Journal Club@Osaka University July 26, 2017

Diving into Traversable Wormholes

Tokiro Numasawa

Osaka University Particle Physics Theory

Based on
arXiv:1704.05333 J. Maldacena, D.Stanford and Z. Yang

see also
"Traversable Wormholes via a Double Trace Deformation"
arXiv:1608.05687, P. Gao , D.Jafferis and A.Wall
(Iizuka-san's JC)

"Conformal symmetry and its breaking in two dimensional Nearly Anti de-Sitter space"
arXiv:1606.01857 J. Maldacena, D.Stanford and Z. Yang

This paper talks about…

- Traversable wormholes in Nearly AdS2 gravity
- Application to information problems

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- Traversable wormholes in Nearly AdS2 gravity
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(1)Introduction/Review of traversable wormholes

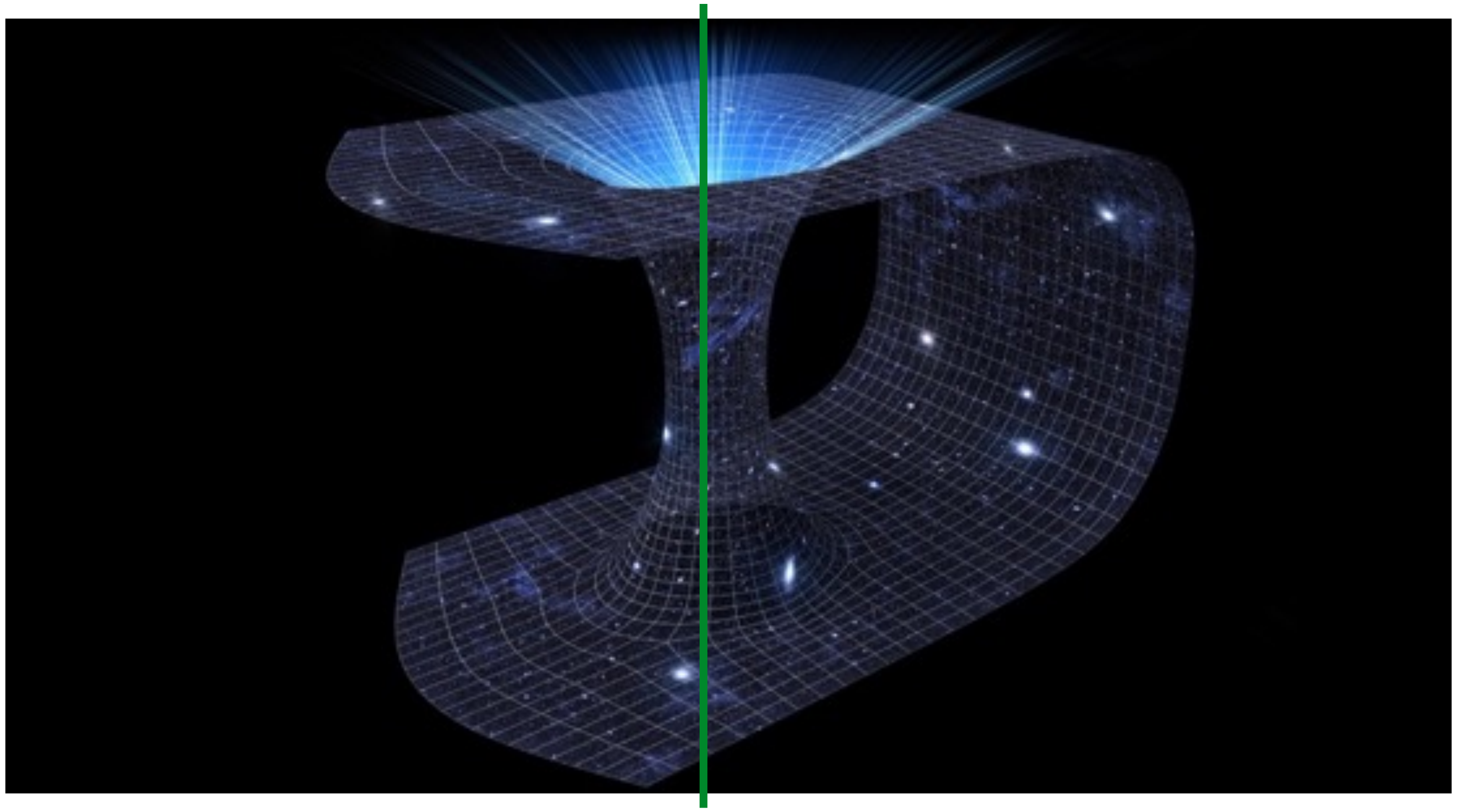
(2)Nearly AdS2 gravity

(3)Traversable wormholes in Nearly AdS2 gravity

(4)Application to information problem

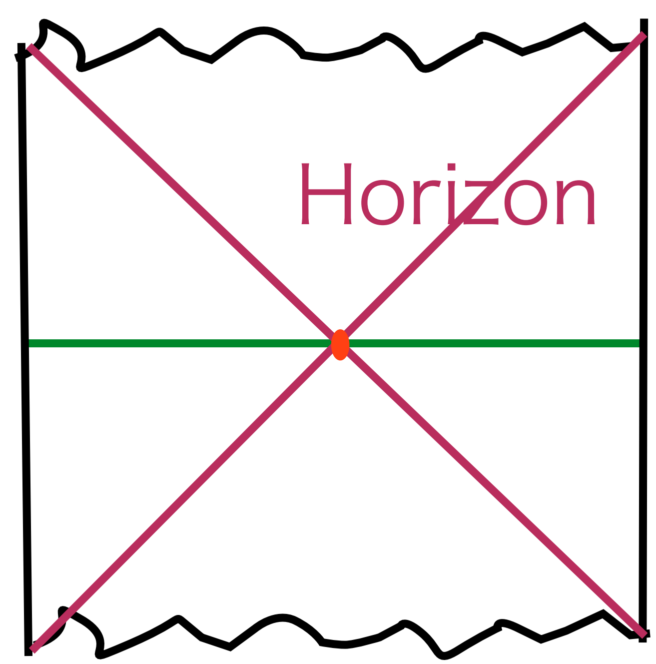
(5)Conclusion

1.Introduction



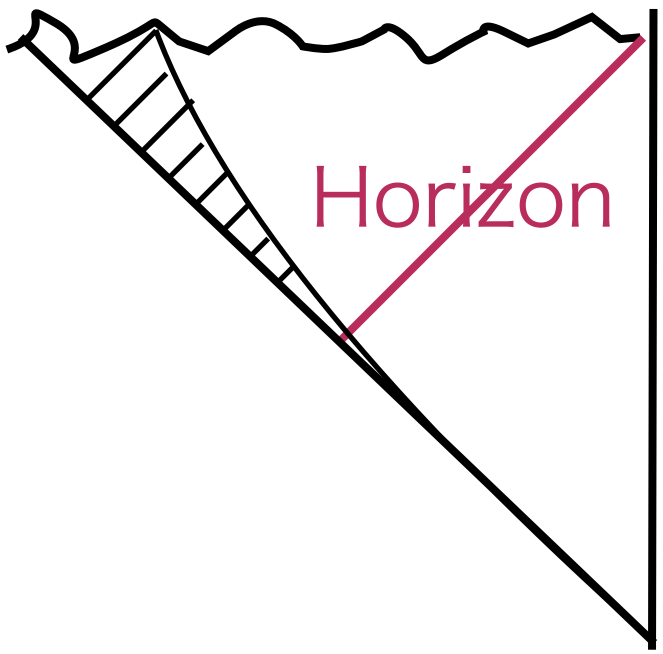
https://www.sciencenews.org/blog/context/new-einstein-equation-wormholes-quantum-gravity

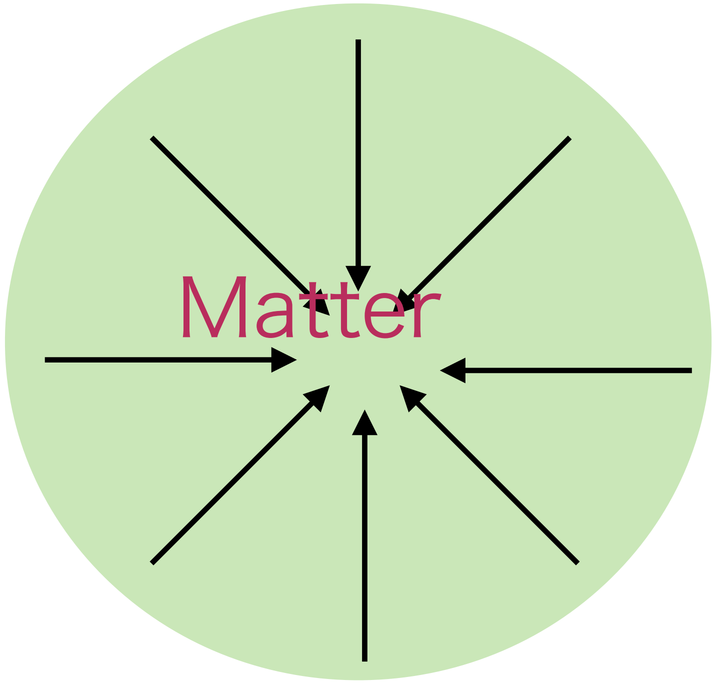
is described by



1.Introduction

Realistic Black Holes are created by collapsing matters.





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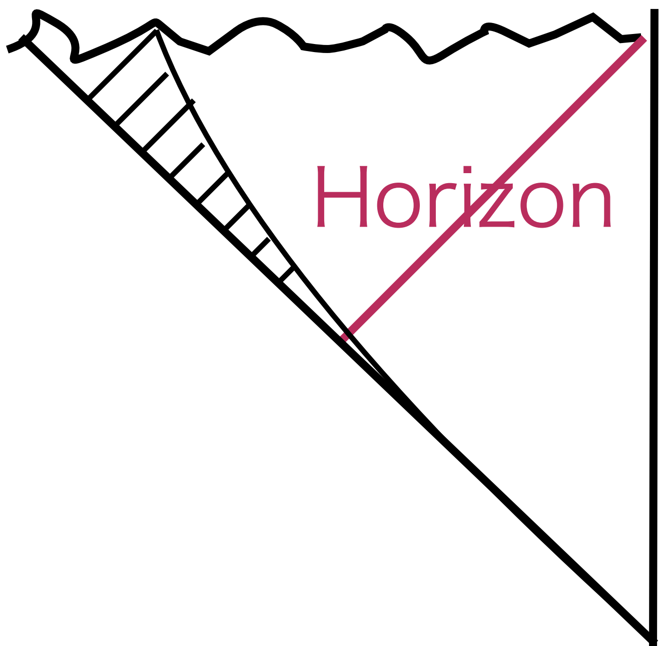

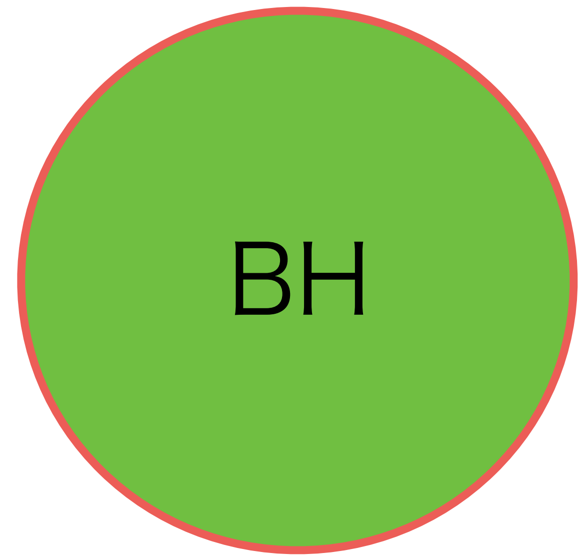

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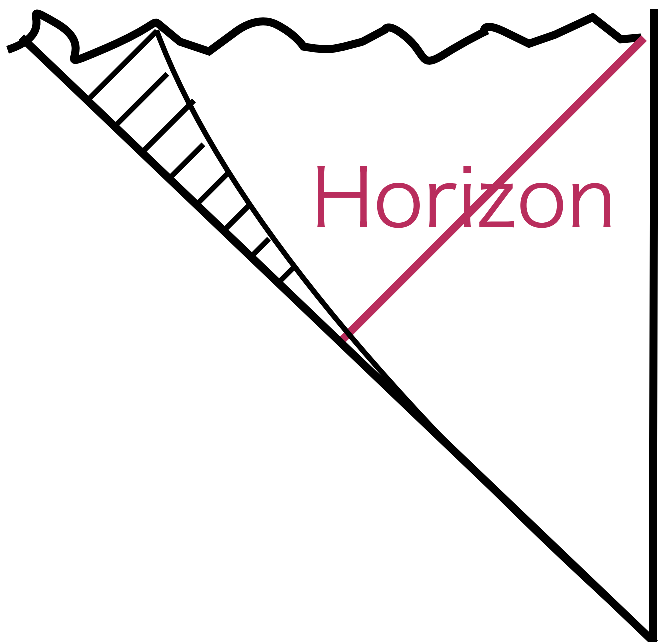
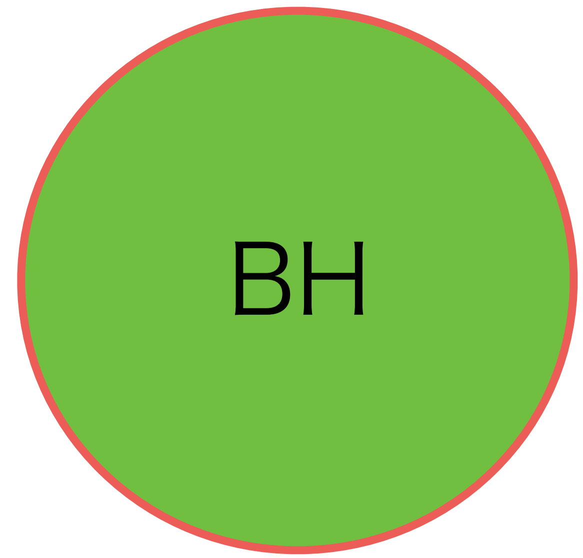
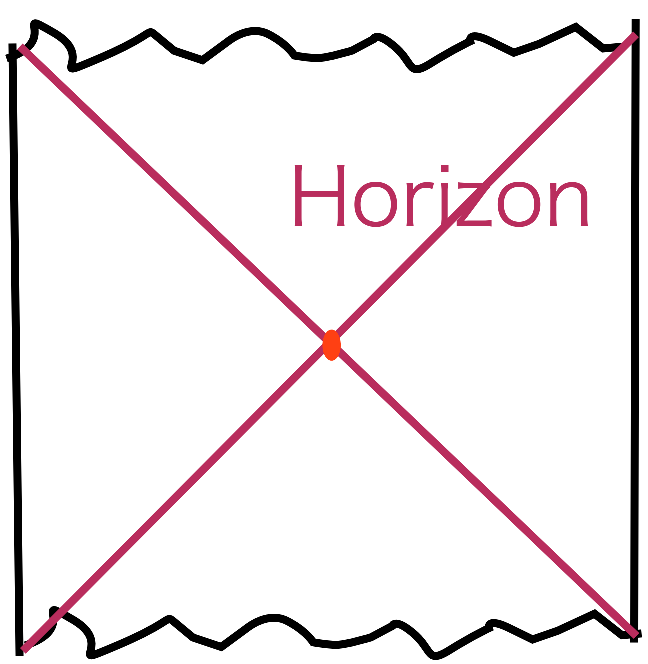




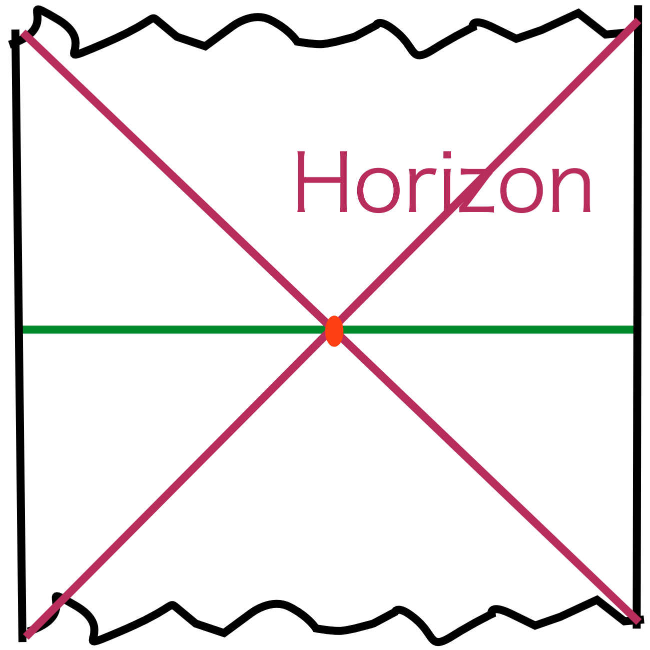
4d Schwarzschild: $ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{1}{1 - \frac{2M}{r}}dr^2 + r^2 d\Omega_2^2$

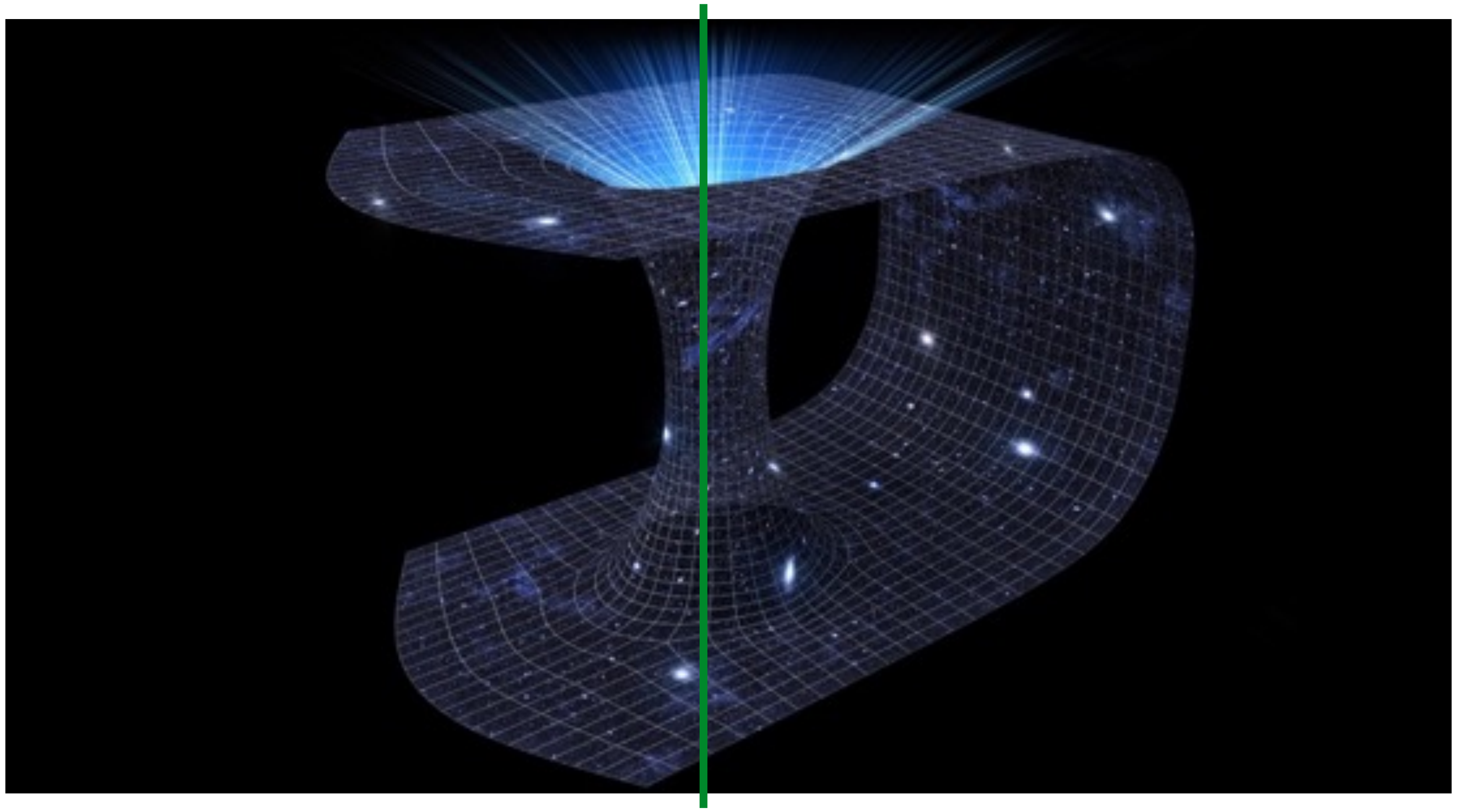
But Einstein eq. also permits eternal Black holes solutions:



$$ds^2 = -\frac{32M^3}{r}e^{-\frac{r}{2M}}dUdV + r^2 d\Omega_2^2$$

:just a coordinate transformation

This  approximately describes



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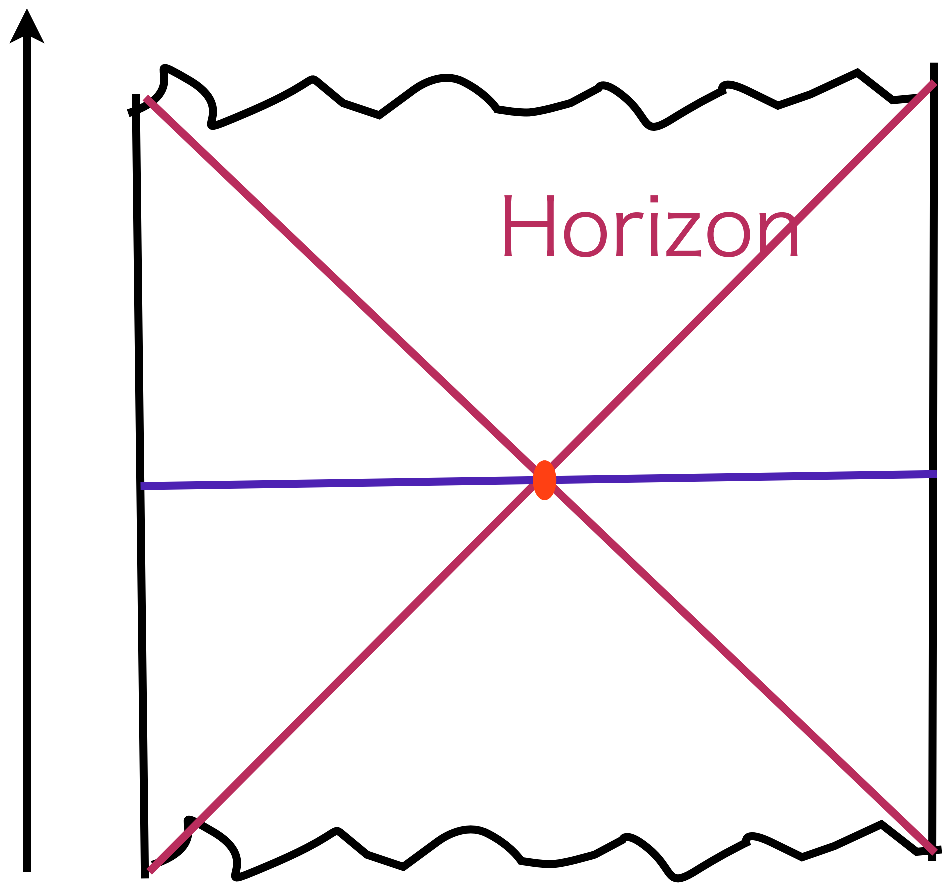

