Higgs Mechanism without Higgs Potential in an Extra Dimension

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Talk at EXTRA DIMENSION 2011 @ Osaka Univ., 2011/01/25, Makoto Sakamoto

Mysteries of Standard Model

- **Symmetry breaking?**

  Higgs mechanism cannot work unless $M_H^2 < 0.$
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▶ Fermion mass hierarchy?

Why do the fermions acquire hierarchical masses?

Purpose

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In the context of **5d gauge theories on an interval**, we clarify whether the mysteries of the Standard Model:

- ▶ **Symmetry breaking?**
- ▶ **Chiral theory?**
- ▶ **Fermion mass hierarchy?**

can be explained naturally or not.

Setting

▶ **5d U(1) gauge theory on an interval**

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- 5d U(1) gauge theory on an interval

- All fields live in the bulk.
- No boundary term
- No fine tuning with $M_H{}^2 > 0$
- General boundary conditions

  compatible with
  - ★ 5d gauge invariance
  - ★ the action principle

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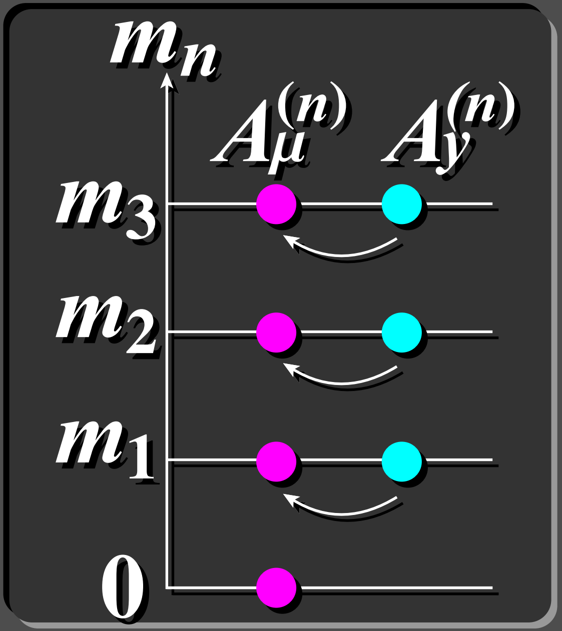
- **4d spectrum**



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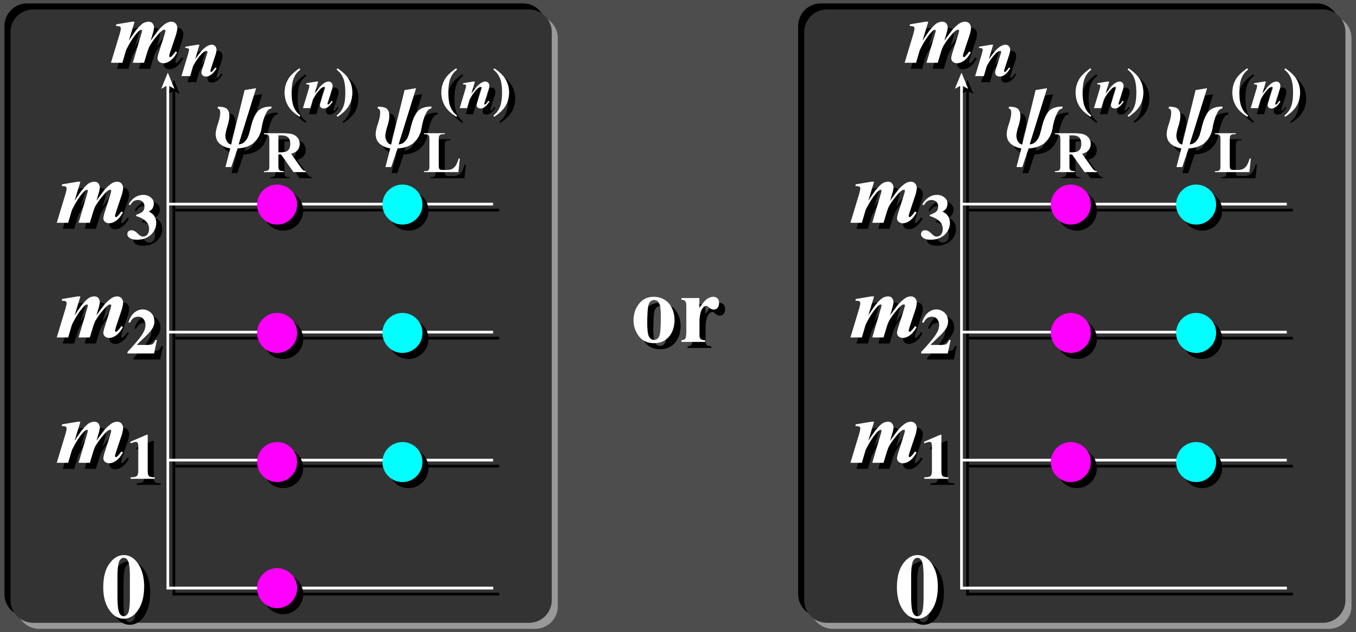

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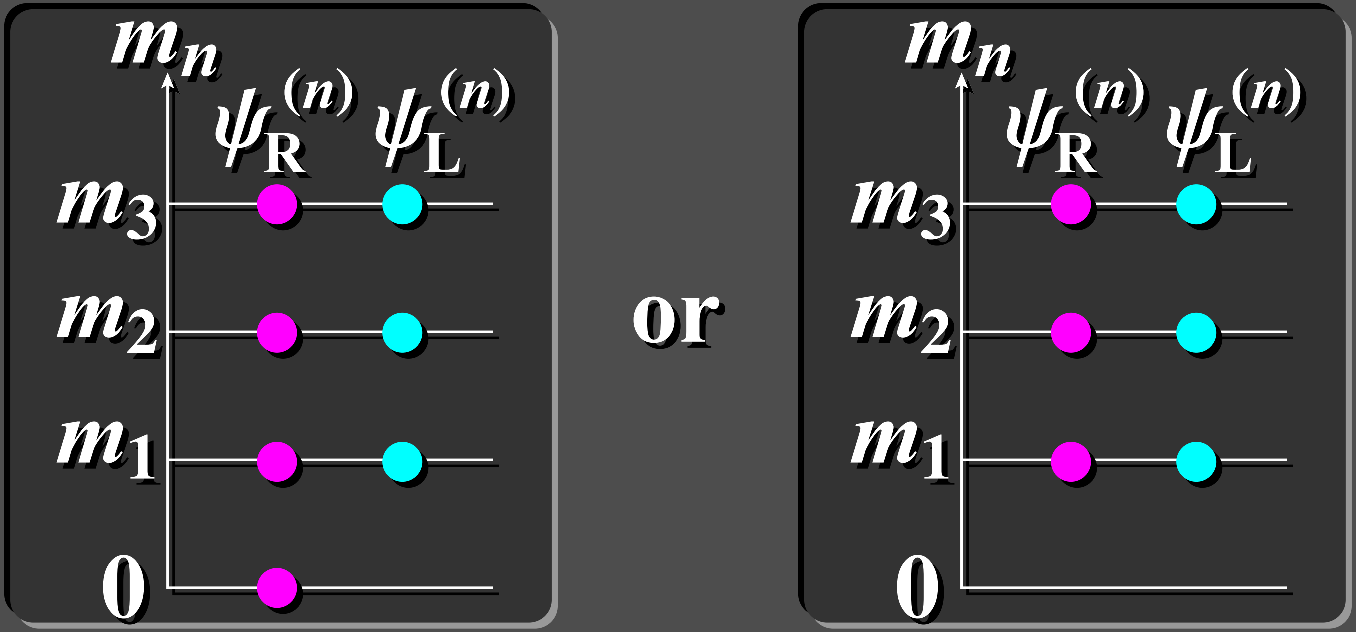
- 4d spectrum

