余剰次元UV問題とmarginal演算子の発見

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余剰次元

場の理論

繰り込み不可能

4次元

$$\lambda\phi^4$$

$$[\phi] = [M^1]$$

$$[\lambda] = [M^0]$$

d次元 (d > 4)

$$\lambda\phi^4$$

$$[\phi] = [M^{(d-2)/2}]$$

$$[\lambda] = [M^{(4-d)}]$$

$$\sum_{\ell=0}^{\infty} c_\ell \ \lambda^\ell \partial^{\ell(d-4)} \ \lambda\phi^4$$ etc

無限個の係数

$$[\lambda^\ell \partial^{\ell(d-4)}] = [M^0]$$

Higher derivative

発散項
高エネルギー効果

取り組む立場

- 弦理論か何か、別のUV completion
- 係数に依存しない物理量同士の関係式
- 形式の発展、4次元理論の手掛かり

etc

問題

余剰次元を持つ場の量子論(場の理論＋量子論)は本当はできるのか？

Wilson流繰り込み

Wilson流繰り込み　　Peskin-Schroeder textbookの方法

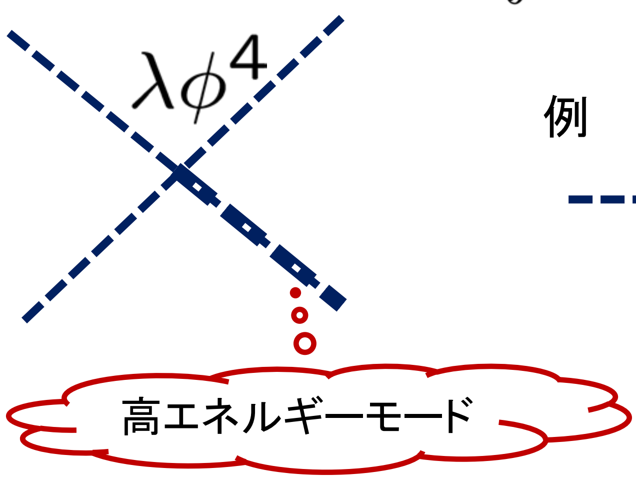
生成汎関数　$$Z = \int [\mathcal{D}\phi] \exp\left(- \int \mathcal{L} \right)$$

高エネルギーモードを積分

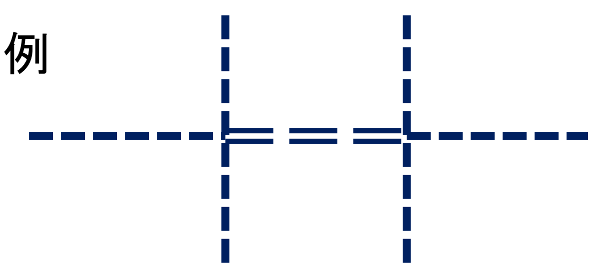
$$b\Lambda \leq |p| < \Lambda$$

$$b < 1$$

$$Z = \int [\mathcal{D}\phi]_{b\Lambda} \exp\left(- \int \mathcal{L}_{\rm eff} \right)$$



例



6点結合　$\frac{1}{\mu_A^2}\lambda^2\phi^6$

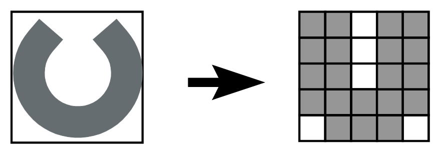
領域Aの
自由度
は積分

Peskin-Schroeder textbook

rescaling

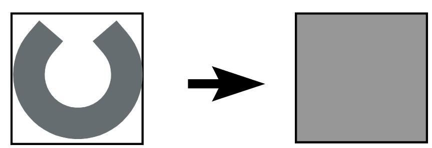
$$x' = xb$$
$$p' = p/b$$
$$|p'| < \Lambda$$

