Formulation of Spacetime Thermodynamics

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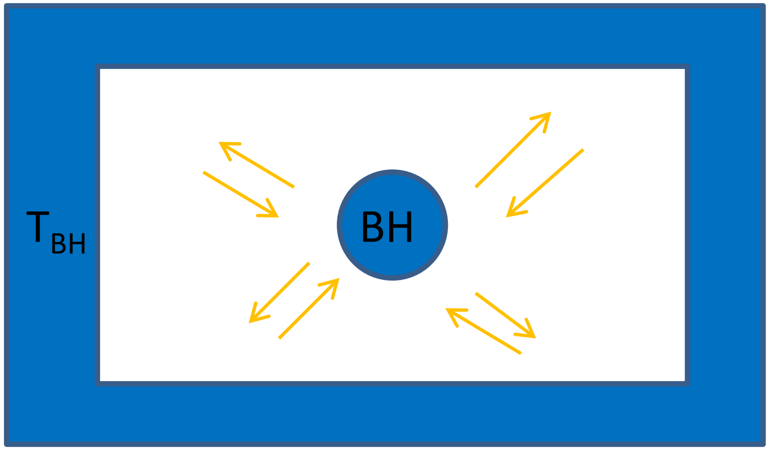
Introduction 1 Black Hole Thermodynamics

Hawking Radiation[Hawking 1975]

⇒Thermal equilibrium

$$< N_\omega >= \frac{\Gamma_\omega}{\exp\left(\frac{\omega}{T_{BH}}\right) - 1}$$



Hawking BH temperature

$$T_{BH} = \frac{1}{8\pi M_{BH}}$$

BH entropy

$$S_{BH} = \frac{1}{4} A_{BH}$$

- 0th law: $T_{BH} = \frac{\kappa_{BH}}{2\pi}$
- 1st law: $\delta M_{BH} = T_{BH}\delta S_{BH} + \Omega_{BH}\delta J$
- 2nd law: Generalized Second Law (GSL)

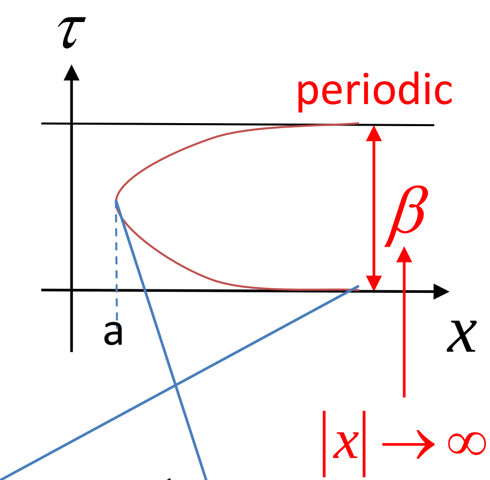
$$\delta S = \delta S_{matter} + \delta S_{BH} \geq 0$$

Introduction 2　Gibbons-Hawking's result

・the partition function of a Schwarzschild BH by Euclidean path integral.

[Gibbons and Hawking 1977]

$$Z = \int d[g] \exp(-I[g]) \cong \exp(-I[g_0])$$



・a vacuum solution of the Einstein eq.

⇒absence of conical singularity

⇒a unique equilibrium energy $M_{BH} = \dfrac{\beta}{8\pi}$

⇒free energy

$$U - TS = F = -\beta^{-1} logZ \cong M_{BH} - \beta^{-1}\frac{1}{4}A_{BH}$$

My motivation:

Why does a single gravitational configuration create the finite statistical entropy?

$$S = \frac{A_{BH}}{4} = log\Omega$$

⇒A gravitational configuration corresponds to a thermodynamic state?

Introduction 3　　Jacobson's idea

- Jacobson showed that the Einstein equation can be regarded as the equation of state for spacetime. [Jacobson 1995]

For a part of any spacetime,

$$T\delta S = \delta Q \Rightarrow \quad R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$$

His assumptions:

