A challenge to the a-theorem in six dimensions

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What is (S)CFT in six dimensions?

Motivations toward 6d SCFT

- Field theory
  - Maximal dimension to admit SCFTs [Nahm '78]
  - New dualities in lower dimensions (AGT, 3d-3d,…)
  - New insights to CFTs, e.g. T_N theory (class $\mathcal{S}$, class $\mathcal{R}$,…)

- String theory
  - Tensionless limit of type IIB compactified on K3 w/ ADE singularity [Witten '95]
  - Effective theory on multiple M5-branes [Strominger '95]
  - To describe M-theory (N^3-scaling,…)
  - Approach to general aspects of AdS/CFT

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Conformal bootstrap, T_N theory, Holography,…

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We would like to know more about 6d SCFTs w/ these tools.

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Central charge ~ Trace anomaly

ex) 2d CFTs on general backgrounds

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{c}{24\pi} R$$

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How about "a-theorem" in 6d?

G. Moore

6.2 Geometry on the space of field theories

The profound results of D. Friedan [122, 123] and A. Zamolodchikov [323] on the geometry of the space of two-dimensional quantum field theories have made it quite clear that we should define a *space* of quantum field theories and explore its geometry. Indeed, Friedan has even proposed to use properties of the space of all two-dimensional models as a starting point for a unified view of physics alternative to the more standard string-theoretic approach [124].

It would be good to give a concrete and rigorous definition of the "space of quantum field theories" both in two and in higher dimensions. One tool, which has been used to explore families of quantum field theories, is the construction of quantities that decay monotonically under renormalization group flow. There are known quantities in two [323], three [55], and four [192] dimensions, generally called c, F, a, respectively. An obvious problem for the future is

Find monotonically decreasing quantities for renormalization group flow of five and six dimensional theories.

As discussed above, we do not understand these higher dimensional theories in the UV very well so this problem is probably out of reach at the moment. A curious aspect of the known monotonically decreasing quantities is that c, a in $2, 4$ dimensions are defined in terms of local correlation functions of the energy-momentum tensor while F is a nonlocal quantity.

Is there a more unified view on the monotonically decreasing quantities? Is there a locally defined quantity in three dimensions that decreases under RG flow?

