# The potential at Fixed Point

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@ M2 Journal 9/25

Reference : Non-minimal coupling in Higgs–Yukawa model with asymptotically safe gravity
arXiv:1510.03734v5 Kin-ya Oda and Masatoshi Yamada
素粒子物理学の非摂動的側面 山田 雅俊
Introduction to the Functional Renormalization Group
P. Kopietz L. Bartosch F. Schu¨tz

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# Introduction

We consider the single-field slow-roll inflation.

$\Rightarrow$ Scalar potential V(ϕ) is important.

Inflation Condition（Slow-roll Condition）

$$\dot{\phi} << 2V(\phi)$$

$$\ddot{\phi} << 3H|\dot{\phi}|$$

$\Rightarrow$ V should be flat.

# Introduction

Higgs Inflation is one of most effective models. So we consider...

$$S = \int d^D x \sqrt{g}(\underline{-F(\phi)R} + \boxed{V(\phi)} + \cdots)$$

scalar potential

But, Einstein-Hilbert action is

$$S_{EH} = 2\kappa^2 \int d^D x \sqrt{g}(\underline{-R} + 2\lambda)$$

$$\boxed{g^E{}_{\mu\nu} = F g_{\mu\nu}} \longrightarrow \begin{cases} \sqrt{g}\underline{F(\phi)R} \sim \sqrt{g^E}\underline{R^E} \\ \sqrt{g}\underline{V(\phi)} \sim \sqrt{g^E}\boxed{\dfrac{V(\phi)}{F^2(\phi)}} \end{cases}$$

# Introduction

Higgs Inflation is one of most effective models. So we consider...

$$S = \int d^D x \sqrt{g}(-F(\phi)R + \boxed{V(\phi)} + \cdots)$$

scalar potential

But, Einstein-Hilbert action is

$$S_{EH} = 2\kappa^2 \int d^D x \sqrt{g}(-R + 2\lambda)$$

$$\boxed{g^E{}_{\mu\nu} = F g_{\mu\nu}} \longrightarrow \begin{cases} \sqrt{g}F(\phi)R \sim \sqrt{g^E}R^E \\ \sqrt{g}V(\phi) \sim \sqrt{g^E}\boxed{\dfrac{V(\phi)}{F^2(\phi)}} \end{cases}$$

$$\boxed{\frac{V(\phi)}{F^2(\phi)} \quad \text{should be flat!}}$$

How do we check it ?

# Introduction

But , usually we can evaluate the potential around small $\phi$

⇦ **how to evaluate the potential around large $\phi$ ?**

Today

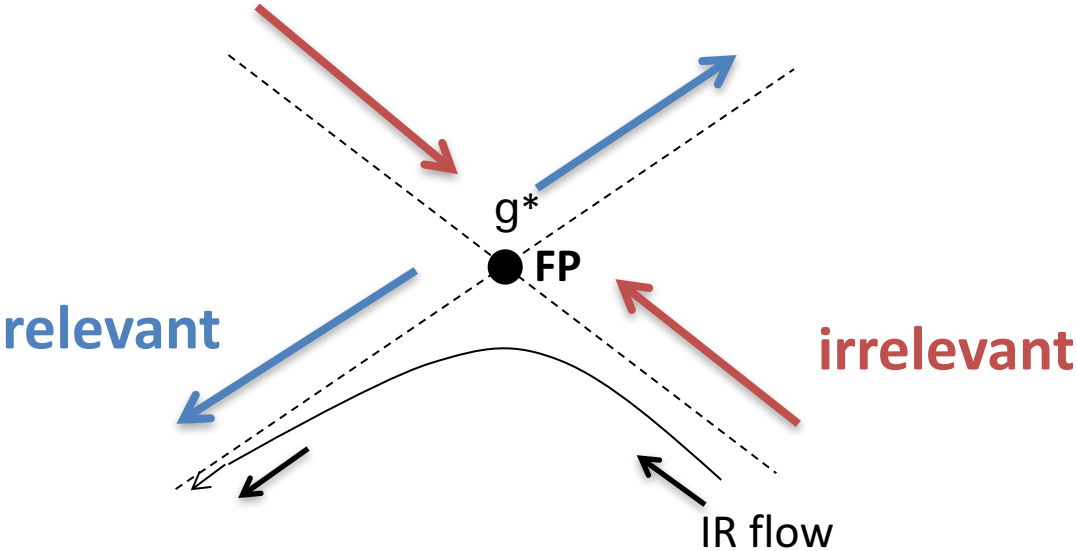
Can we evaluate the potential around large $\phi$
by considering the **"fixed point"** ?

