Gauge-Higgs unification model with S^2/Z_2 extra space based on E_6 gauge theory

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1. Introduction

In the LHC era, It is Interesting to investigate model which provide an origin of Higgs boson

⇒ Gauge-Higgs Unification model

Manton (1979) D.B. Fairlie(1979)

- ●Providing origin of the Higgs field
  - ⇒ Higgs field is identified as **extra-components of gauge field**
- ●The Higgs sector is predictive
  - ⇒ determined by the gauge interaction in higher-dimensions
- ●Possible solution to **hierarchy problem**
  - ⇒ quadratic divergence for Higgs mass correction do not appear because of gauge symmetry

We consider …

A Gauge-Higgs Unification model on $M^4 \times S^2/Z_2$ with back ground gauge field

- Based on six dimensional gauge theory
- Extra space is compact two-sphere orbifold
- Introduce background gauge field (Dirac monopole)

Interesting properties

- Higgs potential can be generated at tree level
- O(100 GeV) Higgs mass is achieved by the potential
- Gauge symmetry can be reduced by background field

We investigate these properties and construct the model based on E_6 gauge theory

Outline

1. Introduction
2. GHU on S^2/Z_2
3. Reduction of six-dimensional theory
4. Higgs potential analysis
5. Summary

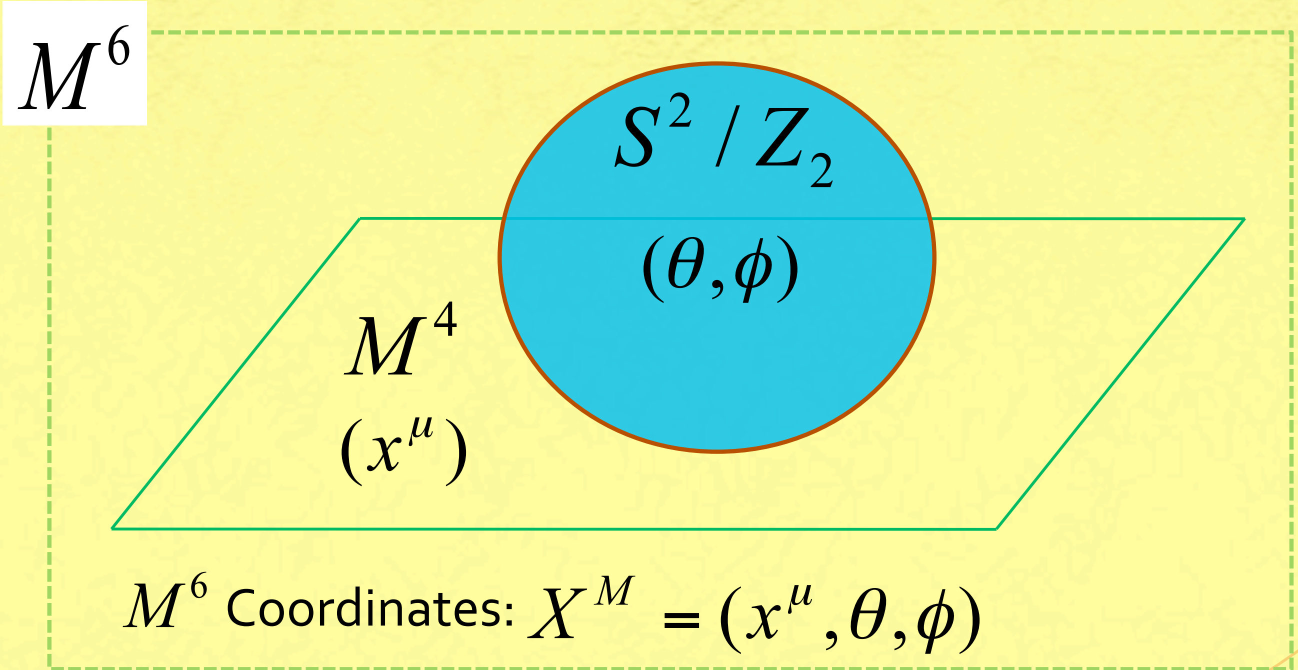
2. GHU on S^2/Z_2

Theory in 6-dim

Gauge theory on 6-dim space-time $M^6 = M^4 \otimes S^2/Z_2$

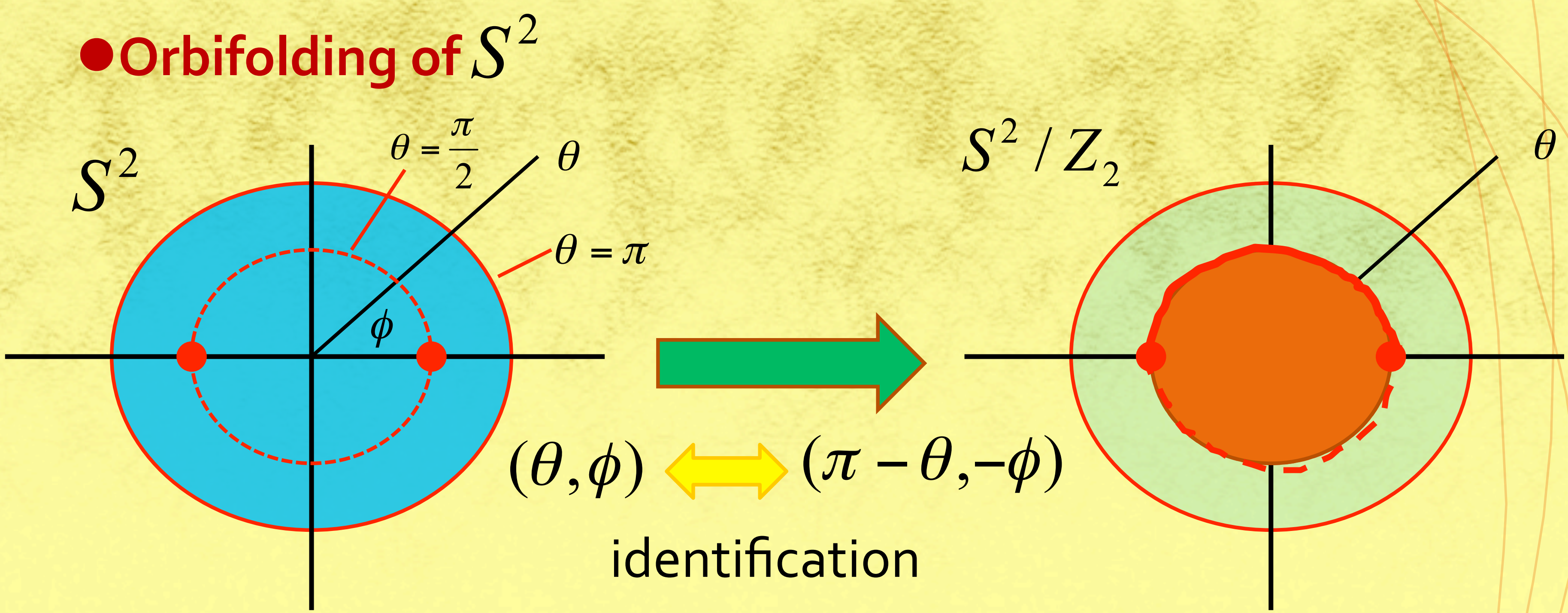
M^4 : 4-dim Minkowski space-time

S^2/Z_2 : 2-sphere orbifold



★orbifolding : $(\theta,\phi) \Longleftrightarrow (\pi-\theta,-\phi)$

● Orbifolding of S^2



Two fixed points: $(\pi / 2, 0)$ $(\pi / 2, \pi)$

By orbifolding

- Non-trivial boundary condition can be defined
- Reduced 4-dim theory is restricted

Set up of the model

- **Gauge symmetry on 6-dim**

  E_6 gauge symmetry

- **Gauge field**

$$A_M(X) = (A_\mu(X), A_\theta(X), A_\phi(X))$$

☆ We introduce a background gauge field

S. Randjbar-Daemi, A. Salam, J. A. Strathdee (1983)

$$A_\phi^B = iQ\frac{\cos\theta \mp 1}{\sin\theta} \left\{ \begin{matrix} - : 0 \le \theta < \pi/2 \\ + : \pi/2 \le \theta \le \pi \end{matrix} \right\} \quad Q \propto U(1)_Z (\subset E_6) \text{ generator}$$

