Chiral susceptibility and axial U(1) anomaly near the (pseudo-)critical temperature

OSAKA UNIVERSITY
Live Locally, Grow Globally

Hidenori Fukaya (Osaka U.)

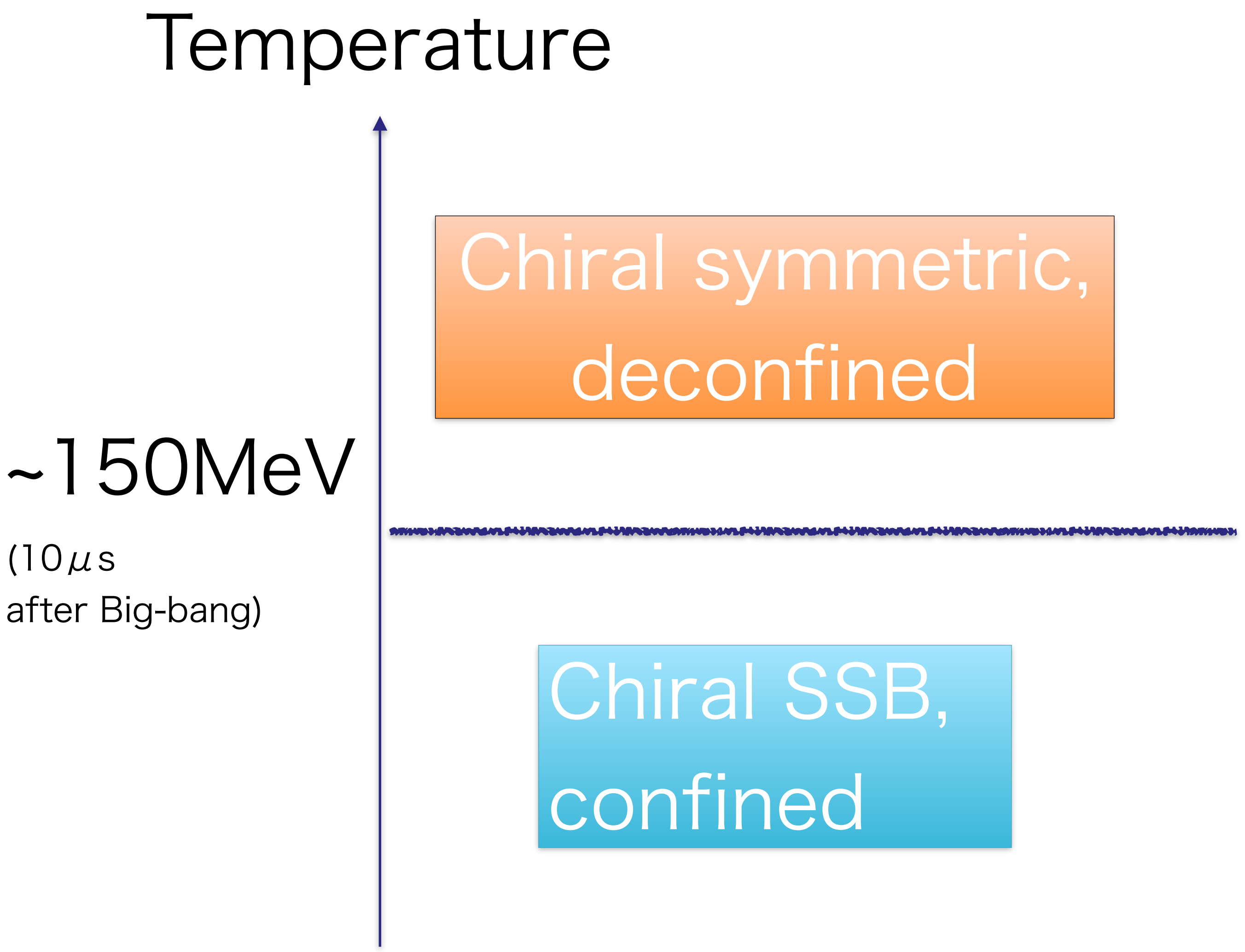
for JLQCD collaboration

[S. Aoki, Y. Aoki, HF, S. Hashimoto, I.Kanamori,
T. Kaneko, Y. Nakamura, K. Suzuki and D. Ward]

Updates from

S. Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki,
PTEP 2022 (2022) 2, 023B05 [2103.05954 [hep-lat]]

QCD phase transition



Chiral condensate (at m=0) probes $SU(2)_L \times SU(2)_R$ symmetry breaking/ restoration :

For T>Tc, $\langle \bar{q} q \rangle = 0$

For T<Tc, $\langle \bar{q} q \rangle \neq 0$

Chiral susceptibility

QCD partition function

A : gluon fields

$$Z(m) = \int [dA] \det(D(A) + m)^{N_f} e^{-S_G(A)}$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

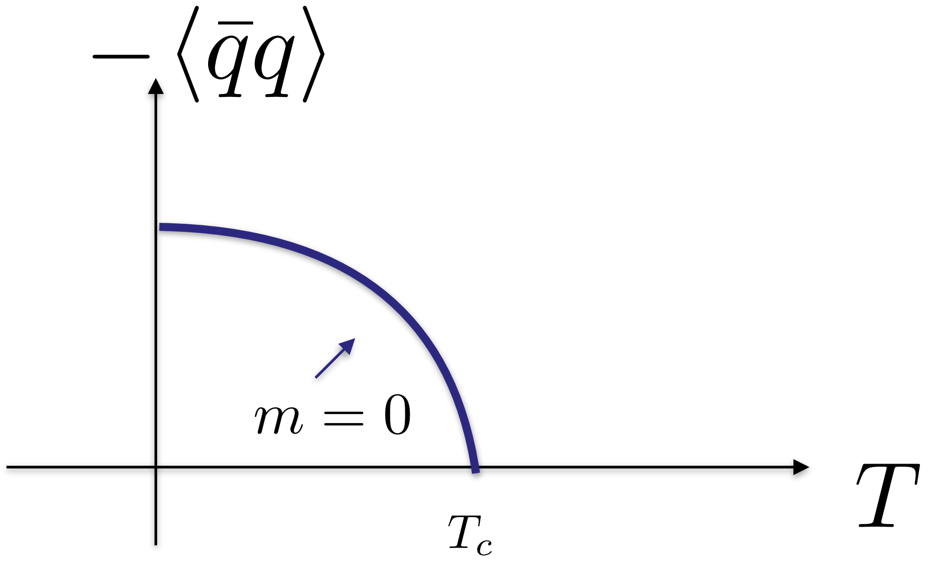
chiral susceptibility

$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle (m)$$

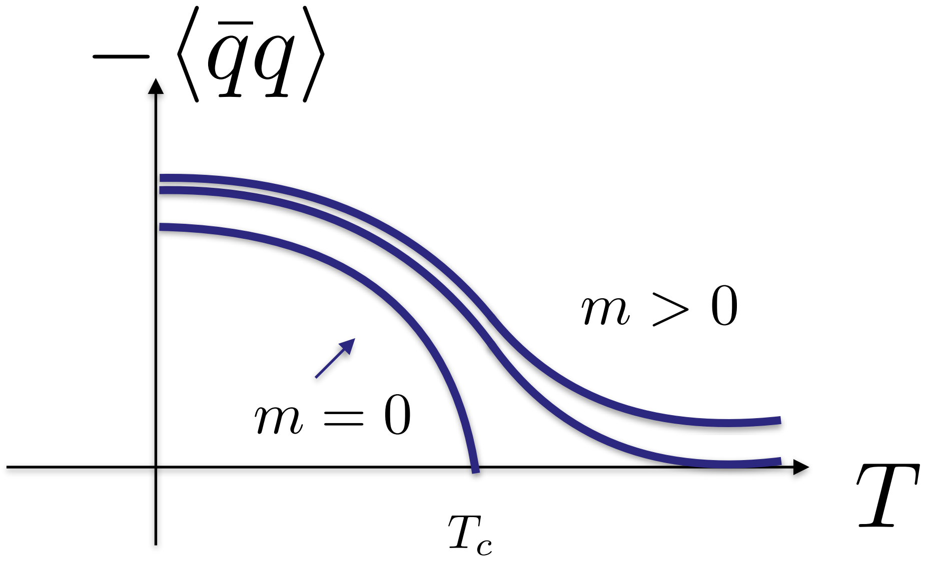
In this talk, $N_f = 2 \quad (m_u = m_d = m)$

* strange quark is just a spectator.