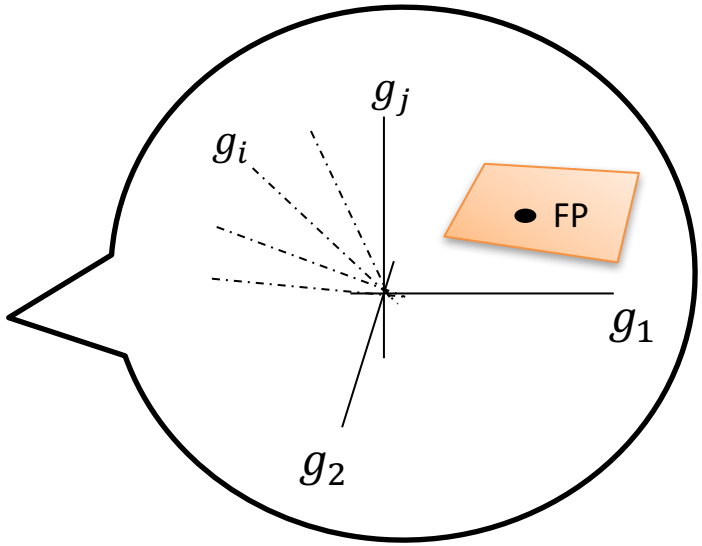
Our position is **"asymptotic safety"**.

For it, **"critical exponent"** is important.

# Fixed Point structure

Fixed Point $\beta_i(g^*_\Lambda) = 0$ $\left( \beta_i(g_\Lambda) = -\Lambda \frac{\partial g_{i,\Lambda}}{\partial \Lambda} \right)$



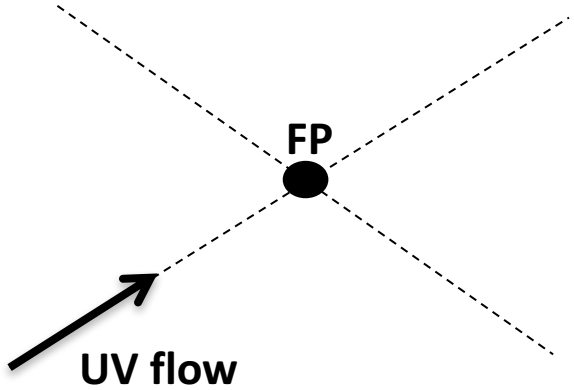




**Relevant** coupling determines the low energy physics.

hypersurface spanned by only relevant couplings

= **UV critical surface**

# Fixed Point structure




If we fix the physics at IR scales , can take $\Lambda \to \infty$.

In 3 dimension scalar theory it's called **"Wilson Fisher FP"**

**Asymptotic safety**

$\Rightarrow$ focusing on the nontrivial fixed point.

generalization of **asymptotic freedom**

focusing on Gaussian FP

$\Rightarrow$ Relevant couplings is renormalizable

# Critical exponent

$$g_\Lambda = g^*_\Lambda + \sum_{j=1}^{\infty} \xi_j \left(\frac{\Lambda_0}{\Lambda}\right)^{\theta_j}$$

**θ : critical exponent**

$Re(\theta) > 0$ ⟶ **relevant**

$Re(\theta) < 0$ ⟶ **irrelevant**

**Re(θ)** can be regarded as **the effective dimension including anomalous dimension** of the coupling

# Critical exponent

hypersurface spanned by only relevant couplings

= **UV critical surface**

↓

a number of $Re(\theta) > 0$ critical exponent

= **the dimension** of **UV critical surface**

↓

**Asymptotic safety**

$\Rightarrow$ When a number of $Re(\theta) > 0$ critical exponent is finite ,

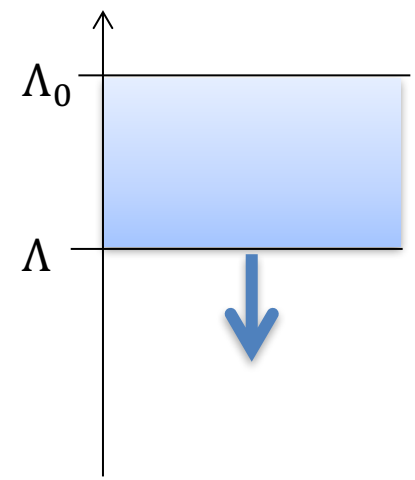
it‘s renormalizable!