☆ Dirac monopole configuration

- Left handed Weyl fermion of SO(1,5) (belongs to 27 rep)

$$\Psi(X) = \begin{pmatrix} \psi_L(X) \\ \psi_R(X) \end{pmatrix}$$

ψ_L :Left handed Weyl fermion of SO(1,3)

ψ_R :Right handed Weyl fermion of SO(1,3)

Action of the theory

$$S = \int dx^4 \sin\theta d\theta d\phi (\Psi i \Gamma^M D_M \Psi - \frac{1}{4g^2} g^{MN} g^{KL} Tr[F_{MK} F_{NL}])$$

$$F^{MN}(X) = \partial_M A_N(X) - \partial_N A_M(X) - [A_M(X), A_N(X)]$$

$$g_{MN} = diag(1,-1,-1,-1,-R^{-2},-R^{-2}\sin^{-2}\theta)$$

: M^6 metric
(R : radius of S^2)

$$\Gamma^M : \begin{cases} \Gamma^\mu = \gamma^\mu \otimes I_2 \\ \Gamma^4 = \gamma^5 \otimes \sigma_1 \\ \Gamma^5 = \gamma^5 \otimes \sigma_2 \end{cases}$$

:6-dim gamma matrix

$$D^M : \begin{cases} D_\mu = \partial_\mu - A_\mu \\ D_\theta = \partial_\theta - A_\theta \\ D_\phi = \partial_\phi - i\frac{\Sigma_3}{2}\cos\theta - A_\phi \end{cases} \qquad (\Sigma_3 = I_4 \otimes \sigma_3)$$

:covariant derivative

Spin connection term (for fermion)

Reduction of the theory to 4-dim effective theory

4-dim theory is restricted by these conditions

- The non-trivial boundary condition of S^2/Z_2

  ⇒ Restricting gauge symmetry and massless particle contents in four-dimensions

- Background gauge field

  ⇒ Gauge symmetry is further restricted by configuration of background gauge field

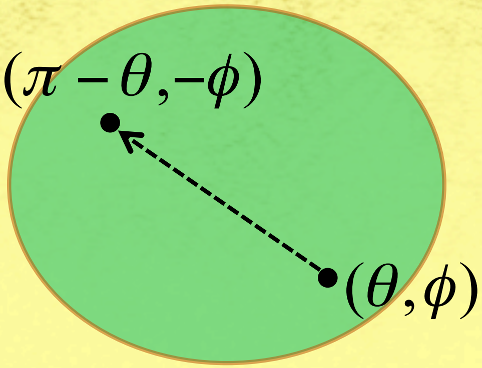
- The condition to obtain massless fermions

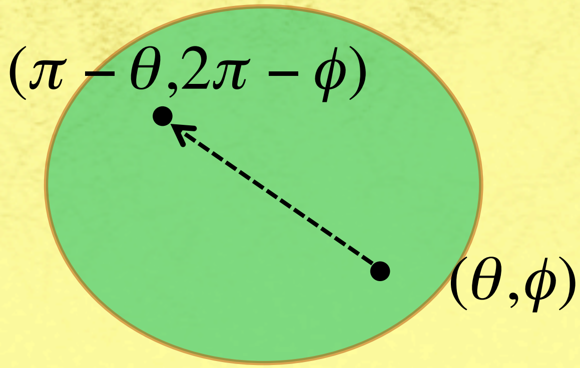
  ⇒ Restricting massless fermion contents in four-dimension

The non-trivial boundary condition of S^2/Z_2

We impose non-trivial boundary condition for

$$(\theta,\phi) \to (\pi-\theta,-\phi) \qquad\qquad (\theta,\phi) \to (\pi-\theta,2\pi-\phi)$$





$(\theta,\phi) \to (\pi-\theta,-\phi)$

$$\Psi(x,\pi-\theta,-\phi) = \gamma_5 P\Psi(x,\theta,\phi)$$
$$A_\mu(x,\pi-\theta,-\phi) = PA_\mu(x,\theta,\phi)P$$
$$A_\theta(x,\pi-\theta,-\phi) = -PA_\theta(x,\theta,\phi)P$$
$$A_\phi(x,\pi-\theta,-\phi) = -PA_\phi(x,\theta,\phi)P$$