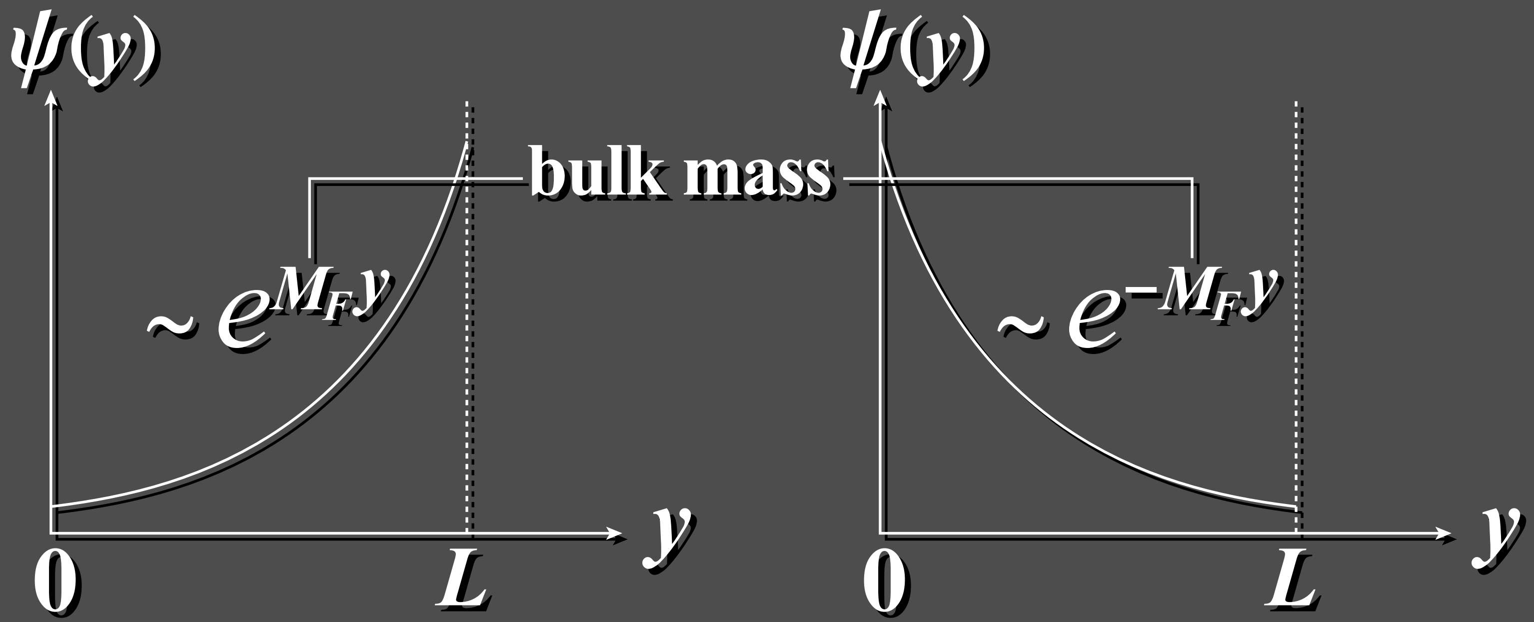


4d low energy effective theory at $E < 1/L$ will be described by a chiral theory, irrespective of the bulk mass.

Our Results – fermion sector –

► Localization

Every chiral zero mode is localized at one of the boundaries.



profiles of chiral zero modes

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- **Possible boundary conditions for $\Phi(y)$**

  two-parameter family:

$$\Phi(0) + L_+ \partial_y \Phi(0) = 0$$
$$\Phi(L) - L_- \partial_y \Phi(L) = 0$$

for $-\infty \leq L_\pm \leq \infty$

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- **Higgs mechanism without Higgs Potential**

  $<\Phi(y)>$ can be non-vanishing even when $M_H{}^2 > 0$.

- **Hierarchical fermion masses**

  If $<\Phi(y)> \neq 0$, $<\Phi(y)>$ inevitably depends on y.

$$m_{ij} \equiv \lambda_{ij}^{(5)} \int_0^L dy\, \bar{f}_i(y) f_j(y) <\Phi(y)>$$

localized fermion profiles

Our Results – scalar sector –

- ► **Non-trivial phase structure**

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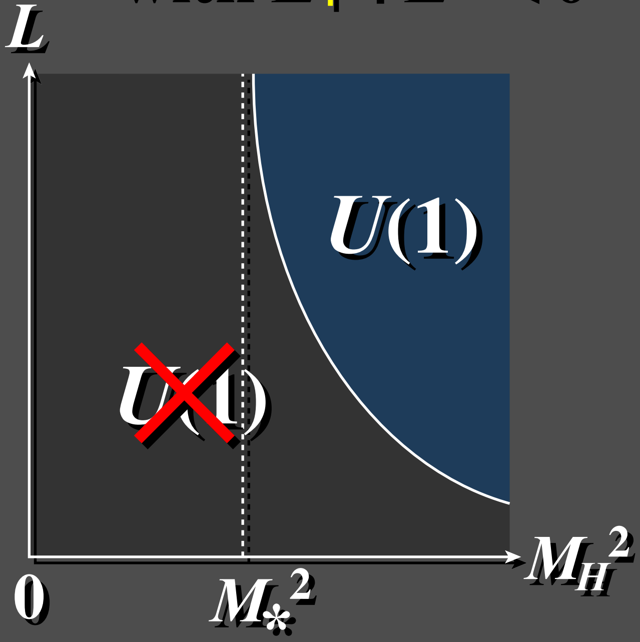
$L_\pm > 0$

or

$L_\pm > 0 > L_\mp$ with $L_+ + L_- < 0$



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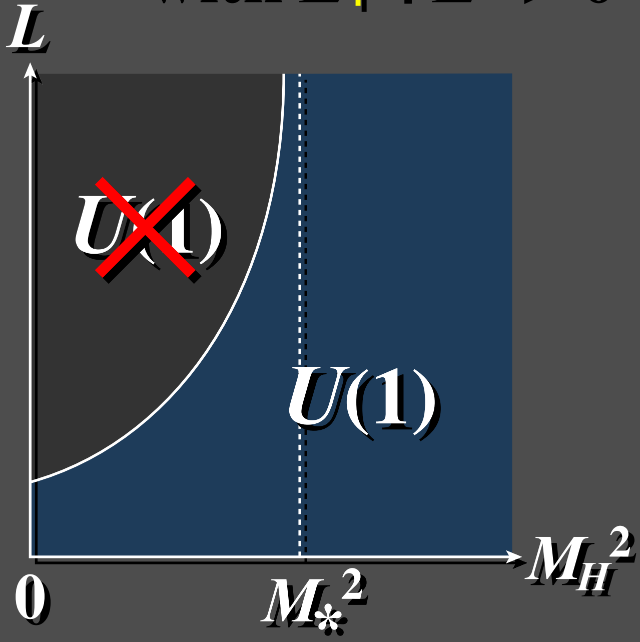

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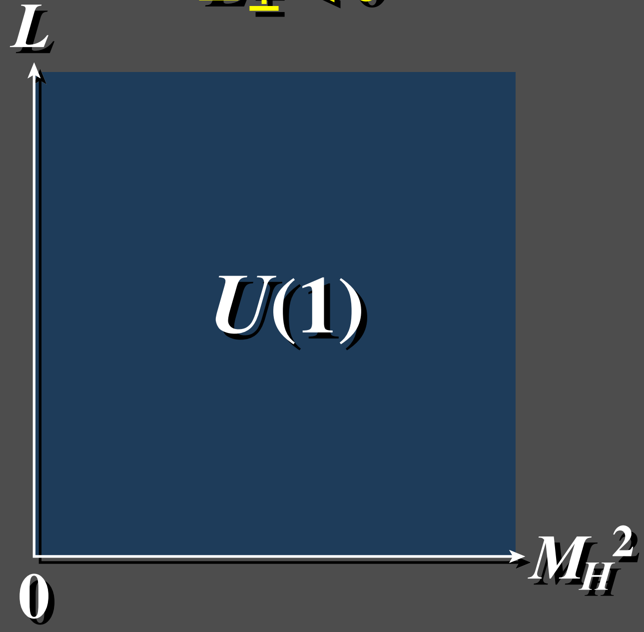

Classification of B.C.

We first clarify all possible boundary conditions on an interval to require any boundary conditions to be compatible with

quantum mechanics

- ▶ the action principle
- ▶ the 5d gauge invariance

unitarity

B.C. for a Fermion

- 4 possible boundary conditions

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$y=0$

$y=L$

$$\boxed{\begin{array}{c}(\partial_y+M_F)\,\Psi_+(0)=0\\ \Psi_-(0)=0\end{array}} \quad \text{or} \quad \left.\boxed{\begin{array}{c}\Psi_+(0)=0\\ (-\partial_y+M_F)\,\Psi_-(0)=0\end{array}}\right\}\begin{array}{l}N=2\\ \text{QM SUSY}\end{array}$$

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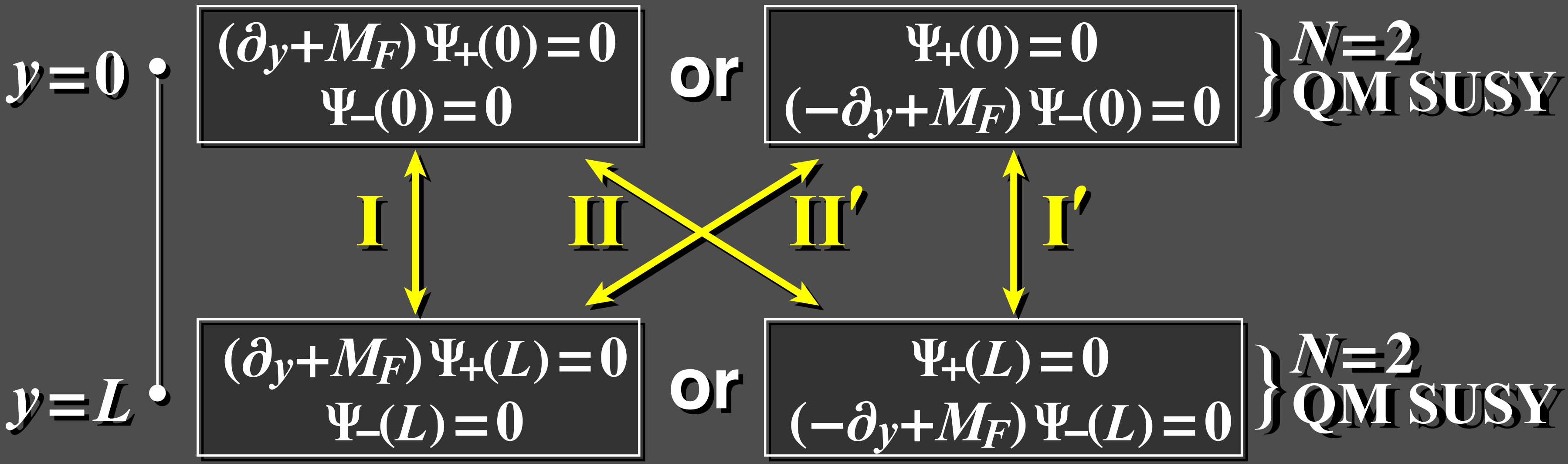

