

# *Phenomenology of Kaluza-Klein gluon in warped extra dimension*

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*Y. Kanehata, N. Maru, K. Yoneyama (in progress)*

- b-quark coupling w/ Z-boson may be one of the good stages to look for the implications of physics beyond the SM
- In this talk, I report an interpretation on a 'puzzle' in the  $Zbb$  coupling in the framework of Warped Extra Dimension model

# Plan

- Introduction: summary of electroweak data
- Puzzles(?) in EW data: b-jet asymmetry
- KK gluon in Warped Extra Dimension model
- Summary

# Introduction

- LEP exp. @CERN (1<sup>st</sup> phase:1989-1995)
  - electron-positron collision on the Z-pole
  - 17 million Z decays
  - precise study of fermion pair production processes in Z decay
  - Test of the Standard Model
  - Window looking for physics beyond the SM

	data	SM	
		prediction	pull
<b>LEP 1</b>			
line-shape & FB asym.:			
$\Gamma_Z$ (GeV)	$2.4952 \pm 0.0023$	2.4957	-0.23
$\sigma_h^0$ (nb)	$41.540 \pm 0.037$	41.477	1.69
$R_l$	$20.767 \pm 0.025$	20.744	0.92
$A_{FB}^{0,l}$	$0.01714 \pm 0.00095$	0.01648	0.69
$\tau$ polarization:			
$A_l(P_\tau)$	$0.1465 \pm 0.0032$	0.14825	-0.54
$b$ and $c$ quark results:			
$R_b$	$0.21629 \pm 0.00066$	0.21586	0.65
$R_c$	$0.1721 \pm 0.0030$	0.172225	-0.04
$A_{FB}^{0,b}$	$0.0992 \pm 0.0016$	0.10393	-2.95
$A_{FB}^{0,c}$	$0.0707 \pm 0.0035$	0.07432	-1.03
jet charge asymmetry:			
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$0.2324 \pm 0.0012$	0.23137	0.86
<b>SLC</b>			
$A_l(SLD) = A_{LR}^0$	$0.1513 \pm 0.0021$	0.14825	1.47
$A_b$	$0.923 \pm 0.020$	0.9350	-0.59
$A_c$	$0.670 \pm 0.027$	0.6684	-0.06
<b>Tevatron + LEP 2</b>			
$m_W$ (GeV)	$80.398 \pm 0.025$	80.380	0.74
$\Gamma_W$ (GeV)	$2.1400 \pm 0.060$	2.0916	0.81
<b>Numerical inputs</b>			
$m_Z$ (GeV)	$91.18756 \pm 0.00021$	91.18737	0.90
$G_F(10^{-5}\text{GeV}^{-2})$	$1.16637 \pm 0.00001$	1.16637	—
<b>Parameters</b>			
$\Delta\alpha_{\text{had}}^{(5)}$	$0.02768 \pm 0.00022$	0.02772	-0.17
$\alpha_s(m_Z)$	$0.118 \pm 0.002$	0.118	-0.25
$m_t$ (GeV)	$172.5 \pm 2.7$	171.1	0.51
$m_H$ (GeV)	—	70.2	—
$\chi_{\text{tot}}^2$ (d.o.f. = 19 - 5)			18.095

**summary of EW precision data**  
*(Phys.Rept.427:257,2006 )*

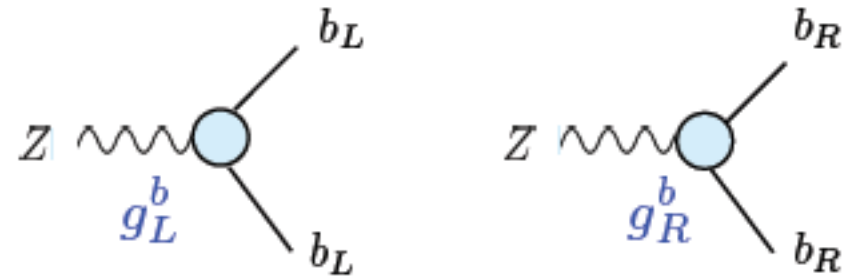
in the SM, there are four parameters which are determined from the data

Puzzles(?) in EW data

# SM is mostly good, but...

LEP electroweak working group, Phys. Rept. 427, 257 (2006)

	data	SM	
<b>LEP1</b>		best fit	pull
line-shape & FB asym.:			
$\Gamma_Z(\text{GeV})$	$2.4952 \pm 0.0023$	2.4965	-0.57
$\sigma_h^0(\text{nb})$	$41.540 \pm 0.037$	41.481	1.59
$R_l$	$20.767 \pm 0.025$	20.739	1.12
$A_{FB}^l$	$0.0171 \pm 0.0010$	0.01642	0.79
b and c quark results:			
$R_b$	$0.21629 \pm 0.00066$	0.21562	1.01
$R_c$	$0.1721 \pm 0.0030$	0.1723	-0.07
$A_{FB}^b$	$0.0992 \pm 0.0016$	0.1037	-2.8
$A_{FB}^c$	$0.07072 \pm 0.0035$	0.07428	-1.02
<b>SLC</b>			
$A_{LR}$	$0.1513 \pm 0.0021$	0.1480	1.51
$A_b$	$0.923 \pm 0.020$	0.9346	-0.58
$A_c$	$0.6702 \pm 0.027$	0.6683	0.07