$(\theta,\phi) \to (\pi-\theta,2\pi-\phi)$

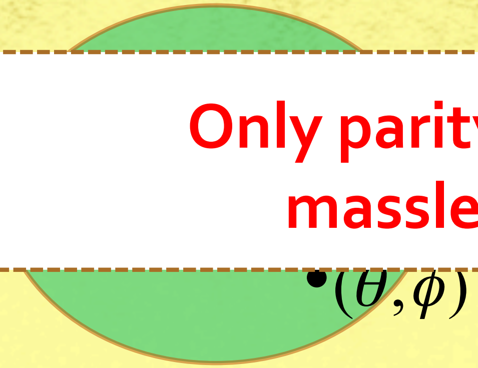
$$\Psi(x,\pi-\theta,2\pi-\phi) = \gamma_5 \bar{P}\Psi(x,\theta,\phi)$$
$$A_\mu(x,\pi-\theta,2\pi-\phi) = \bar{P}A_\mu(x,\theta,\phi)\bar{P}$$
$$A_\theta(x,\pi-\theta,2\pi-\phi) = -\bar{P}A_\theta(x,\theta,\phi)\bar{P}$$
$$A_\phi(x,\pi-\theta,2\pi-\phi) = -\bar{P}A_\phi(x,\theta,\phi)\bar{P}$$

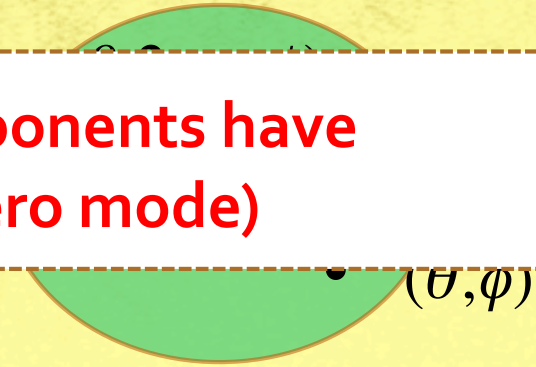
- P, $\bar{P}$: matrices acting on the representation space of gauge group
- Components of P, $\bar{P}$ is +1 or -1 and $P^2 = 1 (\bar{P}^2 = 1)$

The non-trivial boundary condition of S^2/Z_2

We impose non-trivial boundary condition for

$(\theta,\phi) \to (\pi-\theta,-\phi)$ $\qquad\qquad$ $(\theta,\phi) \to (\pi-\theta,2\pi-\phi)$





Only parity even components have massless mode (Zero mode)

$(\theta,\phi) \to (\pi-\theta,-\phi)$

$$\Psi(x,\pi-\theta,-\phi) = \gamma_5 P\Psi(x,\theta,\phi)$$
$$A_\mu(x,\pi-\theta,-\phi) = PA_\mu(x,\theta,\phi)P$$
$$A_\theta(x,\pi-\theta,-\phi) = -PA_\theta(x,\theta,\phi)P$$
$$A_\phi(x,\pi-\theta,-\phi) = -PA_\phi(x,\theta,\phi)P$$

$(\theta,\phi) \to (\pi-\theta,2\pi-\phi)$

$$\Psi(x,\pi-\theta,2\pi-\phi) = \gamma_5 \bar{P}\Psi(x,\theta,\phi)$$
$$A_\mu(x,\pi-\theta,2\pi-\phi) = \bar{P}A_\mu(x,\theta,\phi)\bar{P}$$
$$A_\theta(x,\pi-\theta,2\pi-\phi) = -\bar{P}A_\theta(x,\theta,\phi)\bar{P}$$
$$A_\phi(x,\pi-\theta,2\pi-\phi) = -\bar{P}A_\phi(x,\theta,\phi)\bar{P}$$

- P, $\bar{P}$: matrices acting on the representation space of gauge group
- Components of P, $\bar{P}$ is +1 or -1 and $P^2 = 1(\bar{P}^2 = 1)$

Gauge symmetry reduction by background field

Gauge symmetry is reduced by background gauge field

B.C.

$$A_\phi^B = iQ\frac{\cos\theta \mp 1}{\sin\theta} \quad \left\{ \begin{array}{l} - : 0 \le \theta < \pi/2 \\ + : \pi/2 \le \theta \le \pi \end{array} \right\}$$