# Wetterich Equation

## Wetterich Eq. = Renormalization Group Eq.

Effective action

$$\Gamma_\Lambda = J_\Lambda^\Phi \Phi - W_\Lambda[J_\Lambda^\Phi] - \Delta S_\Lambda[\Phi]$$




$$\Delta S_\Lambda[\Phi] = \frac{1}{2}\int_x \int_y \Phi^T(x) R_\Lambda(x,y)\Phi(y)$$

$$R_\Lambda(x,y) \sim \begin{cases} \Lambda^2 & for \quad p < 0 \\ 0 & for \quad p > 0 \end{cases}$$

Wetterich Eq.

$$\boxed{\frac{d\Gamma_\Lambda[\Phi]}{d\Lambda} = \frac{1}{2}\int_x \int_y \left(\frac{\delta^2 \Gamma_\Lambda}{\delta\Phi(x)\delta\Phi(y)}[\Phi] + R_\Lambda(x,y)\right)^{-1} \frac{dR_\Lambda}{d\Lambda}}$$

# Wetterich Equation

## For Example … (scalar field)

$$\Gamma_\Lambda[\Phi] = \int d^D r [U_\Lambda(\Phi(\mathbf{r})) + \frac{c_0}{2} Z_\Lambda^{-1}(\rho(\mathbf{r})) \sum_{i=1}^{N} (\nabla\Phi(\mathbf{r}))^2 + \frac{c_0}{4} Y_\Lambda(\rho(\mathbf{r}))(\nabla\rho(\mathbf{r}))^2 + \cdots]$$

$$\rho(\mathbf{r}) = \frac{1}{2}\Phi^2(\mathbf{r})$$

$$R_\Lambda(k^2) = c_0(\Lambda^2 - k^2)\Theta(\Lambda^2 - k^2)$$

$$U_\Lambda(\Phi) \equiv \frac{1}{V}\Gamma_\Lambda[\Phi]|_{\Phi(\mathbf{r})=\Phi}$$

$$\Theta(x) = \begin{cases} 0 & (x > 0) \\ 1 & (x \leq 0) \end{cases}$$

approximation

$$U_\Lambda(\Phi) \sim U_\Lambda(\rho)$$

<u>LPA</u>

$$\begin{cases} Z_\Lambda^{-1}(\rho(\mathbf{r})) = 1 \\ Y_\Lambda(\rho(\mathbf{r})) = 0 \end{cases}$$

$$\partial_\Lambda U_\Lambda(\rho) = \frac{c_0 K_D \Lambda^{D+1}}{D} [\frac{1}{c_0\Lambda^2 + U'_\Lambda(\rho) + 2\rho U''(\rho)} + \frac{N-1}{c_0\Lambda + U'_\Lambda(\rho)}]$$

# Analysis

- Potential on fixed point

  1. evaluate the scalar field potential → **today**
  2. apply the way to other potential

finally we want to evaluate $\dfrac{V(\phi)}{F^2(\phi)}$

- Critical Exponent of each points

  1. can check which solution is WF FP
  2. can identify relevant operators

# Analysis

- Evaluate the scalar potential at FP

<u>scalar field renormalization group eq.</u>

$$\partial_\Lambda U_\Lambda(\rho) = \frac{c_0 K_D \Lambda^{D+1}}{D}\left[\frac{1}{c_0\Lambda^2 + U'_\Lambda(\rho) + 2\rho U''(\rho)} + \frac{N-1}{c_0\Lambda + U'_\Lambda(\rho)}\right]$$