$$A_{FB}^b(\text{exp.}) = 0.0992 \pm 0.0016$$

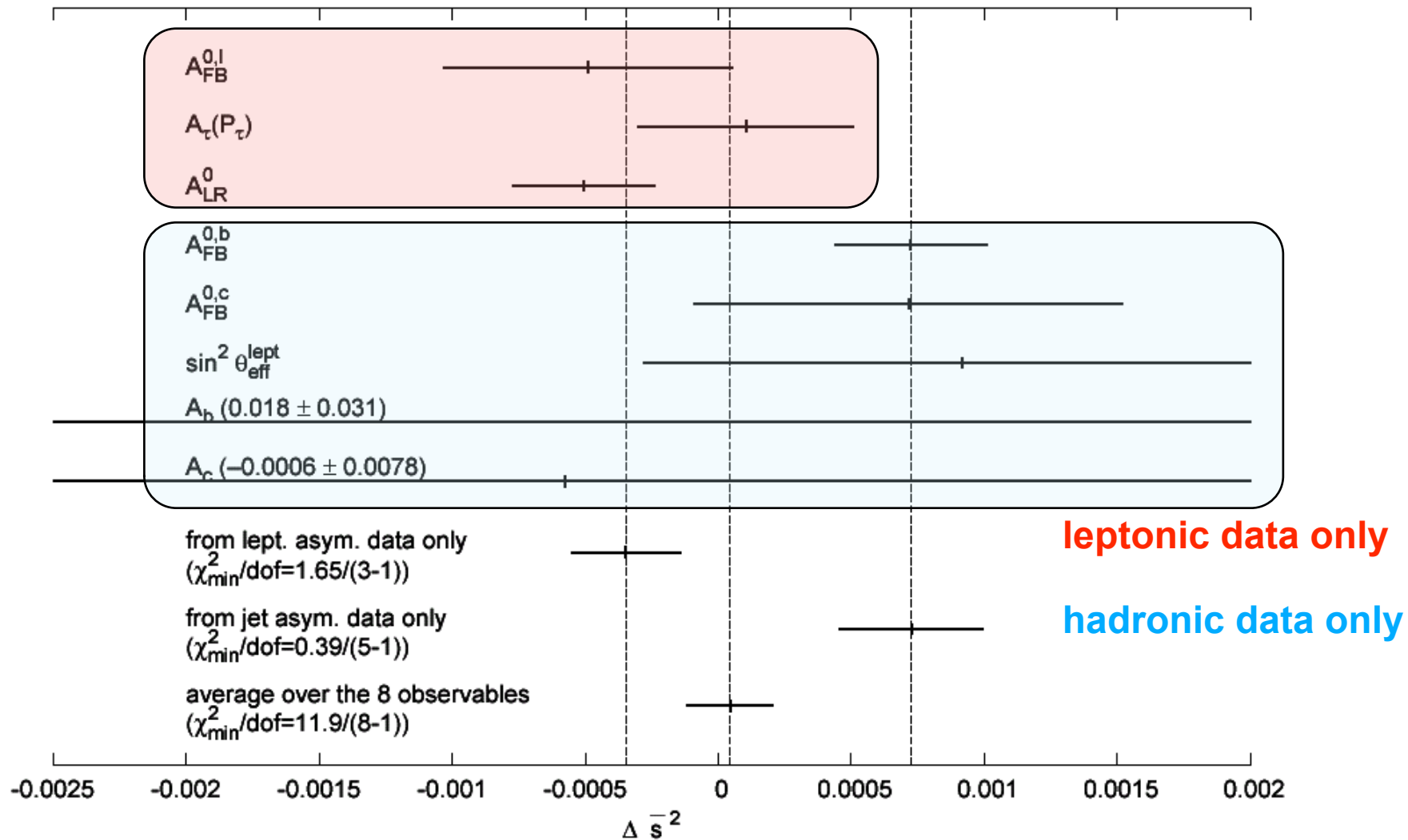
$$A_{FB}^b(\text{SM}) = 0.1037$$

2.8 $\sigma$  deviation

$$A_{FB}^b(\text{new phys.}) = A_{FB}^b(\text{SM}) - 0.0326\Delta g_L^b - 0.1789\Delta g_R^b$$

$$\delta A_{FB}^b = -0.0326\Delta g_L^b - 0.1789\Delta g_R^b = -(45 \pm 16) \times 10^{-4}$$

# effective Weinberg angle vs. jet-asymmetry data





$$\begin{aligned}
\Gamma_Z[\text{GeV}] &= 2.4952 + 4.515\Delta\bar{g}_Z^2 - 2.621\Delta\bar{s}^2 - 1.743\Delta g_L^b + .306\Delta g_R^b + .0017x_s, \\
\sigma_h^0[\text{nb}] &= 41.481 + .047\Delta\bar{g}_Z^2 + 3.878\Delta\bar{s}^2 + 16.41\Delta g_L^b - 2.880\Delta g_R^b - .016x_s, \\
R_l &= 20.738 - .008\Delta\bar{g}_Z^2 - 17.474\Delta\bar{s}^2 - 20.734\Delta g_L^b + 3.638\Delta g_R^b + .020x_s, \\
R_b &= 0.21584 - .00016\Delta\bar{g}_Z^2 + .0382\Delta\bar{s}^2 - .7790\Delta g_L^b + .1367\Delta g_R^b - .000007x_s, \\
R_c &= 0.17222 + .00014\Delta\bar{g}_Z^2 - .0587\Delta\bar{s}^2 + .1711\Delta g_L^b - .0300\Delta g_R^b + .000006x_s,
\end{aligned}$$

$$\begin{aligned}
A_{FB}^{0,l} &= 0.01628 + .000440\Delta\bar{g}_Z^2 - 1.7565\Delta\bar{s}^2, \\
A_{FB}^{0,b} &= 0.10326 + .0014\Delta\bar{g}_Z^2 - 5.612\Delta\bar{s}^2 - .0326\Delta g_L^b - .1789\Delta g_R^b, \\
A_{FB}^{0,c} &= 0.0738 + .0011\Delta\bar{g}_Z^2 - 4.34\Delta\bar{s}^2, \\
A_l &= 0.14731 + .0020\Delta\bar{g}_Z^2 - 7.894\Delta\bar{s}^2, \\
A_b &= 0.93463 + .000027\Delta\bar{g}_Z^2 - .6372\Delta\bar{s}^2 - .2929\Delta g_L^b - 1.608\Delta g_R^b, \\
A_c &= 0.6680 + .0005\Delta\bar{g}_Z^2 - 3.468\Delta\bar{s}^2,
\end{aligned}$$

$$\begin{aligned}
\sin^2\theta_{\text{eff}}^{\text{lept}} &= 0.23148 - .00025\Delta\bar{g}_Z^2 + 1.0033\Delta\bar{s}^2, \\
m_W &= 80.365 - .137x_H - .019x_H^2 + .018x_t - .005x_\alpha - .002x_s.
\end{aligned}$$

$\Delta\bar{g}_Z^2$  and  $\Delta\bar{s}^2$  (also  $\Delta g_L^b$  and  $\Delta g_R^b$ ) include the radiative corrections not only in the SM but also new physics

# an interpretation of $A_{FB}^b$ data

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FB asymmetry can be expressed using LR asymmetry

$$A_{FB}^b = \frac{3}{4} A_e A_b$$

But b-quark LR asym. (@SLD) looks consistent with SM

Thus, reason may be

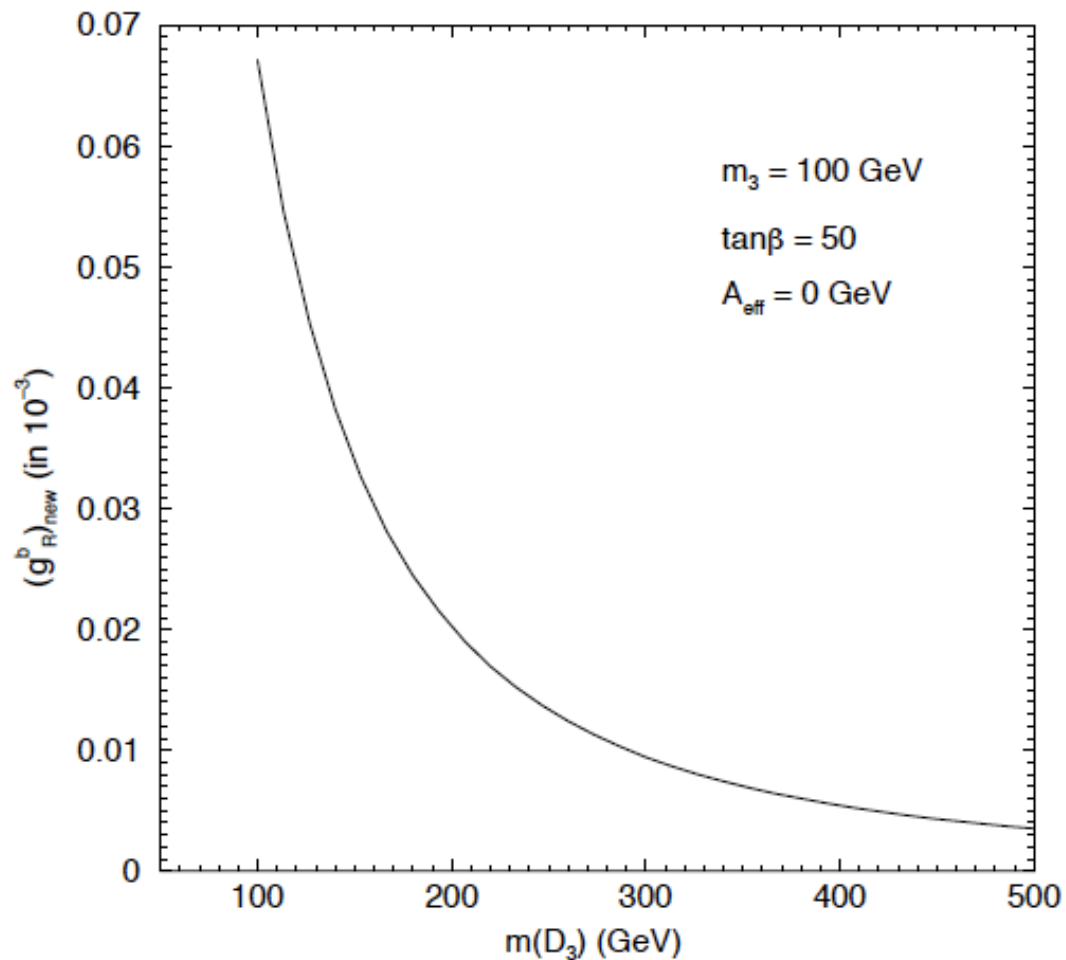
- (1)  $A_e (=A_{LR})$ ?
- (2) b-tagging @LEP?
- (3) statistics?