5x5 の環



単位升目が1x1

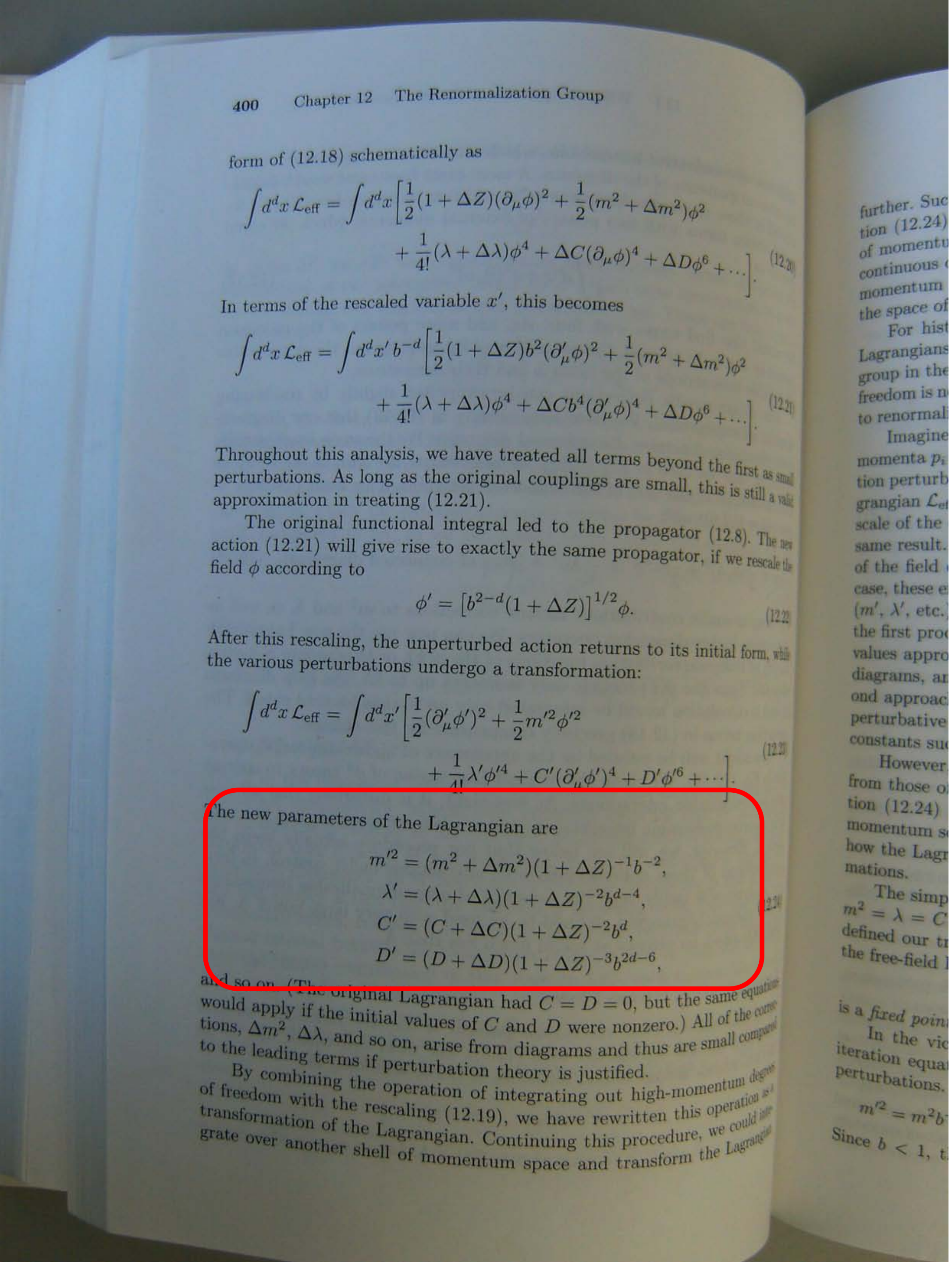
$$x' = (1/5)x$$



x

単位升目が5x5

$b^{負}$	relevant
$b^{正}$	irrelevant
b^{0}	marginal



400 Chapter 12 The Renormalization Group

form of (12.18) schematically as

$$\int d^dx\,\mathcal{L}_{\text{eff}} = \int d^dx\Big[\frac{1}{2}(1+\Delta Z)(\partial_\mu\phi)^2+\frac{1}{2}(m^2+\Delta m^2)\phi^2 + \frac{1}{4!}(\lambda+\Delta\lambda)\phi^4+\Delta C(\partial_\mu\phi)^4+\Delta D\phi^6+\cdots\Big]. \quad (12.20)$$

