Bare Higgs mass at Planck scale

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with H. Kawai & K. Oda

arXiv:1210.2538

2013.1.15 Osaka University

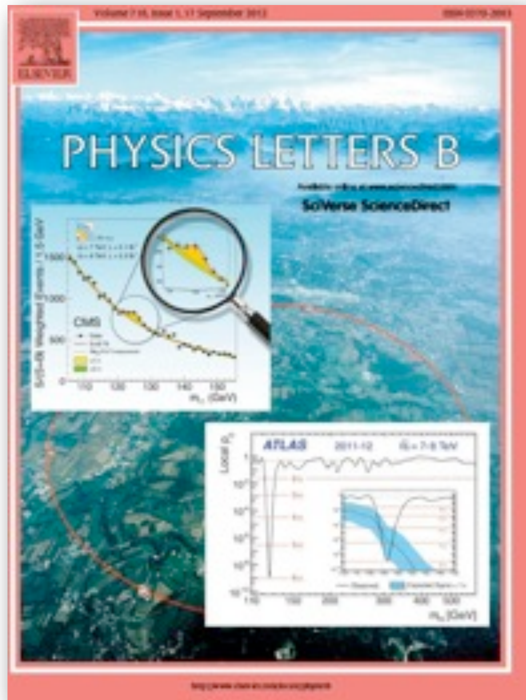
Finally, Higgs-like boson is discovered!!

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

After half century!



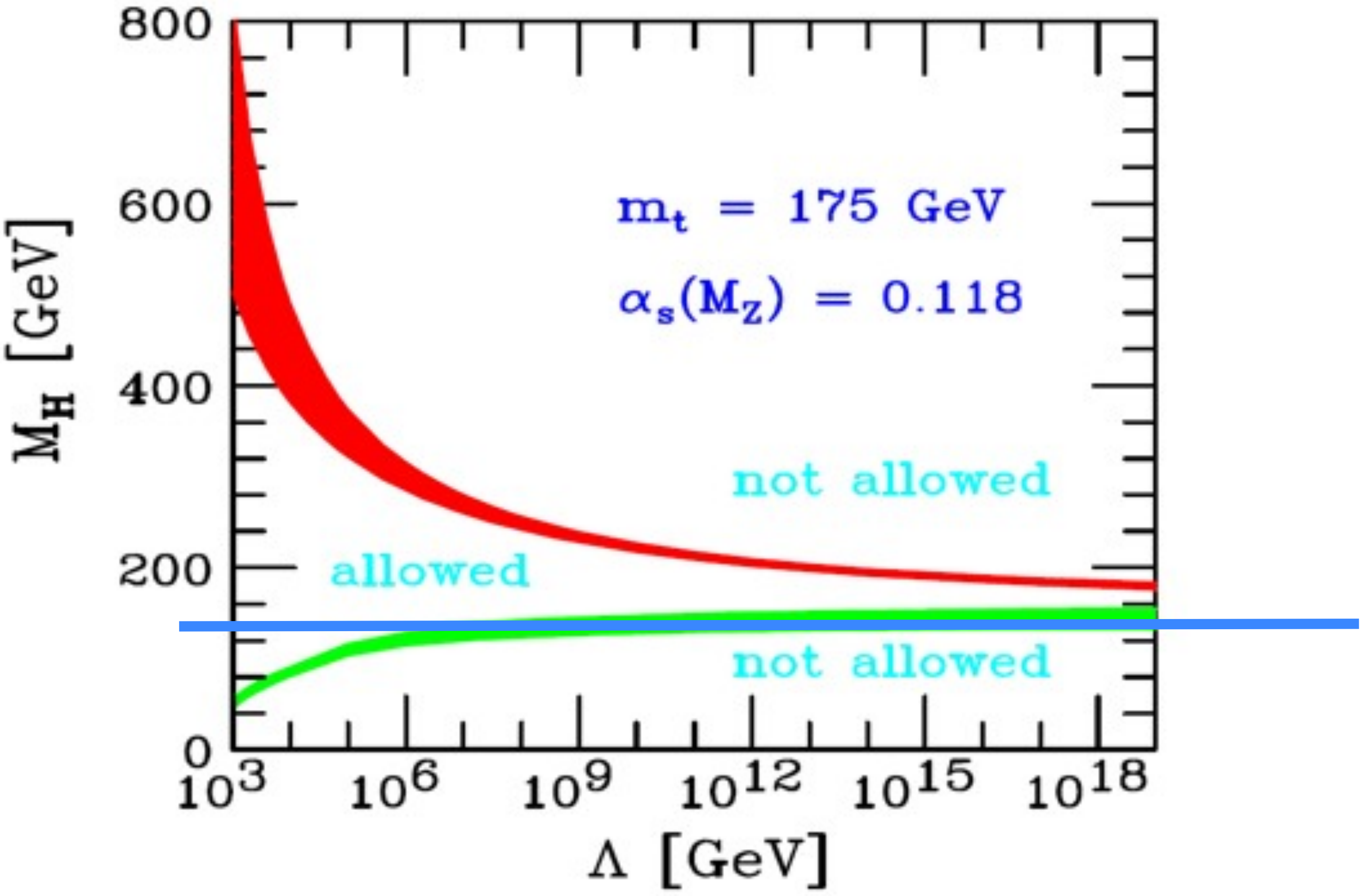
Where is cutoff of SM?