$$A_{FB}^b(\text{new phys.}) = A_{FB}^b(\text{SM}) - 0.0326\Delta g_L^b - 0.1789\Delta g_R^b$$

$$\delta A_{FB}^b = -0.0326\Delta g_L^b - 0.1789\Delta g_R^b = -(45 \pm 16) \times 10^{-4}$$

or **NEW PHYSICS** in b-quark coupling

# Example: MSSM (gluino contribution)



*GCC, Hagiwara (2000)*

relative sign of  $g_b^R$  is good,  
but size is too small to explain  
the puzzle

other new physics approach:

**extra gauge boson**

(He et al, 2002)

**extra vector-like quark**

(Choudhury et al, 2002, etc)

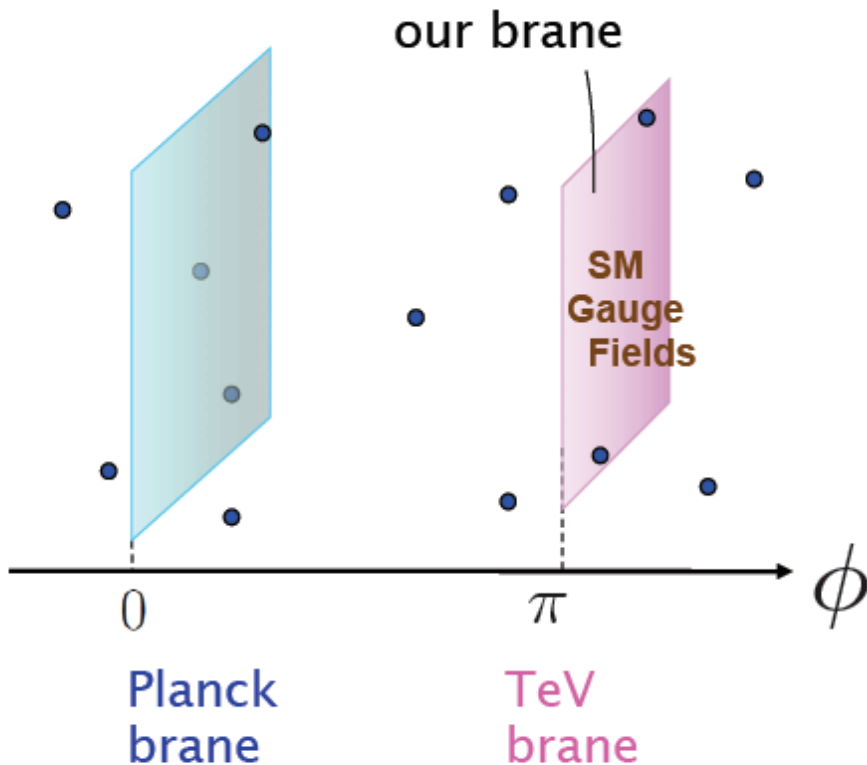
Figure 4: The gluino contribution to  $(g_R^b)_{\text{new}}$ .

KK gluon  
in Warped Extra Dimension model

# Warped extra dimension (RS model)

- one of the solutions to the hierarchy problem

Randall and Sundrum, PRL83,3370('90)



• : gravitons

Set up:

Two 3-branes with  $\Lambda (< 0)$  in the bulk

& 5D metric:

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

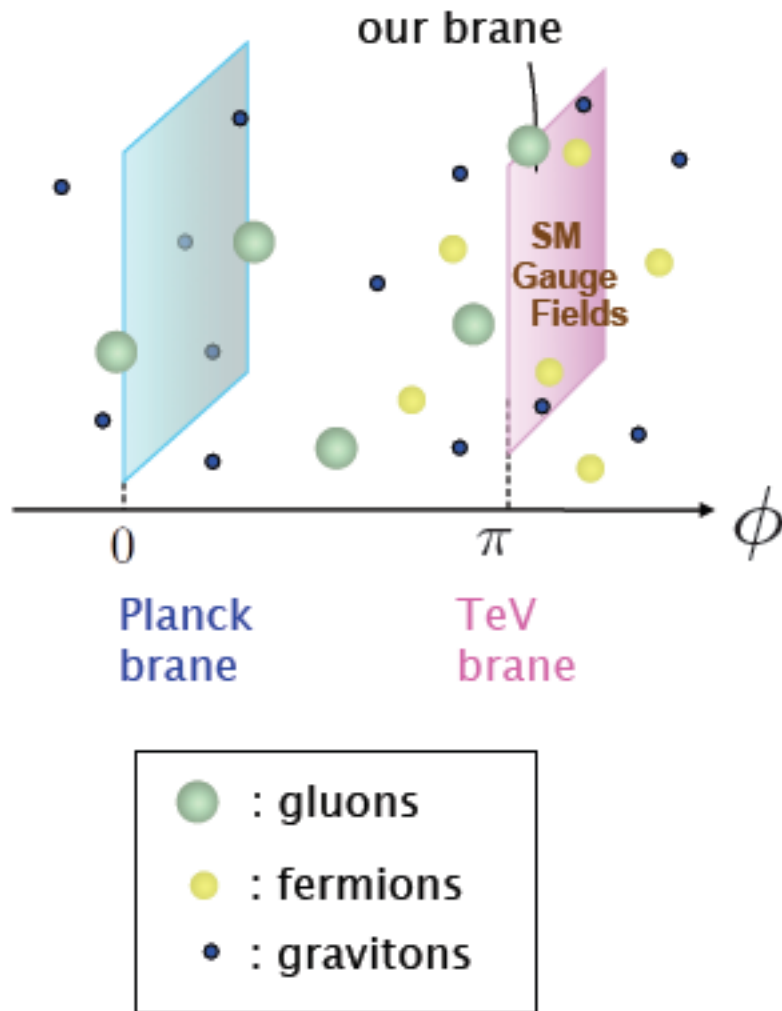
electroweak scale:

$$v \sim O(100\text{GeV}) = e^{-kr_c\pi} \times v_0$$

$$= e^{-kr_c\pi} \times 10^{18}\text{GeV}$$

the hierarchy between Planck and  
EW scales is stable if  $kr_c = 11 - 12$

# Extension of RS model



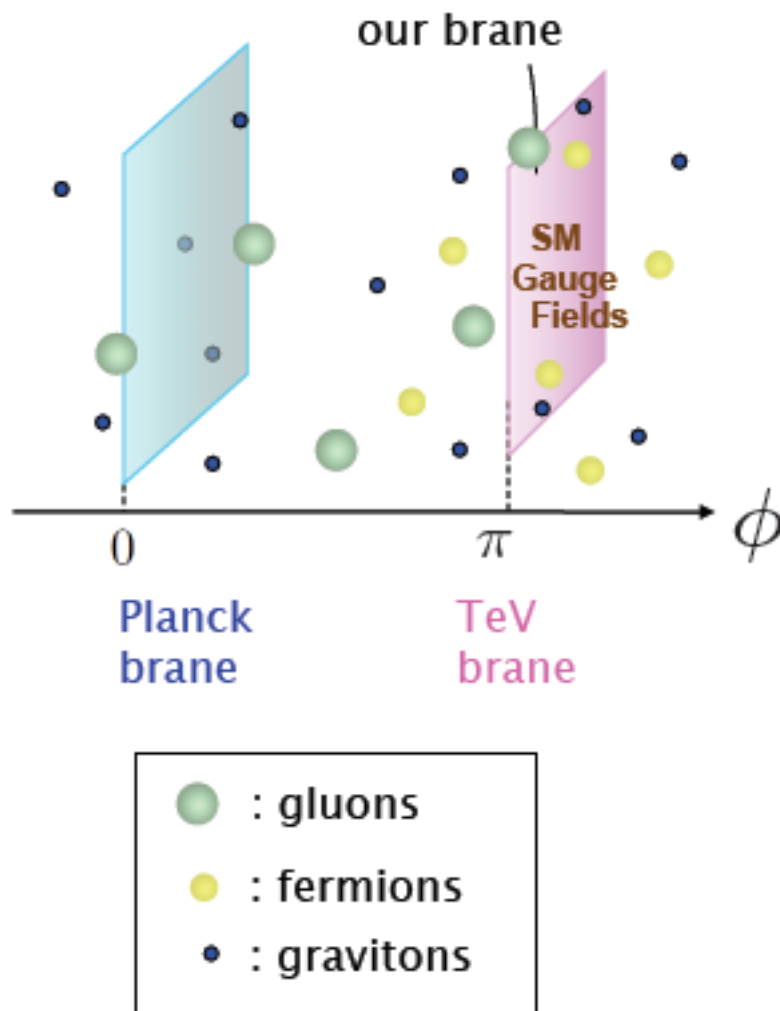
allow that fermions and gauge boson propagate into the bulk

KK(Kaluza-Klein) decomposition

$$A_\mu(x, \phi) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{\chi_A^{(n)}(\phi)}{\sqrt{r_c}}$$

$$\Psi_{L,R}(x, \phi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)}(x) \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \hat{f}_{L,R}^{(n)}(\phi)$$

# Extension of RS model



- KK EW gauge bosons gives too large oblique corrections (corrections to gauge boson propagators) so that the scale of KK mass is strongly constrained as  $\sim O(10\text{TeV})$

- further arrangement on the model is required

Davoudiasl et al, PLB473, 43('00)

Pomarol, PLB486,153('00)

Csaki et al, PRD66, 064021('02)

# possibility of Kaluza-Klein gluon

- no interaction with EW gauge boson → constraints from the oblique corrections are expected to be negligible
- couples only to quarks (not to leptons)
- (KK) gluon may resolve  $A_{fb}^b$  puzzle w/o conflicting the agreement of the lepton sector between exp. and SM
- comments: difficulty from model building (suppression of KK of W, Z?)



# possibility of Kaluza-Klein gluon

interaction of KK gluon and fermions

$$g_{mnq}^{ffA} \sim \int_{-\pi}^{\pi} d\phi e^{\sigma} \hat{f}_L^{(m)*} \hat{f}_L^{(n)} \chi_A^{(q)}$$

$$A_\mu(x, \phi) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{\chi_A^{(n)}(\phi)}{\sqrt{r_c}}$$

$$\Psi_{L,R}(x, \phi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)}(x) \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \hat{f}_{L,R}^{(n)}(\phi)$$

overlapping of wave functions in 5th D

it depends on the bulk mass term of fermion

$$S_f = \int d^4x \int d\phi r_c \sqrt{G} \left[ V_n^M \left( \frac{i}{2} \bar{\Psi} \gamma^n \partial_M \Psi + h.c. \right) - \text{sgn}(\phi) m_\Psi \bar{\Psi} \Psi \right]$$

$$m_\Psi = \nu_\Psi k \epsilon(y)$$

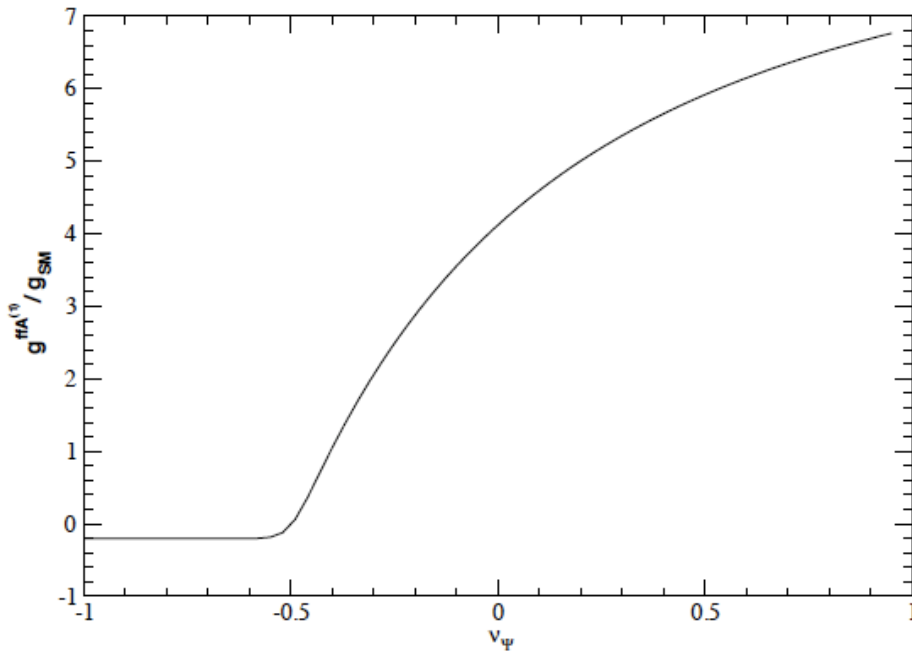
Grossman and Neubert, PLB474,361('00)

the bulk fermion mass terms have been used to explain the hierarchy of Yukawa couplings

Agashe et al, (2003)

