Unitarity and BRST invariance in Dirichlet Higgs Model

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Based on the collaboration with
Kin-ya Oda (Osaka University)
arXiv:1011.0405 [hep-ph]
and work in progress

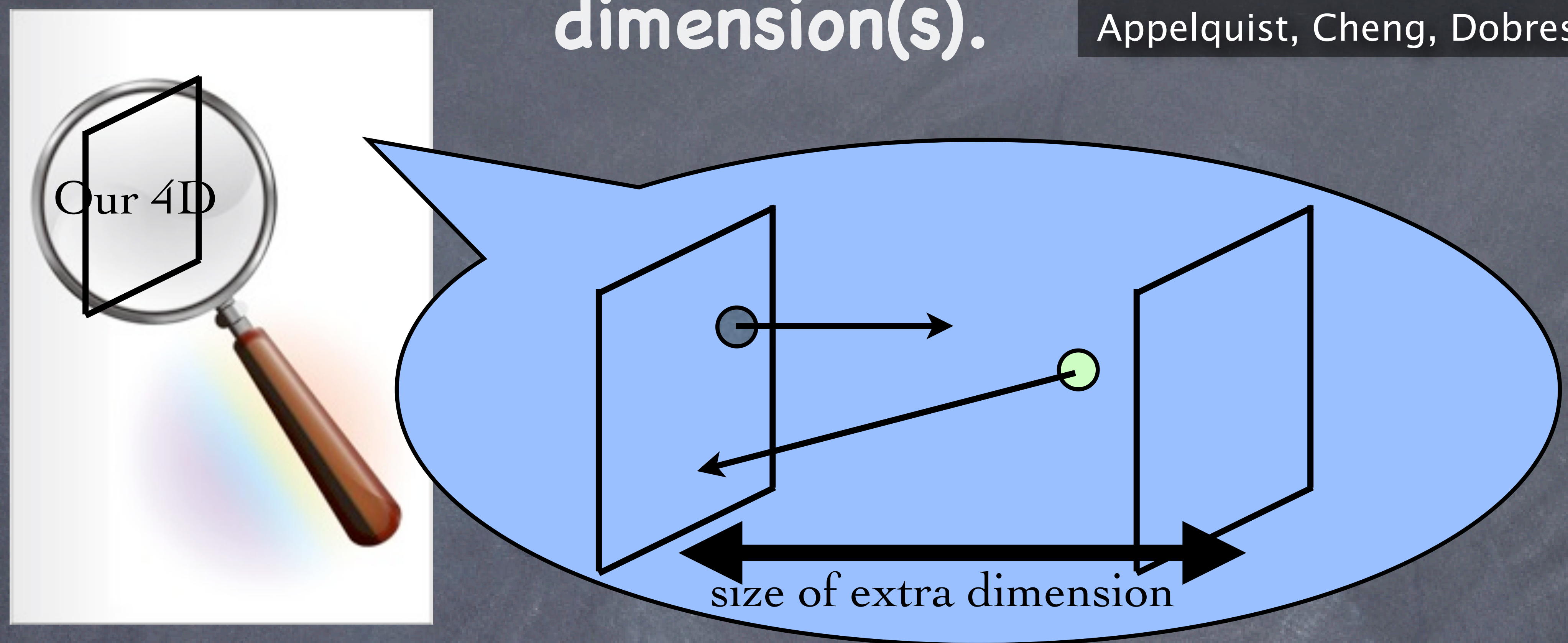
Extra Dimension 2011 @ Osaka University

2011/01/25

Ordinary 5D UED, Review

We consider Standard Model on compact extra dimension(s).

Appelquist, Cheng, Dobrescu, 01



[Interesting points]

- Dark matter candidate = Lightest KK particle
- A few new parameters
- Loose constraint on m_{KK} ← Possibly detectable at the LHC

How EWSB Occurs in Ordinary UED

[Higgs Action]

$$S = \int d^4x \int_0^L dy \left(- |\partial_\mu \Phi|^2 - |\partial_y \Phi|^2 - V(\Phi) \right)$$

[Higgs potential]

$$V(\Phi) = -m^2 |\Phi|^2 + \lambda |\Phi|^4$$

Ordinary Higgs potential

What is the origin of EWSB?
(the source of negative mass squared?)