- There are theoretical bounds on Higgs mass depending on cutoff scale of SM.

– Upper bound: Couplings should be perturbative up to cutoff scale .

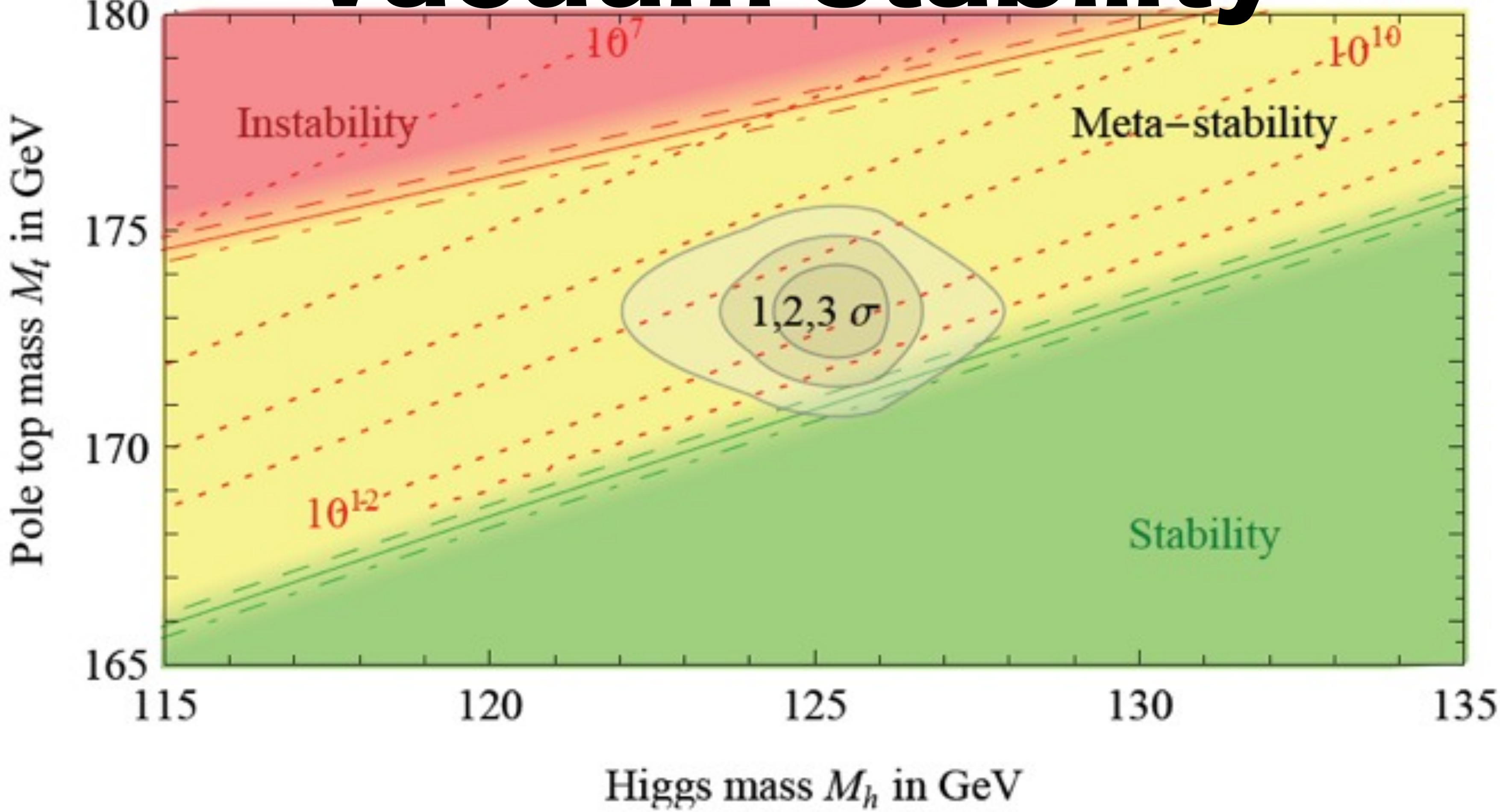
– Lower bound: Current vacuum should be (meta)stable.