1 all energy through the observer's horizon = heat

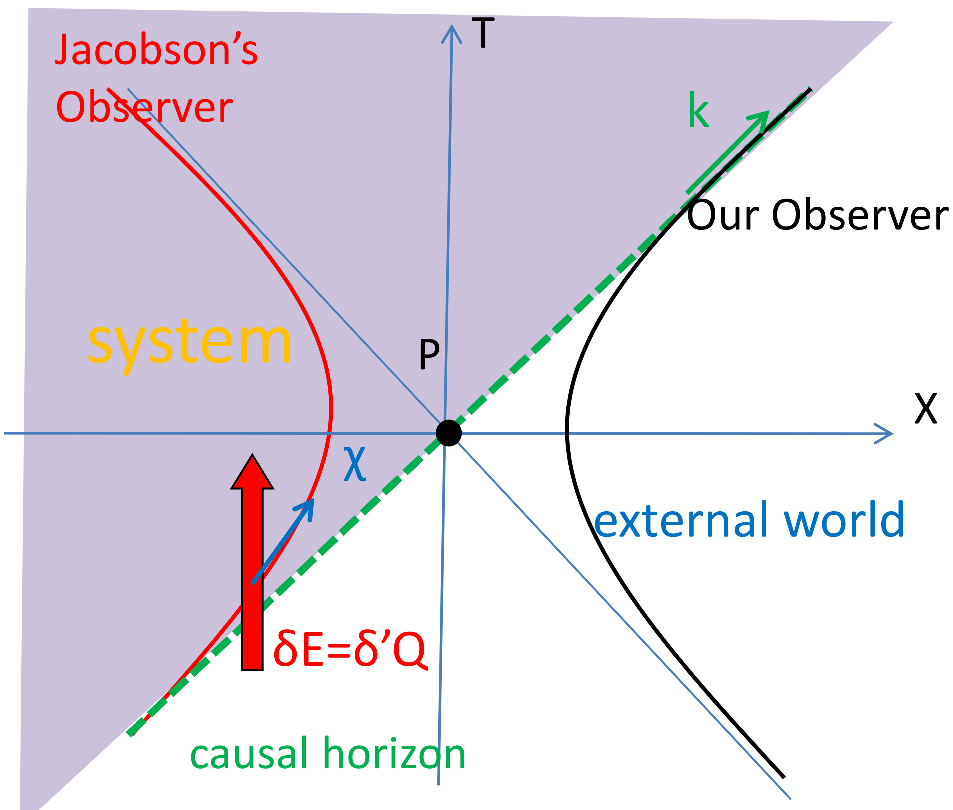
$$\delta E = \delta' Q$$

2 the entropy area law

$$\delta S = \frac{\delta A}{4}$$

3 the Unruh effect

$$T_U = \frac{a}{2\pi}$$



Introduction4

Can a part of any spacetime be really regarded as a thermodynamic system?

⇒probably, no!

But I pointed out unnaturalness of Jacobson's discussion and tried to reconstruct the discussion in the following ways.

What I tried:

- Introduce outside observer, rearrange the discussion and construct the first law for non-equilibrium processes
- Generalize to the f(R) gravity
- Introduce ``work term"

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2-1-1 Hawking's discussion

- Dust ⇒gravitational collapse ⇒Schwarzschild BH

Question:

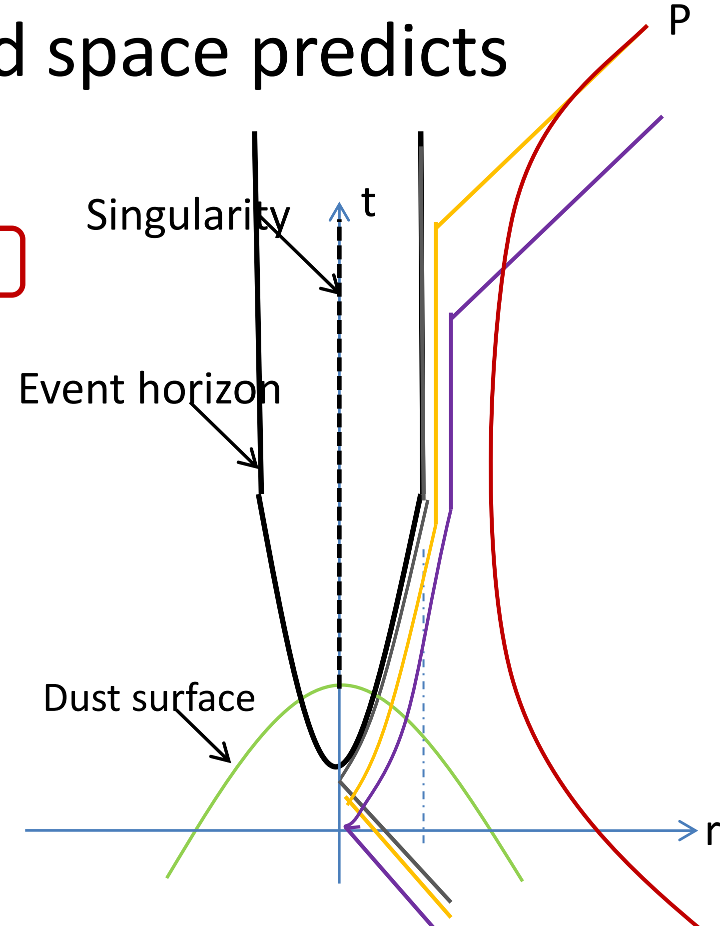
What do we observe in asymptotic flat region?

⇒QFT in time-dependent curved space predicts Hawking radiation!

graybody factor

$$< N_\omega >= \frac{\Gamma_\omega}{\exp\left(\frac{\omega}{T_{BH}}\right) - 1}$$

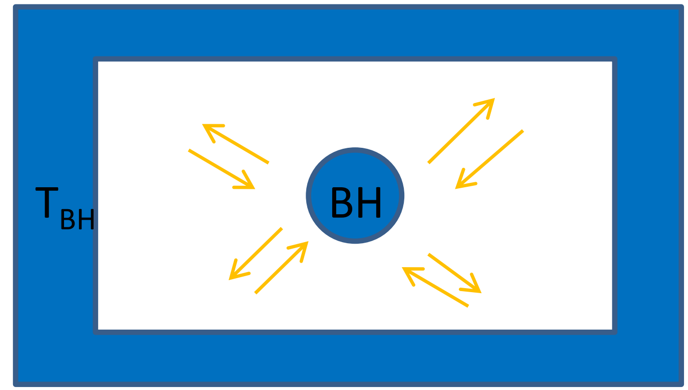
$$T_{BH} = \frac{1}{8\pi M_{BH}}$$



2-1-2 BH entropy from thermodynamic viewpoint

- BH can be equilibrium

with heat bath of temperature T_{BH}.