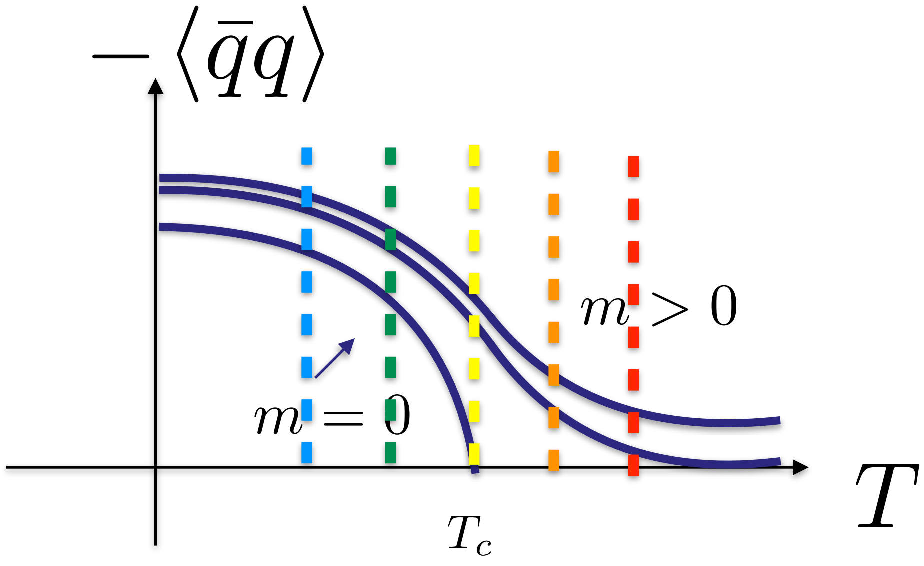
Temperature(T) and mass(m) dependence



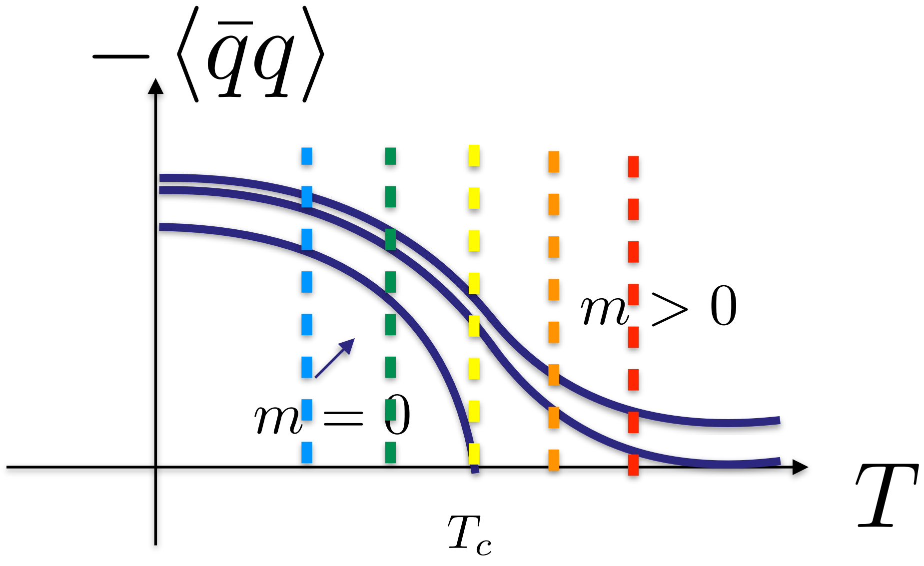
Temperature(T) and mass(m) dependence

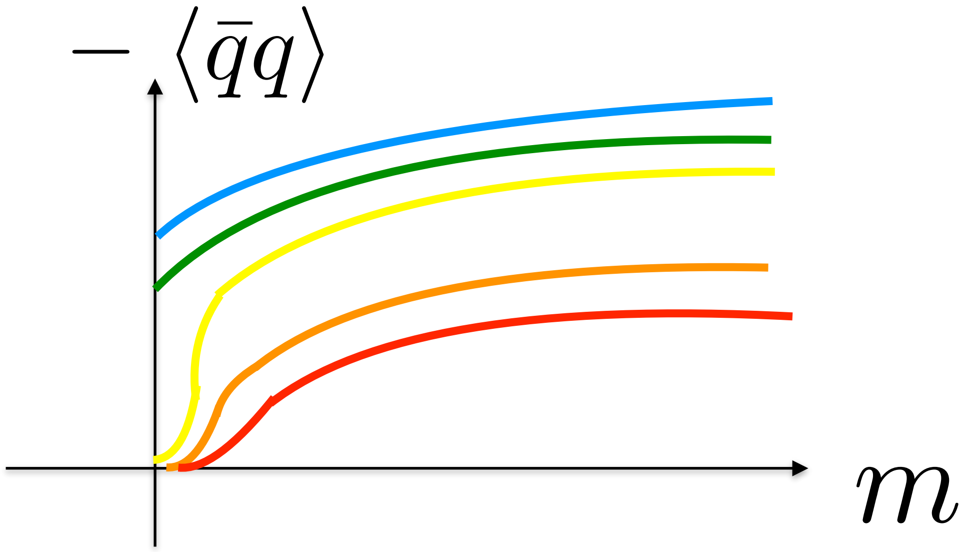


Temperature(T) and mass(m) dependence

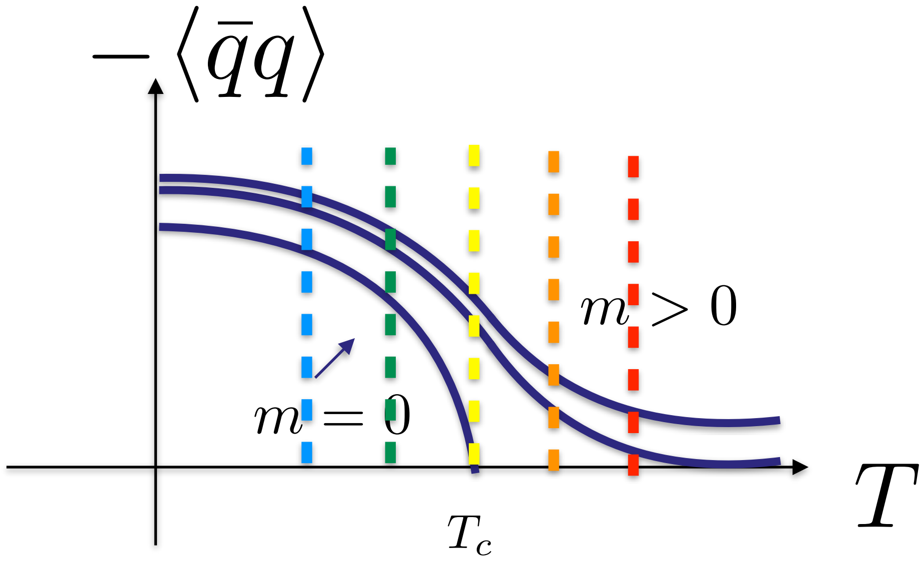


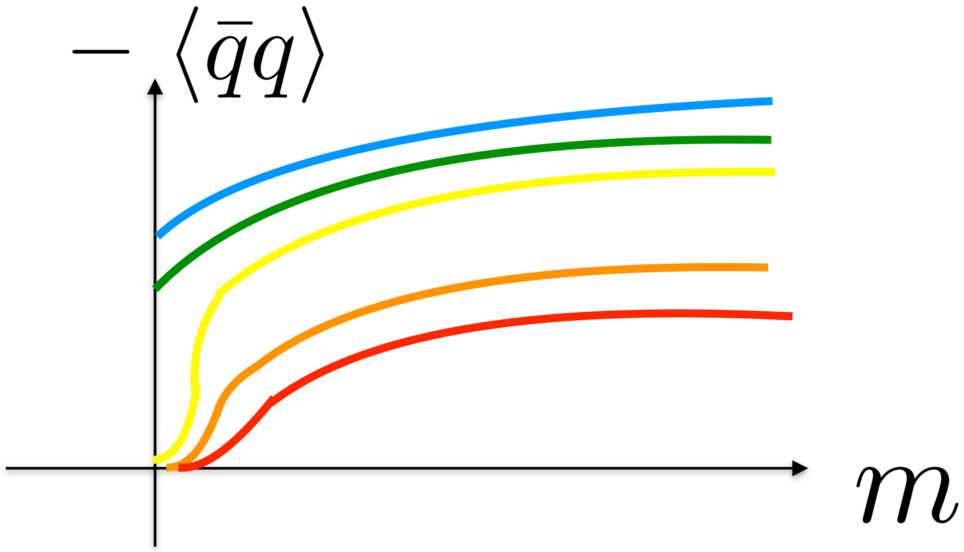
Temperature(T) and mass(m) dependence

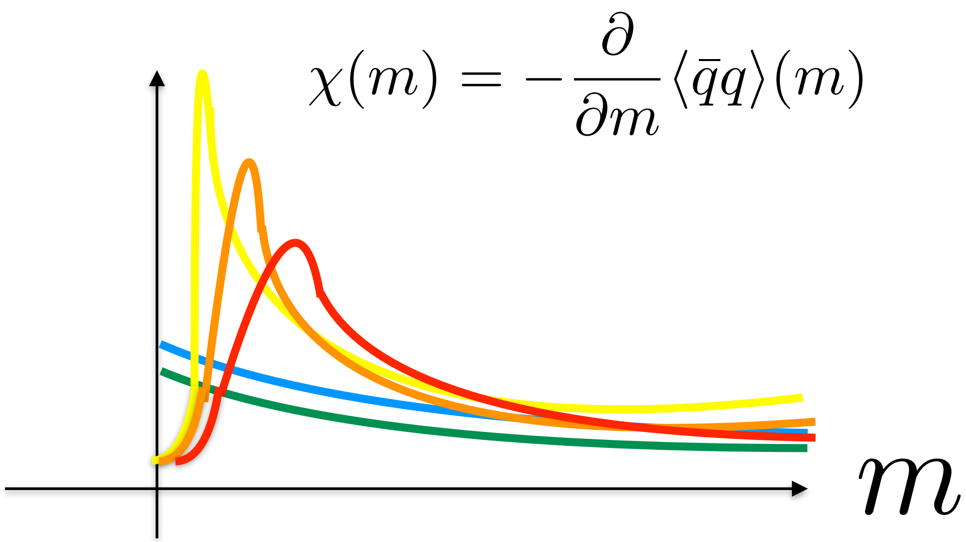




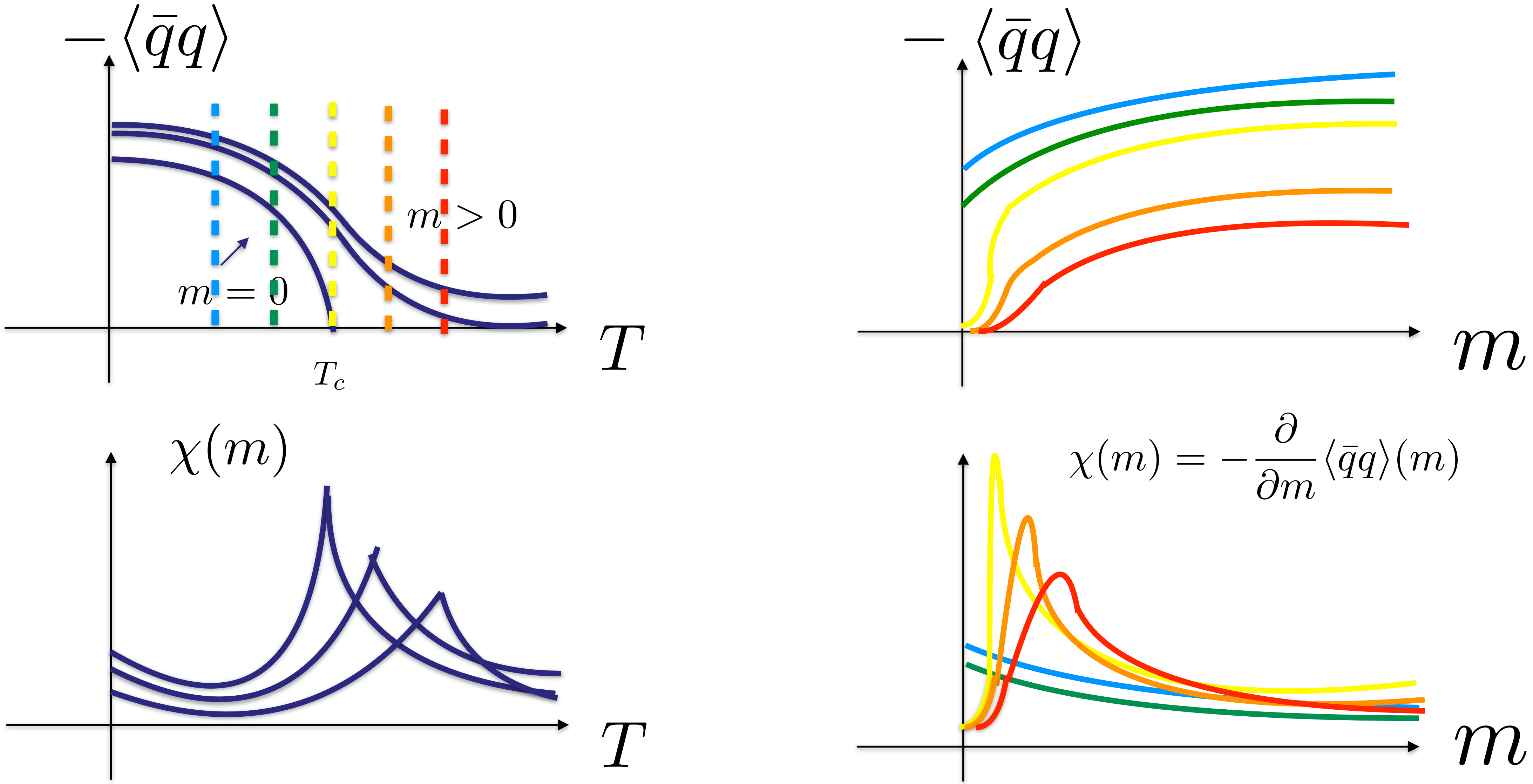
Temperature(T) and mass(m) dependence





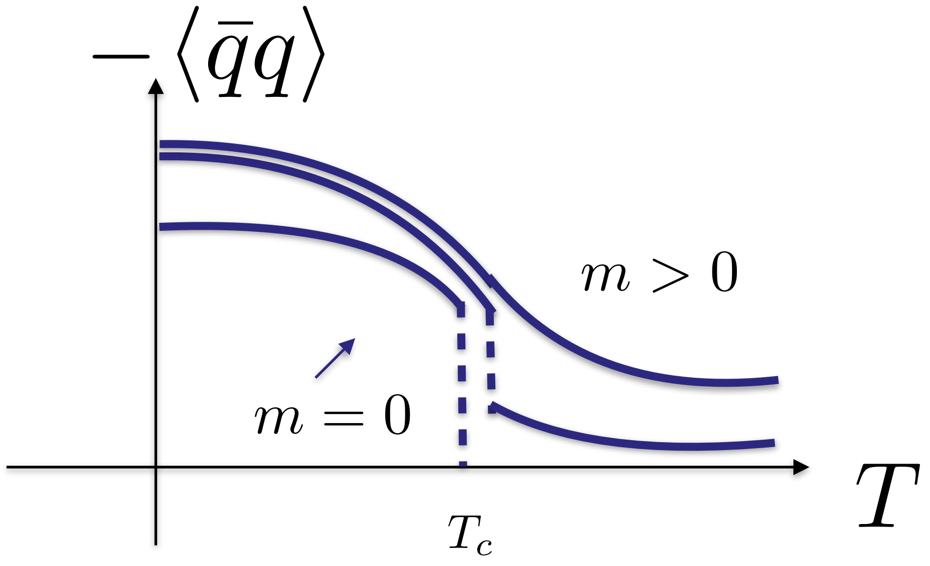