SM can be valid up to Planck scale



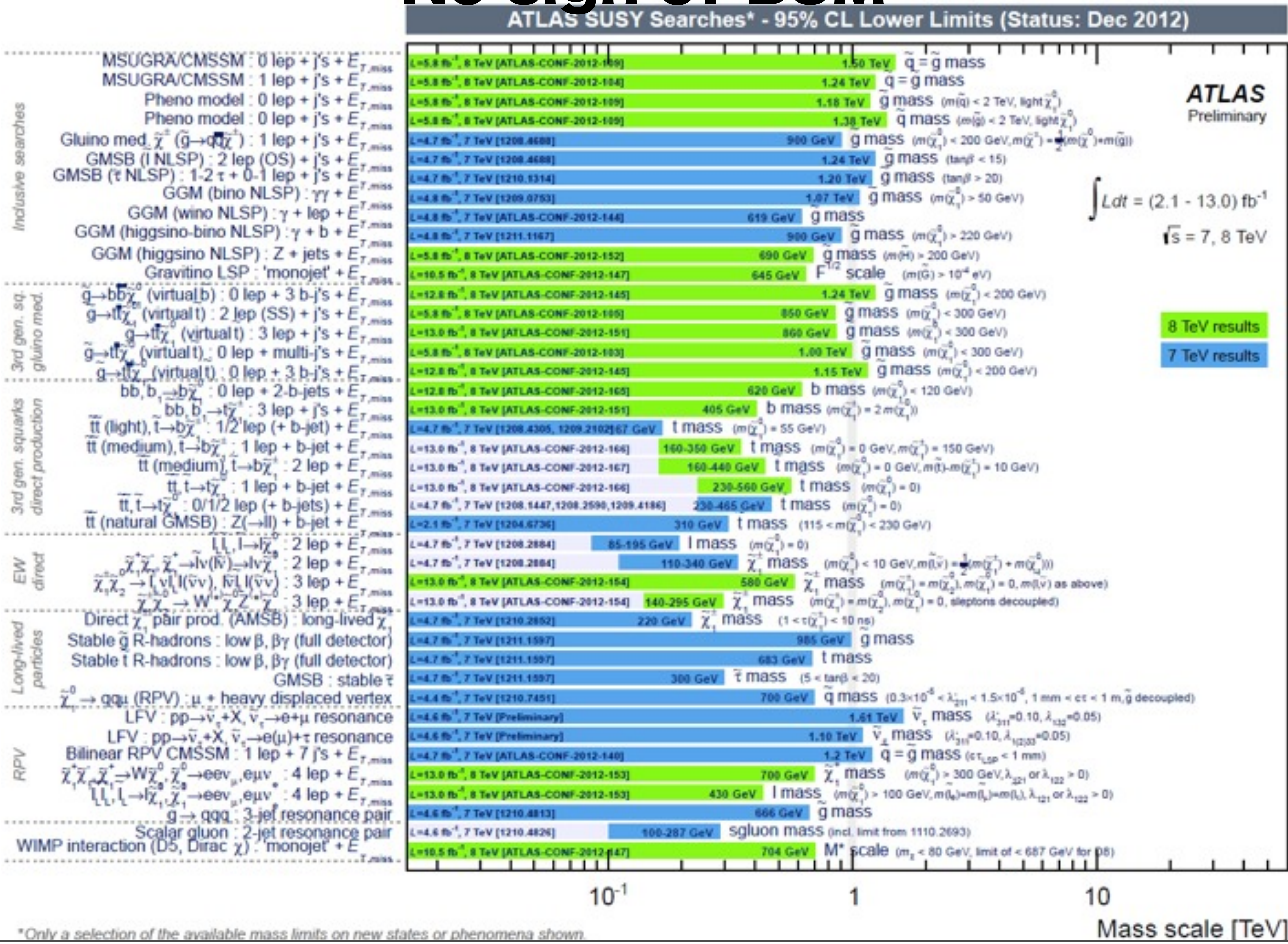
[Hambye & Riesselmann, 1997]

