita,[2] S. Hashimoto,[3] K.-I. Ishikawa,[3] N. Ishizuka,[1,4] Y. Iwasaki,[1,4] K. Kanaya,[1,4] T. Kaneko,[3] Y. Kuramashi,[3] M. Okawa,[3] T. Onogi,[5,6] S. Tominaga,[4] N. Tsutsui,[3] A. Ukawa,[1,4] N. Yamada,[3] and T. Yoshie[1,4]
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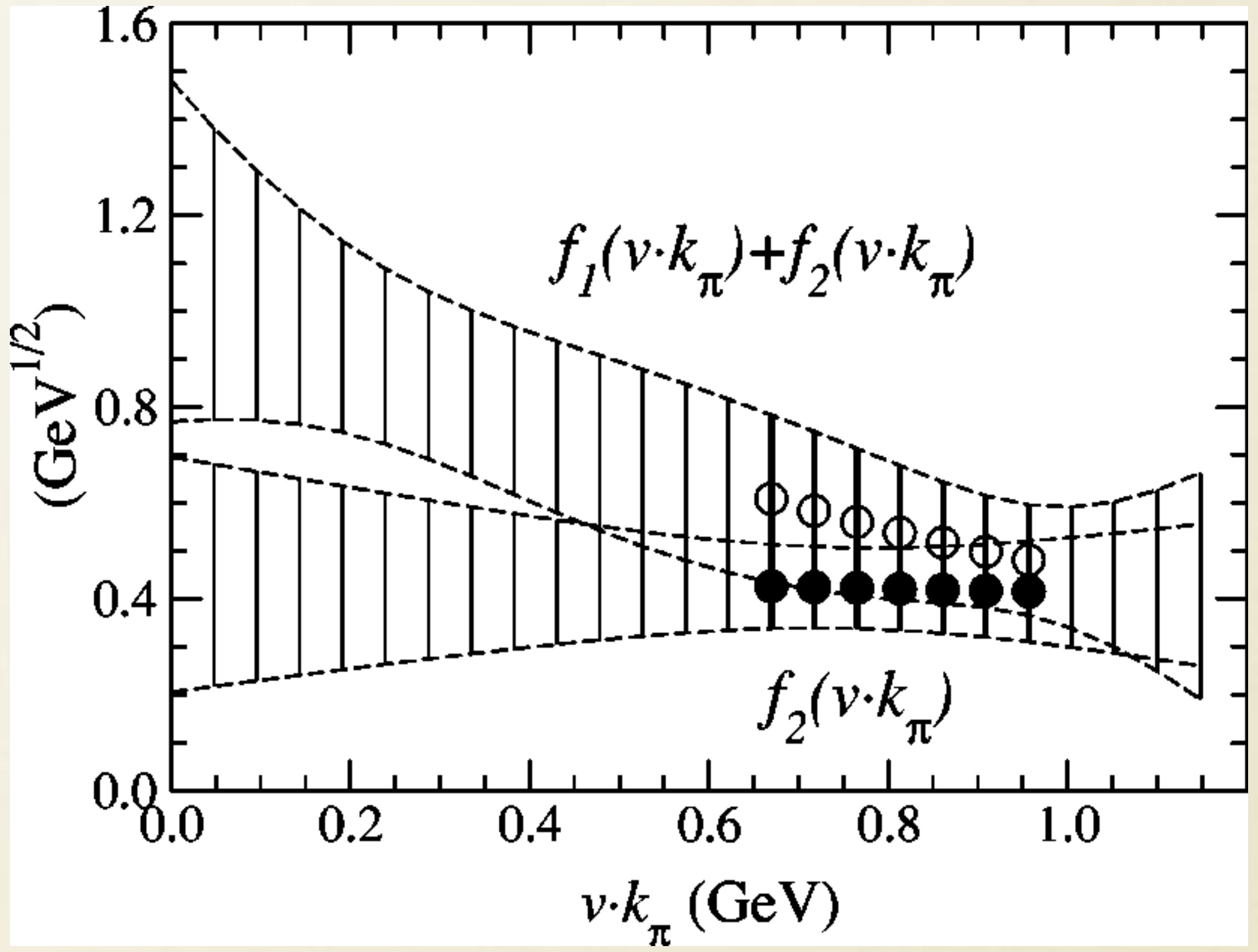
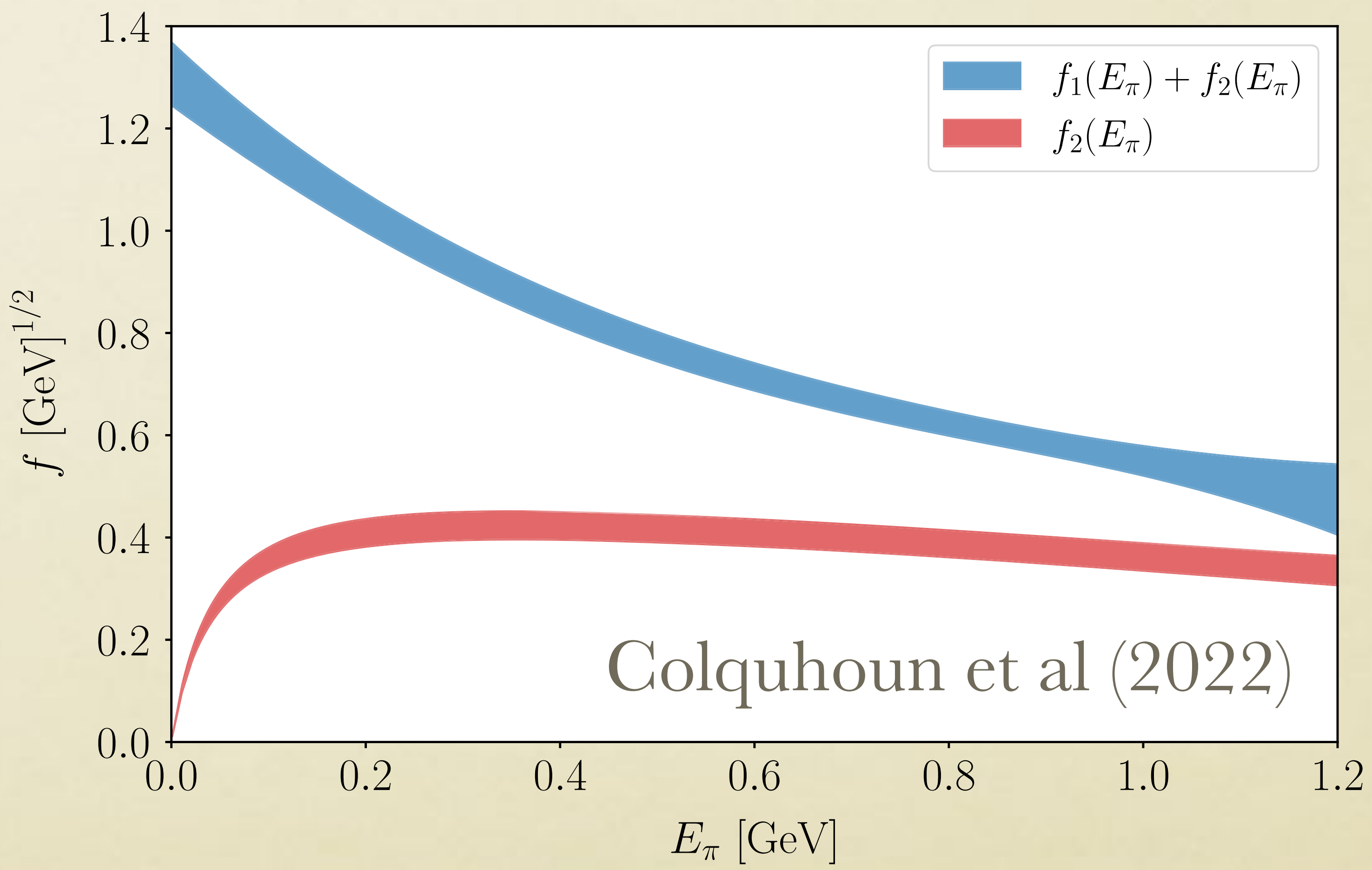
(Received 2 July 2001; published 7 November 2001)

We present a lattice QCD calculation of $B\to\pi l\nu$ semileptonic decay form factors in the small pion recoil momentum region. The calculation is performed on a quenched $16^3\times48$ lattice at $\beta=5.9$ with the nonrelativistic QCD action including the full $1/M$ terms. The form factors $f_1(v\cdot k_\pi)$ and $f_2(v\cdot k_\pi)$ defined in the heavy quark effective theory for which the heavy quark scaling is manifest are adopted, and we find that the $1/M$ correction to the scaling is small for the B meson. The dependence of the form factors on the light quark mass and on the recoil energy is found to be mild, and we use a global fit of the form factors at various quark masses and recoil energies to obtain model independent results for the physical differential decay rate. We find that the B^* pole contribution dominates the form factor $f^+(q^2)$ for small pion recoil energy, and obtain the differential decay rate integrated over the kinematic region $q^2>18$ GeV^2 to be $|V_{ub}|^2\times(1.18\pm0.37\pm0.08$



20 years after



New direction: chiral, topology

- Development of “chiral” lattice fermions (domain-wall, overlap, etc., 1998~) opened new directions.
  - Connection to χPT, simpler renormalization, and of course, topology.
- Yet, its simulation was impractical, due to the cost and the **topology change**.
- Tetsuya (then at YITP) together with his student Hidenori Fukaya, started looking at the problem from the viewpoint of theorist, not a lattice practitioner.