Temperature(T) and mass(m) dependence



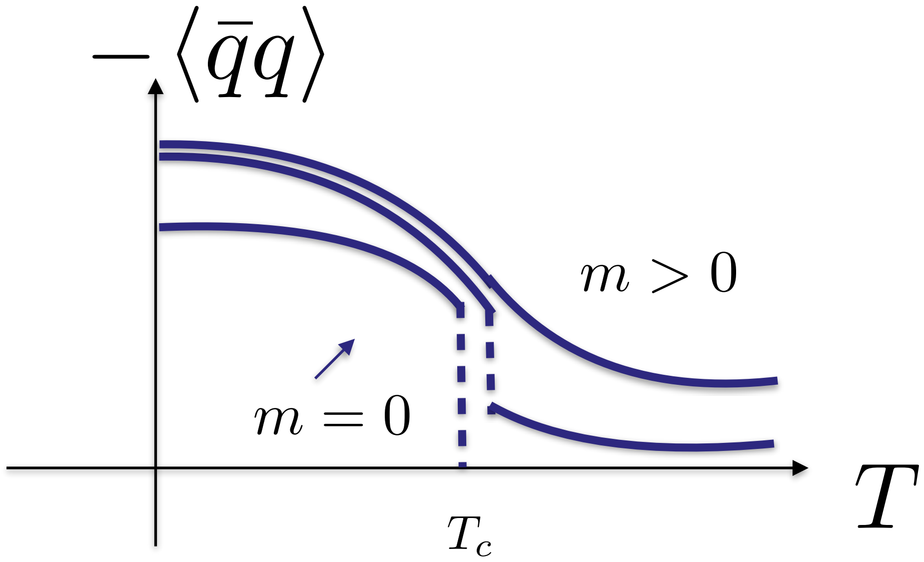
When the transition is 1st order

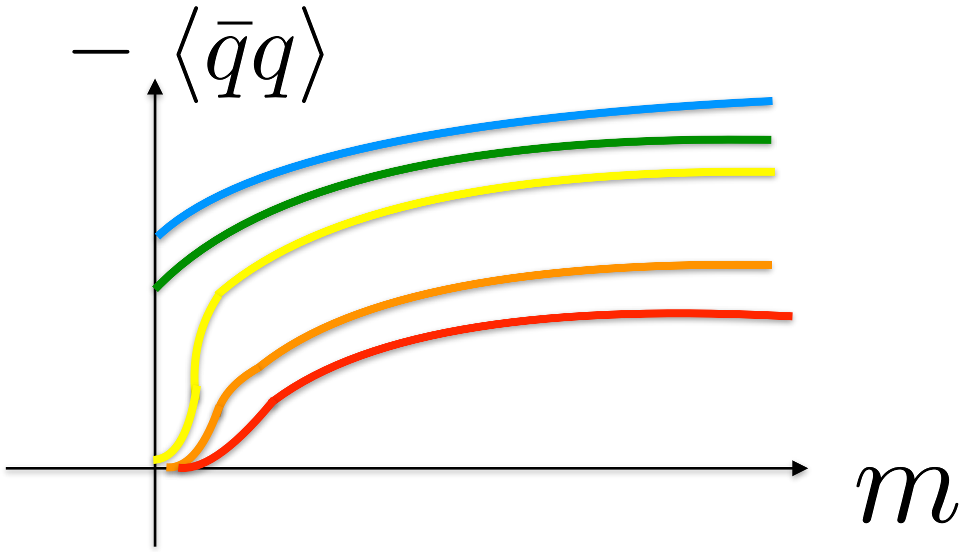
* But finite V effect makes the transition not sharp.



When the transition is 1st order

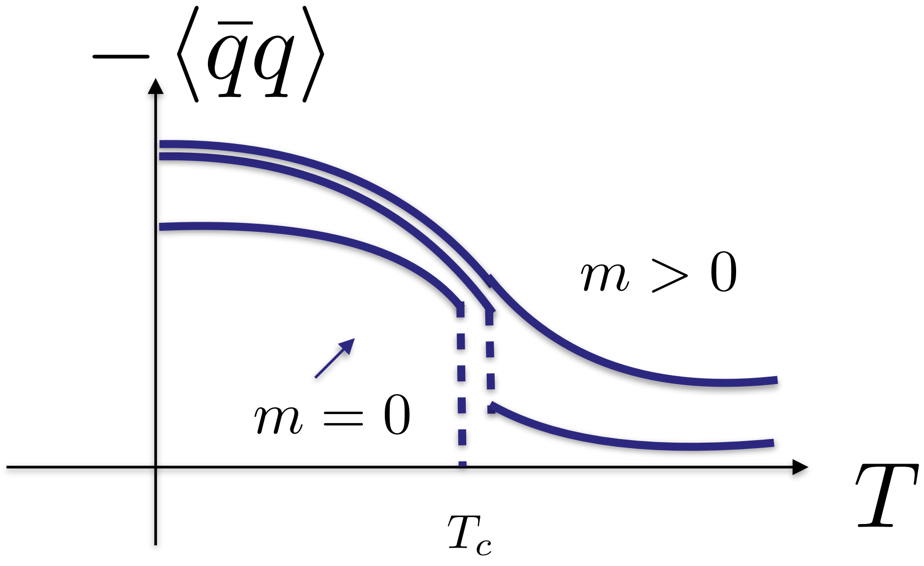
* But finite V effect makes the transition not sharp.

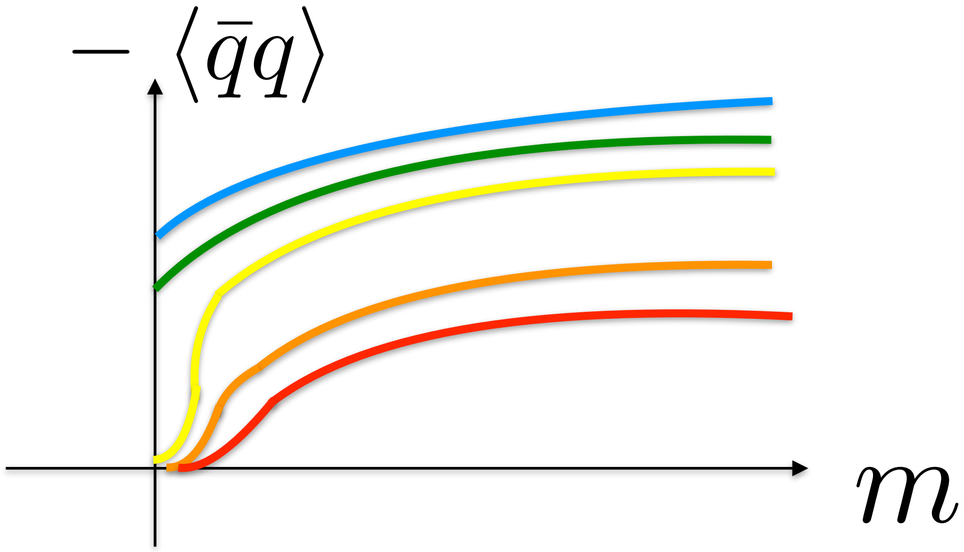


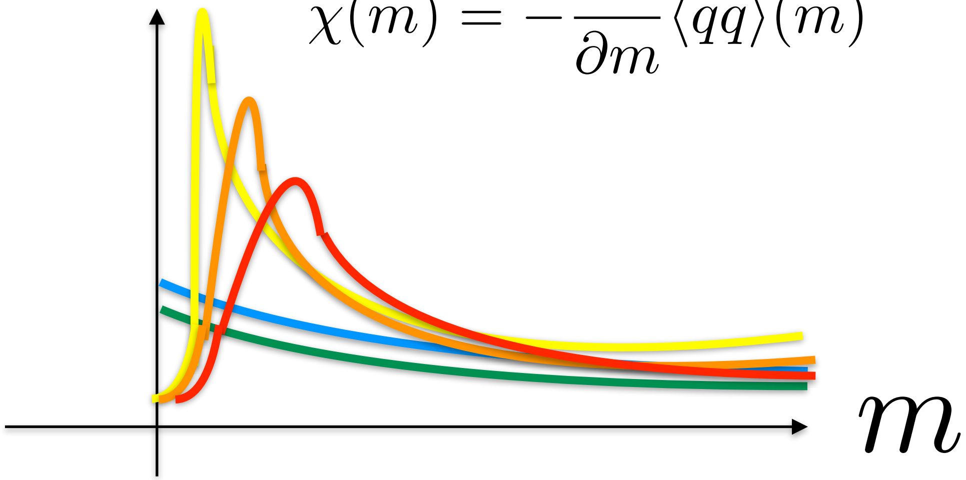


When the transition is 1st order

* But finite V effect makes the transition not sharp.

