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1+1+1 Flavor QCD+QED Simulation at the Physical Point

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Before My Talk

What is Advanced Institute for Computational Science (AICS)?

- Next-Generation Supercomputer (NGS) Project
- New Institute in Kobe
- Strategic Field Program for NGS

Sext-Generation Supercomputer (NGS) Project

Overview of Project

Development of 10 Pflops-class system in Kobe

⇒ named "K computer" by public competition

- Development of grand challenge applications in nano science and life science
- Buildup of a research center in computational science around the 10 Pflops-class system

⇒ Advanced Institute for Computational Science (AICS)

- Project period is from Japanese FY 2006 to 2012
- RIKEN is responsible for the computer development
- High Performance Computing Infrastructure (HPCI)



Site for K computer









SAdvanced Institute for Computational Science

founded on July 1, 2010

Objectives

- Research and development of computational and computer science
- Operation of K computer
- Lead High Performance Computing (HPC) in Japan

Special emphasis on

- Strong collaboration between computational and computer scientists
- Research on future HPC systems after K computer
- Fostering young scientists with expertise in both computational and computer science



Research Division

launched on October 1, 2010

8 research teams lead by Principal Investigators (PI)

Computational Science	Field Theory	Y.Kuramashi
	Computational Molecular Science	T.Nakajima
	Computational Biophysics	Y.Sugita
	Computational Material Science	S.Yunoki
	Computational Climate Science	H.Tomita
Computer Science	System Software	Y.Ishikawa
	Programming Environment	M.Sato
	Processor	M.Taiji

Seeking Research Scientists

http://www.riken.go.jp/r-world/info/recruit/k111118_e_aics.html



Strategic Field Program

For strategic use of K computer

- Government selected 5 strategic fields in science and technology for importance from national view point
- For each field, Government also selected a core institute
- Each core institute is responsible for organizing research and supercomputer resources in the respective field and its community, for which they receive
 - priority allocation of K computer resources
 - funding to achieve the research goals

Strategic Fields and Core Institutes

strategic field

Plan of Talk

- §1. Brief Introduction
- §2. Hadron Spectrum
- §3. Fine Tuning of Quark Masses
- §4. 1+1+1 Flavor QCD+QED
- §5. Summary

§1. Brief Introduction to Lattice QCD

QCD Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} \left[\gamma_{\mu} (\partial_{\mu} - igA_{\mu}) + m_{q} \right] q$$

Only coupling const. g and quark masses m_q are free parameters

Why is numerical analysis necessary for strong interaction? Too strong to investigate with perturbative analysis Characteristic features (confinement etc.) are nonperturbative

Aiming at quantitative analyses on

- hierarchical structures made of quarks
- phase diagram and EOS under finite temperature/density

based on first principle (QCD Lagrangian) calculations

Numerical method (1)

Path integral in 4-dim. (space 3-dim. + time 1-dim.) continuum theory

$$\langle \mathcal{O}[A_{\mu}, q, \bar{q}] \rangle = \frac{1}{Z} \int \mathcal{D}A_{\mu} \mathcal{D}q \mathcal{D}\bar{q} \ \mathcal{O}[A_{\mu}, q, \bar{q}] \ \exp\left\{-\int d^4x \mathcal{L}[A_{\mu}, q, \bar{q}]\right\}$$

Similar to partition function in stat. mechanics \Rightarrow Monte Carlo method Discretize 4-dim. space-time for finite degree of freedom \Rightarrow 4-dim. lattice

Numerical method (2)

Path integral on 4-dim. lattice

$$\langle \mathcal{O}[U_{\mu}, q, \bar{q}] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_{\mu} dq d\bar{q} \ \mathcal{O}[U_{\mu}, q, \bar{q}] \ \exp\left\{-\sum_{n} \mathcal{L}_{\text{latt}}[U_{\mu}, q, \bar{q}]\right\}$$

