Black hole singularities in the boundary theory

Kazuya Higashide

Based on
G. Festuccia and H. Liu, arXiv: 0506202v2
“Excursions beyond the horizon: Black hole singularities in Yang-Mills theories(1)”
and
some research works.

Introduction

[F,L('05)]

We consider a black hole in AdS and assume AdS/CFT.

Then we check

whether **the BH characters** ,especially **the singularity**,

appear

in physical quantities of **the dual boundary theory**.

(Calculations are done in the bulk side.)

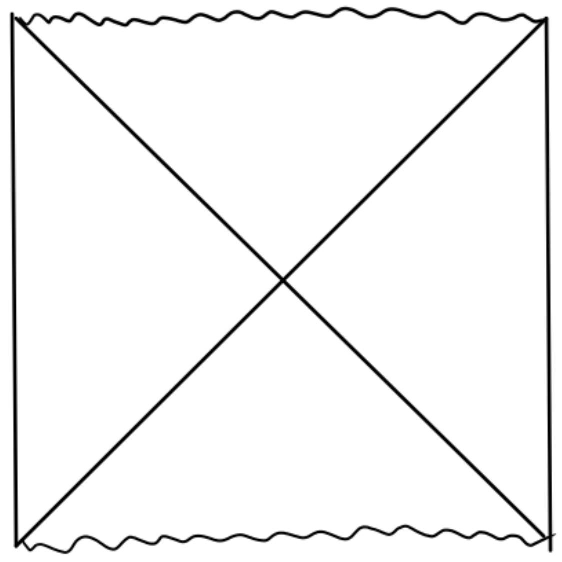
[Maldacena('01)]

Quantum Mechanics

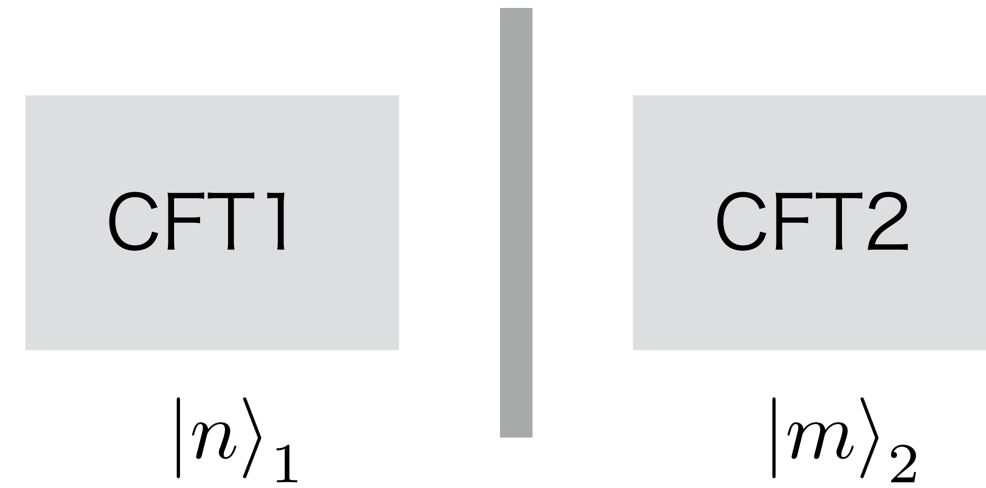
$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_1 |n\rangle_2$$

equiv ⟺

extended BH



$$
\begin{cases}
|\psi\rangle = \dfrac{1}{\sqrt{Z}} \displaystyle\sum_n e^{-\beta E_n/2} |n\rangle_1 |n\rangle_2 \\
\rho = |\psi\rangle \langle\psi|
\end{cases}
$$



- imaginary time evolution

$$
e^{-\beta H/2} = e^{-iH(-i\beta/2)}
$$

- density matrix

$$
\begin{aligned}
\tilde{\rho} &= \mathrm{tr}_2 \rho \\
&= \sum_m \langle m|_2 |\psi\rangle \langle\psi| |m\rangle_2 \\
&= \frac{1}{Z} \sum_n e^{-\beta E_n} |n\rangle_1 \langle n|_1
\end{aligned}
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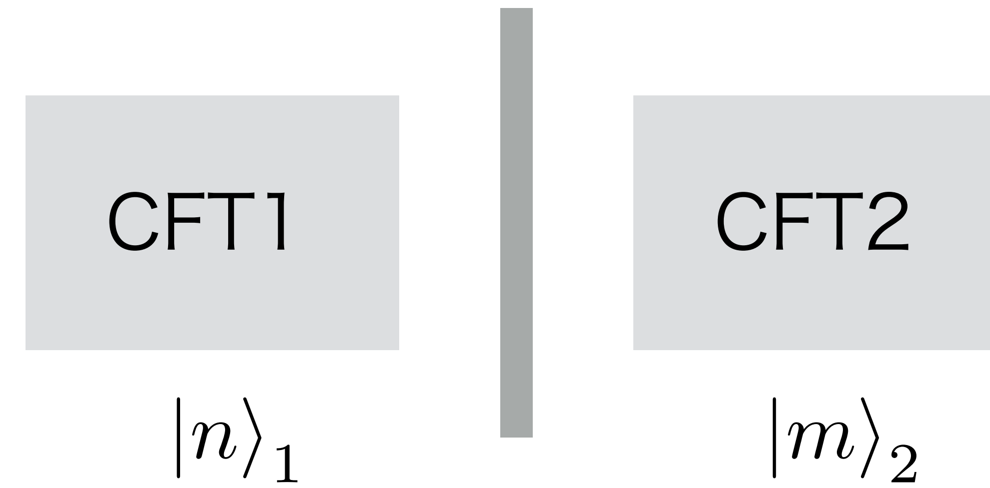

- imaginary time evolution

$$e^{-\beta H/2} = e^{-iH(-i\beta/2)} \longrightarrow t_2 = t_1 - i\frac{\beta}{2}$$

