Gauge-Higgs unification in Randall Sundrum Spacetime at Finite Temperature

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January 24, 2011
Electroweak phase transition and thermal effects

- Our universe: baryon asymmetric (no anti-proton in our daily life)
- Baryogenesis - Sakhalov’s three conditions
  1. $B$ violation process
  2. $C$ and $CP$ symmetry is broken
  3. out of thermal equilibrium
- Electroweak baryogenesis - the 3rd condition requires the first-order phase transition and the expanding bubbles (inside: broken phase)
Introduction

Phase transition @high-T for SM and other Models

1. Standard Model - 2nd order
2. SUSY - 1st order for some models [Funakubo et.al.]
   - Attempts to get EWPT [Ahriche, et.al. (2009-)]
4. In GHU (Hosotani mechanism, flat ExD) - 1st order [Ho-Hosotani (1990), Panico, et.al.(2005)]
5. UED
Introduction

Gauge-Higgs unification (GHU)

- Hierarchy Problem $\Leftrightarrow$ quadratic divergent $\delta m_h^2$

$$m_h^2 = m_{\text{bare}}^2 + (\cdots \sim g \Lambda^2), \quad \Lambda: \text{cutoff} \quad (1)$$

$$m_h = \mathcal{O}(100\,\text{GeV}), \quad m_{\text{bare}}, \Lambda \sim M_{\text{GUT}} \gg m_h \quad (2)$$

$\rightarrow$ fine-tuning between $m_{\text{bare}}$ and $\Lambda$.

- Gauge-Higgs unification [N.S. Manton (1983), ...]:
  - extra-dimensional component of the gauge field = the Higgs field

$$A_M = (A_\mu, A_y = h) \quad (3)$$

- gauge symmetry is spontaneously broken by nonzero $\langle A_y \rangle$
- Effective potential and the Higgs-mass is finite, thanks to the gauge symmetry in the higher-dimensional spacetime
  $\rightarrow$ solve the fine-tuning problem [Inami-Lim-HH (1998)]
Fermions in $S^1/Z_2$ extra space:
- zero-mode wave-function : domain-wall profile due to bulk mass term
- Yukawa-coupling : overlap of wave-functions of fermions and gauge zero modes:

$$H(x)\bar{\psi}_R(x)\psi_L(x) = \int dy \bar{\psi}_R(x,y)A_y(x,y)\psi_L(x,y)$$

$\rightarrow$ lightest-mode mass depends exponentially on the bulk mass parameter!

Higgs effective potential (and Higgs mass) are enhanced [Hosotani et.al,2007, HH 2007].

$$m_h \sim \mathcal{O}(100\text{GeV})$$
Introduction

Randall-Sundrum space-time

- non-factorizable metric:

\[ ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \quad k : AdS_5 \text{ curvature} \quad (6) \]

- circle with identification: \( y \to -y \) fundamental region: \([0, \pi R]\) fixed points: \( y_0 = 0, \quad y_1 = \pi R \)

- Hierarchy

1. **UV (hidden brane) scales**: \( \Lambda, M_5, k, R \)
2. **IR (visible brane) scales**: 
   \[ m_{KK} = \pi ke^{-kR\pi} \frac{1}{1 - e^{-\pi kR}} \]
   \[ kR \simeq 12 \rightarrow e^{kR\pi} \simeq M_{\text{Planck}}/M_{\text{Weak}} \]
Finite-Temperature Effects with non-periodic KK tower

1-loop effective potential (per field degrees of freedom) at temperature $T$ with Kaluza-Klein mass $m_\ell$:

\[
V_{\text{eff}}^{1-\text{loop}} = \frac{(-1)^{2\eta}}{2} \frac{1}{\beta} \int \frac{d^3p}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \sum_{\ell} \ln \left[ \left( \frac{2\pi(n+\eta)}{\beta} \right)^2 + \vec{p}^2 + m_\ell^2 \right],
\]

$\eta = 0$ (boson), $\frac{1}{2}$ (fermion), $\beta \equiv 1/T$.  

(7)

When the extra dimension is compactified on $S^1$ (radius $R$),

\[
m_\ell^2 = \left( \frac{2\pi\ell + \theta}{2\pi R} \right)^2 + \tilde{M}^2, \quad M : \text{bulk mass}
\]

(8)

$\rightarrow$ one may make use of many tricks (Poisson sum formula, etc...)

