Neutrino diffraction: Theory and implications

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Abstract

An interference of a neutrino wave in a decay of high-energy pion is studied and the following four topics will be discussed in this seminar.

topics

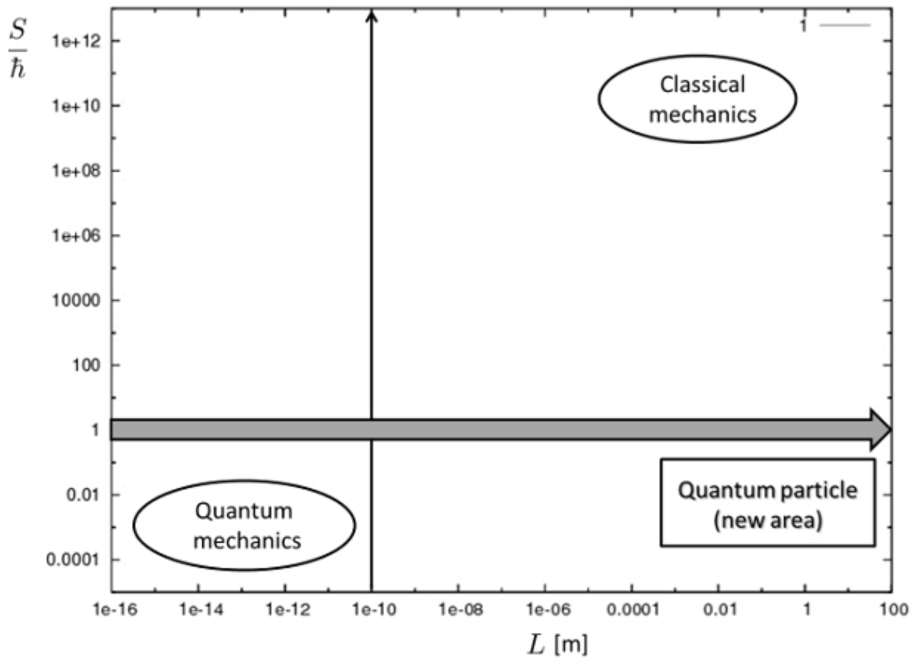
1. Derivation of a diffraction component in a neutrino flux that depends on the absolute neutrino mass.
2. Implicatios of the neutrino diffraction to the LSND anomaly and the two neutrino experiment.
3. Implications of the neutrino diffraction to the absolute neutrino flux and total cross section.(**T2K**)
4. On neutrino's orbit(trajectory)

Refferences:

Ishikawa and Tobita,
1."Neutrino diffraction induced by light-cone singularity and small neutrino mass", arxiv:1106.4968[hep-ph](Derivation of diffraction term),
2."Resolving LSND anomaly by neutrino diffraction",arxiv:1109.3105[hep-ph](LSND and TWN),
3."Diffraction-corrected neutrino flux and total cross section",arxiv: [hep-ph]
4."Quantum particles",in preparation(neutrino orbit and light quanta)
and others

1 Introduction

1-1 quantum vs classical
length L vs action S



1-2 electron mass and Bohr radius

$$m_e = 0.5MeV/c^2 \tag{1}$$
$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = r_e\alpha^{-2} = 0.5\times10^{-10}M(Bohr\ radius)$$

1-3 (1-3-1)Wave functions: a wave length λ vs   a slit size: d

$$(1).\quad e^{Et/i\hbar}\psi_E(\vec{x});\qquad d\approx\lambda;\quad stationary\ state \tag{2}$$

$$(2).\quad \int d\vec{k}e^{(E(\vec{k})t-\vec{k}\vec{x})/i\hbar}f(\vec{k});\quad d\gg\lambda;\quad non-stationary\ staes \tag{3}$$

1-4:**S-matrix in quantum field theory**:

(1)Asymptotic conditions !

LSZ :scattering amplitude is defined with the asymptotic boundary conditions.

Wave packets satisfy the asymptotic boundary conditions and are used to compute a finite size correction of a system of finite size $L = cT$.

(2): A large finite size correction !

Detection probability of the neutrino in a pion decay,

$$\sigma(p_i, L) = \sigma^{(0)}(p_i) + \tilde{g}(\omega_\nu T)\sigma^{(1)}(p_i) \tag{4}$$

$$\omega_\nu = \omega_E - \omega_{dB} = \tfrac{m_\nu^2}{2E_\nu}\ \omega_E = E/\hbar, \omega_{dB} = cp/\hbar$$

$$i\int_0^{\mathrm{T}} dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|}e^{i\omega_\nu \delta t} = \mathrm{T}(\tilde{g}(\mathrm{T}, \omega_\nu) - \frac{\pi}{2}). \tag{5}$$

(2-1) $\sigma^{(0)}$: normal term which is computed with S-matrix of plane waves and satisfy ordinarly conservation laws (enery-momentum, others).

(2-2) $\tilde{g}(\omega_\nu T)\sigma^{(1)}(p_i)$: a finite distance correction which is not computed with S-matrix of plane waves and does not satisfy cconservation laws.

