Numerical simulation of the $\mathcal{N} = (2,2)$ Landau-Ginzburg model

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- Syo Kamata (Rikkyo Univ.) and H.S., arXiv:1107.1367, to appear in Nucl. Phys. B

Non-perturbative study of SUSY theories

- SUSY will be a very fundamental symmetry
  - Beyond the Standard Model
  - Consistency of the String theory
- No manifest SUSY regularization in general
- Available tools
  - Perturbation theory
  - Consistency

  could however be insufficient to investigate non-perturbative phenomena, s.t.,
  - Confinement, bound states, spontaneous chiral symmetry breaking, quantum tunneling, spontaneous SUSY breaking, . . .
- Non-perturbative analysis of SUSY theories from first principles is a challenging problem!

2D $\mathcal{N} = (2,2)$ Wess-Zumino (WZ) model provides...

- Action ($z, \bar{z} = x_0 \pm ix_1$). $W = W(\Phi)$ is the superpotential

$$S = \int d^2x \left[4\partial_z A^* \partial_{\bar{z}} A - F^* F - F^* W'(A^*) - F W'(A) + (\bar{\psi}_{\dot{1}}, \psi_2) \begin{pmatrix} 2\partial_z & W''(A^*) \\ W''(A) & 2\partial_{\bar{z}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \bar{\psi}_{\dot{2}} \end{pmatrix} \right]$$

- SUSY transformations (similarly for $\bar{Q}_{\dot{1}}$ and $\bar{Q}_{\dot{2}}$)

$$\begin{aligned} Q_1 \bar{\psi}_{\dot{1}} &= -2\sqrt{2}\partial_{\bar{z}} A^* & Q_1 A^* &= 0, \\ Q_1 F^* &= 2\sqrt{2}\partial_{\bar{z}} \bar{\psi}_{\dot{2}} & Q_1 \bar{\psi}_{\dot{2}} &= 0 \\ Q_1 A &= \sqrt{2}\psi_1 & Q_1 \psi_1 &= 0 \\ Q_1 \psi_2 &= \sqrt{2} F & Q_1 F &= 0 \end{aligned}$$

$$\begin{aligned} Q_2 \bar{\psi}_{\dot{2}} &= -2\sqrt{2}\partial_z A^* & Q_2 A^* &= 0 \\ Q_2 F^* &= -2\sqrt{2}\partial_z \bar{\psi}_{\dot{1}} & Q_2 \bar{\psi}_{\dot{1}} &= 0 \\ Q_2 A &= \sqrt{2}\psi_2 & Q_2 \psi_2 &= 0 \\ Q_2 \psi_1 &= -\sqrt{2} F & Q_2 F &= 0 \end{aligned}$$

Landau-Ginzburg (LG) model for $\mathcal{N} = (2,2)$ SCFT?

- Conjectured correspondence to the $\mathcal{N} = (2,2)$ unitary discrete series

superpotential W	Lie algebra g	rank μ	Coxeter number N	Coxeter exponents e
Φ^n $(n \geq 3)$	A_{n-1}	$n-1$	n	$1, 2, \ldots, n-1$
$\Phi^n + \Phi\Phi'^2$ $(n \geq 3)$	D_{n+1}	$n+1$	$2n$	$1, 3, 5, \ldots, 2n-1$, and n
$\Phi^3 + \Phi'^4$	E_6	6	12	1, 4, 5, 7, 8, 11
$\Phi^3 + \Phi\Phi'^3$	E_7	7	18	1, 5, 7, 9, 11, 13, 17
$\Phi^3 + \Phi'^5$	E_8	8	30	1, 7, 11, 13, 17, 19, 23, 29

  - Central charge and the conformal dimension of chiral primary fields in the NS sector:

$$c = 3 - \frac{6}{N}, \qquad (h, \bar{h}) = \left(\frac{e-1}{2N}, \frac{e-1}{2N} \right)$$

  - Dimension of the chiral ring and the Witten index Δ is given by μ

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- Simplest case, $W(\Phi) = \lambda\Phi^3/3$,

$$c = 1, \qquad (h, \bar{h}) = \left(\frac{1}{6}, \frac{1}{6} \right), \qquad \Delta = 2$$

Appendix: Argument for a homogeneous superpotential $W = \lambda\Phi^n/n$

- Under the scale transformation, $x \to \lambda x$, $\theta \to \lambda^{1/2}\theta$, $d^2x\,d^2\theta \to \lambda d^2x\,d^2\theta$. Thus
$$\Phi \to \lambda^{-1/n}\Phi$$
leaves F-term intact
- The operator W' would be irrelevant, because
$$d^2x\,d^2\theta\,W' = d^2x\,d^2\theta\,\frac{1}{4}\bar{D}^2\bar{\Phi} \to \lambda^{-1/n}d^2x\,d^2\theta\,\frac{1}{4}\bar{D}^2\bar{\Phi} \to 0 \qquad \text{for } \lambda \to \infty$$
- A, the first component of Φ, is a chiral field, in the sense that
$$\bar{Q}_{\dot{2}}A = 0 \qquad \bar{Q}_{\dot{2}} \Leftrightarrow G^+_{-1/2} \text{ of } \mathcal{N} = 2 \text{ SCA}$$
- Basis of a chiral ring
$$1, A, A^2, \dots, A^{n-2}, A^{n-1} \sim W' \sim 0$$
will possess the conformal dimension
$$h = 0, \frac{1}{2n}, \frac{2}{2n}, \dots, \frac{n-2}{2n}$$
and the $U(1)$ charge
$$q = 0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-2}{n} = \frac{c}{3} \text{ from } \mathcal{N} = 2 \text{ SCA}$$

Landau-Ginzburg description of the $\mathcal{N} = (2,2)$ SCFT

- Various tests:
  - EOM vs the fusion rule (Kastor-Martinec-Shenker (1988))
  - Singularity theory (Vafa-Warner (1988))
  - Chiral ring with $W' \simeq 0$ (Lerche-Vafa-Warner (1989))
  - “Topological” correlation function (Cecotti-Girardello-Pasquinucci (1989))
  - Non-renormalization theorem and renormalization group (Howe-West (1989))
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  - Strong coupling in low energies
  - Massless perturbation theory suffers from IR divergences

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- Very large anomalous dimension. For example, for $W(\Phi) = \lambda\Phi^3/3$,

$$\langle A(x)A^*(0)\rangle \sim \frac{1}{z^{2h}\bar{z}^{2\bar{h}}}, \qquad 1 - h - \bar{h} = 0.666\dots$$

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- Non-perturbative formulation of the 2D WZ model?