Plan

1. c-theorem in 2d

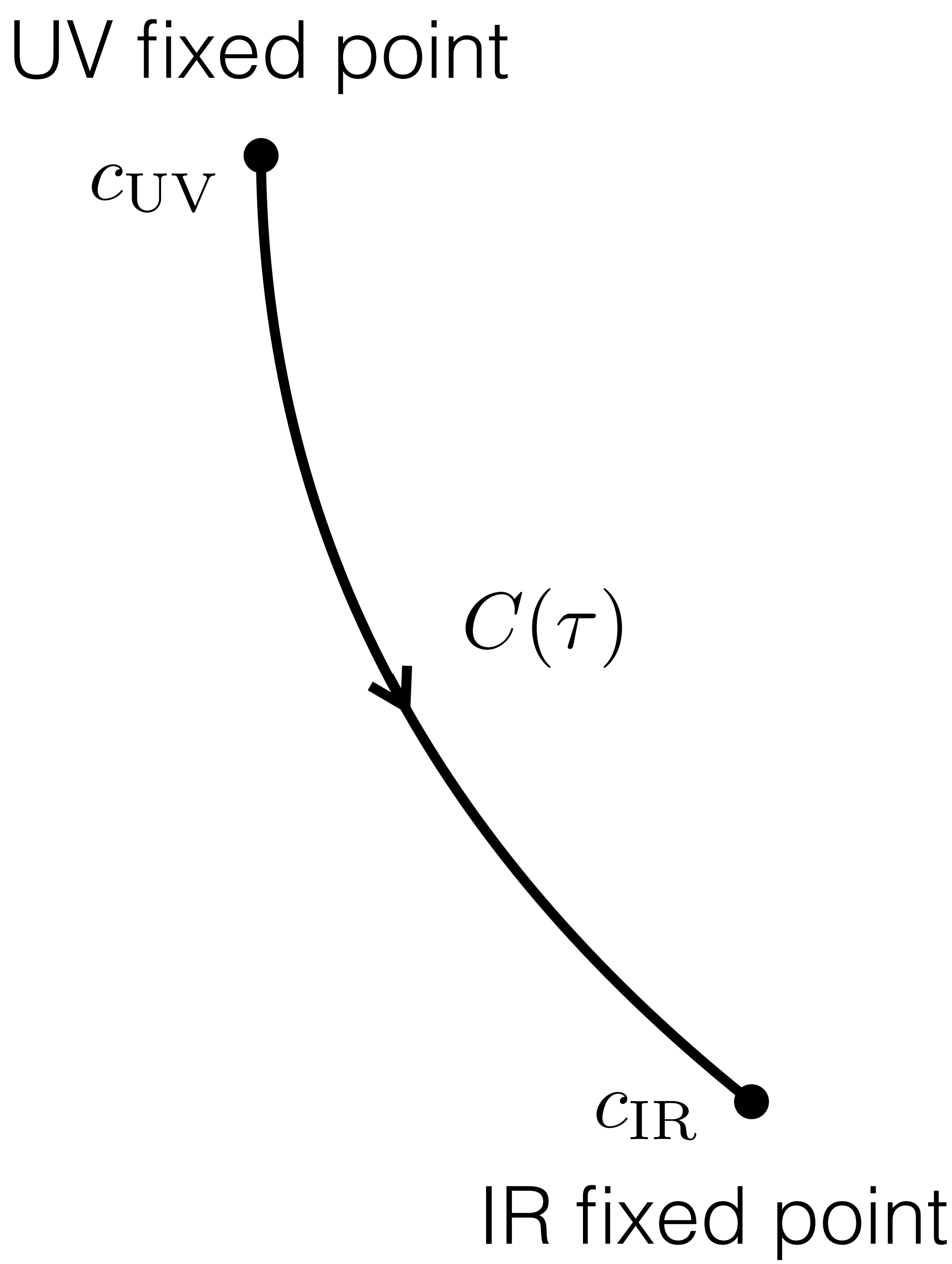
2. a-theorem in 4d

3. "a-theorem" in 6d

4. Summary and discussions

1. c-theorem in 2d

c-theorem in 2d [Zamolodchikov '86]



monotonically decrease along RG flow

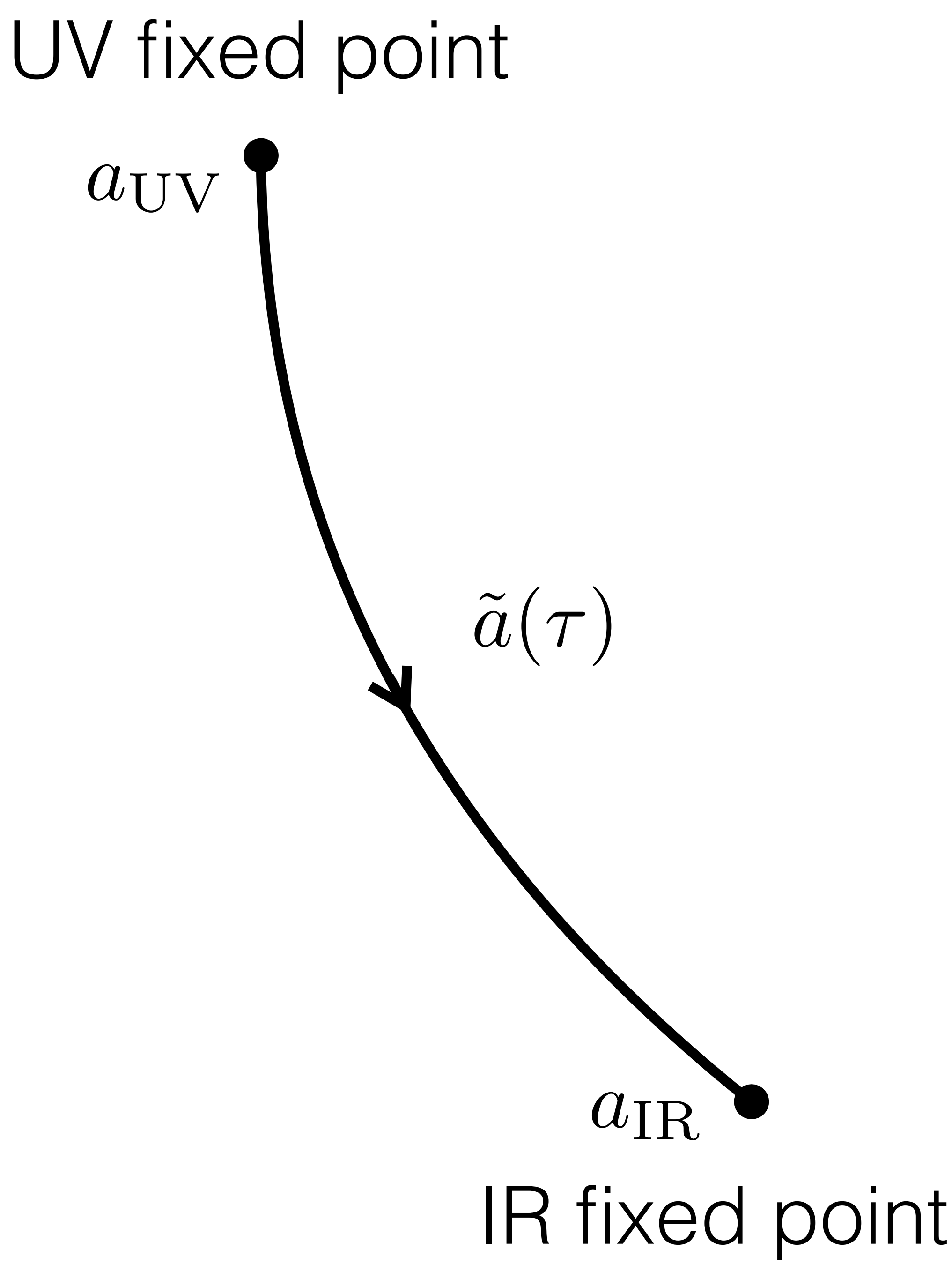
$$\frac{dC(\tau)}{d\tau} < 0 \Rightarrow \boxed{c_{\rm UV} > c_{\rm IR}}$$