Suppress or even prohibit the topology change

one way:

$$S_G = \begin{cases} \beta \sum_P \frac{1-\mathrm{ReTr}P_{\mu\nu}(x)/3}{1-(1-\mathrm{ReTr}P_{\mu\nu}(x)/3)/\epsilon}, & \text{when } 1-\mathrm{ReTr}P_{\mu\nu}(x)/3 < \epsilon, \\ \infty & \text{otherwise} \end{cases},$$

and finally:

$$\det\left[\frac{H_W(m_0)^2}{H_W(m_0)^2 + \mu^2}\right]$$

Formed the basis of our next project: dynamical overlap fermion

Overlap fermion & topology

Exact chiral symmetry realized by

$$D = \frac{1}{\bar{a}}[1 + \gamma_5 \,\mathrm{sgn}(aH_W)],$$

- It has a discontinuity, which poses a problem of locality.
- It occurs on the background gauge field for which the topological charge Q is not well-defined. Chiral symmetry is related to topology as index theorem dictates.
- Can be avoided when Q doesn't change. Any problem?

— "No," it's merely a finite volume effect.

PHYSICAL REVIEW D **76,** 054508 (2007)

Finite volume QCD at fixed topological charge

Sinya Aoki,[1,2] Hidenori Fukaya,[3] Shoji Hashimoto,[4,5] and Tetsuya Onogi[6]

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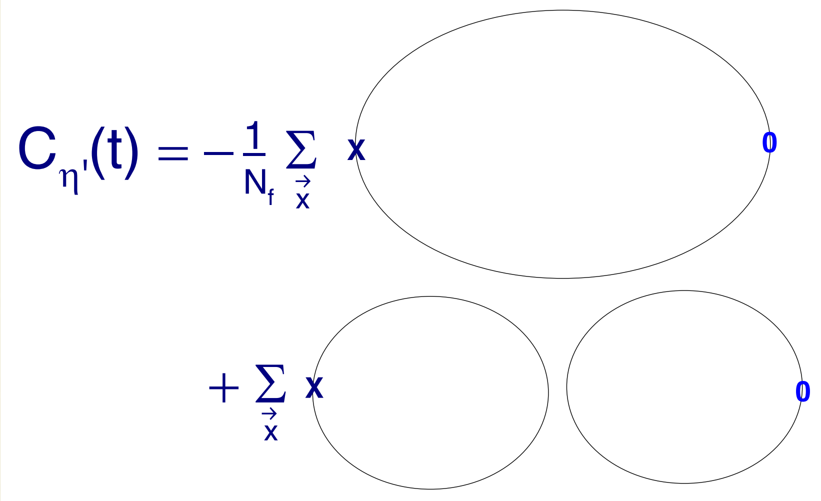
(Received 5 July 2007; published 19 September 2007)

In finite volume the partition function of QCD with a given θ is a sum of different topological sectors with a weight primarily determined by the topological susceptibility. If a physical observable is evaluated only in a fixed topological sector, the result deviates from the true expectation value by an amount proportional to the inverse space-time volume $1/V$. Using the saddle point expansion, we derive formulas to express the correction due to the fixed topological charge in terms of a $1/V$ expansion. Applying this formula, we propose a class of methods to determine the topological susceptibility in QCD from various correlation functions calculated in a fixed topological sector.

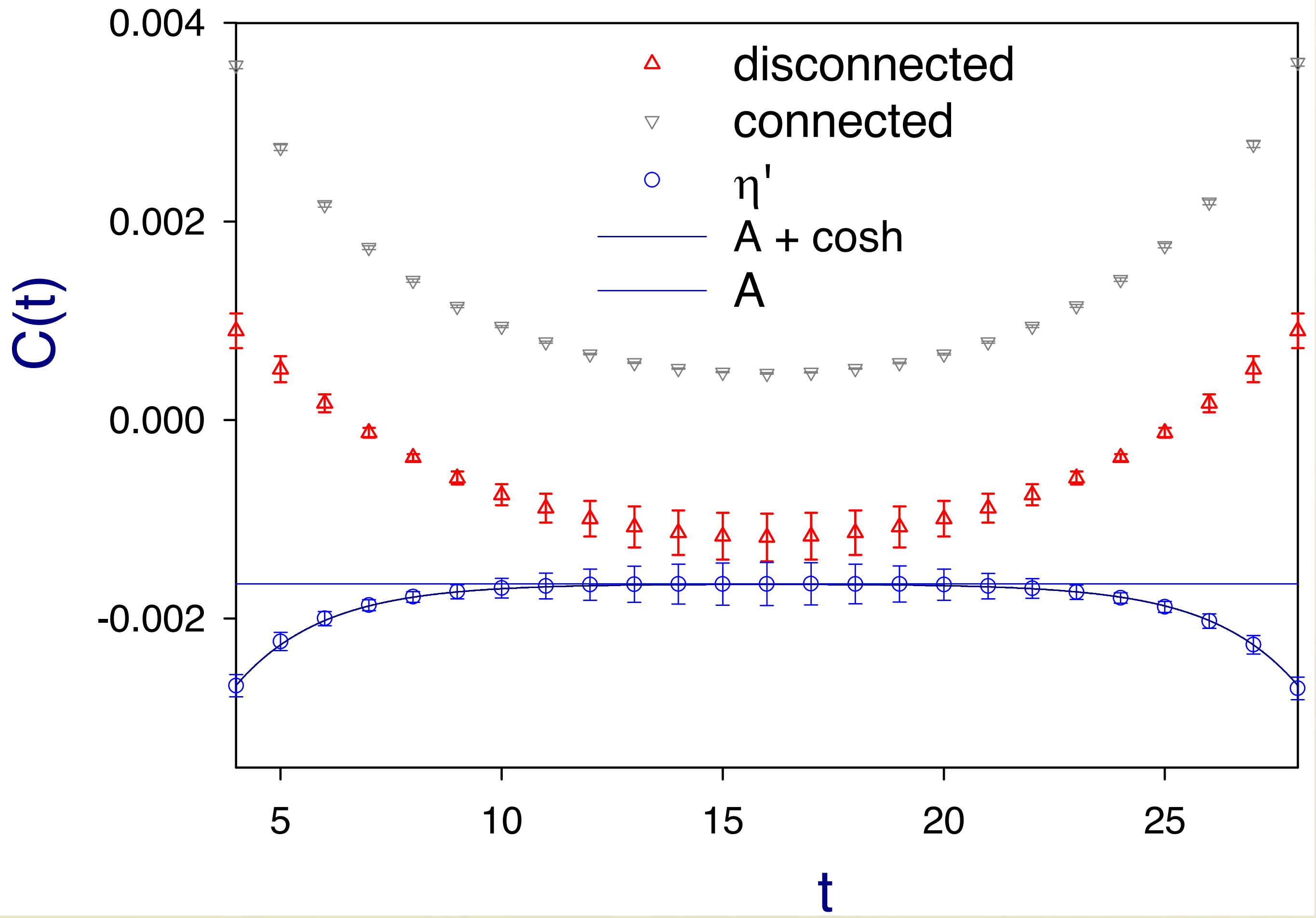
$$G_Q^{\mathrm{even}} = G(0) + G^{(2)}(0)\frac{1}{2\chi_t V}\left[1 - \frac{Q^2}{\chi_t V} - \frac{c_4}{2\chi_t^2 V}\right] + G^{(4)}(0)\frac{1}{8\chi_t^2 V^2} + O(V^{-3}),$$

