Towards a holographic dual of large-N_c QCD

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References: T S Biró, S G Matinyan, B Müller
"Chaos and Gauge Field Theory", World Scientific

My motivation

Calculating Lyapunov exponent of $\bar{q}q$ in QCD

Lyapunov exponent λ is characteristic of chaos.

Change of initial position in phase space

$$x_0 \to x_0 + d(t)|_{t=0}$$

$$\lambda \equiv \lim_{t \to \infty} \lim_{d(0) \to 0} \frac{1}{t} \ln \frac{d(t)}{d(0)}$$

My motivation

Calculating Lyapunov exponent of $\bar{q}q$ in QCD

Needing EOM of meson

But we can not calculate Lyapunov exponent directly.

$\therefore$ Using DBI action in D4/D6 system by AdS/CFT correspondence

Main topic of this talk

- Deriving static solution about D6-brane

$$S_{D6} = -T_{D6} \int d^7\sigma \sqrt{h} \left(1 + \frac{U_{KK}^3}{4\rho^3}\right)^2 \lambda^2 \sqrt{1 + \dot{r}^2}$$

- Deriving spectrum of fluctuation around static solution

  ①Substituting fluctuation for Lagrangian
  ②Calculating Linearised EOM
  ③Deriving about meson spectrum

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- **Introduction**
- D4/D6 system
- D6-brane fluctuation
- Summary

Introduction

D4/D6 system's characteristic

- Breaking of SUSY
  ∵ Anti-periodic boundary condition about gaugino

- Introducing quark mass
  If $r(\lambda)|_{\lambda\to\infty} \neq 0$,quark is not massless.

N $_c$ **D4**	0	1	2	3	4	-	-	-	-	-
N $_f$ **D6**	0	1	2	3	-	5	6	7	-	-

x^μ (0–3) τ (4) λ, Ω_2 (5–7) r, ϕ (8–9)

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The D4-soliton background

$$ds^2_{D4} = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \frac{dU^2}{f(U)} + R^{\frac{3}{2}}U^{\frac{1}{2}}d\Omega_4^2$$

$$f(U) \equiv 1 - \frac{U_{KK}^3}{U^3} \qquad R^3 \equiv \pi g_s N_c l_s^3$$

$$U_{KK} : U_{KK} < U < \infty \qquad \delta\tau = \frac{4\pi}{3}\frac{R^{\frac{3}{2}}}{U_{KK}^{\frac{1}{2}}}$$

$$\delta\tau : 0 \leq \tau < \delta\tau$$

***N_c* D4**	0	1	2	3	4	-	-	-	-	-

x^μ (0–3) $\qquad$ τ (4) $\qquad$ U, Ω_4 (remaining)

The D4-soliton background

$$ds^2_{D4} = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2)$$

$$+ \left(\frac{R}{U}\right)^{\frac{3}{2}} \frac{dU^2}{f(U)} + R^{\frac{3}{2}} U^{\frac{1}{2}} d\Omega_4^2$$

$$U(\rho) = \left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{\frac{2}{3}} \quad K(\rho) = \frac{R^{\frac{3}{2}}U^{\frac{1}{2}}}{\rho^2}$$

$$\rho^2 = \lambda^2 + r^2$$

$$ds^2_{D4} = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2)$$

$$+ K(\rho)(d\lambda^2 + \lambda^2 d\Omega_2^2 + dr^2 + r^2 d\phi^2)$$

Ansatz: $$r = r(\lambda), \;\; \phi = 0$$

Induced metric on the D6-branes

$$ds^2_{D6} = \left(\frac{U}{R}\right)^{\frac{3}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + K(\rho)[(1+\dot{r}^2)d\lambda^2 + \lambda^2 d\Omega_2^2]$$

N_c **D4**	0	1	2	3	4	-	-	-	-	-
N_f **D6**	0	1	2	3	-	5	6	7	-	-

x^μ (0–3) $\quad \tau$ (4) $\quad \lambda, \Omega_2$ (5–7) $\quad r, \phi$ (8–9)

D6-branes action $(N_f = 1)$

$$S_{D6} = -T_{D6} \int d^7\sigma \sqrt{h} \left(1 + \frac{U_{KK}^3}{4\rho^3}\right)^2 \lambda^2 \sqrt{1+\dot{r}^2}$$

EOM

$$\frac{d}{d\lambda}\left[\left(1+\frac{1}{4\rho^3}\right)^2\lambda^2\frac{\dot{r}}{\sqrt{1+\dot{r}^2}}\right]=-\frac{3}{2}\frac{1}{\rho^5}\left(1+\frac{1}{4\rho^3}\right)\lambda^2 r\sqrt{1+\dot{r}^2}$$