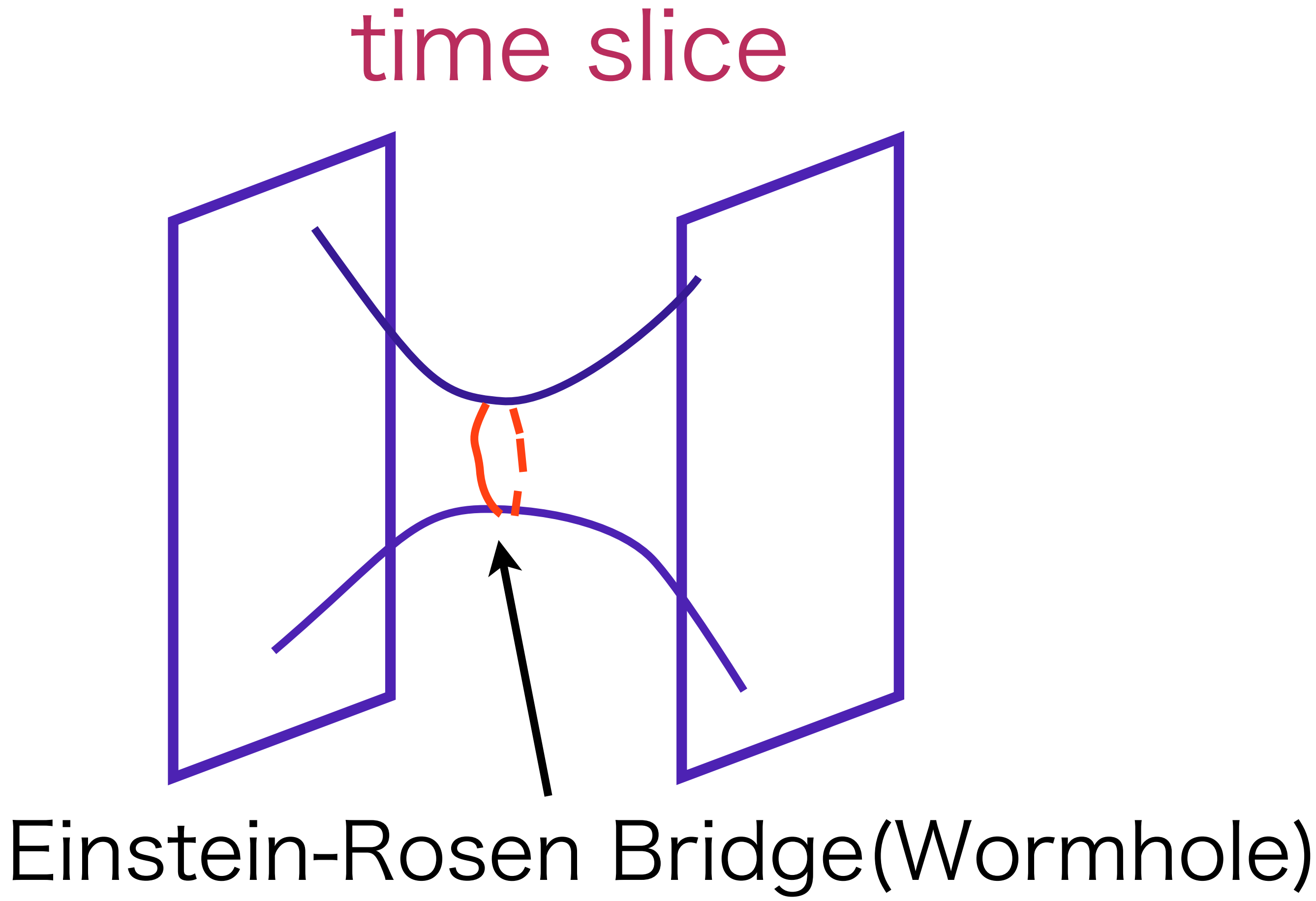

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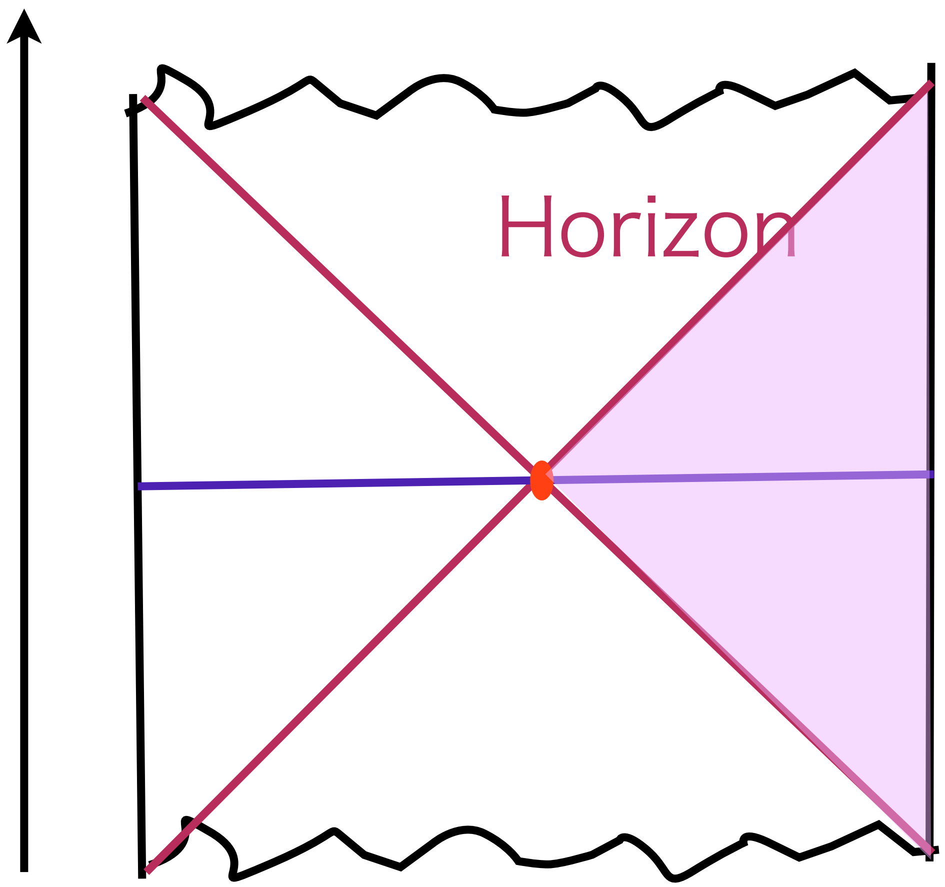

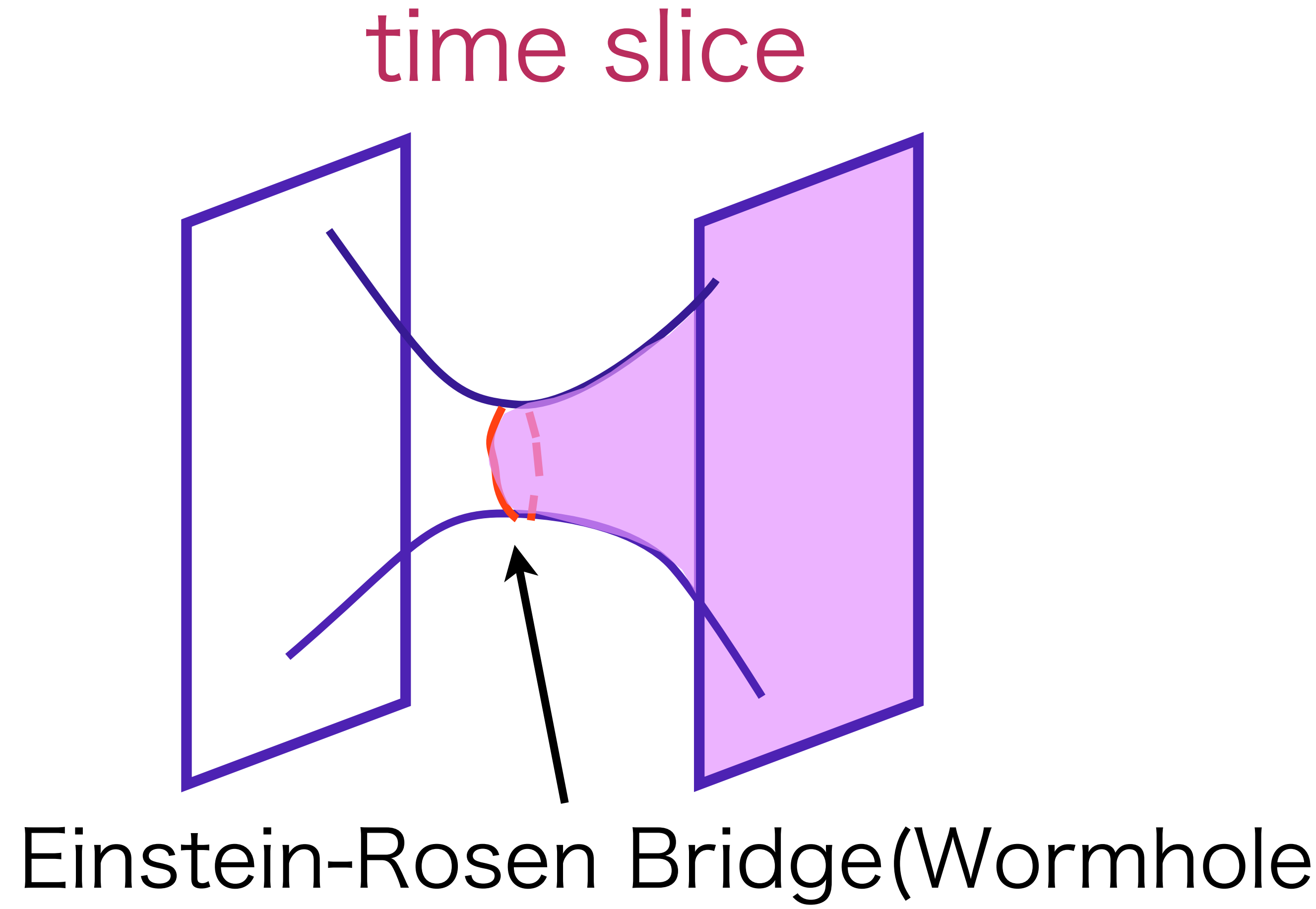

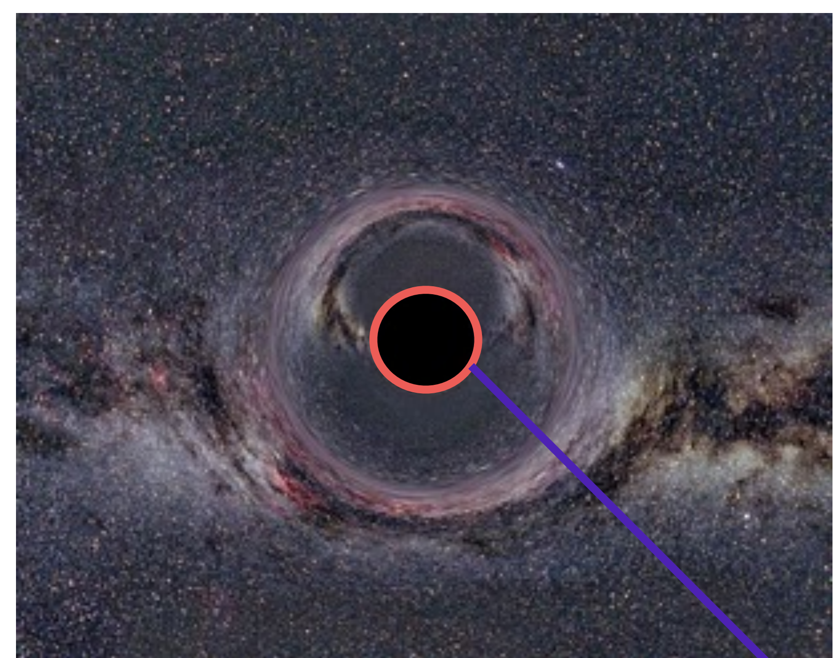

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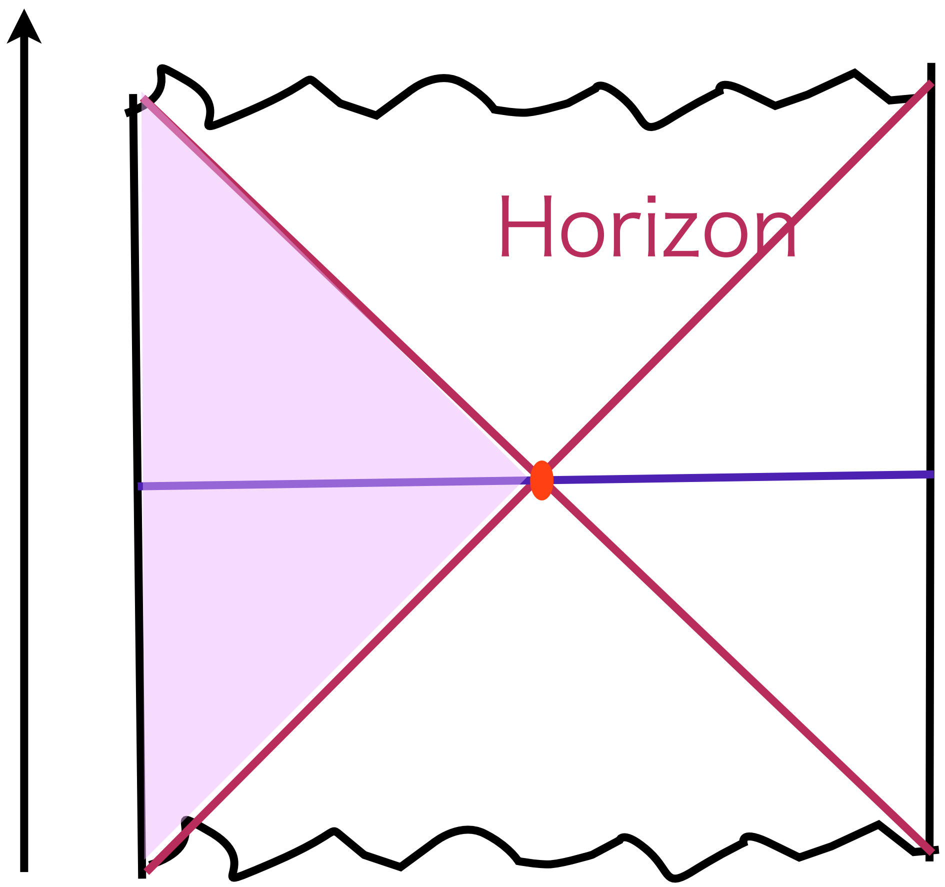

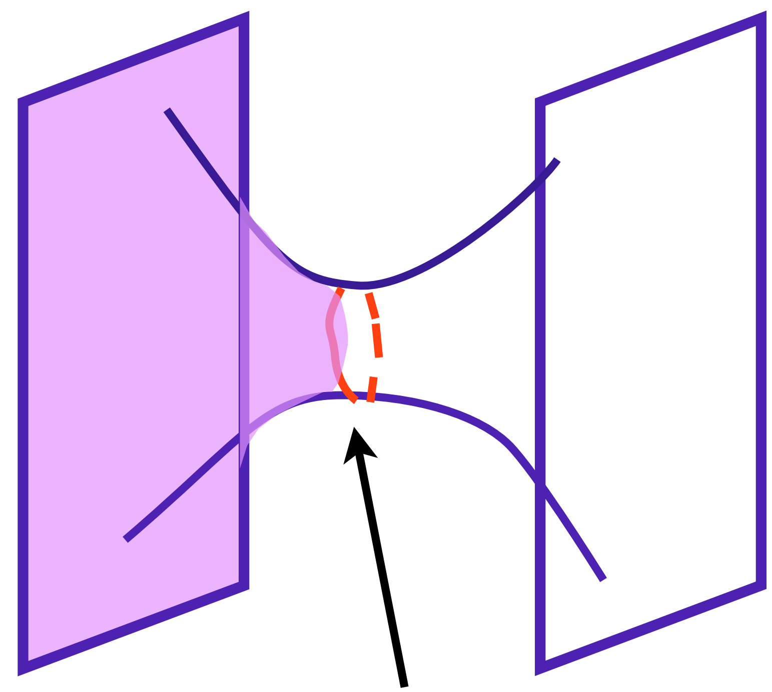

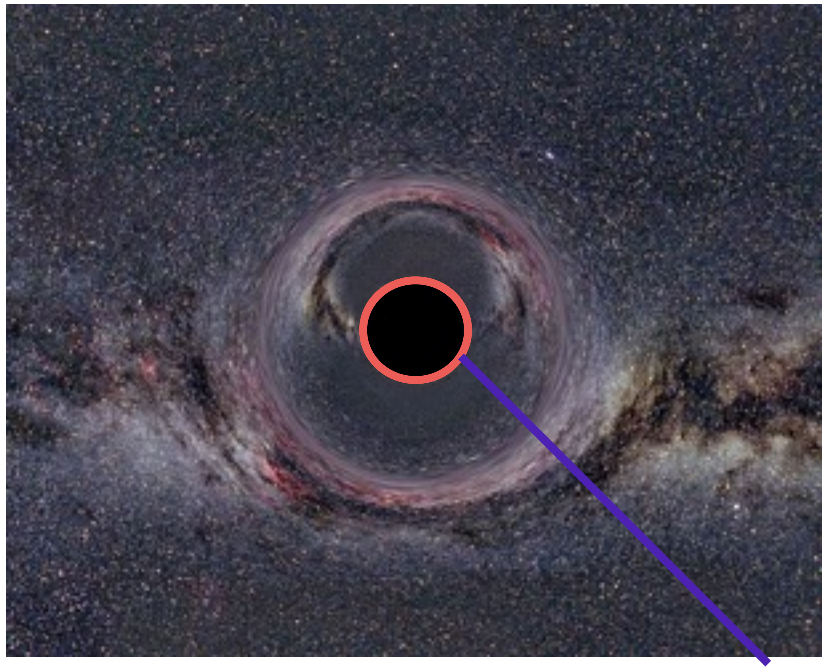

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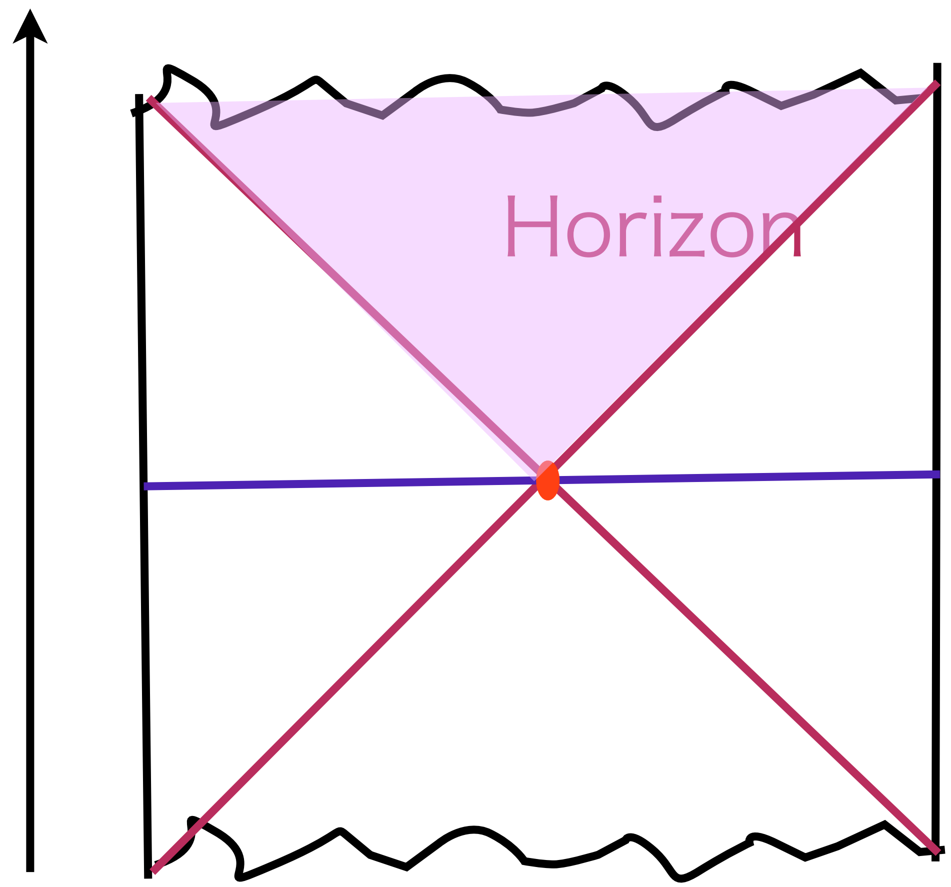