How to evaluate

1. **expand the potential** into polynomial of ρ around $\rho_0$

$$U_\Lambda(\rho) = \sum_n^\infty \lambda_{\Lambda,n}(\rho-\rho_0)^n$$

2. substitute the potential for RGE, and solve **RGE=0**

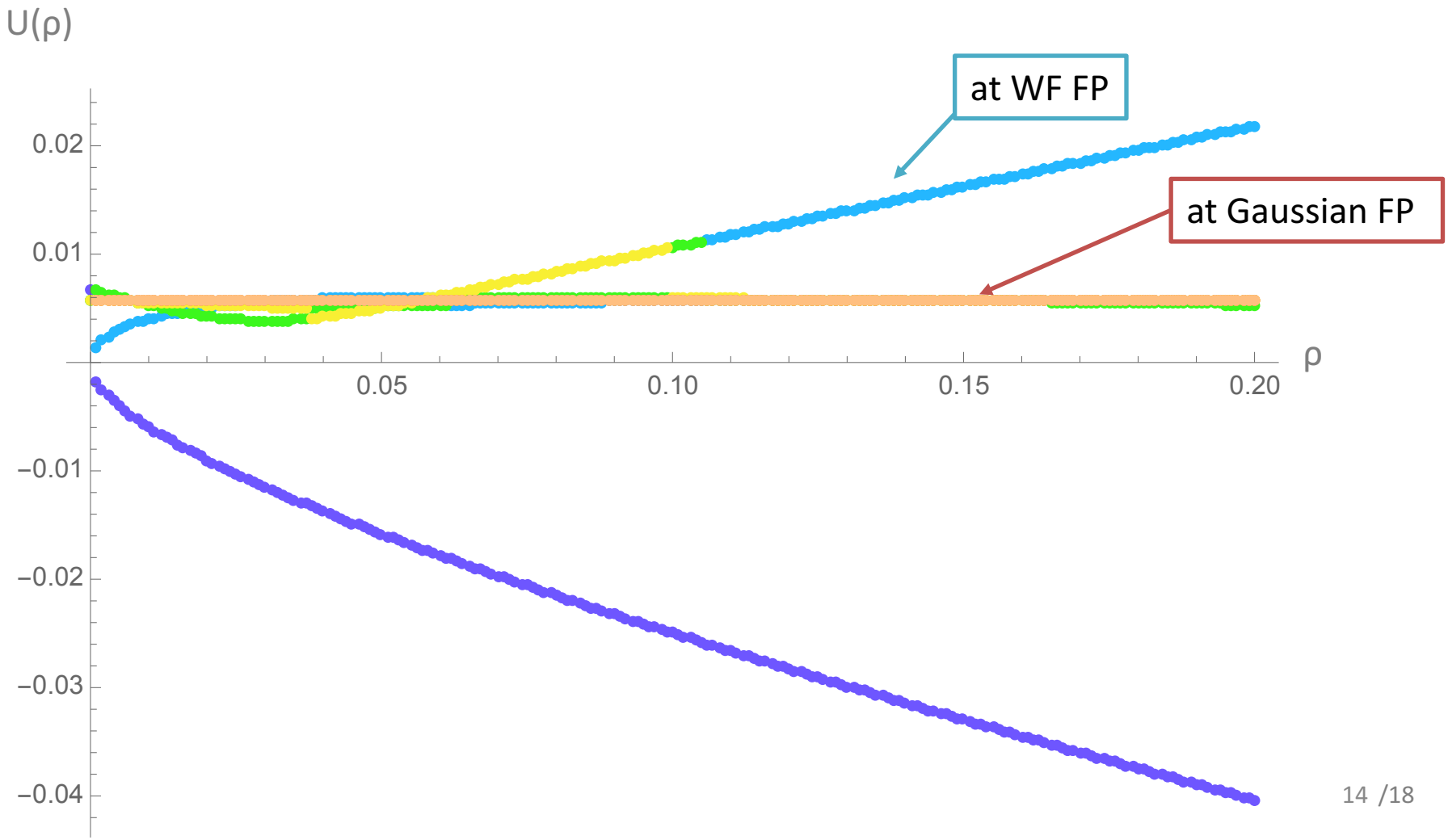
$$\partial_\Lambda U_\Lambda(\rho) = \sum_n^\infty A_n(\lambda_{\Lambda,n}, \lambda_{\Lambda,n-1}, \cdots)(\rho-\rho_0)^n \quad \Rightarrow \quad 0 = A_n(\lambda^*_{\Lambda,n}, \lambda^*_{\Lambda,n-1}, \cdots)$$