- Wilson fermion (Sakai-Sakamoto (1983))
- Overlap fermion (Kikukawa-Nakayama (2002))
  - Local lattice actions are manifestly invariant under

$$Q \equiv -\frac{1}{\sqrt{2}}(\bar{Q}_{\dot{1}} + Q_2)$$

  but cannot be so under all SUSY, because

$$\{Q_1, \bar{Q}_{\dot{1}}\} = -4\partial_{\bar{z}} \qquad \{Q_2, \bar{Q}_{\dot{2}}\} = -4\partial_z$$

- SUSY is restored in the continuum limit
  - This is actually the case without fine-tuning (Fujikawa (2002), Giedt-Poppitz (2004), Kawai-Kikukawa (2010), Kadoh-H.S. (2010))

Lattice formulation of the 2D WZ model

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- Triumph of a lattice formulation! (Kawai-Kikukawa (2010))

$$1 - h - \bar{h} = 0.660(11) \qquad \text{for } W(\Phi) = \lambda\Phi^3/3$$

Supersymmetric formulation (Kadoh-H.S. (2009))

- In terms of momentum modes in a finite box with periodic boundary conditions

$$\varphi(x) = \frac{1}{L_0 L_1} \sum_p e^{ipx} \tilde{\varphi}(p) \qquad p_\mu = \frac{2\pi}{L_\mu} n_\mu \quad (n_\mu = 0, \pm 1, \pm 2, \dots)$$

the SUSY transformations ($p_z, p_{\bar{z}} = (1/2)(p_0 \mp ip_1)$)

$$Q_1 \tilde{\bar{\psi}}_{\dot{1}}(p) = -2\sqrt{2} i p_{\bar{z}} \tilde{A}^*(p) \qquad Q_1 \tilde{A}^*(p) = 0$$
$$Q_1 \tilde{F}^*(p) = 2\sqrt{2} i p_{\bar{z}} \tilde{\bar{\psi}}_{\dot{2}}(p) \qquad Q_1 \tilde{\bar{\psi}}_{\dot{2}}(p) = 0$$
$$Q_1 \tilde{A}(p) = \sqrt{2} \tilde{\psi}_1(p) \qquad Q_1 \tilde{\psi}_1(p) = 0$$
$$Q_1 \tilde{\psi}_2(p) = \sqrt{2} \tilde{F}(p) \qquad Q_1 \tilde{F}(p) = 0$$

etc. are simply linear transformations

- So restriction on momentum modes

$$-\Lambda \leq p_\mu \leq \Lambda \qquad \Lambda\text{: UV cutoff}$$

does not break SUSY

Supersymmetric formulation

- Partition function

$$\mathcal{Z} \equiv \int \prod_{-\Lambda \leq p_\mu \leq \Lambda} \left[d\tilde{A}(p)\, d\tilde{A}^*(p) \prod_{\alpha} d\tilde{\psi}_\alpha(p) \prod_{\dot{\alpha}} d\tilde{\bar{\psi}}_{\dot{\alpha}}(p) d\tilde{F}(p)\, d\tilde{F}^*(p) \right] e^{-S}$$

- Action

$$S = \frac{1}{L_0 L_1} \sum_p \Big[4p_z \tilde{A}^*(-p) p_{\bar{z}} \tilde{A}(p) - \tilde{F}^*(-p)\tilde{F}(p) - \tilde{F}^*(-p) * W'(\tilde{A})^*(p)$$

$$- \tilde{F}(-p) * W'(\tilde{A})(p) + (\tilde{\bar{\psi}}_{\dot{1}}, \tilde{\psi}_2)(-p) \begin{pmatrix} 2ip_z & W''(\tilde{A})^* * \\ W''(\tilde{A}) * & 2ip_{\bar{z}} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\bar{\psi}}_{\dot{2}} \end{pmatrix} (p) \Big]$$

where $*$ denotes the convolution

$$(\tilde{\varphi}_1 * \tilde{\varphi}_2)(p) \equiv \frac{1}{L_0 L_1} \sum_q \tilde{\varphi}_1(q) \tilde{\varphi}_2(p - q)$$

Supersymmetric formulation

- This finite-dimensional integral

$$\mathcal{Z} \equiv \int \prod_{-\Lambda \le p_\mu \le \Lambda} \left[d\tilde{A}(p)\, d\tilde{A}^*(p) \prod_\alpha d\tilde{\psi}_\alpha(p) \prod_{\dot\alpha} d\tilde{\bar{\psi}}_{\dot\alpha}(p) d\tilde{F}(p)\, d\tilde{F}^*(p) \right] e^{-S}$$

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- Linear internal symmetries, such as the $U(1)_R$ symmetry, for $W(\Phi) = \lambda\Phi^n/n$,

$$A \to e^{(2/n)i\theta} A, \quad \psi_\alpha \to e^{-(1-2/n)i\theta}\psi_\alpha, \quad \bar{\psi}_{\dot{\alpha}} \to e^{(1-2/n)i\theta}\bar{\psi}_{\dot{\alpha}}, \quad F \to e^{-(2-2/n)i\theta} F$$

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- Perfect? Sounds too good to be true!