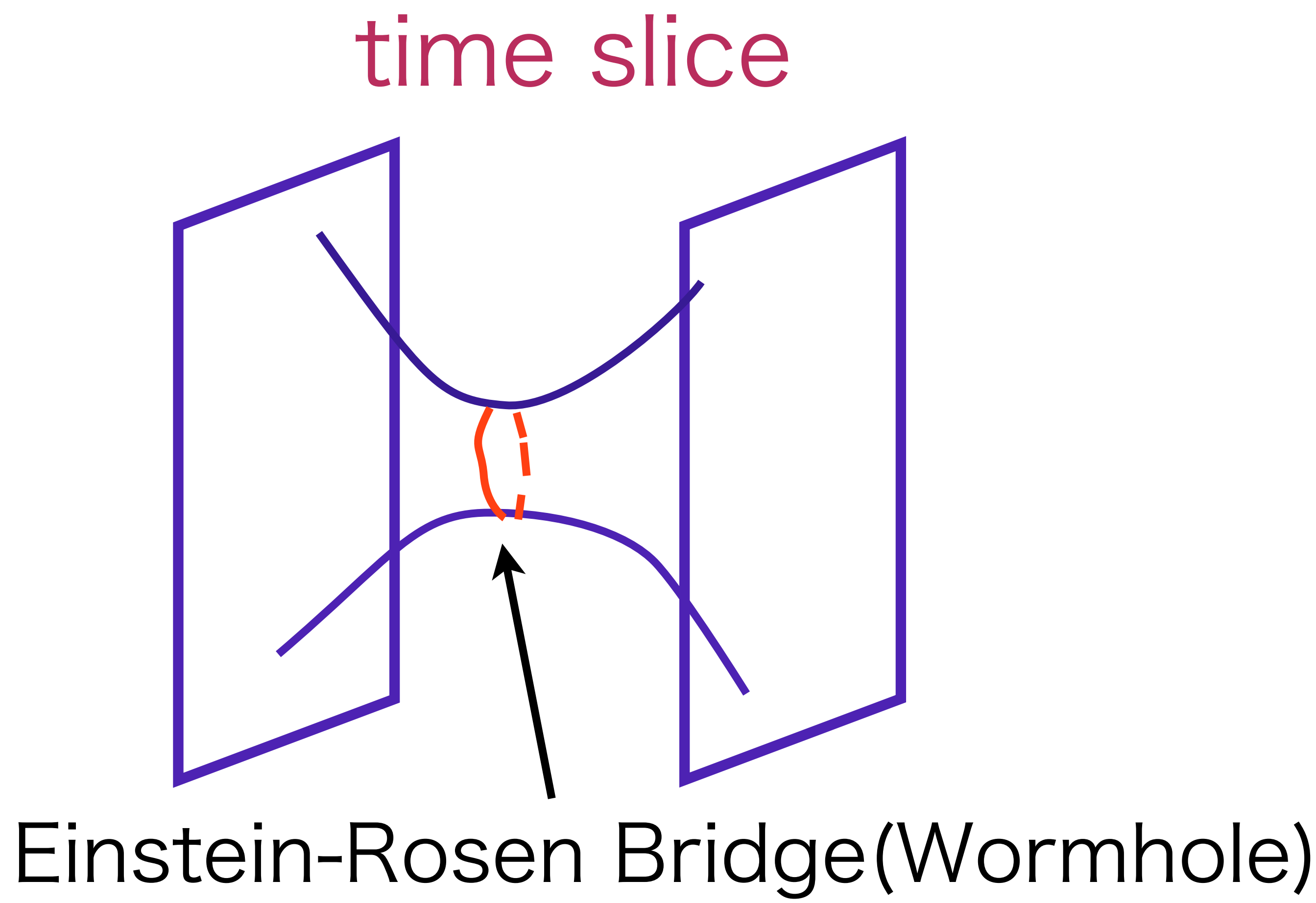

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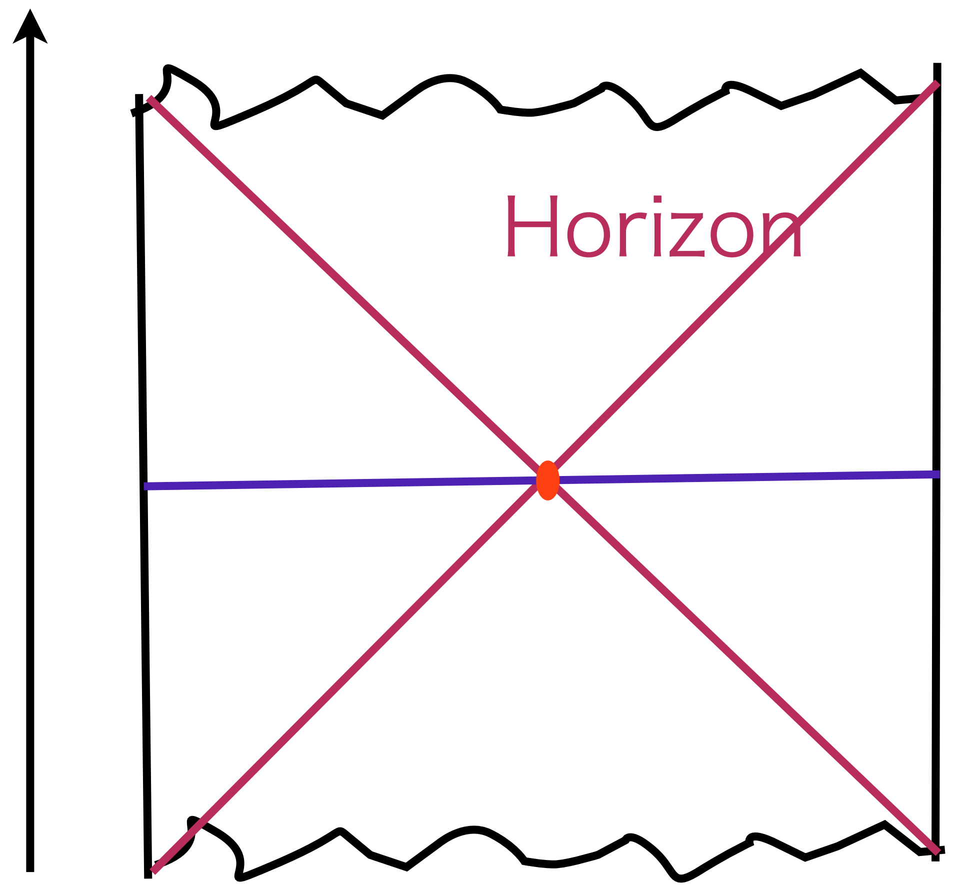
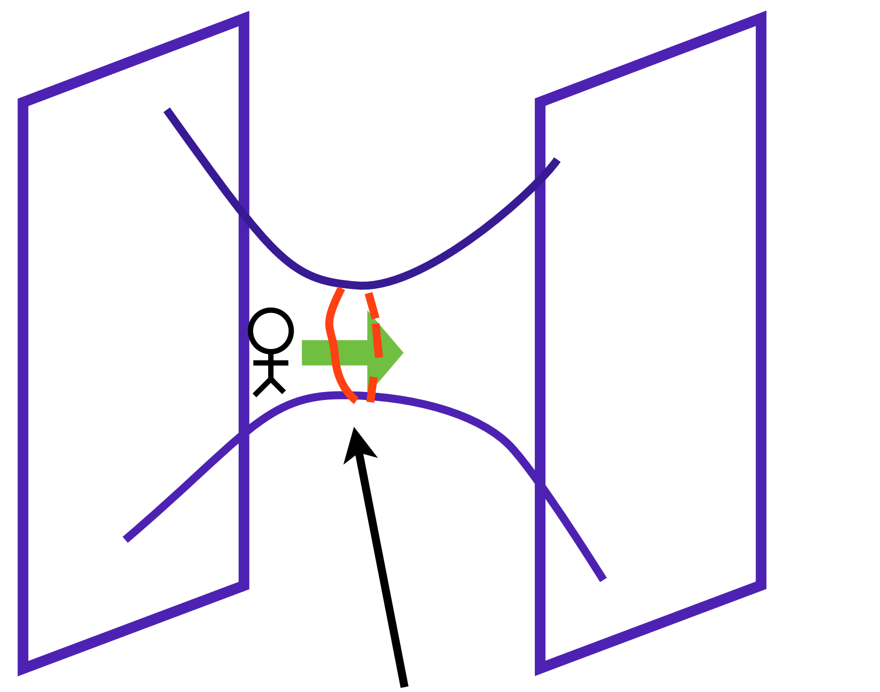
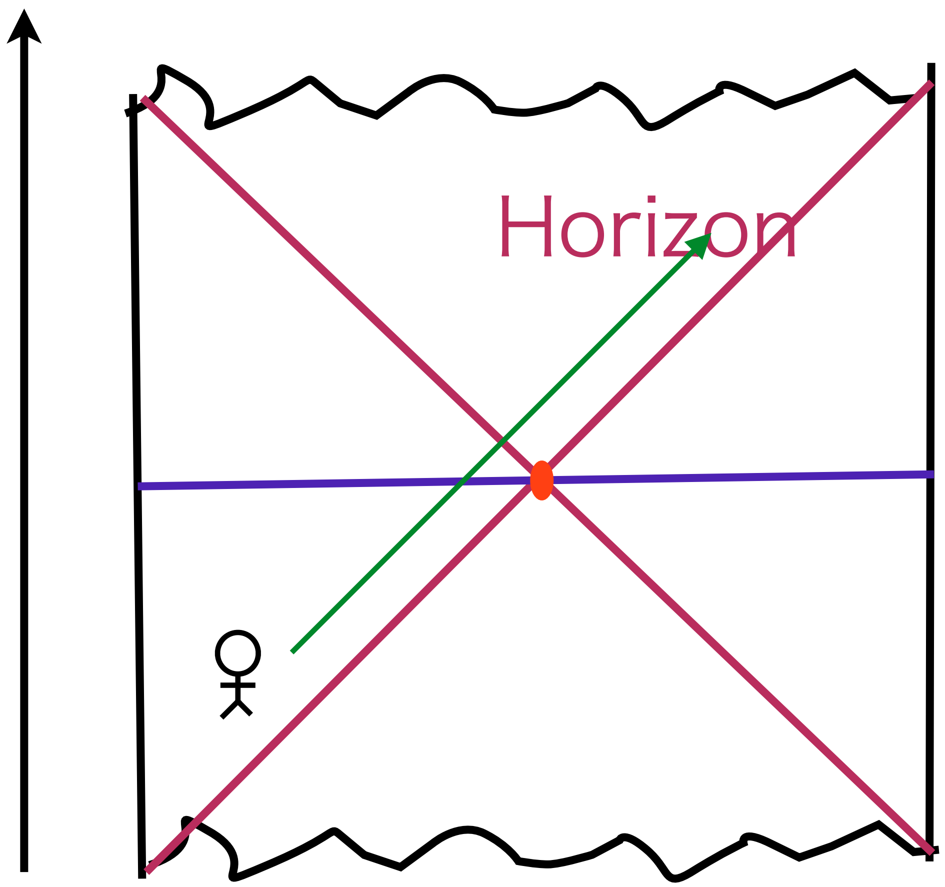
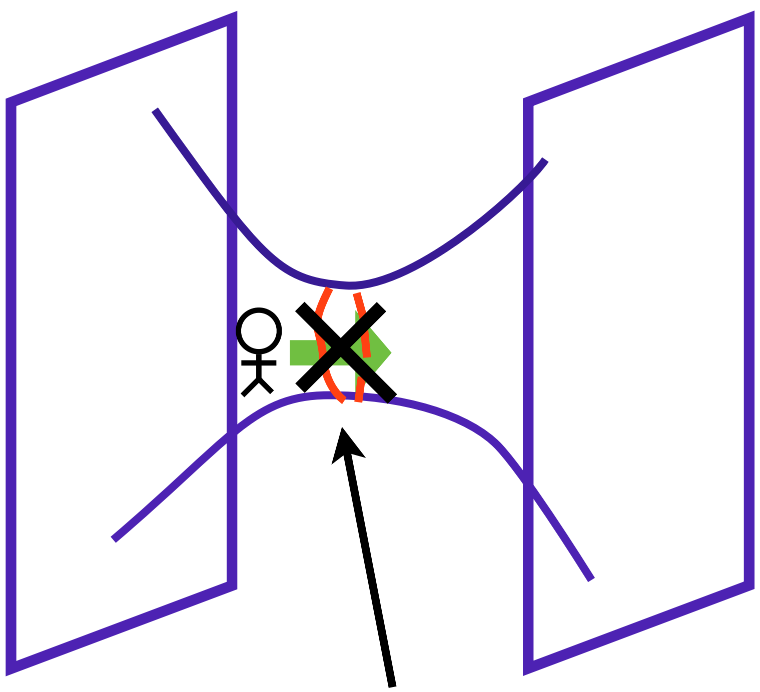




Can we go to the other side?

1.Introduction



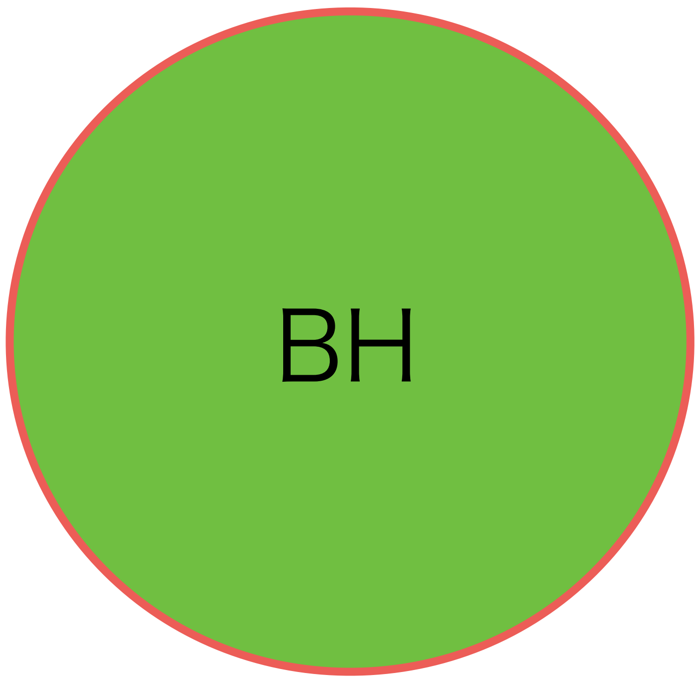


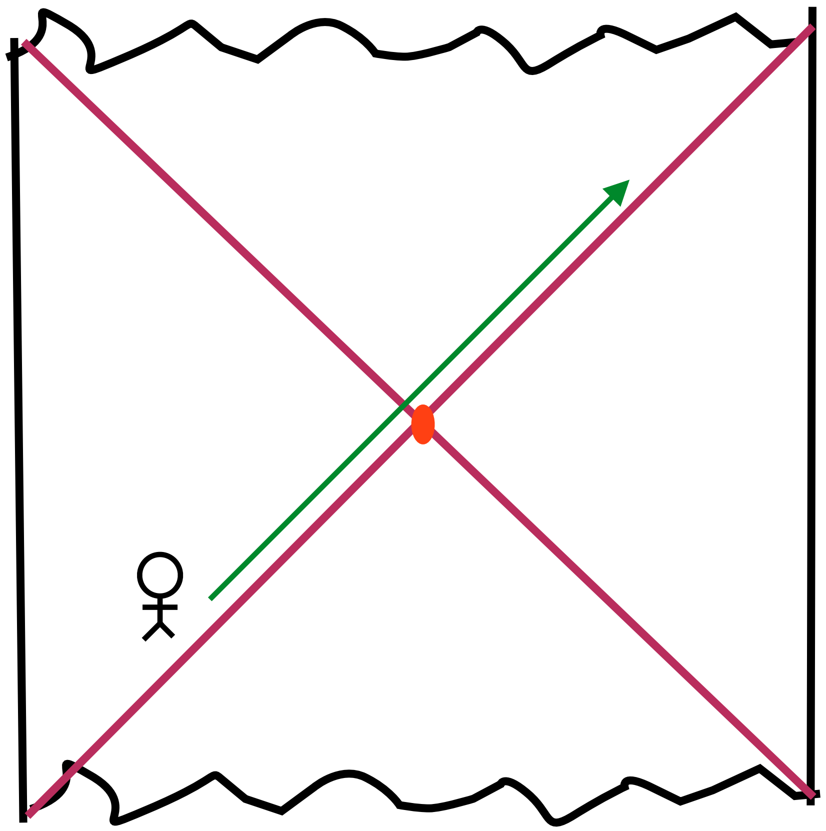
Can we go to the other side?

→No.

If we through matter with negative energy

(violate Averaged Null Energy Condition)





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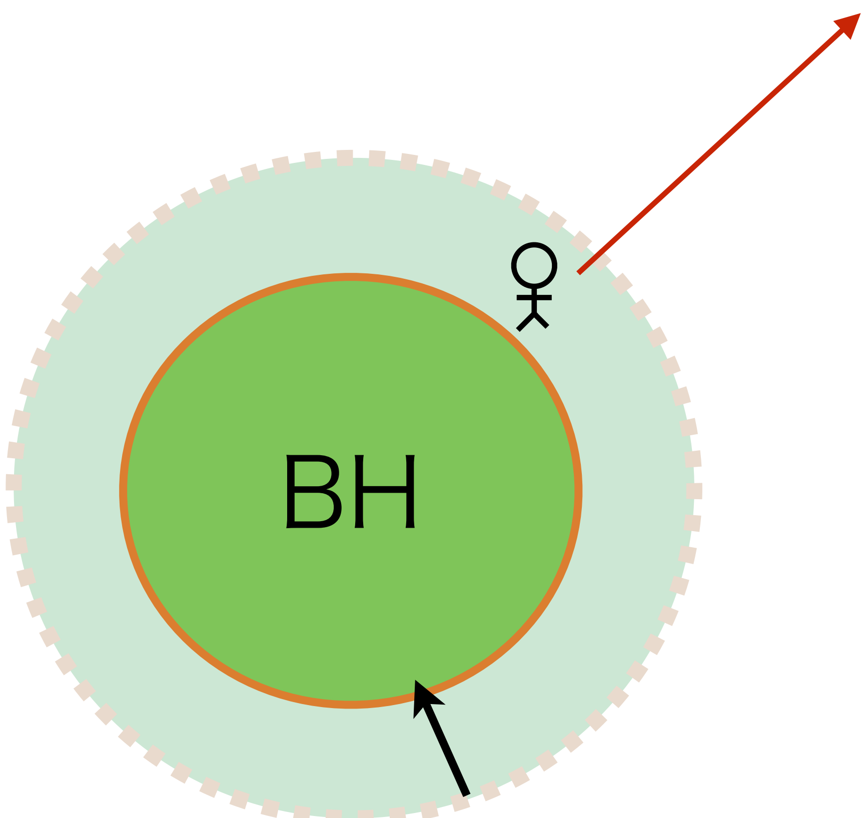
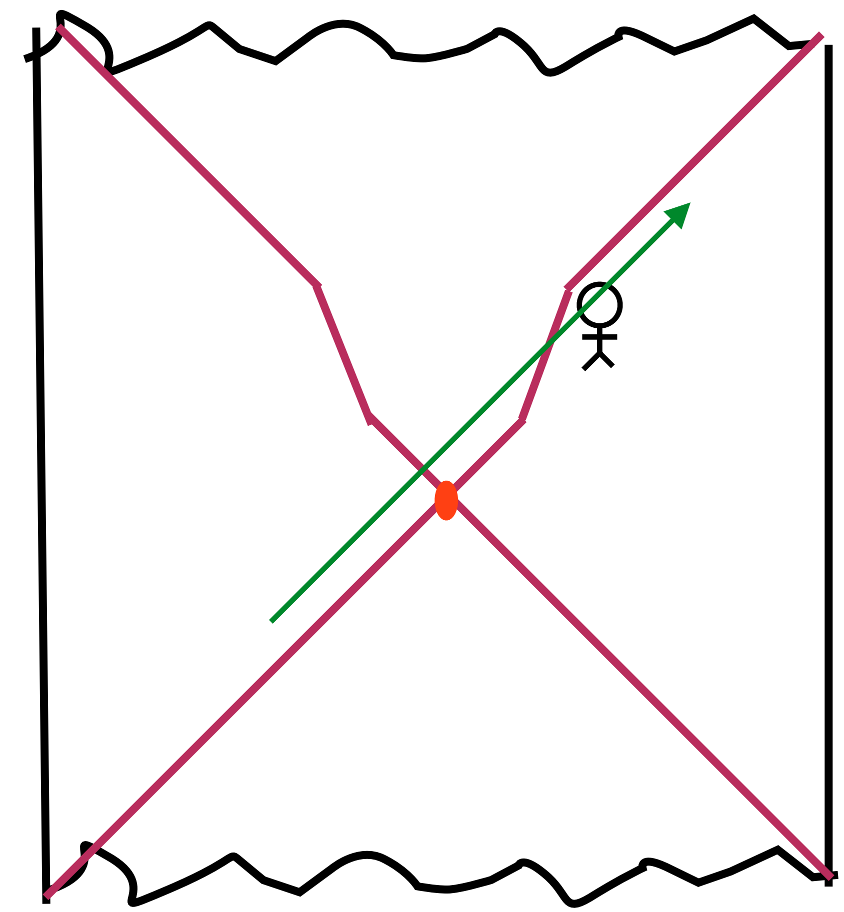


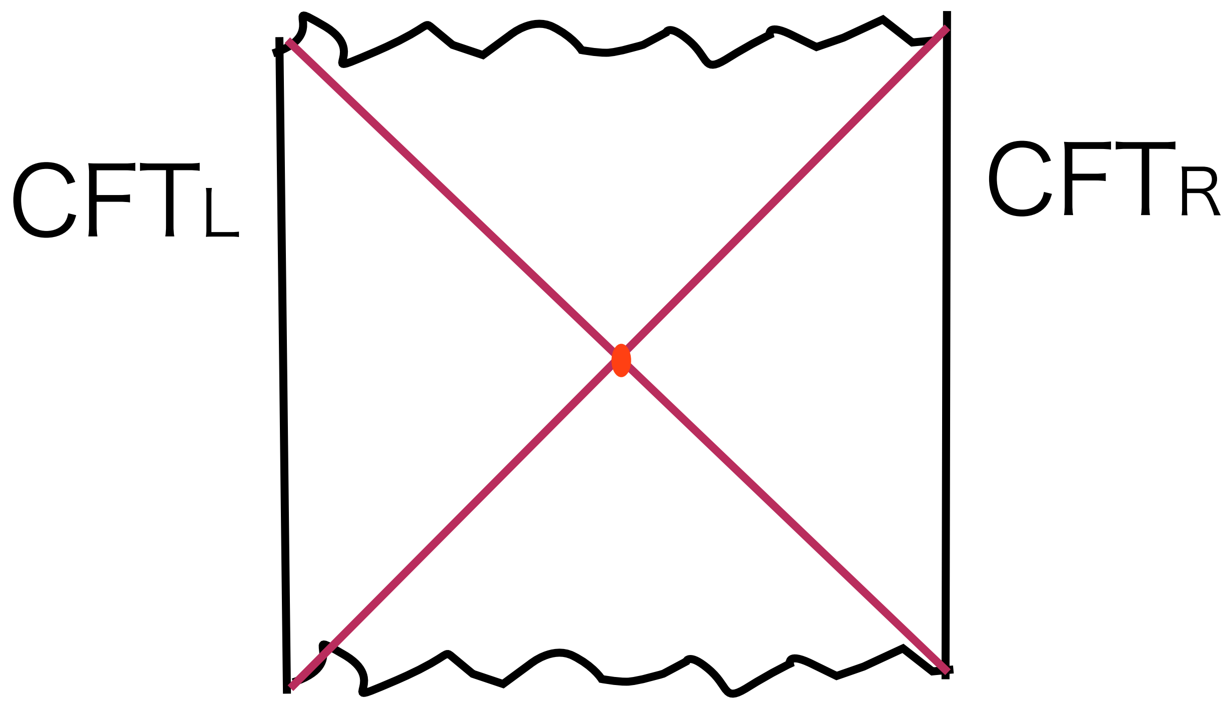


horizon radius decreases

→He can escape from Black Holes

Traversable wormhole via double trace deformation

[Gao-Jefferis-Wall, 16]



- Assume AdS/CFT
- CFTL and CFTR are decoupled:

$$H = H_L + H_R$$

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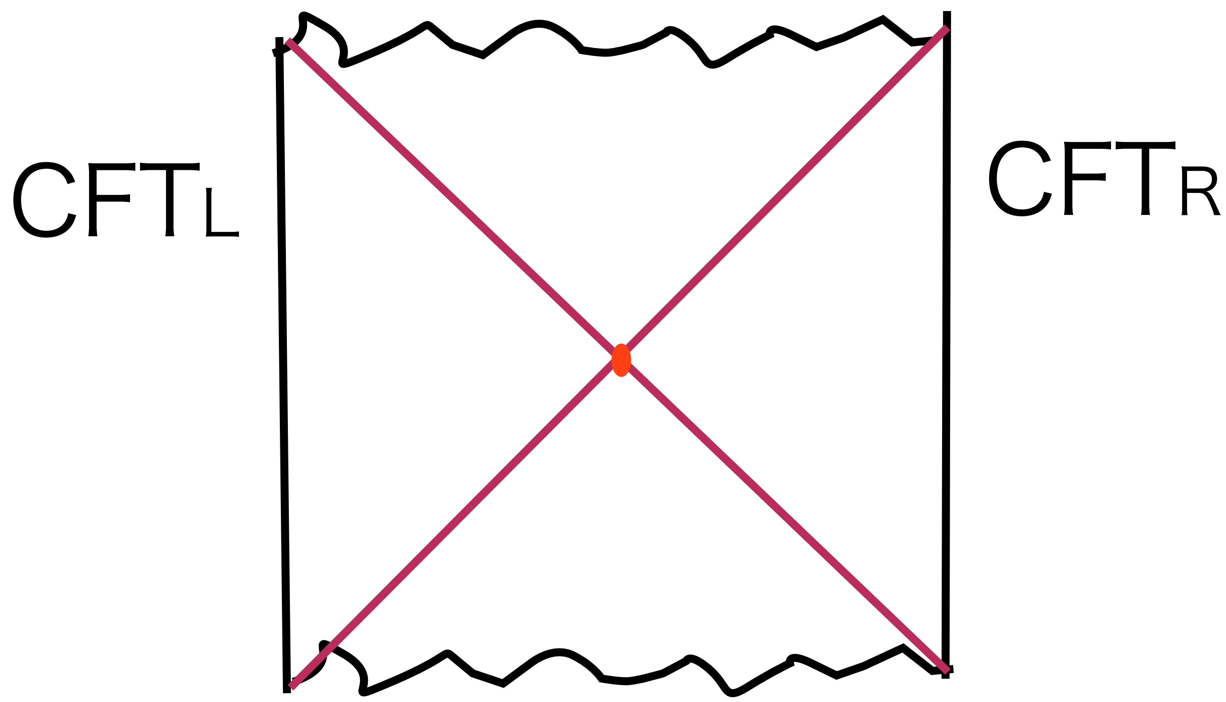

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$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{\frac{\beta}{2} E_n} |E_n\rangle_L \otimes |E_n\rangle_R$$