(2-3)$L_0 = 2cE/m_\nu^2 = afew\ \ 100M$

(i) $ct > L_0$ $\sigma(p_i, L) = \sigma^{(0)}(p_i)$: asymptotic region

(ii)$ct < L_0$ $\sigma(p_i, L) = \sigma^{(0)}(p_i) + \tilde{g}(\omega_\nu T)\sigma^{(1)}(p_i)$: non asymptotic region

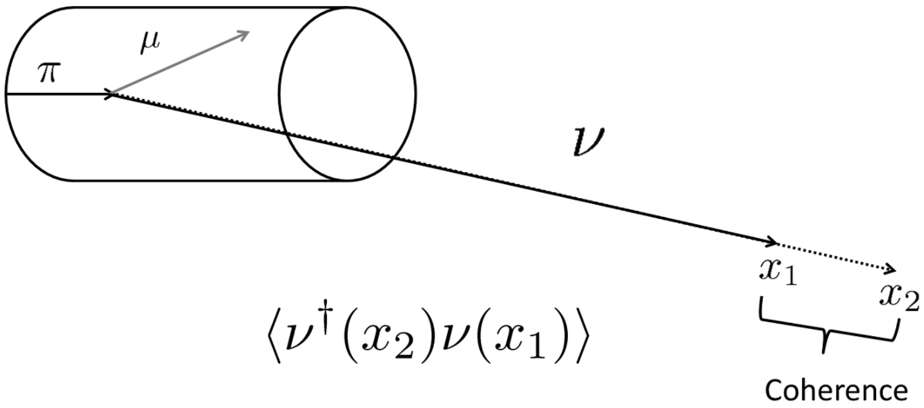
S-matrix in a finite time intereval T, $\qquad S(T) = \Omega_-^\dagger(T)\Omega_+(T)$

$$[S(T), H^0] = (\frac{\partial}{\partial T}\Omega_-(T))^\dagger\Omega_+(T) - \Omega_-^\dagger\frac{\partial}{\partial T}\Omega_+(T) \tag{6}$$

Wave (quantum) nature disappears in $|\vec{x}_1 - \vec{x}_2| > L_0$

$$1.\ L_0 =< 10^{-10}M\ microscopic\ ;\quad normal\quad case; electron, \cdots \tag{7}$$
$$2.\ L_0 = L_0 > 1(200)M\ macroscopic\ ;\quad anomalous\quad case\quad \textbf{neutrino diffraction}$$

Fig.2



2 Neutrino diffraction induced by light cone singularity

1 Neutrino diffraction .

unique features of neutrino

(1). **single neutrino interference** in a distribution of ensemble of events.
(2). Incoherent interaction with nucleus, so the amplitude is that of the **non-stationary state**.
(3). **Lorentz invariance** makes $E = \sqrt{p^2 + m^2}$ and the phase are cancelled at the infinite momentum, which forms a **light-cone singularity**.
(4) . Neutrino has so **small mass**.

The neutrino from a decay of high-energy pion reveals a diffraction pattern of a length scale $2cm_\nu^2/m_\nu^2$.

2 Compute a finite size correction of position dependent probability.

The pion decay amplitude,

$$T = \int d^4x \, \langle \mu, \nu | H_w(x) | \pi \rangle \tag{8}$$

(2-1) The wave packet field theory:

Compute **a finite distance correction** of the neutrino probability with wave packets developed in :Ishikawa-Shimomura,Ishikawa-Tobita-ptp, Ishikawa-Tobita; a complete set $(\vec{X}, \vec{P})$

(different from a single wave packet propagation by : Kayser,Giunti,Nussinov,Kiers,Stodolsky,Lipkin,Asahara,others.)

The state vectors

$$|\pi\rangle = |\vec{p}_\pi, \mathrm{T}_\pi\rangle, \; |\mu, \nu\rangle = |\mu, \vec{p}_\mu; \nu, \vec{p}_\nu, \vec{\mathrm{X}}_\nu, \mathrm{T}_\nu\rangle. \tag{9}$$

The amplitude T

$$T = \int d^4x d\vec{k}_\nu \, N \langle 0|\varphi_\pi(x)|\pi\rangle \bar{u}(\vec{p}_\mu)(1-\gamma_5)\nu(\vec{k}_\nu) \times e^{ip_\mu \cdot x + ik_\nu \cdot (x - \mathrm{X}_\nu) - \frac{\sigma_\nu}{2}(\vec{k}_\nu - \vec{p}_\nu)^2}, \tag{10}$$

where $N = igm_\mu \, (\sigma_\nu/\pi)^{\frac{4}{3}} \, (m_\mu m_\nu / E_\mu E_\nu)^{\frac{1}{2}}$, $\mathrm{T}_\pi \leq t$.

This depends upon $(\mathrm{T}_\nu, \vec{\mathrm{X}}_\nu)$ and T_π, wave packet size, σ_ν, of the neutrino. σ_ν is from a nucleus size.

The Gaussian wave packet in this talk and generalization is OK.

The neutrino momentum $\vec{k}_\nu$ is integrated, the spin summations are made, this amplitude is expressed with the correlation function and the neutrino wave function in the form

$$\begin{aligned}&\int \frac{d\vec{p}_\mu}{(2\pi)^3}\sum_{s_1,s_2}|T|^2\\ &= \frac{C}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i-\vec{x}_i^{\,0})^2}\Delta_{\pi,\mu}(\delta x)e^{i\phi(\delta x)},\end{aligned} \tag{11}$$

where

$$\begin{aligned}&C = g^2 m_\mu^2\,(4\pi/\sigma_\nu)^{\frac{3}{2}}\,V^{-1}, \vec{x}_i^{\,0} = \vec{\mathrm{X}}_\nu + \vec{v}_\nu(t_i - \mathrm{T}_\nu),\\ &\delta x = x_1 - x_2, \phi(\delta x) = p_\nu\cdot\delta x\end{aligned} \tag{12}$$

and the correlation function,

$$\Delta_{\pi,\mu}(\delta x) = \frac{1}{(2\pi)^3}\int \frac{d\vec{p}_\mu}{E(\vec{p}_\mu)}(p_\mu\cdot p_\nu)e^{-i(p_\pi-p_\mu)\cdot\delta x}. \tag{13}$$

The muon momentum is integrated in whole positive energy region.