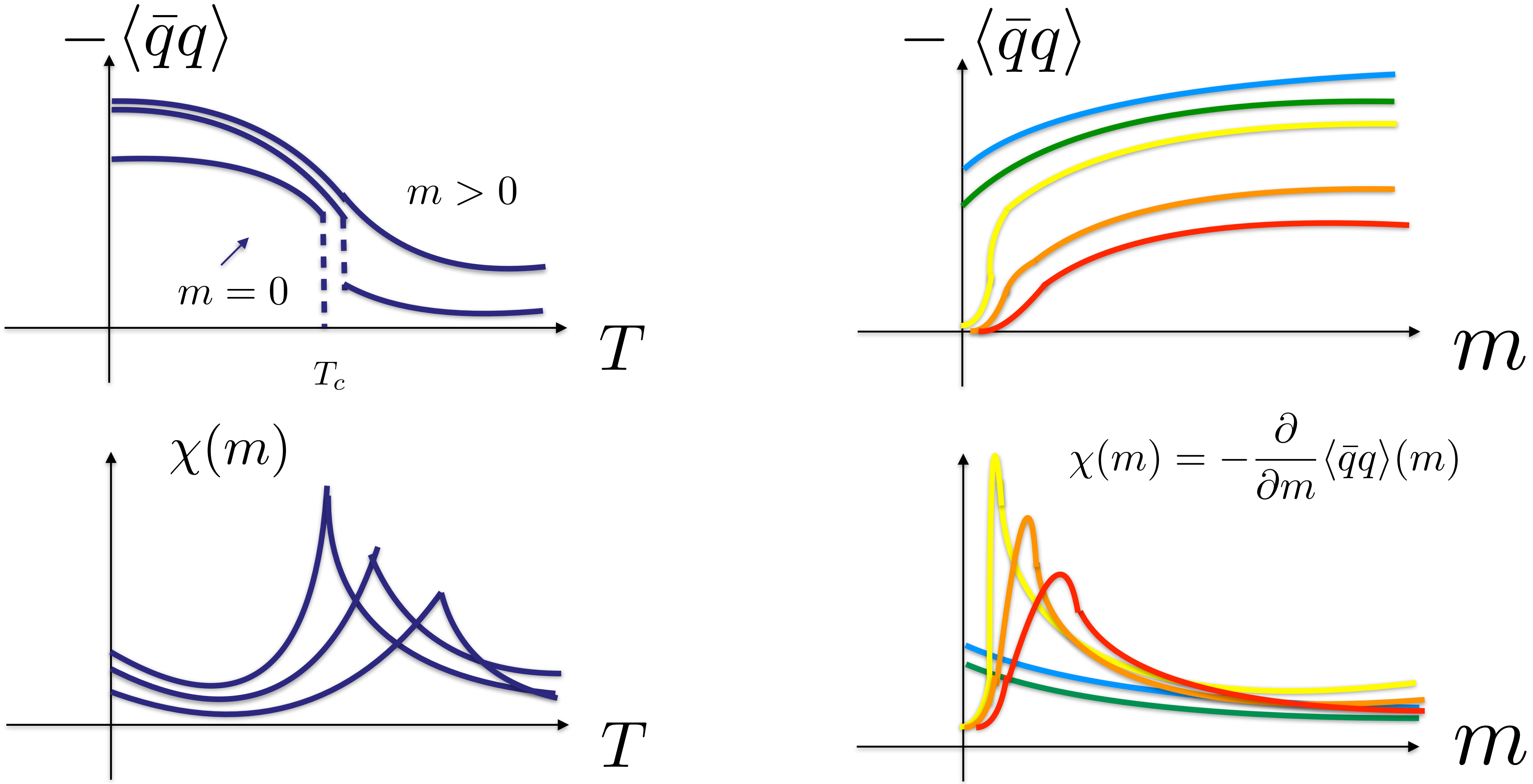






When the transition is 1st order

* But finite V effect makes the transition not sharp.



Chiral phase transition

Chiral condensate probes

$SU(2)_L \times SU(2)_R$ symmetry breaking/restoration :

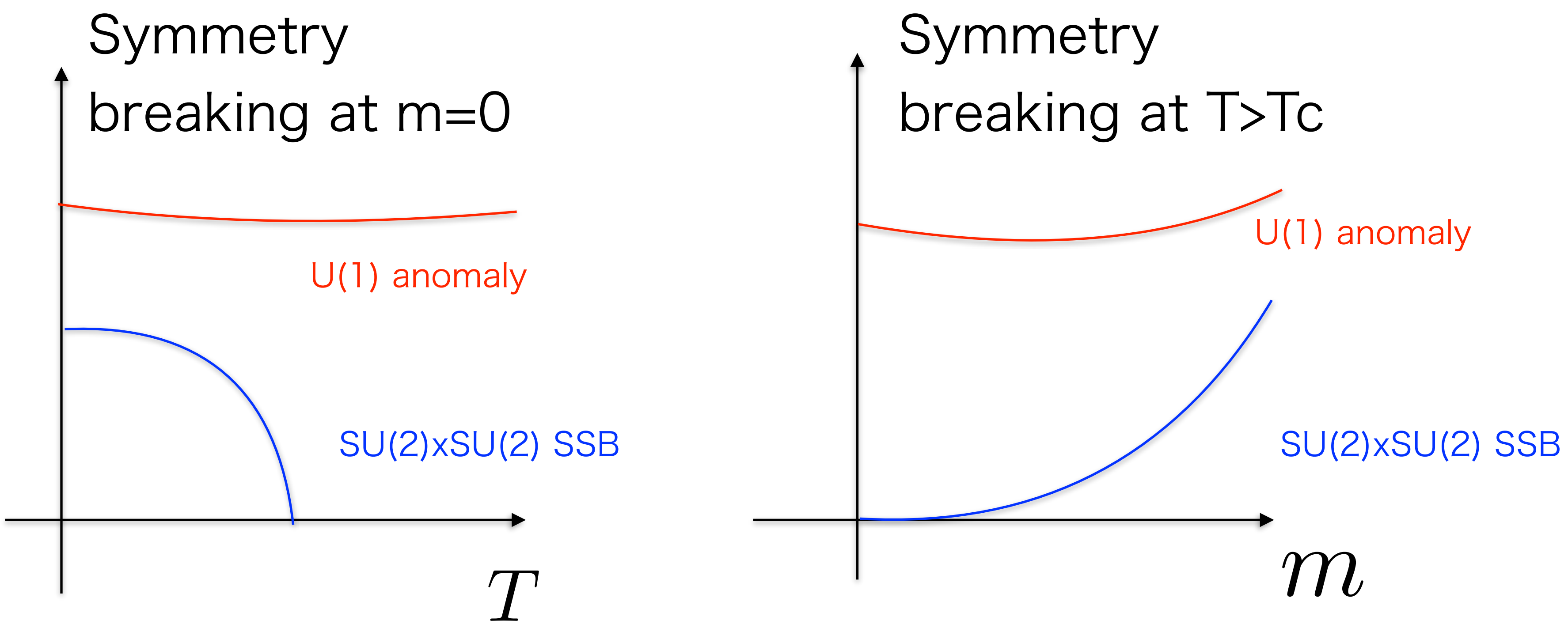
For T<Tc, $\langle \bar{q}q \rangle \neq 0$ For T>Tc, $\langle \bar{q}q \rangle = 0$

But $\langle \bar{q}q \rangle$ also breaks $U(1)_A$ symmetry.

Question:

How much does $U(1)_A$ (anomaly) contribute to the transition?

Naive expectation: U(1) anomaly exists at any energy scale (does not change much)



You may think that T and m dependences of chiral condensate should reflect $SU(2)_L x SU(2)_R$ breaking rather than U(1) anomaly.

But in early days of QCD

QCD founders in 70's and 80's thought

instanton → axial U(1) anomaly → SU(2)xSU(2) breaking.

Callan, Dashen & Gross 1978:

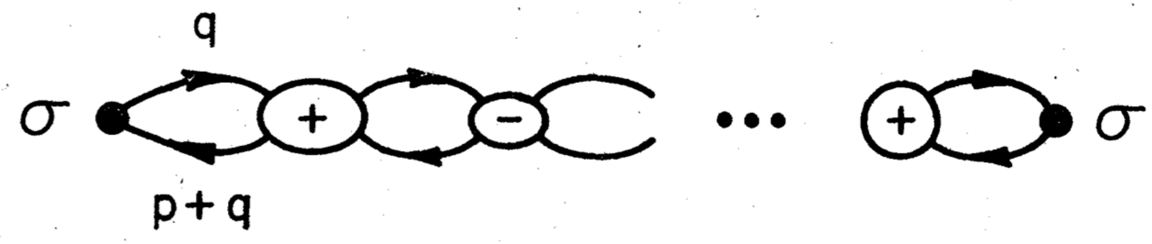


FIG. 9. The structure of the diagrams that produce a tachyon in the σ channel. The + (−) blobs refer to the effective determinantal four-fermion interaction induced by instantons (anti-instantons).

If this inverse is true, we should have

instanton disappears → anomaly disappears → SU(2)xSU(2) restored.

It has been difficult issue.

Analytic method:

Semi-classical QCD instantons are not enough to describe the low-energy dynamics of QCD.

Lattice simulations :

Staggered fermions explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow U(1)_{A'}$

Wilson fermion explicitly breaks

$SU(2)_L \times SU(2)_R \times U(1)_A \rightarrow SU(2)_V$

Moreover, we found that lattice artifacts are enhanced at high temperature (even for domain-wall fermions) [JLQCD 2015, 2016]

Our work

In this work we study chiral condensate and its susceptibility in 2- and 2+1-flavor QCD with chiral symmetric Dirac operator.
We separate the axial U(1) breaking (in particular topological) effect from others in a clean way.