A lot of applications

topological susceptibility

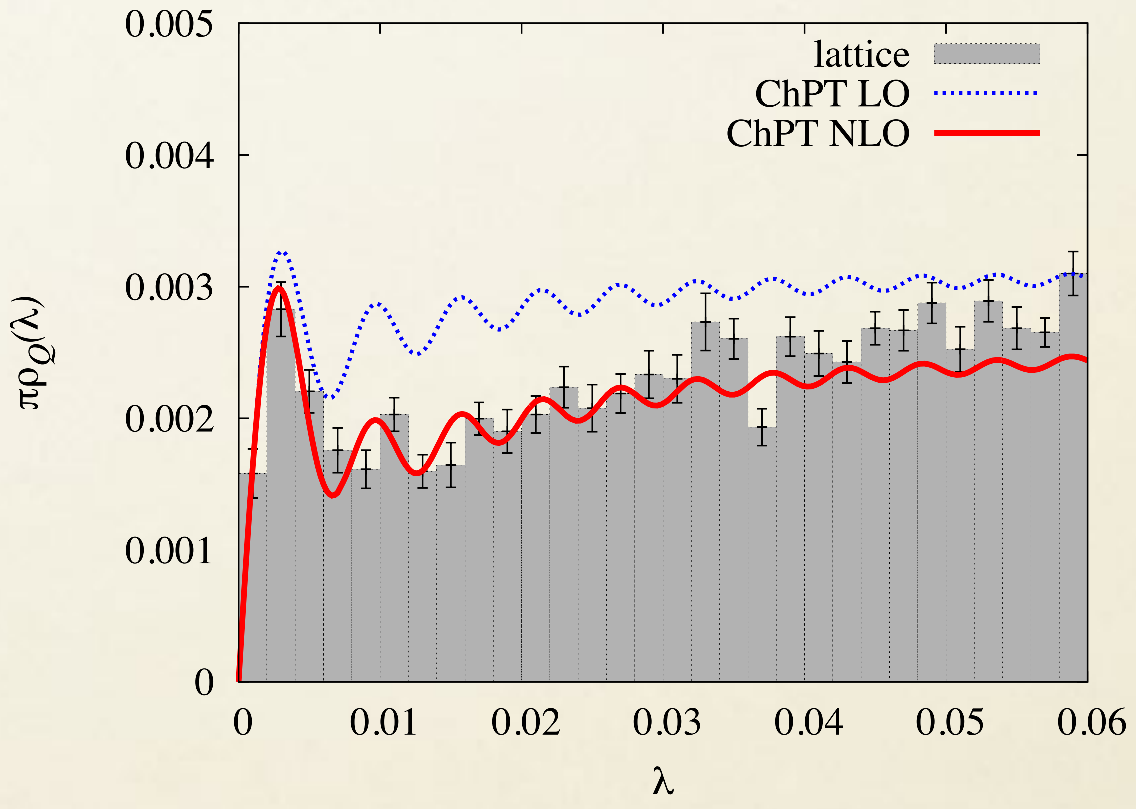


JLQCD&TWQCD (2007)

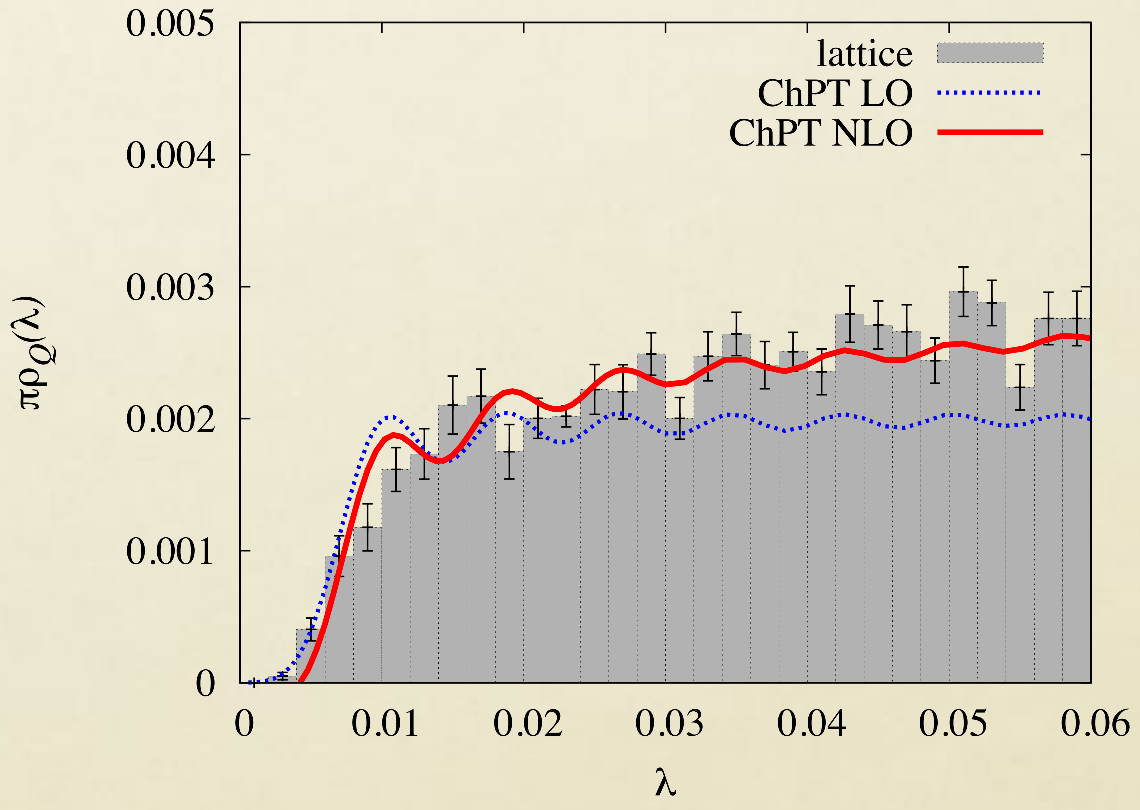


from the Q=0 sector!

Dirac eigenvalue spectrum



JLQCD (2009)



even near m=0!

Nucleon!

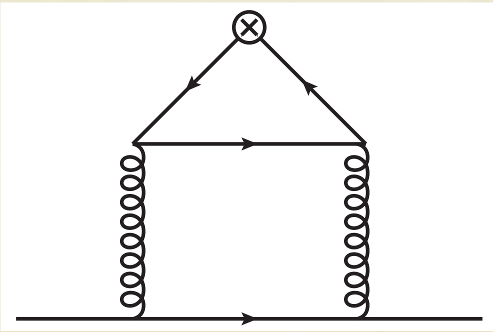
R. Young at Lattice 2012

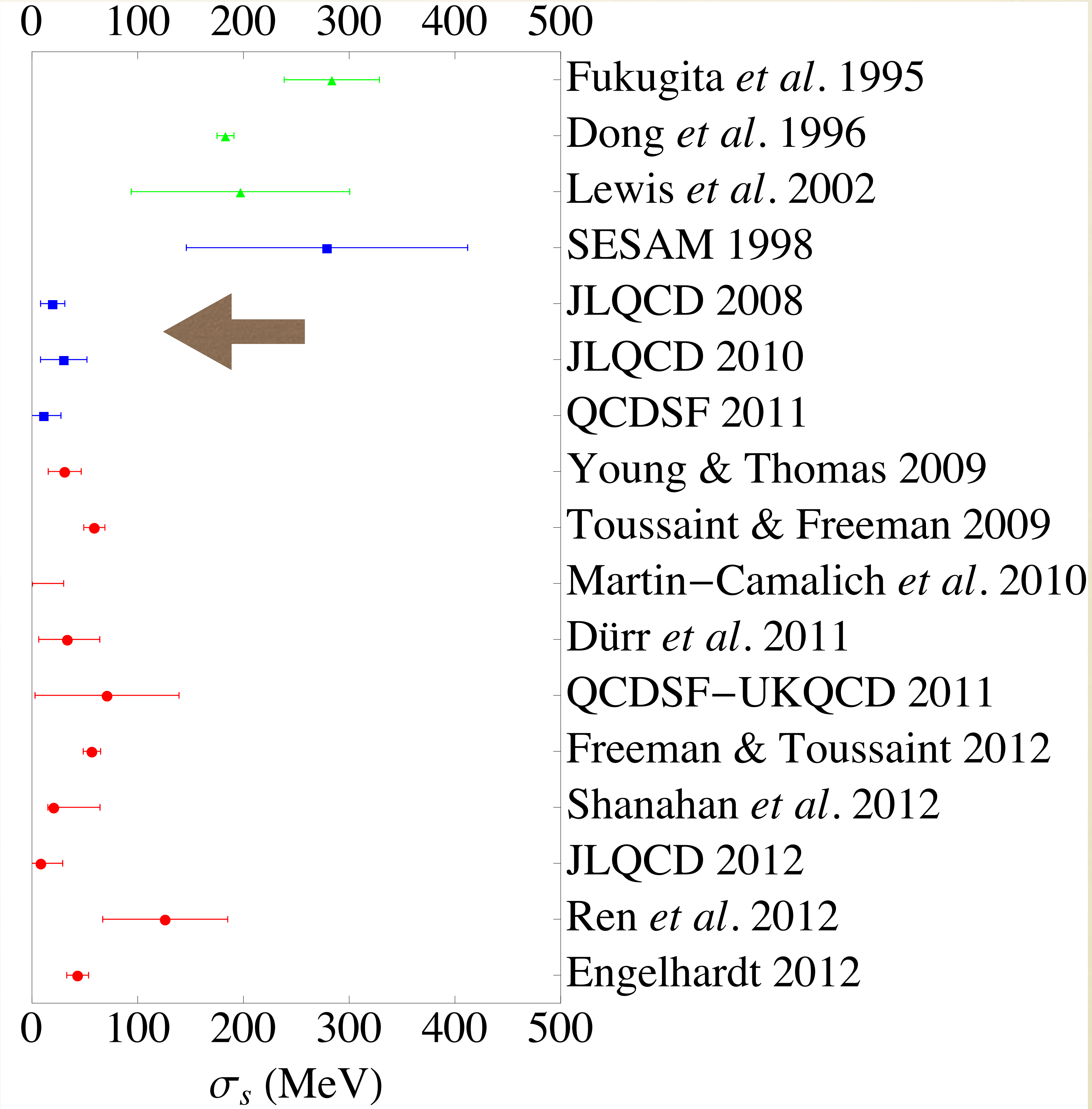
- No additive renormalization with overlap fermion. It enabled another direction of studies: sigma terms, the strange quark content, in particular.

$$y \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle},$$

- Tetsuya, together with his student Hiroshi Ohki, initiated a computation.