3 Light-cone singularity.

Waves of a same phase in $\Delta_{\pi,\mu}(\delta x)$ form a singularity, which is extracted with an integral in a four dimensional form with a new variable $q = p_\mu - p_\pi$ that is conjugate to δx.

$$\Delta_{\pi,\mu}(\delta x) = \frac{1}{4\pi^4}\int d^4q Im[\frac{1}{q^2+2p_\pi\cdot q+\tilde{m}^2-i\epsilon}((q+p_\pi)\cdot p_\nu)e^{-iq\cdot\delta x}. \tag{14}$$

The integral in $0\le q^0$,I_1 ; in $-p_\pi^0\le q^0\le 0$,I_2. The I_1 is written in the form, $(p_\pi\cdot p_\nu - ip_\nu\cdot(\frac{\partial}{\partial\delta x}))\tilde{I}_1$, where

$$\tilde{I}_1 = \int d^4q\,\frac{\theta(q^0)}{4\pi^4}\mathrm{Im}\left[\frac{1}{q^2+2p_\pi\cdot q+\tilde{m}^2-i\epsilon}\right]e^{iq\cdot\delta x}, \tag{15}$$

and $\tilde{m}^2 = m_\pi^2 - m_\mu^2$. The integrand of $\tilde{I}_1$ is expanded in $p_\pi\cdot q$ and the integration leads the light-cone singularity, $\delta(\delta x^2)$, and less singular and regular terms which are described by Bessel functions. The I_2 is written with the momentum $\tilde{q} = q + p_\pi$ and has no singularity.

Thus the correlation function, $\Delta_{\pi,\mu}(\delta x)$, is written in the form

$$\Delta_{\pi,\mu}(\delta x) = 2i \left\{ p_\pi \cdot p_\nu - i p_\nu \cdot \left(\frac{\partial}{\partial \delta x} \right) \right\} \times \left[D_{\tilde{m}} \left(-i \frac{\partial}{\partial \delta x} \right) \left(\frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f_{short} \right) + I_2 \right], \tag{16}$$

where

$$\lambda = (\delta x)^2, D_{\tilde{m}}(-i\frac{\partial}{\partial \delta x}) = \sum_l (1/l!) \big(2p_\pi \cdot (-i\frac{\partial}{\partial \delta x}) \frac{\partial}{\partial \tilde{m}^2} \big)^l, \tag{17}$$

$$f_{short} = -\frac{i\tilde{m}^2}{8\pi\xi} \theta(-\lambda) \{N_1(\xi) - i\epsilon(\delta t) J_1(\xi)\} - \frac{i\tilde{m}^2}{4\pi^2\xi} \theta(\lambda) K_1(\xi), \xi = \tilde{m}\sqrt{\lambda}$$

N_1, J_1, and K_1 are Bessel functions. f_{short} has singularity of the form $1/\lambda$ around $\lambda = 0$ and decrease as $e^{-\tilde{m}\sqrt{|\lambda|}}$ or oscillates as $e^{i\tilde{m}\sqrt{|\lambda|}}$ at large $|\lambda|$. The convergence of the series will be studied later.

Integration of spatial coordinates.

Next, write the probbility.

$$\int \frac{d\vec{p}_\mu}{(2\pi)^3} \sum_{s_1,s_2} |T|^2 = \frac{C}{E_\nu} \int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{x}_i^{\,0})^2} \Delta_{\pi,\mu}(\delta x) e^{i\phi(\delta x)}, \tag{18}$$

1. From the singular term,

$$\Delta_{\pi,\mu} = \frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) \tag{19}$$

$$J_{\delta(\lambda)} = C_{\delta(\lambda)} \epsilon(\delta t) |\delta t|^{-1} e^{i\bar{\phi}_c(\delta t) - \frac{m_\nu^4}{16\sigma_\nu E_\nu^4} \delta t^2}, C_{\delta(\lambda)} = (\sigma_\nu \pi)^{\frac{3}{2}} \sigma_\nu / 2 \tag{20}$$

$$\phi(\delta x) = \bar{\phi}_c(\delta t) = \delta t \ m_\nu^2 / 2E_\nu \tag{21}$$

the interference pattern is determined by $m_\nu^2/2E_\nu$, but not by de Broglie phase.

Singular terms: long-range correlation

The most singular term

$$
\begin{aligned}
J_{\delta(\lambda)} &= \int d\vec{x}_1 d\vec{x}_2 e^{i\phi(\delta x)} e^{-\frac{1}{2\sigma_\nu}\left(\vec{x}_1-\vec{X}_\nu-\vec{v}_\nu(t_1-T_\nu)\right)^2} e^{-\frac{1}{2\sigma_\nu}\left(\vec{x}_2-\vec{X}_\nu-\vec{v}_\nu(t_2-T_\nu)\right)^2} \\
&\times \frac{1}{4\pi}\delta(\lambda)\epsilon(\delta t)
\end{aligned}
\tag{22}
$$

use $X^\mu = \frac{x_1^\mu + x_2^\mu}{2}$ and $\vec{r} = \vec{x}_1 - \vec{x}_2$,

$$
\begin{aligned}
&= \int d\vec{X} d\vec{r}\, e^{i\phi(\delta x)} e^{-\frac{1}{\sigma_\nu}\left(\vec{X}-\vec{X}_\nu-\vec{v}_\nu(X^0-T_\nu)\right)^2} e^{-\frac{1}{4\sigma_\nu}(\vec{r}-\vec{v}_\nu\delta t)^2} \\
&\times \frac{1}{4\pi}\delta(\lambda)\epsilon(\delta t).
\end{aligned}
\tag{23}
$$