Our result shows that
signal of chiral susceptibility is dominated by axial U(1) breaking effect (at T>=Tc),
rather than $SU(2)_L \times SU(2)_R$.

Other finite T talks by JLQCD members

K. Suzuki (Mon): axial U(1) anomaly

D. Ward(Mon) : meson screening mass and symmetries

J. Goswami(Mon) :Quark number susceptibility

Y. Zhang(Tue) : 3-flavor QCD phase transition

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_\lambda (i\lambda(A)+m)^{N_f} e^{-S_G(A)}$$

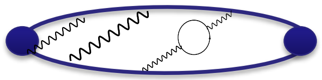
O(100) eigenvalues can be computed on the lattice.

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V} \left\langle \sum_\lambda \frac{1}{i\lambda(A)+m} \right\rangle,$$

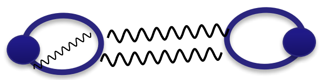
chiral susceptibility

$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}} \langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$



$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}} \langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$



Chiral rotations (with angle π)

$$S^0(x) = \bar{q}q(x)$$

$SU(2)_L \times SU(2)_R$ ↔ $\exp(i\pi\gamma_5\tau_a/2)$

$$P^a(x) = \bar{q}i\gamma_5\tau^a q(x)$$

$U(1)_A$ ↕ $\exp(i\pi\gamma_5/2)$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$$S^a(x) = \bar{q}\tau^a q(x)$$

isospin singlet

isospin triplet

Relation to scalar susceptibility

$$L_{\mathrm{QCD}} = \left[\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}\left(\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) + m\right)q\right]$$

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2}\ln Z(m, \theta = 0)$$

$$= -\sum_x \langle S^0(x) S^0(0)\rangle - V\langle S^0\rangle^2$$

$$S^0(x) = \bar{q}q(x)$$

Relation to pseudoscalar susceptibility

$$Z(m,\theta) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)+i\theta Q(A)}$$

$$= \int [dA] \det(D(A)+me^{i\gamma_5\theta/N_f})^{N_f} e^{-S_G(A)}$$

← U(1)$_A$ rotation

$$\chi_{\mathrm{top.}}(m) = -\frac{1}{N_f V}\frac{\partial^2}{\partial\theta^2}\ln Z(m,\theta)|_{\theta=0} = m\left[\frac{\partial}{\partial\theta}\langle\bar{q}i\gamma_5 e^{i\gamma_5\theta/N_f}q\rangle\right]|_{\theta=0}$$

⇨

$$\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) = -\sum_x \langle P^0(x)P^0(0)\rangle - \frac{\langle\bar{q}q\rangle(m)}{m}.$$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$$*N_f = 2$$

Connected/disconnected pseudoscalar susceptibilities

From a Ward-Takahashi identity $0 = \langle \delta^a_{SU(2)} P^a(0) \rangle - \langle \delta^a_{SU(2)} S P^a(0) \rangle,$

we have

$$m \sum_x \langle P^a(x) P^a(0) \rangle + \langle S^0 \rangle = 0.$$

Therefore,

$$\frac{N_f}{m^2} \chi_{\rm top.}(m) = -\sum_x \langle P^0(x) P^0(0) \rangle - \frac{\langle S(0) \rangle}{m}$$

$$= \sum_x \langle P^a(x) P^a(0) \rangle - \sum_x \langle P^0(x) P^0(0) \rangle$$

Symmetry structure of scalar/pseudoscalar susceptibilities

$\chi(m)$

$SU(2)_L \times SU(2)_R$

$$\sum_x \langle S^0(x) S^0(0) \rangle \longleftrightarrow \sum_x \langle P^a(x) P^a(0) \rangle$$

iso-singlet

$U(1)_A$ $\updownarrow$ $\updownarrow$ $U(1)_A$

iso-triplet

$$\sum_x \langle P^0(x) P^0(0) \rangle \longleftrightarrow \sum_x \langle S^a(x) S^a(0) \rangle$$

$SU(2)_L \times SU(2)_R$

$$-\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) - \frac{-\langle \bar{q}q \rangle(m)}{m}$$

See also LLNL/RBC Collaboration 2014, Nicola & Elvira 2018, Nicola 2020.

Separating U(1)A breaking part

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle|Q(A)|\rangle}{m^2 V}}_{\text{U(1)A breaking contribution}} \underbrace{- \frac{-\langle\bar{q}q\rangle_{\mathrm{sub.}}(m)}{m}}_{\text{mixed}}$$

* quadratic divergence is subtracted using the data at reference quark mass mref=0.005.

$$\chi^{\mathrm{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m)}\underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

where $\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0)\rangle$ axial U(1) susceptibility

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0)\rangle \qquad \Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0)\rangle$$

Lattice formulas

Using

$$\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$$

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

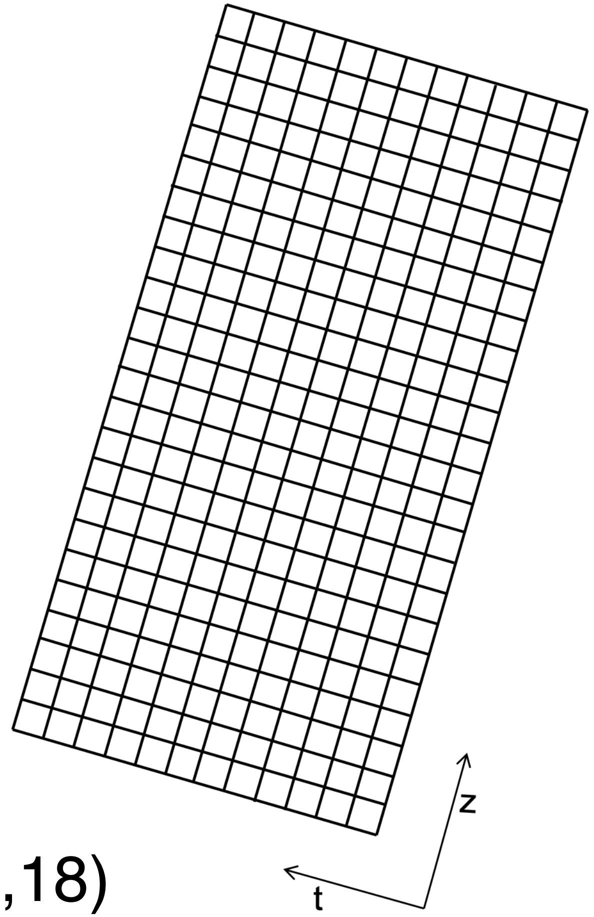
$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle \bar{q}q\rangle^{lat}|^2 V^2\right].$$

Remark.1 eigen functions do not matter.

Remark.2 chiral symmetry is essential for this decomposition.

Simulation setup (Nf=2)



Nf=2 flavor QCD
1/a = 2.6 GeV (0.075fm)
Symanzik gauge action
L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)
Mobius domain-wall fermions with m_{res}<1MeV
(and reweighted overlap fermion)
Quark mass from 3MeV (< phys. pt. ~4MeV) to 30MeV.
T=147, 165 (~Tc), 195, 220, 260, 330 MeV (Lt=8,10,12,14,16,18)
Tc is estimated to be around 175MeV (from Polyakov loop)

Simulation codes : Irolro++ (https://github.com/coppolachan/Irolro)
Grid (https://github.com/paboyle/Grid)
Bridge++([https://bridge.kek.jp/Lattice-code/)](https://bridge.kek.jp/Lattice-code/)

Simulation setup (Nf=2+1)

Nf=2+1 flavor QCD

1/a = 2.453GeV

L=32 (2.58 fm), 40 (3.22 fm), 48(3.9fm)

Mobius domain-wall fermion with m_{res}<1MeV

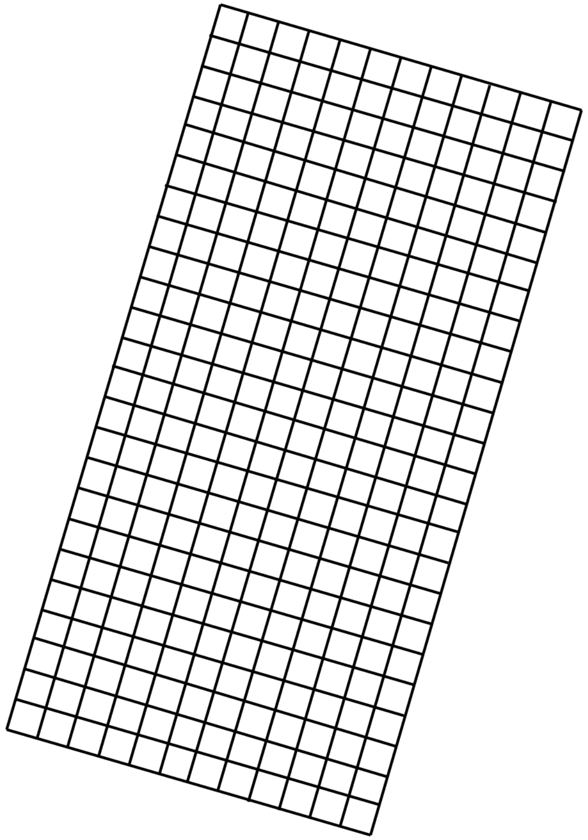
(and reweighted overlap fermion)

up-down quark mass from

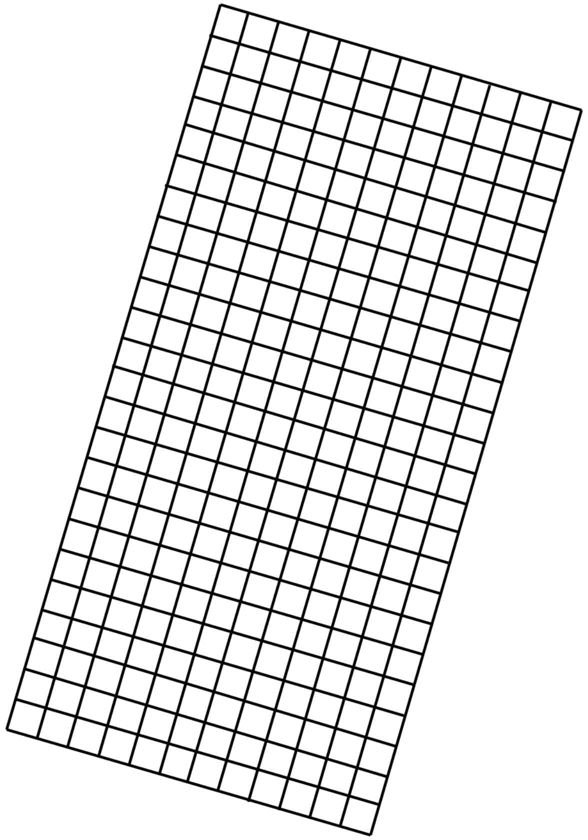
phys. pt. ~4MeV to 30MeV.

strange quark mass at phys.pt.