- density matrix

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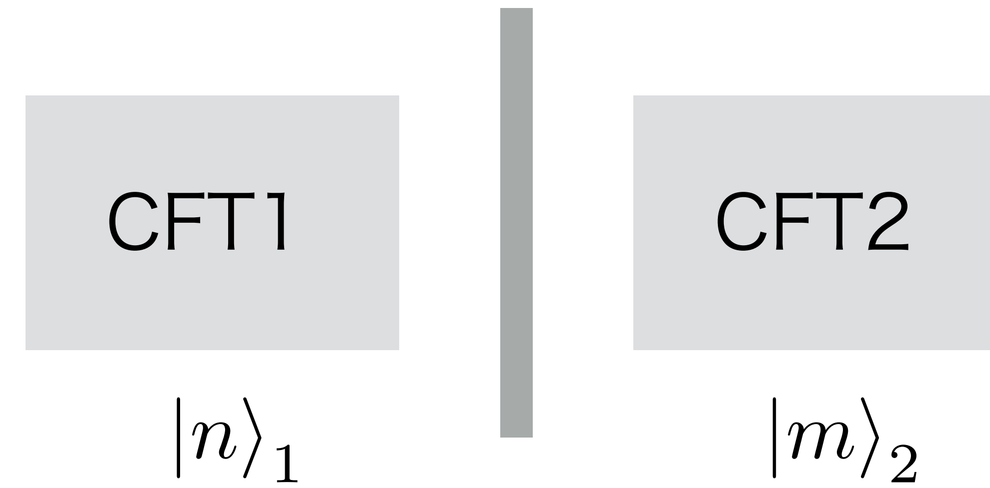

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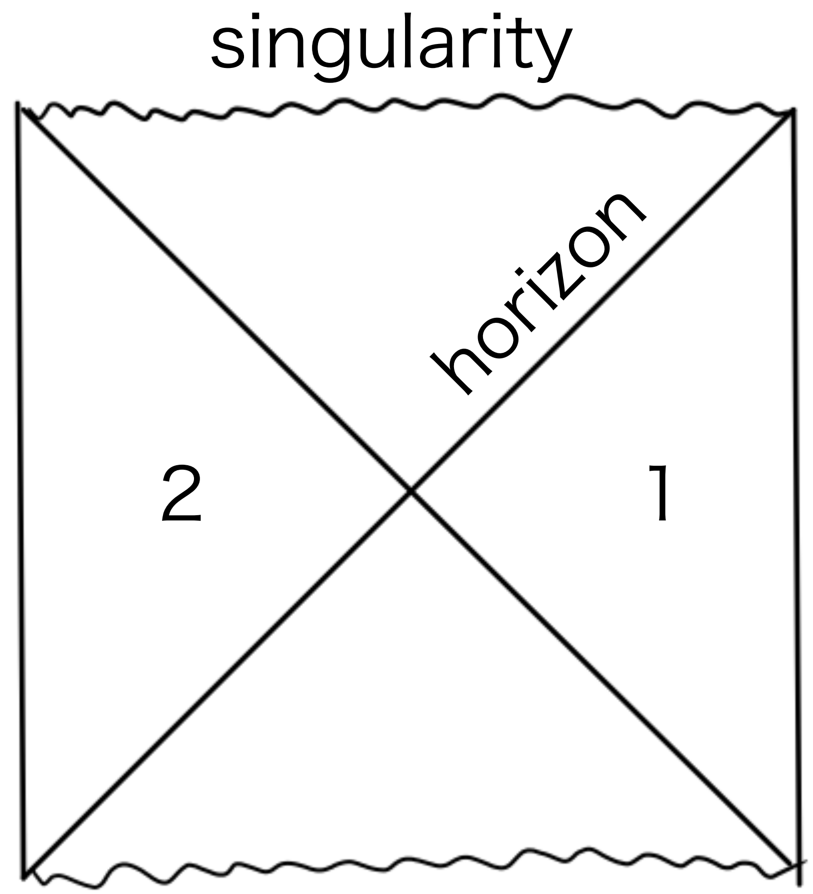
$\longrightarrow$ finite temperature



bdry(AdS) ⟷ CFT

- $t_2 = t_1 - i\dfrac{\beta}{2}$ (analytic continuation)
- $[\mathcal{O}_1, \mathcal{O}_2] = 0$
- finite temperature
- two copy of CFT

[Maldacena('01)]