$\vec{X}$ is integrated easily. $J_{\delta(\lambda)}$ becomes the integral of $(\vec{r}_T, r_l)$,

$$
= \epsilon(\delta t)(\sigma_\nu \pi)^{\frac{3}{2}} \int d\vec{r}_T dr_l\, e^{i\phi(\delta t,\vec{r}) - \frac{1}{4\sigma_\nu}(\vec{r}_T^{\,2} + (r_l - v_\nu \delta t)^2)} \frac{1}{4\pi}\delta(\delta t^2 - \vec{r}_T^{\,2} - \vec{r}_l^{\,2}).
\tag{24}
$$

$$
\begin{aligned}
&= (\sigma_\nu\pi)^{\frac{3}{2}} \frac{\sigma_\nu}{2}\frac{1}{|\delta t|}\epsilon(\delta t) e^{i\bar{\phi}_c(\delta t) - \frac{m_\nu^4}{16\sigma_\nu E_\nu^4}\delta t^2} \\
&\approx (\sigma_\nu\pi)^{\frac{3}{2}} \frac{\sigma_\nu}{2}\frac{1}{|\delta t|}\epsilon(\delta t) e^{i\bar{\phi}_c(\delta t)}
\end{aligned}
\tag{25}
$$

Theorem:General cases of non-Gaussian form with spreading effect The singular part $J_{\delta(\lambda)}$ has the slow phase and the magnitude that is inversely proportional to the time difference, of the form Eq. (25). The phase is given by the sum of $\bar{\phi}_c(\delta t)$ and a small correction, which is proportional to $1/E_\nu$ in general systems and becomes $1/E_\nu^2$ if the neutrino wave packet is invariant under the time inversion and is real.

(Proof.)

The wave function $\psi(\vec{x} - \vec{v}t)$ in the general wave packet $\psi_k(k_l, \vec{k}_T)$ with the spreading effect is expressed in the following form

$$
\begin{aligned}
\psi(\vec{x} - \vec{v}t) = \int dk_l d\vec{k}_T\, & e^{ik_l(x_l - v_\nu t) + i\vec{k}_T\cdot\vec{x}_T + iC_{ij}k_T^i k_T^j t} \\
& \times \psi_k(k_l,\ \vec{k}_T),\ C_{ij} = \delta_{ij}/2E,
\end{aligned}
\tag{26}
$$

$$
J_{\delta(\lambda)} = \pi e^{-\frac{1}{2}} w_0 (1+\gamma)\frac{\epsilon(\delta t)}{2\delta t} e^{i\bar{\phi}_c(\delta t)(1+\delta)}
\tag{27}
$$

, where

$$w_0 = \int dk_l |\psi_k(k_l,0)|^2 \tag{28}$$

$$\delta = \frac{d_1}{E} + \frac{d_2}{2E^2},\ \gamma = \frac{d_1}{2E} + \frac{d_2}{2!}\left(\frac{1}{2E}\right)^2 - (1-v_\nu)^2\delta t^2,$$
$$d_1 = \frac{1}{w_0}\int dk_l k_l |\psi_k(k_l,0)|^2,\ d_2 = \frac{1}{w_0}\int dk_l k_l^2 |\psi_k(k_l,0)|^2.$$

The regular terms are short-range.

2. First term, $\tilde{L}_1$, is from f_{short} in Eq.(16) and is expressed with Bessel functions. At a large $|\delta t|$ and $\lambda < 0$, introduce

$$r_l = v_\nu \delta t + \tilde{r}_l \tag{29}$$
$$\lambda \approx -2v_\nu \tilde{r}_l \delta t - \tilde{r}_l^2 - \vec{r}_T^2. \tag{30}$$

use the asymptotic expression of the Bessel function

$$\tilde{L}_1 = C_1 |\delta t|^{-\frac{3}{4}} e^{i(E_\nu - p_\nu v_\nu)\delta t - \sigma_\nu p_\nu^2 + i\tilde{m}\sqrt{|2v_\nu \sigma_\nu p_\nu \delta t|}}, \tag{31}$$

where

$$C_1 = i\frac{\sigma_\nu}{4}\left(\frac{\sigma_\nu \tilde{m}}{2}\right)^{\frac{1}{2}} (4v_\nu \sigma_\nu p_\nu)^{-\frac{3}{4}} \tag{32}$$

, and oscillates as fast as

$$e^{i\tilde{m}b_1\sqrt{|\delta t|}} \tag{33}$$

$e^{i\tilde{m}b_1\sqrt{|\delta t|}}$ where b_1 is determined by p_ν and σ_ν and is of macroscopic magnitude.

Second term, $\tilde{L}_2$, is from I_2 and is approximately the integral of

$$e^{-i\left(E_\pi - E_\nu - \sqrt{q^2+m_\mu^2}\right)\delta t} \tag{34}$$

over the momentum $\vec{q}$ in the range $1/\sqrt{\sigma_\nu}$. This becomes a short-range correlation of the length, $2\sqrt{\sigma_\nu}$, in the time direction. So the L_2's contribution to the total probability comes from the small δt region, and agrees to that of the standard calculation.

4 Time-dependent probability.

$$\int \frac{d\vec{p}_\mu}{(2\pi)^3} \sum_{s_1,s_2} |T|^2 = N_1 \int dt_1 dt_2 \Bigg[e^{i\frac{m_\nu^2}{2E_\nu}\delta t} \times \frac{\epsilon(\delta t)}{|\delta t|} + 2D_{\tilde{m}}(p_\nu)\frac{\tilde{L}_1}{\sigma_\nu} - \frac{2i}{\pi}\left(\frac{\sigma_\nu}{\pi}\right)^{\frac{1}{2}} \tilde{L}_2 \Bigg], \tag{35}$$

where $N_1 = ig^2 m_\mu^2 \pi^3 \sigma_\nu (8p_\pi \cdot p_\nu / E_\nu) V^{-1}$.

