A few topics in Gauge Higgs Unification - flavor and the relation to Little Higgs

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I. Flavor Physics in GHU

(w./ **Y. Adachi, N. Kurahashi and N. Maru**, JHEP1011:150('10))

"Gauge-Higgs unification"
(N.S. Manton, Hosotani, Hatanaka-Inami-L., …)

$$A_M = (A_\mu, A_y) \quad (5D) \qquad A_y^{(0)}(x) = H(x) : \ \text{Higgs}$$

To achieve flavor violation is non-trivial in GHU, since Yukawa couplings are originated from gauge coupling: universal.

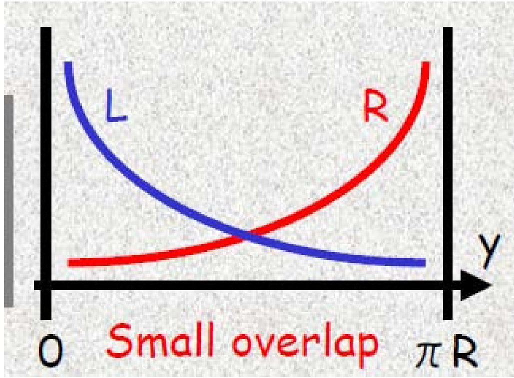
A new feature of higher dimensional model with Z_2-orbifolding, Z_2-odd bulk masses

$$\epsilon(y) M_i \bar{\psi}_i \psi_i \quad (\epsilon(y) : \text{sign function})$$

are allowed , with M_i being different depending on each flavor.

→ new source of the violation of flavor symmetry, specific to higher dimensional model.

The bulk masses lead to the localization of Weyl fermions and, therefore, exponentially suppressed Yukawa couplings,



$$\sim g e^{-\pi M_i R} \ (R: \text{the radius in the case of } S^1)$$

(N.B.)

In this scenario, the observed hierarchical masses of light fermions are naturally realized by the exponential factor without fine tuning .

Flavor Changing Neutral Current (FCNC) Processes

Flavor violation $\rightarrow$ FCNC (rare) processes,

$$K^0 \leftrightarrow \bar{K}^0, \quad K_L \rightarrow \mu\bar{\mu}, \quad B^0 \leftrightarrow \bar{B}^0, \ldots$$

(Natural flavor conservation in GHU)

It has been playing a crucial role in order to check the viability of New Physics (SUSY, etc.).

We ask if "natural flavor conservation" is met in GHU, i.e. if FCNC processes at tree level are forbidden (to make them "rare").

FCNC processes in pure zero-mode sector at the tree level are strictly forbidden , as the sector recovers ordinary SM. In other words, the mode functions for zero-mode gauge bosons are “flat” in extra-dimension

$\rightarrow$ their gauge interactions are universal for all flavors.

$$g \int \text{const.} \times |f_{iL}(y)|^2 dy$$

$$\propto \int |f_{iL}(y)|^2 dy = 1 : \quad \text{universal for all } i$$

$\longrightarrow$ no FCNC (@ tree) even after the flavor mixing

But, the new sauce of flavor violation , characteristic to higher dimensional theory, i.e. non-degenerate bulk masses, is known to lead to FCNC due to the exchanges of non-zero KK modes of gauge bosons already at the tree level.

(GIM-like mechanism)

Interestingly, however, in our analysis "GIM-like mechanism" is found to operate:
For large bulk mass M_i , fermions almost perfectly localize at the fixed point, and for them the mode function of gauge boson look like constant $\rightarrow$ gauge coupling becomes almost universal !

In fact, we found exponential suppression factor of FCNC in the effective lagrangian (of L-R type) in $K^0 \leftrightarrow \bar{K}^0$ mixing :

$$-\frac{\pi^2}{2}\left(e^{-2\pi M^1 R} + e^{-2\pi M^2 R}\right) - \frac{\pi}{2R}\frac{(M^1)^2 - M^1 M^2 + (M^2)^2}{M^1 M^2 (M^1 - M^2)}$$
$$\times \left(e^{-2\pi M^1 R} - e^{-2\pi M^2 R}\right)$$

(disappears when $M_1 = M_2$)

Note that $\left(\frac{m_{q_i}}{M_W}\right)^2 \sim \exp(-2\pi M_i R)$,

and the suppression is similar to that of GIM mechanism by $\frac{m_c^2 - m_u^2}{M_W^2}$.