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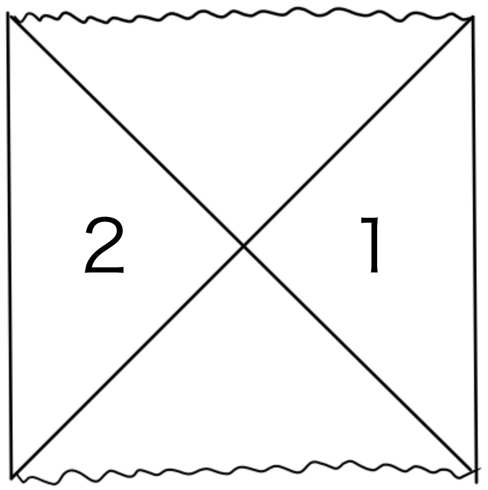
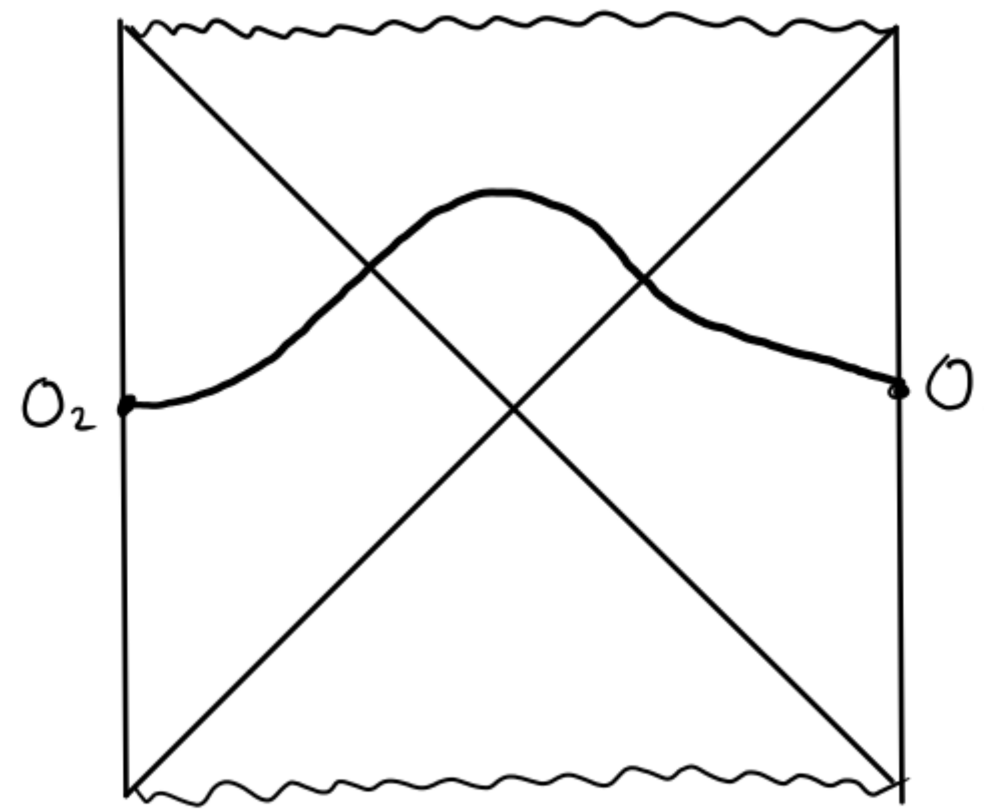


- two copy of CFT
- $t_2 = t_1 - i\frac{\beta}{2}$
  (time evolution)
- no-interaction
- finite temperature

⟺

- two copy of CFT
- $t_2 = t_1 - i\frac{\beta}{2}$
  (analytic continuation)
- $[\mathcal{O}_1, \mathcal{O}_2] = 0$
- finite temperature

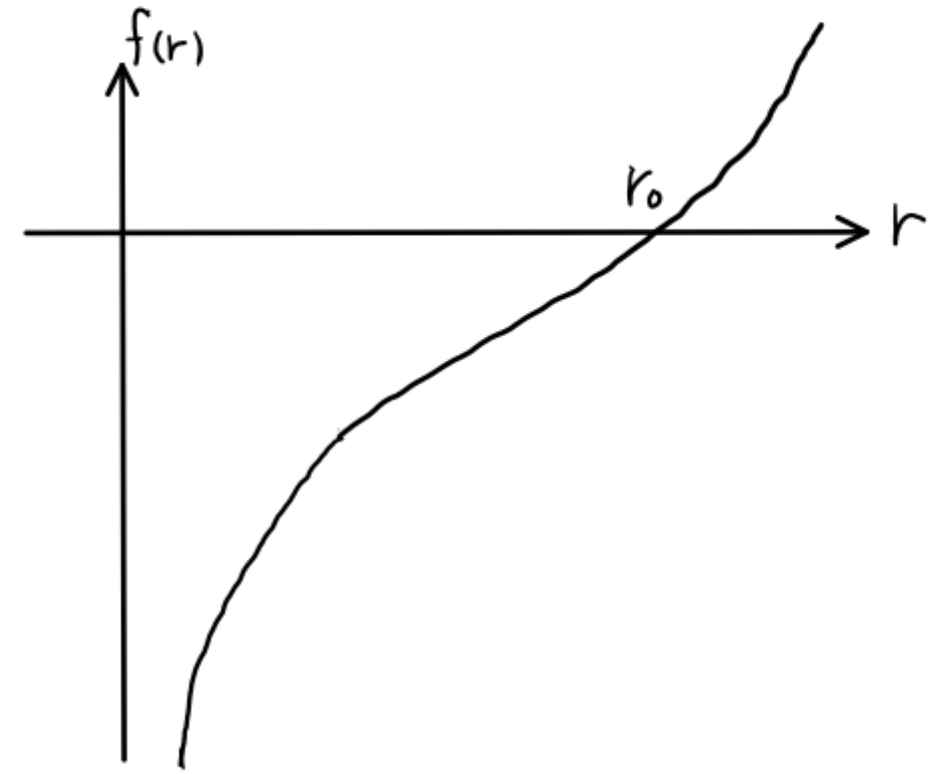
physical quantity in the boundary → **Green function** $\langle 0|\mathcal{O}_1\mathcal{O}_2|0\rangle$



metric

AdS5 Schwarzschild BH

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_3^2$$



$$\begin{aligned} f(r) &= r^2 - \frac{\mu}{r^2} + 1 \\ &= \frac{1}{r^2}(r^2 - r_0^2)(r^2 + r_1^2) \end{aligned}$$

[Gbuser,Klebanov,Polyakov('98),Witten('98)]

physical quantity in the boundary → **Green function**

$$G_{bulk} = \langle 0|\phi(t,r)\phi(t',r')|0\rangle$$

$$G_{bdry} \propto \lim_{r,r'\to\infty}(rr')^{\Delta}G_{bulk}$$

→ Let's consider the EOM of ϕ

K-G Eq $$(\Box - m^2)\phi = 0$$

- ansatz $$\phi(t,r) = e^{-iu\nu t} r^{-3/2} e^{\nu S(r)}$$

- large mass(WKB) $\nu \to \infty$

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- ansatz $\phi(t,r) = e^{-iu\nu t} r^{-3/2} e^{\nu S(r)}$
- large mass(WKB) $\nu \to \infty$