- Followed by other groups, then. Many of them were problematic, though.

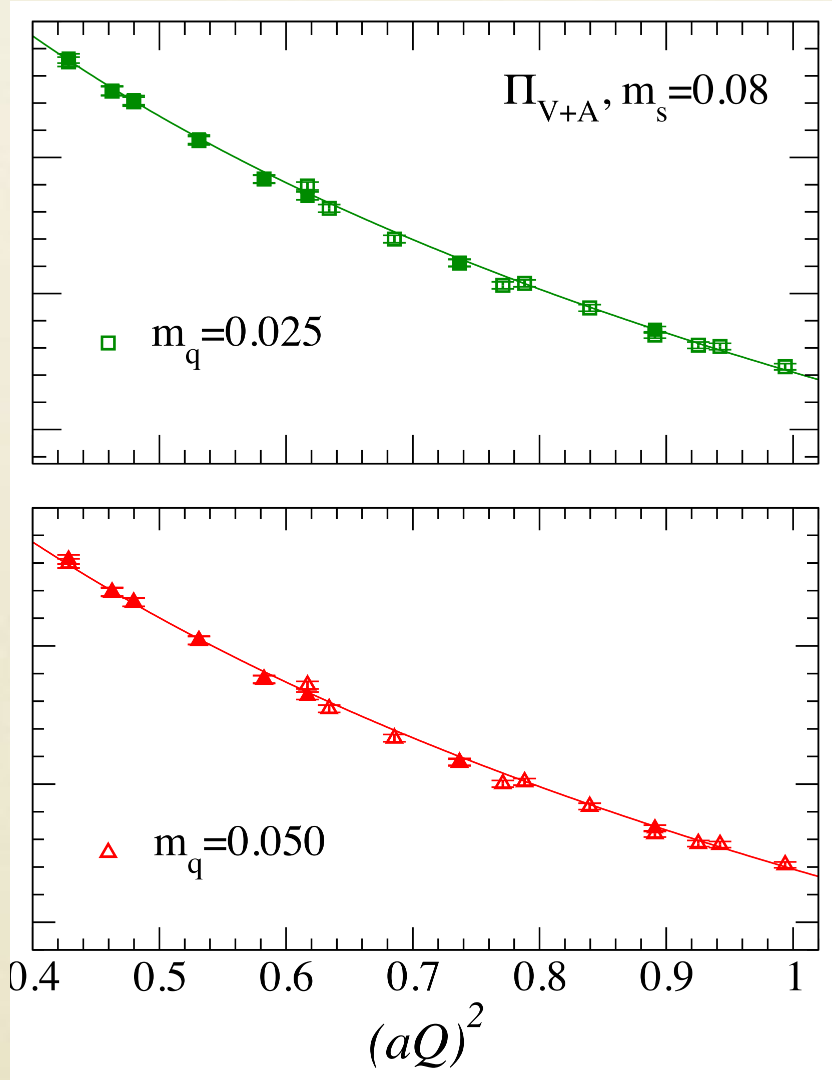




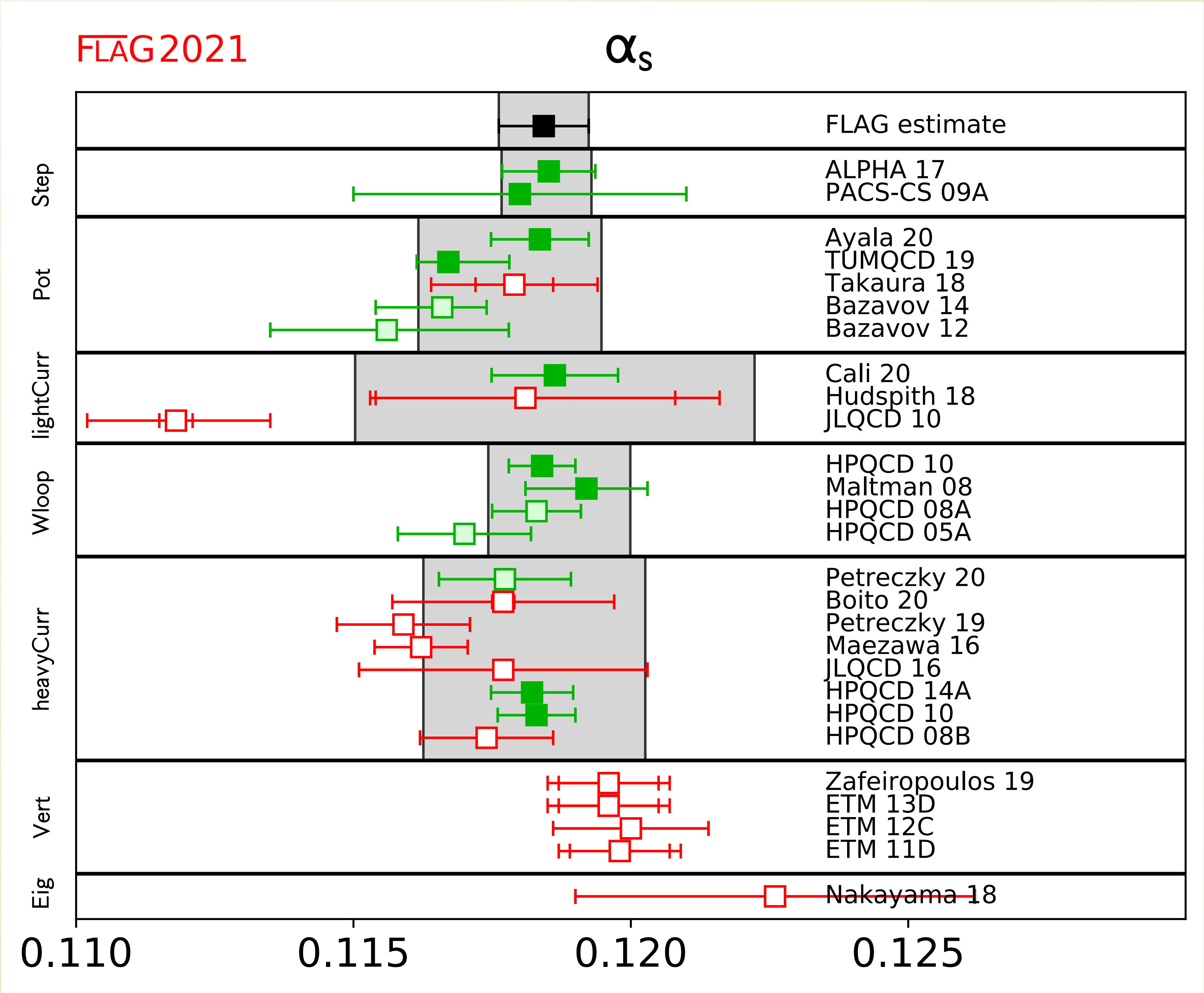
Strong coupling constant

- Mostly done (at the time) through the QCD potential (or the Wilson-loop), but one can use any perturbative quantities, say the Adler function.
- Discussion started at Tetsuya's office.

$$
\begin{aligned}
\Pi_{V+A}|_{\rm OPE}(Q^2,\alpha_s) \;=\;& c + C_0(Q^2,\mu^2,\alpha_s) \\
&+ C_m^{V+A}(Q^2,\mu^2,\alpha_s)\frac{\bar{m}^2(Q)}{Q^2} \\
&+ \sum_{q=u,d,s} C_{\bar{q}q}^{V+A}(Q^2,\alpha_s)\frac{\langle m_q\bar{q}q\rangle}{Q^4} \\
&+ C_{GG}(Q^2,\alpha_s)\frac{\langle(\alpha_s/\pi)GG\rangle}{Q^4} + \mathcal{O}(Q^{-6})
\end{aligned}
$$