II. Relation to the Little Higgs Model
(w./ Adachi, Hasegawa, Kurahashi, Tanabe)

Possible solutions to the hierarchy problem (at quantum level) in the SM ,

$$\delta m_H^2 \propto \Lambda^2 \quad ,$$

invoking symmetries,

a. supersymmetry
b. gauge symmetry, i.e. Higgs as a gauge boson
   → Gauge-Higgs Unification (GHU)
c. global symmetry , i.e. Higgs as a Nambu-Goldstone (NG) boson
   → Litlle Higgs

Interestingly, there seems to be close relation between GHU and Little Higgs scenarios.

(The circumstantial evidence)

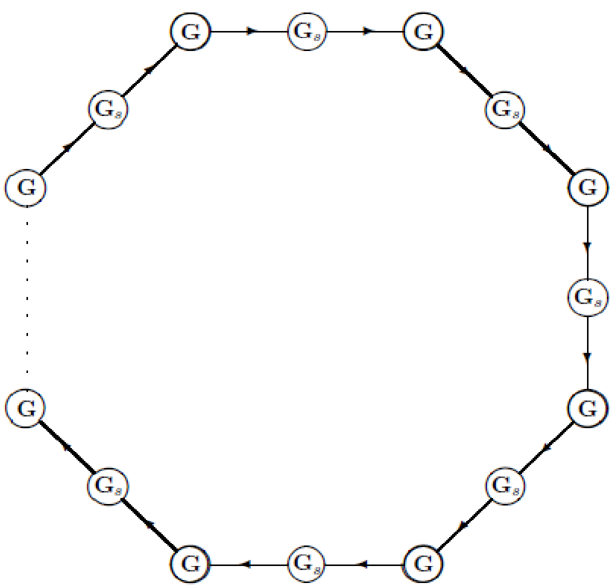
(i) In both scenarios, gauge group of SM is enlarged to group G, and Higgs is identified with NG boson of G/H in Little Higgs , and A_y of G/H in GHU.
(ii) The coupling of A_y to fermions in 5D GHU takes a form
$g\bar{\psi}(i\gamma_5)\psi \cdot A_y$, which mimics the pseudo scalar coupling of mesons.
(iii) Both have sift symmetries, $A_y \to A_y + \partial_y \lambda, \quad G \to G + \text{const.}$

(N.B.)
How global and local symmetries can be identified ?
Little Higgs: repetition of global symmetry $\lambda_i \quad (i = 1, 2, \ldots N)$
GHU: higher dimensional local gauge symmetry $\lambda(y)$

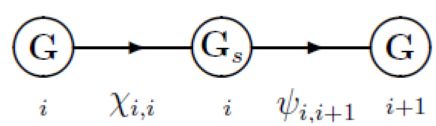
The idea of "dimensional deconstruction " and its close relation to GHU (N. Arkani-Hamed, A.G. Cohen and H. Georgi)

- Higgs : pseudo-NG boson, a bound state of fermions, just as pions in QCD
- Repetition of gauge symmetries : $(G \times G_s)^N \quad (G = SU(m),\ G_s = SU(n))$ $(\Lambda \ll \Lambda_s)$

(moose diagram)



The strong force causes a condensate and form a bound state, a la pion:



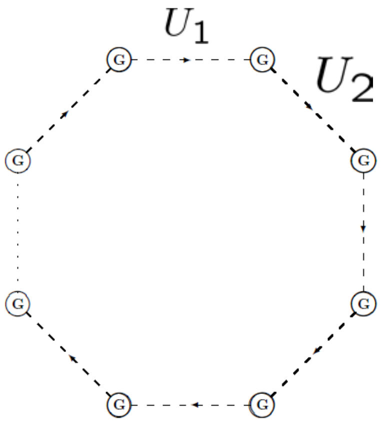
$\rightarrow$

$$U_i = e^{i\frac{\pi_i^a T_a}{f}}$$

$(m, \bar{m}) \quad (SU_i(m), SU_{i+1}(m))$

: non-linear sigma model

Below Λ_s , effective theory is a gauged non-linear sigma model:



$$S = \int d^4x \, (-\frac{1}{2g^2} \sum_{j=1}^{N} tr F_j^2 + f^2 \sum_{j=1}^{N} tr[(D_\mu U_j)^\dagger (D^\mu U_j)]),$$

$$D_\mu U_j = \partial_\mu U_j - i A_\mu^j U_j + i U_j A_\mu^{j+1}$$

This is nothing but 5D pure gauge theory, with extra-space being latticized: U_j is the link variable ("Wilson line").

The theory has a gauge symmetry

$$SU(m)^N$$

, under which

$$U_i \quad \to \quad g_i U_i g_{i+1}^\dagger \quad (g_i : \text{elements of } SU(m)).$$

The symmetry is spontaneously broken to $SU(m)$:

$$\langle U_i \rangle = 1 \quad \to \quad g_i \langle U_i \rangle g_{i+1}^\dagger = \langle U_i \rangle, \ \text{when } g_i = g_{i+1}$$

Thus among $U_i \quad (i = 1, 2, \ldots N)$, N-1 pieces are "eaten" by Higgs mechanism, and there remains one physical (pseudo-) NG boson.

The remaining pseudo- NG boson, identified with Higgs, is given by

$$Tr(U_1 U_2 \ldots U_N),$$

which is invariant under the gauge transformation ,

$$U_1 U_2 \ldots U_N \quad \to \quad g_1 (U_1 U_2 \ldots U_N) g_1^\dagger \quad (g_{N+1} = g_1).$$

and therefore cannot be "gauged away".

In Abelian case, $U_1 U_2 \ldots U_N = e^{\frac{i}{f}(\pi_1 + \ldots \pi_N)}$ $\left(\phi = \frac{\pi_1 + \pi_2 + \ldots \pi_N}{\sqrt{N}}\right)$

ϕ corresponds to the zero-mode of A_y, or Wilson-loop, in GHU:

$$W = exp\{ig \int A_y dy\}.$$

For $N \geq 3$ the potential for ϕ is finite:

$$V(\phi) = -\frac{9}{4\pi^2} g^4 f^4 \sum_1^\infty \frac{\cos(\frac{2n\sqrt{N}\phi}{f})}{n(n^2N^2 - 1)(n^2N^2 - 4)} + \text{constant}$$

and reduces to that in GHU (Hosotani, Kubo-L.- Yamashita, …) for $N \to \infty$

$$V(A_y) = \frac{9}{4\pi^2} \frac{1}{(2\pi R)^4} \sum_{n=1}^\infty \frac{\cos(ngA_y 2\pi R)}{n^5}$$

“Little Higgs ”

Purpose:

To construct a 4D theory including SM, where Higgs is a pseudo-NG boson, while the quadratic divergence of the mass cancels without use of SUSY. ← inspired by Dimensional Deconstruction

In this approach, the NG boson does not need to be a bound state .

（”The simplest Little Higgs” (M. Schmaltz, JHEP 0408(2004)056) ）

Consider $SU(3) \times U(1)_X$ model with triplet scalar ϕ. The VEV $\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ causes a breaking $SU(3) \rightarrow SU(2)$, and NG bosons are written as

$$\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & h \\ 0 & -\eta/2 & \\ \hline \multicolumn{2}{c|}{h^\dagger} & \eta \end{array}\right)$$

(h: Higgs doublet)

"Corrective breaking"

Introduce two copies of triplet scalars $\phi_1, \ \phi_2$ and add SU(3) invariant covariant derivatives for both

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 \quad (D_\mu = \partial_\mu - igA_\mu)$$

$$\phi_1 = e^{i\frac{\pi_1}{f}} \begin{pmatrix} 0 \\ f \end{pmatrix}, \quad \phi_2 = e^{i\frac{\pi_2}{f}} \begin{pmatrix} 0 \\ f \end{pmatrix}.$$

- Each sector of two scalars has global SU(3) symmetry.