Gauge boson gets mass from interaction term $[A_\mu, A_\phi^B]^2$

Mass contribution for lowest mode from background gauge field

$$m_B^2 = \frac{Q^2}{4\pi R^2}\int d\Omega \frac{(\cos\theta \mp 1)^2}{\sin^2\theta} \quad \left\{ \begin{array}{l} - : 0 \le \theta < \pi/2 \\ + : \pi/2 \le \theta \le \pi \end{array} \right\}$$

Gauge symmetry in 4-dim should commute with $U(1)_Z$

The condition to obtain massless fermions

Positive curvature of S^2 → Masses of fermions in four-dim

The condition to obtain massless fermions

Positive curvature of S^2 → Mass… …ions in… ← The background gauge field A_ϕ^B

cancel

The condition to obtain massless fermions

Positive curvature of S^2 → Mas … ions in … ← The background gauge field A_ϕ^B

cancel

Spin connection term should be canceled by background gauge field

$$Q\Psi(X) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\Psi(X)$$

→ $U(1)_Z$ **charges of the 4-dim massless fermion contents are restricted**

We chose Q to provide SM fermions as zero mode

The reduced 4-dim theory

Particle contents and gauge symmetry in four-dimensions are restricted by….

The parity assignment for non-trivial boundary condition

+

The configuration of background gauge field

+

The condition for massless fermion (for fermions)

3. Reduction of six-dim theory

Reducing the E_6 gauge symmetry

• Reduction by background gauge field

We chose background gauge field to belong $U(1)_Z$ as

$$
\begin{aligned}
E_6 &\supset SO(10)\times U(1)_Z \\
&\supset SU(5)\times U(1)_X\times U(1)_Z \\
&\supset SU(3)\times SU(2)\times U(1)_Y\times U(1)_X\times U(1)_Z
\end{aligned}
$$

B.C.

$$
A_\phi^B = iQ\frac{\cos\theta\mp 1}{\sin\theta} \quad \begin{cases} - : 0\le\theta<\pi/2 \\ + : \pi/2\le\theta\le\pi \end{cases}
$$

$Q \propto U(1)_Z$ charge

$$
\begin{aligned}
78 = {} & (8,1)(0,0,0)+(1,3)(0,0,0)+(1,1)(0,0,0)+(1,1)(0,0,0)+(1,1)(0,0,0) \\
& +(3,2)(-5,0,0)+(\bar{3},2)(5,0,0)+(3,2)(1,4,0)+(\bar{3},2)(-1,-4,0) \\
& +(3,1)(4,-4,0)+(\bar{3},1)(-4,4,0)+(1,1)(-6,-4,0)+(1,1)(6,4,0) \\
& +(3,2)(1,-1,-3)+(\bar{3},2)(-1,1,3)+(3,1)(4,1,3)+(\bar{3},1)(-4,-1,-3) \\
& +(3,1)(-2,-3,3)+(\bar{3},1)(2,3,-3)+(1,2)(-3,3,-3)+(1,2)(3,-3,3) \\
& +(1,1)(-6,1,3)+(1,1)(6,-1,-3)+(1,1)(0,-5,-3)+(1,1)(0,5,3)
\end{aligned}
$$

Reducing the E_6 gauge symmetry

- Reduction by background gauge field

We chose background gauge field to belong $U(1)_Z$ as

$$E_6 \supset SO(10)\times U(1)_Z$$
$$\supset SU(5)\times U(1)_X \times U(1)_Z$$
$$\supset SU(3)\times SU(2)\times U(1)_Y \times U(1)_X \times U(1)_Z$$

B.C.

$$A^B_\phi = iQ\frac{\cos\theta \mp 1}{\sin\theta} \quad \left\{ \begin{array}{l} - : 0 \le \theta < \pi/2 \\ + : \pi/2 \le \theta \le \pi \end{array} \right\}$$