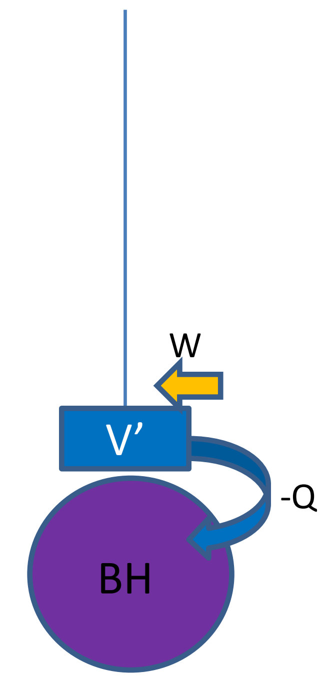


⇒BH can work as heat bath of T_{BH} in Carnot cycle.

$$\delta S_{BH} = \frac{\delta Q}{T_{BH}}$$

⇒BH has the thermodynamic entropy.

$$S_{BH}[M_{BH}] = \int dM \frac{1}{T_{BH(M)}} = \frac{1}{4} A_{BH} = log\Omega(M_{BH})$$



(Hawking's picture is micro-canonical viewpoint.)

2-2-1 Gibbons-Hawking's discussion

- Image a BH in heat bath of T.

⇒Canonical viewpoint

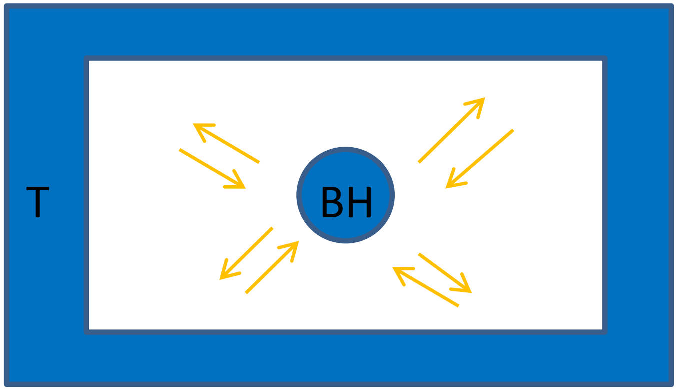
$$\Rightarrow F(T;M) = -T log Z(T;M)$$

⇒equilibrium condition $\frac{dF}{dM} = 0$

$$\Rightarrow M = \frac{1}{8\pi T}$$

$$\Rightarrow F[T] = \frac{1}{16\pi T}$$

$$\Rightarrow S(T) = -\frac{\partial F}{\partial T} = \frac{1}{16\pi M^2} = \frac{1}{4} A_{BH}(T)$$



2-2-2 Gibbons-Hawking's derivation

HOW TO?

- Euclidean QFT
- WKB approximation
- Pure gravity

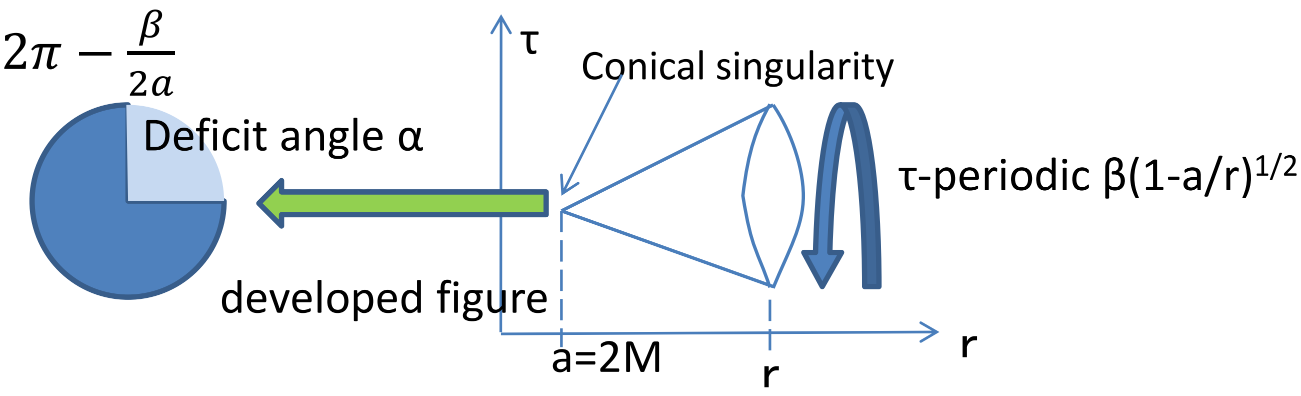
$$Z = \int d[g] \exp(-I[g]) \cong \exp(-I[g_0])$$

$$F(T;M) = -T log Z(T;M) \cong T I[g_0(T;M)]$$

$$I[g_0(T;M)] = -\frac{1}{16\pi}\int_V d^4x\sqrt{g}R - \frac{1}{8\pi}\int_{\partial V} d^3y\sqrt{h}\,(K-K_0) - \frac{1}{16\pi}2\alpha A$$

Einstein-Hilbert Gibbons-Hawking Gauss-Bonnet

deficit angle : $\alpha = 2\pi - \frac{\beta}{2a}$



2-2-3 BH entropy from statistical viewpoint

- In equilibrium

$$S_{BH}(T) = \frac{A_{BH}}{4} = S[M_{BH}] = kBlog\Omega(M_{BH}) > 0$$

A single gravitational configuration g_0 creates!

Finite statistical entropy!

⇒A gravitational configuration corresponds to a thermodynamic state?

2-2-4 Gravity has a duality?(1)

<Other interaction>

effective theory of finite temperature QCD (equation of state) ≠ low energy effective theory of QCD (T=0) (Chiral lagrangian)

⇒A single classical configuration of finite temperature chiral lagrangian dose not create finite entropy.