Latest result of vacuum stability



[arXiv:1205.6497, Degrassi et al.]

No sign of BSM



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Given current situation,
it is important to examine scenario
in which SM is valid
towards Planck scale.
This talk assumes such situation.

Bare mass and coupling at Planck scale cutoff

- Because of Higgs discovery, we can discuss SM **bare Lagrangian** at Planck scale.
  - Bare Lagrangian is important because it reflects Planck scale physics.
  - We evaluate **bare** Higgs mass/coupling (Note: This is **not** MS-bar running mass).
  - We compute **quadratic divergence** in **bare** Higgs mass up to **2-loop** orders.
- We find $m_B^2=0$, $\lambda_B=0$ is possible.

Plan

1. Now we can evaluate bare mass
2. Quartic coupling can take zero at Planck scale
3. Bare Higgs mass can take zero at Planck scale

Now we can evaluate bare mass

"We compute quadratic divergence in bare Higgs mass up to 2-loop orders."

ϕ^4 example

- We explain our procedure by taking concrete evaluation for ϕ^4 theory.
- Bare Lagrangian with cutoff Λ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_B)^2 - \frac{m_B^2}{2}\phi_B^2 - \frac{\lambda_B}{4!}\phi_B^4$$

- Our analysis corresponding to the case

$$m_{\rm phys}^2 \ll \Lambda^2$$

- Quadratic divergence is dominant.

Bare mass determined to fix m_{phys}=0

- Bare mass consists of quadratic divergent part and logarithmic divergent part which is proportional to m_{phys}^2.