Shintani et al. (2010)



Not competitive in precision…

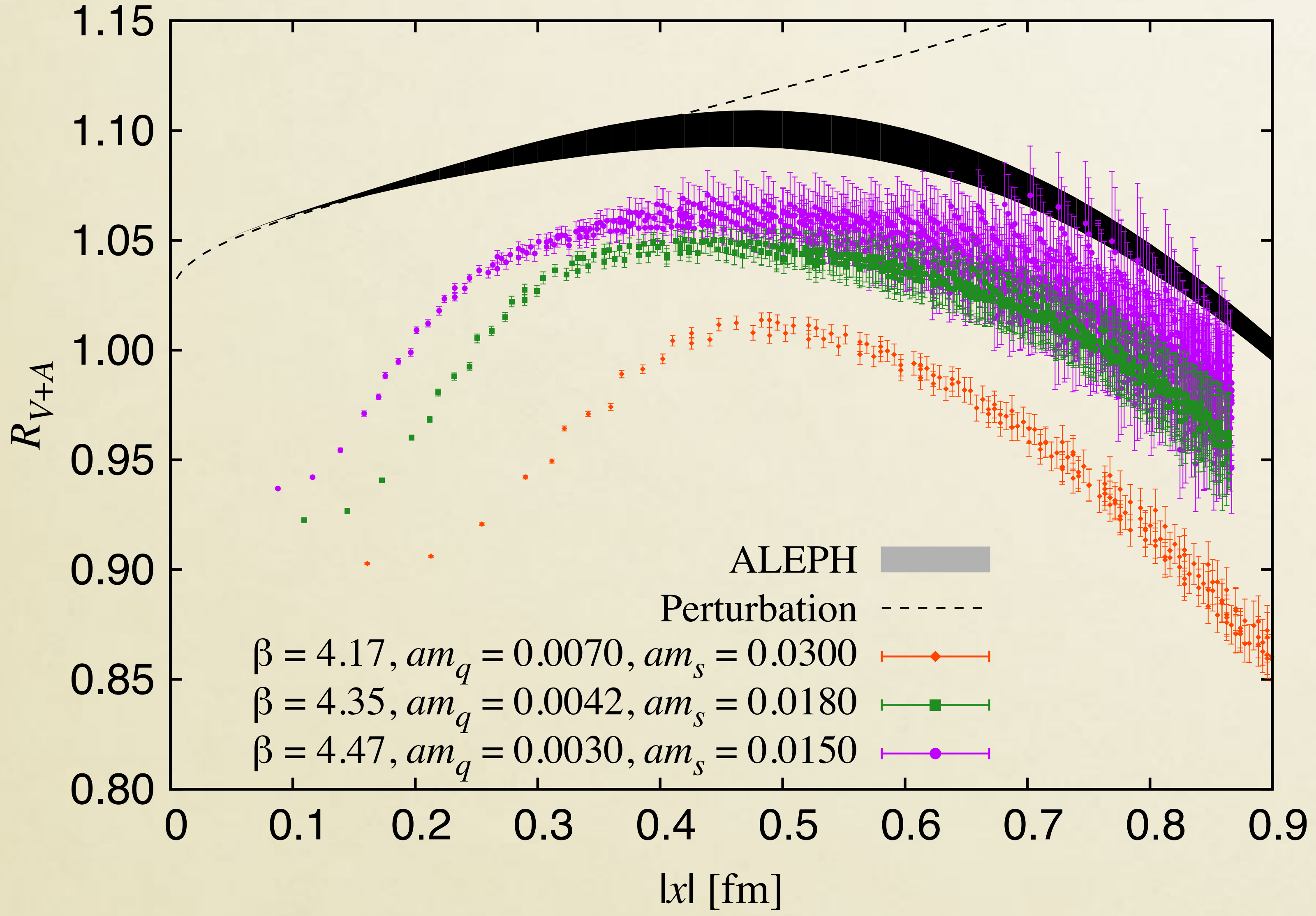
I learned a lot of things from our decades-long collaborations.

- LGT, LQCD in particular, is not just a toy to play on computers. In fact, it includes everything.
- Chiral symmetry is central to χSB, index theorem, topological susceptibility… It works on the lattice! All new.
- Perturbative at <u>short distances</u>, even on the lattice, of course!