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or

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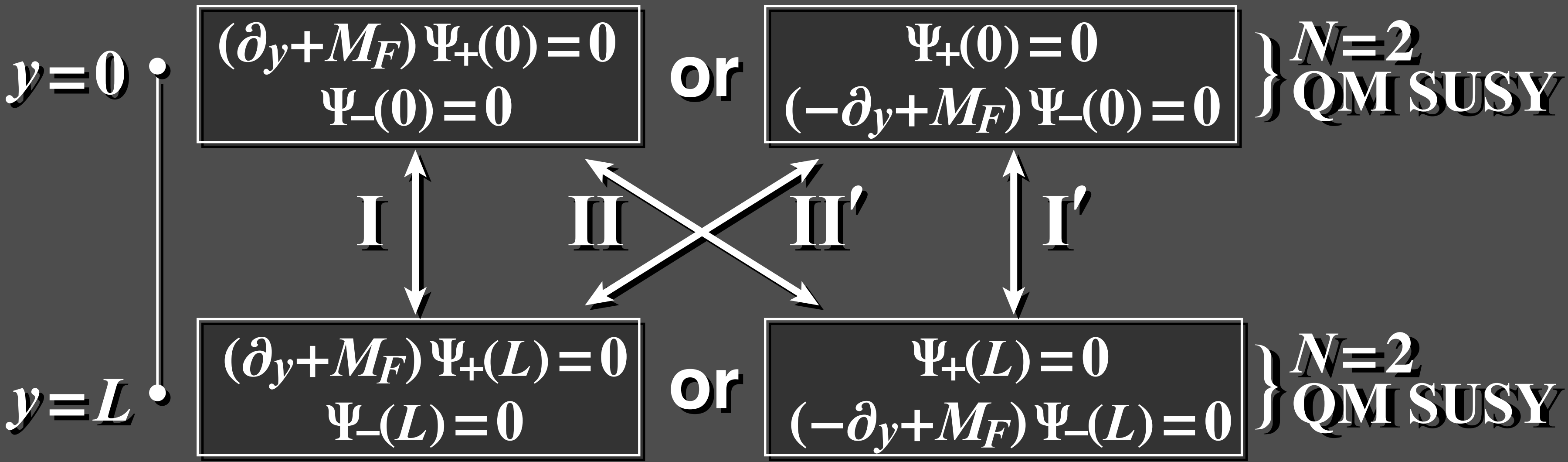

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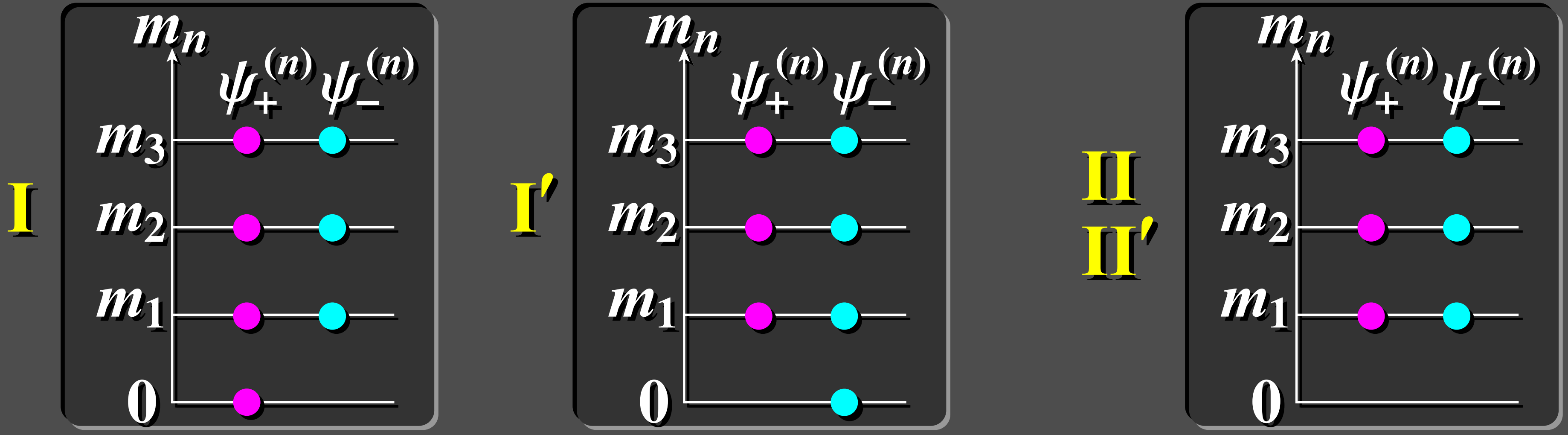
- ▶ **4d mass spectrum**



B.C. for U(1) Gauge Fields

- The 5d gauge invariance restricts the boundary condition for the U(1) gauge fields uniquely:

$$\begin{aligned} \partial_y A_\mu(x,y) &= 0, \\ A_y(x,y) &= 0, \quad \text{at } y = 0, L \end{aligned}$$

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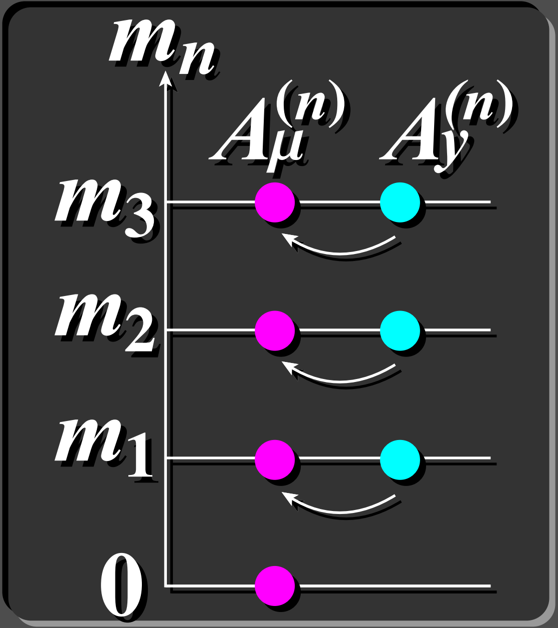

B.C. for a Scalar

$$S = \int d^4x \int_0^L dy \left\{ \Phi^* (\partial_\mu^2 + \partial_y^2) \Phi - V(\Phi) \right\}$$

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eq. of motion $\oplus$ constraint on b.c.

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2-parameter family of b.c.

$$\Phi(0) + L_+\partial_y\Phi(0) = 0$$
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for $-\infty \leq L_\pm \leq \infty$

Vacuum Expectation Value <$\Phi(y)$>

How to determine <$\Phi(y)$> ?

Vacuum Expectation Value $<\Phi(y)>$

How to determine $<\Phi(y)>$?

Minimize

$$E(\Phi) \equiv \int_0^L dy \left\{ \Phi^*(-\partial_y^2)\Phi + \underbrace{M_H^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4}_{V(\Phi)} \right\}$$

with the boundary conditions.

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$$\Updownarrow$$

Find solutions to $\delta E(\Phi)=0$, i.e.

$$-\partial_y^2\Phi(y) + M_H^2\Phi(y) + \frac{\lambda}{2}|\Phi(y)|^2\Phi(y) = 0$$

with the boundary conditions.