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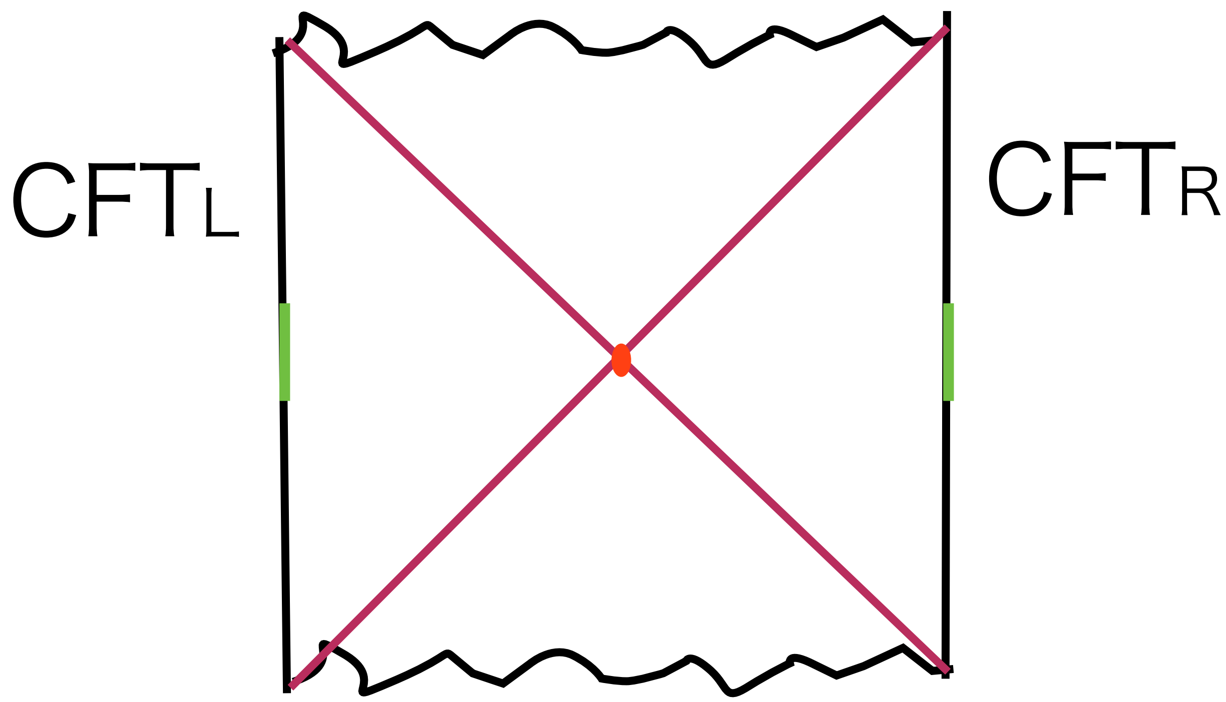

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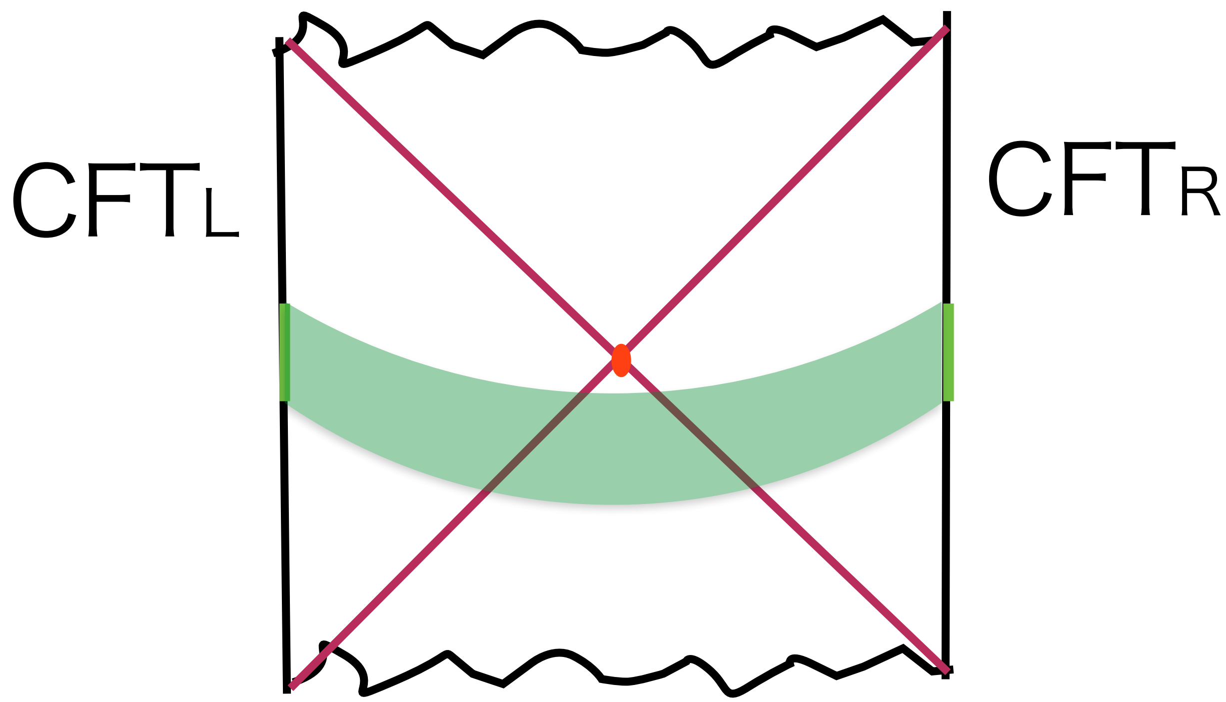

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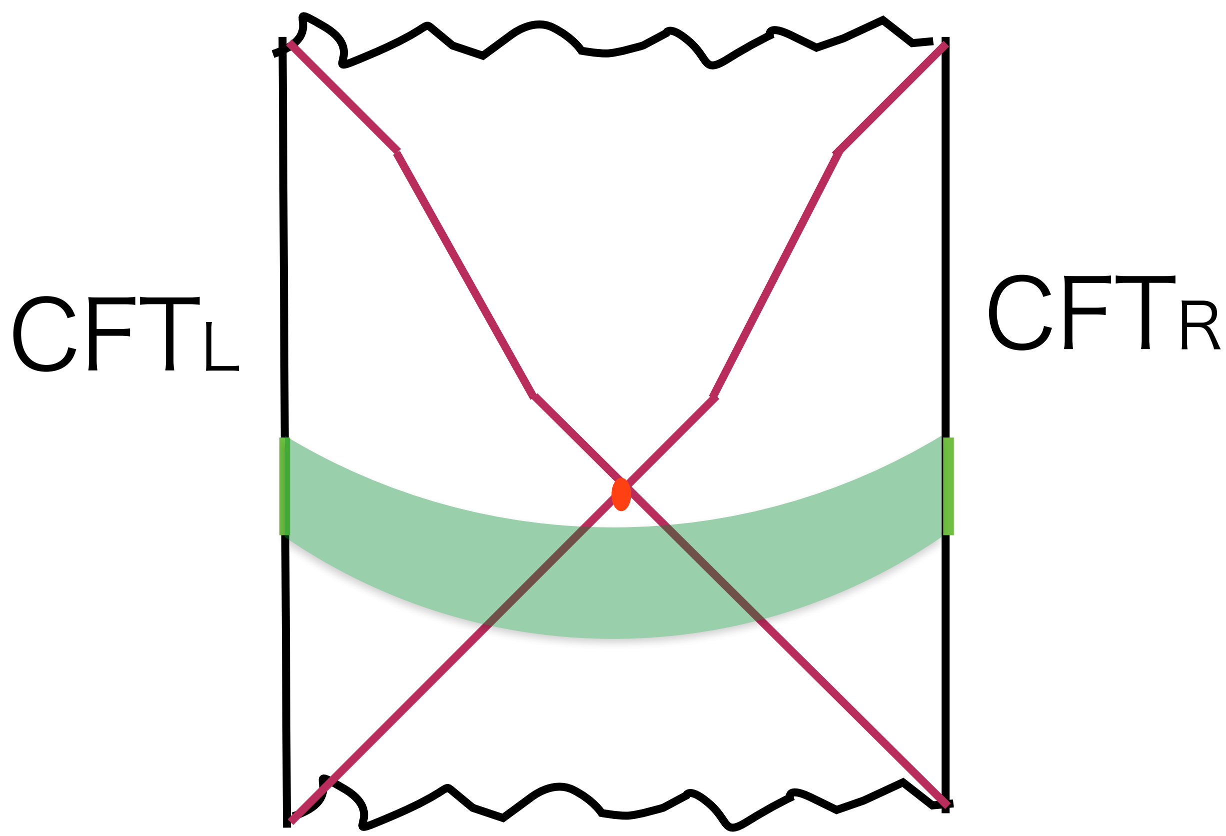
→negative energy by Casimir effect

(positive/negative depend on the sign of g)

Traversable wormhole via double trace deformation



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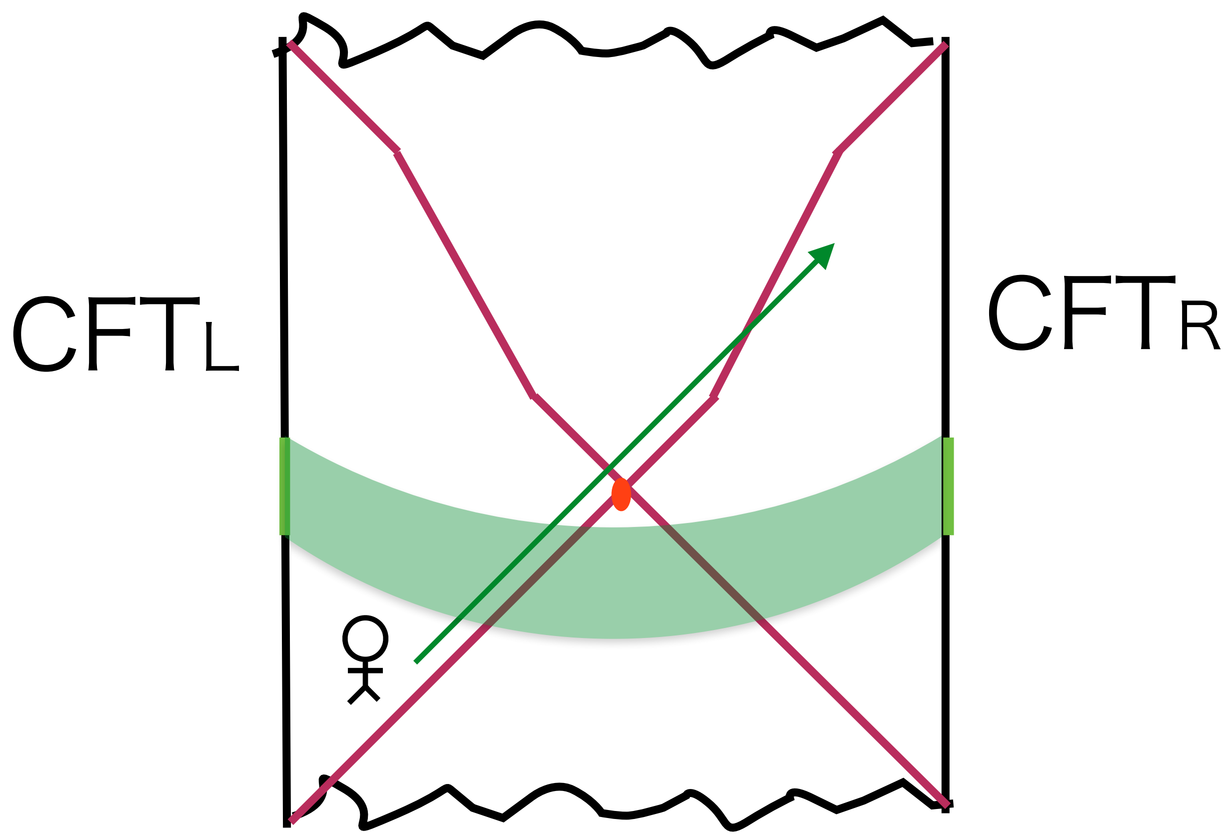

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(2)Nearly AdS2 gravity

(3)Traversable wormholes in Nearly AdS2 gravity

(4)Application to information problem

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2.Nearly AdS2 gravity dynamics

- exact AdS2 does not permit finite energy excitation

For example, $$\int \sqrt{g}R + S_{matter} \quad \rightarrow \quad T_{\mu\nu} = 0$$

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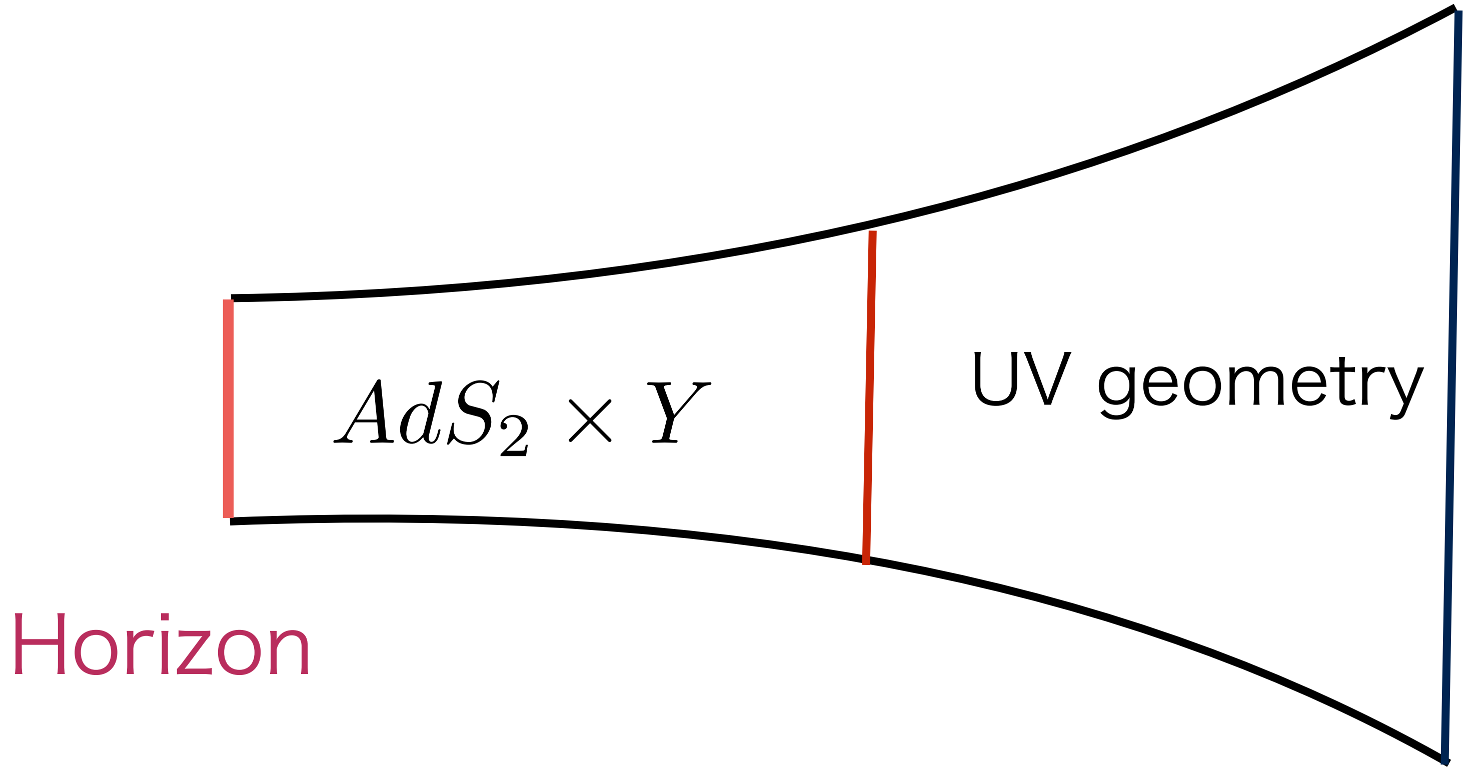

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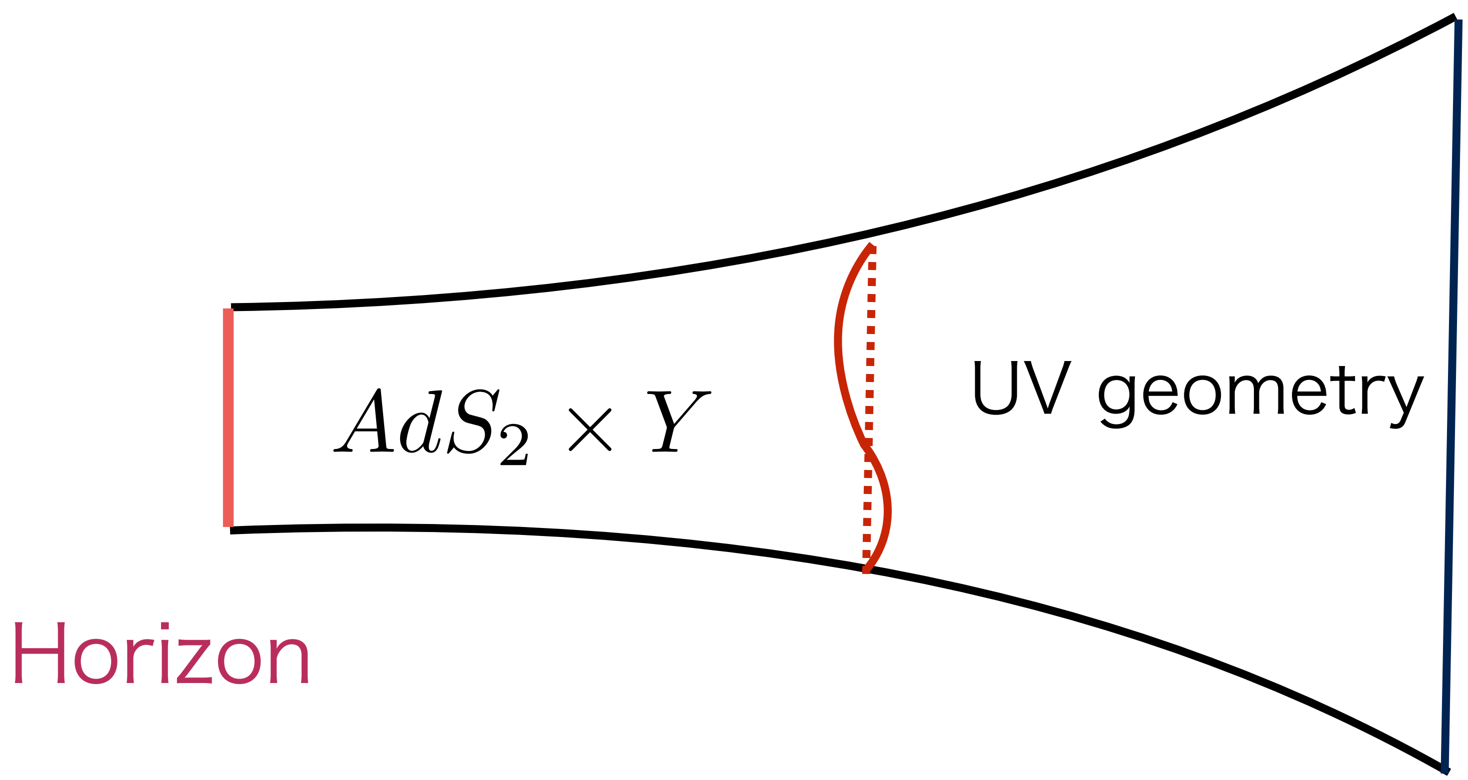
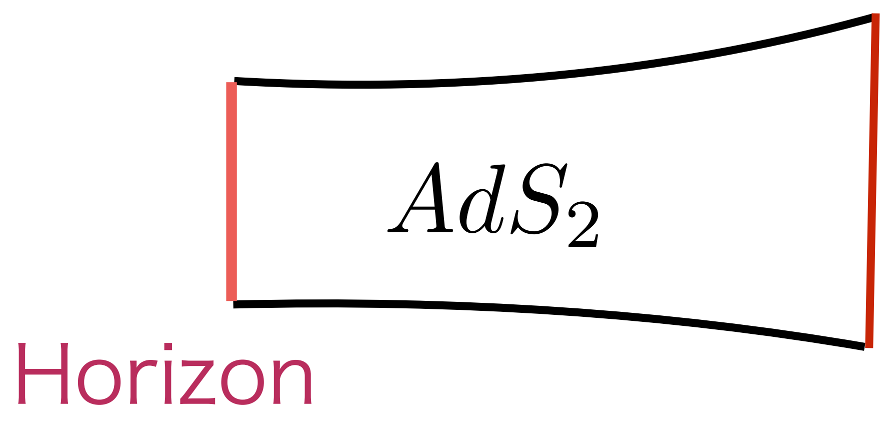
- middle geometry can be changed by finite energy excitation



- Consider a model of following part.



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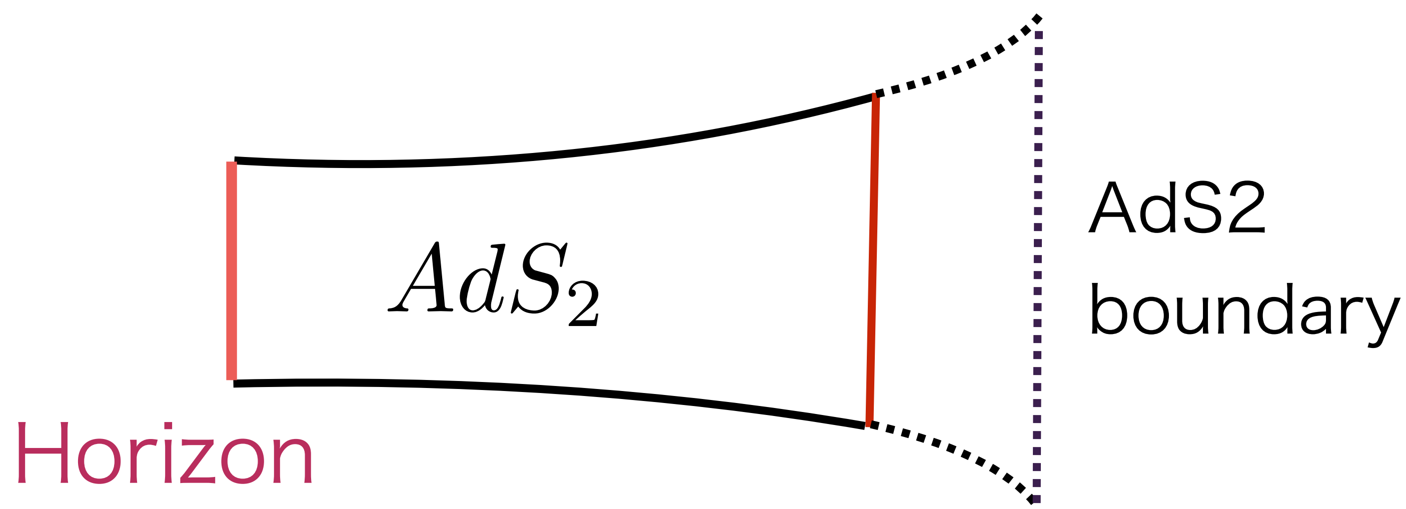

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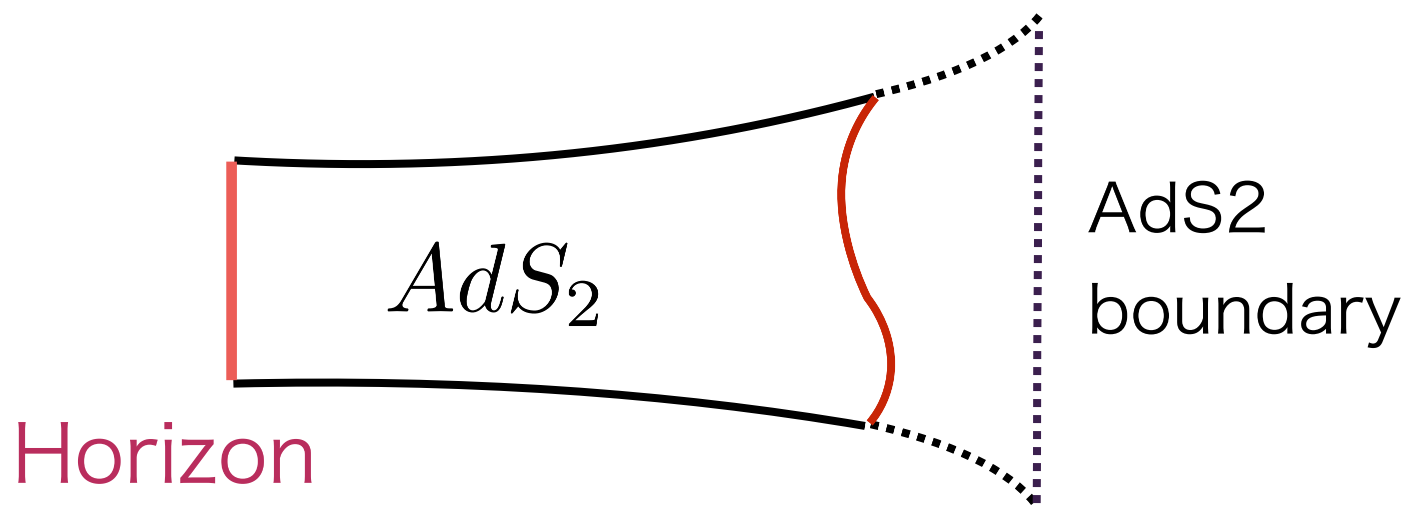