T=136, 153(~Tc), 175, 220 MeV



Mass reweighting



For the data below the physical points, we perform the mass reweighting.

Nf=2 : m=0.0002(1/5 physical mass), 0.0005, 0.0015 from m=0.001

Nf=2+1: m=0.001(1/2 physical mass) from m=0.002

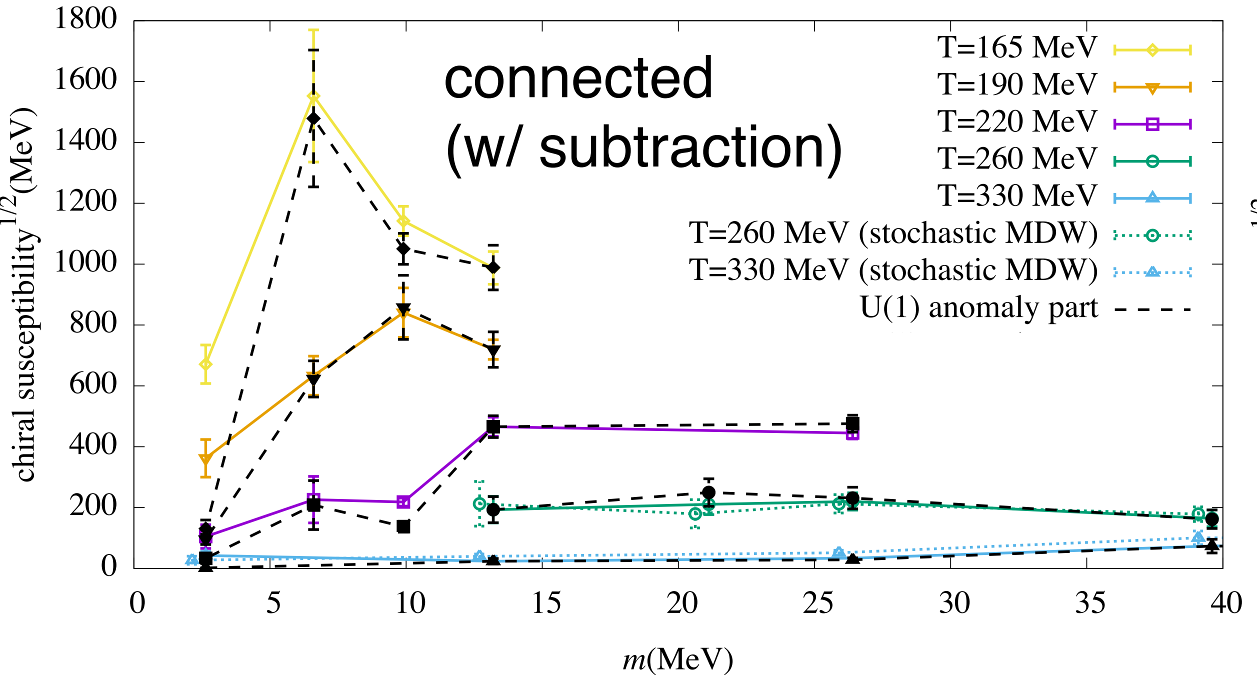
Computational resources

- Fugaku (hp200130, hp210165, hp210231, hp220279)
- Oakforest-PACS [JCAHPC]
  HPCI projects : hp170061, hp180061, hp190090,
  hp200086, hp210104,
  MCRP in CCS, U. Tsukuba : xg17i032 and xg18i023
- Wisteria/BDEC-01 [HPCI: hp220093, MCRP: wo22i038]
- Polarie/Grand Chariot (hp200130)
- Flow at Nagoya U.
- SQUID at Osaka U.
- Program for Promoting Researches on the Supercomputer Fugaku, Simulation for basic science: from fundamental laws of particles to creation of nuclei Joint
- Institute for Computational Fundamental Science (JICFuS)

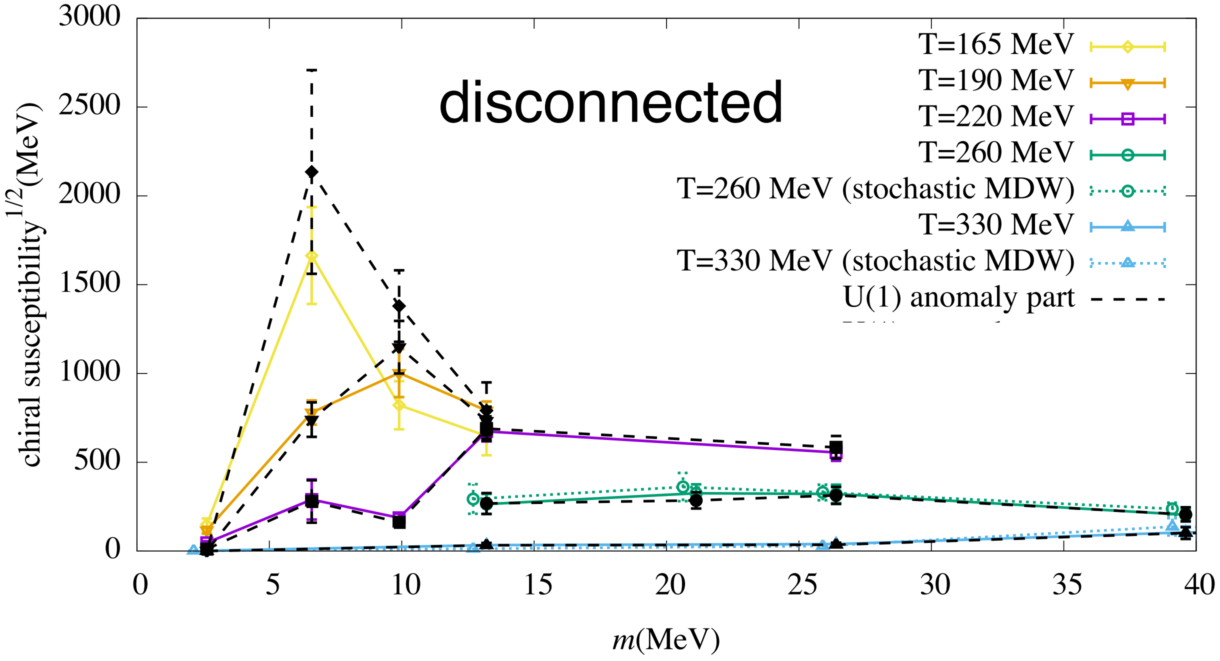
Previous Nf=2 results at higher Ts

S.Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki [JLQCD collaboration] PTEP2022 (2022) 2, 023B05 [arXiv:2103.05954]

(-chiral susceptibility)$^{1/2}$ (connected, subtracted)



(chiral susceptibility)$^{1/2}$ (disconnected)



The dominance by axial U(1) anomaly is seen at 5 different Ts.

$$\chi^{\rm con.}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)}_A \text{ breaking}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\rm sub.}(m)}{m}}_{\text{mixed}}$$

$$\chi^{\rm dis.}(m) = \underbrace{\frac{N_f}{m^2}\chi_{\rm top.}(m)}_{\text{U(1)}_A \text{ breaking}} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