# flavor dependent coupling

$$\xi = \frac{g_{001}^{ffA}}{g_{\text{SM}}} \sim \int_{-\pi}^{\pi} d\phi e^{\sigma} \hat{f}_L^{(m)*} \hat{f}_L^{(n)} \chi_A^{(q)}$$
$$\approx \begin{cases} -0.2 & (\nu_{\Psi} < -0.5) \\ 4.0 + 5.2\nu_{\Psi} - 4.6\nu_{\Psi}^2 & (\nu_{\Psi} > -0.5) \end{cases}$$



flavor dependent coupling of quarks with KK gluon

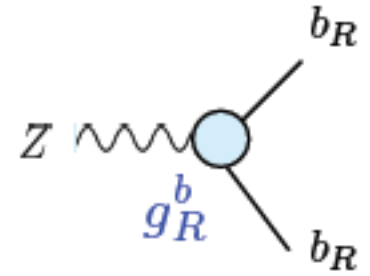
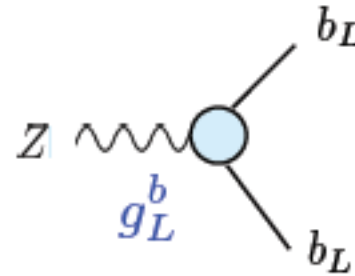
the coupling of KK gluon could be a few times larger than the QCD coupling

# recall $A_{FB}^b$ puzzle

$$A_{FB}^b(\text{exp.}) = 0.0992 \pm 0.0016$$

$$A_{FB}^b(\text{SM}) = 0.1037$$

$$\delta A_{FB}^b = -45 \pm 16 \quad (\times 10^{-4})$$



evaluate KK gluon  
contribution to  $Zbb$  couplings

$$g_\alpha^b = g_\alpha^b(\text{SM}) + \Delta g_\alpha^b \quad (\alpha = L, R)$$

$$\begin{aligned} A_{FB}^b &= A_{FB}^b(\text{SM}) + \delta A_{FB}^b \\ &= A_{FB}^b(\text{SM}) - 0.0326 \Delta g_L^b - 0.1789 \Delta g_R^b \end{aligned}$$

# KK gluon contribution in 1-loop

The diagram illustrates the 1-loop contribution of KK gluons to the  $Z b_{L,R} b_{L,R}$  vertex. The left side shows the tree-level vertex. The right side shows two loop diagrams: one with a KK gluon  $g^{(n)}$  and one with a KK gluon  $g^{(n)}$  and a KK quark  $b_{L,R}^{(n)}$ .

$$\Delta g_{L,R}^b(m_{g^{(n)}}, \xi) = \frac{1}{\sqrt{4\sqrt{2}G_F m_z^2}} \left( g_{L,R}^{bbZ} \Sigma'(0) - \Gamma(m_z^2) \right)$$

(Note)

UV divergence of loop integral  $\rightarrow$  cancel between two diagrams

However, infinity comes from sum of infinite KK towers.

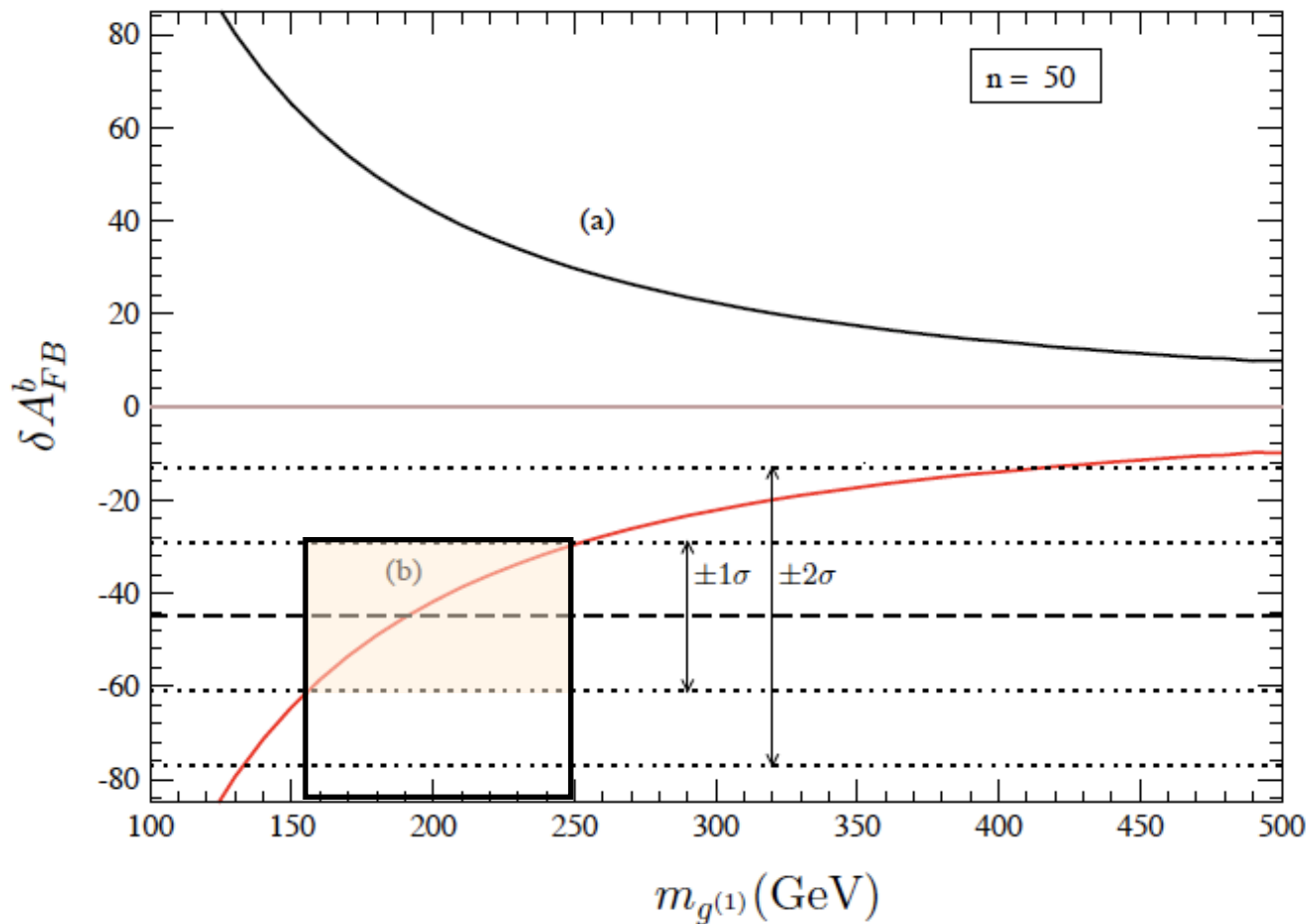
The following result is obtained for  $n=50$