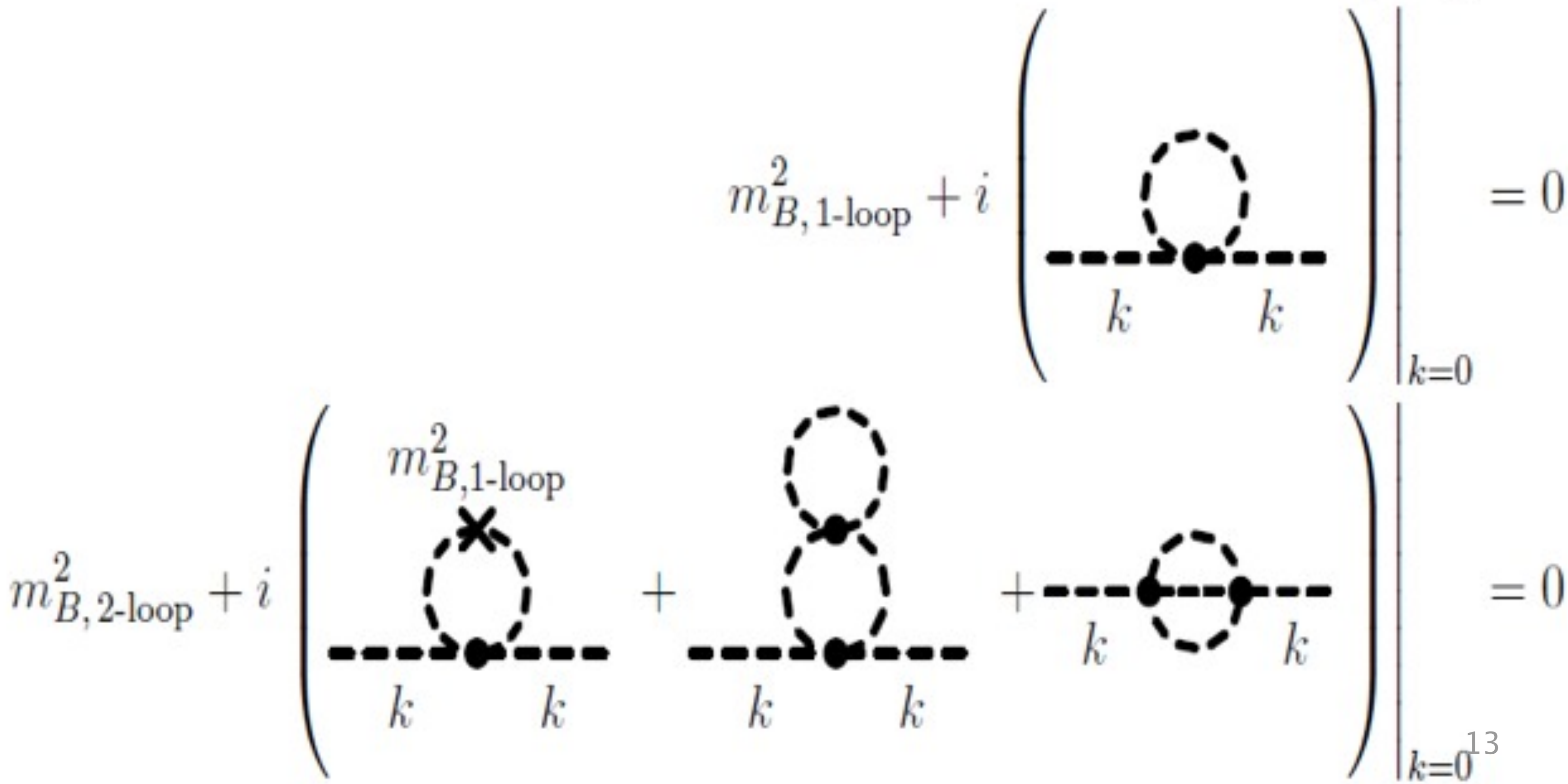
$$m_B^2 = a\Lambda^2 + b m_{phys}^2 \log(\Lambda^2/m_{phys}^2)$$

- In order to obtain quadratic divergence in m_B^2, we determine m_B^2 order by order so that physical mass is zero

$$m_B^2 = m^2_{B,\,\text{0-loop}} + m^2_{B,\,\text{1-loop}} + m^2_{B,\,\text{2-loop}} + \cdots$$