In terms of the rescaled variable x', this becomes

$$\int d^dx\,\mathcal{L}_{\text{eff}} = \int d^dx'\,b^{-d}\Big[\frac{1}{2}(1+\Delta Z)b^2(\partial'_\mu\phi)^2+\frac{1}{2}(m^2+\Delta m^2)\phi^2 + \frac{1}{4!}(\lambda+\Delta\lambda)\phi^4+\Delta C b^4(\partial'_\mu\phi)^4+\Delta D\phi^6+\cdots\Big]. \quad (12.21)$$

Throughout this analysis, we have treated all terms beyond the first as small perturbations. As long as the original couplings are small, this is still a valid approximation in treating (12.21).

The original functional integral led to the propagator (12.8). The new action (12.21) will give rise to exactly the same propagator, if we rescale the field ϕ according to

$$\phi' = \left[b^{2-d}(1+\Delta Z)\right]^{1/2}\phi. \quad (12.22)$$

After this rescaling, the unperturbed action returns to its initial form, while the various perturbations undergo a transformation:

$$\int d^dx\,\mathcal{L}_{\text{eff}} = \int d^dx'\Big[\frac{1}{2}(\partial'_\mu\phi')^2+\frac{1}{2}m'^2\phi'^2 + \frac{1}{4!}\lambda'\phi'^4+C'(\partial'_\mu\phi')^4+D'\phi'^6+\cdots\Big]. \quad (12.23)$$

The new parameters of the Lagrangian are

$$m'^2 = (m^2+\Delta m^2)(1+\Delta Z)^{-1}b^{-2},$$
$$\lambda' = (\lambda+\Delta\lambda)(1+\Delta Z)^{-2}b^{d-4},$$
$$C' = (C+\Delta C)(1+\Delta Z)^{-2}b^{d},$$
$$D' = (D+\Delta D)(1+\Delta Z)^{-3}b^{2d-6}, \quad (12.24)$$

and so on. (The original Lagrangian had $C = D = 0$, but the same equations would apply if the initial values of C and D were nonzero.) All of the corrections, Δm^2, $\Delta\lambda$, and so on, arise from diagrams and thus are small compared to the leading terms if perturbation theory is justified.

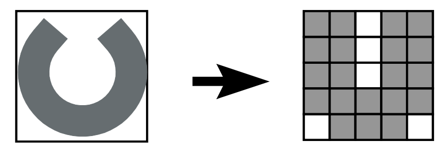
By combining the operation of integrating out high-momentum degrees of freedom with the rescaling (12.19), we have rewritten this operation as a transformation of the Lagrangian. Continuing this procedure, we could integrate over another shell of momentum space and transform the Lagrangian

Peskin-Schroeder textbook

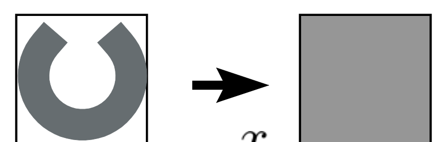
rescaling

$$x' = xb$$
$$p' = p/b$$
$$|p'| < \Lambda$$

5x5 の環　　単位升目が1x1



$$x' = (1/5)x$$



単位升目が5x5

$b^{負}$ relevant

$b^{正}$ irrelevant

b^{0} marginal

renormalization group transformation.

This criterion is met whenever the initial conditions for the renormalization group flow are adjusted so that the trajectory passes very close to a fixed point. In principle, the flow could begin far away, along the direction of an irrelevant operator. The original value of m^2 need not be particularly small, as long as this original value is canceled by corrections arising from the diagrammatic contributions to $\mathcal{L}_{\text{eff}}$. Thus we could imagine constructing a scalar field theory in $d > 4$ by writing a complicated nonlinear Lagrangian, but adjusting the original m^2 so the trajectory that begins at this Lagrangian eventually passes close to the free-field fixed point $\mathcal{L}_0$. In this case, the effective theory at momenta small compared to the cutoff should be extremely simple: It will be a free field theory with negligible nonlinear interaction. As will be discussed in the next chapter, this remarkable prediction has been verified in mathematical models of magnetic systems in more than four dimensions: Even though the original model is highly nonlinear, the correlation function of spins near the phase transition has the free-field form given by the higher-dimensional analogue of Eq. (12.2).