EWSB by Higgs Boundary Condition

$$S = \int d^4x \int_0^L dy \left(- |\partial_\mu \Phi|^2 - |\partial_y \Phi|^2 - V(\Phi) \right)$$

$$V(\Phi) = \cancel{-m^2|\Phi|^2} + \cancel{\lambda|\Phi|^4}$$

$$\Phi = \Phi^c + \Phi^q$$

classical part(VEV) quantum fluctuation around VEV

Surface term: $\int d^4x \, [-\delta\Phi^c \partial_y \Phi^{c\dagger}]_{y=0}^{y=L}$

Dirichlet: $\underline{\delta\Phi^c|_{bd} = 0}$ rather than $\underline{\Phi^c|_{bd} = 0}$!!

$\underline{\Phi^c|_{bd} = \text{const.}}$ suffices!! (EWSB by boundary condition)

($\Phi^q|_{bd} = 0$: Ordinary Dirichlet boundary condition)

EWSB by Non-Zero Dirichlet B.C.

- A proposal: Dirichlet Higgs Model on an interval
  - ★ Put non-zero Dirichlet b.c. on Higgs! (No SM Higgs)
    - ✳ (with KK-parity being assumed)

Haba, Oda, Takahashi, 09, 10

$$\Phi\big|_{y=0,L} = \begin{bmatrix} \phi_D^1 \\ \phi_D^2 \end{bmatrix} \xrightarrow{\text{field red.}} \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

Classical(VEV) profile

$$\frac{v}{\sqrt{2}} := \sqrt{|\phi_D^1|^2 + |\phi_D^2|^2}$$

$$\cancel{V(\Phi)} = \cancel{-m^2|\Phi|^2} + \cancel{\frac{\hat{\lambda}}{\Lambda}|\Phi|^4}$$

- Merit:
  - ★ No need of <u>negative mass-squared</u>, nor <u>quartic coupling</u>.
  - ★ Fewer number of free parameters.
  - ★ Deviation in Higgs interaction (interesting phenomenologically).

But, there remain two questions...

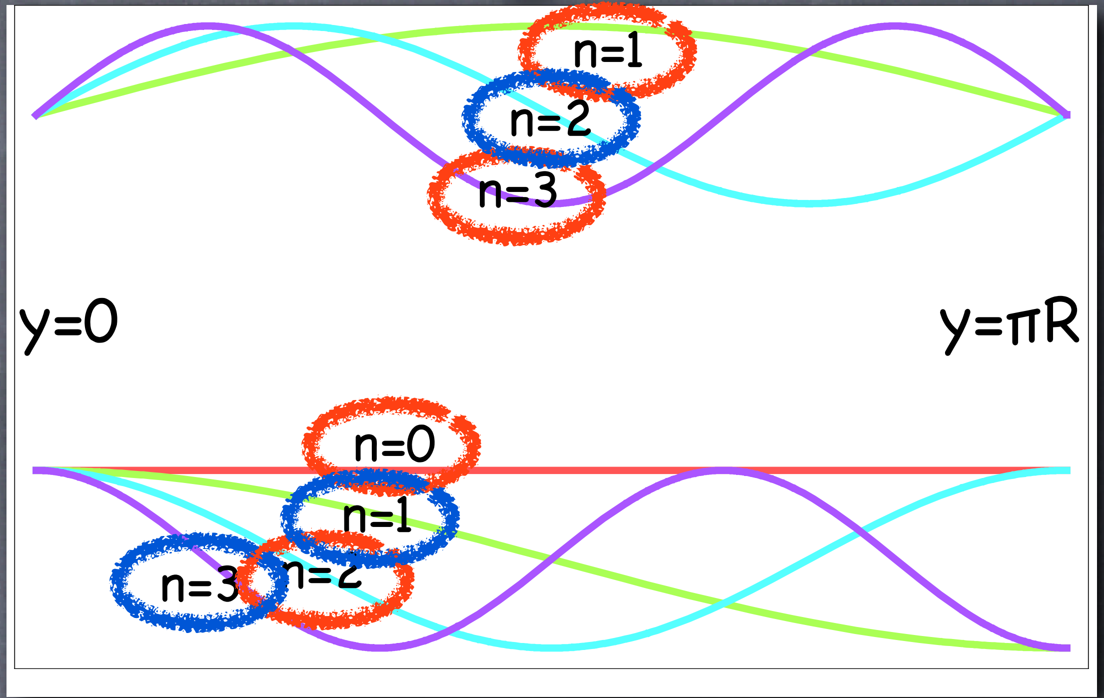
1. EWSB by b.c. $\underline{\varphi^c|_{bd} = \text{Const}}$ looks explicit (well-defined as quantum theory?).
2. Who unitarizes the $W_L W_L$-scattering?

To answer the *first* question

1. EWSB by b.c. $\varphi^c|_{bd} = \text{Const}$ looks explicit (well-defined as quantum theory?).
2. Who unitarizes the $W_L W_L$-scattering?