What sacrificed is the . . . locality

- Delta function is modified as

$$\delta(x) \to \frac{1}{L} \sum_{-\Lambda \leq p \leq \Lambda} e^{ipx} = \frac{1}{L} \frac{\sin\left((N+1)\pi(x-y)/L\right)}{\sin\left(\pi(x-y)/L\right)}$$

  where we have set

$$\Lambda = \pi \frac{N}{L} = \frac{\pi}{L/N}$$

- There is an issue of universality
- For odd N, the above formulation is identical to a lattice formulation based on the SLAC derivative (Bartels-Bronzan (1983), Bergner (2009))

Locality is OK within the perturbation theory

- The above prescription reproduces correct Feynman integrals, at least for 2D WZ model and 3D WZ model with a cubic W, because...
- The superficial degrees of divergence of a superdiagram Γ

$$\omega(\Gamma) = d - 2 - \frac{1}{2}(d-2)E + \sum_n \left[\frac{1}{2}(d-2)n + 1 - d\right] V_n - C$$

E: external lines V_n: Φ^n-vertices C: internal lines

is strictly negative for these models

- All Feynman integrals are absolutely UV convergent; these models are ignorant of the UV regularization (as far as the latter preserves SUSY)
- Of course, non-perturbative validity is a different issue

Appendix: Possible problem for 4D

- Cutoff function

$$\Theta(p;\Lambda)=\begin{cases}1 & \text{for } -\Lambda\leq p_\mu\leq\Lambda\\ 0 & \text{otherwise}\end{cases}$$

- Modified Feynman rule

$$\frac{1}{p^2+m^2}\longrightarrow\frac{\Theta(p;\Lambda)}{p^2+m^2}$$

- One-loop correction to the D-term $\tilde{\Phi}^\dagger\tilde{\Phi}$:

$$\int\frac{d^4p}{(2\pi)^4}\frac{\Theta(p;\Lambda)}{p^2+m^2}\frac{\Theta(p+k;\Lambda)}{(p+k)^2+m^2}$$

$$=\int\frac{d^4p}{(2\pi)^4}\Theta(p;\Lambda)\Theta(p+k;\Lambda)\frac{p^2-(p+k)^2}{(p^2+m^2)^2[(p+k)^2+m^2]}\xrightarrow{\Lambda\to\infty}\text{finite part}$$

$$+\int\frac{d^4p}{(2\pi)^4}\Theta(p;\Lambda)\frac{1}{(p^2+m^2)^2}\xrightarrow{\Lambda\to\infty}\text{wave function renormalization}$$

$$-\int\frac{d^4p}{(2\pi)^4}\Theta(p;\Lambda)\,[1-\Theta(p+k;\Lambda)]\frac{1}{(p^2+m^2)^2}\xrightarrow{\Lambda\to\infty}0\text{ for a fixed }k$$

- In the two-loop level, k can become an integration variable and the last term would survive as a non-local term

- After integrating over $\tilde{F}$ and $\tilde{F}^*$,

$$\mathcal{Z} = \int \prod_{-\Lambda \leq p_\mu \leq \Lambda} \left[d\tilde{A}(p)\, d\tilde{A}^*(p) \prod_\alpha d\tilde{\psi}_\alpha(p) \prod_{\dot{\alpha}} d\tilde{\bar{\psi}}_{\dot{\alpha}}(p) \right] e^{-S}$$

- Action

$$S = \frac{1}{L_0 L_1} \sum_p \left[\tilde{N}^*(-p)\tilde{N}(p) + (\tilde{\bar{\psi}}_{\dot{1}}, \tilde{\psi}_2)(-p) \begin{pmatrix} 2ip_z & W''(\tilde{A})^{*}* \\ W''(\tilde{A})* & 2ip_{\bar{z}} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\bar{\psi}}_{\dot{2}} \end{pmatrix} (p) \right]$$

where

$$\tilde{N}(p) \equiv 2ip_z \tilde{A}(p) + W'(\tilde{A})^*(p) \qquad \tilde{N}^*(-p) \equiv \tilde{N}(p)^*$$

- We note

$$S = \frac{1}{L_0 L_1} \sum_p \left[\tilde{N}^*(-p)\tilde{N}(p) + (\tilde{\bar{\psi}}_{\dot{1}}, \tilde{\psi}_2)(-p) \begin{pmatrix} \frac{\partial \tilde{N}(p)}{\partial \tilde{A}(p)} & \frac{\partial \tilde{N}(p)}{\partial \tilde{A}^*(p)} * \\ \frac{\partial \tilde{N}^*(p)}{\partial \tilde{A}(p)} * & \frac{\partial \tilde{N}^*(p)}{\partial \tilde{A}^*(p)} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\bar{\psi}}_{\dot{2}} \end{pmatrix} (p) \right]$$

. . . on the basis of the Nicolai mapping (Nicolai (1980))

- After integrating over fermions

$$\mathcal{Z} = \int \prod_{-\Lambda \leq p_\mu \leq \Lambda} \left[d\tilde{A}(p)\, d\tilde{A}^*(p) \right] \exp\left[-\frac{1}{L_0 L_1} \sum_p \tilde{N}(p)^* \tilde{N}(p) \right] \det \frac{\partial(\tilde{N}, \tilde{N}^*)}{\partial(\tilde{A}, \tilde{A}^*)}$$

- Change of integration variables from $(\tilde{A}, \tilde{A}^*)$ to $(\tilde{N}, \tilde{N}^*)$:

$$\mathcal{Z} = \int \prod_{-\Lambda \leq p_\mu \leq \Lambda} \left[d\tilde{N}(p)\, d\tilde{N}^*(p) \right] \exp\left[-\frac{1}{L_0 L_1} \sum_p \tilde{N}(p)^* \tilde{N}(p) \right]$$

$$\times \sum_i \operatorname{sign} \det \left. \frac{\partial(\tilde{N}, \tilde{N}^*)}{\partial(\tilde{A}, \tilde{A}^*)} \right|_{\tilde{A}=\tilde{A}_i, \tilde{A}^*=\tilde{A}_i^*}$$

where $(\tilde{A}_i, \tilde{A}_i^*)$ $(i = 1, 2, \ldots)$ are solutions of

$$2ip_z \tilde{A}(p) + W'(\tilde{A})(-p)^* = \tilde{N}(p)$$

$$-2ip_{\bar{z}} \tilde{A}(p)^* + W'(\tilde{A})(-p) = \tilde{N}(p)^*$$

Simulation algorithm using the Nicolai mapping (Beccaria-Curci-D'Ambrosio (1998))