<Gravity>

effective theory of finite temperature string theory (General relativity) = low energy effective theory of string theory (T=0) (General relativity)

⇒A single classical configuration of finite temperature general relativity creates finite entropy.

2-2-4 Gravity has a duality?(2)

viewpoint A:
low energy effective theory
Gravity = fundamental interaction
Einstein eq.=eq. of motion

viewpoint B:
thermodynamic effective theory
Gravity = entropic force
Einstein eq.=eq. of state

⇒Gravity has two different properties simultaneously.
⇒A duality?

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3-1 Jacobson's idea

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His assumptions:

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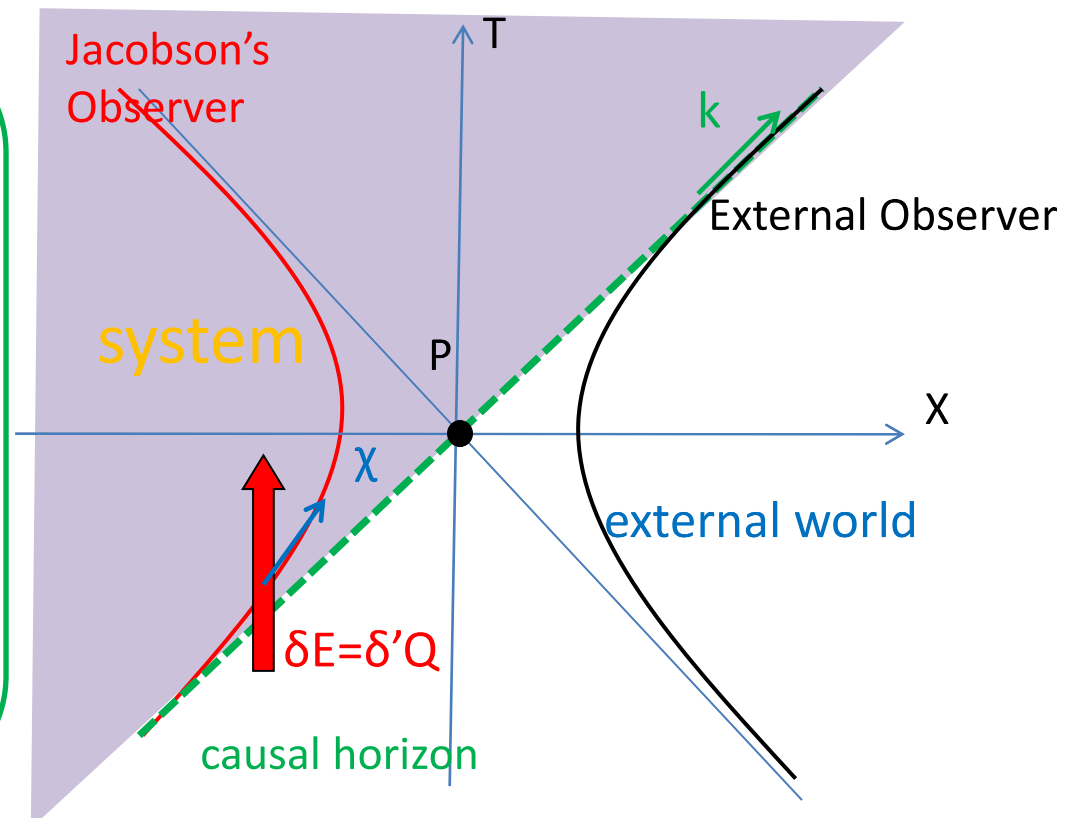


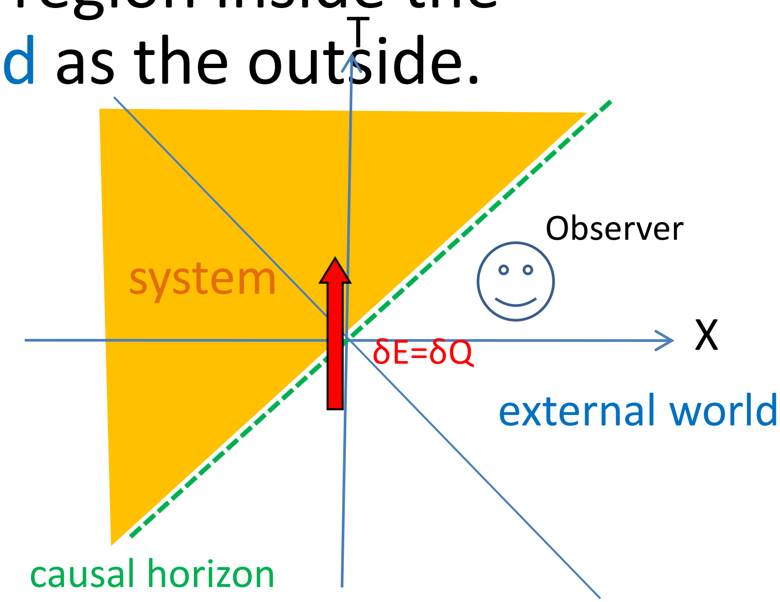
3-2 System, External world, and Heat

- In general, heat is transfer of energy which cannot be identified and controlled by an external observer.

- ⇒ In spacetime thermodynamics, heat can be defined as energy flow through any causal horizon.

- ⇒The system is defined as the region inside the horizon, and the external world as the outside.

- ⇒A conventional observer is defined as an observer in the external world, who measures the thermodynamic quantities.