The first term of Eq. (35) is oscillating extremely slowly and the rests oscillate or decrease rapidly. They are separated in a clear manner.

$$\sum_n (-2p_\pi \cdot p_\nu)^n \frac{1}{n!} \left(\frac{\partial}{\partial \tilde{m}^2}\right)^n \tilde{L}_1, \tag{36}$$

converges in

$$2p_\pi \cdot p_\nu \leq \tilde{m}^2 \tag{37}$$

.

The power series then oscillates with $\sqrt{|\delta t|}$ rapidly as

$$S_2 = e^{i\tilde{m}\sqrt{|2v_\nu \sigma_\nu p_\nu \delta t|}\left(1 - \frac{p_\pi p_\nu}{\tilde{m}^2}\right)} \tag{38}$$

. So the separation of the light-cone singular term from $\tilde{I}_1$ is valid in the region $2p_\pi \cdot p_\nu \leq \tilde{m}^2$, and the light-cone singularity exists only in this kinematical region.

In the outside of this region, the power series diverges, and $\Delta_{\pi,\mu}(\delta x)$ has no light-cone singularity.

time integration

1.Integrating t_1 and t_2 in the finite $\mathrm{T} = \mathrm{T}_\nu - \mathrm{T}_\pi$, we have the slowly decreasing term

$$i \int_0^{\mathrm{T}} dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i\omega_\nu \delta t} = \mathrm{T} g(\mathrm{T}, \omega_\nu),\ \omega_\nu = \frac{m_\nu^2}{2E_\nu}. \tag{39}$$

The slope of $g(\mathrm{T}, \omega_\nu)$ at $\mathrm{T} = 0$ is $\frac{\partial}{\partial \mathrm{T}} g(\mathrm{T}, \omega_\nu)|_{\mathrm{T}=0} = -\omega_\nu$. At $\mathrm{T} = \infty$, $g(\mathrm{T}, \omega_\nu)$ becomes $g(\infty, \omega_\nu) = -\pi$ that is cancelled with the short-range term $\tilde{L}_1$ of Eq. (35). So it is convenient to subtract the asymptotic value from $g(\mathrm{T}, \omega_\nu)$ and to define $\tilde{g}(\mathrm{T}, \omega_\nu) = g(\mathrm{T}, \omega_\nu) - g(\infty, \omega_\nu)$. $\tilde{g}(\mathrm{T}, \omega_\nu)$ is the diffraction term.

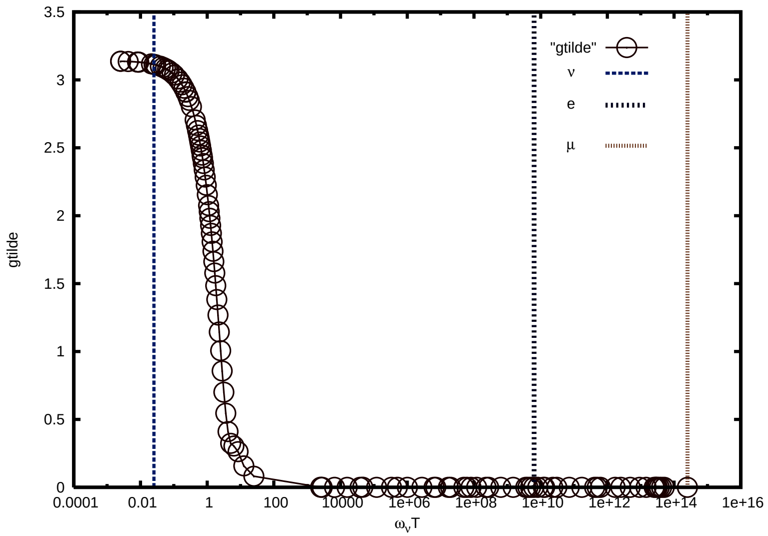


Figure 1: Values of $\tilde{g}(T,\omega)$ for ν, e and μ at $L = 10$M and $E = 1$ GeV,and masses $m_\nu c^2 = 1$ eV, $m_e c^2 = 0.5$ MeV,and $m_\mu c^2 = 100$MeV.

2. The short-range term

$$\frac{2}{\pi}\sqrt{\frac{\sigma_\nu}{\pi}}\int dt_1 dt_2 \tilde{L}_2(\delta t) = \mathrm{T}G_0 \tag{40}$$

$\delta t \approx 0$, and G_0 is constant in T. This term is independent from σ_ν and agrees to the standard method of using plane waves. the integration of neutrino's coordinates $\vec{\mathrm{X}}_\nu$ and this is cancelled with V^{-1}, the normalization of the initial pion state.

3. The final form of **The total probability**(the neutrinoflux) is

$$P = N_2 \int \frac{d^3 p_\nu}{(2\pi)^3}\frac{p_\pi \cdot p_\nu}{E_\nu}\left[\tilde{g}(\mathrm{T},\omega_\nu) + G_0\right], \tag{41}$$

At $\mathrm{T} \to \infty$, $\tilde{g} \to 0$, and P agrees to G_0, the normal term of standard calculation using plane waves. At finite T, the probability has the diffraction component, which is stable under the varivation of the pion's momentum.