1. Generate gaussian random numbers $(\tilde{N}(p), \tilde{N}(p)^*)$
2. Find all the solutions $(\tilde{A}(p)_i, \tilde{A}(p)_i^*)$ $(i = 1, 2, \ldots)$ of
$$2ip_z\tilde{A}(p) + W'(\tilde{A})(-p)^* = \tilde{N}(p)$$
$$-2ip_{\bar{z}}\tilde{A}(p)^* + W'(\tilde{A})(-p) = \tilde{N}(p)^*$$
3. Compute
$$\sum_i \text{sign}\det \frac{\partial(\tilde{N}, \tilde{N}^*)}{\partial(\tilde{A}, \tilde{A}^*)}\bigg|_{\tilde{A}=\tilde{A}_i, \tilde{A}^*=\tilde{A}_i^*}$$
and
$$\sum_i \text{sign}\det \frac{\partial(\tilde{N}, \tilde{N}^*)}{\partial(\tilde{A}, \tilde{A}^*)}\mathcal{O}(\tilde{A}, \tilde{A}^*)\bigg|_{\tilde{A}=\tilde{A}_i, \tilde{A}^*=\tilde{A}_i^*}$$
4. Repeat the steps from 1 and take an average over the random numbers

Simulation algorithm using the Nicolai mapping

- Advantage
  - No autocorrelation, no critical slowing down!
  - "Normalized partition function"

$$\Delta \equiv \frac{\left\langle \sum_i \text{sign}\det \frac{\partial(\tilde{N},\tilde{N}^*)}{\partial(\tilde{A},\tilde{A}^*)}\Big|_{\tilde{A}=\tilde{A}_i,\tilde{A}^*=\tilde{A}_i^*} \right\rangle_N}{\langle 1 \rangle_N}$$

  gives rise to the Witten index
- Disadvantage
  - It is impossible to be sure whether all solutions of

$$2ip_z\tilde{A}(p) + W'(\tilde{A})(-p)^* = \tilde{N}(p)$$
$$-2ip_{\bar{z}}\tilde{A}(p)^* + W'(\tilde{A})(-p) = \tilde{N}(p)^*$$

  are found or not
  - Applicable to periodic boundary conditions only (not to the thermal one, for example)

Simulation parameters

- All dimensionful quantities are measured in the "lattice spacing" *a* defined by

$$\Lambda = \frac{\pi}{a}$$

- Cubic superpotential

$$W = \frac{\lambda}{3}\Phi^3 \qquad a\lambda = 0.3 \text{ fixed (same as Kawai-Kikukawa)}$$

- Configurations (357.4 CPU · day on the Intel Xeon 2.93GHz)

$N_0 = N_1$	16	18	20	22	24	26
$(2+)$	1276	1273	1275	1271	1271	1273
$(3+, 1-)$	4	7	5	9	9	7
$(1+)$	0	0	0	0	0	0
$(3+)$	0	0	0	0	0	0
$(4+, 1-)$	0	0	0	0	0	0
$(4+2-)$	0	0	0	0	0	0
Δ	2	2	2	2	2	2
δ [%]	$0.2(2)$	$0.1(1)$	$-0.2(1)$	$0.1(1)$	$-0.1(1)$	$-0.1(1)$

$N_0 = N_1$	28	30	32	34	36
$(2+)$	1264	1261	1250	1254	1221
$(3+, 1-)$	16	17	24	17	26
$(1+)$	0	1	2	6	31
$(3+)$	0	0	4	2	2
$(4+, 1-)$	0	1	0	0	0
$(4+, 2-)$	0	0	0	1	0
Δ	2	$2.000(1)$	$2.002(2)$	$1.997(2)$	$1.977(4)$
δ [%]	$-0.0(1)$	$0.1(2)$	$0.0(2)$	$-0.1(3)$	$0.0(5)$

- Newton-Raphson method from 100 initial random guesses

- One-point relation (see Table)

$$\delta \equiv \frac{\langle S_B \rangle}{(N_0+1)(N_1+1)} - 1 = 0$$

where

$$S_B \equiv \frac{1}{L_0 L_1} \sum_p \tilde{N}(p)^* \tilde{N}(p)$$

- From $\langle Q_1(\tilde{A}(p)\tilde{\bar{\psi}}_{\dot{1}}(-p))\rangle = 0$,

$$p_0 \left\langle \tilde{A}(p)\tilde{A}^*(-p) \right\rangle = -\operatorname{Im} \left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p) \right\rangle$$

- From $\langle Q_2(\tilde{F}^*(p)\tilde{\psi}_1(-p))\rangle = 0$,

$$\left\langle \tilde{F}(p)\tilde{F}^*(-p) \right\rangle = -p_1 \operatorname{Re} \left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p) \right\rangle + p_0 \operatorname{Im} \left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p) \right\rangle$$

In particular,

$$\left\langle \tilde{F}(0)\tilde{F}^*(0) \right\rangle = -\frac{1}{\sqrt{2}} \left\langle Q_2(\tilde{\psi}_1(0)\tilde{F}^*(0)) \right\rangle = 0$$

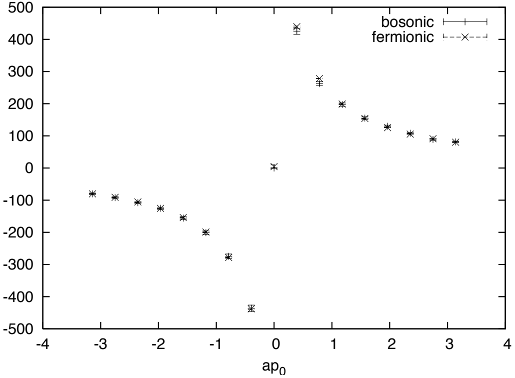


Figure: $p_0 \left\langle \tilde{A}(p)\tilde{A}^*(-p) \right\rangle$ and $-\,\mathrm{Im}\left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_1(-p) \right\rangle$. 16×16, along the line $ap_1 = 0$