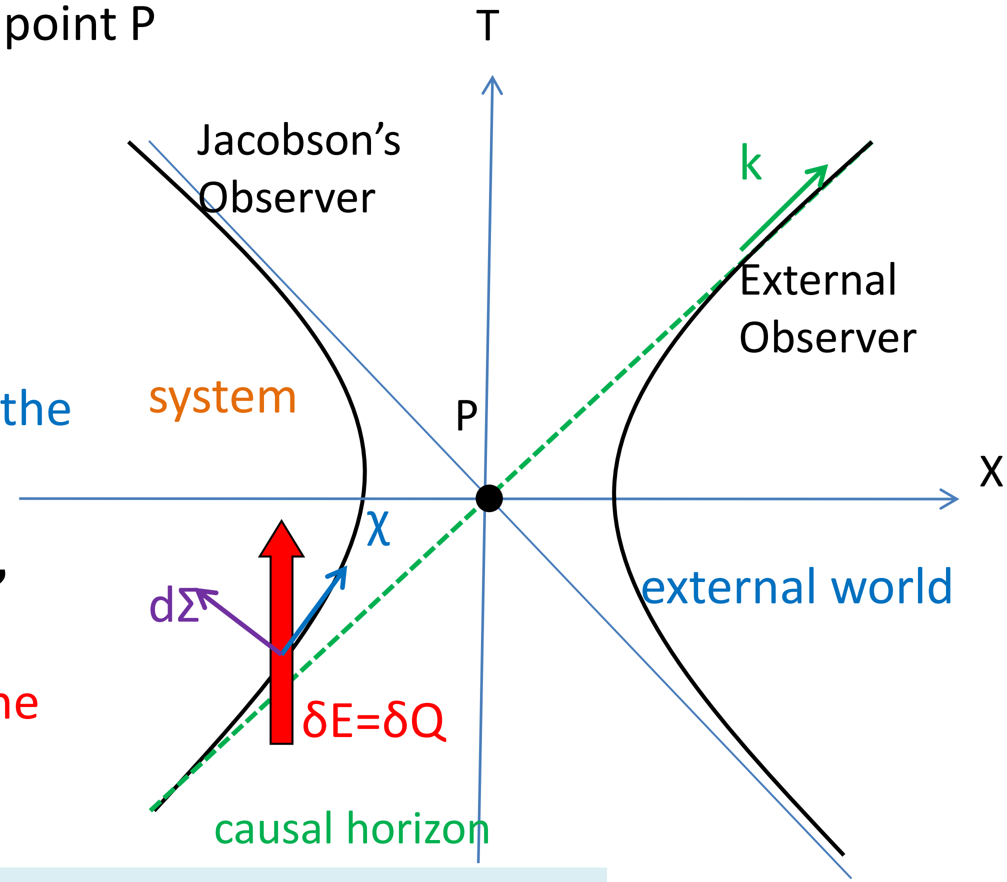
3-3 Jacobson's setup

- Take a local inertial frame near any point P

⇒uniformly accelerating observer

⇒Rindler horizon for him

⇒spacetime thermodynamic system



▪By using an accelerating observer χ in the system:

$$T = x\cosh t\,, X = -x\sinh t\,,$$

estimate the energy flow δE with affine parameter λ:

(Near horizon limit $x \to 0, \chi \to k$)

$$\delta Q = \delta E = \int T_{ab}\chi^a d\Sigma^b \approx -x^{-1}\int T_{ab}k^a k^b \lambda d\lambda\, dA$$

▪Unruh temperature

$$T_U = \frac{x^{-1}}{2\pi}$$

$$\frac{\delta Q}{T} \approx -2\pi \int T_{ab}k^a k^b \lambda d\lambda\, dA$$

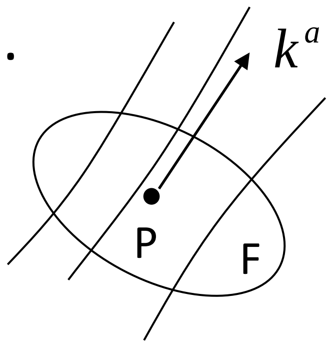
3-4 Jacobson's derivation1

- Entropy change=area change

$$S = \frac{A}{4G} \quad \Rightarrow \quad \delta S = \frac{\delta A}{4G} \qquad \delta A = \int_H \theta \, d\lambda \, dA$$

Expansion $\theta = \frac{1}{\Delta A}\frac{d\Delta A}{d\lambda}$

- The affine-parameterized Raychaudhuri eq.



$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b$$

Assumption:
Local equilibrium
$\theta = \sigma = 0$

$$\theta = -\lambda R_{ab}k^a k^b \quad \Rightarrow \quad \delta A = -\int_H R_{ab}k^a k^b \, \lambda \, d\lambda \, dA$$

3-4 Jacobson's derivation2

$$\delta S = \frac{\delta Q}{T_U} \Longrightarrow -\frac{1}{4G}\int d\lambda dA \lambda R_{ab} k^a k^b = -2\pi \int d\lambda\, dA \lambda T_{ab} k^a k^b$$

- This holds for any point.

$\Rightarrow$
$$R_{ab} k^a k^b = 8\pi G T_{ab} k^a k^b$$

- This holds for any null vector.