3. read coefficient , and it is **the potential around $\rho_0$**

$$U_\Lambda(\rho) = \sum_n^\infty \lambda^*_n(\rho-\rho_0)^n$$

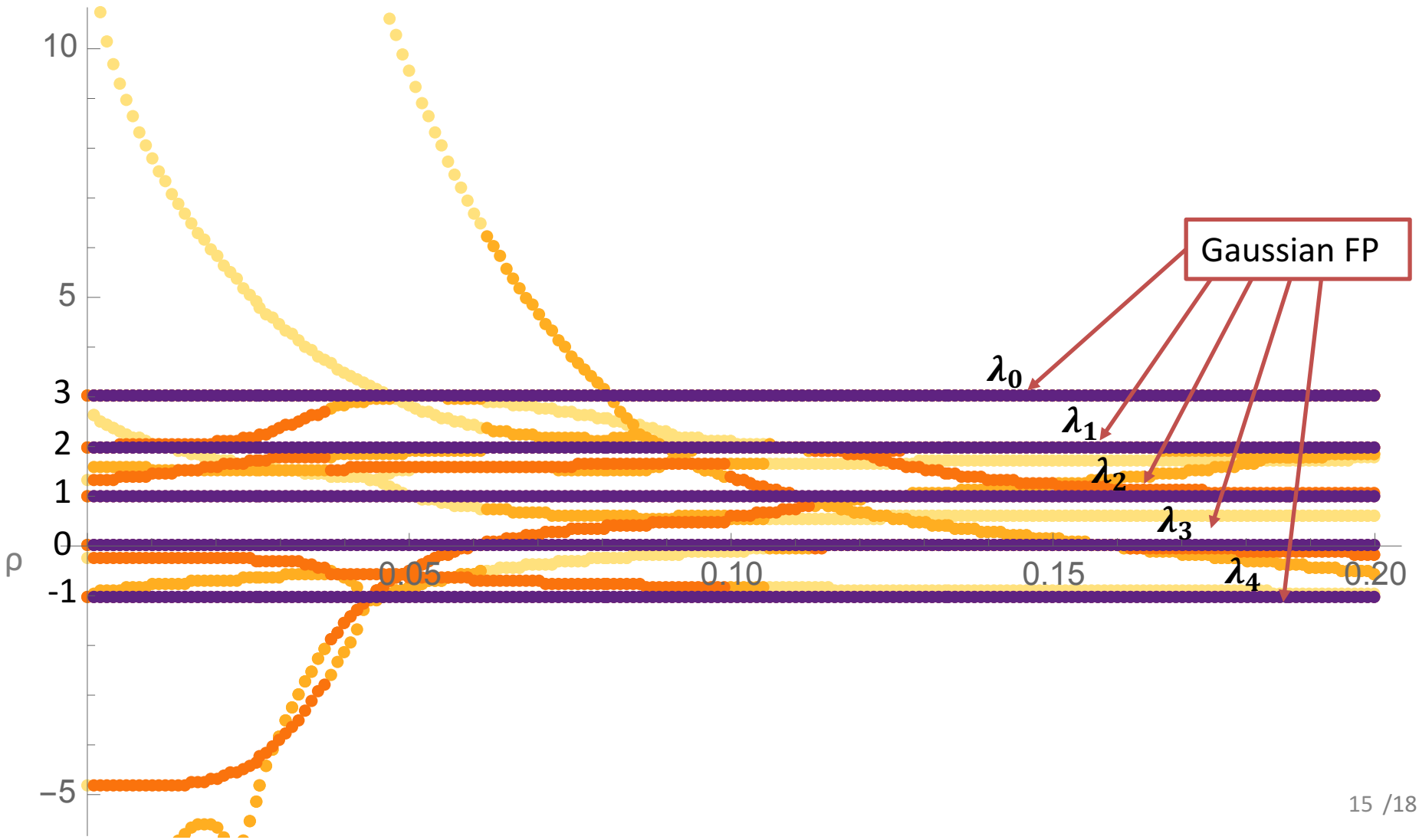
# Analysis

For Example … [scalar field , consider until $(\rho - \rho_0)^4$]