For non-periodic KK modes (e.g. warped compactification) we needs another way of summation.
Poisson re-summation only for Matsubara modes gives

\[ V_{\text{eff}}^{1\text{-loop}} = V_{\text{eff}}^{T=0} + 2(-1)^{2\eta-1} \sum_{\ell} \sum_{\tilde{n}=1}^{\infty} (-1)^{2\eta \tilde{n}} \frac{(\tilde{n}/\beta |m_\ell|)^2 K_2(\tilde{n}/\beta |m_\ell|)}{(\sqrt{2\pi \tilde{n}/\beta})^4} \]  

\[ (9) \]

- \( x^d K_d(x) \equiv B_d(x) \) is a super-convergent function of \( x \):

\[ \rightarrow \text{By summing up the KK-mode masses from small to large, numerically we can obtain the finite-temperature correction in } V_{\text{eff}}. \]
(Review) – $SU(2)$ on $M^3 \times S^1$


- Wilson line phase

$$ W = \exp(i\theta \sigma_3) = \begin{cases} 
1_2 & \theta = 0, \text{ } SU(2) \text{ unbroken} \\
-1_2 & \theta = \pi, \text{ } SU(2) \text{ unbroken} \\
\text{diag}(e^{i\theta}, e^{-i\theta}) & \text{otherwise, } SU(2) \rightarrow U(1) 
\end{cases} \quad (10) $$

- Contributions from fundamental fermion, adjoint fermion, and gauge+ghost fields (blue (cold) $\leftrightarrow$ red (hot))
SU(2) on $M^3 \times S^1$ (continued)

- **gauge+ghost+ 1 fundamental fermion:**

\[
\begin{align*}
\text{SU}(2) & : \text{unbroken} \\
& \quad \text{(11)}
\end{align*}
\]

- **gauge+ghost+1 adjoint fermion:**

\[
\begin{align*}
\text{SU}(2) \rightarrow \begin{cases} 
\text{SU}(2) & T > T_c \sim 0.81/L \\
\text{U}(1) & T < T_c 
\end{cases}
\end{align*}
\]
**GHU in RS**

- **KK modes** [Hosotani-Noda et al, 2005]:

\[
m_n = k \lambda_n, \quad G(\alpha, \theta, \lambda_n) = 0, \quad (13)
\]

\[
G(\alpha, \theta, \lambda_n) \equiv \lambda_n^2 z_1 F_{\alpha-1,\alpha-1}(\lambda_n, z_1) F_{\alpha,\alpha}(\lambda_n, z_1) - \frac{4}{\pi^2} \sin^2 \frac{\theta}{2}, \quad (13)
\]

\[
\alpha = \begin{cases} 
\frac{1}{2} \pm (\frac{M}{k}) & \text{fermion} \\
0, 1 & \text{gauge-ghost-higgs fields}
\end{cases}
\]

\[
\theta : \text{Wilson-line phase},
\]

\[
F_{\alpha,\beta}(\lambda, z) \equiv Y_{\beta}(\lambda) J_{\alpha}(\lambda z) - J_{\beta}(\lambda) Y_{\alpha}(\lambda z), \quad z_1 = e^{\pi k R}, \quad (15)
\]


\[
a \equiv e^{-k \pi R}
\]

\[
V^{T=0}(\alpha, \theta) = \frac{k^4 a^4}{16 \pi^4} \int_0^\infty dt \, t^3 \, \text{Re} \ln \left[ \frac{G(\alpha, \theta, it)}{G_{\text{asymp}}(\alpha, it)} \right], \quad (16)
\]

\[
G(\alpha, \theta, it) \xrightarrow{t \to \infty} G_{\text{asymp}}(\alpha, it) \quad (17)
\]
$SU(2)$ model on RS(central)

- gauge + ghost + 1 fermion (bulk mass $M = ck$)
- $(SU(2)$ is broken to $U(1)_3$ by orbifold b.c.)
- gauge field + fundamental fermion : $U(1)_3$ unbroken
- gauge field + adjoint fermion :

\[ T_c/(ka) = \frac{12.0}{U(1)_3} \]

$V_{\text{wall}}/(ka)^4 \times 10^4$}

$\text{c}_{\text{crit}}$
$SO(5) \times U(1)$ GHU model (preliminary)

As application to the particle physics, we study the finite-temperature effect on the model proposed by Hosotani et.al (2008-),