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- We want to include the effect of finite energy excitation

→ include the term that perturb from AdS2

$$\frac{1}{16\pi G}\int \phi\sqrt{g}(R+2)+\frac{1}{8\pi G}\int_{bdy}\phi_b K$$

[Teitelboim 83]
[Jackiw 85]
[Almheiri-Polchinski 14]

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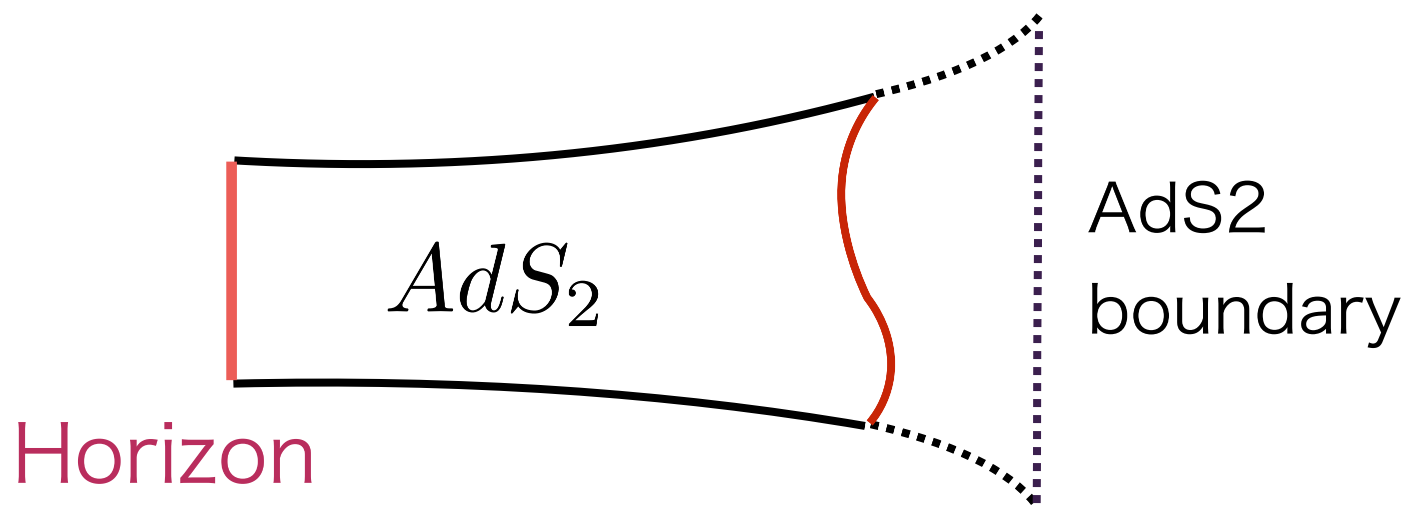

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- We call this nearly AdS2 gravity

Nearly AdS2 gravity

$$\frac{1}{16\pi G}\int \phi\sqrt{g}(R+2)+\frac{1}{8\pi G}\int_{bdy}\phi_b K$$

Schwarzian action: $S=-C\int du\{f(u),u\}$

$$\{f(u),u\}=\frac{f'''}{f'}-\frac{3}{2}\Big(\frac{f''}{f'}\Big)^2$$

describes the dynamics of boundary cutoff curve

Dynamics are encoded the motion of boundary curve !

Nearly AdS2 gravity

$$\frac{1}{16\pi G}\int \phi\sqrt{g}(R+2)+\frac{1}{8\pi G}\int_{bdy}\phi_b K$$

SYK model

(1d Nearly CFT)

low energy

Schwarzian action: $S=-C\int du\{f(u),u\}$

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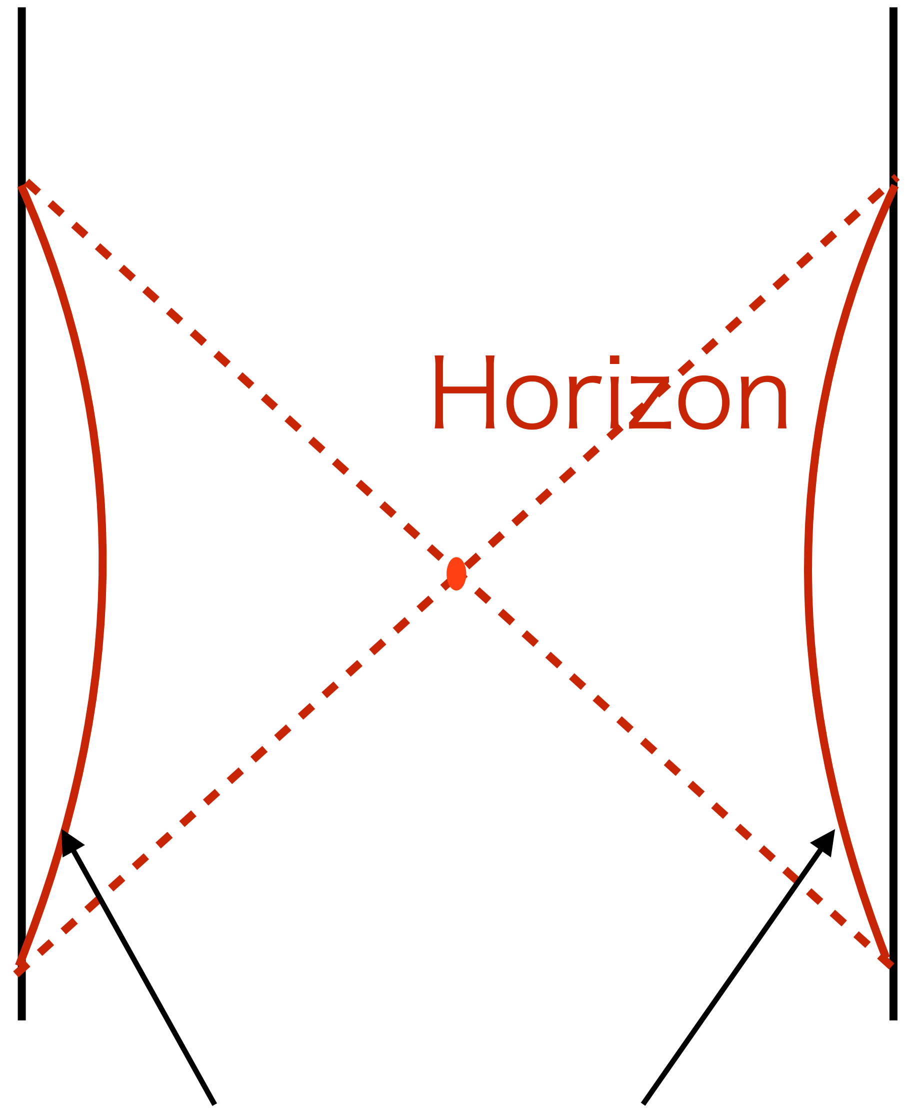
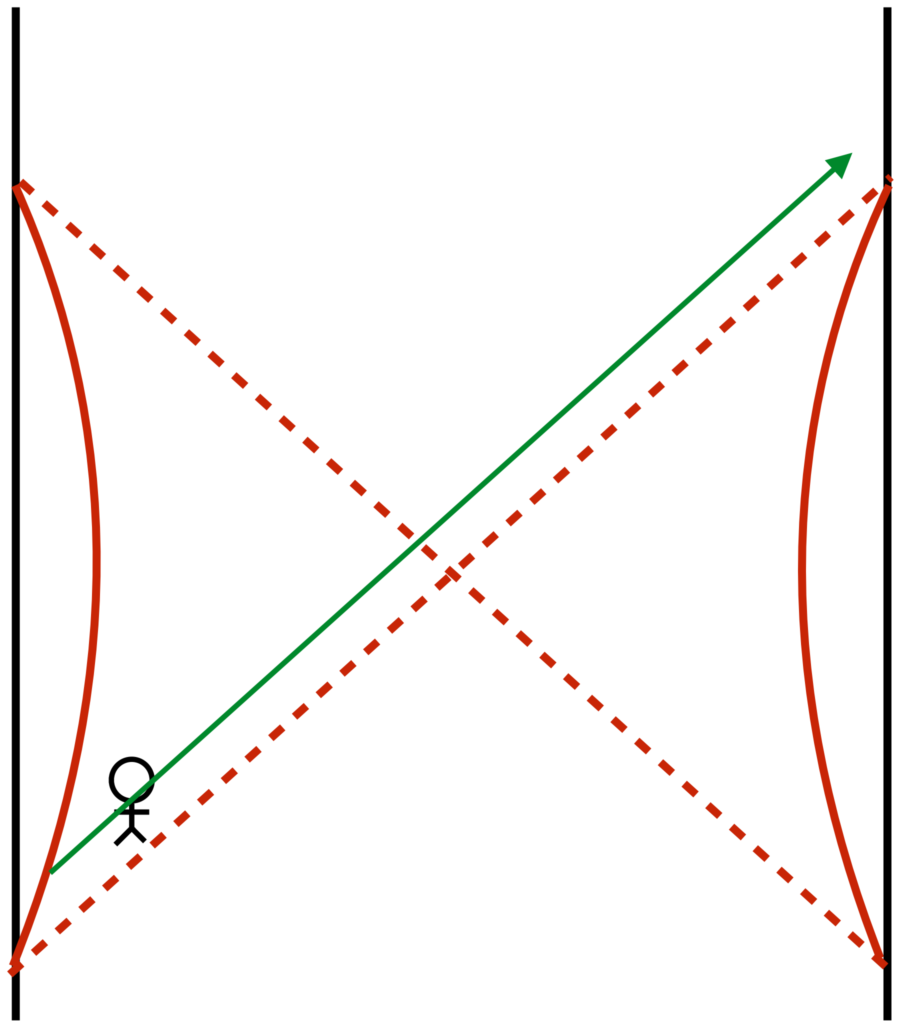
3.Traversable wormhole in Nearly AdS2

BH solution
(BH/Rindler patch)



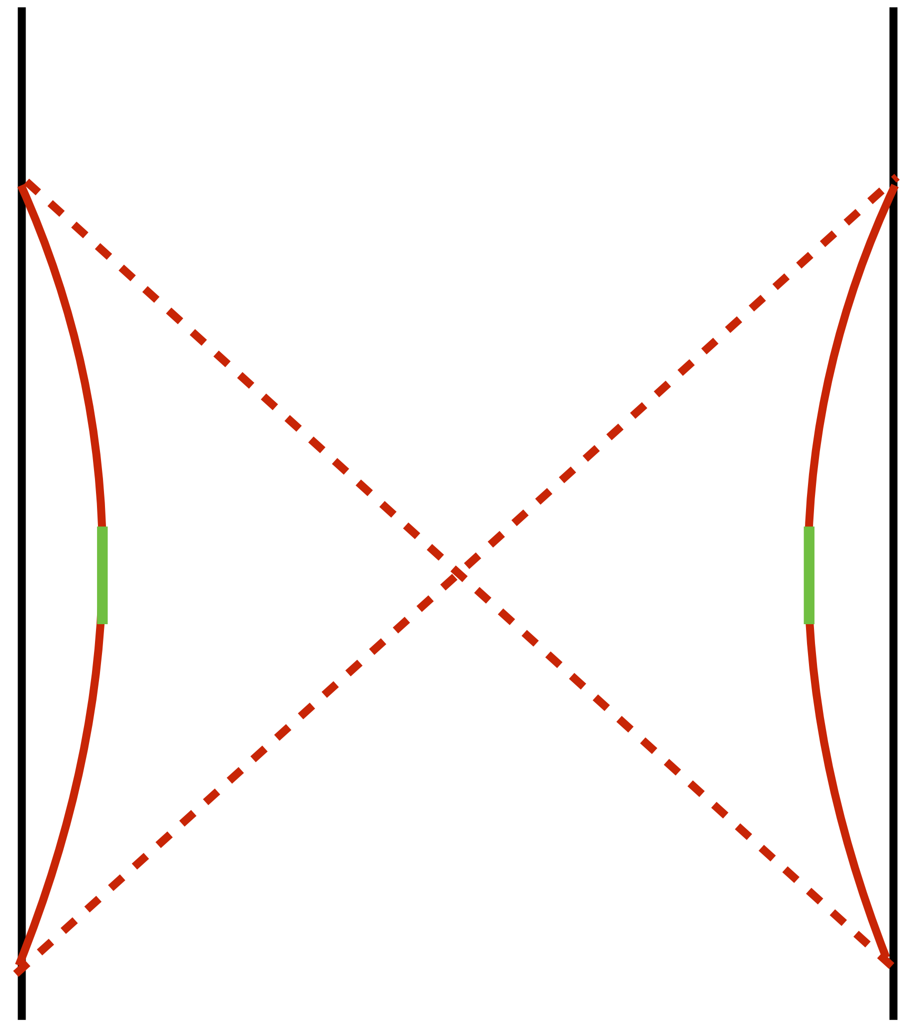
cutoff curve in global AdS2
(described by Schwarzian action)

What we need is to see the dynamics of boundary curves



BH solution

not traversable



put two side interaction

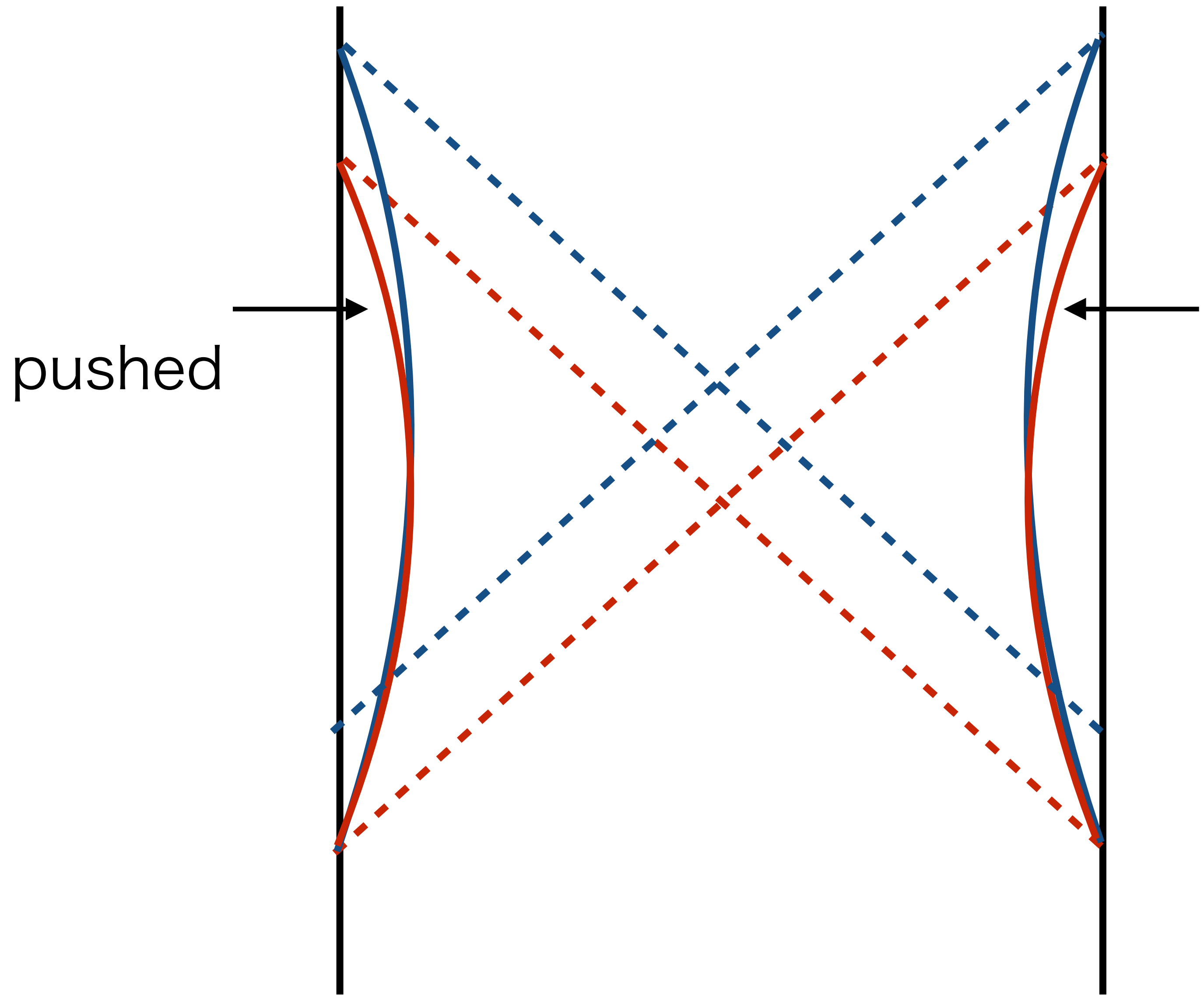
$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

$$\downarrow$$

$$e^{ig\langle\phi_L(t_L)\phi_R(t_R)\rangle}$$
$$\sim e^{-igV_{pot}}$$

$$V_{pot} \sim e^{-m\rho}$$

ρ :AdS distance



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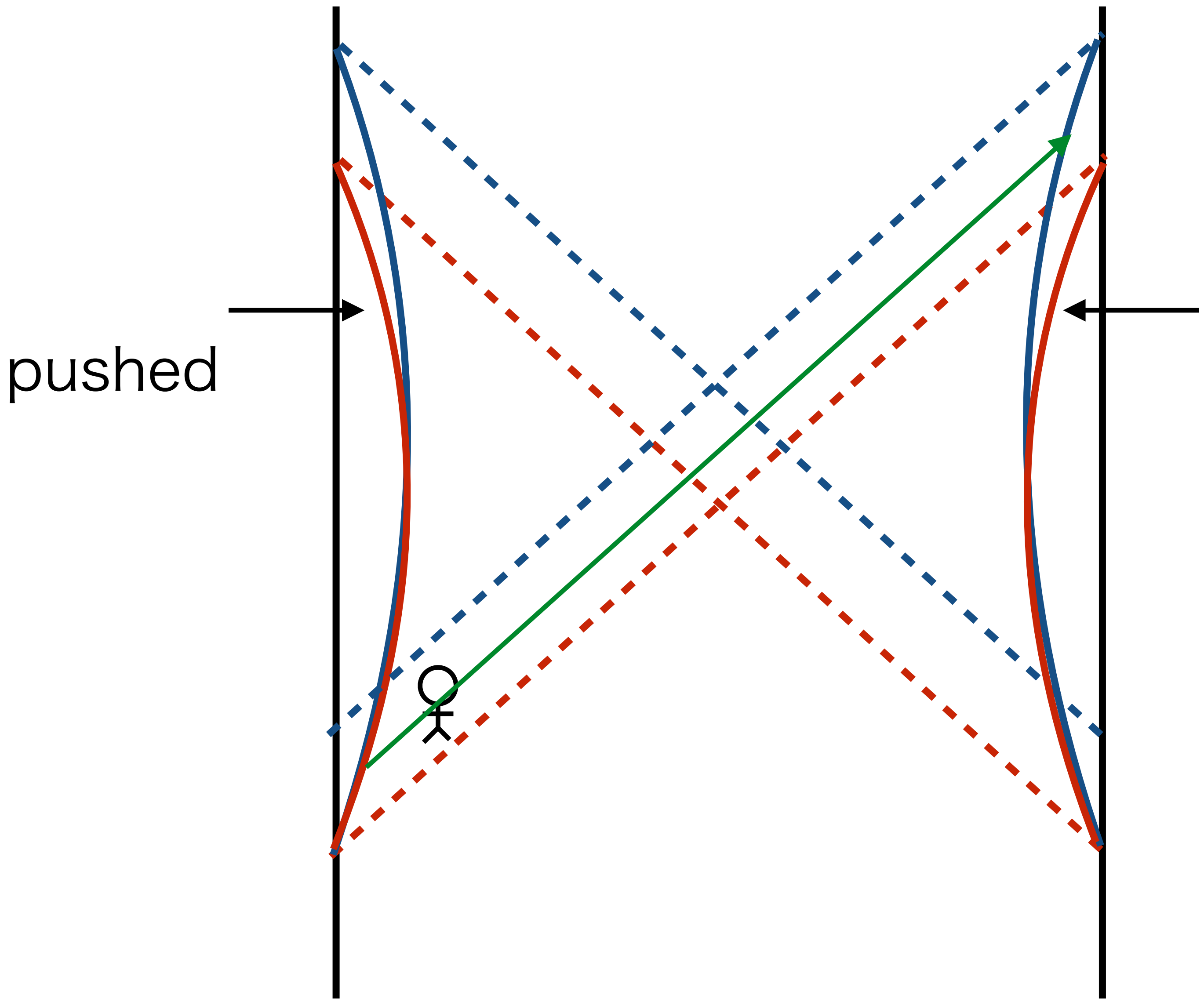
g > 0: attractive force (wormhole becomes traversable)



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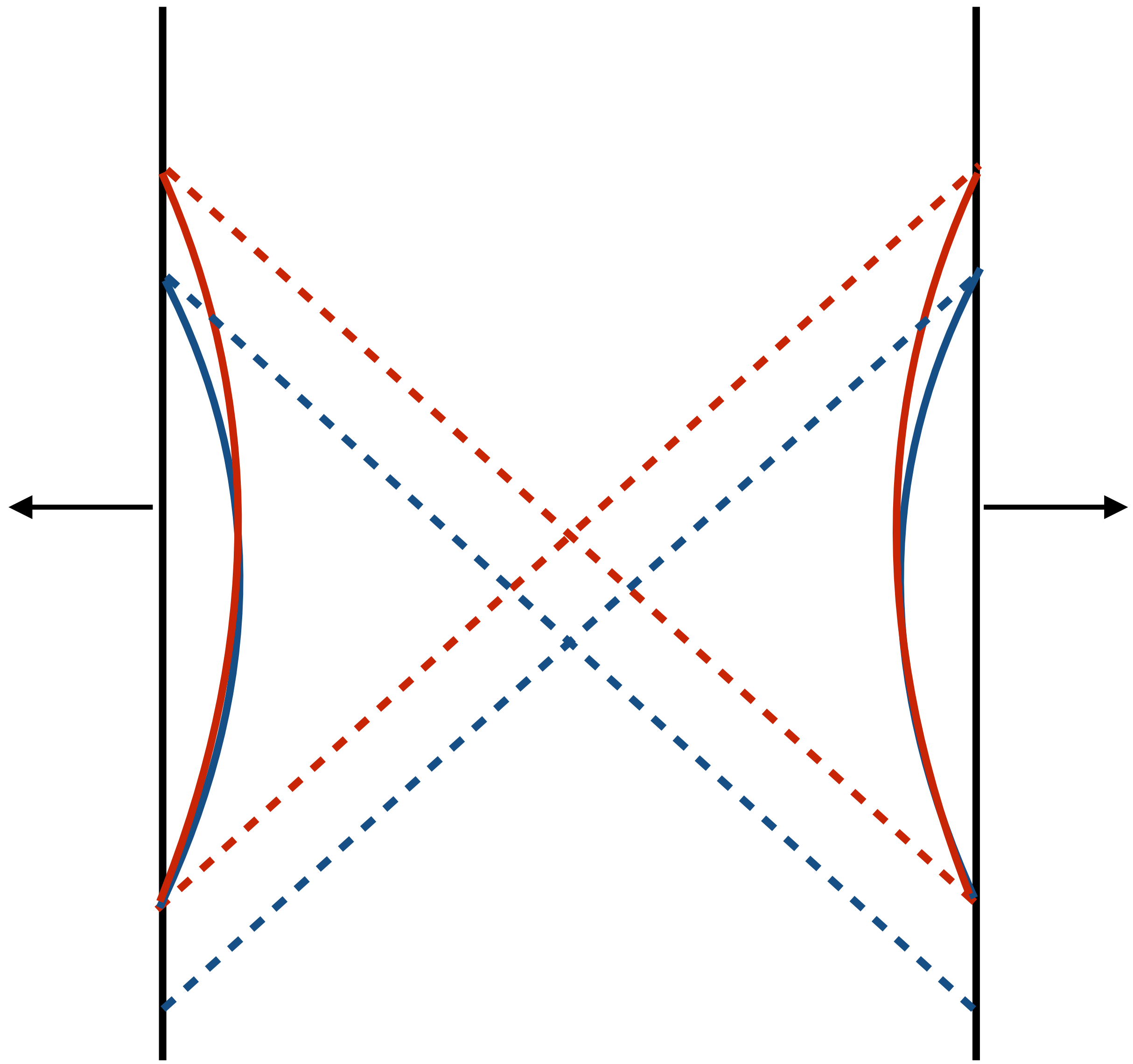