Scaling

$$\lambda\to U_{KK}\lambda,\ r\to U_{KK}r,\ \rho\to U_{KK}\rho$$

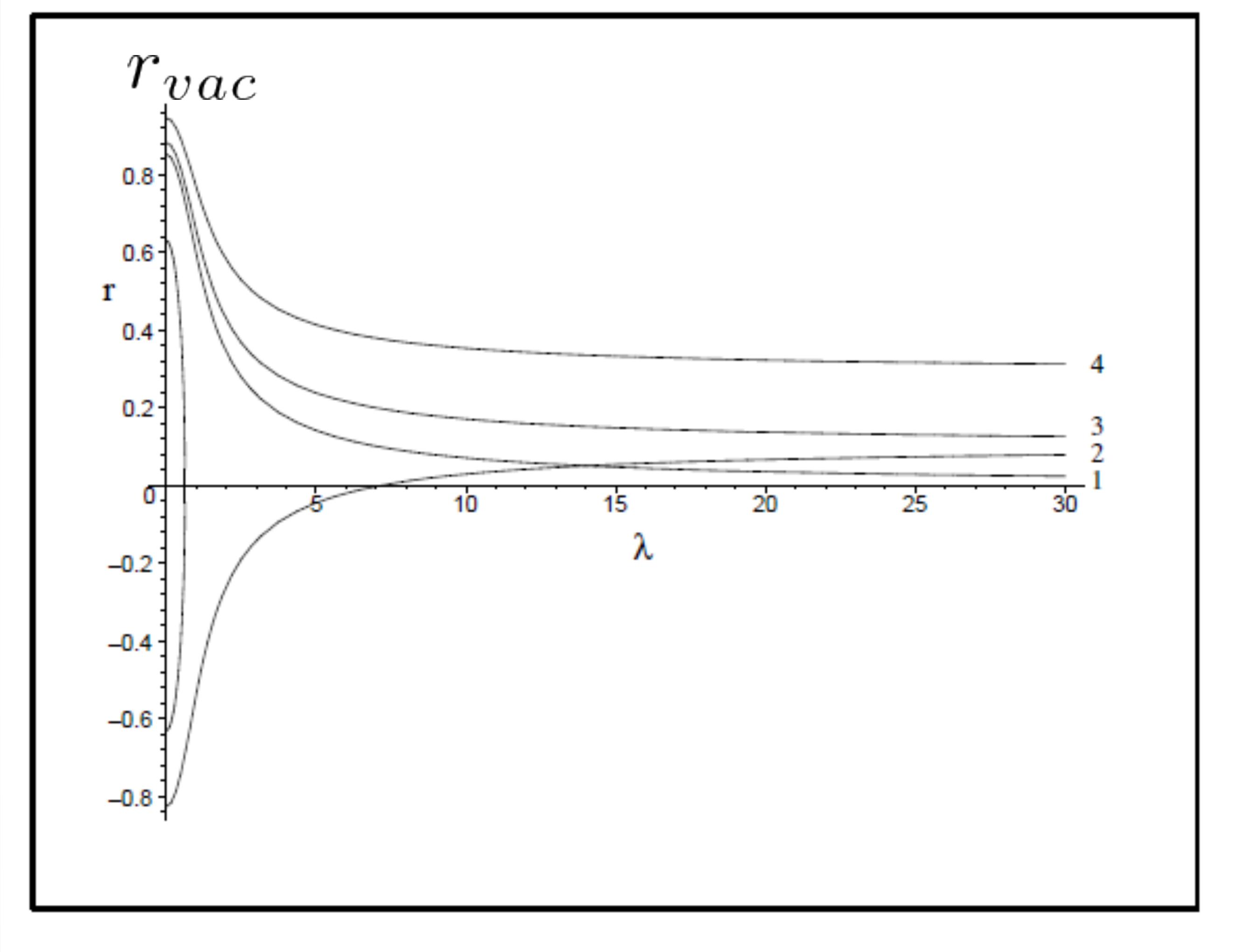

Figure: Numerical solution of EOM

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Fluctuation around static solution

$$\phi = 0 + \delta\phi, \;\; r = r_{vac}(\lambda) + \delta r$$

Metric

$$ds^2_{D6} = \left(\frac{U}{R}\right)^{\frac{3}{2}} \eta_{\mu\nu} dx^\mu dx^\nu$$
$$+ K(\rho)[(1 + \dot{r}^2_{vac}) d\lambda^2 + \lambda^2 d\Omega_2^2 + 2\dot{r}_{vac}\partial_a(\delta r) d\lambda dx^a]$$
$$+ K[\partial_a(\delta r)_b(\delta r) dx^a dx^b + (r_{vac} + \delta r)^2 \partial_a(\delta\phi)_b(\delta\phi) dx^a dx^b]$$