(cf. Hosotani-san’s Talk)

- $SO(5) \times U(1) \rightarrow SU(2)_L \times SU(2)^c_R \times U(1) \rightarrow U(1)_{\text{em}}$
- $m_h$: 70GeV $\sim$ 140GeV
- The model have “H-parity”
  - $P_H = -1$ is assigned for $h$ and $+1$ for other SM fields
  - All $P_H$-odd interactions ($hWW, hZZ, h\bar{f}f, hhh$) vanish.
    → the model can avoid the LEP constraint ($m_h \leq 114$GeV)
- $h$ is stable and can be the candidate of dark matter (higgs dark-matter).
Effective potential - we adopt the following approximation:

\[
V_{\text{eff}} \simeq V_W + V_Z + V_{\text{top}},
\]

\[
V_W + V_Z \sim 3V_W = 3 \cdot 3V(1,2\theta),
\]

\[
V_{\text{top}} = -3_{\text{col}} \cdot 4V(0.063,2\theta)
\]

other fermions’ contribution (b, c, s, d, u, e, \(\mu\), \(\tau\), and non-SM heavy particles) are negligible.

Result for \(kR = 12\) (preliminary)

- \(m_{\text{KK}}(\simeq \pi ka) \sim 1.5\text{TeV}\)
- critical temperature : \(\beta_c \sim 1.5/ka \rightarrow T_c \sim 320\text{GeV}\)
- height of potential barrier : \(2 \times 10^{-4}(ka)^4 \sim (60\text{GeV})^4\)
- \(V_c \equiv V_{\text{eff}}(T = T_c) \simeq V_{\text{eff}}(T = 0) \simeq 250\text{GeV}\)
  \(\rightarrow\) unfortunately, not satisfy Shaposhnikov’s criteria \((V_c/T_c > 1)\)
- $kR = 4$
  
- Changes
  
  - $KK$ scale lowered: $ka \sim 300\text{GeV}$ ($M_{KK} \sim 1.2\text{TeV}$)
  
  - Top bulk mass for top quark becomes smaller ($c_{\text{top}} = [0.294, 0.325]$ for $\pi/2 \leq \theta \leq \pi$)
  
- Result for $kR = 4$ (preliminary)
  
  - Critical temperature: $\beta c \sim 1.5/ka \rightarrow T_c \sim 200\text{GeV}$
  
  - Height of potential barrier: $2 \times 10^{-4} (ka)^4 \sim (50\text{GeV})^4$
  
  - $V_c/T_c \geq 1$ : satisfy Shaposhnikov’s criteria
Summary

- Numerically studied the Hosotani mechanism on RS at finite-temperature correction from the zero-temperature is obtained by summing up hundreds of Kaluza-Klein masses
- obtain critical temperature and the height of the potential wall
- Apply to the gauge-Higgs unification ($SO(5) \times U(1)$ model) [preliminary]

<table>
<thead>
<tr>
<th></th>
<th>$kR = 12$</th>
<th>$kR = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\exp(\pi kR)$</td>
<td>$2.4 \times 10^{16}$</td>
<td>$2.9 \times 10^{5}$</td>
</tr>
<tr>
<td>$m_{KK}, [ka]$</td>
<td>$1.5\text{TeV, [\sim 500}\text{GeV}]$</td>
<td>$1.2\text{TeV, [\sim 300}\text{GeV}]$</td>
</tr>
<tr>
<td>$c_{top}$</td>
<td>0.437</td>
<td>0.294 – 325</td>
</tr>
<tr>
<td>$T_c$</td>
<td>$330\text{GeV}$ ((V_c/T_c &lt; 1))</td>
<td>$200\text{GeV}$ ((V_c/T_c &gt; 1))</td>
</tr>
<tr>
<td>$V_{\text{barrier}}$</td>
<td>$\sim (60\text{GeV})^4$</td>
<td>$\sim (50\text{GeV})^4$</td>
</tr>
<tr>
<td>order of PT</td>
<td>1st</td>
<td>1st</td>
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</tbody>
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Perspective

- Spharelon process in higher-dimensional space-time
- Flavor mixing, CP violation phase in GHU
Figure: Bulk mass parameter for top quark $c_{top}$ versus warp-index $kR$. Solid (dashed) line is for $\theta_H = \pi/2$ ($\theta_H = \pi$).