$Q \propto U(1)_Z$ charge

$$\begin{aligned} 78 = {} & (8,1)(0,0,0) + (1,3)(0,0,0) + (1,1)(0,0,0) + (1,1)(0,0,0) + (1,1)(0,0,0) \\ & +(3,2)(-5,0,0) + (\bar{3},2)(5,0,0) + (3,2)(1,4,0) + (\bar{3},2)(-1,-4,0) \\ & +(3,1)(4,-4,0) + (\bar{3},1)(-4,4,0) + (1,1)(-6,-4,0) + (1,1)(6,4,0) \\ & +(3,2)(1,-1,-3) + (\bar{3},2)(-1,1,3) + (3,1)(4,1,3) + (\bar{3},1)(-4,-1,-3) \\ & +(3,1)(-2,-3,3) + (\bar{3},1)(2,3,-3) + (1,2)(-3,3,-3) + (1,2)(3,-3,3) \\ & +(1,1)(-6,1,3) + (1,1)(6,-1,-3) + (1,1)(0,-5,-3) + (1,1)(0,5,3) \end{aligned}$$

No zero mode

Reducing the E_6 gauge symmetry

- Reduction by non-trivial boundary condition

We chose following Parity assignment

$$A_\mu(x) \to P A_\mu(x) P,\ A_\mu(x) \to \overline{P} A_\mu(x) \overline{P} \quad \text{under } (\theta,\phi) \to (\pi-\theta,-\phi)((\theta,\phi) \to (\pi-\theta,2\pi-\phi))$$

$$\begin{aligned}
78_{4\,\mathrm{dim}} = {} & (8,1)(0,0,0)^{(+,+)} + (1,3)(0,0,0)^{(+,+)} + (1,1)(0,0,0)^{(+,+)} + (1,1)(0,0,0)^{(+,+)} + (1,1)(0,0,0)^{(+,+)} \\
& +(3,2)(-5,0,0)^{(-,+)} + (\bar{3},2)(5,0,0)^{(-,+)} + (3,2)(1,4,0)^{(+,-)} + (\bar{3},2)(-1,-4,0)^{(+,-)} \\
& +(3,1)(4,-4,0)^{(-,-)} + (\bar{3},1)(-4,4,0)^{(-,-)} + (1,1)(-6,-4,0)^{(-,-)} + (1,1)(6,4,0)^{(-,-)} \\
& +(3,2)(1,-1,-3)^{(+,+)} + (\bar{3},2)(-1,1,3)^{(+,+)} + (3,1)(4,1,3)^{(-,+)} + (\bar{3},1)(-4,-1,-3)^{(-,+)} \\
& +(3,1)(-2,-3,3)^{(+,-)} + (\bar{3},1)(2,3,-3)^{(+,-)} + (1,2)(-3,3,-3)^{(-,-)} + (1,2)(3,-3,3)^{(-,-)} \\
& +(1,1)(-6,1,3)^{(-,+)} + (1,1)(6,-1,-3)^{(-,+)} + (1,1)(0,-5,-3)^{(+,-)} + (1,1)(0,5,3)^{(+,-)}
\end{aligned}$$

No zero mode

Reducing the E_6 gauge symmetry

• Reduction by non-trivial boundary condition

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$$+(3,2)(-5,0,0)^{(-,+)} + (\bar{3},2)(5,0,0)^{(-,+)} + (3,2)(1,4,0)^{(+,-)} + (\bar{3},2)(-1,-4,0)^{(+,-)}$$

$$+(3,1)(4,-4,0)^{(-,-)} + (\bar{3},1)(-4,4,0)^{(-,-)} + (1,1)(-6,-4,0)^{(-,-)} + (1,1)(6,4,0)^{(-,-)}$$

No zero mode

$$+(3,2)(1,-1,-3)^{(+,+)} + (\bar{3},2)(-1,1,3)^{(+,+)} + (3,1)(4,1,3)^{(-,+)} + (\bar{3},1)(-4,-1,-3)^{(-,+)}$$

$$+(3,1)(-2,-3,3)^{(+,-)} + (\bar{3},1)(2,3,-3)^{(+,-)} + (1,2)(-3,3,-3)^{(-,-)} + (1,2)(3,-3,3)^{(-,-)}$$

$$+(1,1)(-6,1,3)^{(-,+)} + (1,1)(6,-1,-3)^{(-,+)} + (1,1)(0,-5,-3)^{(+,-)} + (1,1)(0,5,3)^{(+,-)}$$