$$A_{FB}^b = A_{FB}^b(\text{SM}) + \delta A_{FB}^b$$

$$= A_{FB}^b(\text{SM}) - 0.0326 \Delta g_L^b - 0.1789 \Delta g_R^b$$

(a) :  $(\xi_L, \xi_R) = (6, 0.2)$

(b) :  $(\xi_L, \xi_R) = (0.2, 6)$



$$g_\alpha^b \sim I_{3b} - Q_b \sin^2 \theta_W$$

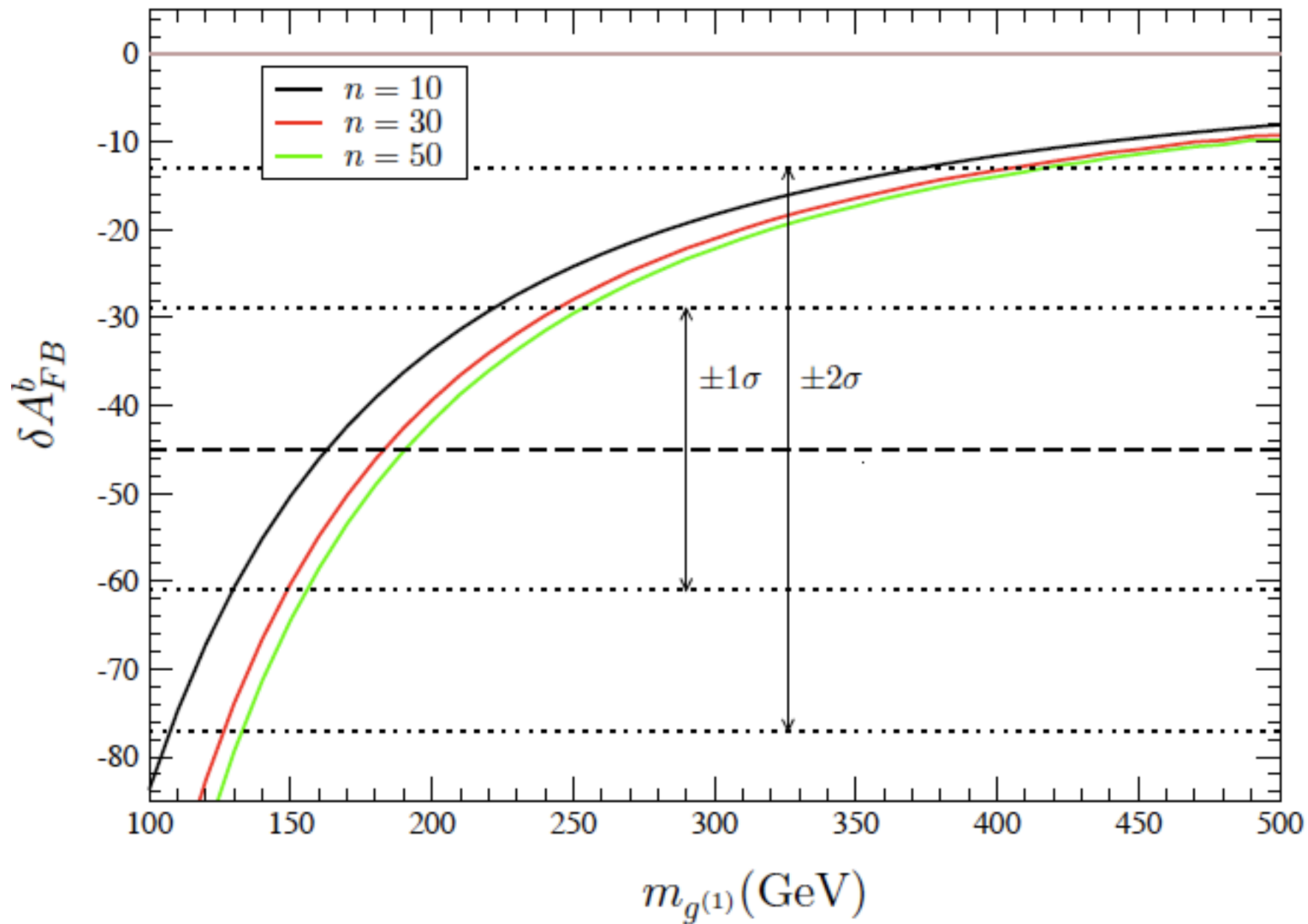
$$g_L^b < 0, \quad g_R^b > 0$$

favored scenario:  
 right-handed b-quark couples  
 strongly to KK gluon

KK gluon w/  $m \sim 200 \text{ GeV}$   
 explains  $A_{FB}^b$  puzzle in  $1\text{-}\sigma$   
 level

mass of KK mode (for  $n=50$ )  
 $\sim 13 \text{ TeV}$

# KK mode dependence



# Correlation w/ other obs.

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# Correlation w/ other obs.

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$A_b$	$0.923 \pm 0.020$	0.935	-0.6

$$A_{FB}^b(\text{NP}) = [A_{FB}^b]_{\text{SM}} - 0.0326\Delta g_L^b - 0.1789\Delta g_R^b$$

$$R_b(\text{NP}) = [R_b]_{\text{SM}} - 0.78\Delta g_L^b + 0.14\Delta g_R^b$$

$$A_b(\text{NP}) = [A_b]_{\text{SM}} - 0.30\Delta g_L^b - 1.63\Delta g_R^b$$



# Correlation w/ other obs.

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 A_b(\text{NP}) &= [A_b]_{\text{SM}} - 0.30\Delta g_L^b - 1.63\Delta g_R^b
 \end{aligned}$$

	$A_{FB}^b$	$R_b$	$A_b$	$\chi^2$
pull(SM)	-2.8	1.0	-0.6	9.7
	↓	↓	↓	↓
pull	-1.0	-2.4	0.7	7.5
$m_{g(1)}$	$\simeq 250\text{GeV}$ for $\xi_R = 6$			

# phenomenology at hadron colliders

- KK gluon couples to (only) left-handed (t,b) quarks
- production process:  $b\bar{b} \rightarrow g^{(1)}$
- no gluon fusion channels

more detail, see Yoneyama's talk  
(next)

compare

$$\sigma_0 \equiv \sigma(p\bar{p} \rightarrow g^{(1)} + X) \times \text{Br}(g^{(1)} \rightarrow b\bar{b}) \quad (\xi_L, \xi_R) = (0.2, 6)$$

and Tevatron results

(CDF) arXiv: 0812.4036, 0808.3347

# Tevatron constraints

lower bound in  $2\sigma$

$$m_{g(1)} > 157\text{GeV}$$

more detail, see Yoneyama's talk  
(next)

→ consistent with  $4\text{fb}$  in  $1-\sigma$

*3 jets event@LEP2 may increase the lower bound*

# Summary

- a possibility to explain b-jet asymmetry data within the framework of Warped Extra Dim.
- Enhancement of right handed b-quark coupling is favored from the data
- 3rd gen. quarks could strongly couple to KK gluon
- KK gluon w/  $m \sim 200$  GeV is favored
- safe from Tevatron bound since production in p-pbar collision is suppressed (Yoneyama's talk)
- signal at LHC?

Back up

# cut-off

- Naive Dimensional Analysis (NDA)

Manohar and Georgi (1984)

- cut-off  $\Lambda \rightarrow$  limit of energy scale where 4D gauge couplings are not perturbative

- in case of RS model

$$\Lambda \sim 24\pi^2/g_5^2$$

$$g_5 = g_4\sqrt{\pi r_c}$$

$$g_4^2 \approx 16\pi^2$$

$$\Lambda \approx 1/r_c \sim k/10$$

interval of mass

$$m_n \simeq n\pi k e^{-\pi k r_c}$$

number of KK mode is  
another model parameter  
(we take  $n=50$ )

number of KK mode

$$\# = \Lambda/m_n \sim 10^{15}$$

## 2. Framework

- EW observables in Z decay experiments can be expressed in terms of the effective couplings

$$g_{\alpha}^f = a_{\alpha}^f + b_{\alpha}^f \Delta \bar{g}_Z^2 + c_{\alpha}^f \Delta \bar{s}^2 + \Delta g_{\alpha}^f$$

f: flavor,  $\alpha$ : chirality

$\bar{g}_Z^2, \bar{s}^2$  : obtained from the gauge boson propagator corrections

$\Delta g_{\alpha}^f$  : vertex corrections

---

$\Delta$ : shift from the SM reference values

$$(m_t, m_H, \Delta \alpha_{\text{had}}^{(5)}(m_Z^2), \hat{\alpha}_s(m_Z)) = (172, 100, 0.0277, 0.118)$$