⇕

#(dof) decrease along RG flow

2. a-theorem in 4d

a-theorem in 4d [Cardy '88][Komargodski-Schwimmer '11]



Anomalies in 4d

$$\langle T^{\mu}{}_{\mu}\rangle = cW^2 - aE_4 \quad \begin{cases} W_{\mu\nu\rho\sigma} : \text{Weyl tensor} \\ E_4 : \text{Euler density} \end{cases}$$

<u>Process of proof</u>

- Spontaneously broken conformal sym
- Effective action of dilaton (NG boson)
- 4-pt scattering amplitude for dilaton

$\tilde{a}(\tau)$ monotonically decrease along RG flow

$$\boxed{a_{\rm UV} > a_{\rm IR}}$$

3. “a-theorem” in 6d

Strategy [Osborn '91][Grinstein-Stergiou-Stone '13]

- Wess-Zumino consistency condition [Wess-Zumino '71]

$$[\Delta^a, \Delta^b]W = i f^{abc} \Delta^c W$$

$$\begin{cases} W : \text{generating functional of connected Green functions} \\ f^{abc} : \text{structure constant} \end{cases}$$

- Weyl rescaling

$$\Delta_\sigma h^{\mu\nu}(x) = 2\sigma(x) h^{\mu\nu}(x)$$
$$\Delta_\sigma g^I(x) = \sigma(x)\beta^I(x)$$

$$\begin{cases} h^{\mu\nu} : \text{spacetime metric} \\ g^I : \text{couplings} \\ \beta^I : \text{beta functions} \end{cases}$$

$\Rightarrow \ [\Delta_\sigma, \Delta_{\sigma'}]W = 0$ (Weyl consistency condition)

Strategy [Osborn '91][Grinstein-Stergiou-Stone '13]

- Anomalies

$$\Delta_\sigma W[h^{\mu\nu}, g^I] = \int d^d x \sqrt{-h}\sigma \sum_i \left(\underline{a_i A_i[h^{\mu\nu}]} + b_i B_i[h^{\mu\nu}, g^I] + c_i C_i[g^I]\right)$$

↑ include a (and Euler density)

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χ_{IJ} : "metric" in the space of couplings

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$$\frac{d\tilde{a}}{d\log\mu} = \frac{1}{6}\chi_{IJ}\beta^I\beta^J$$

χ_{IJ} : "metric" in the space of couplings

We need computing χ_{IJ} to determine the behavior of $\tilde{a}$.

(c-theorem and a-theorem can be proven by this method.)