→

$$fS'^2 + \frac{1}{f}u^2 = 1 + \mathcal{O}(\nu^{-1})$$

$$G_{bdry} \propto \lim_{r,r' \to \infty} (rr')^{\Delta} \langle 0|\phi\phi|0\rangle$$
$$\sim e^{2\nu S}$$

physical meaning of S

in flat case

$$\phi(t,r) = e^{-iu\nu t} e^{ipr}$$
$$= e^{-iu\nu t} e^{i\nu(p/\nu)r}$$

$\longrightarrow$ $S \propto r$: distance

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in flat case

$$\phi(t,r) = e^{-iu\nu t}e^{ipr}$$
$$= e^{-iu\nu t}e^{i\nu(p/\nu)r}$$

$\longrightarrow$ $S \propto r$: distance

in curved case

$S \propto$ **proper length(or time)**

[F,L('05)]

EOM of S $$fS'^2 + \frac{1}{f}u^2 = 1$$

$$\phi \propto e^{-iu\nu t} e^{\nu S}$$

[F,L('05)]

EOM of S $$fS'^2 + \frac{1}{f}u^2 = 1$$

$$\phi \propto e^{-iu\nu t} e^{\nu S}$$

time-like geodesic

$$-\frac{1}{f}\dot{r}^2 + \frac{1}{f}E^2 = 1$$

S' : pure imaginary

$$u = E$$

[F,L('05)]