- EW observables in Z decay experiments can be expressed in terms of the effective couplings

$$g_{\alpha}^f = a_{\alpha}^f + b_{\alpha}^f \Delta \bar{g}_Z^2 + c_{\alpha}^f \Delta \bar{s}^2 + \Delta g_{\alpha}^f$$

e.g.,

$$g_L^{\nu} = 0.50199 + 0.45250 \Delta \bar{g}_Z^2 + 0.00469 \Delta \bar{s}^2$$

$$(g_L^f \sim I_3 - Q_f \sin^2 \theta_W)$$



$$\begin{aligned}
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\sigma_h^0[\text{nb}] &= 41.481 + .047\Delta\bar{g}_Z^2 + 3.878\Delta\bar{s}^2 + 16.41\Delta g_L^b - 2.880\Delta g_R^b - .016x_s, \\
R_l &= 20.738 - .008\Delta\bar{g}_Z^2 - 17.474\Delta\bar{s}^2 - 20.734\Delta g_L^b + 3.638\Delta g_R^b + .020x_s, \\
R_b &= 0.21584 - .00016\Delta\bar{g}_Z^2 + .0382\Delta\bar{s}^2 - .7790\Delta g_L^b + .1367\Delta g_R^b - .000007x_s, \\
R_c &= 0.17222 + .00014\Delta\bar{g}_Z^2 - .0587\Delta\bar{s}^2 + .1711\Delta g_L^b - .0300\Delta g_R^b + .000006x_s, \\
A_{FB}^{0,l} &= 0.01628 + .000440\Delta\bar{g}_Z^2 - 1.7565\Delta\bar{s}^2, \\
A_{FB}^{0,b} &= 0.10326 + .0014\Delta\bar{g}_Z^2 - 5.612\Delta\bar{s}^2 - .0326\Delta g_L^b - .1789\Delta g_R^b, \\
A_{FB}^{0,c} &= 0.0738 + .0011\Delta\bar{g}_Z^2 - 4.34\Delta\bar{s}^2, \\
A_l &= 0.14731 + .0020\Delta\bar{g}_Z^2 - 7.894\Delta\bar{s}^2, \\
A_b &= 0.93463 + .000027\Delta\bar{g}_Z^2 - .6372\Delta\bar{s}^2 - .2929\Delta g_L^b - 1.608\Delta g_R^b, \\
A_c &= 0.6680 + .0005\Delta\bar{g}_Z^2 - 3.468\Delta\bar{s}^2, \\
\sin^2\theta_{\text{eff}}^{\text{lept}} &= 0.23148 - .00025\Delta\bar{g}_Z^2 + 1.0033\Delta\bar{s}^2, \\
m_W &= 80.365 - .137x_H - .019x_H^2 + .018x_t - .005x_\alpha - .002x_s.
\end{aligned}$$

$\Delta\bar{g}_Z^2$  and  $\Delta\bar{s}^2$  (also  $\Delta g_L^b$  and  $\Delta g_R^b$ ) include the radiative corrections not only in the SM but also new physics

# summary of radiative corrections

## gauge boson propagator corrections

$$\text{Diagram: } \gamma \text{ wavy line} \rightarrow \text{circle} \rightarrow \gamma \text{ wavy line} \sim \bar{e}^2(q^2) = \hat{e}^2 [1 - \text{Re}\bar{\Pi}_{T,\gamma}^{\gamma\gamma}(q^2)]$$

$$\text{Diagram: } \gamma \text{ wavy line} \rightarrow \text{circle} \rightarrow Z \text{ wavy line} \sim \bar{s}^2(q^2) = \hat{s}^2 [1 + \frac{\hat{c}}{\hat{s}} \text{Re}\bar{\Pi}_{T,\gamma}^{\gamma Z}(q^2)]$$

$$\text{Diagram: } Z \text{ wavy line} \rightarrow \text{circle} \rightarrow Z \text{ wavy line} \sim \bar{g}_Z^2(q^2) = \hat{g}_Z^2 [1 - \text{Re}\bar{\Pi}_{T,Z}^{ZZ}(q^2)]$$

$$\text{Diagram: } W \text{ wavy line} \rightarrow \text{circle} \rightarrow W \text{ wavy line} \sim \bar{g}_W^2(q^2) = \hat{g}^2 [1 - \text{Re}\bar{\Pi}_{T,W}^{WW}(q^2)]$$

Hagiwara, Haidt, Kim, Matsumoto (1994)

independent of external fermions  
(process independent corrections)

## Zff vertex corrections

$$\text{Diagram: } Z \text{ wavy line} \rightarrow \text{circle} \rightarrow f_\alpha \text{ fermion} \sim \Delta g_\alpha^f$$

## vertex/box corrections in muon decay

$$\text{Diagram 1: } \mu \text{ line} \rightarrow \text{circle} \rightarrow \nu_\mu \text{ line} \quad \text{Diagram 2: } \mu \text{ line} \rightarrow \text{circle} \rightarrow \nu_\mu \text{ line} \quad \text{Diagram 3: } \nu_e \text{ line} \rightarrow \text{circle} \rightarrow e \text{ line}$$

$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \sim \Delta \bar{\delta}_G$$

process dependent corrections

# gauge boson propagator corrections

$\bar{g}_Z^2, \bar{s}^2 \leftrightarrow S, T, U$  parameters (Peskin-Takeuchi, 1990)

$$\begin{aligned}\Delta\bar{g}_Z^2 &= 0.00412\Delta T_Z \\ \Delta\bar{s}^2 &= 0.00360\Delta S_Z - 0.00241\Delta T_Z + 0.00011x_\alpha\end{aligned}$$

$$T_Z = T + 1.49R - \frac{\Delta\bar{\delta}_G}{\alpha}$$

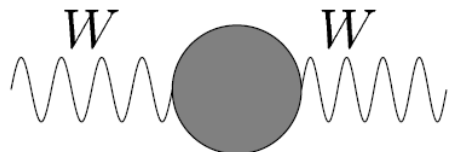
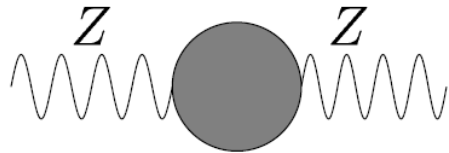
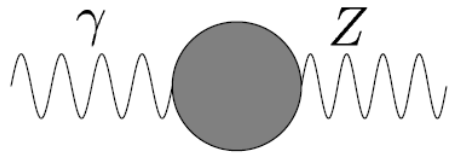
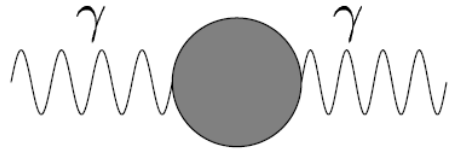
$$S_Z = S + R - 0.064x_\alpha$$

$$\frac{4\pi}{\bar{g}_Z^2(m_Z^2)} - \frac{4\pi}{\bar{g}_Z^2(0)} = -\frac{1}{4}R$$