Quark fields are Grassmann (anticommuting) numbers $\Rightarrow \text{ analytically integrated}$ $\langle \bar{\mathcal{O}}[U_{\mu}] \rangle = \frac{1}{Z} \int \prod_{n,\mu} dU_{\mu} \ \bar{\mathcal{O}}[U_{\mu}] \ \exp\left\{-S_{\text{latt}}^{\text{eff}}[U_{\mu}]\right\}$

Average over the configurations gives expectation value

$$\langle \bar{\mathcal{O}}[U_{\mu}] \rangle = \frac{1}{N} \sum_{i=1}^{N} \bar{\mathcal{O}}[U_{\mu}^{(i)}] + O\left(\frac{1}{\sqrt{N}}\right)$$

statistical error

Simulation Parameters

Few parameters

- •4-dim. volume: V=NX•NY•NZ•NT
- lattice spacing: a (as function of bare coupling g)
- •quark masses: m_q (q=u,d,s,c,b,t)

Major Systematic Errors

• Finite volume effects

⇒ larger V=NX•NY•NZ•NT

- Finite lattice spacing effects
 Λ_{QCD} ≪ 1/a, currently m_b > 1/a
 ⇒ smaller a
- Quenched approximation (det D_q=1)
 ⇒ simulations including the effects of det D_q
- Chiral extrapolation

⇒ simulations at physical quark masses (physical point)

Need heavier computational cost to diminish the systematic errors $cost \propto (physical vol.)^{1.25} \cdot (lattice spacing)^{-6} \cdot (quark mass)^{-2} \cdot (add mass)$

§2. Hadron Spectrum

Fundamental quantities both in physical and technical senses

physical side physical input ⇒ $m_u, m_d, m_s, ...$ ⇒ reproduce hadron spectrum? (ex. $m_{\pi}, m_{K}, m_{\Omega}$) validity of QCD / determination of m_{α}

technical side

hadron correlators in terms of quark fields

 $\left\langle \mathcal{O}_{h}(t)\mathcal{O}_{h}^{\dagger}(0)\right\rangle \overset{t\gg0}{\sim} C\exp\left(-m_{h}t\right) \Rightarrow \text{extract } \mathbf{m}_{\mathbf{h}} \text{ by fit}$

quark diagrams from Wick contractions

History of Hadron Spectrum Calculation

1981 first calculation of hadron masses in quenched approx.
 Hamber-Parisi
 demonstrate the possibility of first principle calculations

1996~2000 precision measurement in quenched approx. CP-PACS clear deviation from the experiment

initiate 2+1 flavor QCD simulations
 CP-PACS/JLQCD, MILC, ...
 incorporate det D_{u,d,s} effects
 reduce ud quark mass toward physical value

Hadron Spectrum in Quenched QCD

physical input m_{π} , m_{κ} or m_{ϕ} , $m_{\rho} \Rightarrow m_{u}=m_{d}$, m_{s} , a

 \sim 10% deviation from experimental values

Hadron Spectrum in 2+1 Flavor QCD

physical input m_{π} , m_{K} , $m_{\Omega} \Rightarrow m_{u}=m_{d}$, m_{s} , a

consistent within $2 \sim 3\%$ error bars

similar results are obtained by other groups

Chiral Behavior (1)

reduction of ud quark mass \Rightarrow check chiral properties as a by-product

logarithmic curvature expected from ChPT

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nonunvial curvature observeu

ChPT fit gives reasonable LECs

What Next?