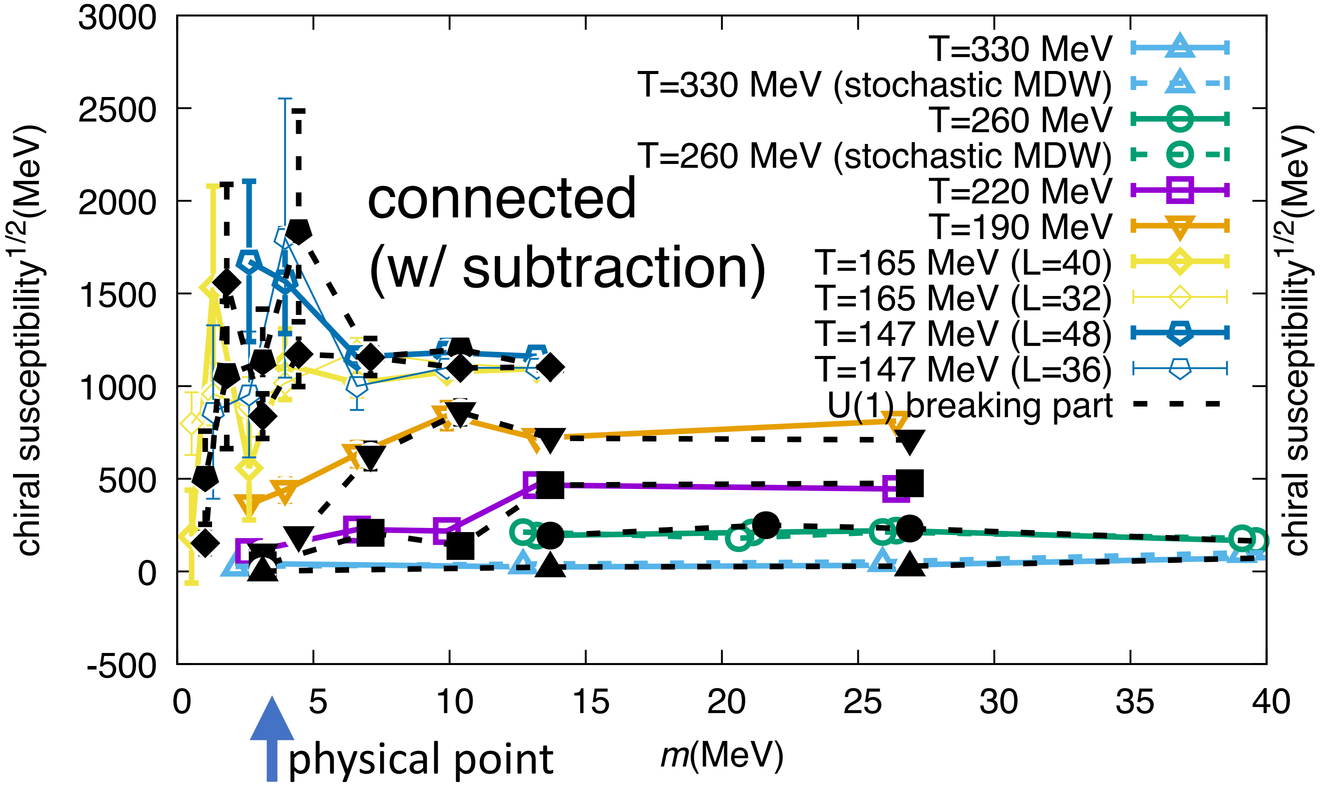
U(1)A breaking

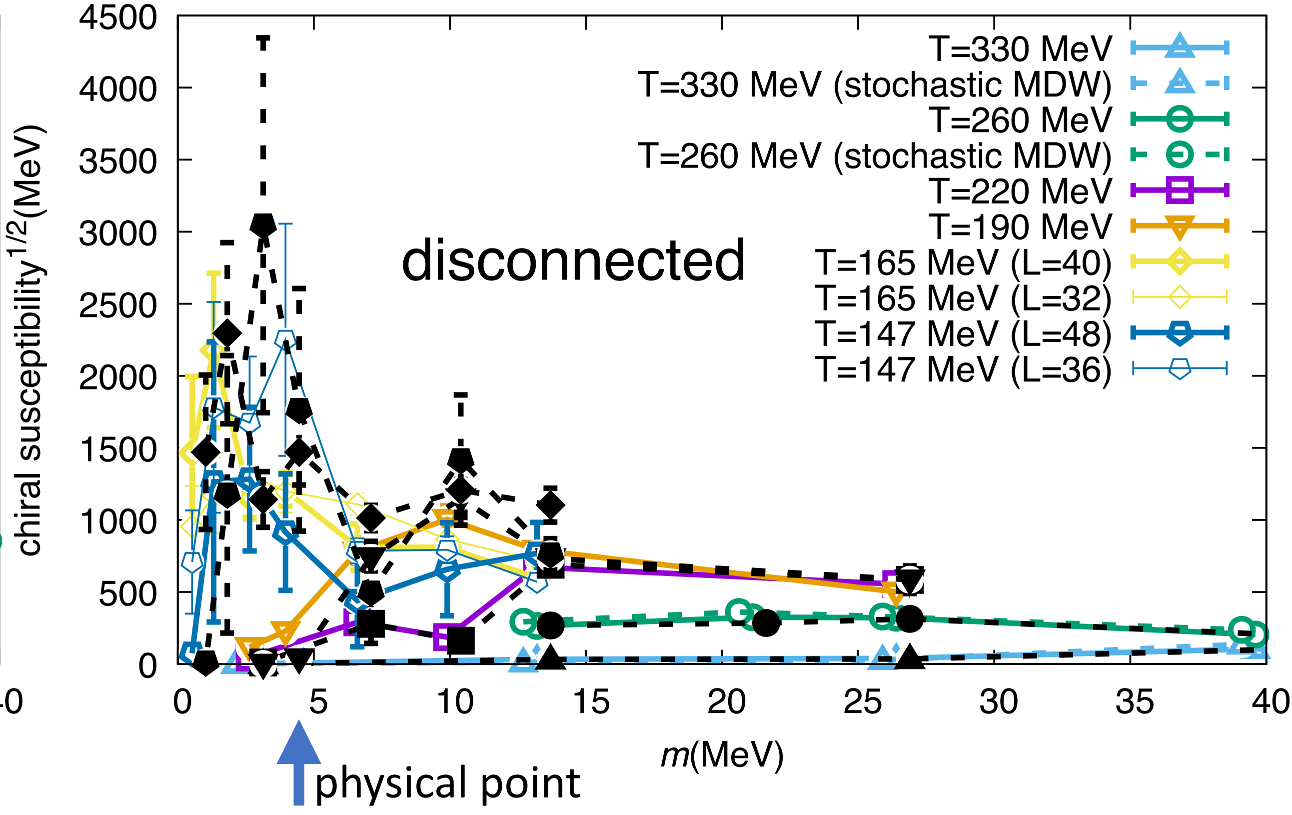
mixed

U(1)A breaking

SU(2)xSU(2)
breaking

Nf=2 QCD updates (w/ lower T and m and larger V)





Down to 0.9Tc and 1/5 physical quark mass the axial U(1) dominance is still seen.

connected part ~ U(1) susceptibility

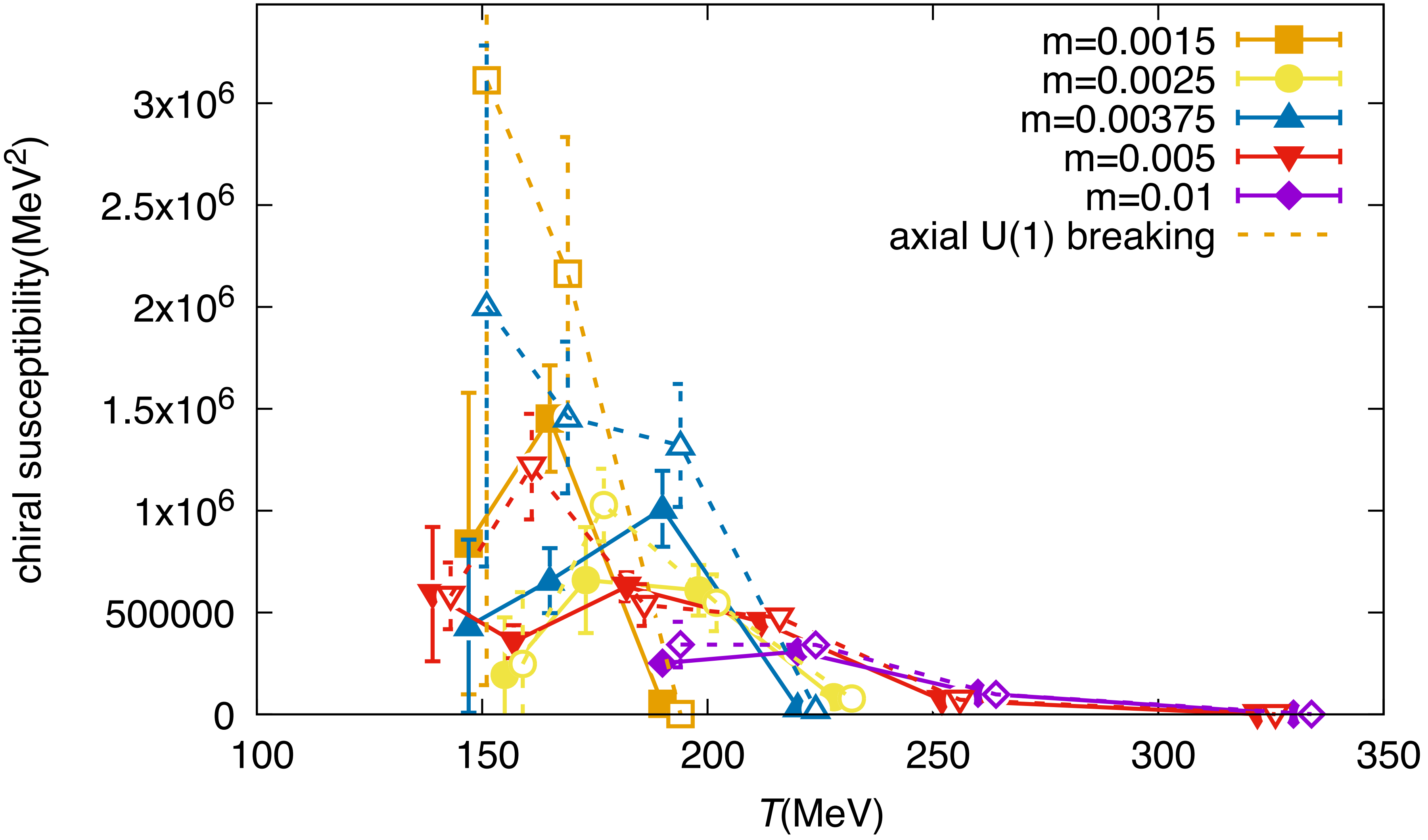
disconnected ~ topological susceptibilityx2/m².

Colored open symbols: data for chiral susceptibility

Black filled symbols: axial U(1) anomaly part

Finite V effects look under control.