Next consider the case $d = 4$. For this case, Eq. (12.26) does not give enough information to tell us whether the ϕ^4 interaction is important or unimportant at large distances. So we must go back to the complete transformation law (12.24). The leading contribution to $\Delta\lambda$ is given by Eq. (12.15). The leading contribution to ΔZ is of order λ^2 and can be neglected. (This is just what happened with the first correction to δ_Z in Section 10.2.) Thus we find the transformation

$$\lambda' = \lambda - \frac{3\lambda^2}{16\pi^2}\log(1/b). \qquad (12.28)$$

This says that λ slowly decreases as we integrate out high-momentum degrees of freedom.

The diagram contributing to the correction $\Delta\lambda$ has the same structure as the one-loop diagram

d > 4

free field theory

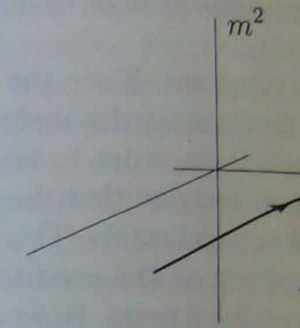


Figure 12.2. Renormalization gr
scalar field theory: $d < 4$.

The theory thus flows away from t
grees of freedom; at large distances
important. However, when λ beco
as that displayed in Eq. (12.28) m
specific effect in $d < 4$, we find the

$$\lambda' = \Big[\lambda - \frac{3\lambda^2}{(4\pi)^{d/2}}$$

This equation implies that there is
to rescaling is compensated by the
At this value, λ is unchanged when
corresponding Lagrangian is a secon
flow. In the limit $d \to 4$, the flow (12
point merges with the free field fixe
new fixed point will share with $\mathcal{L}_0$ th
increased by the iteration. Then the
near the new fixed point, so that th
the form shown in Fig. 12.2.

Peskin-Schroeder textbook

rescaling

$$x' = xb, \qquad y' = yb$$

を仮定した

rescaling

$$x' = xb, \qquad y' = yb$$

を仮定した

Randall-Sundrum metric

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - \frac{1}{k^2}dz^2\right)$$