**G.C.C, Hagiwara ('98)**

# *gauge boson propagator corrections*

*(overview of Peskin-Takeuchi's formalism (1990))*



- there are four form factors (function of momentum transfer  $q^2$ )
- at  $q^2 = 0$  and  $m_Z^2$ , eight form factors in total
- at  $q^2=0$ , two form factors including photon leg vanish due to the gauge invariance
- three of six form factors could be replaced by  $(\alpha, m_Z, G_F)$
- then, gauge boson propagator corrections can be evaluated by three parameters (S,T,U)

# *gauge boson propagator corrections*

*(overview of Peskin-Takeuchi's formalism (1990))*

assuming the running of the pi-functions between  $q^2 = 0$  and  $q^2 = m_Z^2$  can be described by the SM, and the scale of new physics is much larger than  $m_Z$ , the three independent parameters can be chosen as follows

$$\begin{aligned} S &= 16\pi \text{Re} \left[ \Pi_{T,\gamma}^{3Q}(m_Z^2) - \Pi_{T,Z}^{33}(0) \right] \\ T &= \frac{4\sqrt{2}G_F}{\alpha} \left[ \Pi_T^{33}(0) - \Pi_T^{11}(0) \right] \\ U &= \left[ \Pi_{T,Z}^{33}(0) - \Pi_{T,W}^{11}(0) \right] \end{aligned}$$

# *gauge boson propagator corrections*

$\bar{g}_Z^2, \bar{s}^2 \leftrightarrow S, T, U$  parameters (Peskin-Takeuchi, 1990)

$$\begin{aligned}\Delta\bar{g}_Z^2 &= 0.00412\Delta T_Z \\ \Delta\bar{s}^2 &= 0.00360\Delta S_Z - 0.00241\Delta T_Z + 0.00011x_\alpha\end{aligned}$$

$$\begin{aligned}T_Z &= T + 1.49R - \frac{\Delta\bar{\delta}_G}{\alpha} \\ S_Z &= S + R - 0.064x_\alpha\end{aligned}$$

$$\frac{4\pi}{\bar{g}_Z^2(m_Z^2)} - \frac{4\pi}{\bar{g}_Z^2(0)} = -\frac{1}{4}R$$

- the parameter “R” accounts for the new physics contributions to the running of Z-boson self energy between 0 and  $m_Z$

- SUSY contributions to R is important

$$m_W = 80.402 - 0.288\Delta S + 0.418\Delta T + 0.337\Delta U + 0.012x_\alpha - 0.126\frac{\Delta\bar{\delta}_G}{\alpha}$$

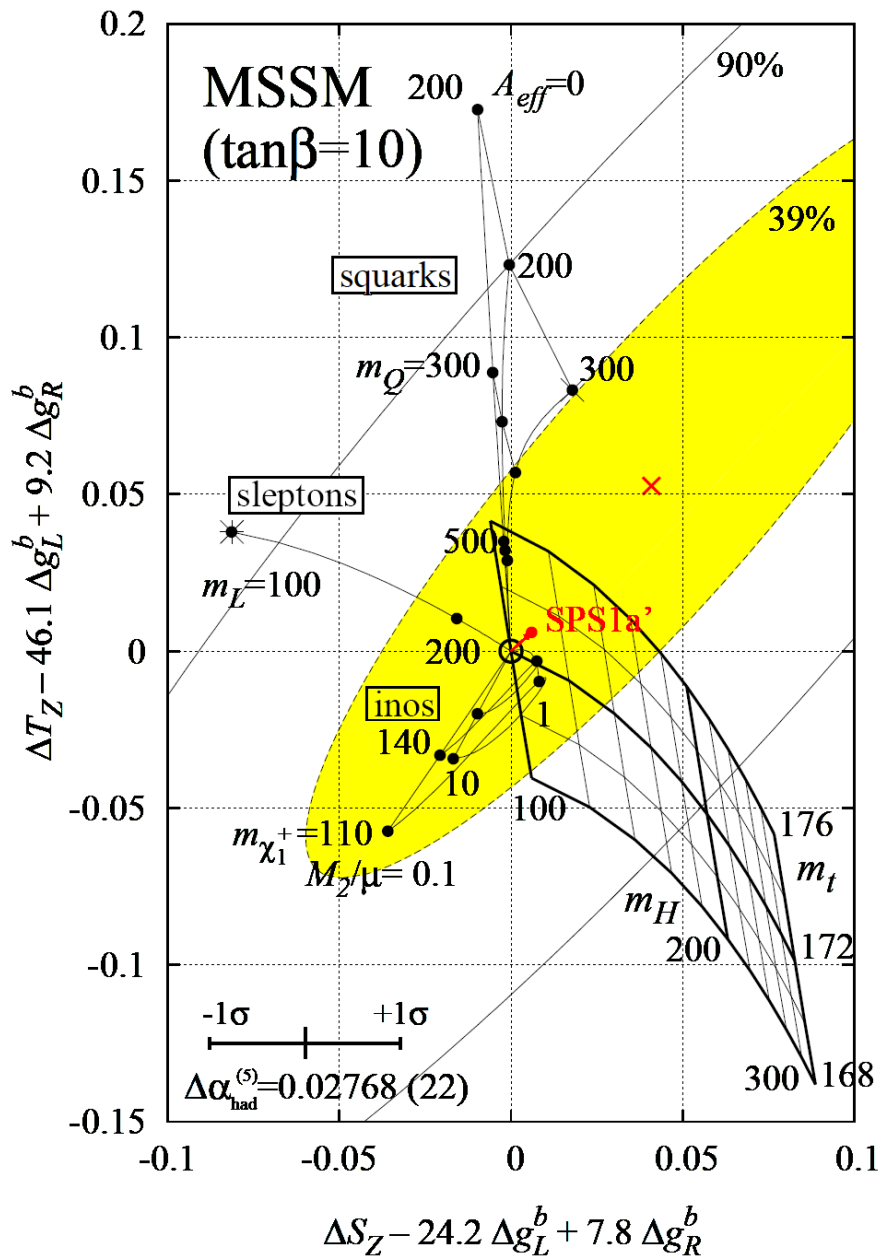
**G.C.C, Hagiwara ('98)**

- we can perform model independent analysis using (Sz, Tz and Zbb vertex corrections) and mW.

$$S_Z, T_Z, g_L^b, g_R^b$$

$$\left. \begin{aligned} \Delta S_Z &= 0.041 + 24.2\Delta g_L^b - 7.8\Delta g_R^b \pm 0.101 \\ \Delta T_Z &= 0.053 + 46.1\Delta g_L^b - 9.2\Delta g_R^b \pm 0.125 \end{aligned} \right\}, \quad \rho_{\text{corr}} = 0.90$$

$$\chi_{\text{min}}^2 = 15.7 + (\text{squared term of } \Delta g_L^b, g_R^b)$$



yellow: 1-sigma allowed region of  $(S_z, T_z, g_L^b, g_R^b)$

- (1) SM prediction ( $m_t, m_H$ )
- (2) light squarks gives significantly large  $T_z$
- (3) sleptons ( $\sim 100\text{GeV}$ ) gives large  $S_z$
- (4) chargino-neutralino

\* SPS1a'  $\rightarrow$  a SUSY parameter set (benchmark) used in the linear collider community

GCC, Hagiwara, Matsumoto, Nomura (2008)



# effective Weinberg angle vs. jet-asymmetry data