$\tilde{g}(\mathrm{T},\omega_\nu)$, at finite L in the kinematical region,

$$|\vec{p}_\nu|(E_\pi - |\vec{p}_\pi|) \leq p_\pi \cdot p_\nu \leq \tilde{m}^2/2 \tag{42}$$

In G_0, $\quad p_\pi \cdot p_\nu = \tilde{m}^2/2$, and the cosine of the angle satisfies

$$1 - \cos\theta = \frac{\tilde{m}^2}{2|\vec{p}_\pi||\vec{p}_\nu|} - \frac{m_\pi^2}{2|\vec{p}_\pi|^2} \tag{43}$$

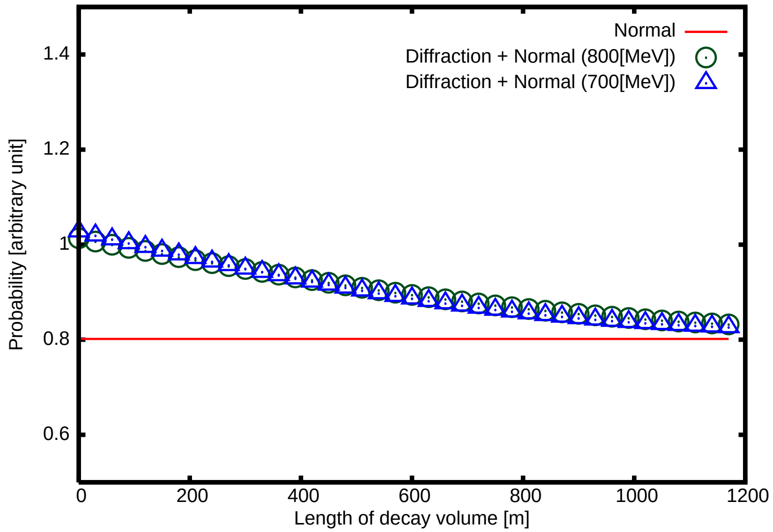


Figure 2: The total probability per time at a finite distance L is given. The constant shows the normal term and the diffraction term is written on top of the normal term. The horizontal axis shows the distance in [m] and the probability is of arbitrary unit. The excess is seen in the distance below 1200m. The neutrino mass, pion energy, neutrino energy are 1.0 [eV/c^2], 4 [GeV], and 700(△) and 800(◯) [MeV].

. The transition rate agrees with the value of the ordinary method.

the wave packet size of the neutrino,

$$\sigma_\nu = A^{\frac{2}{3}}/m_\pi^2 \tag{44}$$

$\sigma_\nu = 6.4/m_\pi^2$ fo ^{16}O nucleus.

An excess of the flux at L $<$ 600 [m],$0.2-0$ of the normal term, has the slope at L $=0$, ω_ν, and is slowly varying with both the distance and energy.

L_0 is L_0 [m] $= 2E_\nu\hbar c/m_\nu^2 = 400\times E_\nu[\mathrm{GeV}]/m_\nu^2[\mathrm{eV}^2/c^4]$.

Within Neutrino's energy uncertainty $\Delta E_\nu \approx 0.1\times E_\nu$, which is about 100 [MeV] for 1 [GeV] and the diffraction components are the same from Fig. 2.

5 Summary and implications.

The neutrino diffraction ;

(1) manifests the neutrino features,

(2) caused by light-cone singularity of $\Delta_{\pi,\mu}(\delta x)$ and the slow phase $\bar{\phi}_c(\delta t)$ "**diffraction**.

(3) decreases with the distance extremely slowly and is added to the normal component. " $\omega = \omega_{\mathbf{E}} - \omega''_{\mathbf{dB}}$

(4)The length scale of the pattern is not determined with de Broglie wave length but with $\mathbf{2cE_\nu/m_\nu^2}$.

(5)Diffraction component has a sizable magnitude and is stable under the change of the parameters of initial state.

Implications to existing neutrino anomalies.

1. Excesses of the neutrino flux at near detectors of ground experiments of $10-20$ **percent**.
2. LSND anomaly
3. Total neutrino cross section: E-dependence
4. determination of the absolute neutrino mass.

Inside of length L_0 quantume mechanical natures remain and particle pictures do not hold

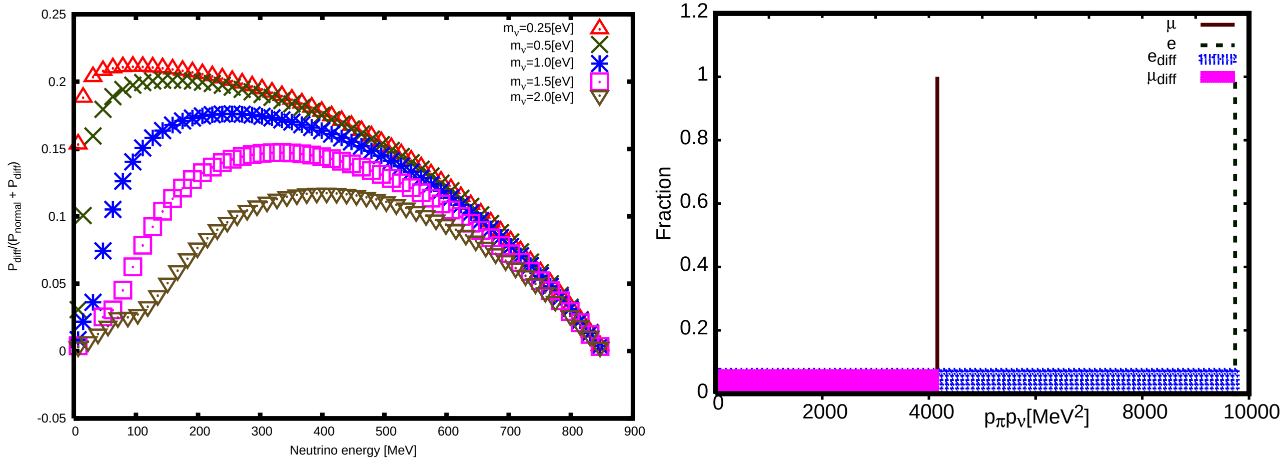


Figure 3: The neutrino energy and $p_\pi p_\nu$ dependences of the fraction of P_{diff} are given in 1-a and 1-b. The horizontal axis shows E_ν in [MeV]. $m_\nu = 0.25 - 2.0$ [eV/c^2], $E_\pi = 2000$ [MeV], and L = 50 [m]. $p_\pi p_\nu$ dependence of the diffraction term is broad and that of the normal term is narrow.