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S' : pure imaginary

$$u = E$$

space-like geodesic

$$\frac{1}{f}\dot{r}^2 - \frac{1}{f}E^2 = 1$$

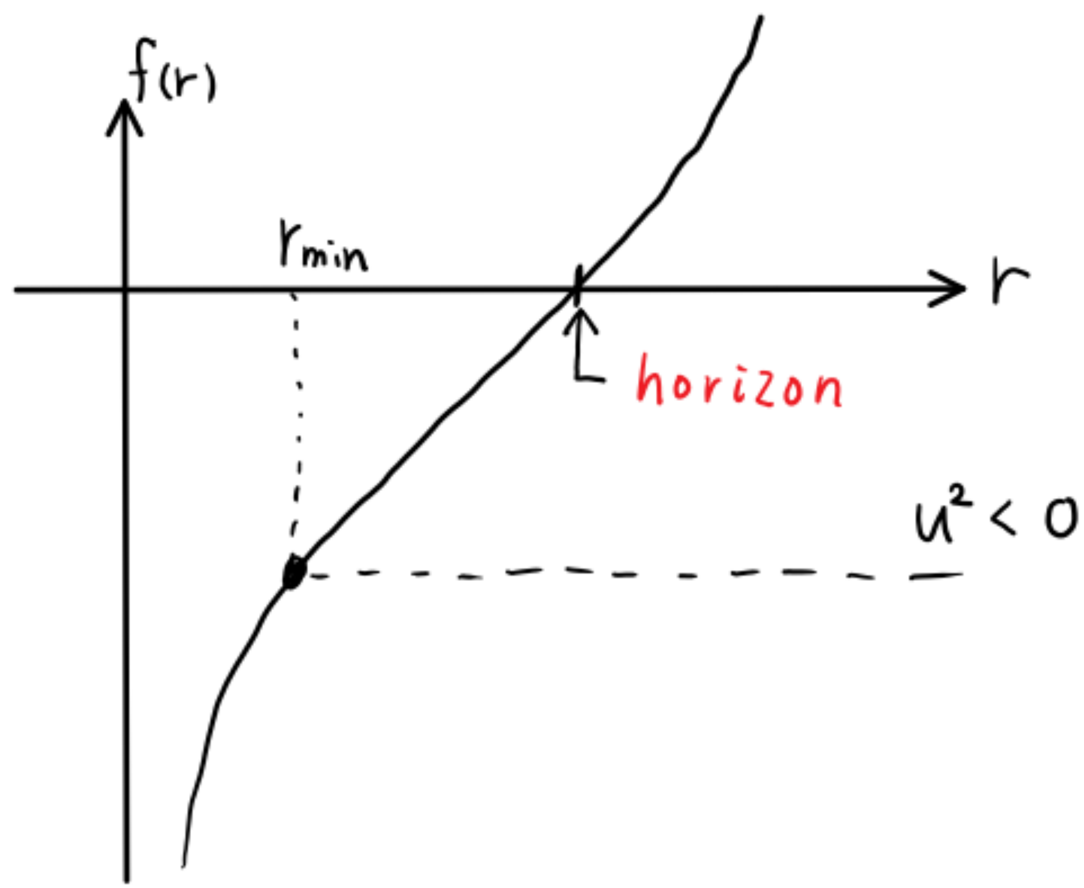
S' : real < 0

$$u = -iE$$

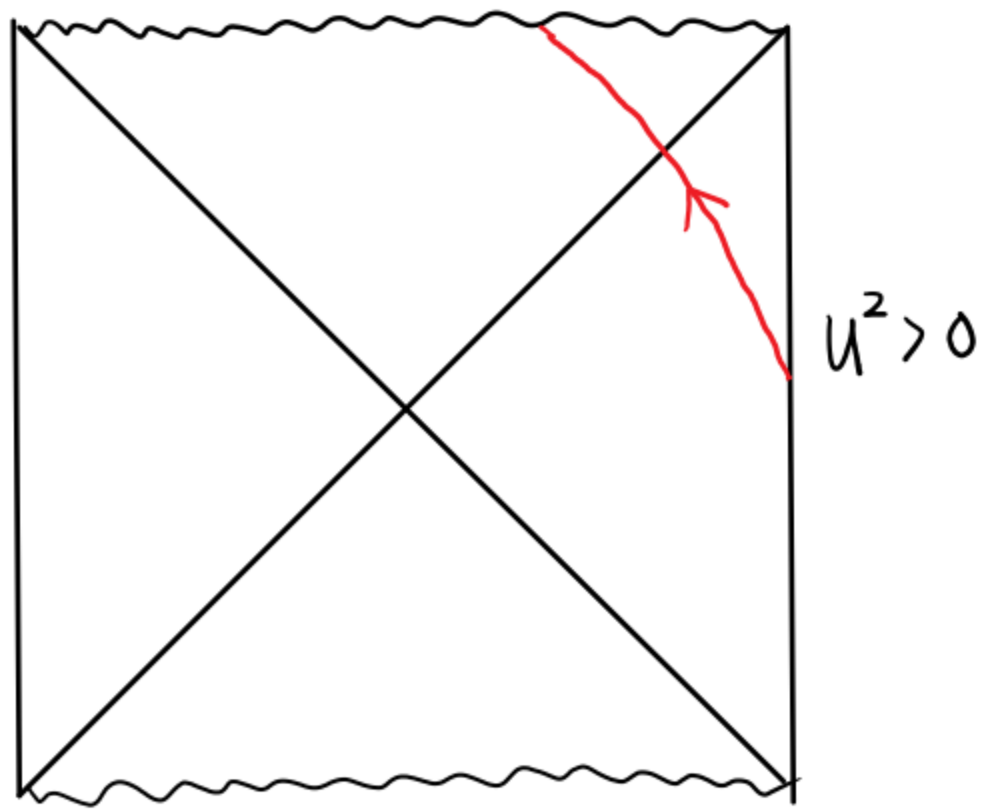
space-like geodesic

$$\dot{r}^2 = f - u^2, \quad u^2 < 0$$

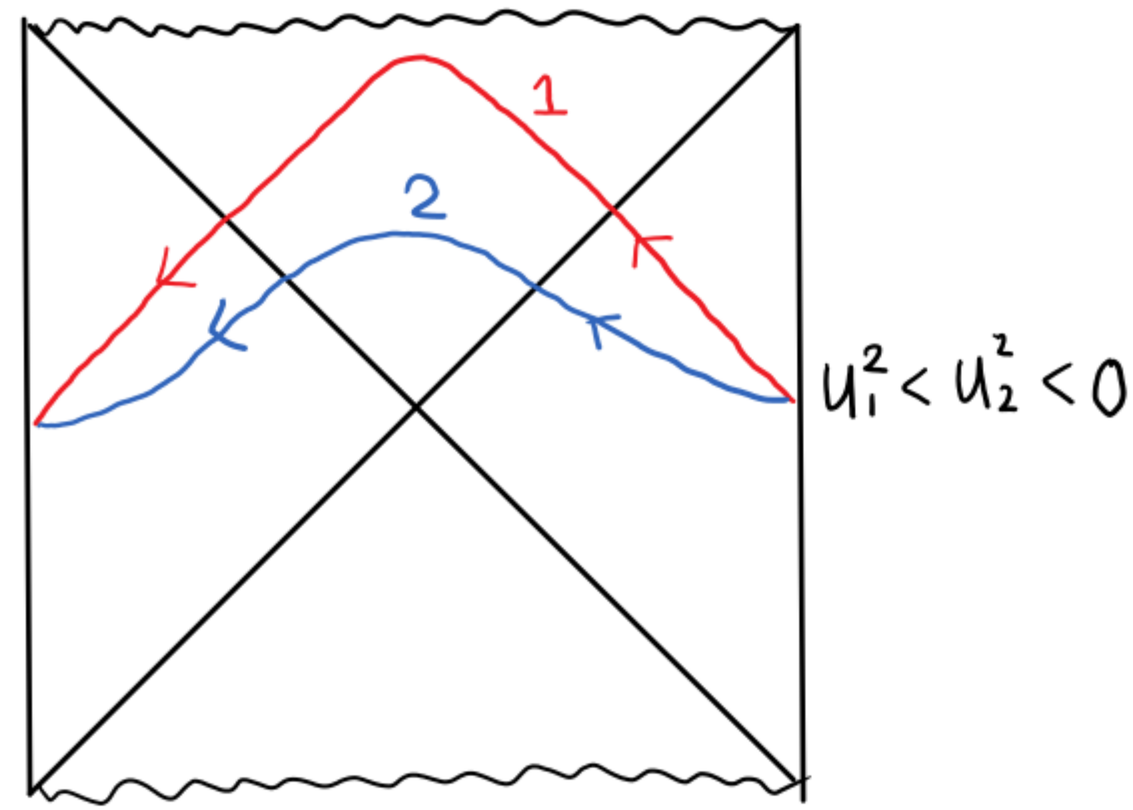
terning point : $\dot{r} = 0 \longrightarrow f = u^2$



time-like geodesic



space-like geodesic



In order to see the singularity, we focus on $u^2 \to -\infty$

$(u \to i\infty)$

What we learned are

$S(u) \sim$ - geodesic length

$u^2 \to -\infty$ → the singularity

$(u \to i\infty)$

Let's solve the EOM of S.

EOM of S

$$fS'^2 + \frac{1}{f}u^2 = 1$$

$$S'^2 = \frac{1}{f}(1 - \frac{u^2}{f})$$

$$S(u) = -\int_{r_{min}}^{r} \frac{\sqrt{f - u^2}}{f} dr \quad \text{with} \;\; u \to i\infty$$

$$
\begin{aligned}
S &= -\int_{r_{min}}^{\infty} \frac{\sqrt{f-u^2}}{f} dr \\
&= \int_{r_{min}}^{\infty} \left(\frac{u^2}{f\sqrt{f-u^2}} - \frac{1}{\sqrt{f-u^2}} \right) dr \\
&= iut_0 + \cdots
\end{aligned}
$$

$$
t_0 = \frac{\pi r_1}{r_1^2 + r_0^2} \qquad \text{for} \quad u = i \times (\text{positive})
$$

$$
\begin{aligned}
G_{bdry} &\sim e^{2\nu S(u)} \\
&\sim e^{2i\nu u t_0} \to 0 \qquad \text{for} \ \ u \to i\infty
\end{aligned}
$$