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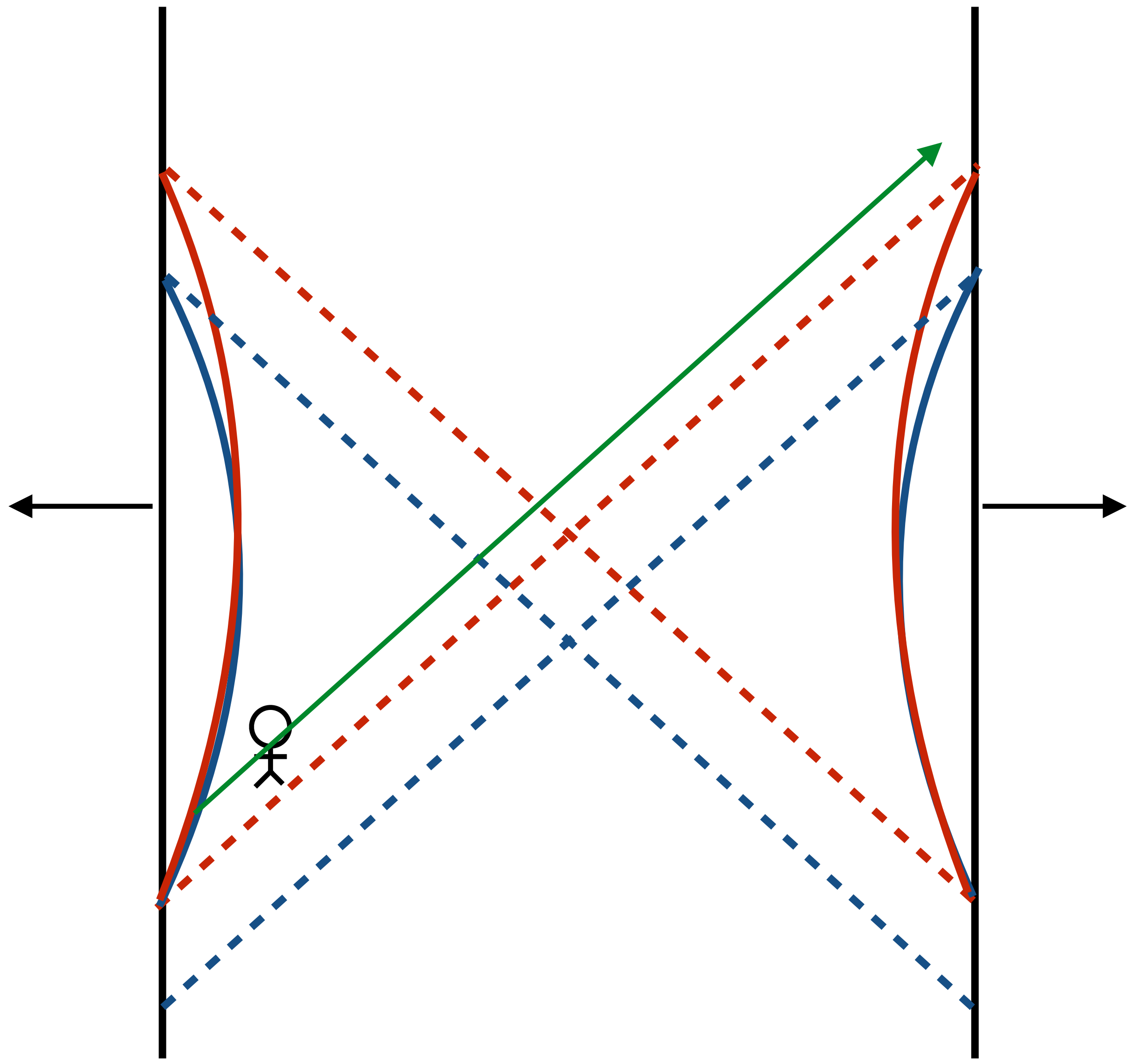
g < 0: repulsive force (not traversable)



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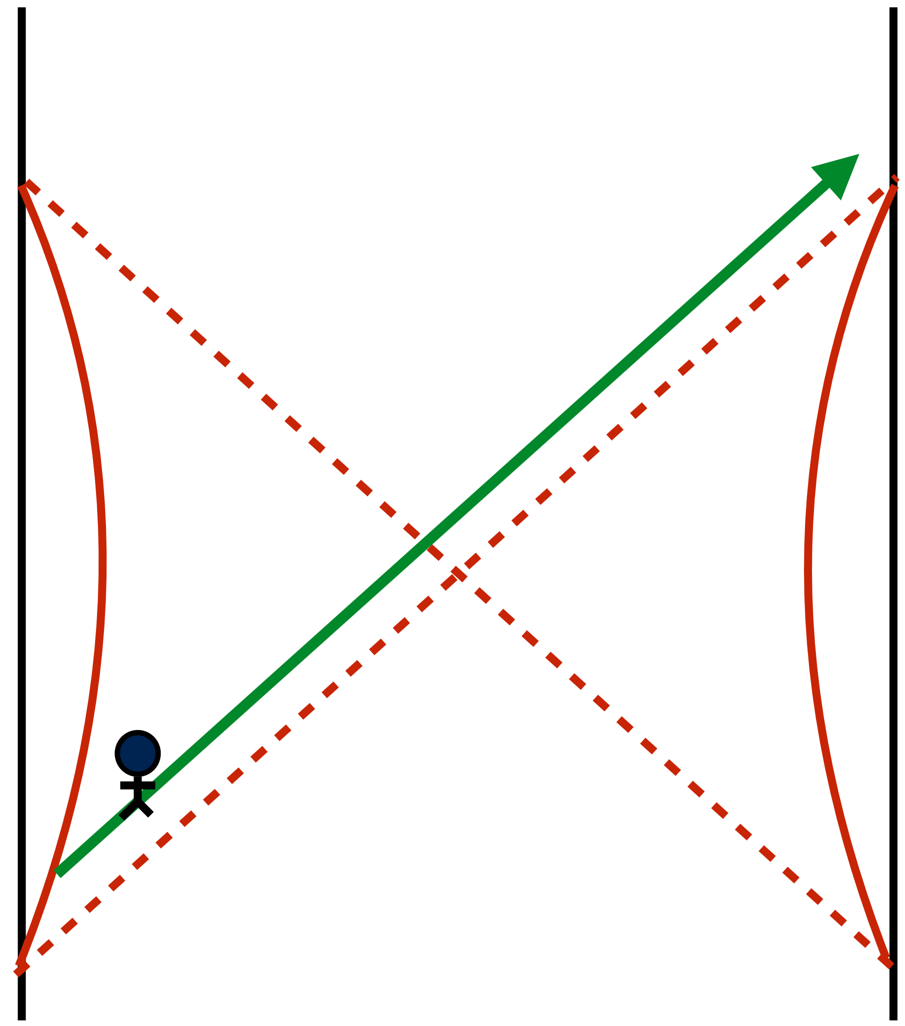
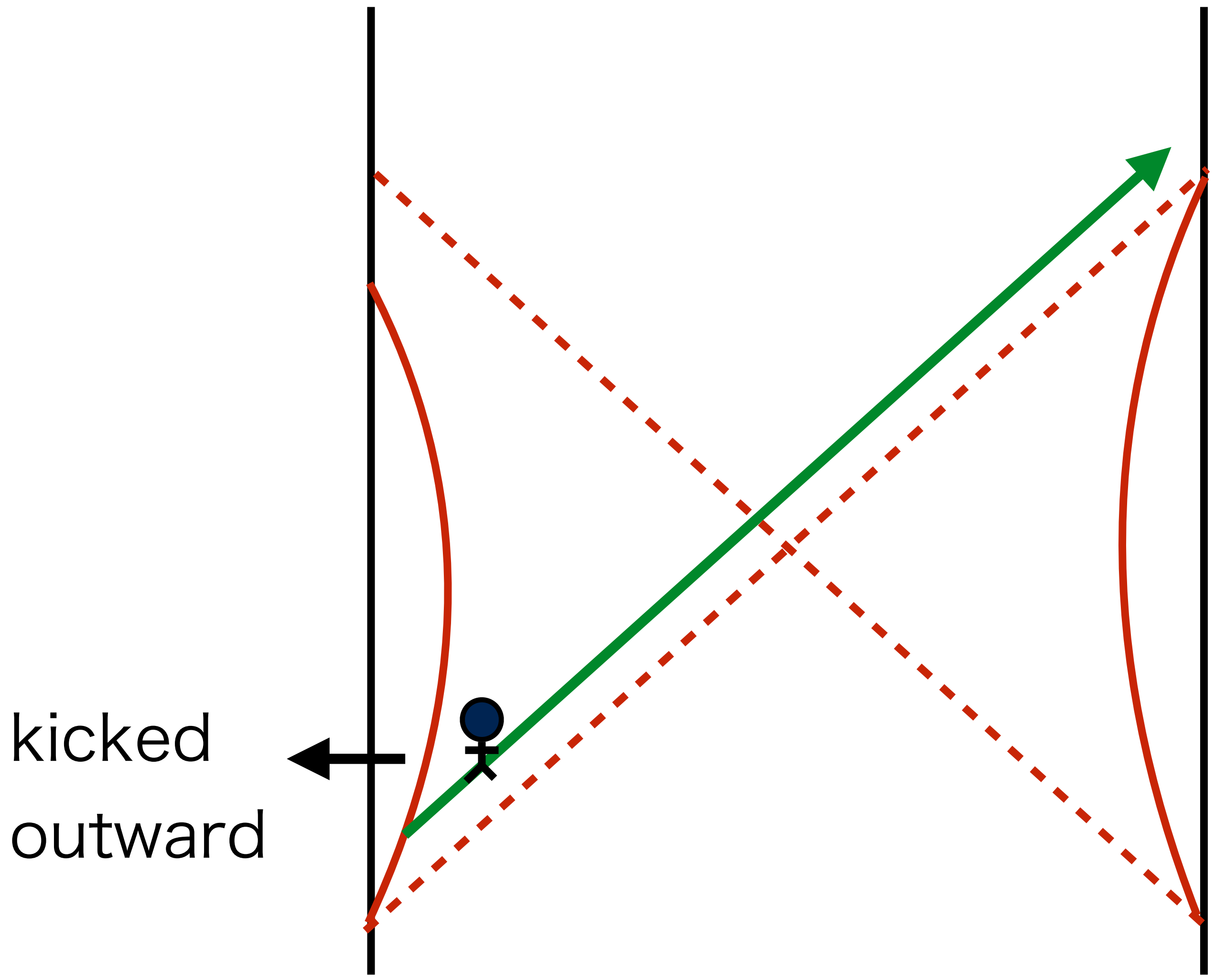
g < 0: repulsive force (not traversable)

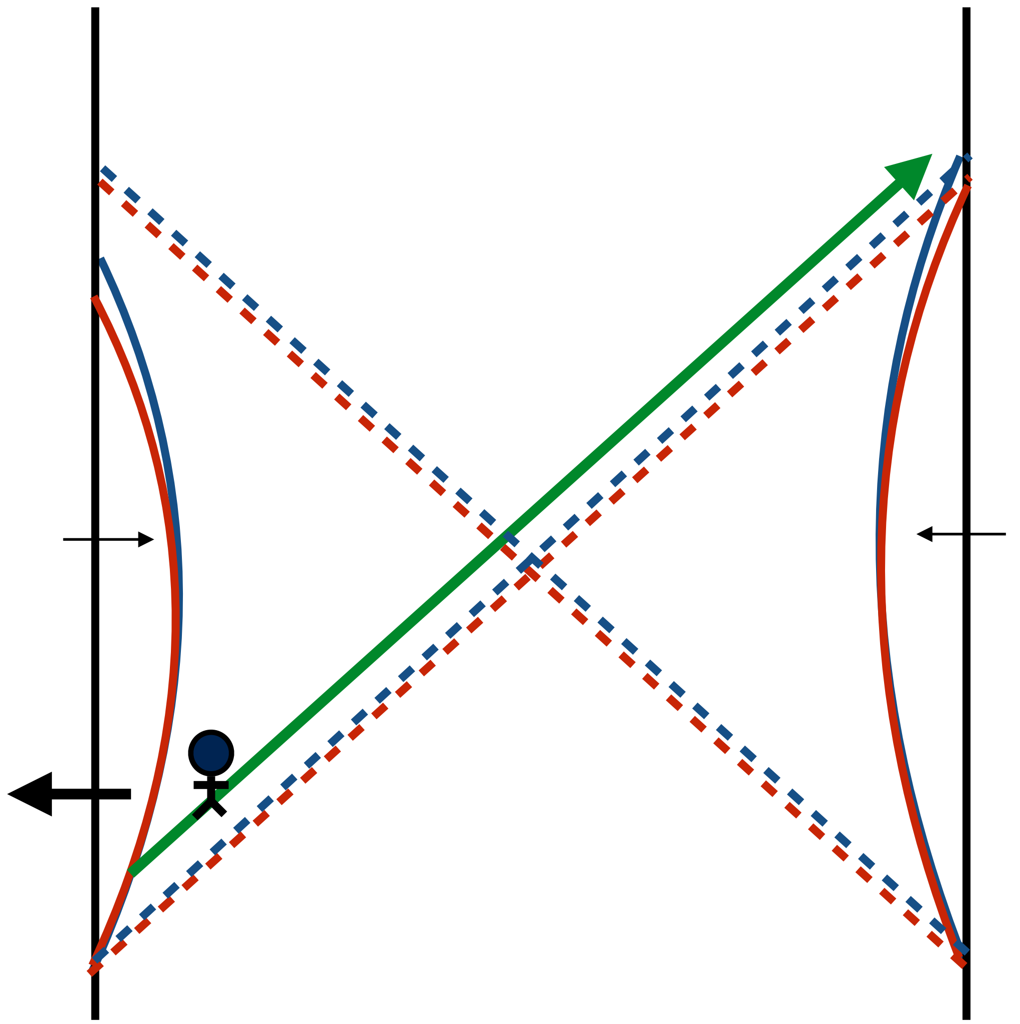
Back reaction



Back reaction



Back reaction



$$V_{pot} \sim e^{-m\underline{\rho}}$$

$\underline{\rho}$:AdS distance
→becomes long by back reaction

(1)Introduction/Review of traversable wormholes

(2)Nearly AdS2 gravity

(3)Traversable wormholes in Nearly AdS2 gravity

(4)Application to information problem

(5)Conclusion

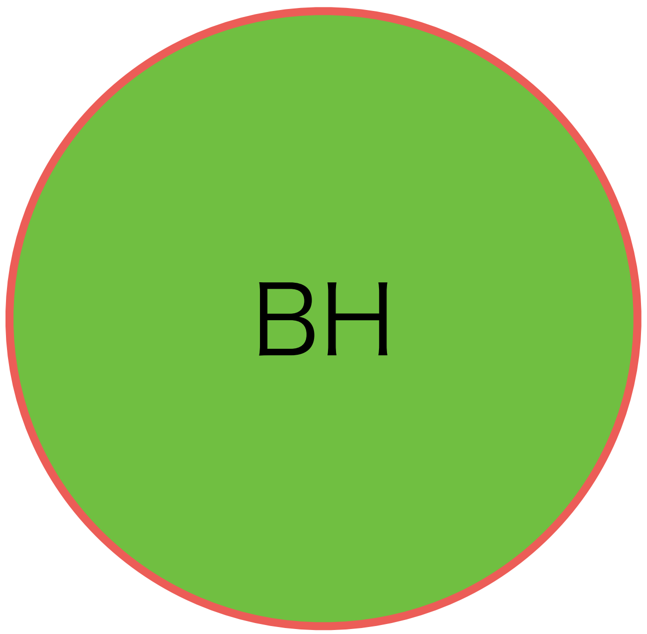
4.Application to information problem

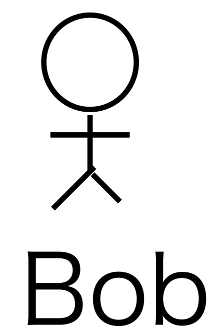
Problem: Can we extract the information behind the horizon from Hawking radiations?

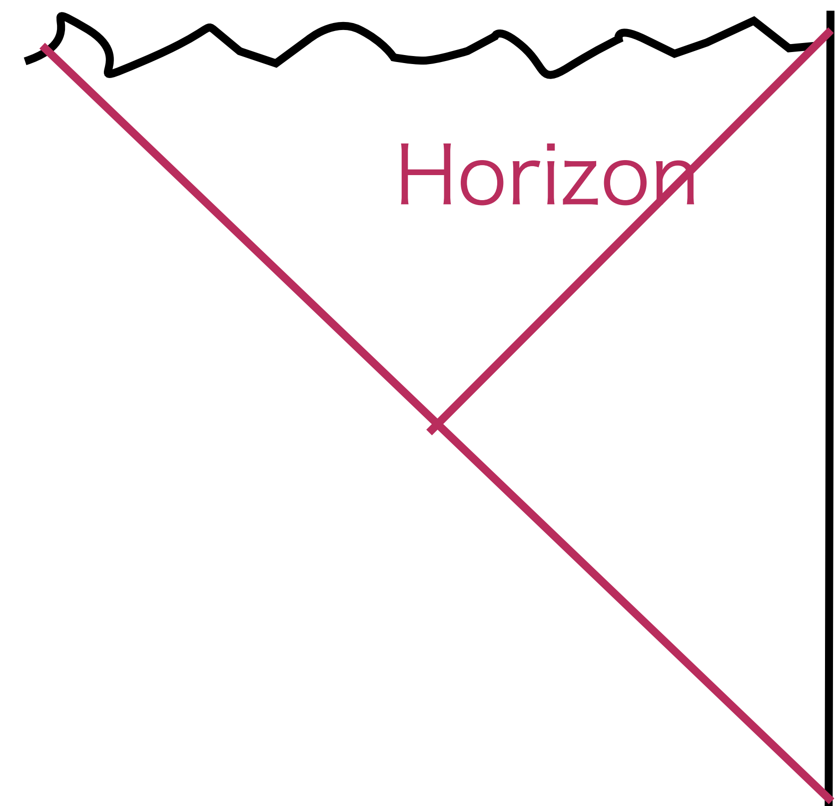
[Hayden-Preskill,07]