Vacuum Expectation Value $<\Phi(y)>$

The existence of any non-trivial solution to eq. of motion implies that $\Phi=0$ is *not* the vacuum but

$$<\Phi(y)> = \text{non-vanishing} \ \ \& \ y\text{-dependent}$$

Vacuum Expectation Value $\langle\Phi(y)\rangle$

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trivial solution

▶ $E(\Phi) \geq 0$ for constant Φ

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The problem turns out to be equivalent to solve $y \leftrightarrow t, \quad \Phi(y) \leftrightarrow x(t)$

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1-dim. classical mechanics

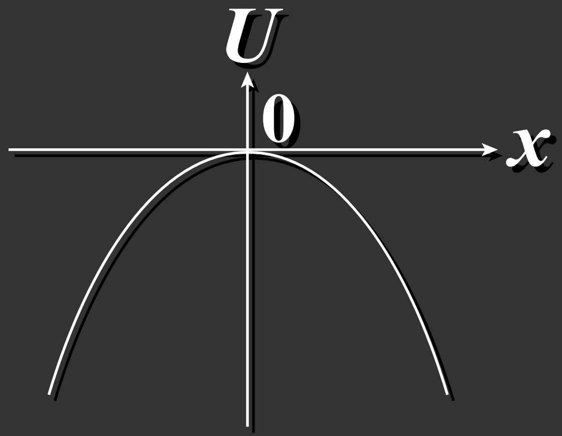
$$\frac{d^2x(t)}{dt^2} = -\frac{\partial U(x)}{\partial x}$$

$$U(x) = -V(x) = -M_H^2 x^2 - \frac{\lambda}{4} x^4$$

with the "initial" conditions

$$x(0) + L_+\dot{x}(0) = 0 \quad \text{at } t=0$$

$$x(L) - L_-\dot{x}(L) = 0 \quad \text{at } t=L$$

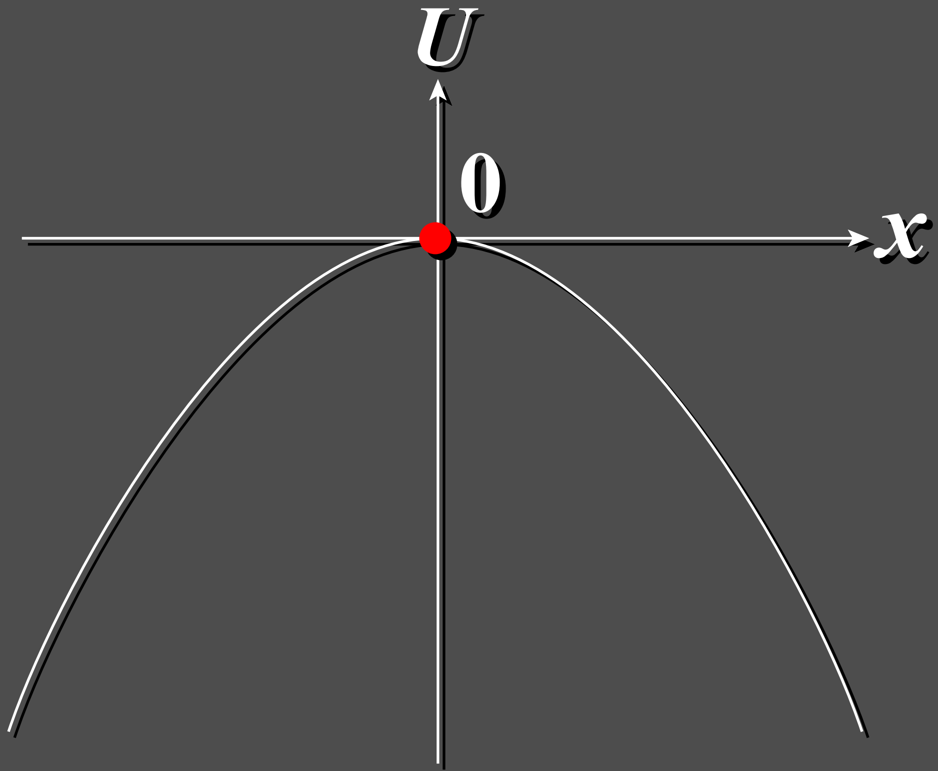


Vacuum Expectation Value $<\Phi(y)>$

No non-trivial solution for

Dirichlet b.c.: $x(0) = x(L) = 0 \quad (\Leftarrow L_{\pm} = 0)$

Neumann b.c.: $\dot{x}(0) = \dot{x}(L) = 0 \quad (\Leftarrow L_{\pm} = \infty)$



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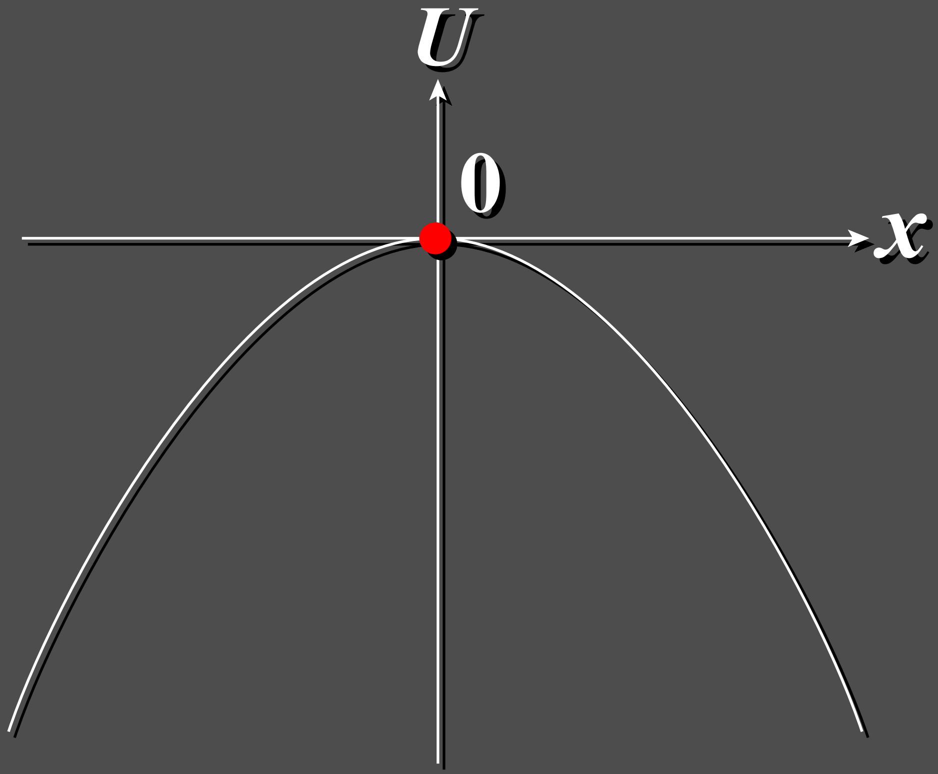

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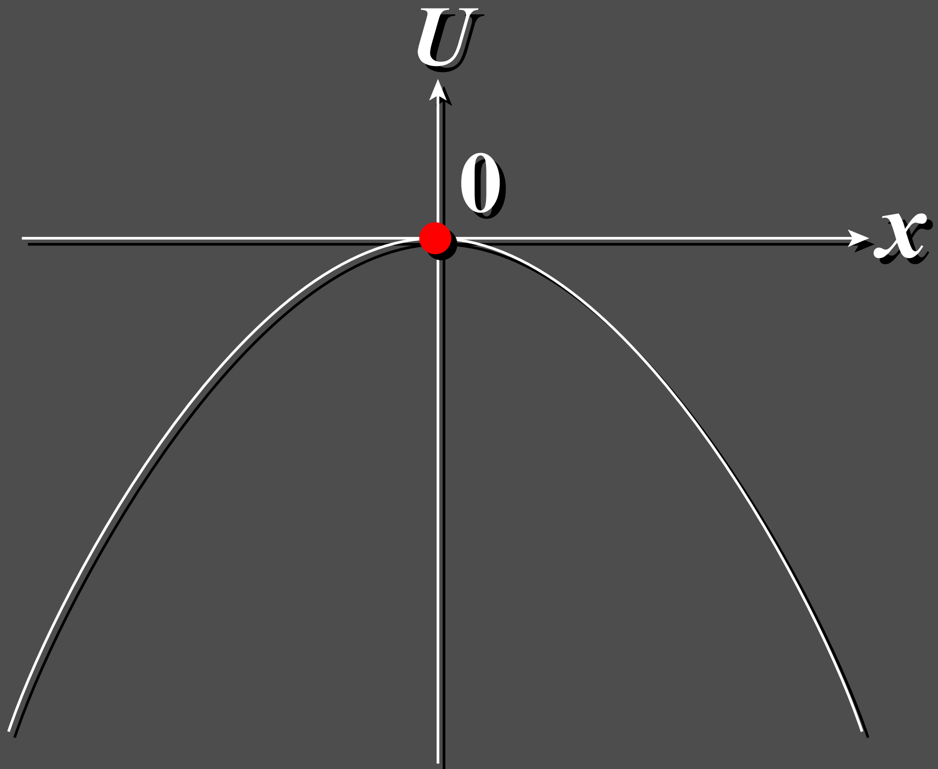