Lagrangian(quadratic order)

$$\mathcal{L} = \mathcal{L}_0 - T_{D6} U_{KK}^3 \lambda^2 \sqrt{h} \sqrt{1 + \dot{r}_{vac}^2}$$

$$\left\{ \left(\frac{3(7r_{vac}^2 - \lambda^2)}{16\rho_{vac}^{10}} \right) + \left(\frac{3(4r_{vac}^2 - \lambda^2)}{4\rho_{vac}^{7}} \right) (\delta r)^2 \right.$$

$$- \frac{3r_{vac}}{4\rho_{vac}^5} \left(1 + \frac{1}{4\rho^3} \right) \frac{\dot{r}_{vac} \partial_\lambda (\delta r^2)}{1 + \dot{r}_{vac}^2}$$

$$\left. + \left(1 + \frac{1}{4\rho_{vac}^3} \right)^2 \sum_a \frac{K}{2g_{aa}} \left(\frac{[\partial_a (\delta r)]^2}{1 + \dot{r}_{vac}^2} + r_{vac}^2 [\partial_a (\delta \phi)^2] \right) \right\}$$

Linearised EOM($\delta\phi$)

$$\frac{9}{4M_{KK}^2}\frac{r_{vac}^2}{\rho_{vac}^3}\left(1+\frac{1}{4\rho_{vac}^3}\right)^{\frac{4}{3}}\partial_\mu\partial^\mu(\delta\phi)+\frac{r_{vac}^2}{\lambda^3}\left(1+\frac{1}{4\rho_{vac}^3}\right)^2\nabla^2(\delta\phi)$$

$$+\frac{1}{\lambda^2\sqrt{1+\dot{r}_{vac}^2}}\frac{d}{d\lambda}\left\{\frac{\lambda^2 r_{vac}^2}{\sqrt{1+\dot{r}_{vac}^2}}\left(1+\frac{1}{4\rho_{vac}^3}\right)^2\partial_\lambda(\delta\phi)\right\}=0$$

$\boldsymbol{N_f}$ **D6**	0	1	2	3	-	5	6	7	-	-

x^μ (0–3), τ (-), λ, Ω_2 (5–7), r, ϕ (-, -)

Linearised EOM(δr)

$$\frac{\lambda^2}{\sqrt{1+\dot{r}_{vac}^2}}\left[\frac{9}{4M_{KK}^2}\frac{1}{\rho_{vac}^3}\left(1+\frac{1}{4\rho_{vac}^3}\right)^{\frac{4}{3}}\partial_\mu\partial^\mu(\delta r)+\frac{1}{\lambda^3}\left(1+\frac{1}{4\rho_{vac}^3}\right)^2\nabla^2(\delta r)\right]$$

$$+\frac{d}{d\lambda}\left\{\frac{\lambda^2}{(1+\dot{r}_{vac}^2)^{\frac{3}{2}}}\left(1+\frac{1}{4\rho_{vac}^3}\right)^2\partial_\lambda(\delta r)\right\}$$

$$-\frac{d}{d\lambda}\left[\frac{3\lambda^2 r_{vac}\dot{r}_{vac}}{2\rho_{vac}^5\sqrt{1+\dot{r}_{vac}^2}}\left(1+\frac{1}{4\rho_{vac}^3}\right)\right]\delta r$$

$$-\lambda^2\sqrt{1+\dot{r}_{vac}^2}\left(\frac{3(7r_{vac}^2-\lambda^2)}{8\rho_{vac}^{10}}+\frac{3(4r_{vac}^2-\lambda^2)}{2\rho^7}\right)\delta r=0$$

N_f D6	0	1	2	3	-	5	6	7	-	-

x^μ $\quad\tau\quad$ λ,Ω_2 $\quad r,\phi$

Ansatz:

$$\delta\phi = \mathcal{P}(\lambda) e^{ik_\phi \cdot x} Y_{l_\phi m_\phi}(S^2)$$

$$\delta r = \mathcal{R}(\lambda) e^{ik_r \cdot x} Y_{l_r m_r}(S^2)$$

+

Normalizability condition

We can get a meson spectrum
by AdS/CFT correspondence

$$M_\phi^2 = -k_\phi^2,\ M_r^2 = -k_r^2$$

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Summary

- Deriving static solution about D6-brane

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- Deriving spectrum of fluctuation around static solution

①Substituting fluctuation for Lagrangian
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Future work

Now we get meson spectrum.

Calculating Lyapunov exponent of $\bar{q}q$ in QCD

$$\lambda \equiv \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \ln \frac{d(t)}{d(0)}$$