$\Rightarrow$
$$R_{ab} + f g_{ab} = 8\pi G T_{ab}$$

- Energy conservation $\quad \nabla_b T^{ab} = 0$
- Bianchi id. $\quad \nabla_b \left(R^{ab} - \frac{1}{2} g^{ab} R \right) = 0$

$\Rightarrow$Einstein eq.
$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$$

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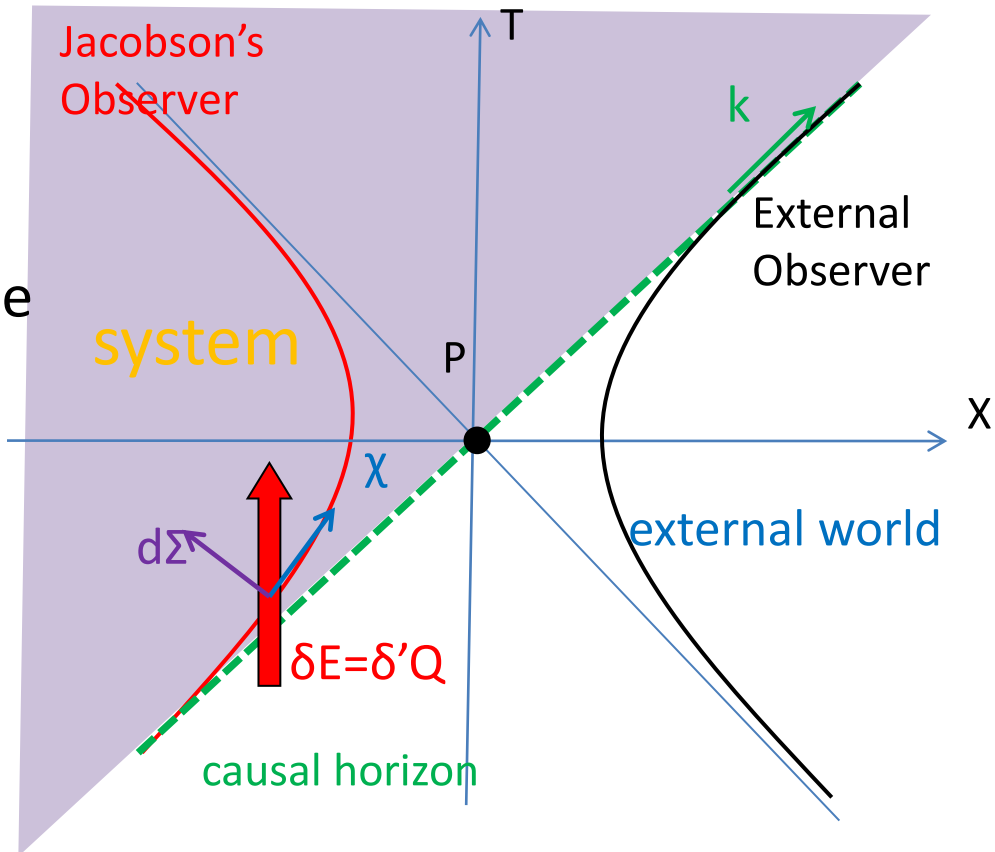
4-1 unnatural observer

Jacobson's observer = observer in the system

Observer in thermodynamics = observer out of the system

Jacobson's formulation cannot be applied to BH thermodynamics.

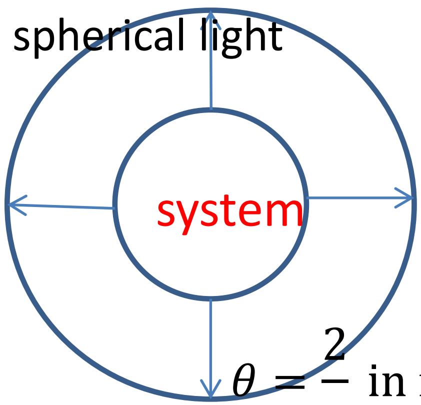


4-2 What is the entropy?

Jacobson's system = open system with $\theta = 0$

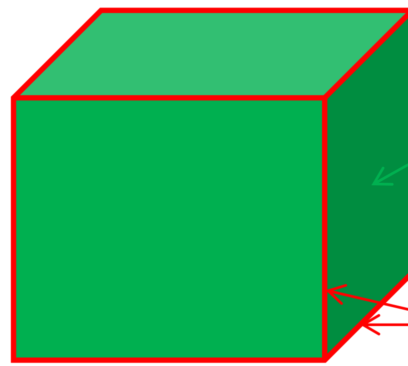
If closed system, $\theta \neq 0$. ⇒non-stationary!

Ex1. spherical light



$\theta = \frac{2}{r}$ in flat spacetime

Ex2. "light box"



- The Carnot cycle cannot be constructed.

⇒Thermodynamic entropy cannot be introduced!