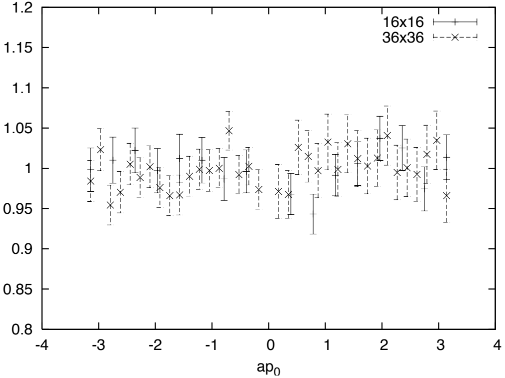


Figure: The ratio $\frac{p_0\langle \tilde{A}(p)\tilde{A}^*(-p)\rangle}{-\operatorname{Im}\left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_1(-p)\right\rangle}$ along the line $ap_1 = 0$

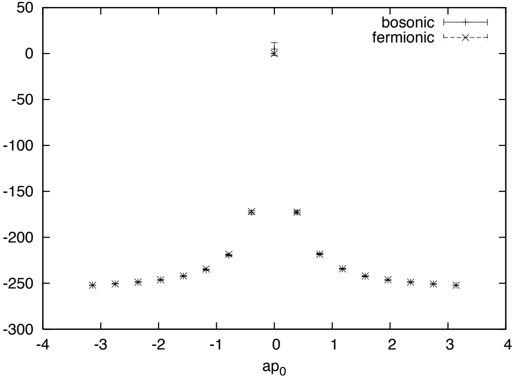


Figure: $\left\langle \tilde{F}(p)\tilde{F}^*(-p) \right\rangle$ and $-p_1 \operatorname{Re} \left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p) \right\rangle + p_0 \operatorname{Im} \left\langle \tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p) \right\rangle$. 16×16, along the line $ap_1 = 0$

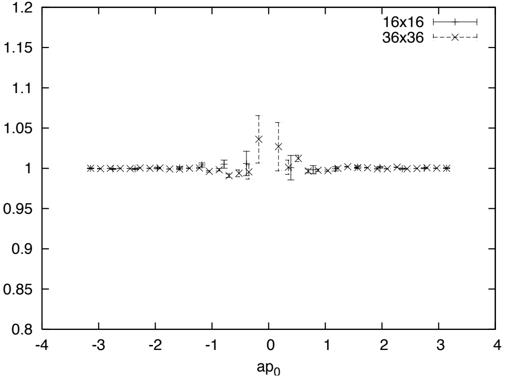


Figure: The ratio $\dfrac{\langle \tilde{F}(p)\tilde{F}^*(-p)\rangle}{-p_1\,\mathrm{Re}\langle\tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p)\rangle + p_0\,\mathrm{Im}\langle\tilde{\psi}_1(p)\tilde{\bar{\psi}}_{\dot{1}}(-p)\rangle}$ along the line $ap_1 = 0$

Scaling dimension

- Susceptibility of the scalar field

$$\chi_\phi \equiv \frac{1}{a^2}\int_{L_0 L_1} d^2x \, \langle A(x)A^*(0)\rangle = \frac{1}{a^2 L_0 L_1}\left\langle \left|\tilde{A}(0)\right|^2\right\rangle$$

- If $A(x)$ behaves as a primary field with the conformal dimensions $(h, \bar{h})$,

$$\chi_\phi \propto \frac{1}{a^2}\int_{L_0 L_1} d^2x \, \frac{1}{x^{2(h+\bar{h})}} \propto (a^{-2} L_0 L_1)^{1-h-\bar{h}}$$

that is

$$\ln(\chi_\phi) = (1 - h - \bar{h}) \ln(a^{-2} L_0 L_1) + \text{const.}$$

- or, a UV subtracted version

$$\chi_\phi \equiv \frac{1}{a^2}\int_{L_0 L_1 - n_0 n_1 a^2} d^2x \, \langle A(x)A^*(0)\rangle$$

$$= \frac{1}{a^2 L_0 L_1}\left\langle \left|\tilde{A}(0)\right|^2\right\rangle - \frac{1}{a^2 (L_0 L_1)^2}\sum_p \frac{2}{p_0}\sin\left(\frac{a p_0 n_0}{2}\right)\frac{2}{p_1}\sin\left(\frac{a p_1 n_1}{2}\right)\left\langle \left|\tilde{A}(p)\right|^2\right\rangle$$

Susceptibility (no UV subtraction)

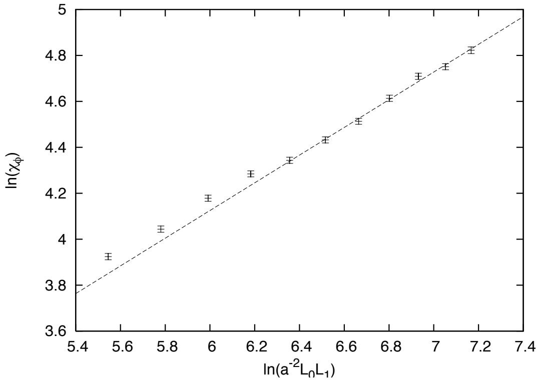


Figure: $\ln(\chi_\phi)$ vs $\ln(a^{-2}L_0L_1)$, no UV subtraction

Susceptibility (with 3×3 UV subtraction)