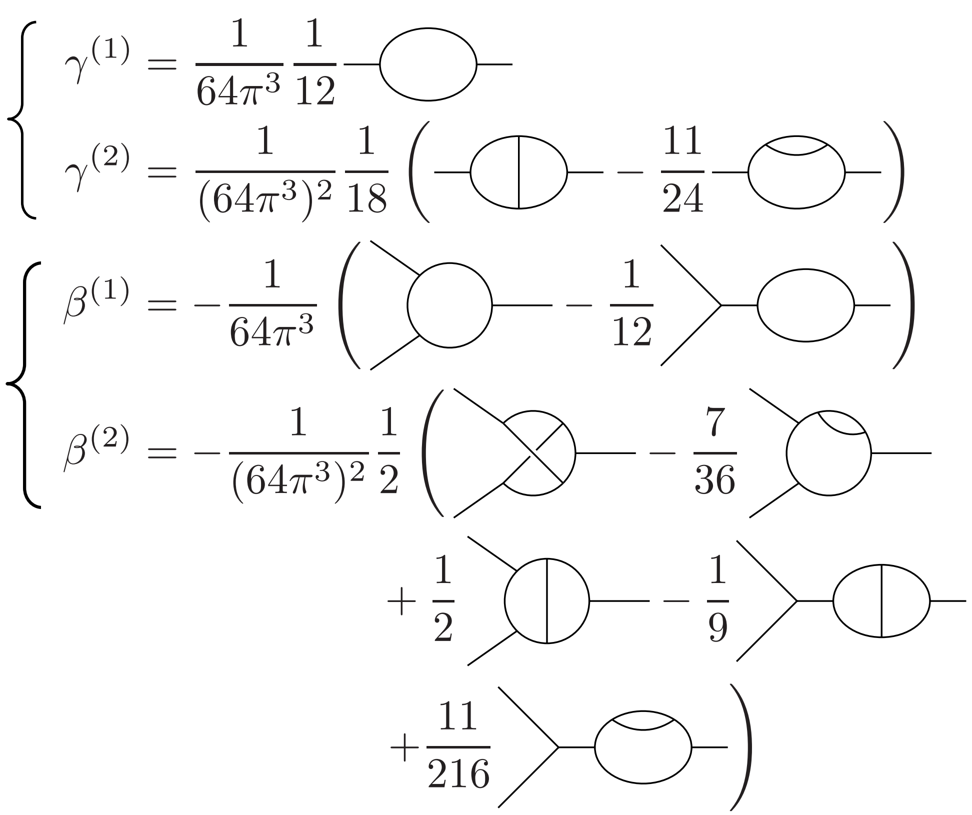
“a-theorem” in 6d [1406.3626]

- conformally-coupled ϕ^3 theory with flavors

$$\mathcal{L} = \frac{1}{2} h^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi^i + \frac{1}{5} R \phi_i \phi^i + \frac{1}{3!} g_{ijk} \phi^i \phi^j \phi^k$$

anomalous dimensions for ϕ^i

beta functions for g_{ijk}



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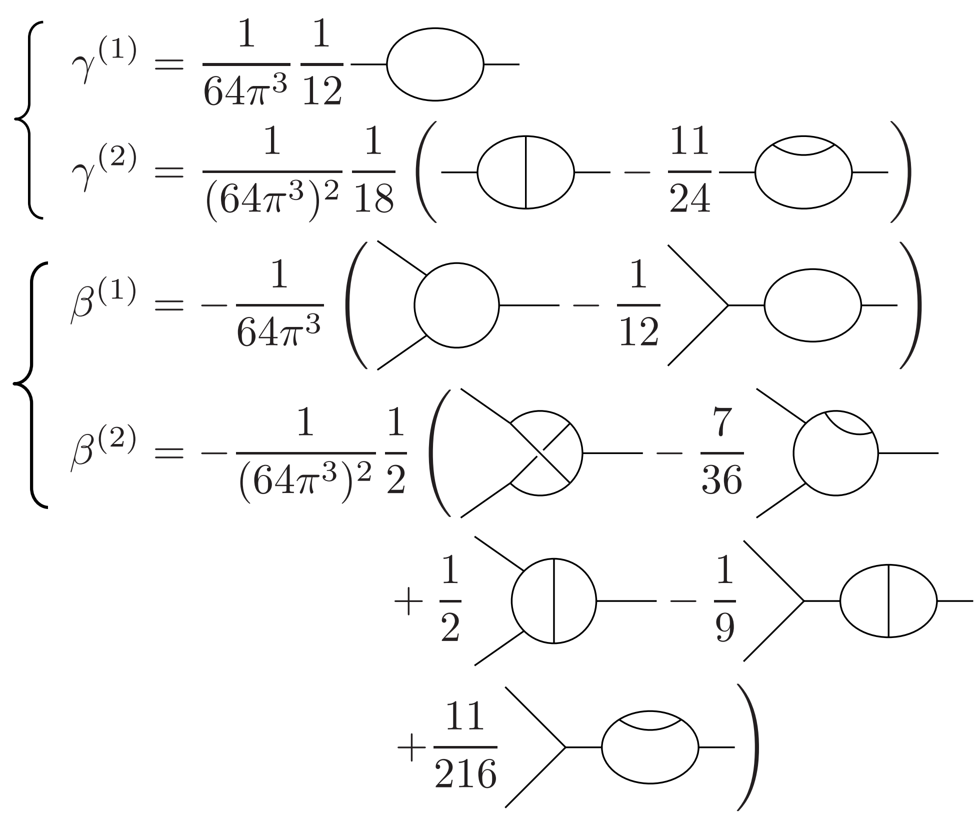