- If Information entropy or entanglement entropy

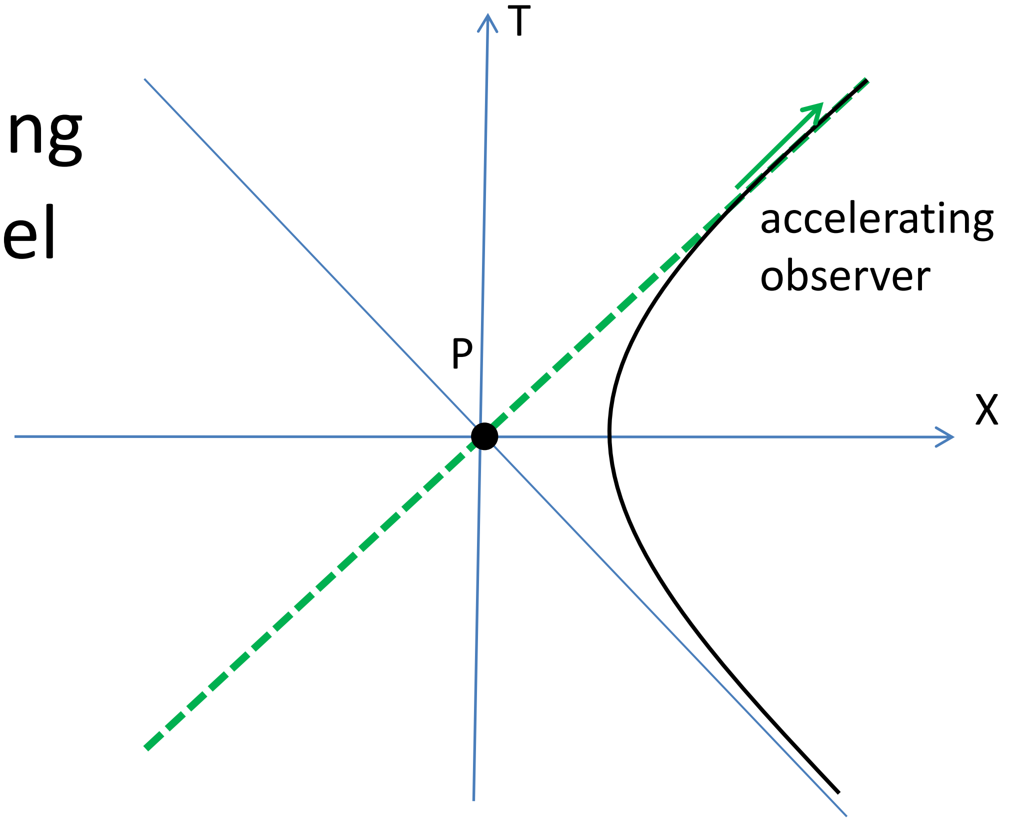
⇒Entropy can diverge!

4-3 Is the Unruh effect true?

- If an observer is accelerating uniformly forever, he can feel the Unruh temperature.
- However, such an observer does not exist!

- Cf. A uniformly rotating observer dose not feel the Unruh effect.

[Davies, Dray, and Manogue 1996]



4-4 Why is there no work term?

Thermodynamic 1st law: $\delta U = \delta' Q + \delta' W$

Thermodynamic 1st law: $\delta M_{BH} = T_{BH} \delta S_{BH} + \Omega_{BH} \delta J$

Jacobson's assumption: $\delta E = \delta' Q + ?$

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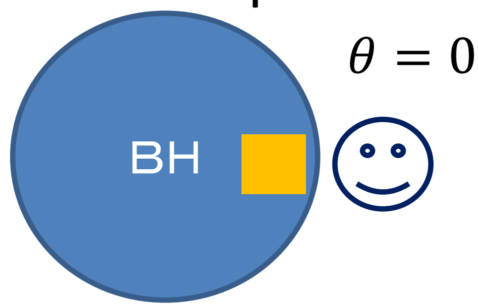
5-1 an observer out of the local system near the BH event horizon

- Stationary BH($\theta = 0$) = equilibrium thermodynamic system

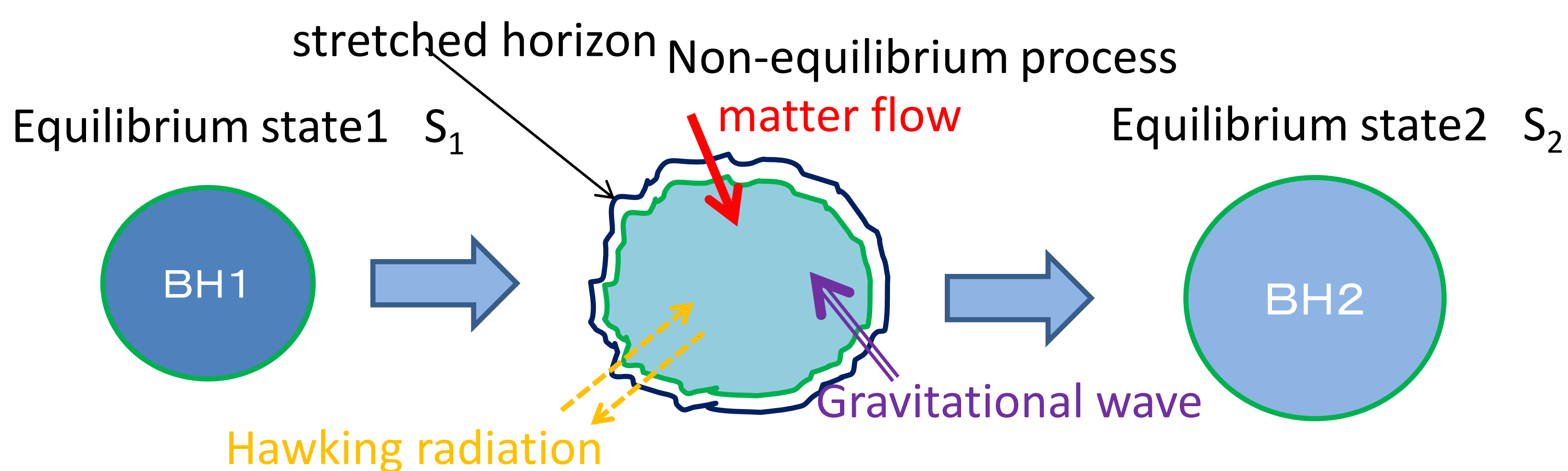
⇒a surface patch = a local thermodynamic system

⇒an external observer near the patch = an natural observer



<question>

What form dose the thermodynamic 1st law of the local system take in the following process?