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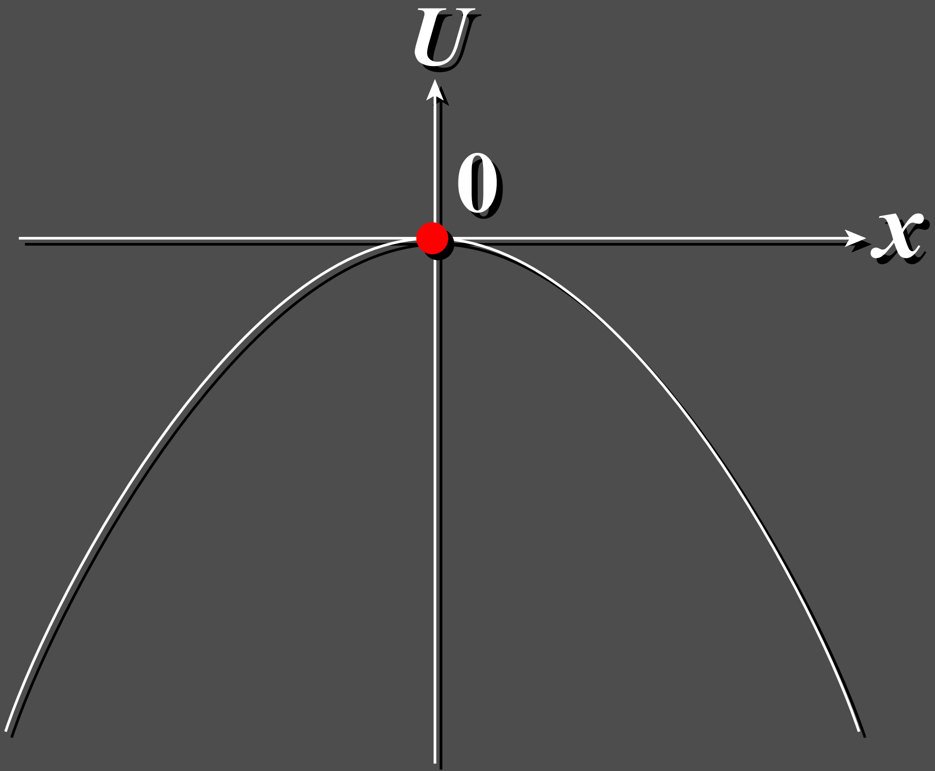

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symmetry unbroken!

Vacuum Expectation Value $<\Phi(y)>$

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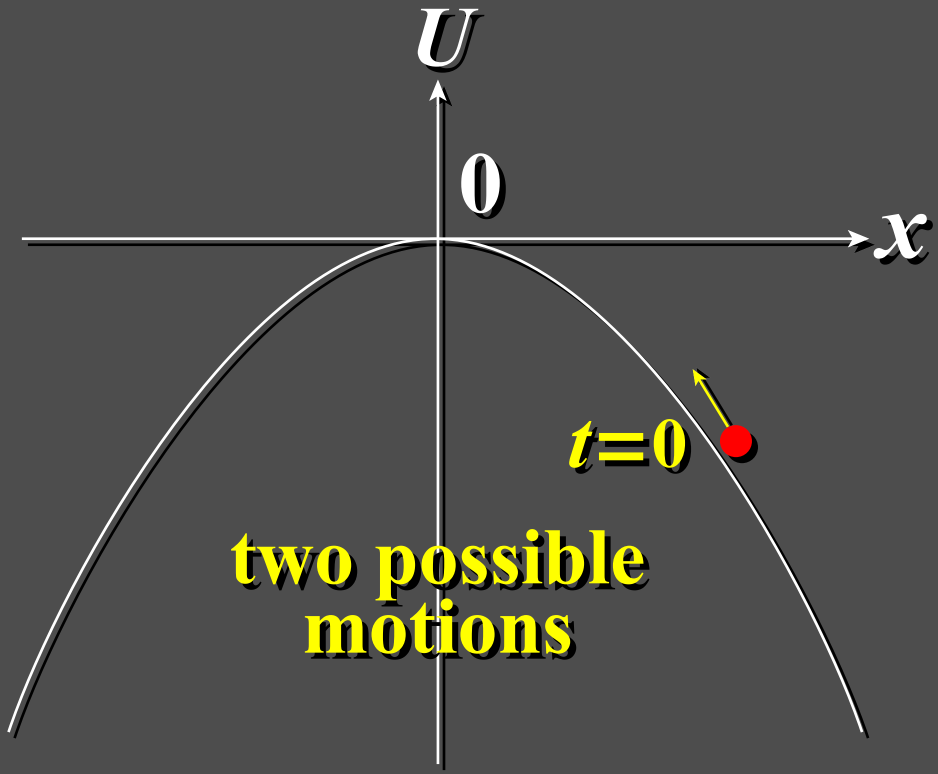

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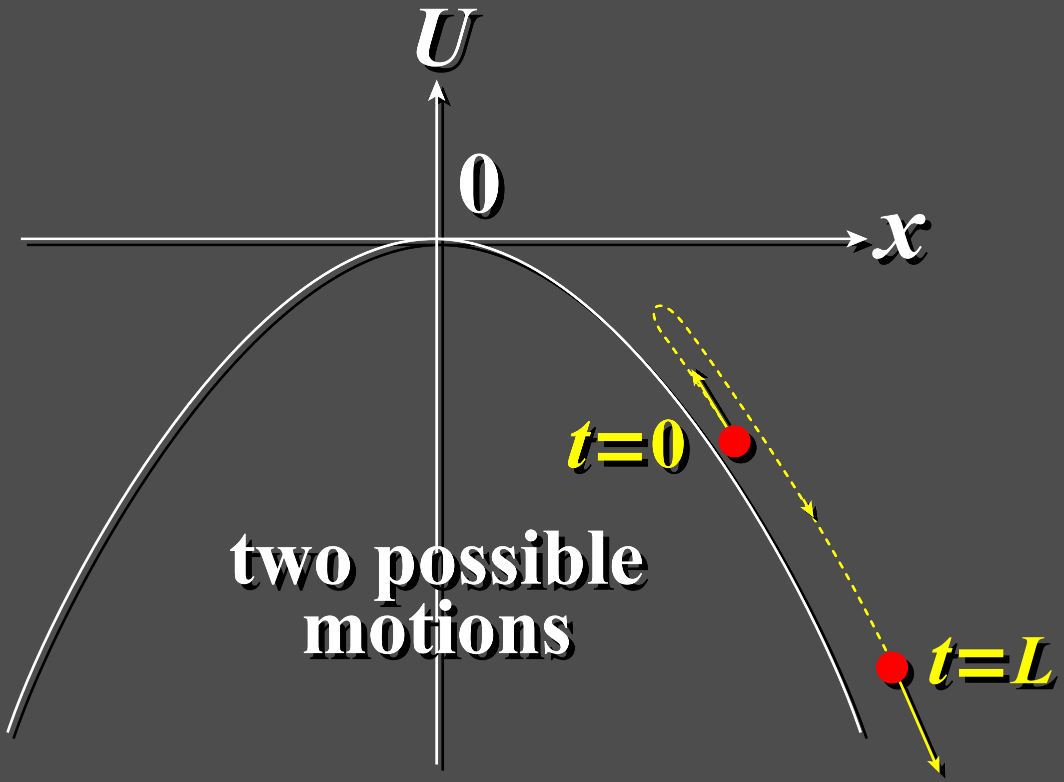

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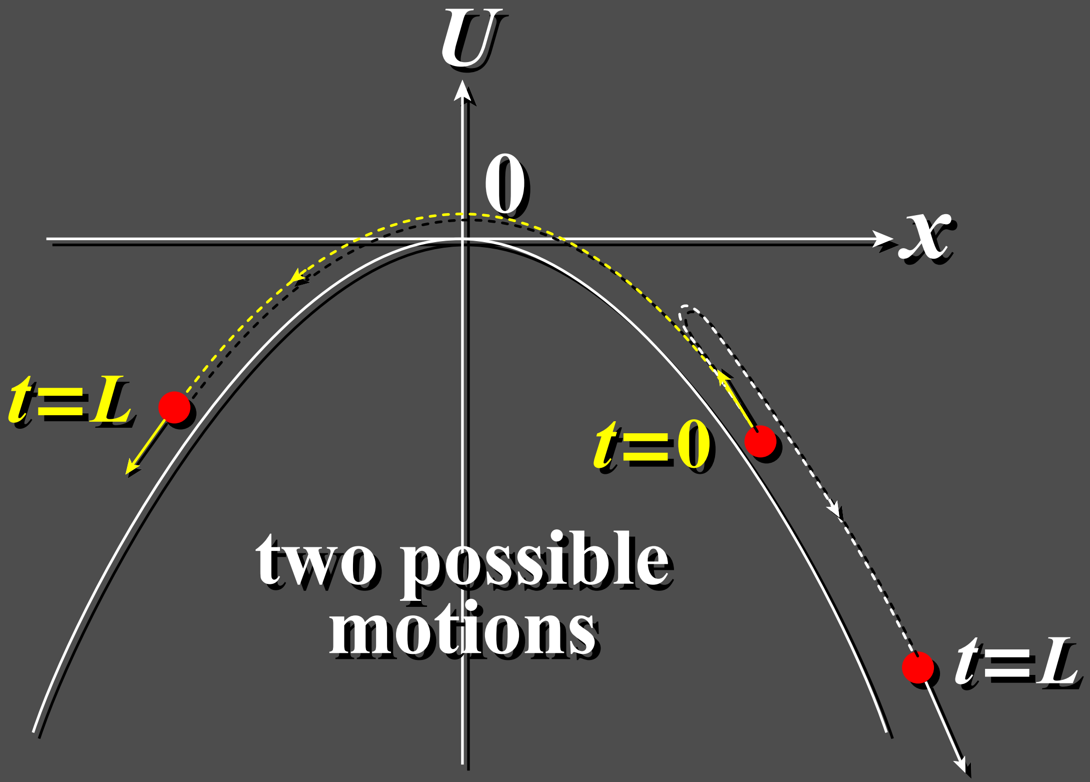