4.Application to information problem

Consider one side BH

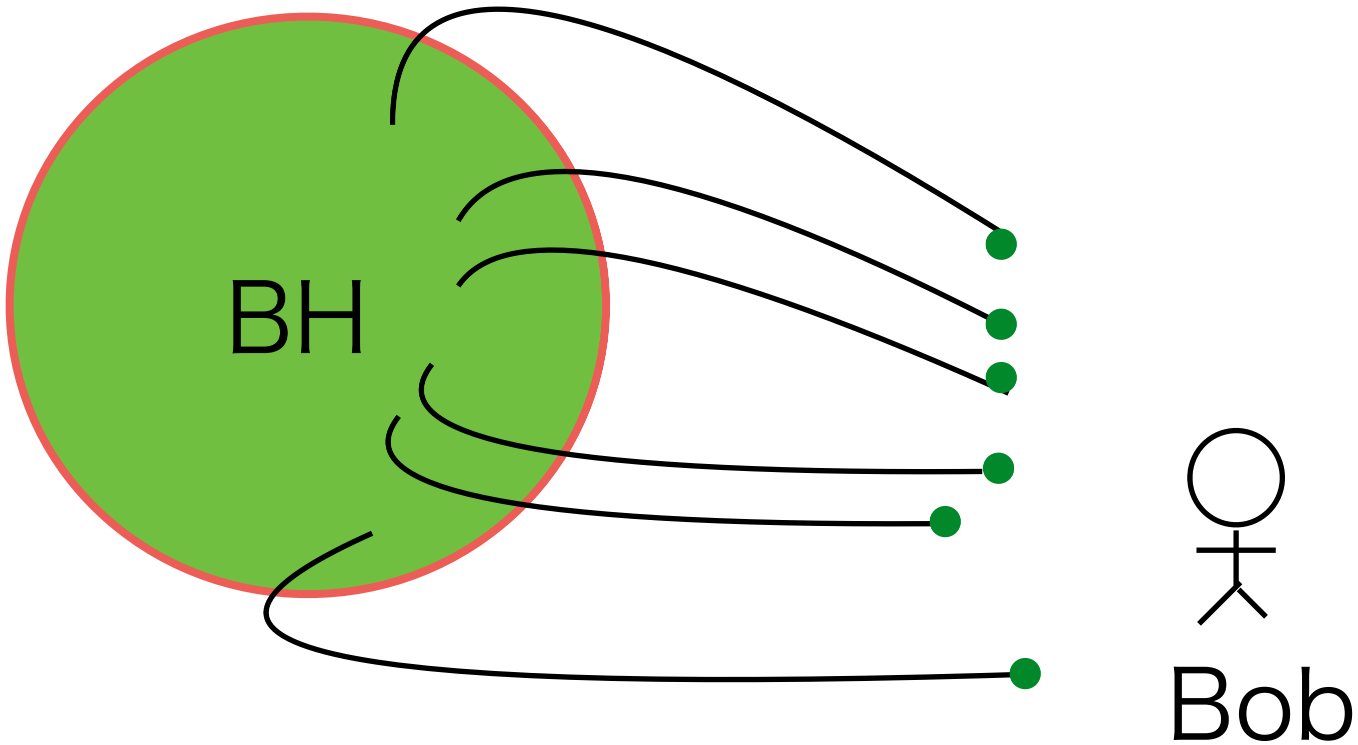


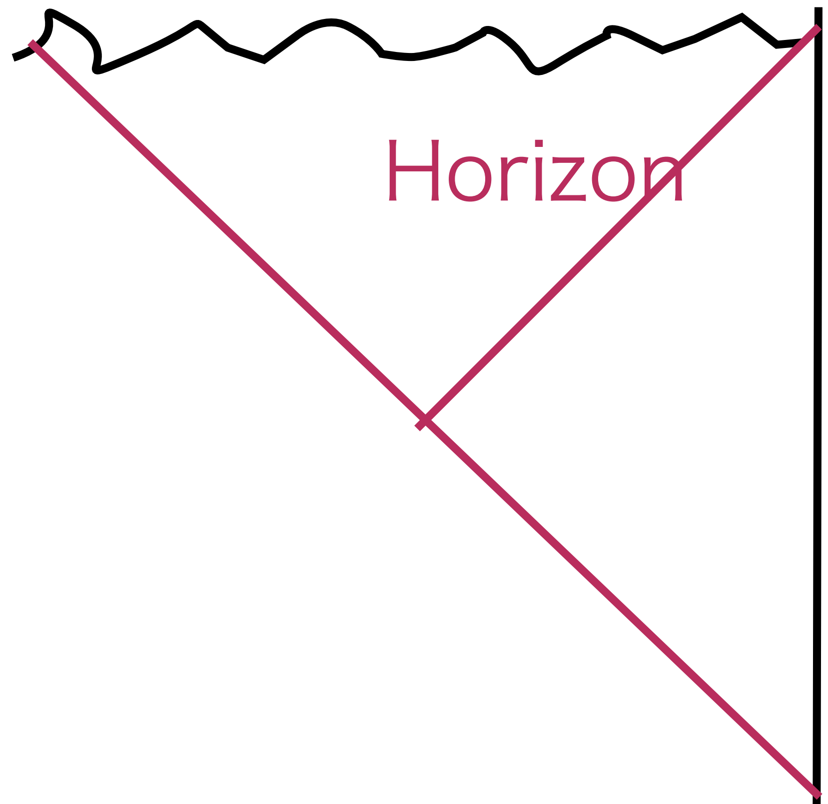




4.Application to information problem

Consider one side BH

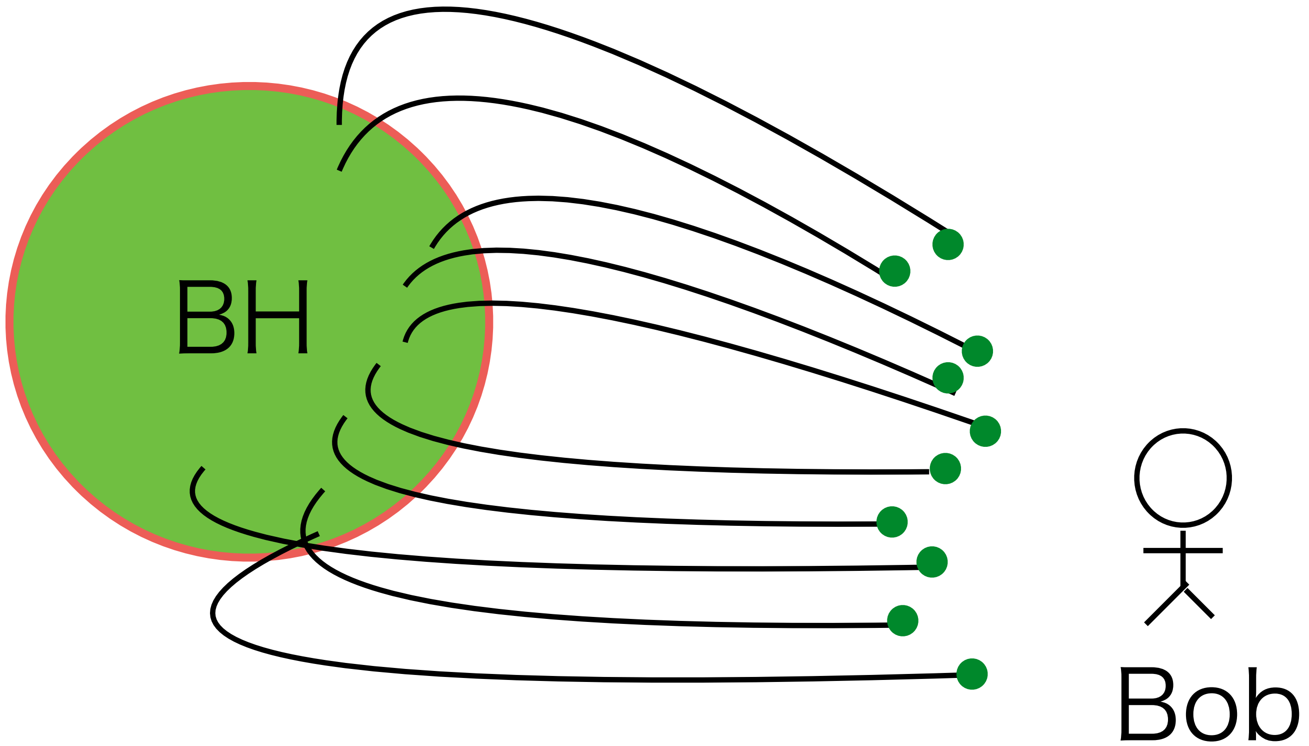


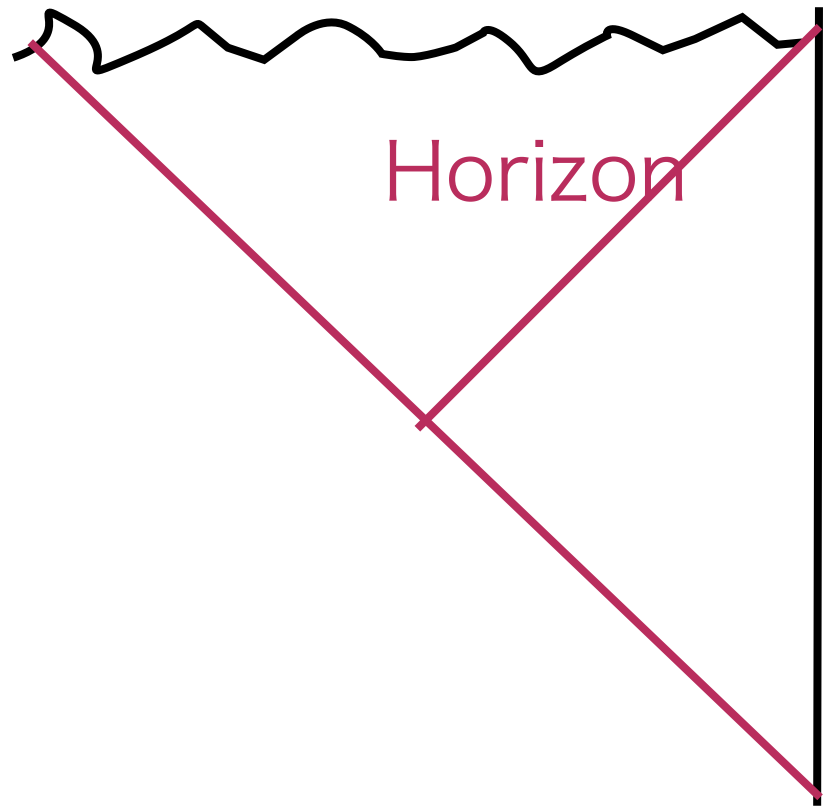


Bob collects Hawking radiations (entangled with BH)···

4.Application to information problem

Consider one side BH



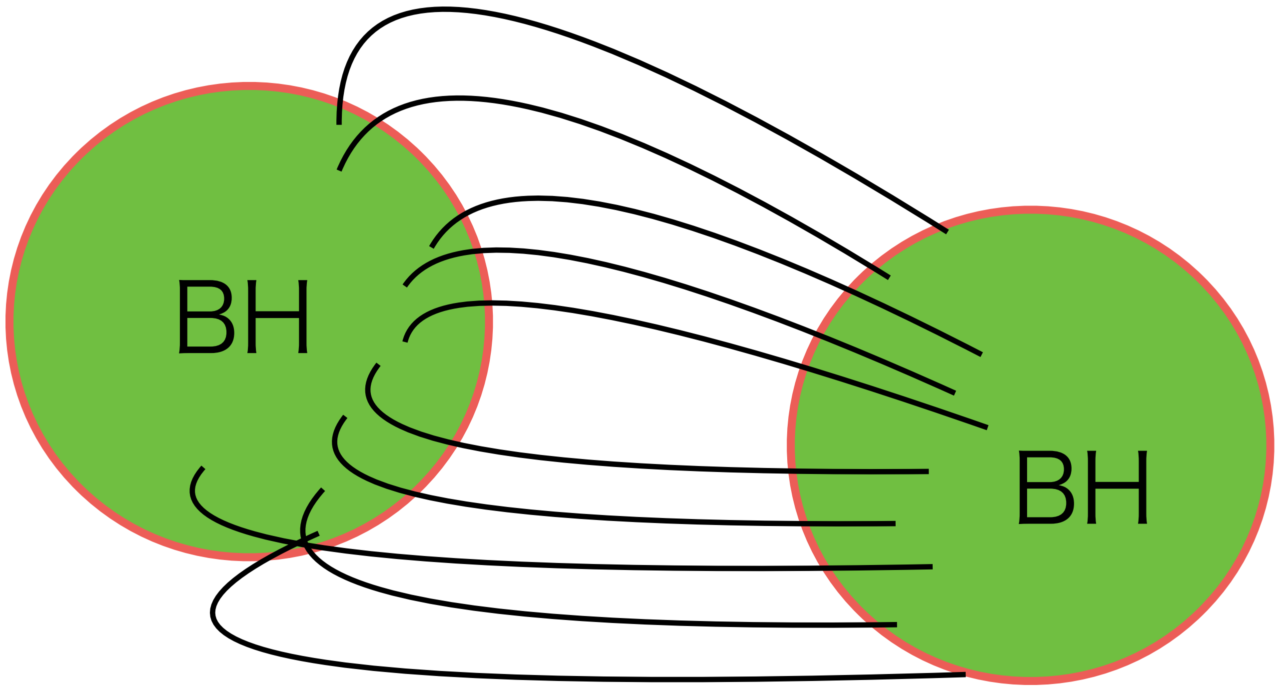


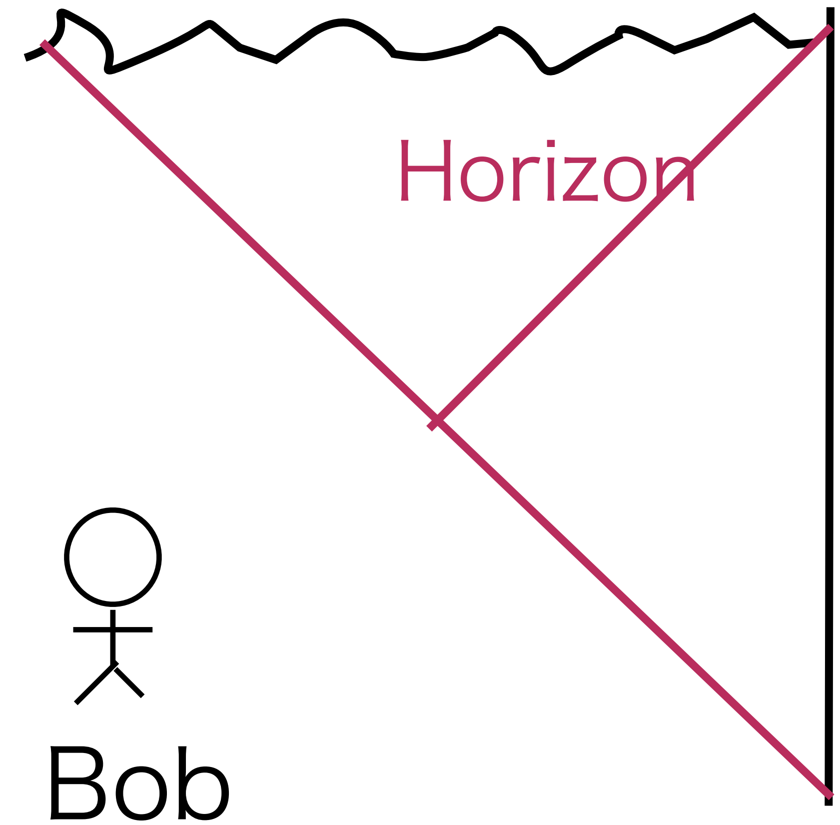
Bob collects Hawking radiations (entangled with BH)…

After half evaporation…

4.Application to information problem

Consider one side BH





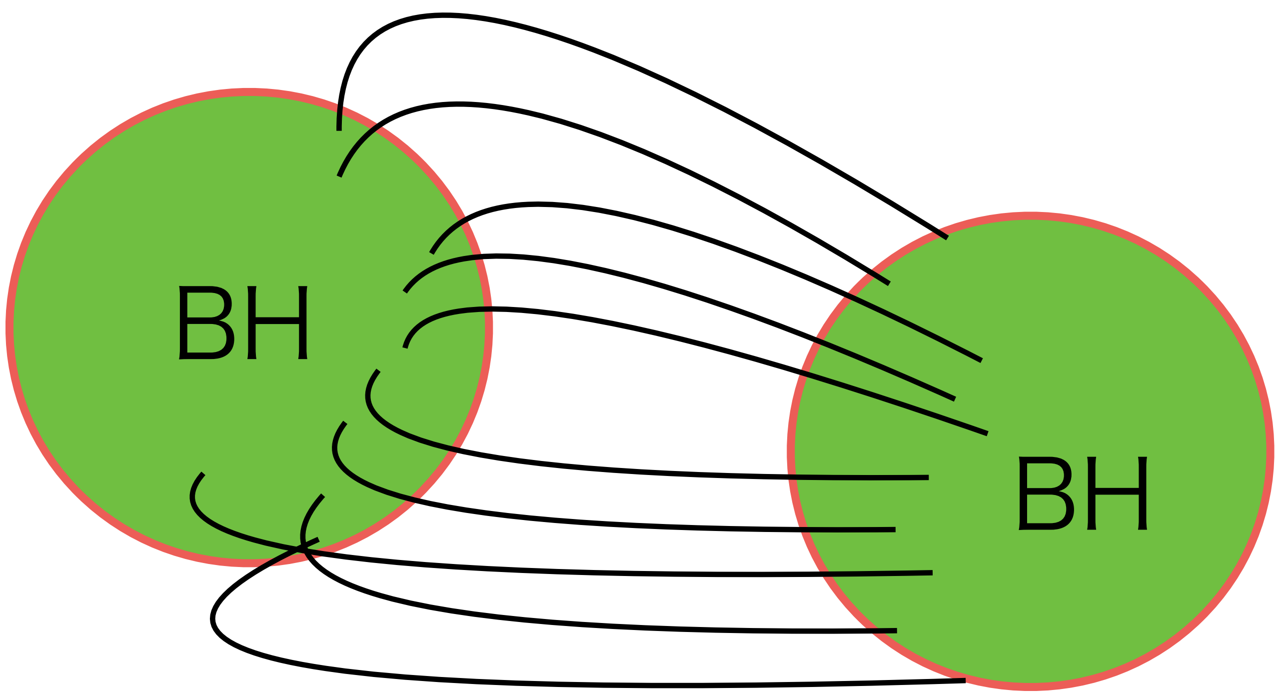
Bob collects Hawking radiations (entangled with BH)···

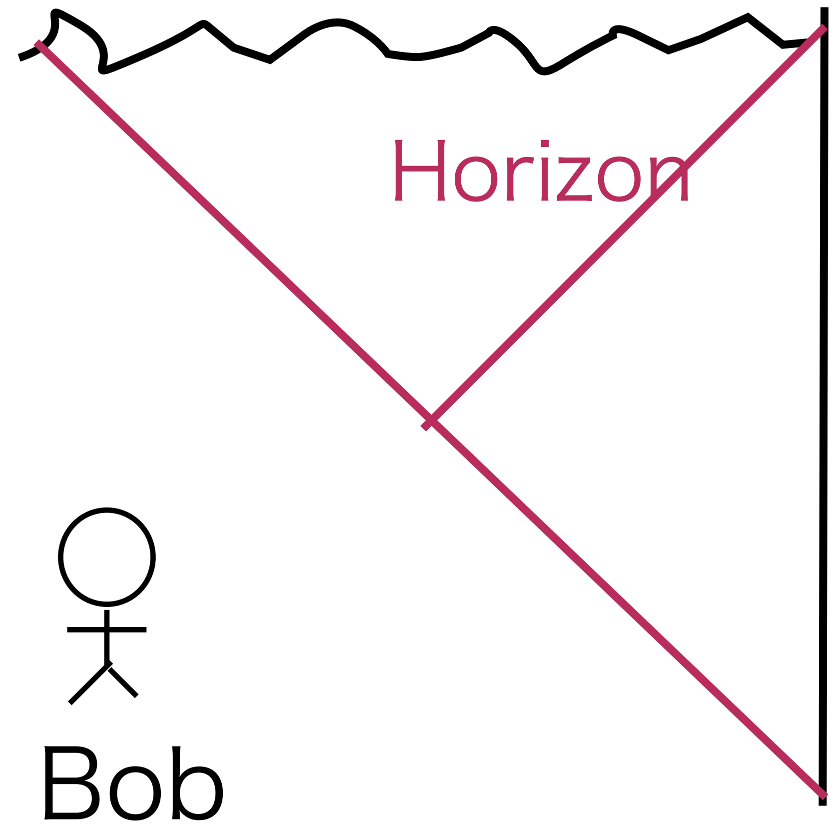
After half evaporation···

Bob create 2nd BH by radiations

4.Application to information problem

Consider one side BH





Bob collects Hawking radiations (entangled with BH)···

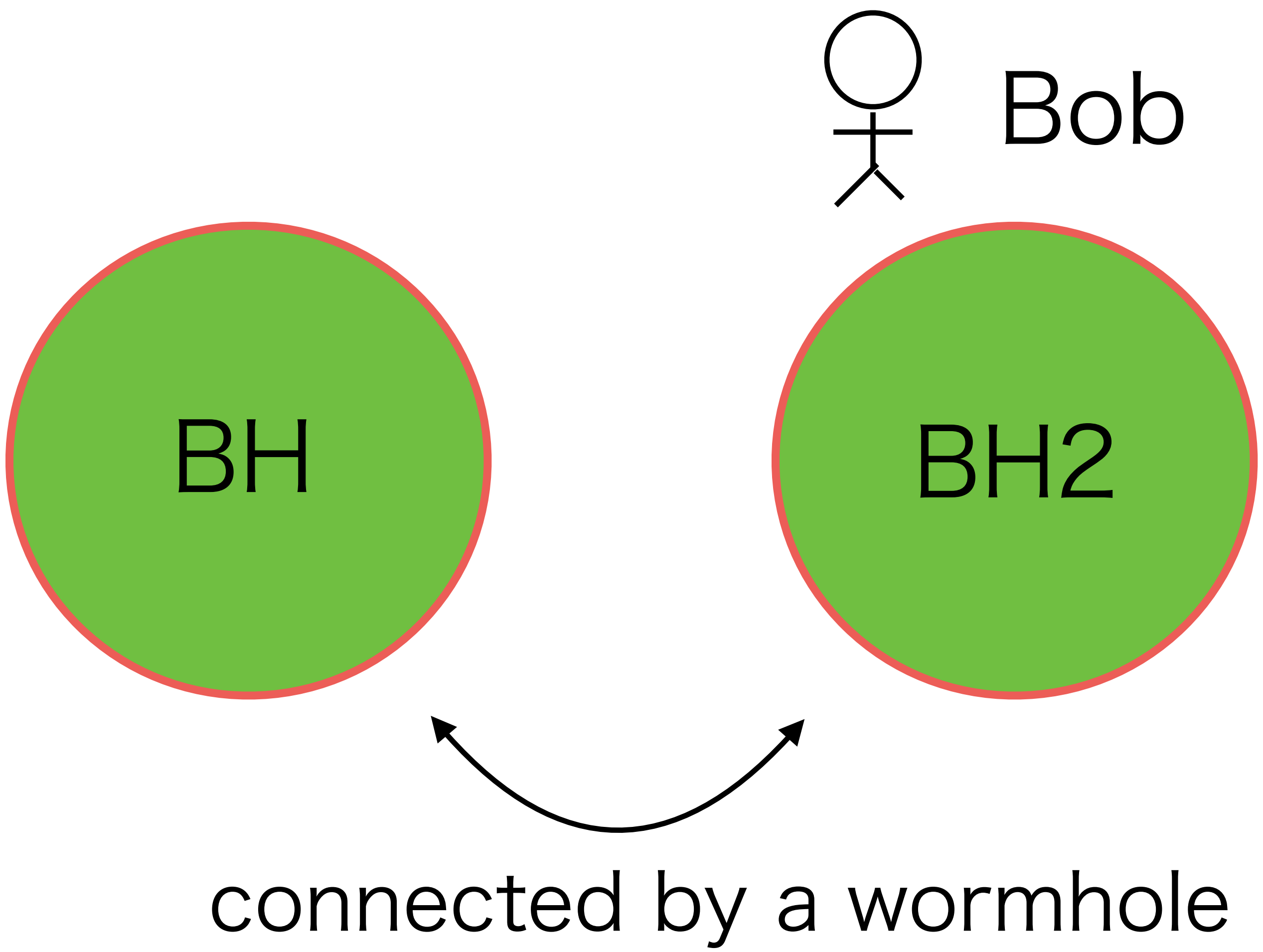
After half evaporation···

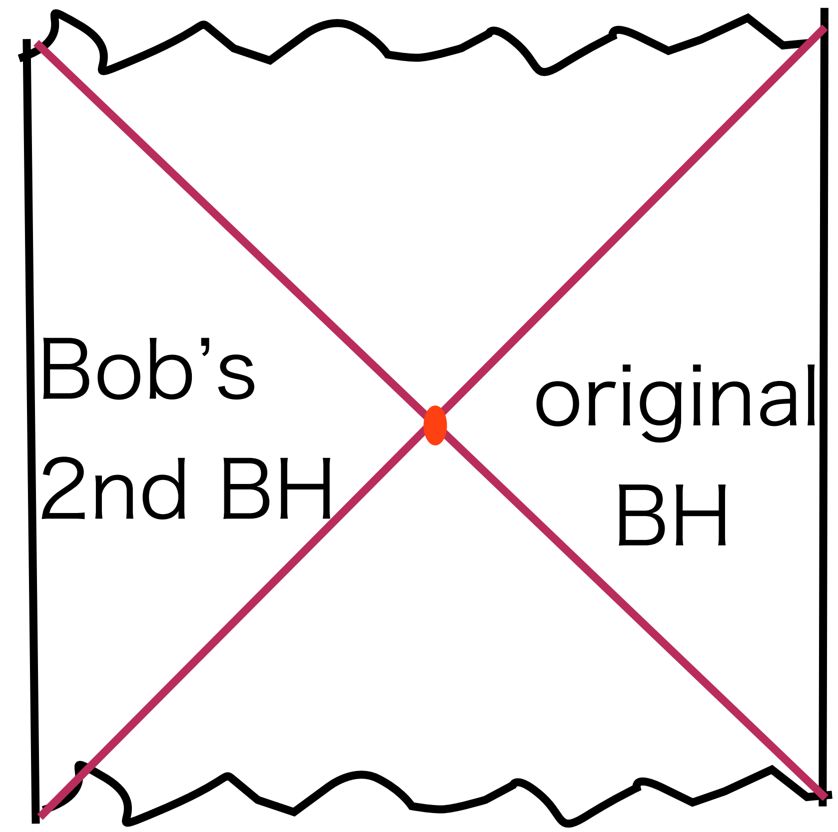
Bob create 2nd BH by radiations

Assume EPR=ER (entanglement = wormhole) [Maldacena-Susskind,13]