Mode Functions & Some Integrals

- Dirichlet: $$S_n(y) := \frac{1}{\sqrt{2\pi R}} \sin\left[\frac{ny}{R}\right]$$ $$S_{-n}(y) = -S_n(y)$$
- Neumann: $$C_n(y) := \frac{1}{\sqrt{2\pi R}} \cos\left[\frac{ny}{R}\right]$$ $$C_{-n}(y) = C_n(y)$$



Mode functions are the same with these of S^1/Z_2.

But there are differences.

$$\varepsilon_n := \begin{cases} 0 & \text{for } n\text{: even,} \\ \frac{2}{n\pi} & \text{for } n\text{: odd,} \end{cases}$$

$$\mathrm{CS}_{n,m} := \int_0^L dy\, C_n(y) S_m(y) = \frac{1}{4}\left[-\varepsilon_{n-m} + \varepsilon_{n+m}\right]$$

$$\int_0^L dy\, S_n(y) = \sqrt{\frac{\pi R}{2}}\, \varepsilon_n$$

These properties are important at Dirichlet Higgs Model.

- KK-Parity: (even) (odd)

Background Field Method

$$\Phi = \Phi^c + \Phi^q$$

classical part(VEV) quantum fluctuation around VEV

Two (gauge) transformations:

Redefinition of VEV

- Background gauge transformation

  $\delta\Phi^c = ig\varepsilon\Phi^c$, $\delta\Phi^q = ig\varepsilon\Phi^q$, (others)

- True gauge transformation

  $\delta\Phi^c = 0$, $\delta\Phi^q = ig\varepsilon(\Phi^c+\Phi^q)$, (others)

The bulk action is invariant under the transformations without new surface term.

→BRST

(By use of $\boxed{\Phi^c = \text{Const}}$ and $\boxed{\Phi^q|_{bd} = 0.}$)

Classical solution of Φ^c Ordinary Dirichlet BC

Our Proposal

K.N., Oda, 2010

True gauge transformation → BRST

Gauge fixing term:

$$S_{\rm GF} = \int d^4x \int_0^L dy \left[-\frac{1}{2\xi} f^a f^a \right],$$

$$f^a = \partial_\mu A^{a\mu} + \xi \partial_5 A^{a5} + \xi \left(ig \left(\Phi^{\rm q} \right)^\dagger T^a \Phi^{\rm c} + {\rm h.c.} \right)$$

$s\Phi^c = 0$, $s\Phi^q = ig\omega(\Phi^c + \Phi^q)$, (others)

(s:BRST operator, ω:ghost)

We can proof that

$s(S + S_{GF} + S_{ghost}) = 0.$

(S: Initial action)

This model is well defined as quantum theory!

BRST Transformation for scalar

- In 5D (Bulk) picture (Abelian case)

$$s^2\Phi^q = i(s\omega)(\Phi^c + \Phi^q) - i\omega(s\Phi^q)$$

$$= -\omega^2(\Phi^c + \Phi^q) + \omega^2(\Phi^c + \Phi^q) = 0$$

Nilpotent!

$$s\Phi^q = ig\boxed{\omega(\Phi^c}+\Phi^q)$$

$$s\Phi^q|_{bd} = ig\omega(\Phi^c+\Phi^q)|_{bd} = ig\boxed{\omega\Phi^c|_{bd}}$$

Neumann-like

What does it mean?

In 4D (KK) picture (Abelian case)

$$s\Phi^{q}_{n}(x) = ig\Phi^{c}\sum_{\ell}\omega_\ell(x)\,\mathrm{CS}_{\ell,n} + ig\sum_{\ell,m}\omega_\ell(x)\Phi^{q}_{m}(x)\,\mathrm{CSS}_{\ell,mn}$$

$s^2\Phi^q{}_n = 0$

Nilpotent!

just numbers
(overlap integrals of
mode functions)

- No $\Phi^q{}_0$ → $s\Phi^q{}_0 = 0$
- $\Phi^c\omega_l$ performs like Neumann.

→ On an interval, mode functions act like as non-orthonormal.

$$\mathrm{CS}_{n,m} := \int_0^L dy\, C_n(y) S_m(y) = \frac{1}{4}\left[-\varepsilon_{n-m} + \varepsilon_{n+m}\right]$$

Natural properties on an interval.