two possible motions

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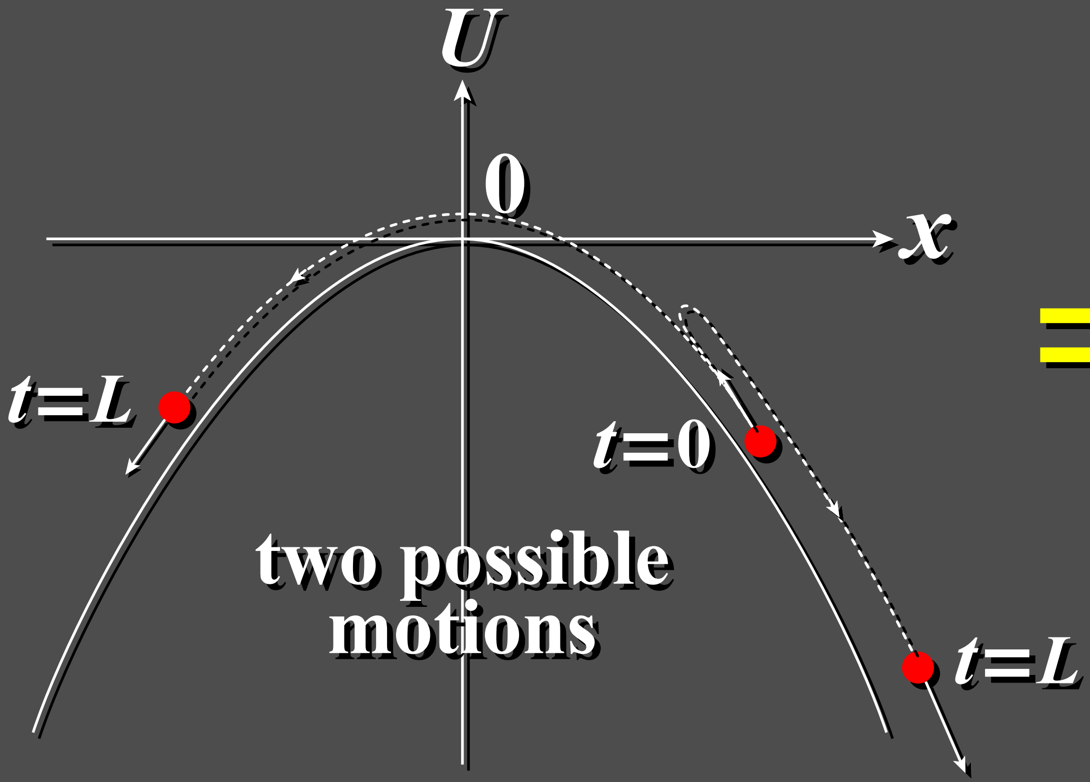

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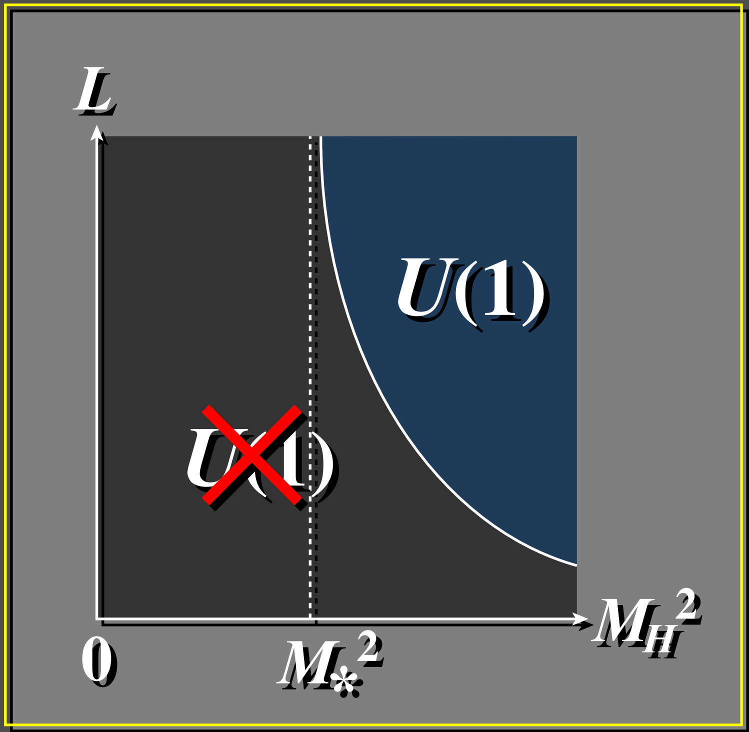

Vacuum Expectation Value <Φ(y)>

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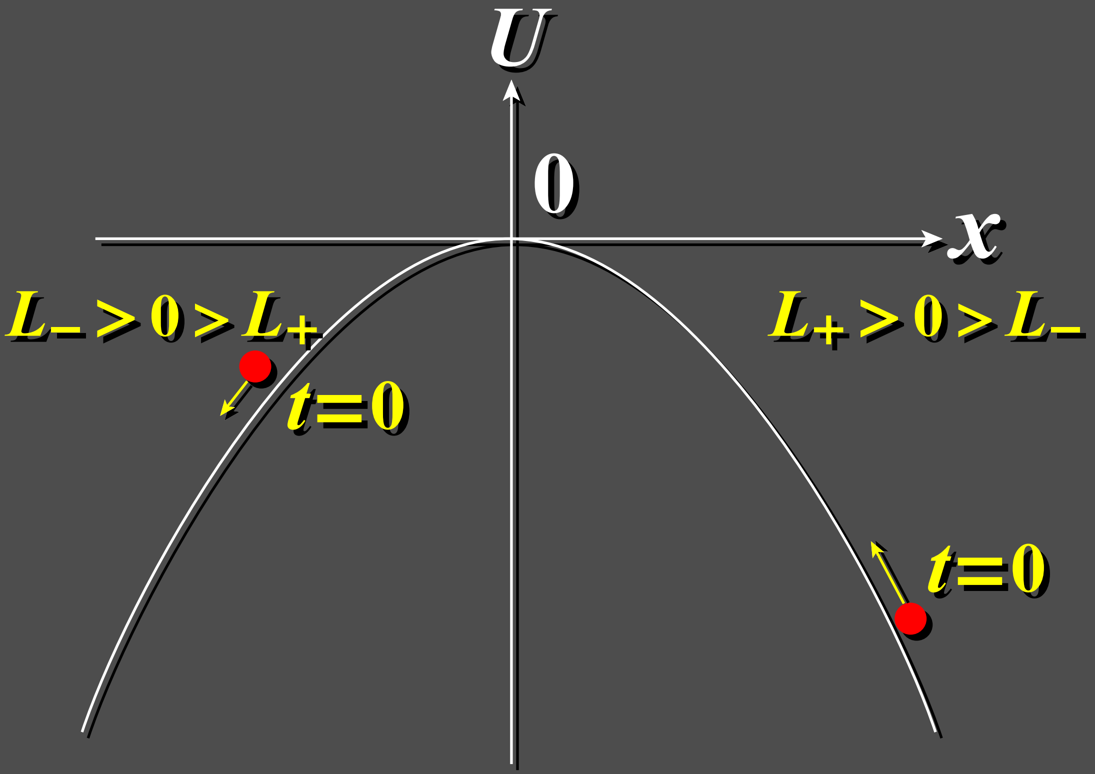