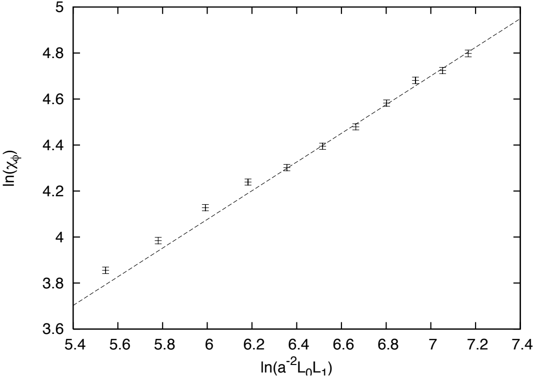


Figure: $\ln(\chi_\phi)$ vs $\ln(a^{-2}L_0L_1)$, with 3×3 UV subtraction

Susceptibility (with 3 × 3 UV subtraction)

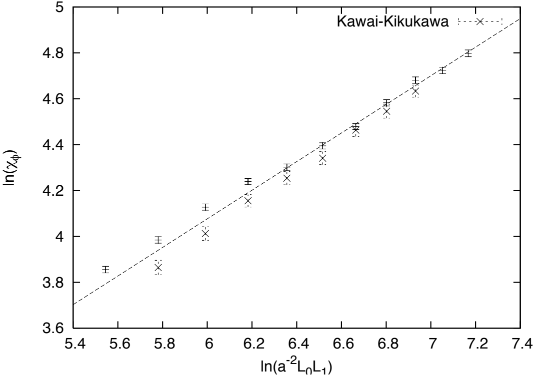


Figure: $\ln(\chi_\phi)$ vs $\ln(a^{-2}L_0L_1)$, with 3×3 UV subtraction

Conformal dimension

- Summary

UV subtraction	fitting region	χ^2/d.f.	$1-h-\bar{h}$
0×0	from 24×24 to 36×36	0.904	0.603(19)
0×0	from 26×24 to 36×36	1.088	0.603(25)
3×3	from 24×24 to 36×36	0.910	0.624(20)
3×3	from 26×24 to 36×36	1.108	0.629(26)

- Our estimate

$$1-h-\bar{h}=0.616(25)(13)$$

is acceptably consistent with the conjecture ($1-h-\bar{h}=0.666\ldots$)

- cf. Kawai-Kikukawa

$$1-h-\bar{h}=0.660(11)$$

Central charge

- Central charge c, defined by the Virasoro anomaly

$$\langle T_{zz}(x) T_{zz}(0) \rangle = \frac{1}{8\pi^2} \frac{c/2}{z^4}$$

  is a fundamental characteristic of CFT
- Naturally, we want to measure the value of c in the IR region by employing the present technique
- A very preliminary study showed that the above function is very noisy
- Thus here we use instead the relation in the $\mathcal{N} = 2$ SCFT

$$\left\langle G_z^+(x) G_z^-(0) \right\rangle = \frac{1}{8\pi^2} \frac{2c/3}{z^3}$$

  where $G_z^{\pm}$ is the superconformal currents in the holomorphic sector

Supercurrents $S_\mu^\pm$ and the superconformal currents $G_z^\pm$

- Since present formulation possesses exact SUSY, we can define a conserved supercurrent
- Localized SUSY transformations

$$\delta\tilde{\varphi}(p) = \frac{1}{L_0 L_1}\sum_q \left[\tilde{\xi}^\alpha(q) Q_\alpha \tilde{\varphi}(p-q) - \tilde{\bar{\xi}}^{\dot{\alpha}}(q)\bar{Q}_{\dot{\alpha}}\tilde{\varphi}(p-q)\right]$$

- The supercurrents $S_\mu^\pm$ and $\bar{S}_\mu^\pm$ are defined by

$$\delta S \equiv \frac{1}{L_0 L_1}\sum_p (-2)\Big[\tilde{\bar{\xi}}^{\dot{2}}(-p)(-ip_\mu)\tilde{S}_\mu^+(p) + \tilde{\xi}^2(-p)(-ip_\mu)\tilde{S}_\mu^-(p)$$
$$+ \tilde{\bar{\xi}}^{\dot{1}}(-p)(-ip_\mu)\tilde{\bar{S}}_\mu^+(p) + \tilde{\xi}^1(-p)(-ip_\mu)\tilde{\bar{S}}_\mu^-(p)\Big]$$

- These currents are correctly normalized in the sense that, for example,

$$p_\mu \left\langle \tilde{S}_\mu^+(p)\tilde{\varphi}_1(q_1)\ldots\tilde{\varphi}_n(q_n)\right\rangle = \frac{i}{2}\sum_{i=1}^n \langle\tilde{\varphi}_1(q_1)\ldots\bar{Q}_{\dot{2}}\tilde{\varphi}_i(q_i+p)\ldots\tilde{\varphi}_n(q_n)\rangle$$

Supercurrents $S_\mu^\pm$ and the superconformal currents $G_z^\pm$

- Explicit forms (the gamma-traceless condition was imposed for $W' = 0$)

$$\tilde{S}_z^+(p) = \frac{1}{L_0 L_1} \sum_q \sqrt{2} i (p-q)_z \tilde{A}(p-q) \tilde{\bar{\psi}}_{\dot{2}}(q)$$

$$\tilde{S}_{\bar{z}}^+(p) = \frac{1}{L_0 L_1} \sum_q \frac{1}{\sqrt{2}} W'(\tilde{A})(p-q) \tilde{\psi}_1(q)$$

$$\tilde{S}_z^-(p) = -\frac{1}{L_0 L_1} \sum_q \sqrt{2} i (p-q)_z \tilde{A}^*(p-q) \tilde{\psi}_2(q)$$

$$\tilde{S}_{\bar{z}}^-(p) = \frac{1}{L_0 L_1} \sum_q \frac{1}{\sqrt{2}} W'(\tilde{A})^*(p-q) \tilde{\bar{\psi}}_{\dot{1}}(q)$$

- We then postulate following correspondence in the IR region:

$$\tilde{S}_z^+(p) \to \tilde{G}_z^+(p) \qquad \tilde{S}_z^-(p) \to \tilde{G}_z^-(p)$$