No zero mode

Gauge symmetry reduction

$E_6 \supset SO(10)\times U(1)_Z$

$\supset SU(5)\times U(1)_X \times U(1)_Z$

$\supset SU(3)\times SU(2)\times U(1)_Y \times U(1)_X \times U(1)_Z$

Scalar particle contents in 4-dim

Scalar contents (extra component gauge field)

- Do not have zero mode in 4-dim
- Getting negative mass term contribution from interaction with background gauge field

N.Maru, TN, J.Sato, M.Yamanaka(2009)
H.Dohi and K.Oda(2010)

$$
\begin{aligned}
78_{scalar} = {} & (8,1)(0,0,0)^{(-,-)} + (1,3)(0,0,0)^{(-,-)} + (1,1)(0,0,0)^{(-,-)} + (1,1)(0,0,0)^{(-,-)} + (1,1)(0,0,0)^{(-,-)} \\
& + (3,2)(-5,0,0)^{(+,-)} + (\bar{3},2)(5,0,0)^{(+,-)} + (3,2)(1,4,0)^{(-,+)} + (\bar{3},2)(-1,-4,0)^{(-,+)} \\
& + (3,1)(4,-4,0)^{(+,+)} + (\bar{3},1)(-4,4,0)^{(+,+)} + (1,1)(-6,-4,0)^{(+,+)} + (1,1)(6,4,0)^{(+,+)} \\
& + (3,2)(1,-1,-3)^{(-,-)} + (\bar{3},2)(-1,1,3)^{(-,-)} + (3,1)(4,1,3)^{(+,-)} + (\bar{3},1)(-4,-1,-3)^{(+,-)} \\
& + (3,1)(-2,-3,3)^{(-,+)} + (\bar{3},1)(2,3,-3)^{(-,+)} + \boxed{(1,2)(-3,3,-3)^{(+,+)} + (1,2)(3,-3,3)^{(+,+)}} \\
& + (1,1)(-6,1,3)^{(+,-)} + (1,1)(6,-1,-3)^{(+,-)} + (1,1)(0,-5,-3)^{(-,+)} + (1,1)(0,5,3)^{(-,+)}
\end{aligned}
$$

$A_{\theta,\phi}(x) \to -PA_{\theta,\phi}(x)P, A_{\theta,\phi}(x) \to -\bar{P}A_{\theta,\phi}(x)\bar{P}$ under $(\theta,\phi) \to (\pi-\theta,-\phi)((\theta,\phi) \to (\pi-\theta,2\pi-\phi))$

Scalar particle contents in 4-dim

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& + (3,2)(1,-1,-3)^{(-,-)} + (\bar{3},2)(-1,1,3)^{(-,-)} + (3,1)(4,1,3)^{(+,-)} + (\bar{3},1)(-4,-1,-3)^{(+,-)} \\
& + (3,1)(-2,-3,3)^{(-,+)} + (\bar{3},1)(2,3,-3)^{(-,+)} + \boxed{(1,2)(-3,3,-3)^{(+,+)} + (1,2)(3,-3,3)^{(+,+)}} \\
& + (1,1)(-6,1,3)^{(+,-)} + (1,1)(6,-1,-3)^{(+,-)} + (1,1)(0,-5,-3)^{(-,+)} + (1,1)(0,5,3)^{(-,+)}
\end{aligned}
$$

We identify these doublet components as Higgs field in 4-dim

Other unwanted modes can be made to get positive KK mass by boundary condition or fixed point localized terms.

Fermion contents in 4-dim

Introduce SO(1,5) Weyl fermions which belongs 27 rep of E_6

Components of 27 under $E_6 \supset SO(10)\times U(1)_Z$

$$27 = 16(1) + 10(-2) + 1(4)$$

BG field belongs to this U(1)

We chose the normalization of background gauge field to make 16(1) have zero mode

$$Q\Psi(X) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\Psi(X) \qquad \Psi(X) = \begin{pmatrix} \psi_L(X) \\ \psi_R(X) \end{pmatrix}$$

Upper component satisfy the condition

$16(1)_L$ can have zero mode

SM fermions can be obtained with proper boundary condition

Fermion contents in 4-dim

Introduce SO(1,5) Weyl fermions which belongs 27 rep of E_6

U(1)

We can obtain particle contents of SM
by choosing proper background gauge field
and boundary conditions