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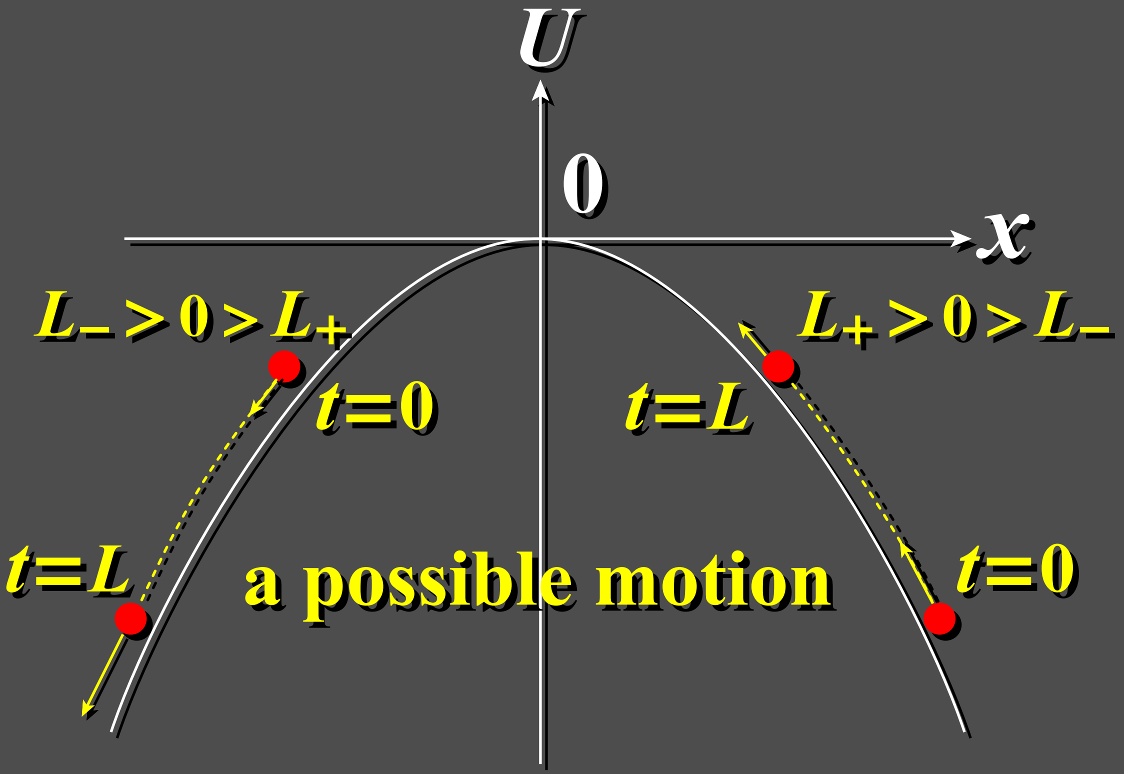

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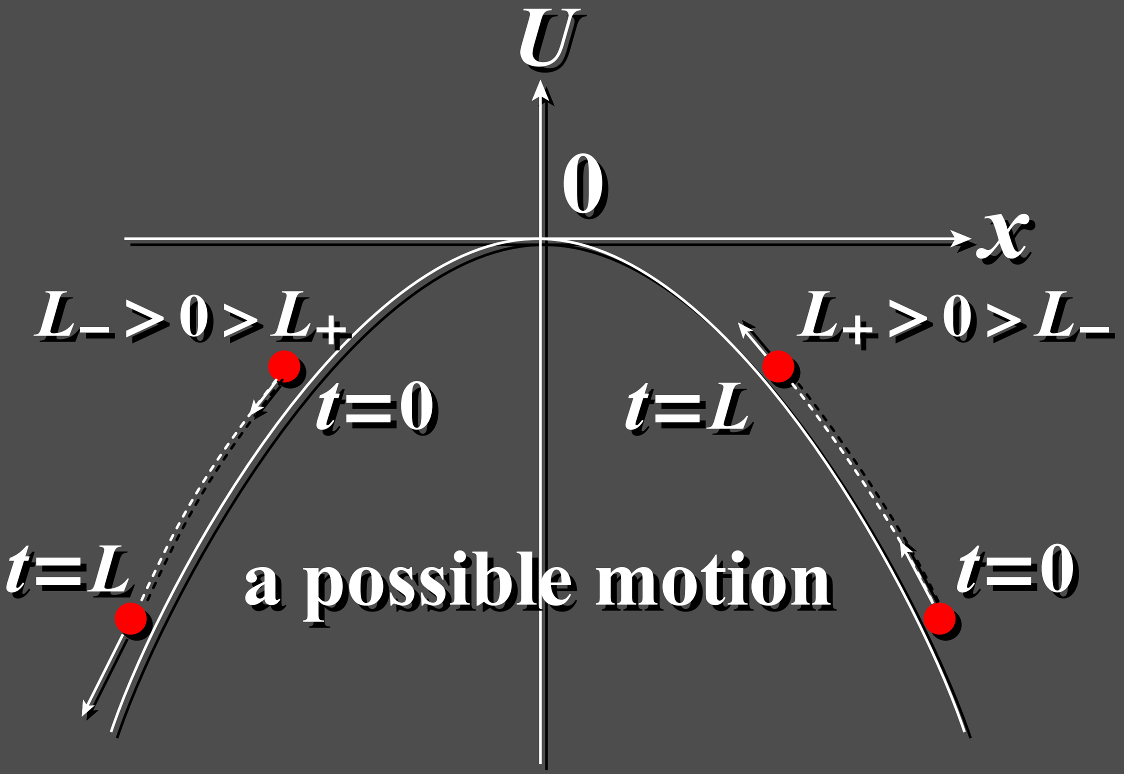

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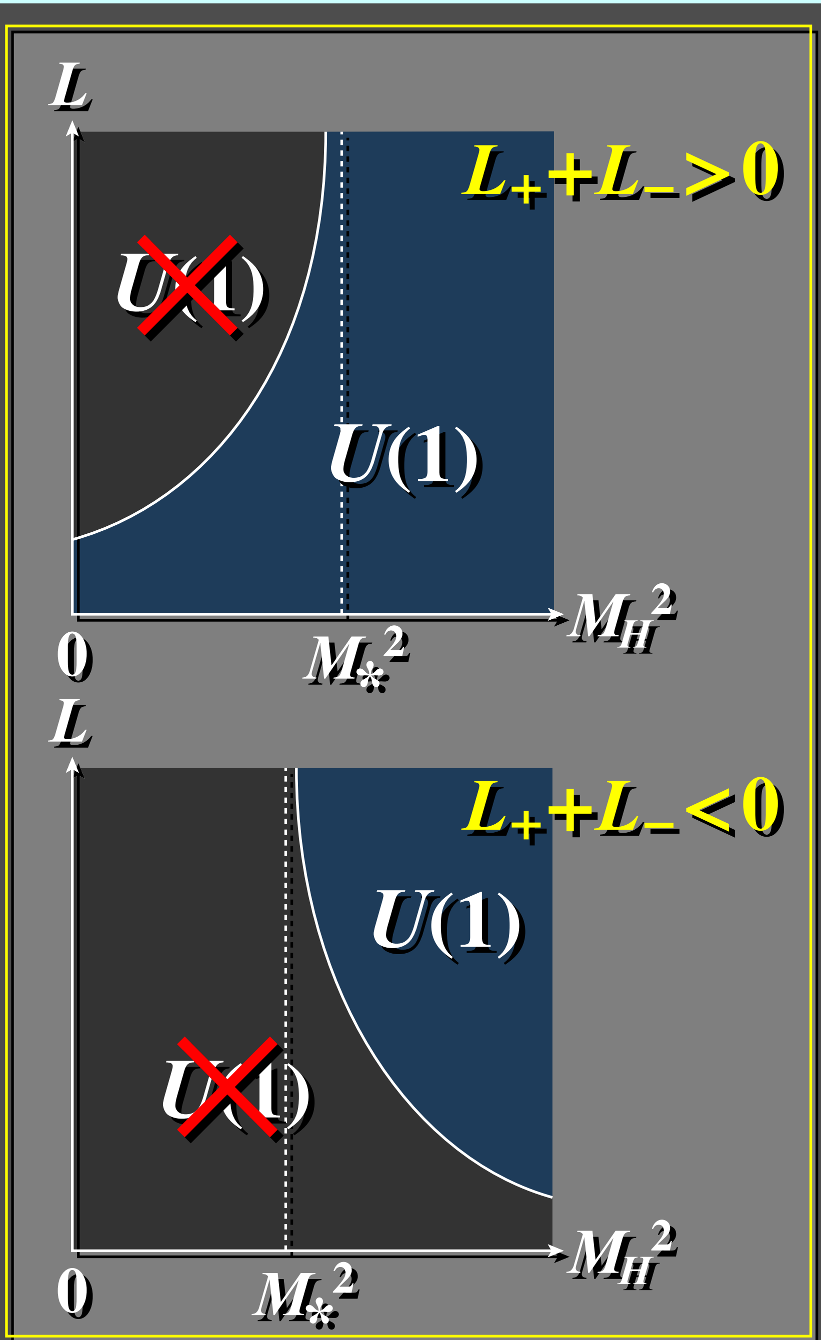