- Our reasoning:
  - Conservation laws $\partial_{\bar{z}} S_z^\pm(x) + \partial_z S_{\bar{z}}^\pm(x) = 0$ would imply that $S_z^\pm(x)$ are holomorphic in IR
  - $S_z^\pm$ possess correct $U(1)$ charges (± 1) as $G_z^\pm$, by identifying the $U(1)_R$ with a $U(1)$ symmetry in the $\mathcal{N} = 2$ SCA
  - Normalization of the supercurrent $S_\mu^\pm$ has been fixed from a matching in the massless free theory ($W' = 0$) which itself is an $\mathcal{N} = (2,2)$ SCFT with $c = 3$

Central charge from a supercurrent correlator

- We measure

$$\left\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\right\rangle$$

- Since, in $\mathbb{R}^2$ (δ is a point splitting)

$$\left\langle \tilde{G}_z^+(p)\tilde{G}_z^-(-p)\right\rangle = L_0 L_1 \int d^2x\, e^{-ipx} \left\langle G_z^+(x) G_z^-(0)\right\rangle$$

$$= L_0 L_1 \int d^2x\, e^{-ipx} \frac{1}{8\pi^2}\frac{2c/3}{z^3} = L_0 L_1 \frac{-ic}{48\pi}\frac{\partial^3}{\partial p_{\bar{z}}^3}\frac{p^2}{\delta^2}K_2(|p|\delta)$$

$$\xrightarrow{|p|\delta\to 0} L_0 L_1 \frac{ic}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$$

we compare the correlation function with

$$L_0 L_1 \frac{i}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$$

in the low-momentum region. The proportionality constant would provide the central charge c

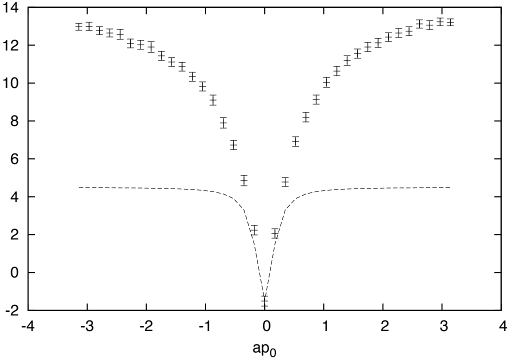


Figure: On the 36 × 36 lattice, the real part along the line $ap_1 = 2\pi/36 \sim 0.17$. The broken line is $L_0 L_1 \frac{ic}{24\pi}\frac{p_z^2}{p_{\bar z}}$ with $c = 1$

The supercurrent correlator $\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\rangle$

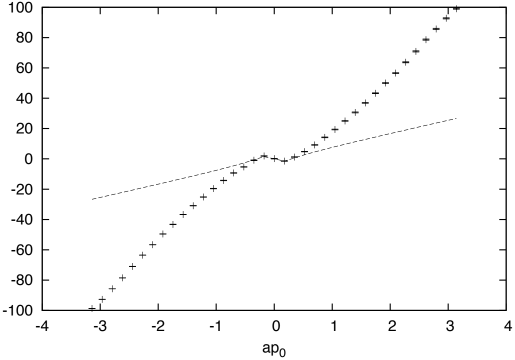


Figure: On the 36×36 lattice, the imaginary part along the line $ap_1 = 2\pi/36 \sim 0.17$. The broken line is $L_0 L_1 \frac{ic}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ with $c = 1$

Fitting for c

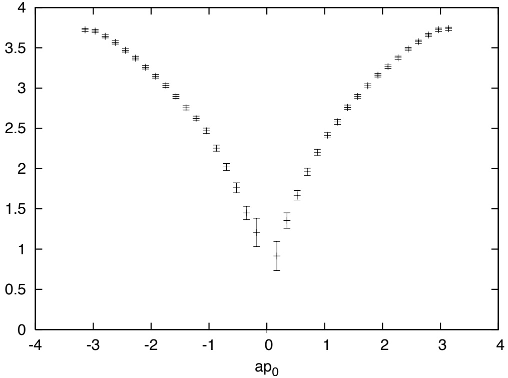
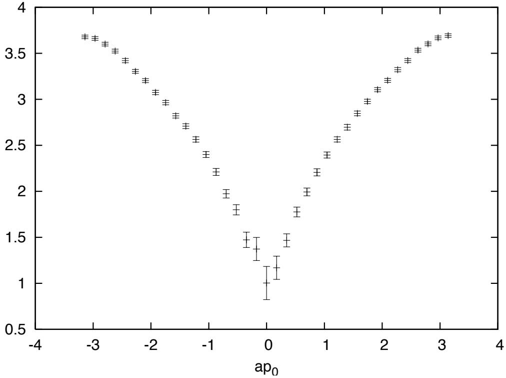


Figure: On the 36×36 lattice, the ratio $\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\rangle / L_0 L_1 \frac{i}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ along the line $ap_1 = 0$

Fitting for c



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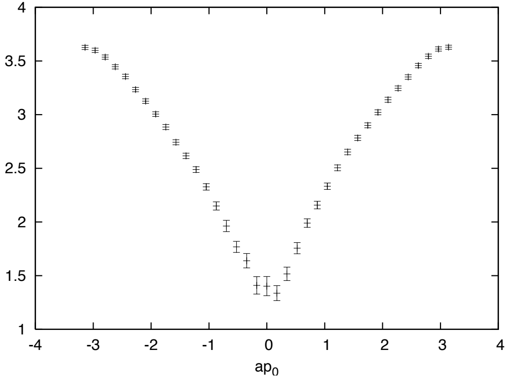


Figure: On the 36 $\times$ 36 lattice, the ratio $\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\rangle / L_0 L_1 \frac{i}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ along the line $ap_1 = 2(2\pi/36) \sim 0.35$

Central charge c

- Summary

fitting region	number of data points	χ^2/d.f.	c
$R(1,1)$	8	1.93	1.09(14)
$R(1,2)$	24	3.50	1.40(8)

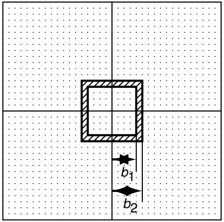


Figure: The fitting region $R(b_1, b_2)$

- Our estimate

$$c = 1.09(14)(31)$$

remarkably reproduces the conjectured value $c = 1$!