two-loop order for “metric” $\chi^{(2)}_{IJ} = -\frac{1}{(64\pi^3)^2} \frac{1}{3240} \delta_{IJ}$

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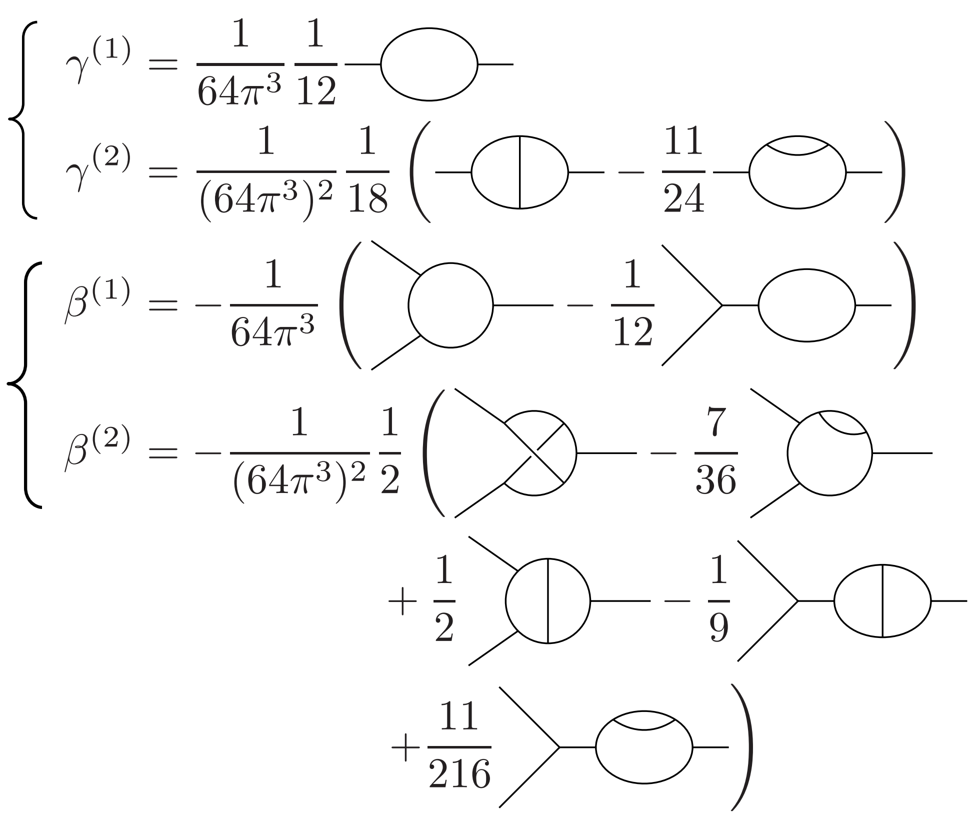

two-loop order for "metric" $\chi^{(2)}_{IJ} = -\frac{1}{(64\pi^3)^2}\frac{1}{3240}\delta_{IJ}$

$$\Rightarrow \frac{d\tilde{a}}{d\log\mu} < 0$$

monotonically increase!

$\tilde{a}_{\rm UV} < \tilde{a}_{\rm IR}$

4. Summary and discussions

∘ Summary

- perturbative computation in 6d conformally-coupled ϕ^3 theory

⇨ "a-theorem" in 6d ; monotonically increases $\tilde{a}_{\rm UV} < \tilde{a}_{\rm IR}$

contrary to c-theorem $(c_{\rm UV} > c_{\rm IR})$ in 2d and a-theorem $(a_{\rm UV} > a_{\rm IR})$ in 4d

∘ Discussions

- non-perturbative?
- more general models?
- physical intuitions for monotonically-increasing behavior?
- holographic RG?
- How about 5d QFTs?

cf. 5d N=1 SYM has a UV non-trivial fixed point [Seiberg '95]