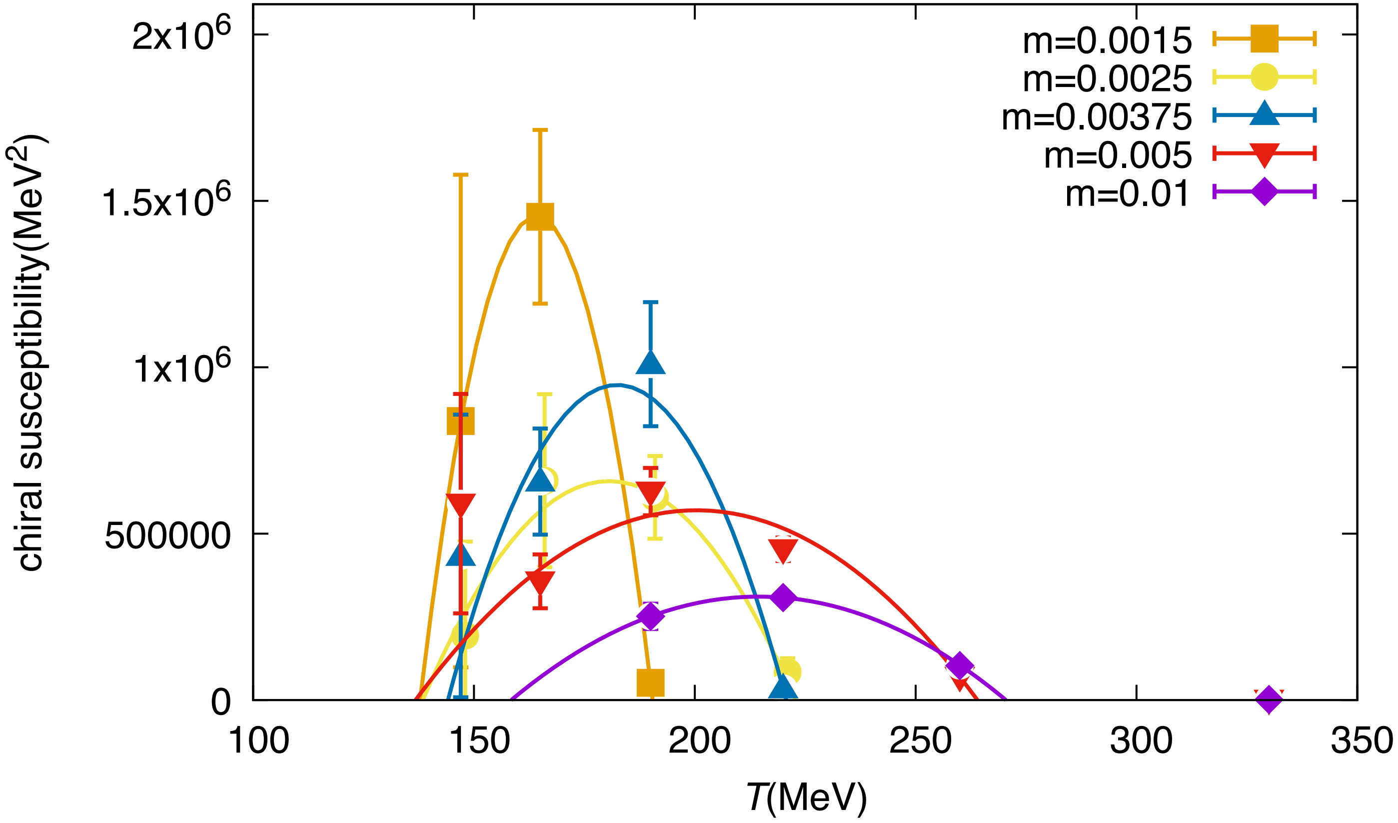
T dependence of disconnected part

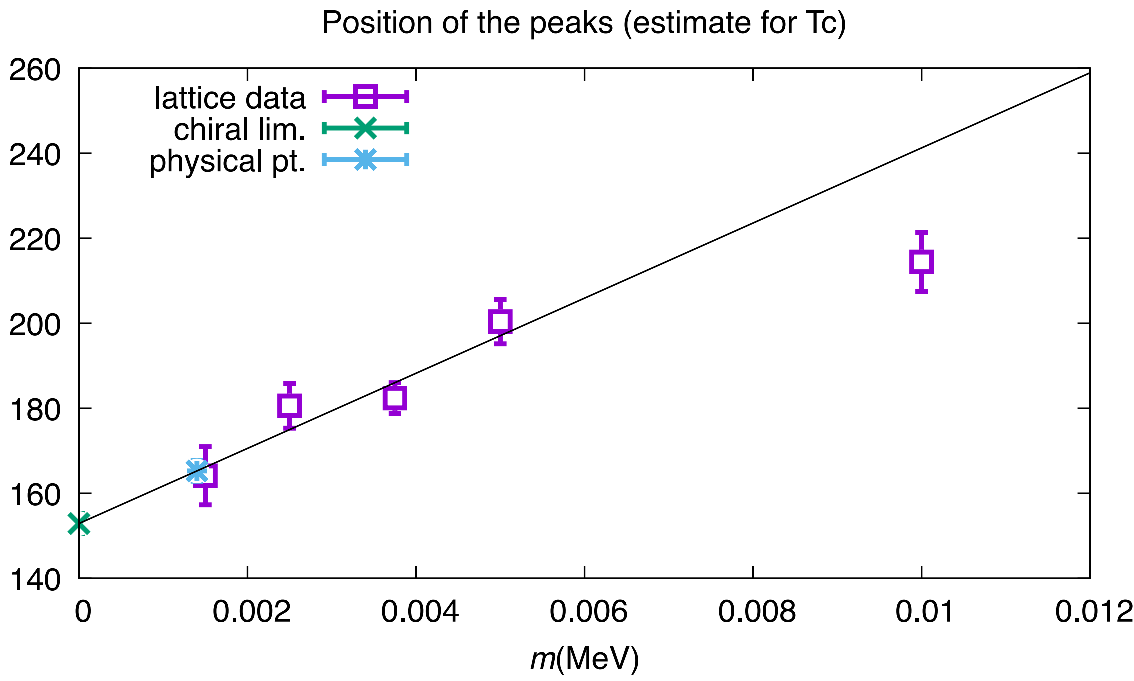


At higer T than the peaks, axial U(1) breaking is dominant.

But it deviates at lower Ts.

Determination of Tc (very preliminary)

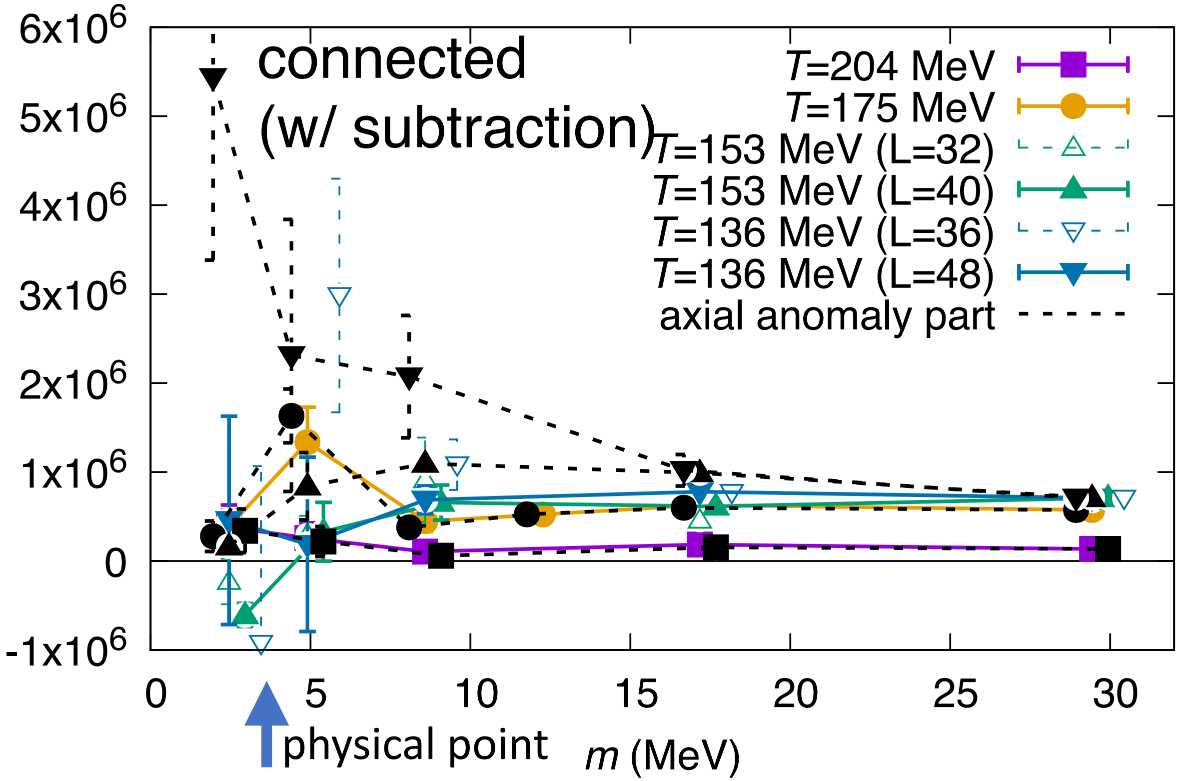


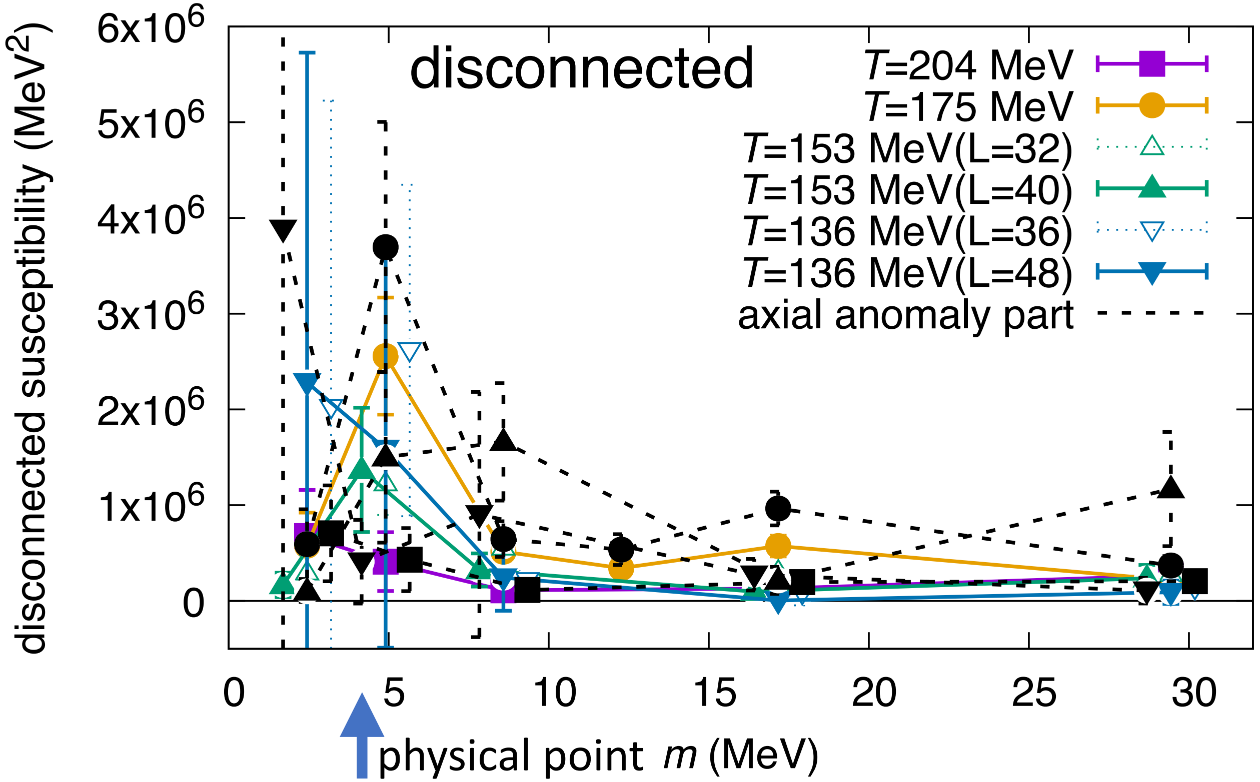


From a quadratic estimate for the position of the peak, we obtain Tc (physical pt.) = 165(3)MeV, Tc(chiral limit)=153(3) MeV.

Nf=2+1results

Colored open symbols: data for chiral susceptibility
Black filled symbols: axial U(1) anomaly part





Axial U(1) dominance is seen.
However, statistically noisy.
Different Vs are consistent.
At the physical point m~4MeV, pseuco-critical T is estimated to be 140-150MeV.

Summary

1. We simulate Nf=2 and 2+1 lattice QCD at high temperatures.
2. Chiral condensate and susceptibility are related to both SU(2)xSU(2) and $U(1)_A$.
3. In the spectral decomposition of the Dirac operator with exact chiral symmetry, we can separate the purely U(1) anomaly effect.
4. Connected/disconnected susceptibilities are dominated by U(1) breaking at T>=Tc.

    Connected part ~ axial U(1) susceptibility.

    Disconnected part ~ top. susceptibility x $2/m^2$

    But for larger mass and lower T, the deviation becomes sizable.

Axial U(1) anomaly may play more important role in the QCD phase diagram than expected.