To answer the second question

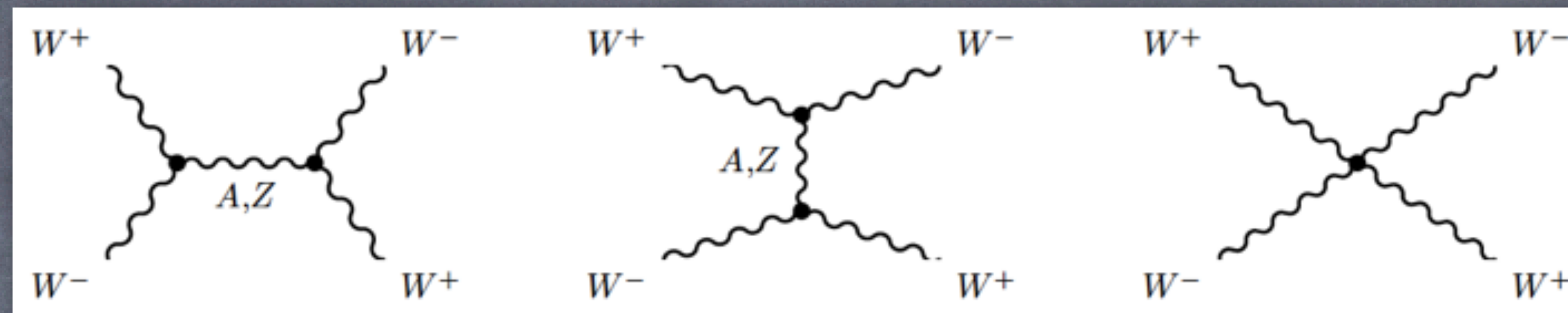
1. EWSB by b.c. $\varphi|_{bd} = \text{Const}$ looks explicit (well-defined as quantum theory?).
2. Who unitarizes the $W_L W_L$-scattering?

$W_L W_L$–Scattering in SM

Unitarity violation occurs if M grows as: <u>$M \propto s$</u>.

(M: scattering amplitude, s: (total energy)^2)

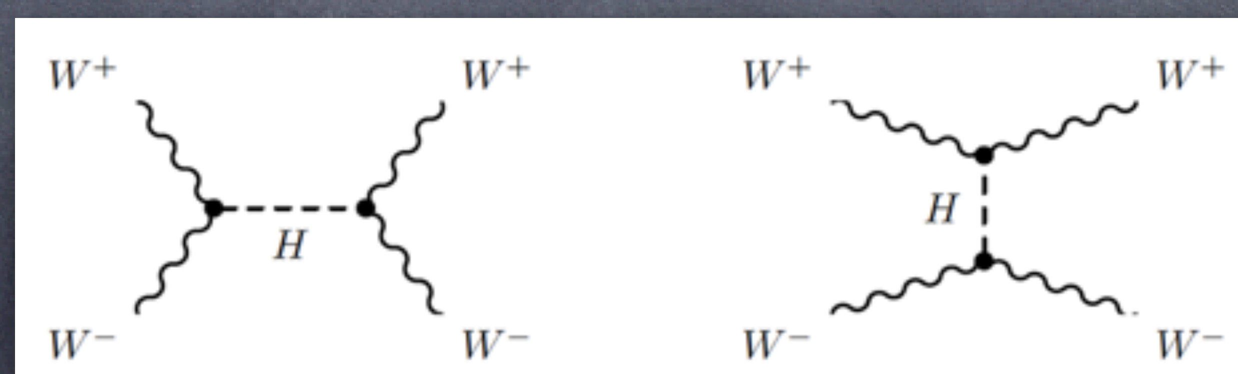
Gauge boson contribution:



Unitarity Violation!

$$\mathcal{M}^{\text{SM gauge only}}_{W_L^+W_L^-\to W_L^+W_L^-} = \frac{s(1-\cos\theta)}{2v_{\text{EW}}^2} + \mathcal{O}(s^0),$$