$x \to xb$

k は曲率

$$\frac{1}{z^2}\left(b^2\eta_{\mu\nu}dx^\mu dx^\nu - \frac{1}{k^2}dz^2\right)$$

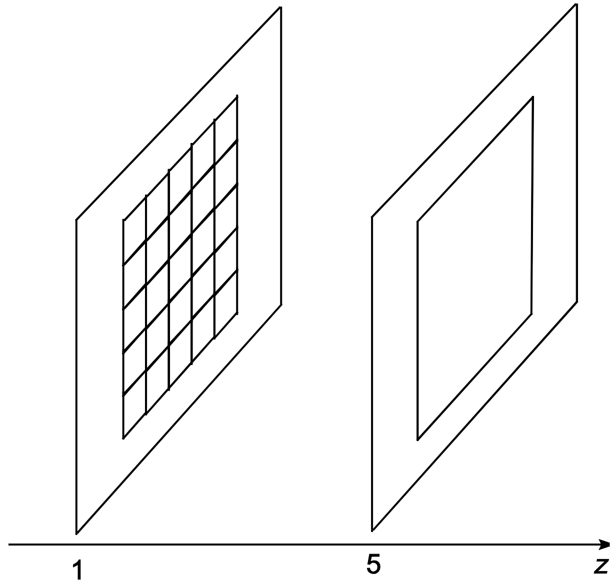
$z \to z/b$ と同じ効果

rescaling $x' = xb, \quad z' = z/b$

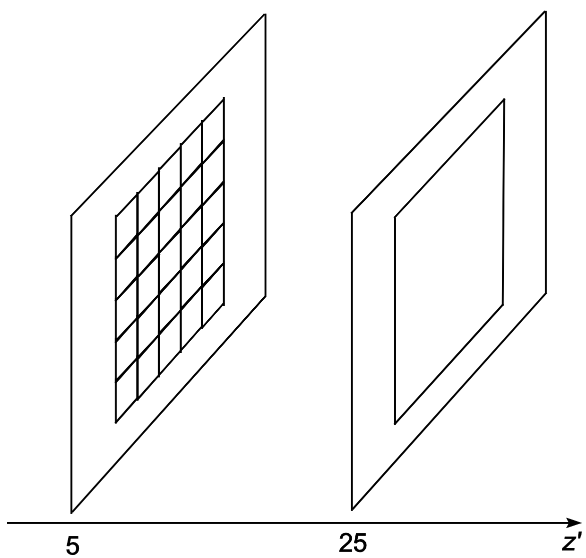
rescaling

$$x' = xb,$$

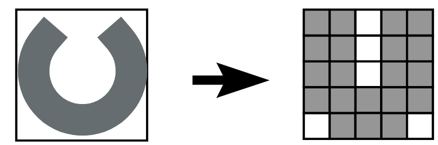
$$z' = z/b$$



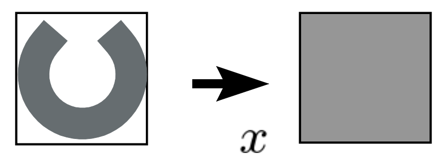
$$z' = 5z$$



5x5 の環　単位升目が1x1



$$x' = (1/5)x$$



単位升目が5x5

$$\int d^4x dz \sqrt{\det g_{MN}} \left[\frac{1}{2} Z (\partial_\mu \phi)(\partial_\nu \phi) g^{\mu\nu} + \frac{1}{2} Z_5 (\partial_z \phi)^2 g^{zz} \right.$$
$$\left. + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 + C((\partial_\mu \phi)(\partial_\nu \phi) g^{\mu\nu})^2 + C_5((\partial_z \phi)^2 g^{zz})^2 + D\phi^6 \right]$$

rescaling $x' = xb,$ $z' = z/b$

$$\int d^4x' \frac{dz'}{kz'} \left[\frac{1}{2z'^2} (\partial'_\mu \phi')^2 - \frac{k^2}{2z'^2} Z'_5 (\partial'_z \phi')^2 \right.$$
$$\left. + \frac{1}{2z'^4} m'^2 \phi'^2 + \frac{1}{4! z'^4} \lambda' \phi'^4 + C'(\partial'_\mu \phi')^4 + C'_5 k^4 (\partial'_z \phi')^4 + \frac{1}{z'^4} D' \phi'^6 \right]$$

$$\phi' = b^{-2} Z^{1/2} \phi, \qquad Z'_5 = b^{-4} Z^{-1} Z_5, \qquad m'^2 = b^{-4} Z^{-1} m^2, \qquad \lambda' = b^0 Z^{-2} \lambda,$$
$$C' = b^8 Z^{-2} C, \qquad C'_5 = b^0 Z^{-2} C_5, \qquad D' = b^4 Z^{-3} D$$

NU arXiv:1008.0487[hep-ph]

$$\int d^4x dz \sqrt{\det g_{MN}} \Big[\frac{1}{2} Z(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu} + \frac{1}{2} Z_5 (\partial_z\phi)^2 g^{zz} + \frac{1}{2} m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 + C((\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu})^2 + C_5((\partial_z\phi)^2 g^{zz})^2 + D\phi^6\Big]$$

rescaling $x' = xb,$ $z' = z/b$

$$\int d^4x' \frac{dz'}{kz'} \Big[\frac{1}{2z'^2}(\partial'_\mu\phi')^2 - \frac{k^2}{2z'^2} Z'_5 (\partial'_z\phi')^2 + \frac{1}{2z'^4} m'^2\phi'^2 + \frac{1}{4! z'^4}\lambda'\phi'^4 + C'(\partial'_\mu\phi')^4 + C'_5 k^4 (\partial'_z\phi')^4 + \frac{1}{z'^4} D'\phi'^6\Big]$$

$\phi' = b^{-2} Z^{1/2}\phi,$ $Z'_5 = b^{-4} Z^{-1} Z_5,$ $m'^2 = b^{-4} Z^{-1} m^2,$ $\lambda' = b^0 Z^{-2}\lambda,$