5-2 introduce external observer

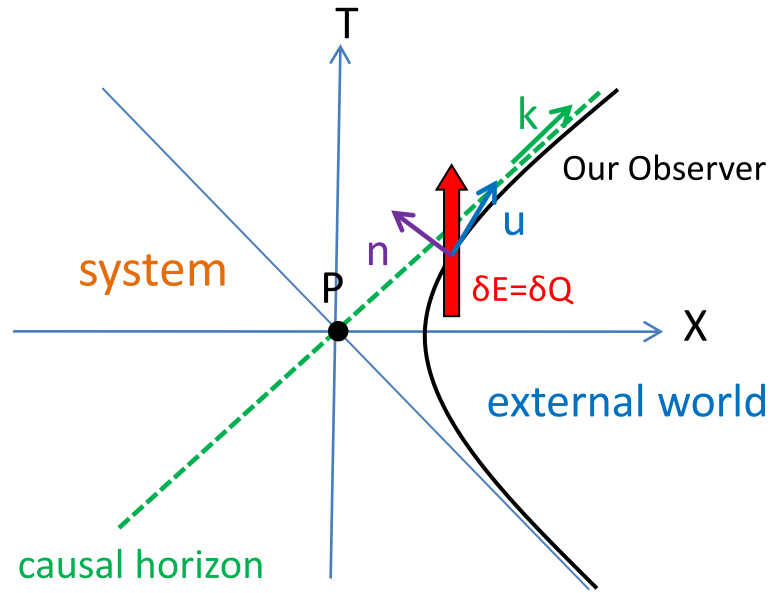
- The external observer:

$$T = x\cosh t\,,X = x\sinh t \qquad u = \frac{\partial}{\partial\tau} = \frac{1}{x}\frac{\partial}{\partial t}$$

- Affine parameter$\lambda \Rightarrow$proper time $\tau = xt$

$$\frac{d\bar{\theta}}{d\tau} = x^{-1}\bar{\theta} - \frac{1}{2}\bar{\theta}^2 - \bar{\sigma}^2 - R_{ab}\bar{k}^a\bar{k}^b$$

Expansion $\bar{\theta} = \frac{1}{\Delta A}\frac{d\Delta A}{d\tau}$



$$\delta E = \int_{\tau 1}^{\tau 2} d\tau \int_{S(\tau)} dAT_{ab}u^a n^b \approx x^{-1}\int_{t1}^{t2} dt \int_{S(t)} dAT_{ab}k^a k^b$$

stretched horizon

$$T_U = \frac{x^{-1}}{2\pi}$$

Local equilibrium
$\bar{\theta} = \bar{\sigma} = 0$ at $\tau = \tau_1, \tau_2$

5-3 1st law for Einstein's gravity

External local temperature

・event horizon⇒time-asymmetry
・σ〜dynamical gravitational effect
⇒dissipation

The negative coefficient only for dynamical processes

$$T_U \delta S = \int_{\tau_1}^{\tau_2} d\tau \int_{S(\tau)} dA \left(\frac{1}{16\pi} 2\bar{\sigma}_{\mu\nu}\bar{\sigma}^{\mu\nu} \right) + \int_{\tau_1}^{\tau_2} d\tau \int_{S(\tau)} dA \left(\frac{-1}{16\pi} \bar{\theta}^2 \right) + \delta E = \delta M_{ADM'}$$

δE＝δ'Q (matter only)

``local ADM energy "= gravitational energy + matter energy

$$T^{(ex)}\delta S = \delta' D + \delta' Q = \delta U$$

1st law $\delta U = \delta' Q + \delta' W = \delta' Q + \delta' D$

2nd law $T^{(ex)}\delta S \geq \delta' Q \Leftrightarrow T^{(ex)}\delta S = \delta' Q + \delta' D, \;\; \delta' D \geq 0$

5-4 1st law for f(R) gravity

< f(R) gravity >

- Action $$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,f(R)$$
- BH entropy $$S = \frac{1}{4}\int_{\mathcal{S}} dA\ f'(R)$$

$$T_U\delta S = \int_{\tau_1}^{\tau_2} d\tau \int_{\mathcal{S}(\tau)} dA \left(\frac{f'(R)}{16\pi}2\bar{\sigma}_{\mu\nu}\bar{\sigma}^{\mu\nu}\right)$$
$$-\frac{1}{4\pi}\int_{\tau_1}^{\tau_2} d\tau \int_{\mathcal{S}(\tau)} dA\ \bar{\theta}\frac{df'(R)}{d\tau} + \int_{\tau_1}^{\tau_2} d\tau \int_{\mathcal{S}(\tau)} dA \left(\frac{-f'(R)}{16\pi}\bar{\theta}^2\right) + \delta E$$
$$= \delta M_{ADM'}$$

Additional new term

The coefficients depend on spacetime points.

$$T^{(ex)}\delta S = \delta' D + \delta' Q = \delta U$$

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6 Summery and Discussions

- BH can be equilibrium with heat bath by Hawking radiation.

  ⇒BH has thermodynamic entropy.

- Gibbons-Hawking's result indicates that a gravitational configuration corresponds to a thermodynamic state.

  ⇒Gravity has a duality?

- Jacobson showed that the Einstein equation can be regarded as the equation of state for spacetime.

  ⇒ However, probably, a part of any spacetime cannot be regarded as a thermodynamic system.

What I tried:

- Introduce outside observer, rearrange the discussion and construct the first law for non-equilibrium processes
- Generalize to the f(R) gravity
- Introduce ``work term"

Outlook

- Micro counting finite temperature BH entropy
- Understanding Gibbons-Hawking's result
- Information Paradox

$\Rightarrow$Back reaction from Hawking radiation is important.

Thank You!