Effective c in intermediate energies?

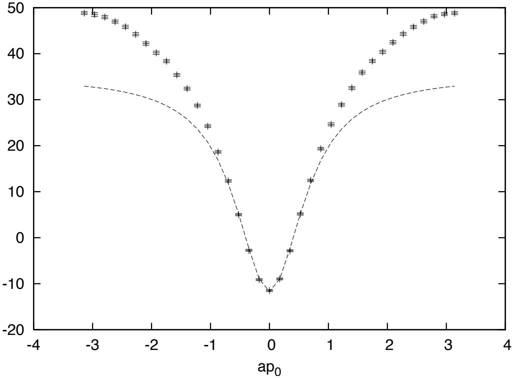
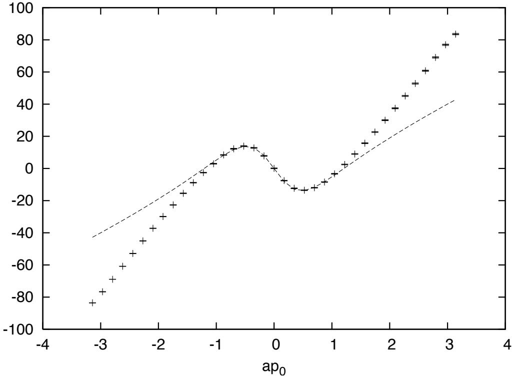


Figure: On the 36×36 lattice, the real part of $\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\rangle$ along the line $ap_1 = 4(2\pi/36) \sim 0.70$. The broken line is $L_0 L_1 \frac{ic}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ with $c = 1.953$

Effective c in intermediate energies?



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Even at the boundary of the Brillouin zone…

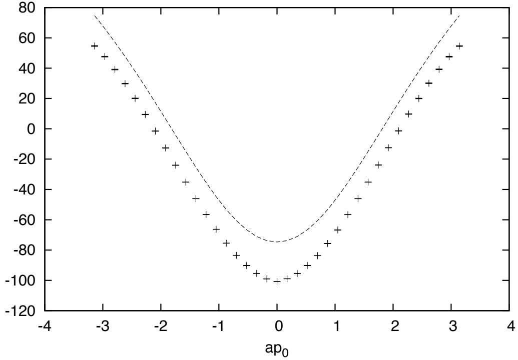

Figure: On the 36 $\times$ 36 lattice, the real part of $\langle \tilde{S}_z^+(p)\tilde{S}_{\bar{z}}^-(-p)\rangle$ along the line $ap_1 = 18(2\pi/36) \sim 3.14$. The broken line is $L_0 L_1 \frac{ic}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ with $c = 2.766$

Effective c for whole energy region?

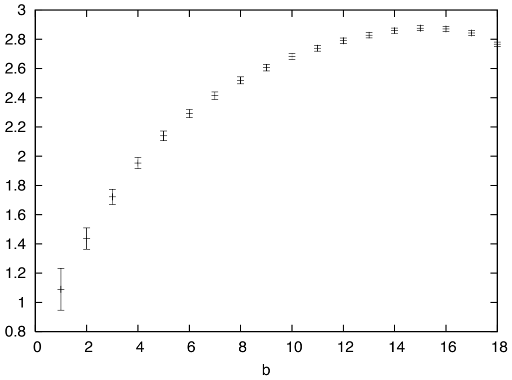

Figure: Result of the constant fit of the ratio $\langle \tilde{S}_z^+(p)\tilde{S}_z^-(-p)\rangle / L_0 L_1 \frac{i}{24\pi}\frac{p_z^2}{p_{\bar{z}}}$ as a function of the fit range $R(b,b)$

- (Supersymmetric analogue of) the Zamolodchikov C-function?

SUSY C-function (?)

- Set
$$W(x) \equiv S_z^+(x) + S_z^-(x), \qquad Q(x) \equiv -S_{\bar{z}}^+(x) - S_{\bar{z}}^-(x)$$
- General forms ($\tau \equiv \ln(\bar{z}z)$)
$$\langle W(x)W(0)\rangle = \frac{F(\tau)}{z^3}$$
$$\langle W(x)Q(0)\rangle = \langle Q(x)W(0)\rangle = \frac{H(\tau)}{z^2\bar{z}}$$
$$\langle Q(x)Q(0)\rangle = \frac{G(\tau)}{z\bar{z}^2}$$
- Conservation laws imply, for $x \neq 0$,
$$\frac{d}{d\tau}(F - H) = -2H, \qquad \frac{d}{d\tau}(H - G) = H - G$$
and
$$\frac{d}{d\tau}(F + H - 2G) = -2G$$
- The reflection positivity will presumably imply $G \geq 0$ and thus
$$C \equiv 6\pi^2(F + H - 2G)$$
is a monotonically decreasing function along the RG flow which becomes c for $H = G = 0$

Summary

- We carried out numerical non-perturbative study of IR physics of the 2D WZ model with $W = \lambda\Phi^3/3$
  - Scaling dimension $1 - h - \bar{h} = 0.616(25)(13)$
  - Central charge $c = 1.09(14)(31)$
- Results are consistent with the emergence of SCFT
- Our non-perturbative formulation seems to be working (i.e., correct universality, although locality is not manifest)

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- Results are consistent with the emergence of SCFT
- Our non-perturbative formulation seems to be working (i.e., correct universality, although locality is not manifest)
- Numerical study can be complementary to analytical investigations even in this SUSY field theory
- More precise measurement of the scaling dimension etc., taking finite volume effects (roughly $1/gL \sim 10\%$) into account (work in progress)
- Observation of the SUSY C-function along a RG flow
- Higher critical models, multi superfields, . . . , the Calabi-Yau compactification on the computer?