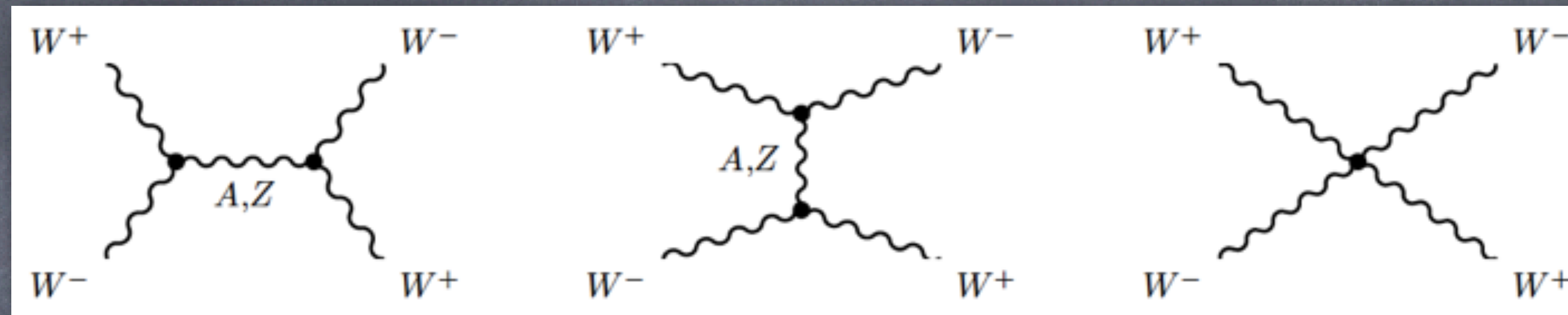
Higgs contribution:



In the SM, Unitarity Violation do not occur because of Higgs.

$W_L W_L$–Scattering in Our Model

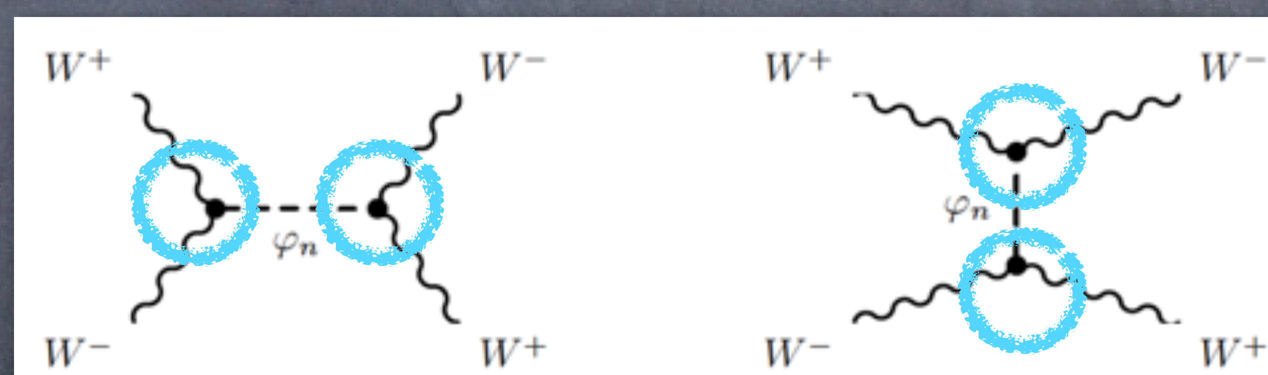
Gauge boson contribution:



$$\mathcal{M}^{\text{SM gauge only}}_{W_L^+ W_L^- \to W_L^+ W_L^-} = \frac{s\,(1-\cos\theta)}{2v_{\text{EW}}^2} + \mathcal{O}(s^0),$$

There is no (Zero-mode) Higgs contribution...

KK Higgs contribution:



KK number violating

$$\int_0^L dy\, S_n(y) = \sqrt{\frac{\pi R}{2}}\,\varepsilon_n$$

- Only n=1,3,5,... contributes. (∵KK-parity)

5D Nature Appears

K.N., Oda, 2010

Haba, Oda, Takahashi, 09, 10

In High energy limit,

$$\mathcal{M}^{\text{KK Higgs exchange}}_{W_L^+W_L^-\to W_L^+W_L^-} \to -\frac{s(1+\cos\theta)}{2v_{\text{EW}}^2} - \frac{\sqrt{2s}}{v_{\text{EW}}^2\pi R}\left(\sqrt{2}+\sqrt{1-\cos\theta}\right) + \mathcal{O}(s^0)$$

Cancels with SM-gauge contribution

In our model, $M \propto \sqrt{s}$.

- This is five-dimensional effect. $[g_5]=-1/2$
  (Naive Dimensional Analysis: $M \sim {g_5}^2 \sqrt{s}$)
- Unitarity limit is lower than that of the SM.
  (Natural Result)

m_{KK} = 430-500GeV (S-T favored),

we get the limit as 6.7-5.7 TeV.

Summary

- EWSB without Higgs potential
  - ★ Non-zero Dirichlet theory is consistent: BRST symmetry
  - ★ KK scale at 500GeV
- WW unitarized by infinite KK modes
  - ★ 5D gauge, perturbative up to 5～6TeV
- What is the longitudinal d.o.f. of zero mode gauge bosons?

Thank You
For
Your Attention