→ excited states, and inclusive processes
our (more recent) works

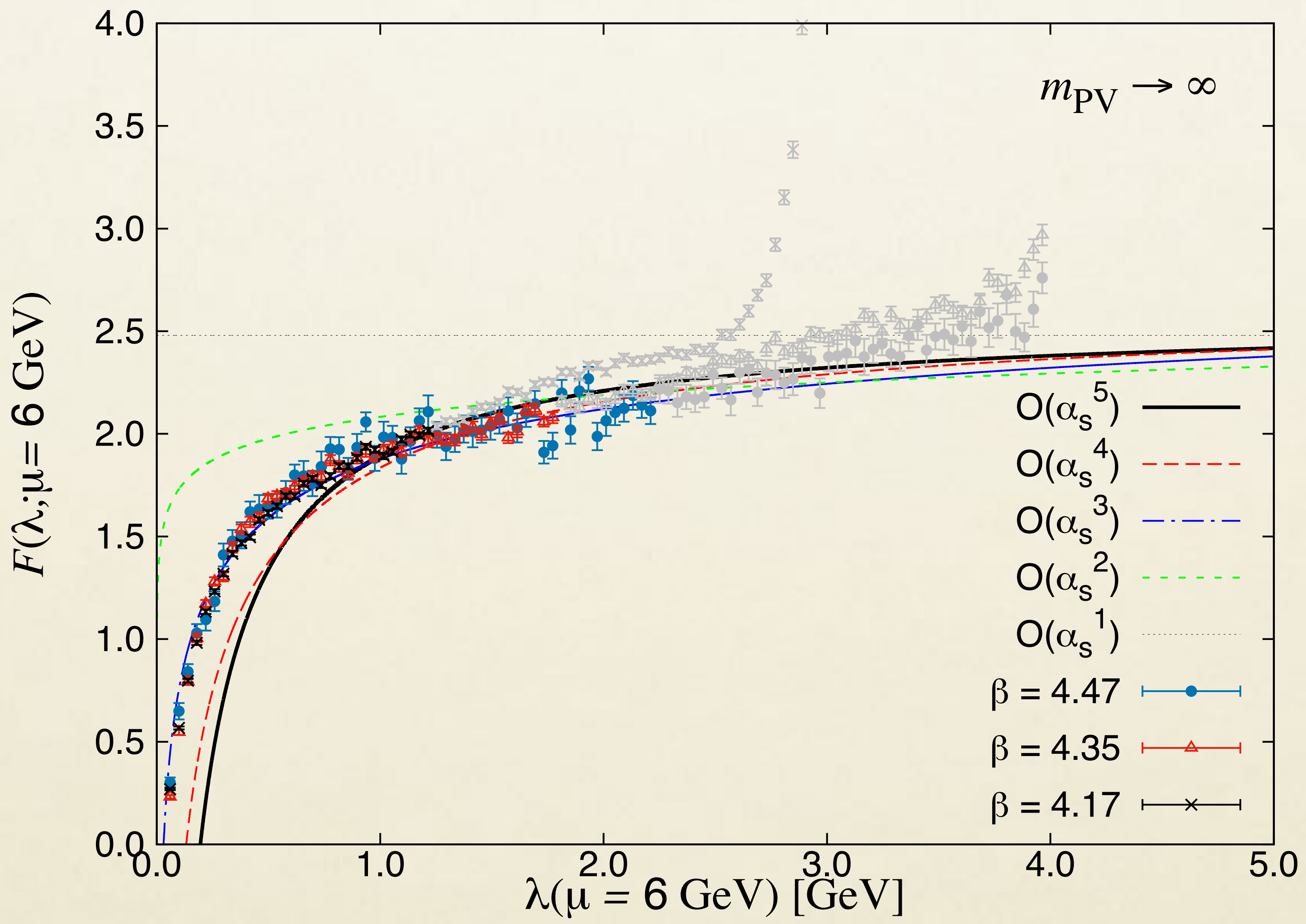
Short distances

Current correlator



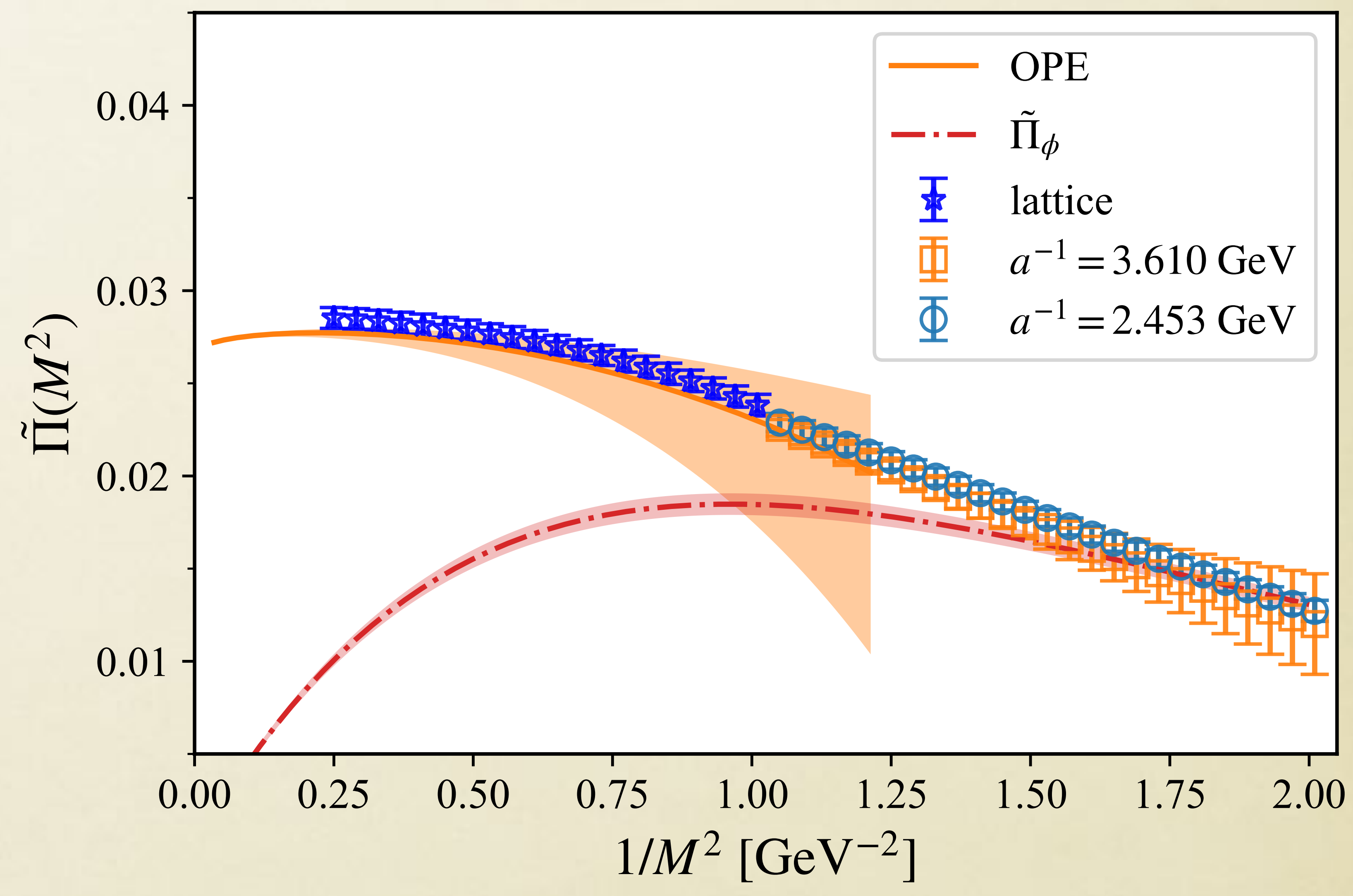
Tomii et al. (2017)

Dirac spectrum



Nakayama et al. (2018)

Borel transformation



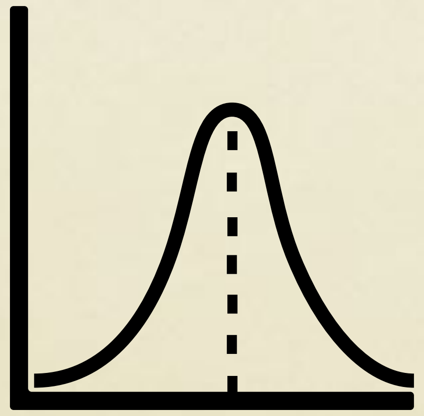
Ishikawa et al. (2021)

Smeared spectrum

Correlation function: $C(t) = \int_0^\infty d\omega\, \rho(\omega) e^{-\omega t}$ $\sim \langle 0 | J\, e^{-\hat{H}t}\, J | 0 \rangle$

Spectral function: $\rho(\omega) \propto \sum_X \delta(\omega - E_X) |\langle X | J | 0 \rangle|^2$ $\sim \langle 0 | J\, \delta(\omega - \hat{H})\, J | 0 \rangle$

Smearing: $\bar{\rho}_\Delta(\omega) = \int_0^\infty d\omega'\, S_\Delta(\omega, \omega') \rho(\omega)$ $\sim \langle 0 | J\, S_\Delta(\omega, \hat{H})\, J | 0 \rangle$