 2+1 (m_u=m_d≠m_s) flavor simulation at the physical point PACS-CS 10

avoid chiral extraporation

1+1+1 (m_u≠m_d≠m_s) flavor simulation at the physical point

 electromagnetic interactions
 quenched study: Eichten et al. 96, Blum et al. 07,10

quenched study: Eichten et al. 96, Blum et al. 07,10 MILC@lat10, BMW@lat1<u>0</u>

- u-d quark mass difference

 Direct treatment of resonances in lattice QCD decay width of resonance states

§3. Fine Tuning of Quark Masses

$$\begin{split} & \text{Reweighting method} \\ & \text{original: } (\kappa_{\text{ud}}, \kappa_{\text{s}}) \Rightarrow \text{target: } (\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) \text{ assuming } \rho_{q} \equiv \kappa_{q} / \kappa_{q}^* \approx 1 \\ & \langle \bar{\mathcal{O}}[U](\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) \rangle_{(\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*)} = \frac{\int \mathcal{D}U \bar{\mathcal{O}}[U](\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) |\det[D_{\kappa_{\text{ud}}^*}[U]]|^2 \det[D_{\kappa_{\text{s}}^*}[U]]e^{-S_g[U]}}{\int \mathcal{D}U |\det[D_{\kappa_{\text{ud}}^*}[U]]|^2 \det[D_{\kappa_{\text{s}}^*}[U]]e^{-S_g[U]}} \\ & = \frac{\int \mathcal{D}U \bar{\mathcal{O}}[U](\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) \left|\det\left[\frac{D_{\kappa_{\text{ud}}^*}[U]}{D_{\kappa_{\text{ud}}}[U]}\right]\right|^2 \det\left[\frac{D_{\kappa_{\text{s}}^*}[U]}{D_{\kappa_{\text{s}}[U]}}\right] |\det[D_{\kappa_{\text{ud}}}[U]]|^2 \det[D_{\kappa_{\text{s}}}[U]]e^{-S_g[U]}} \\ & = \frac{\int \mathcal{D}U \bar{\mathcal{O}}[U](\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) \left|\det\left[\frac{D_{\kappa_{\text{ud}}^*}[U]}{D_{\kappa_{\text{ud}}}[U]}\right]\right|^2 \det\left[\frac{D_{\kappa_{\text{s}}^*}[U]}{D_{\kappa_{\text{s}}[U]}}\right] |\det[D_{\kappa_{\text{ud}}}[U]]|^2 \det[D_{\kappa_{\text{s}}}[U]]e^{-S_g[U]}} \\ & = \frac{\langle \bar{\mathcal{O}}[U](\kappa_{\text{ud}}^*, \kappa_{\text{s}}^*) R_{\text{ud}}[U]R_{\text{s}}[U]\rangle_{(\kappa_{\text{ud}}, \kappa_{\text{s}})}}, \end{split}$$

Reweighting factors

$$R_{\rm ud}[U] = |\det[W[U](\rho_{\rm ud})]|^2, \quad R_{\rm s}[U] = \det[W[U](\rho_{\rm s})]$$

where

$$W[U](\rho_q) \equiv \frac{D_{\kappa_q^*}[U]}{D_{\kappa_q}[U]}$$
²³

Evaluation of R_{ud}

Introduce a complex bosonic field $\boldsymbol{\eta}$

$$R_{\rm ud}[U] = \left| \det \left[W[U](\rho_{\rm ud}) \right] \right|^2$$
$$= \left\langle e^{-|W^{-1}[U](\rho_{\rm ud})\eta|^2 + |\eta|^2} \right\rangle_{\eta}$$

Given a set of $\eta^{(i)} \, (i{=}1,{\dots},N_\eta)$ with Gaussian distribution

$$R_{\rm ud}[U] = \lim_{N_\eta \to \infty} \frac{1}{N_\eta} \sum_{i=1}^{N_\eta} e^{-|W^{-1}[U](\rho_{\rm ud})\eta^{(i)}|^2 + |\eta^{(i)}|^2}$$

R_s is obtained in a similar way

Tuning to the Physical Point PACS-CS 10

0.20 $(m_h/m_\Omega)_{lat} / (m_h/m_\Omega)_{exp} - 1$ 0.10 Ŧ ∙ --● 0.00 **T** Ī original • target input • -0.10 $K^* \phi$ $\Sigma *$ Κ η_{ss} Ξ Ν Σ $\Xi*$ þ Λ Δ Ω π