- But, once gauge interaction is switched on, the global symmetry is explicitly broken to "diagonal" SU(3): when each of ϕ_1 and ϕ_2 transforms as

$$\phi_1 \ \rightarrow \ U_1 \phi_1, \quad \phi_2 \ \rightarrow \ U_2 \phi_2$$

gauge fields should transform as

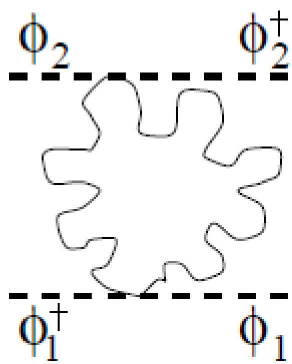
$$A_\mu \ \rightarrow \ U_1 A_\mu U_1^\dagger, \quad A_\mu \ \rightarrow \ U_2 A_\mu U_2^\dagger.$$

Thus we get $U_1 = U_2.$

The NG boson corresponding to this diagonal SU(3) remains exactly massless , but is “eaten”. The orthogonal one , h , is identified as Higgs and acquires a mass, since the relevant symmetry is broken explicitly by gauge interaction.

(N.B.) To break the global symmetry, gauge couplings to both of U_1, U_2 are necessary → “corrective breaking”

b.)



→ $$\frac{g^4}{(4\pi)^2}\log\frac{\Lambda^2}{\mu^2}|\phi_1^\dagger\phi_2|^2$$

, giving rise to the Higgs mass

No quadratic divergence !

III. Summary

- Dimensional deconstruction may be regarded as latticized scenario of Gauge-Higgs unification (GHU).
- Little Higgs scenario, which stems from the idea of dimensional deconstruction, therefore has a close relation with GHU.
- In order for the collective breaking to operate, the repetition of fields was essential, which seems to correspond to (finite number of) KK modes in GHU.
- Quadratic divergence of Higgs mass disappears as the result of the repetition of fields in Little Higgs model , just as in the case of GHU, where the sum over all KK modes provides a finite Higgs mass (H. Hatanaka, T. Inami and C.S.L., Mod.Phys.Lett.A13('98)2601).

Remaining issues

Still, Little Higgs seems to have different aspects from GHU (or dimensional deconstruction):

- Why not SU(3) → SU(2) x U(1) ? Why is the extra $U(1)_X$ necessary ?
- In GHU, higher dimensional gauge symmetry by $\lambda(y)$ is not broken, but in Little Higgs global symmetry by λ_i is explicitly broken.
- (Maybe,) the mechanism of corrective breaking is not the same as what we expect in GHU:

  Little Higgs with N scalars ϕ_i : local operator $|\phi_i^\dagger \phi_j|^2$ is allowed .

  GHU: only non-local operator $|Tr(U_1 U_2 \ldots U_N)|^2$ is allowed.
- This may be due the fact that gauge boson and triplet fermion are not doubled . If gauge bosons are doubled, global symmetry is not broken, though NG bosons are completely eaten.
- The fact that Higgs mass gets quadratic divergence at 2-loop level, may be related to that the doubling is incomplete.
- It will be interesting to reconsider Little Higgs scenario from the viewpoint of GHU.
- AdS/CFT correspondence ?

(Dream)

・All of 3 scenarios may be related
Quantum Mechanical SUSY in GHU (T. Nagasawa, M. Sakamoto, H. Sonoda and C.S.L., Phys. Rev. D72('05) 064006)

・It will be great if some unified theory of three scenarios is realized !
("Unified Theory of Theories").