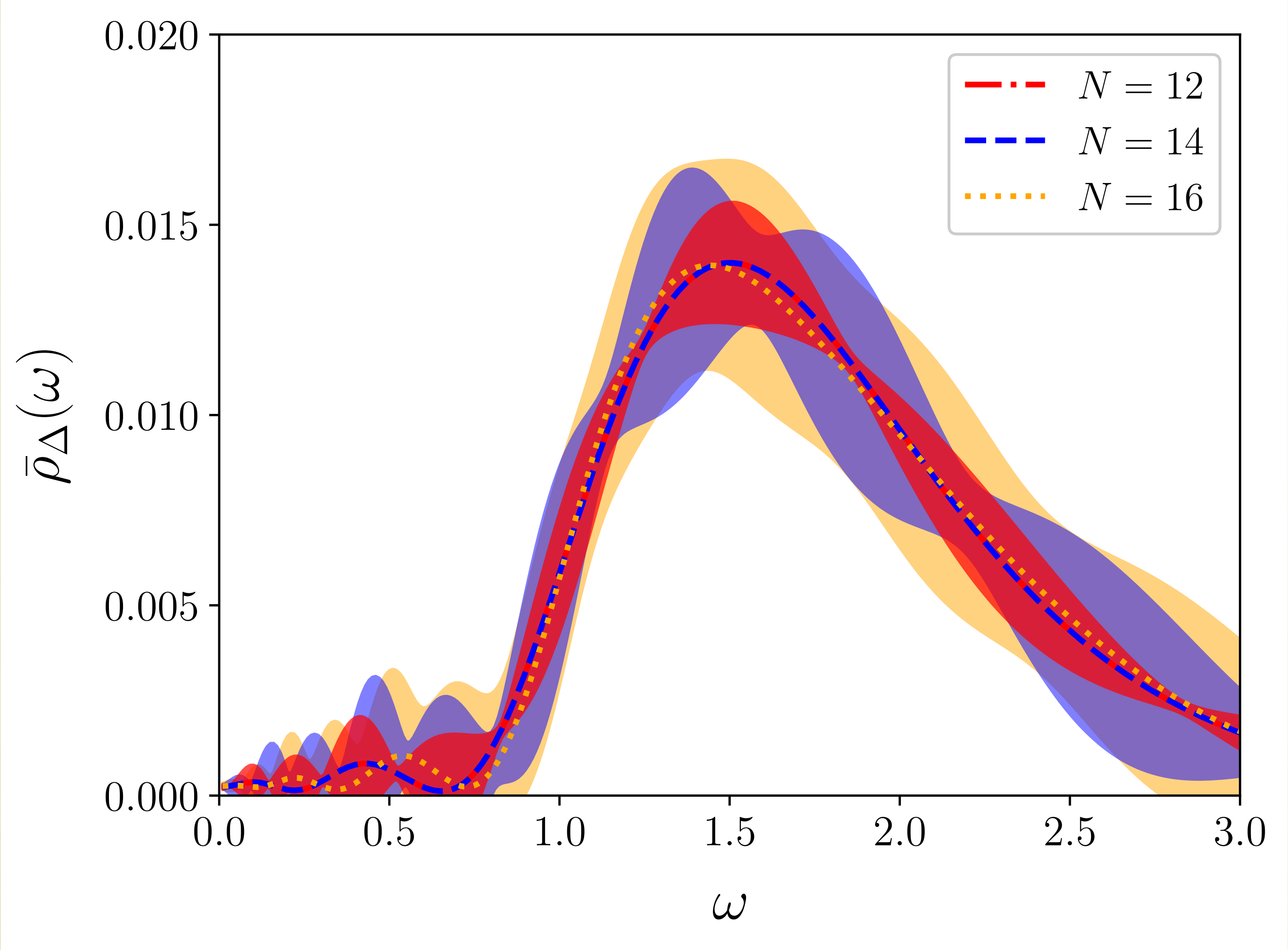


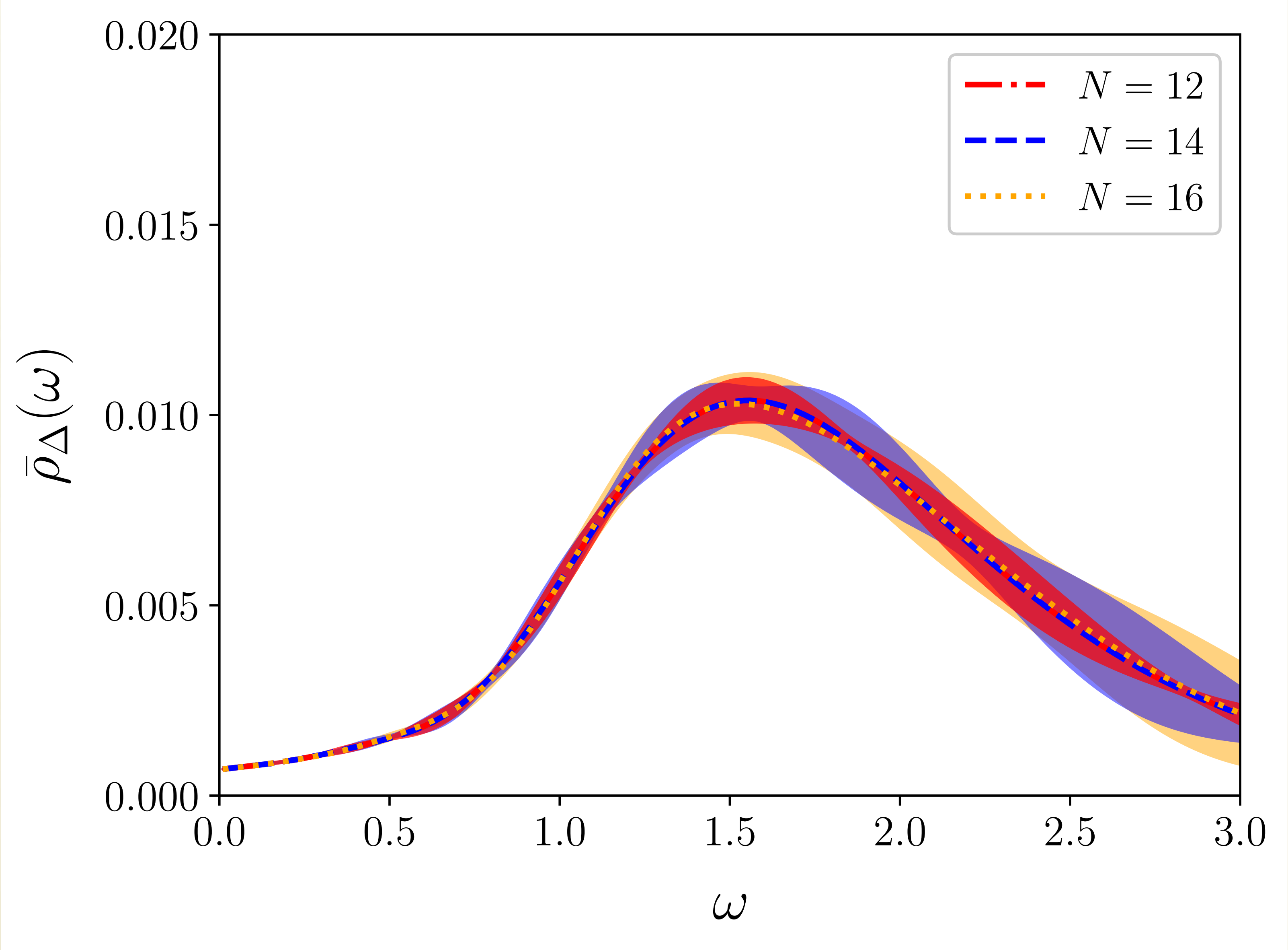
approx: $S_\Delta(\omega, \hat{H})$ by $e^{-\hat{H}t}$?

$\simeq k_0 + k_1 e^{-\hat{H}} + k_2 e^{-2\hat{H}} + \cdots + k_N e^{-N\hat{H}}$

Bailas, SH, Ishikawa, PTEP 2020, 4, 043B07 (2000); arXiv:2001.11779

narrow when the smearing kernel is ⋯ wide



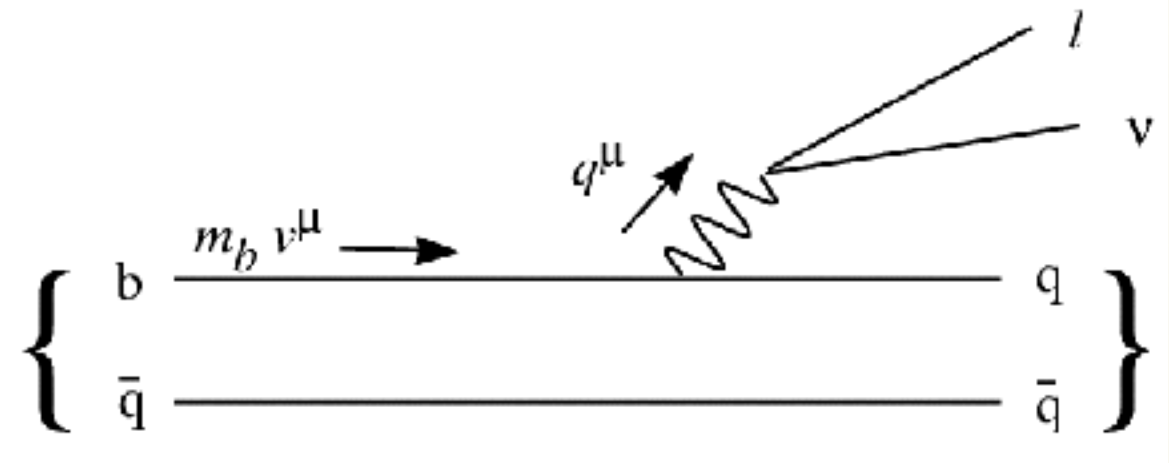


Inclusive processes

Gambino, SH (2020)

Differential decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$



Structure function (or hadronic tensor):

$$W_{\mu\nu} = \sum_X (2\pi)^2 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$\rightarrow \langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t)\ \delta(\omega - \hat{H})\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

Total decay rate:

$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega - \hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

kinematical (phase-space) factor

Energy integral to be evaluated:

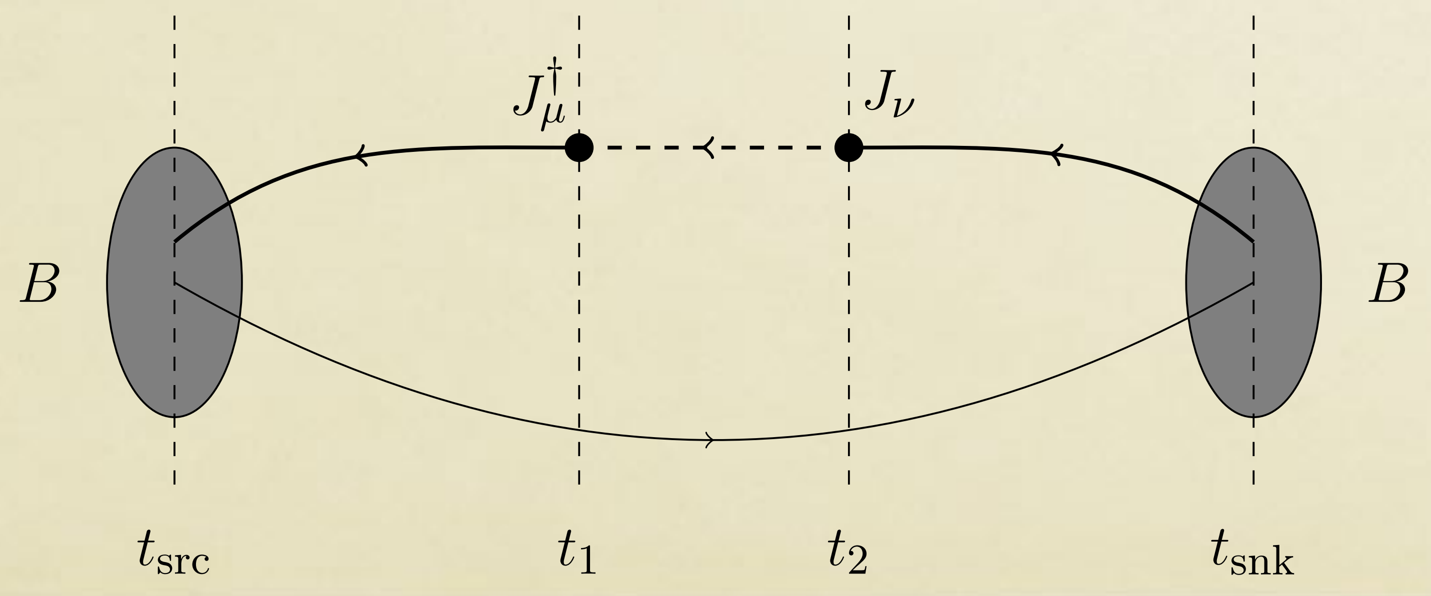
$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

$$= \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})K(\hat{H};\boldsymbol{q}^2)\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

smeared spectrum!