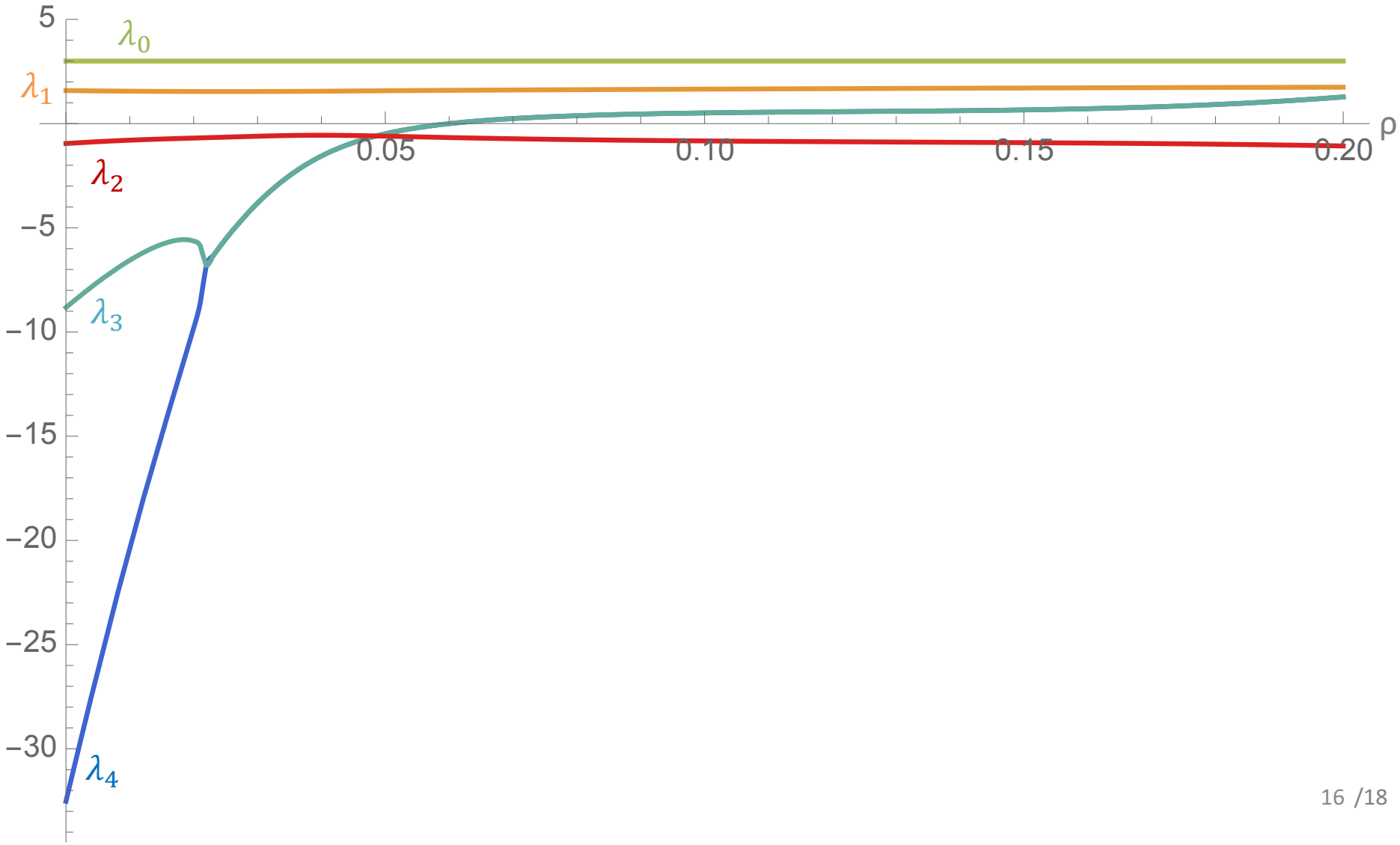
# Analysis

Critical Exponent(Re(θ)) —— Gaussian FP



# Analysis

Critical Exponent(Re(θ)) —— WF FP



# Summary

- By considering **the potential at FP ,**
  we can evaluate large ϕ scalar potential

- **Re(θ)>0 critical exponent** = the dimension of the coupling
  A number of them = the dimension of **UV critical surface**

- The dimension of UV critical surface is finite
  = **Asymptotic safety**

# Next Study

- the theory including gravity

- check F and V renormalization flow

# 予備

# β(g) flow & Re(θ) = dimension

expand β(g) around g*(fixed point)

$$\beta_i[g] = \beta[g*] + \sum_j^n \boxed{\frac{\partial \beta_i}{\partial g_j}|_{g=g*}}(g - g*) + \cdots$$

the eigenvalue of this matrix = θ

$$\begin{cases} \beta_1 = 2g_1 + \frac{c}{2}\frac{g_2}{1+g_1} \\ \beta_2 = (4-d)g_2 - \frac{3}{2}c\frac{g_2^2}{(1+g_1)^2} + \frac{c}{2}\frac{g_3}{1+g_1} \\ \beta_3 = (6-2d)g_3 + 15c\frac{g_2^3}{(1+g_1)^3} - \frac{15c}{2}\frac{g_2 g_3}{(1+g_1)^2} \end{cases}$$

canonical dimension