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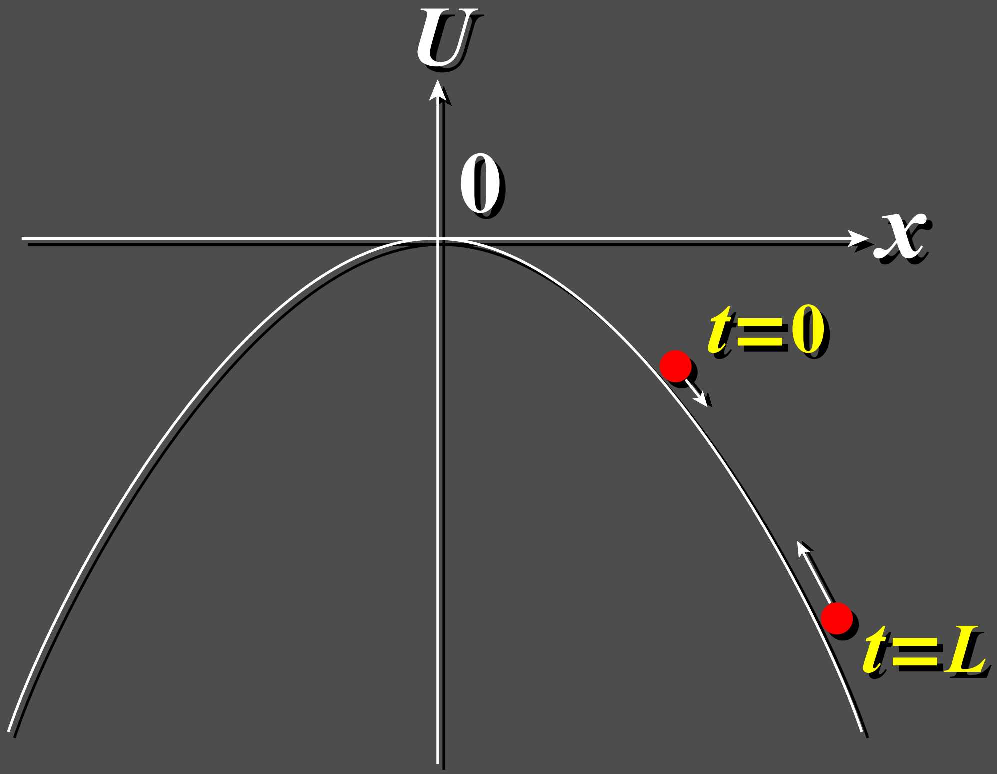

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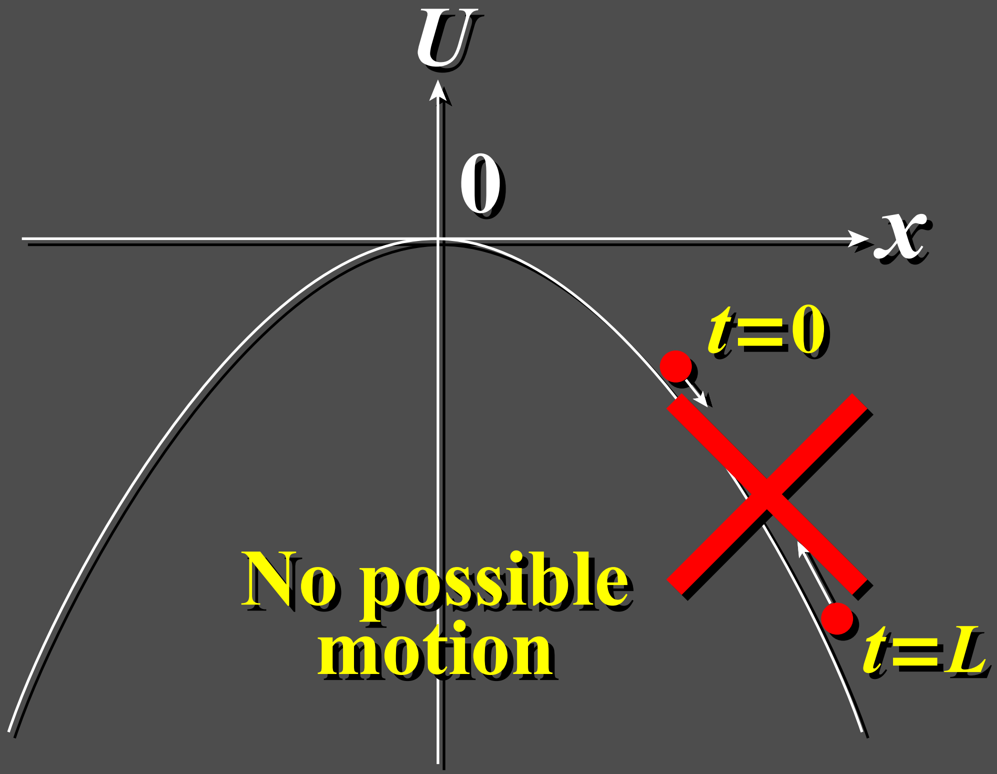

No possible motion

Vacuum Expectation Value <Φ(*y*)>

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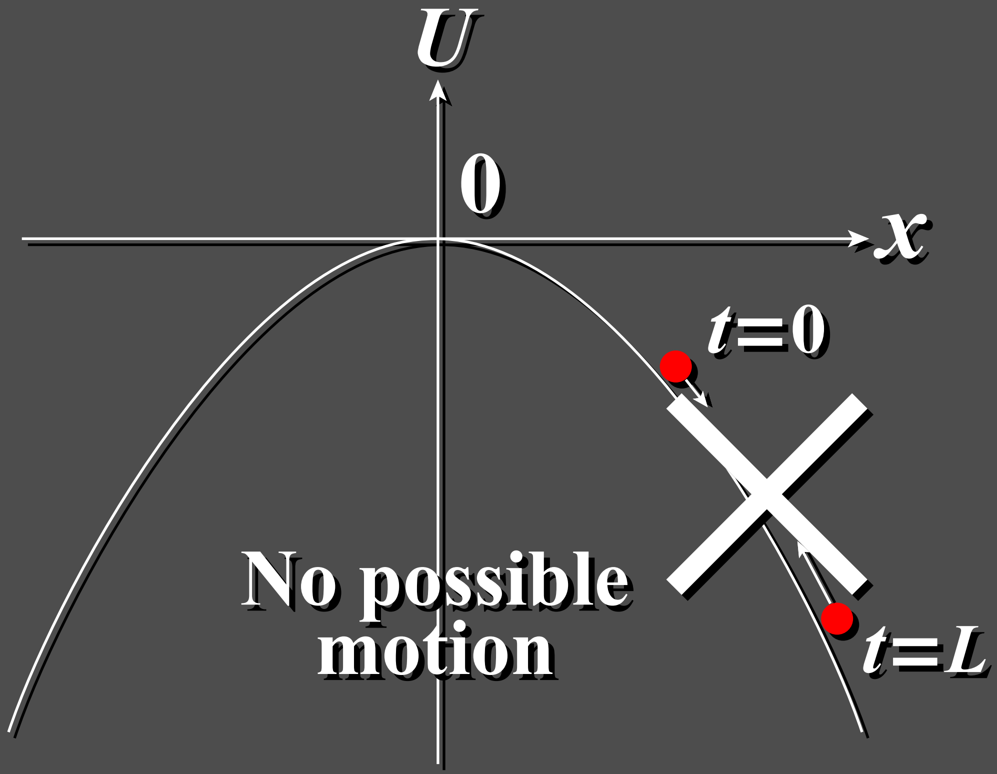
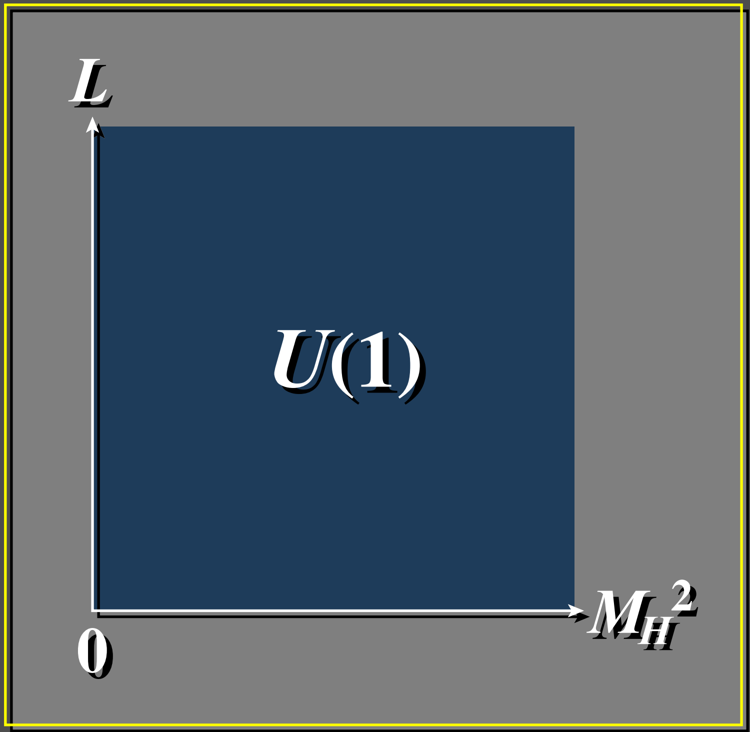
$$x(0) + L_+ \dot{x}(0) = 0$$
$$x(L) - L_- \dot{x}(L) = 0$$



No possible motion

⇒



Conclusions and Discussions

- In the context of 5d gauge theories on an interval, chiral theories with hierarchical fermion masses and a Higgs mechanism, just like SM, naturally appear as low energy effective theories.

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- The properties we found will hold for non-abelian gauge theories with a wider class of allowed boundary conditions.