$C' = b^8 Z^{-2} C,$ $C'_5 = b^0 Z^{-2} C_5,$ $D' = b^4 Z^{-3} D$

Relevant operator b 負

Mass term

NU arXiv:1008.0487[hep-ph]

$$\int d^4x dz \sqrt{\det g_{MN}} \Big[\frac{1}{2}Z(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu} + \frac{1}{2}Z_5(\partial_z\phi)^2 g^{zz} + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 + C((\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu})^2 + C_5((\partial_z\phi)^2 g^{zz})^2 + D\phi^6\Big]$$

rescaling $x' = xb,$ $z' = z/b$

$$\int d^4x' \frac{dz'}{kz'} \Big[\frac{1}{2z'^2}(\partial'_\mu\phi')^2 - \frac{k^2}{2z'^2}Z'_5(\partial'_z\phi')^2 + \frac{1}{2z'^4}m'^2\phi'^2 + \frac{1}{4!z'^4}\lambda'\phi'^4 + C'(\partial'_\mu\phi')^4 + C'_5 k^4(\partial'_z\phi')^4 + \frac{1}{z'^4}D'\phi'^6\Big]$$

$$\phi' = b^{-2}Z^{1/2}\phi, \quad Z'_5 = b^{-4}Z^{-1}Z_5, \quad m'^2 = b^{-4}Z^{-1}m^2, \quad \lambda' = b^0 Z^{-2}\lambda,$$
$$C' = b^8 Z^{-2} C, \quad C'_5 = b^0 Z^{-2} C_5, \quad D' = b^4 Z^{-3} D$$

Irrelevant operator b正

Higher 4-derivative, 6点

NU arXiv:1008.0487[hep-ph]

$$\int d^4x dz \sqrt{\det g_{MN}} \left[\frac{1}{2} Z (\partial_\mu \phi)(\partial_\nu \phi) g^{\mu\nu} + \frac{1}{2} Z_5 (\partial_z \phi)^2 g^{zz} + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 + C((\partial_\mu \phi)(\partial_\nu \phi) g^{\mu\nu})^2 + C_5 ((\partial_z \phi)^2 g^{zz})^2 + D\phi^6\right]$$

rescaling $x' = xb,$ $z' = z/b$

$$\int d^4x' \frac{dz'}{kz'} \left[\frac{1}{2z'^2} (\partial'_\mu \phi')^2 - \frac{k^2}{2z'^2} Z'_5 (\partial'_z \phi')^2 + \frac{1}{2z'^4} m'^2 \phi'^2 + \frac{1}{4! z'^4} \lambda' \phi'^4 + C' (\partial'_\mu \phi')^4 + C'_5 k^4 (\partial'_z \phi')^4 + \frac{1}{z'^4} D' \phi'^6\right]$$

$$\phi' = b^{-2} Z^{1/2} \phi, \quad Z'_5 = b^{-4} Z^{-1} Z_5, \quad m'^2 = b^{-4} Z^{-1} m^2, \quad \lambda' = b^0 Z^{-2} \lambda,$$
$$C' = b^8 Z^{-2} C, \quad C'_5 = b^0 Z^{-2} C_5, \quad D' = b^4 Z^{-3} D$$

Marginal operator b^0

4点，Higher z-derivative

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NU arXiv:1002.1904[hep-ph]

Brane 項

$$\int d^4x dz \sqrt{\det(g_{KL})}\, \mathcal{L}_{\text{bulk}}$$
$$+ \int d^4x dz \sqrt{\det(g_{KL})}\, \mathcal{L}_{\text{brane}}\, kz\delta(z-z_i)$$

rescaling

$$z\delta(z-z_i) \to b^0 z\delta(z-z_i)$$

のため

Bulk 項のrelevant, irrelevant, marginal の分類がそのままBrane項に適用できる

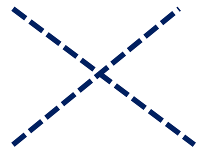
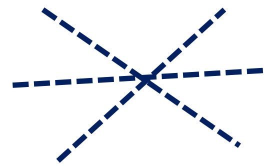
marginal

Brane kinetic term,　4点coupling

flat $x' = xb, y' = yb$

SUMMARY

warped $x' = xb, z' = z/b$

flat		warped
irrelevant		marginal
irrelevant		irrelevant
irrelevant		marginal
irrelevant		marginal
irrelevant		marginal
irrelevant	∂_μ higher derivative	irrelevant

free theory

interacting theory

ただし、 $g^{zz}\partial_z^2 = k^2 z^2 \frac{\partial^2}{\partial z^2} \to b^0$

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