3 Resolving LSND anomaly by neutrino diffraction

A diffraction component leads large electron neutrino fluxes at short base-line experiments, and resolves anomalies of LSND and two neutrino experiment (TWN).

The elctron neutrino flux :

(1) short distance ≈ the neutrino diffraction $= \bar{m}^2$. LSND and TWN

(2)long distance ≈ the normal component or flavor oscillation $= \delta m^2$. Solar,reactor,longbase line.

The (V-A)(V-A) interaction:the normal term G_0 and the diffraction term $\tilde{g}(\mathrm{T}, \omega_\nu)$,

$$P = N_3 \int \frac{d^3 p_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu (m_\pi^2 - 2 p_\pi \cdot p_\nu)}{E_\nu} \left[\tilde{g}(\mathrm{T}, \omega_\nu) + G_0\right], \tag{45}$$

The neutrino probability in pion decay in a macroscopic distance, (where charged leptons are unobserved), is

$$P = P_{normal} + P_{diff}. \tag{46}$$

where P_{normal} is the normal term and

$$P_{diff} = CT \sum_i \tilde{g}(T, \omega_i) |U_{i,e}|^2 =, CT \tilde{g}(T, \bar{\omega}) |U_{i,e}|^2 \tag{47}$$

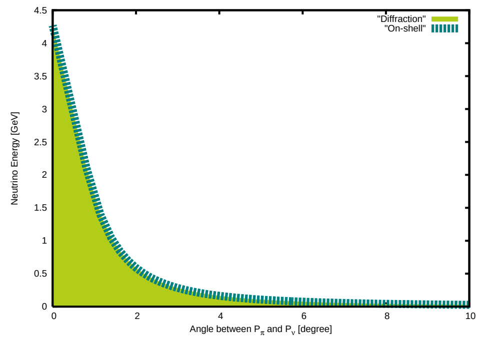


Figure 4: The angle between p_π and p_ν dependences of the neutrino energy is given. The horizontal axis shows the angle and the vertical axis shows the neutrino energy at $E_\pi = 10$ [GeV]. The normal term has a value along the boundary and the diffraction term has a value in broad area below the normal term.

is the new term. Due to this new term neutrino flux is written in the form

$$f = f_{normal} + f_{diff}. \tag{48}$$

Experiments
(1)TWN and LSND :from f_{diff}. $\bar{m}^2$
(2)Long baseline,solar,atmosphere,reactor :from f_{normal}. δm^2 Comparisons diffraction component, which is stable under the variation of the pion's

Experiment	P_{ν_e}/P_{ν_μ} (Exp)	P_{diff}/P_{normal} (Th)
TWN	0.18	0.17
LSND	$0.0026^{\pm}$	0.0028
CDHSW(N)	?	0.2-0.5
CDHSW(F)	?	$0.02 - 0.05$
MiniBooNE, KARMEN	0	$\approx 10^{-5}$

Table 1: Experimental values and theoretical values

momentum.

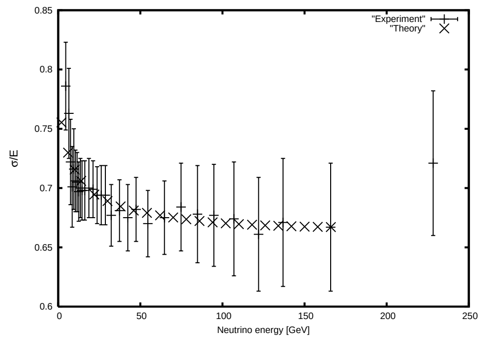


Figure 5: NOMAD

4 Diffraction-corrected Neutrino flux and total cross section

Neutrino diffraction modifies the neutrino fluxes. Using corrected flux, total cross section is obtained. Slowly decreasing νN total cross sections, which are hard to understand in standard model, become energy-independent with the diffraction term of absolute neutrino mass less than $2eV/c^2$.

The total cross sections for the neutrino and anti-neutrino are written as

$$\sigma^{\nu} = \frac{M_N E_\nu G_F^2}{\pi}(Q + 1/3\bar{Q}) \tag{49}$$
$$\sigma^{\bar{\nu}} = \frac{M_N E_\nu G_F^2}{\pi}(\bar{Q} + 1/3Q)$$

using integrals of quark-parton distribution functions $q(x)$ and $\bar{q}(x)$, $Q = \int_0^1 dxxq(x), \bar{Q} = \int_0^1 dxx\bar{q}(x)$. Both are proportional to the neutrino energy and the current values are $\sigma_\nu/E = 0.67 \times 10^{-38}[cm^2/GeV], \sigma_{\bar{\nu}}/E = 0.34 \times 10^{-38}[cm^2/GeV]$. So experiments seem consistent with However, the recent experiments show that the ratio σ_ν/E_ν is decreasing with E_ν very slowly.

Conversly the false cross section is written as

$$\sigma(E)^{false}/E = (1 + r_{diff})(\sigma(E)^{true}/E). \tag{50}$$

$(\sigma(E)^{true}/E)$ is believed constant so the E-dependence of $\sigma(E)^{false}/E$ is due to E-dependence of r_{diff}.