> $m_{\pi}/m_{\Omega}, m_{K}/m_{\Omega}$ are properly tuned Δm_{ud} ~1MeV, Δm_{s} ~3MeV

Distribution is slightly moved toward larger values Still almost degenerate

§4. 1+1+1 Flavor QCD+QED

Isospin symmetry breaking

- electromagnetic interactions

 Q_u =+2/3e, Q_d = Q_s =-1/3e, e= $\sqrt{4\pi/137}$

- u-d quark mass difference

 $m_u = m_d \neq m_s (2+1 \text{ flavor}) \Rightarrow m_u \neq m_d \neq m_s (1+1+1 \text{ flavor})$

Physical inputs:

 $m_{\pi^+}(ud), m_{K0}(ds), m_{K^+}(us), m_{\Omega^-}(sss)$

Outputs:

 $m_u, m_d, m_s, a(lattice spacing), m_n-m_p, \dots$

Strategy

- Generate 2+1 flavor QCD configs near the physical point
- Generate U(1) gauge configs independent of QCD ones

 non-compact A_µ ⇒ compactification to U_µ^e=exp(iQ_qA_µ)
- Incorporate EM effects by reweighting method

 Q_u=0 ⇒ Q_u=+2/3e for u quark
 Q_{d,s}=0 ⇒ Q_{d,s}=-1/3e for d, s quark
- Tune $(\kappa_u, \kappa_d, \kappa_s)$ to the physical point such that $m_{\pi^+}(ud)$, $m_{K0}(ds)$, $m_{K^+}(us)$, $m_{\Omega^-}(sss)$ are reproduced

In the following, all the results are preliminary

Reweighting factor

$$R_{\rm uds}[U_{\mu}] = \det\left[\frac{D_{\kappa_{\rm u}^{*}}[U_{\mu}\exp(iQ_{u}A_{\mu})]}{D_{\kappa_{\rm u}}[U_{\mu}]}\frac{D_{\kappa_{\rm d}^{*}}[U_{\mu}\exp(iQ_{d}A_{\mu})]}{D_{\kappa_{\rm d}}[U_{\mu}]}\frac{D_{\kappa_{\rm s}^{*}}[U_{\mu}\exp(iQ_{s}A_{\mu})]}{D_{\kappa_{\rm s}}[U_{\mu}]}\right]$$

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N_{η} dependence

Physical inputs: $m_{\pi^+}(ud)$, $m_{K0}(ds)$, $m_{K^+}(us)$, $m_{\Omega^-}(sss)$

Mass Splittings among Isospin Multiplets

SQuark Masses with NP Renormalization Factor

Physical inputs:

 $\begin{array}{l} m_{\pi^+}(ud) = 139.7(15.5) \ [MeV] \\ m_{K0}(ds) = 497.6(8.1) \ [MeV] \\ m_{K^+}(us) = 492.4(8.1) \ [MeV] \\ m_{K0}(ds) - m_{K^+}(us) = 3.21(57) \ [MeV] \end{array}$

exp: 139.6 [MeV] exp: 497.6 [MeV] exp: 493.7 [MeV] exp: 3.937(28) [MeV]

Quark Masses in MSbar at μ =2 GeV: m_u =1.97(67) [MeV] m_d =4.31(83) [MeV] m_s =90.32(67) [MeV] $(m_u+m_d)/2$ =3.14(72) [MeV] m_u/m_d =0.457(93) $2m_s/(m_u+m_d)$ =28.8(6.6)

§5. Summary

- 2+1 ($m_u = m_d \neq m_s$) flavor QCD \Rightarrow 1+1+1 ($m_u \neq m_d \neq m_s$) flavor QCD+QED
- Incorporate QED effects by reweighting method
- Next step is to enlarge the lattice volume