Compton amplitude obtained on the lattice:

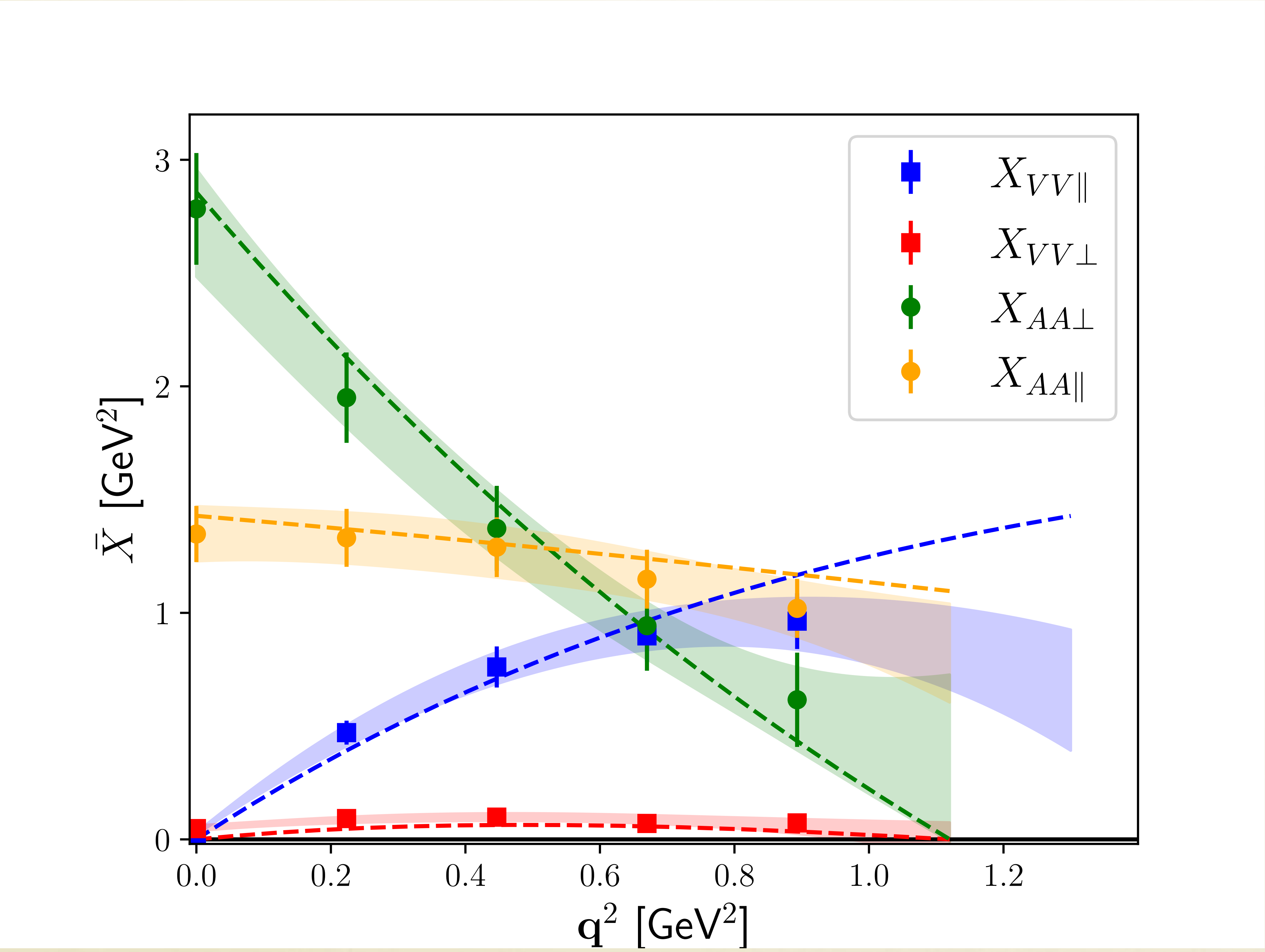
$$\langle B(\mathbf{0})|\tilde{J}^\dagger_\mu(-\boldsymbol{q};t)\;\; \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle \longrightarrow \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

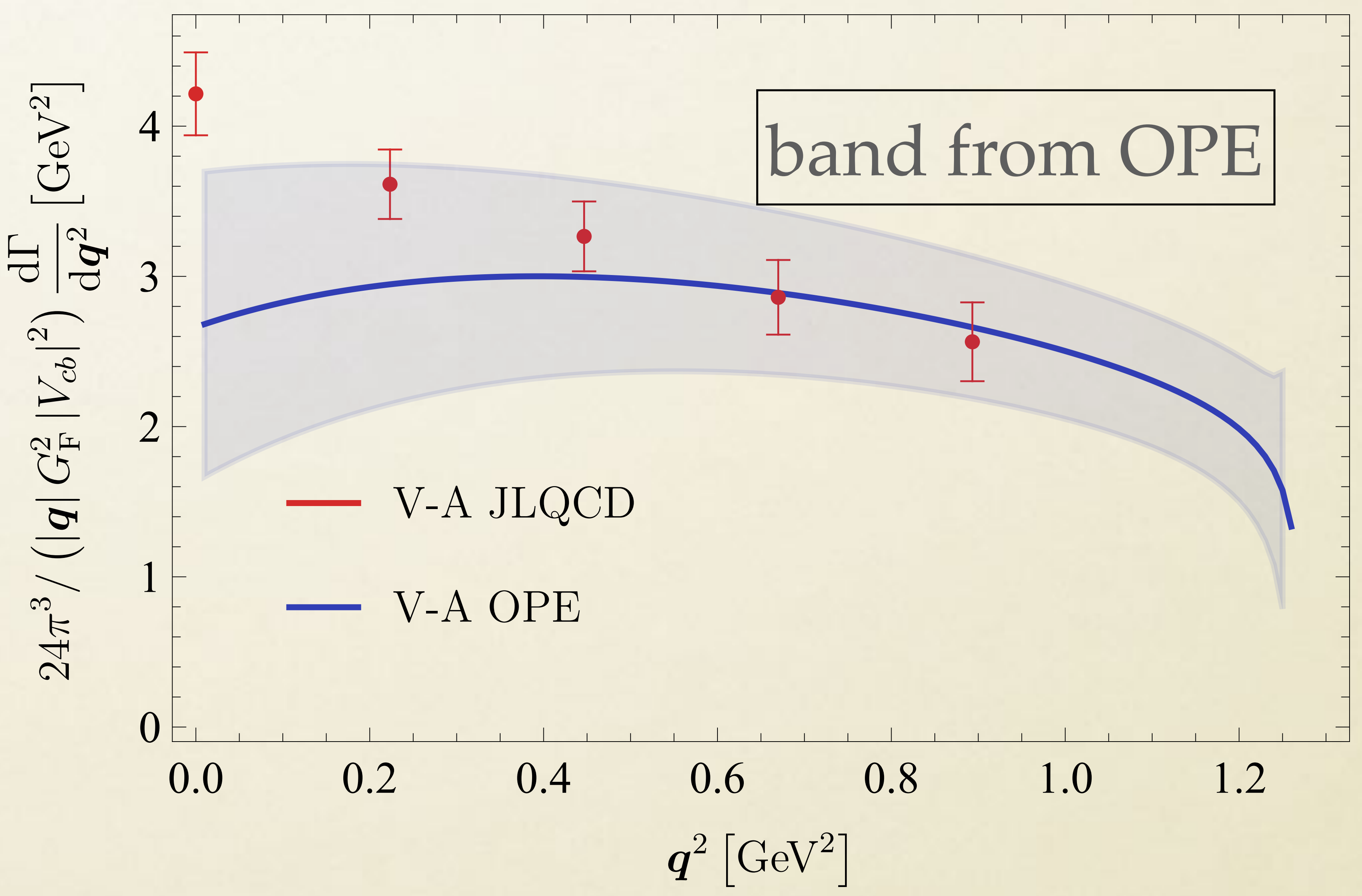


Approx :

$$K(\hat{H}) = k_0 + k_1 e^{-\hat{H}} + k_2 e^{-2\hat{H}} + \cdots + k_N e^{-k_N \hat{H}}$$

Gambino et al., 2203.11762





More challenges ahead

- QCD seems to be **the** theory at all distances: short and long. But, harder to treat in the middle. Can we go without the factorization, which is the basis of PDF, say?
- Related: Can we approach the light-cone from Euclidean lattice?
- (smeared) Spectral function: more uses, including hadronic B decays.

We need Tetsuya's insight on QFT, one more time!