4.Application to information problem

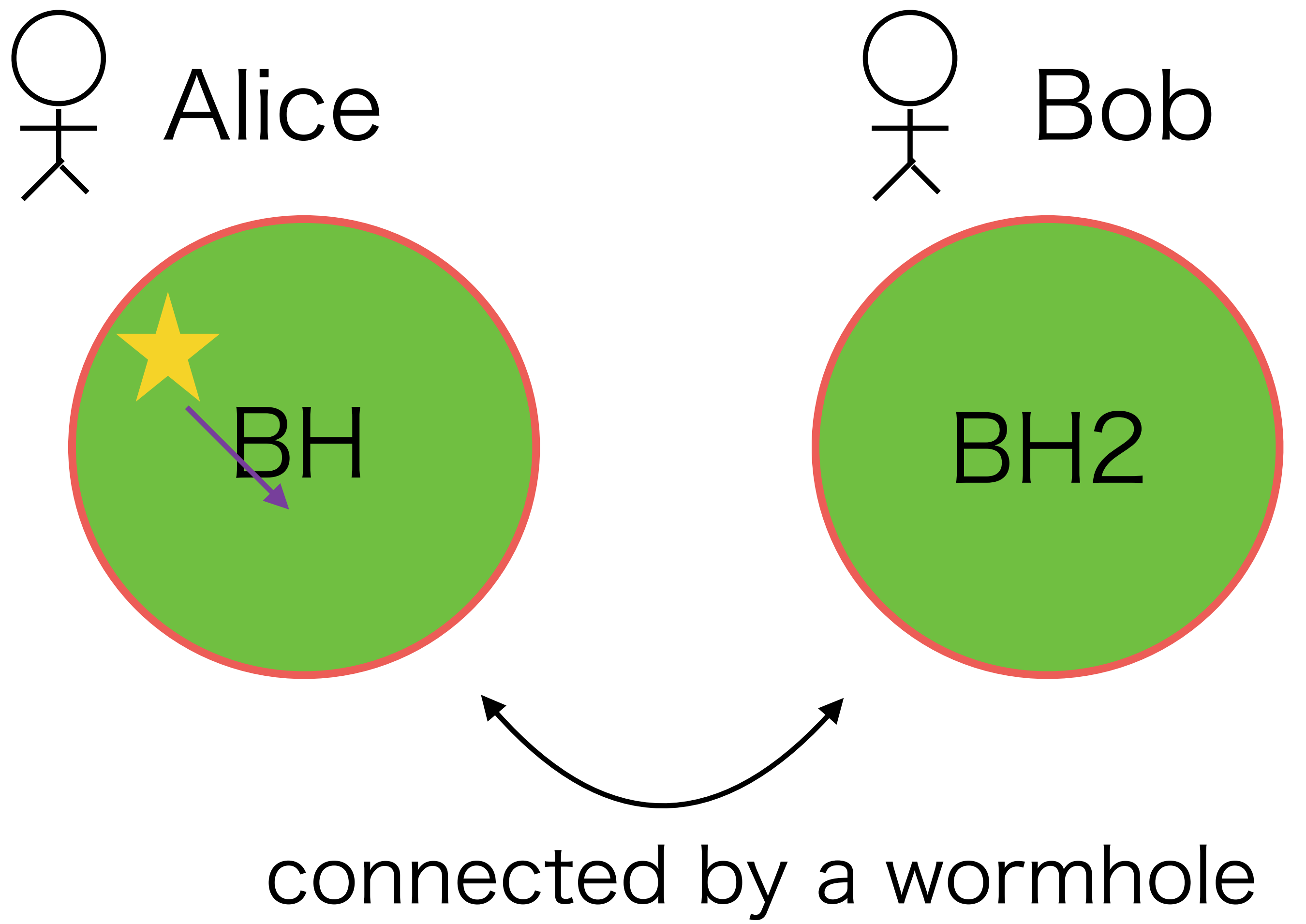
Then, we obtain

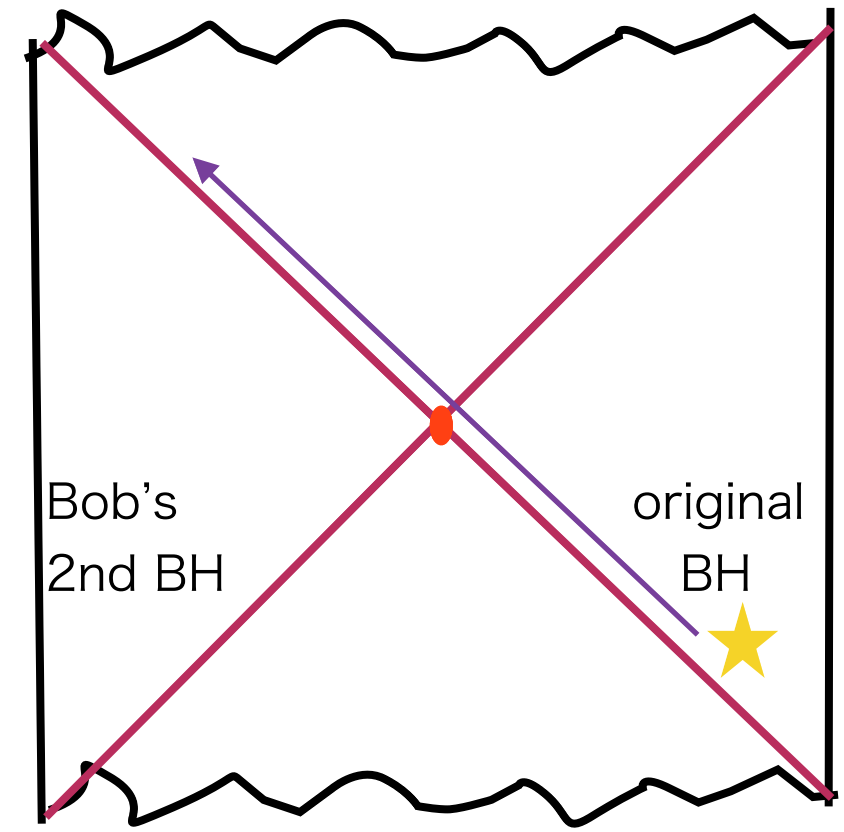




4.Application to information problem

Then, we obtain

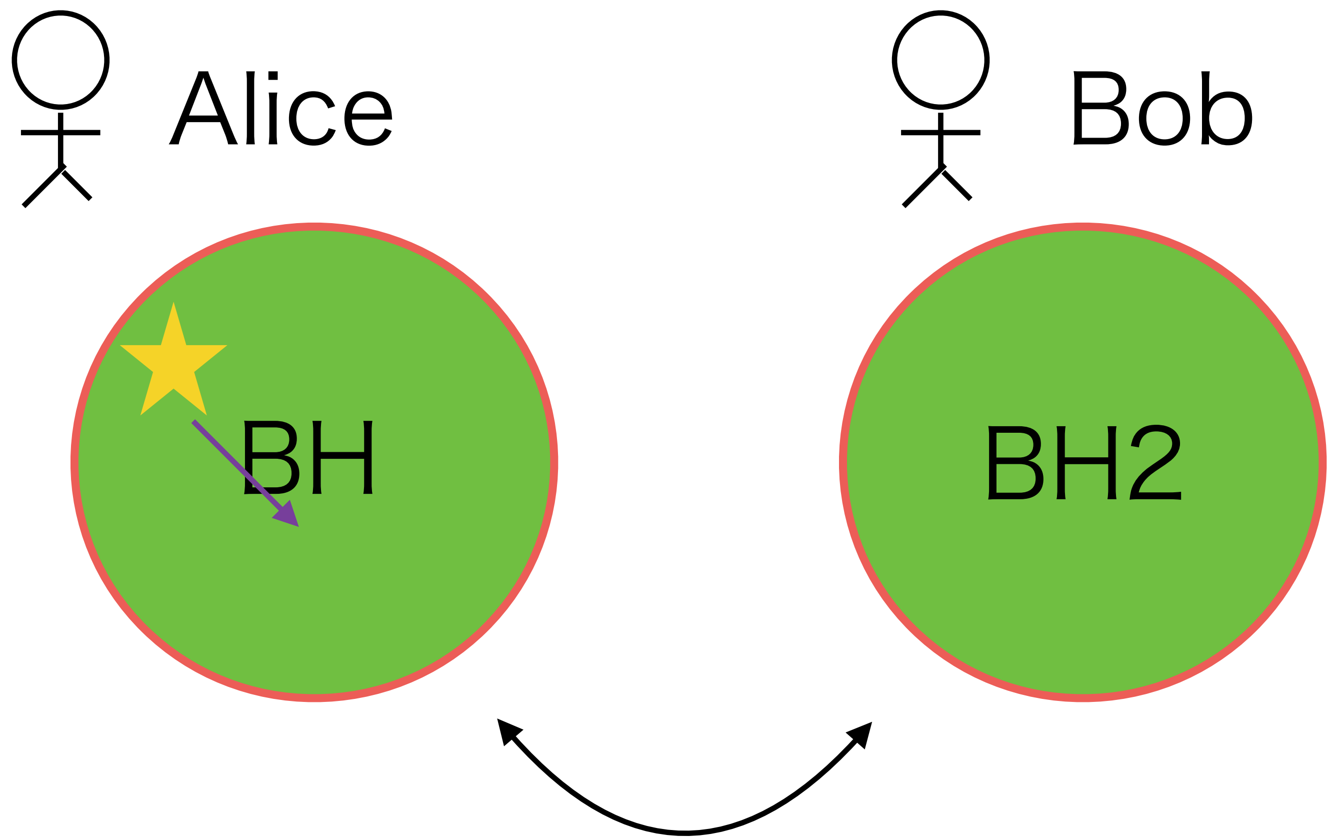




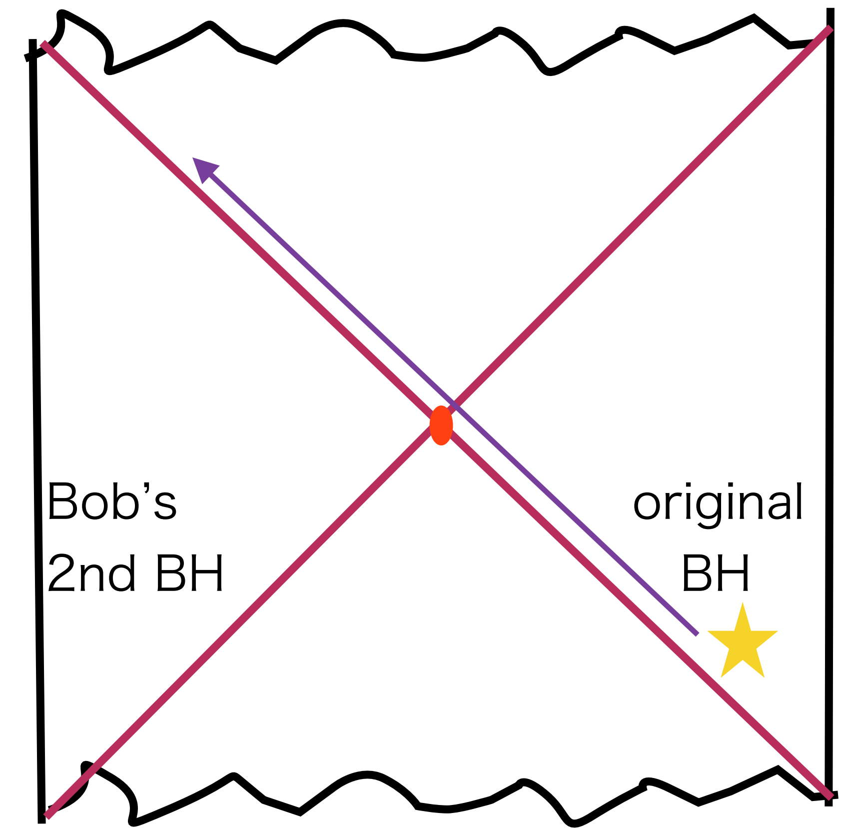
Now, Alice throws her message in the original BH

4.Application to information problem

Then, we obtain



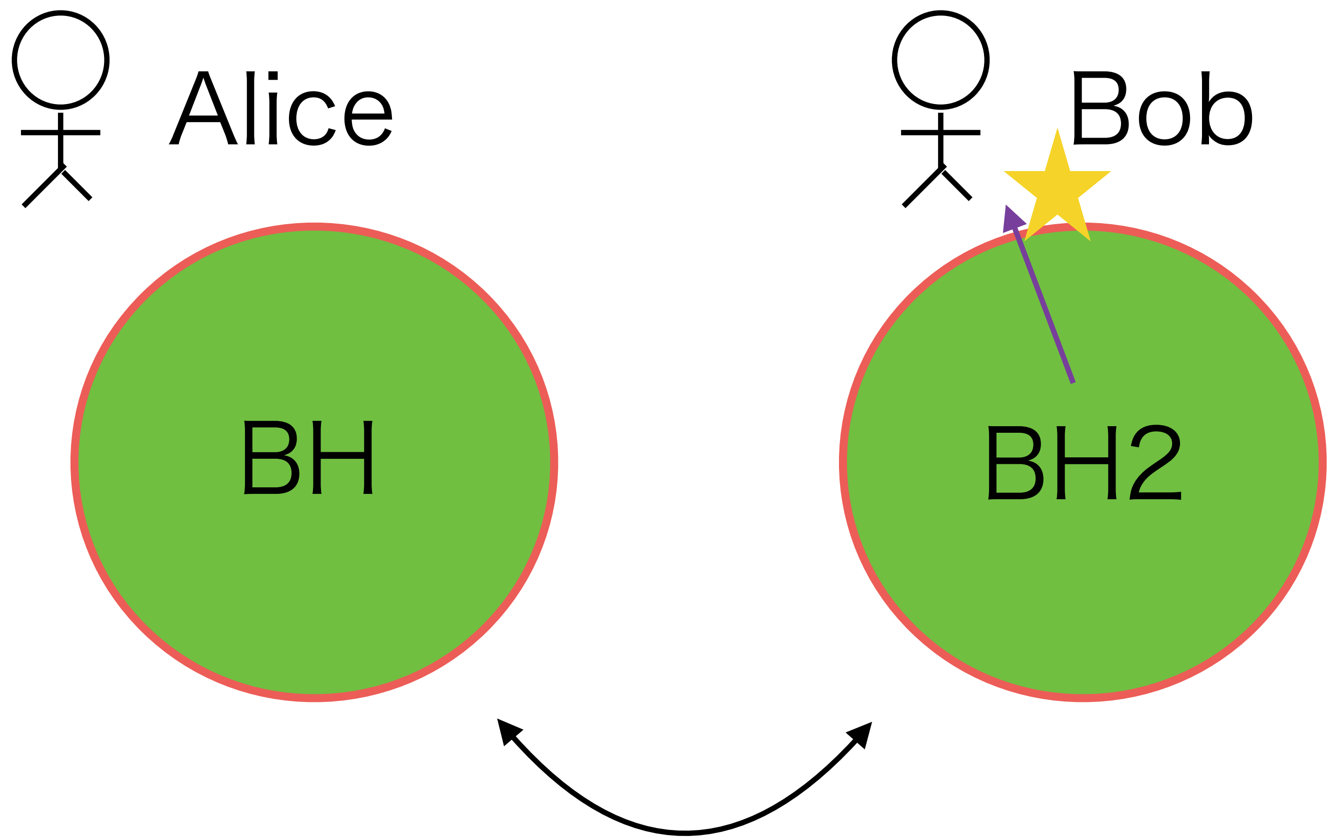
connected by a wormhole



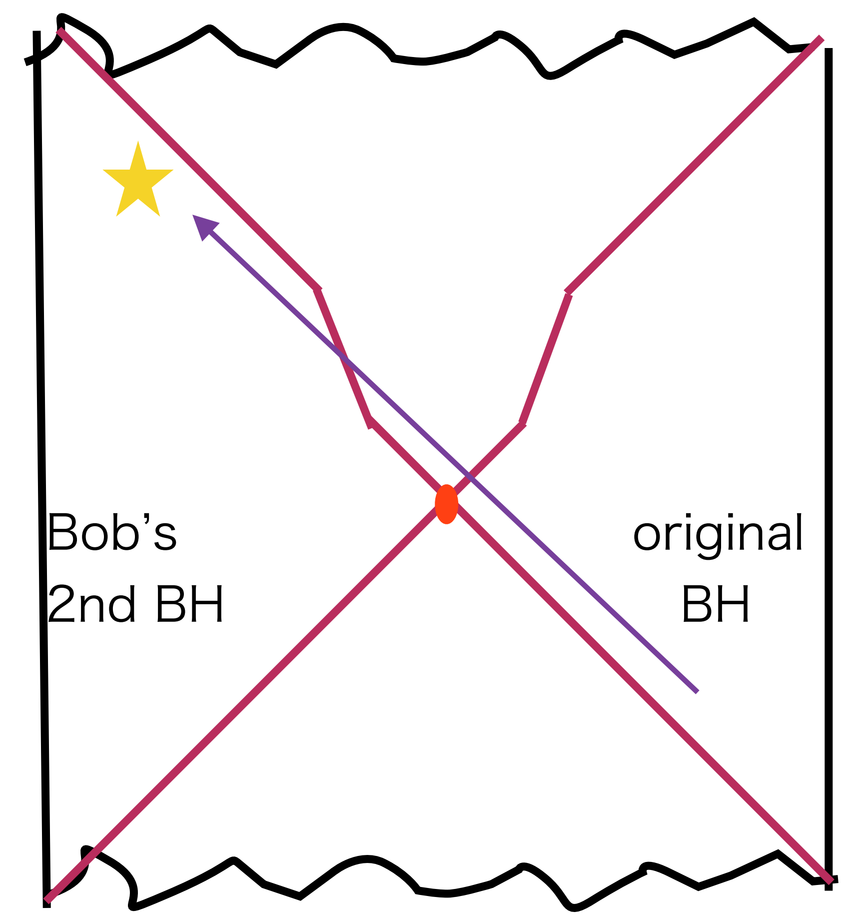
Now, Alice throws her message in the original BH

4.Application to information problem

Then, we obtain



connected by a wormhole



Now, Alice throws her message in the original BH

By traversable wormhole protocol, Bob can extract Alice's message from radiations(2nd BH)

(1)Introduction/Review of traversable wormholes

(2)Nearly AdS2 gravity

(3)Traversable wormholes in Nearly AdS2 gravity

(4)Application to information problem

(5)Conclusion

5.Conclusion

- They studied traversability of wormholes in nearly AdS2 gravity
- Because of the simple dynamics in nearly AdS2 gravity, we can also see the back reaction of message.
- Assuming ER=EPR, traversable wormholes are used to extract information behind the horizon from Hawking radiations

Appendix

Averaged Null Energy Condition(ANEC) and Traversability

BH metric: $ds^2 = -\frac{r^2 - r_h^2}{l^2} dt^2 + \frac{l^2}{r^2 - r_h^2} dr^2 + r^2 d\Omega_{d-2}^2$

After perturbation $T_{\mu\nu} \sim O(\epsilon)$, $h_{\mu\nu} = \delta g_{\mu\nu} \sim O(\epsilon)$ satisfies

$$\frac{d-2}{4}[(d-3)r_h^{-2} + (d-1)l^{-2}(h_{UU} + \partial_U(U h_{UU})) - 2r_h^{-2}\partial_U^2 h_{\phi\phi}] = 8\pi G_N T_{UU}$$

at V = 0 in Kruskal Coordinate

$$8\pi G_N \int dU T_{UU} = \frac{d-2}{4}((d-3)r_h^{-2} + (d-1)l^{-2}) \int dU h_{UU}$$

$$V(U) = -(2g_{UV}(0)) \int_{-\infty}^{U} dU \; h_{UU}$$

→ANEC = traversability

AdS2 form Higher dim

$$L = \frac{1}{16\pi G_N}\sqrt{-g}\Phi^2 R + \lambda(\nabla\Phi)^2 - U(\Phi)$$

- CGHS model

$$\lambda = 4 \qquad U(\Phi) = -A\Phi^2$$

UV: near extremal diatonic BH at 4 or 5 dim

- Magnetic brane

$$\lambda = 2 \qquad U(\Phi) = \frac{B}{\Phi^2} - A\Phi^2$$

UV: $ds_4^2 = g_{\mu\nu}dx^\mu dx^\nu + \Phi^2(x)(dy_1^2 + dy_2^2)$

[cf: Bennett-Brassard-Crepeau-Jozsa-Peres-Wootters 93]

Quantum Teleportation

Initial state:

$$|\psi\rangle_A \otimes |EPR\rangle_{BC} = \frac{1}{2}\sum_{k=1}^{4} |\Psi_k\rangle_{AB} \otimes |\psi_k\rangle$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

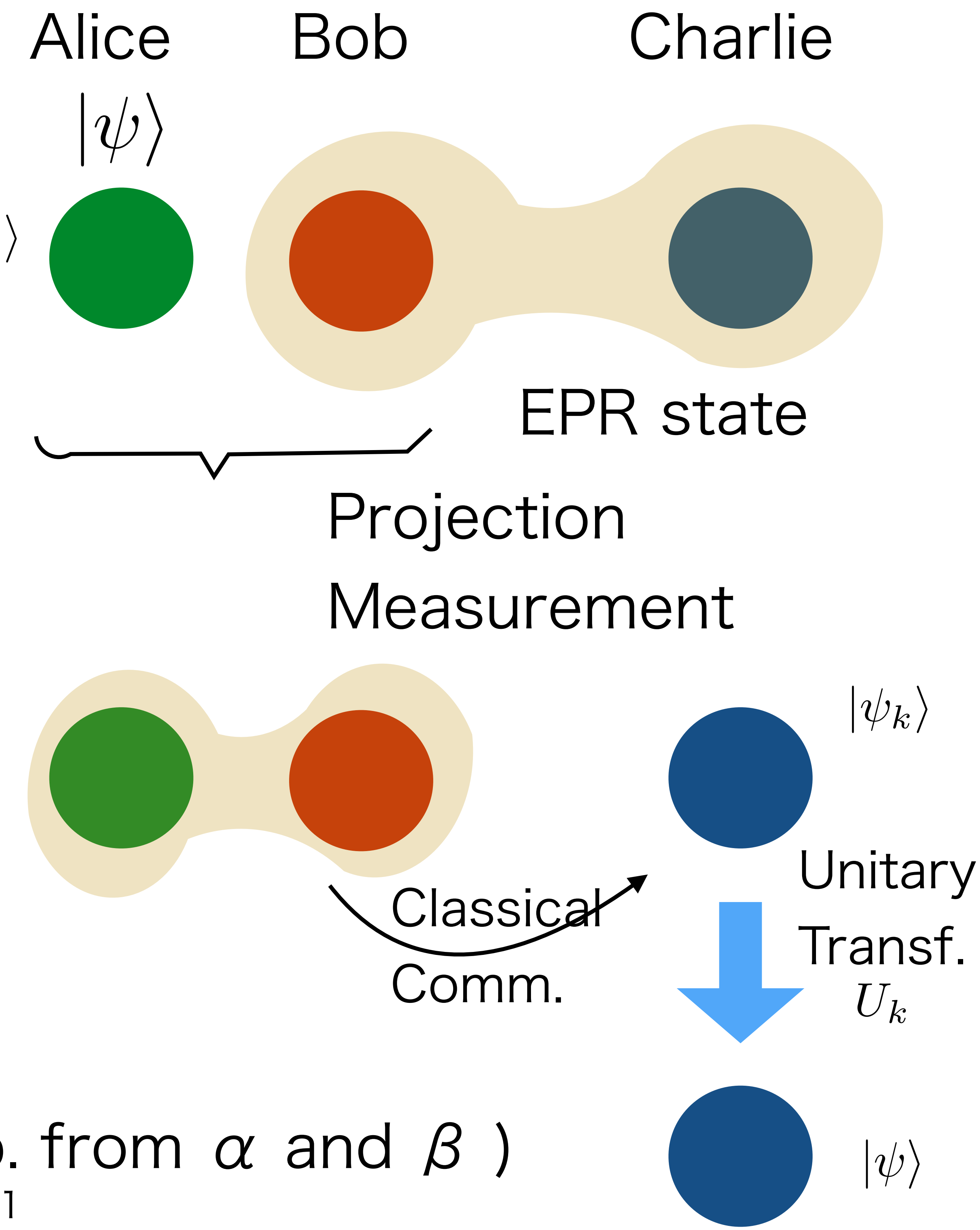
where

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle\,|1\rangle - |1\rangle\,|0\rangle) \qquad |\Psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle\,|1\rangle + |1\rangle\,|0\rangle)$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle\,|0\rangle - |1\rangle\,|1\rangle) \qquad |\Psi_4\rangle = \frac{1}{\sqrt{2}}(|0\rangle\,|0\rangle + |1\rangle\,|1\rangle)$$

$$|\psi_1\rangle = \alpha\,|1\rangle - \beta\,|0\rangle \qquad |\psi_2\rangle = \alpha\,|1\rangle + \beta\,|0\rangle$$

$$|\psi_3\rangle = \alpha\,|0\rangle - \beta\,|1\rangle \qquad |\psi_4\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$



Projection: $P_k = |\Psi_k\rangle\,\langle\Psi_k|$, $\sum_k P_k = 1$

U_k: k dependent Unitary (indep. from α and β)

CFT_1

In 1d, QFT becomes quantum mechanics

Conformal symmetry $\rightarrow$ $T^{\mu}_{\mu} = 0$

In 1d , $T^0_0 = H = 0$ and there are no finite energy excitation