Experiments

Comparisons

NOMAD

MINOS

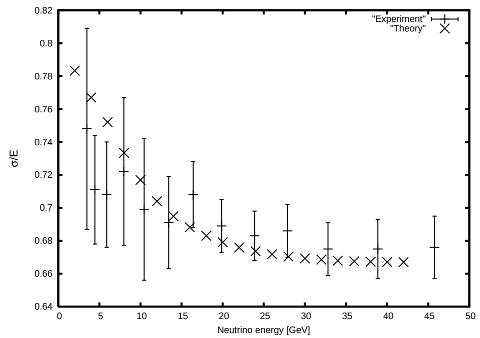


Figure 6: MINOS

T2K:electron neutrino spectrum is sensitive to neutrino mass at around $0.2eV/c^2$

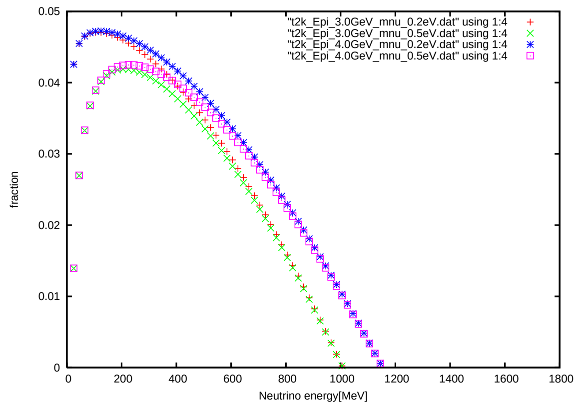


Figure 7: T2K off-axis: E dependence of the electron neutrino;m=0.2,0.5

5 Macroscopic trajectory and energy-momentum conservation of quantum particles

K I and Shimomura,PTP,114(2005),1201,K I and Tobita,PTP,122(2009),1111

generalized vertices of arbitrary wave packets

The product of the wave packets at $(t, \vec{x})$,

$$\begin{aligned}
&\Pi_j N_j^* e^{-\frac{1}{2\sigma_j}(\vec{x}-\vec{X}_j-\vec{v}_j(t-T_j))^2+iE(\vec{p}_j)(t-T_j)-i\vec{p}_j(\vec{x}-\vec{X}_j)} \\
&\times \Pi_l N_l e^{-\frac{1}{2\sigma_l}(\vec{x}-\vec{X}_l-\vec{v}_l(t-T_l))^2-iE(\vec{p}_l)(t-T_l)+i\vec{p}_l(\vec{x}-\vec{X}_l)} \\
&= \Pi_j N_j^* \Pi_l N_l e^{-\frac{1}{2\sigma_s}(\vec{x}-\vec{x}_0(t))^2-\frac{1}{2\sigma_t}(t-t_0)^2} e^{R+i\phi}.
\end{aligned} \tag{51}$$

Wave packet sizes in the spatial and temporal directions are

$$\frac{1}{\sigma_s} = \Sigma_j \frac{1}{\sigma_j}, \frac{1}{\sigma_t} = \Sigma_j \frac{1}{\sigma_j} \vec{v}_j^2 - \frac{1}{\sigma_s} \vec{v}_0^2 \tag{52}$$

and the central values of the space-time coordinates are

$$\vec{x}_0(t) = \vec{v}_0 t + \vec{x}_0(0), \vec{v}_0 = \sigma_s \Sigma_j \frac{1}{\sigma_j} \vec{v}_j, \tag{53}$$

$$\vec{x}_0(0) = \sigma_s(\Sigma_j \frac{1}{\sigma_j} \tilde{\vec{X}}_j - i(\Sigma_j(\pm)\vec{p}_j)) \tag{54}$$

$$t_0 = \sigma_t(\frac{1}{\sigma_s}\vec{v}_0 \cdot \vec{x}_0 - \Sigma_j \frac{1}{\sigma_j} \vec{v}_j \cdot \tilde{\vec{X}}_j + i\Sigma_j(\pm)E(\vec{p}_j)) \tag{55}$$

$$\tilde{\vec{X}}_j = \vec{X}_j - \vec{v}_j T_j. \tag{56}$$

The real part → magnitude → the trajectory and energy-momentum .

$$R = R_{trajectory} + R_{momentum}, \tag{57}$$

$$R_{trajectory} = -\Sigma_j \frac{1}{2\sigma_j} \tilde{\vec{X}}_j^{\,2} + 2\sigma_s(\Sigma_j \frac{1}{2\sigma_j} \tilde{\vec{X}}_j)^2 + 2\sigma_t(\Sigma_j(\vec{v}_0 - \vec{v}_j)\tilde{\vec{X}}_j)^2, \tag{58}$$

$$R_{momentum} = -\frac{\sigma_t}{2}(\Sigma_j(\pm)(E(\vec{p}_j) - \vec{v}_0\vec{p}_j))^2 - \frac{\sigma_s}{2}(\Sigma_j(\pm)\vec{p}_j)^2. \tag{59}$$

The phase factor:

$$\phi = \phi_0 + \phi_1, \tag{60}$$

$$\phi_0 = \Sigma_j(\pm)(\vec{p}_j\vec{X}_j - E(\vec{P}_j)T_j), \tag{61}$$

$$\phi_1 = -2\sigma_t(\Sigma_j\frac{1}{2\sigma_j}(\vec{v}_0 - \vec{v}_j)\tilde{\tilde{\vec{X}}}_j)(\Sigma(\pm)\vec{v}_0(\vec{P}_j - E(\vec{p}_j)))$$

$$-2\sigma_s(\Sigma_j(\pm)\vec{p}_j)(\Sigma_j\frac{1}{2\sigma_j}\tilde{\tilde{\vec{X}}}_j). \tag{62}$$

6 Summary

1.Neutrino's (quantum mechanical) wave is extended in huge area of few 100 meters and gives the large finite size correction to the detection and interaction probability.

2.The diffraction component has peculiar properties that are different from naive particle pictures because they are finite size corrections. The energy and momentum are not conserved in the diffraction terms and the helicity suppression of the lepton modes get modified. They can explain various anomalies of the previous experiments.

3.Neutrino diffractions give a new signal which depends on the absolute neutrino mass and their confirmations would be made at T2K.