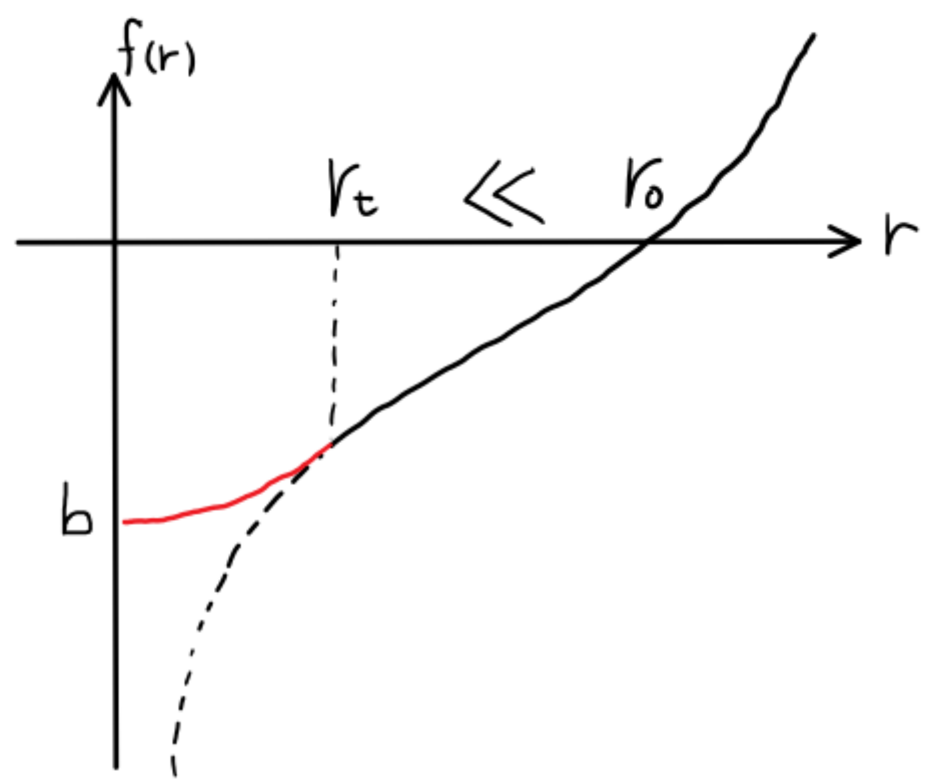
The authors say

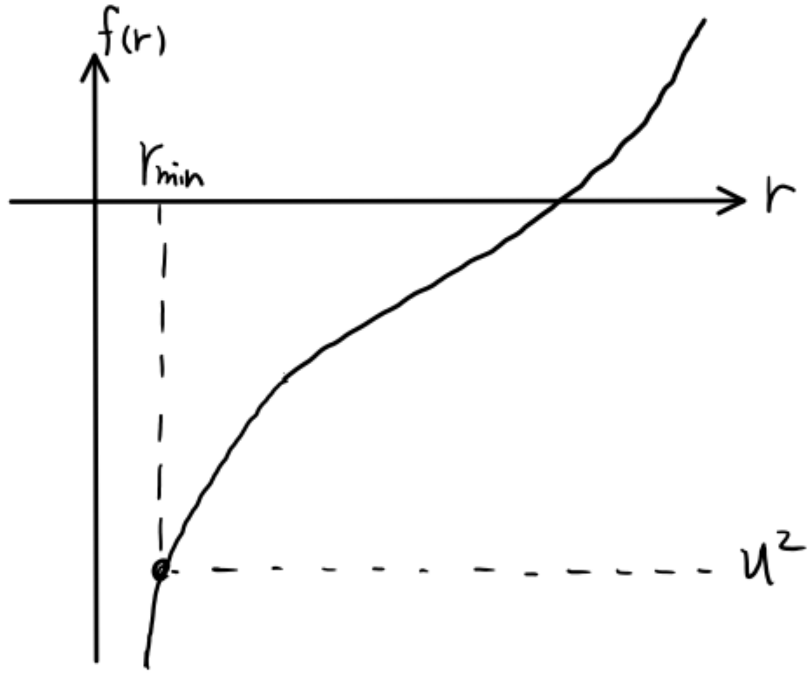
the character of the singularity appears as **decaying of Green fn**

To check this statement, we do the same calculations using a geometry which has **no singularities**.

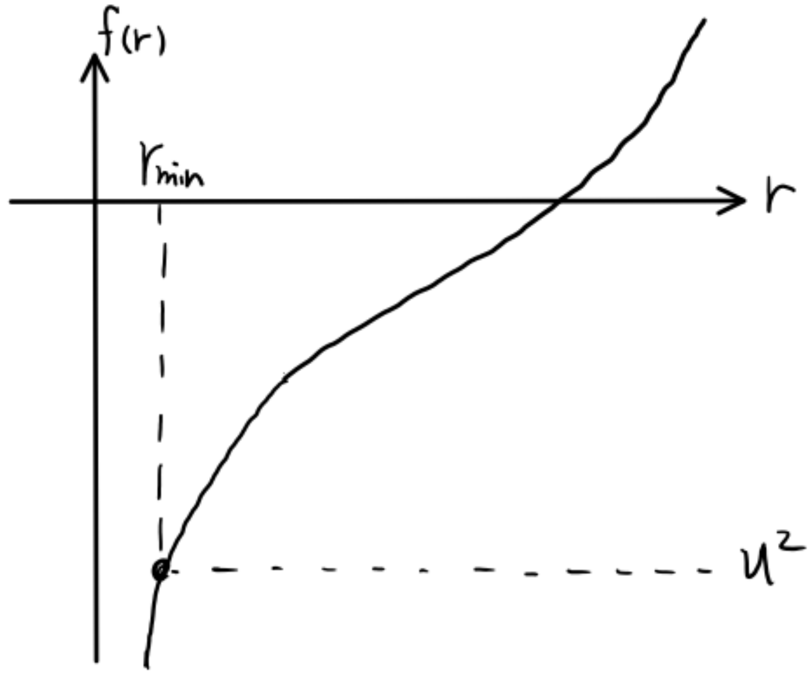
Finally, **we see the decaying** in this case as well.