U

$16(1)_L$ can have zero mode

SM fermions can be obtained with proper boundary condition

4. Higgs potential analysis

Higgs potential is obtained from extra component gauge sector

$$V = \frac{1}{2g^2R^2}\int d\Omega \frac{1}{\sin^2\theta} Tr[F_{\theta\phi}F_{\theta\phi}]$$

$$= \frac{1}{2g^2R^2}\int d\Omega Tr\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\tilde{A}_\phi + \sin\theta A^B_\phi) - \frac{1}{\sin\theta}\partial_\phi A_\theta - i[A_\theta, \tilde{A}_\phi + A^B_\phi]\right]$$

$$\left(\tilde{A}_\phi = \frac{A_\phi}{\sin\theta}\right)$$

$(A_\theta, \tilde{A}_\phi)$ are expanded in terms of derivative of spherical harmonics

We find (l=1,|m|=1) mode has negative mass term ⇒ Higgs field

$$A_\theta = -\frac{1}{\sqrt{2}}\left[\Phi_1(x)\partial_\theta Y^-_{11}(\theta,\phi) + \Phi_2(x)\frac{1}{\sin\theta}\partial_\phi Y^-_{11}(\theta,\phi)\right] + \cdots$$

$$\tilde{A}_\phi = \frac{1}{\sqrt{2}}\left[\Phi_2(x)\partial_\theta Y^-_{11}(\theta,\phi) - \Phi_1(x)\frac{1}{\sin\theta}\partial_\phi Y^-_{11}(\theta,\phi)\right] + \cdots$$

$$\left(Y^-_{11} = -\frac{1}{\sqrt{2}}[Y_{11} + Y_{1-1}]\right)$$

Taking (l=1,|m|=1) mode

Integrating out extra-space we obtain Higgs potential

$$V = -\frac{7+9(7/4-3\ln 2)}{8R^2}|\phi|^2 + \frac{3g^2}{40\pi R^2}|\phi|^4$$

{ ϕ **:corresponding to Higgs doublet** }

This potential leads electroweak symmetry breaking!

VEV of the Higgs field

$$<\phi>= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad v = \frac{1}{6}\sqrt{\frac{35+45(7/4-3\ln 2)}{2}}\frac{1}{R}$$

VEV is expressed by radius of S^2

- Higgs mass and W-boson mass

$$\begin{cases} m_W = \dfrac{g_2}{2} v \approx \dfrac{0.53}{R} \qquad \left\{ g_2 = \dfrac{g}{\sqrt{6\pi R^2}} \right\} \\[2ex] m_H = \sqrt{\dfrac{3}{20\pi}} \dfrac{gv}{R} = 3\sqrt{\dfrac{2}{5}} m_W \end{cases}$$

They are related !

⇒

$$R^{-1} \approx 152 GeV$$

$$m_H \approx 150 GeV$$

- Weinberg angle

$$\sin^2 \theta_W = \frac{3}{8}$$

Same as SU(5) GUT case

Summary

- We analyzed Gauge Higgs Unification for gauge theory on S^2/Z_2 with background gauge field

- We construct E_6 model and obtained
  - One generation of SM fermions
  - Higgs potential which cause SSB
  - Relation between Higgs boson mass and W-boson mass

$$m_H = 3\sqrt{\frac{2}{5}}m_W$$

- Power divergence could be removed by parity invariance on S^2/Z_2

Comment on divergence in Higgs potential

Generally 6-dim case would provide power divergence

Operator $F_{\theta\phi}(x)$ would be allowed

But it could be eliminated by assuming the parity invariance on S^2/Z_2

$$\theta \to \pi - \theta$$

$$A_\mu(x,\theta,\phi) \to A_\mu(x,\pi-\theta,\phi)$$

$$A_\theta(x,\theta,\phi) \to -A_\theta(x,\pi-\theta,\phi)$$

$$A_\phi(x,\theta,\phi) \to A_\phi(x,\pi-\theta,\phi)$$

$$\Psi(x,\theta,\phi) \to \pm\Gamma^4\Psi(x,\pi-\theta,\phi)$$

Compatible with orbifold boundary conditions

Dangerous operator is forbidden since $F_{\theta\phi}(x,\theta,\phi) \to -F_{\theta\phi}(x,\pi-\theta,\phi)$