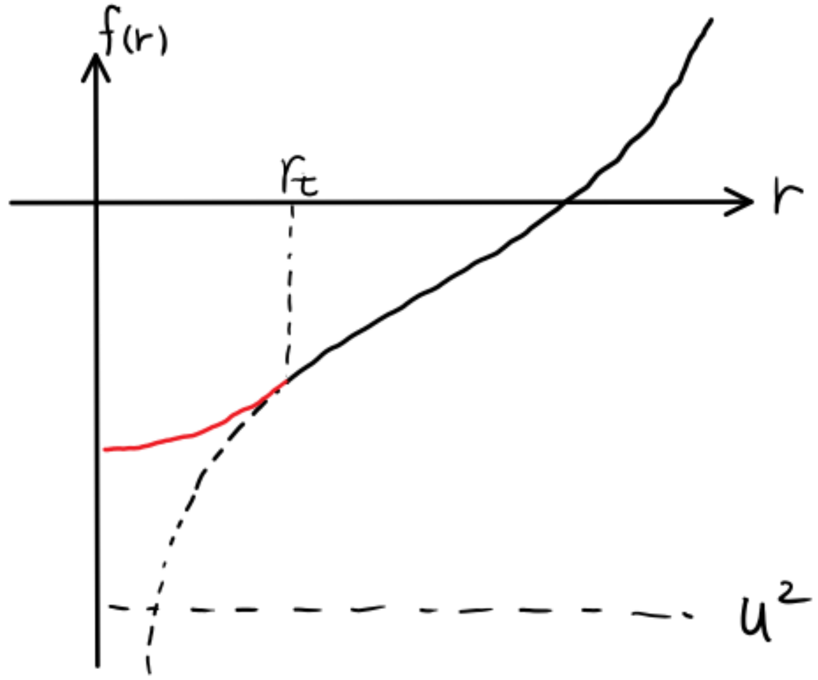
trial function $f(r) = \begin{cases} ar^2 + b & r \leq r_t \\ \frac{1}{r^2}(r^2 - r_0^2)(r^2 + r_1^2) & r > r_t \end{cases}$





$$S = \int_{r_{min}}^{\infty} \frac{\sqrt{f^2 - u^2}}{f} dr$$
$$= iut_0 + \cdots$$





$$S = \int_{\color{red}{r_{min}}}^{\infty} \frac{\sqrt{f^2 - u^2}}{f} dr$$
$$= iut_0 + \cdots$$

$$\tilde{S} = \int_{\color{red}{0}}^{\infty} \frac{\sqrt{f^2 - u^2}}{f} dr$$
$$= iu\tilde{t}_0 + \cdots$$

If $\tilde{t}_0 = t_0 + \Delta t > 0$ $\longrightarrow$ $G_{bdry} \sim e^{2i\nu u\tilde{t}_0} \underset{u \to i\infty}{\longrightarrow} 0$

$$r_0, r_1 \sim \mathcal{O}(1), \quad r_t \ll \mathcal{O}(1)$$

$$\begin{cases} \Delta t = \dfrac{r_t}{(r_0 r_1)^2} \left[\sqrt{2} \ln(\sqrt{2} - 1) + \dfrac{2}{3} \right] \ll \mathcal{O}(1) \\ t_0 = \dfrac{\pi r_1}{r_1^2 + r_0^2} \sim \mathcal{O}(1) \end{cases}$$

$$\longrightarrow \quad \tilde{t}_0 = t_0 + \Delta t > 0$$

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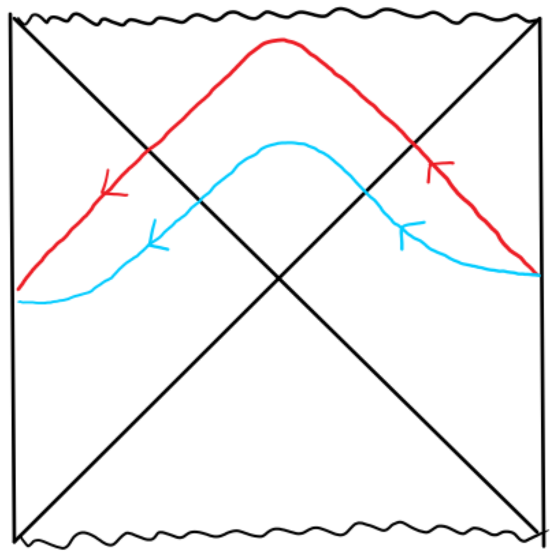
$$\longrightarrow \quad \tilde{t}_0 = t_0 + \Delta t > 0$$

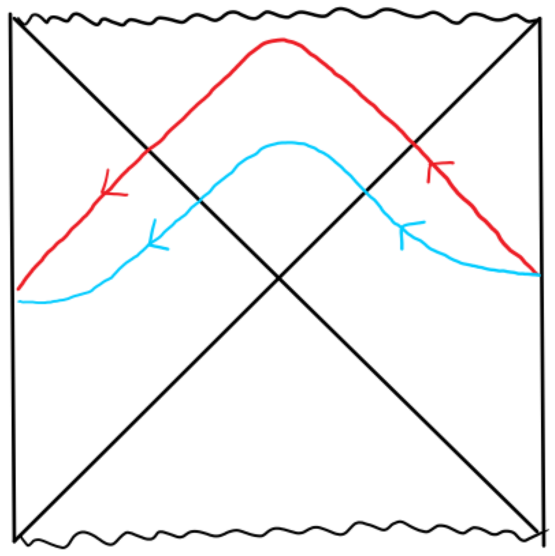
$\longrightarrow$ **Green fn decays without the BH singularity**

physical interpretation



The larger we take u, the deeper the geodesic goes inside.

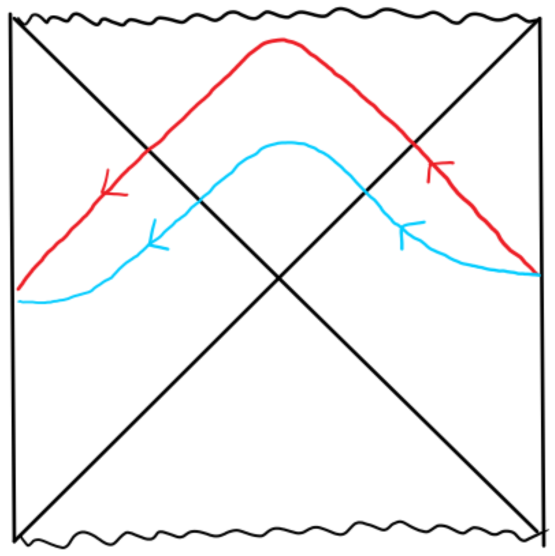
physical interpretation



The larger we take u, the deeper the geodesic goes inside.

$\longrightarrow$ the geodesic length $\sim S(u) \to -\infty$

physical interpretation

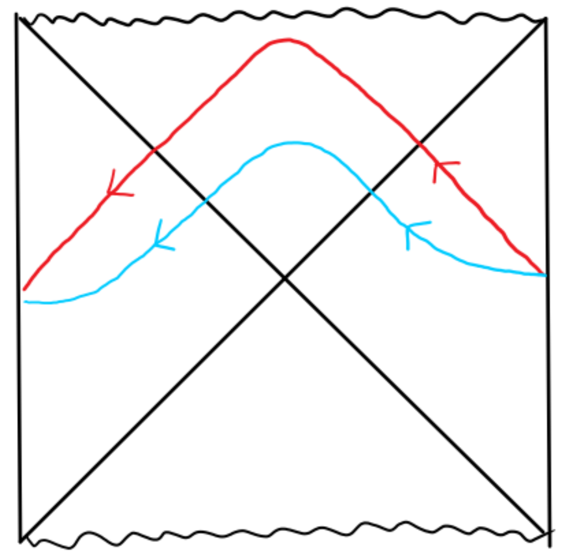


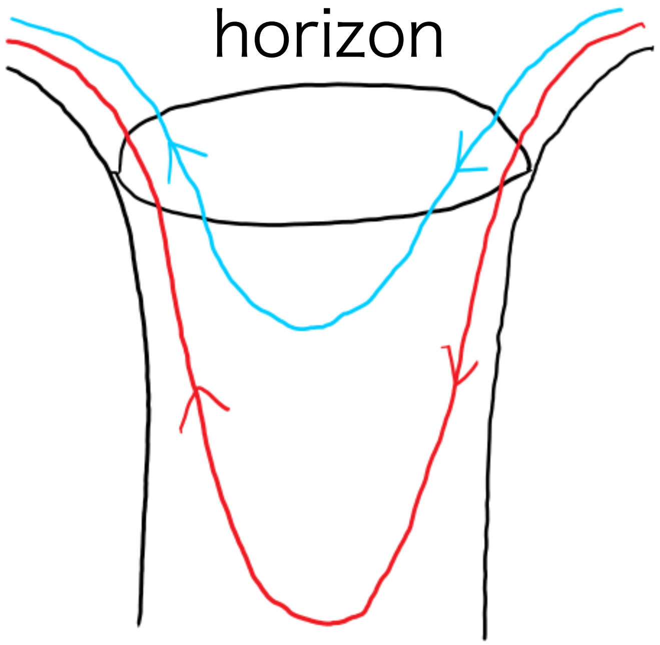
The larger we take u, the deeper the geodesic goes inside.

⟶ the geodesic length $\sim S(u) \to -\infty$

⟶ Green fn $G_{bdry} \sim e^{2\nu S(u)}$ decays.

physical interpretation





The larger we take u, the deeper the geodesic goes inside.

⟶ the geodesic length $\sim S(u) \to -\infty$

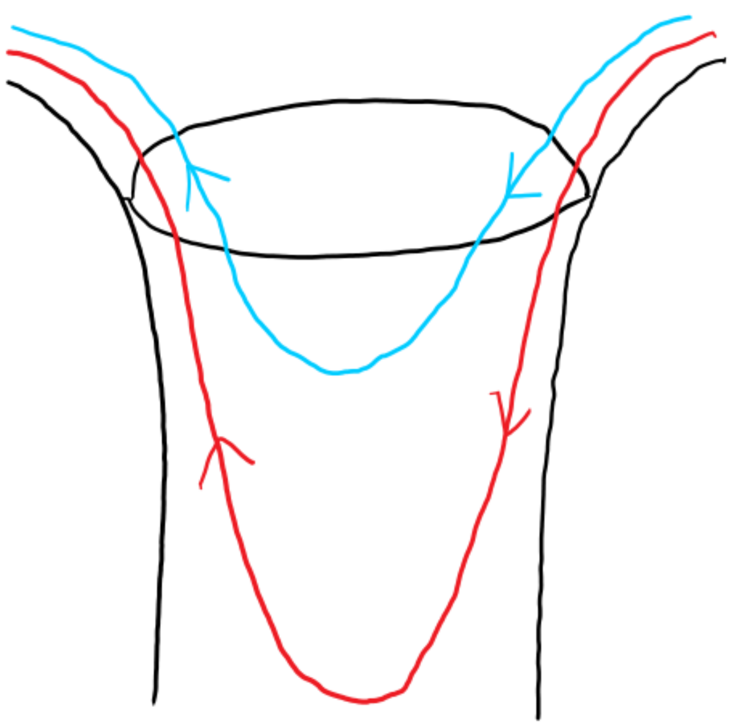
⟶ Green fn $G_{bdry} \sim e^{2\nu S(u)}$ decays.

⟶ throat-like structure appears(**global structure**)

Summary

We consider a black hole in AdS and assume AdS/CFT and check whether we can get any information about the BH from the boundary Green function.

⟶ As a result, we can see the signal of **the global structure**.



Summary

We consider a black hole in AdS and assume AdS/CFT and check whether we can get any information about the BH from the boundary Green function.

But

no information about the singularity

Future work

We want **local information** rather than global one.

→ I'm studying

how to reconstruct **local** bulk operators

from the boundary operators.

("bulk reconstruction" or "HKLL")

[A.Hamilton, D.Kabat, G.Lifschytz, D.Lowe('06)]