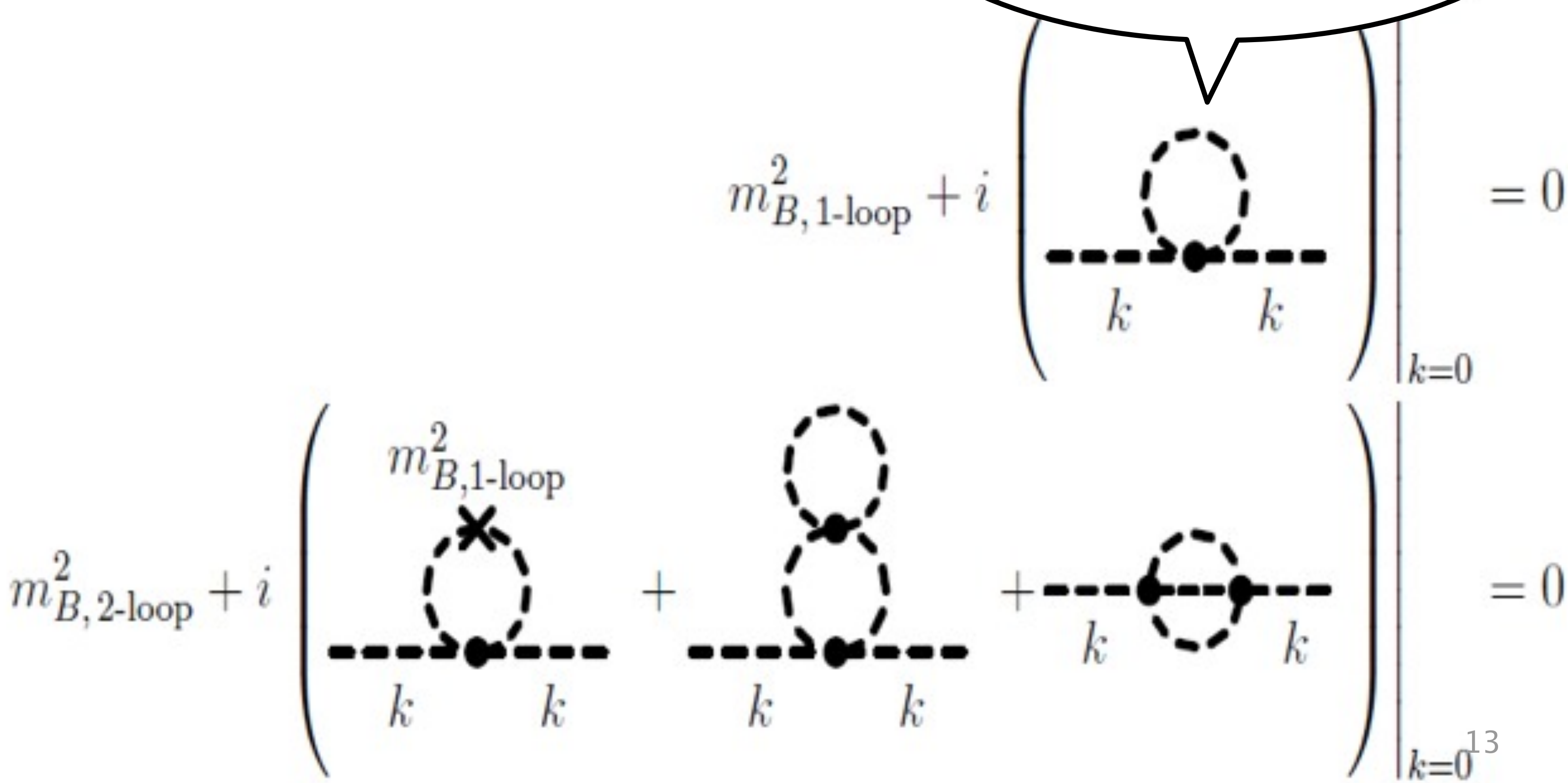
No IR divergences

$$m^2_{B,\,0\text{-loop}} = 0$$



No IR divergences

$$I_1 := \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} \propto \Lambda^2$$

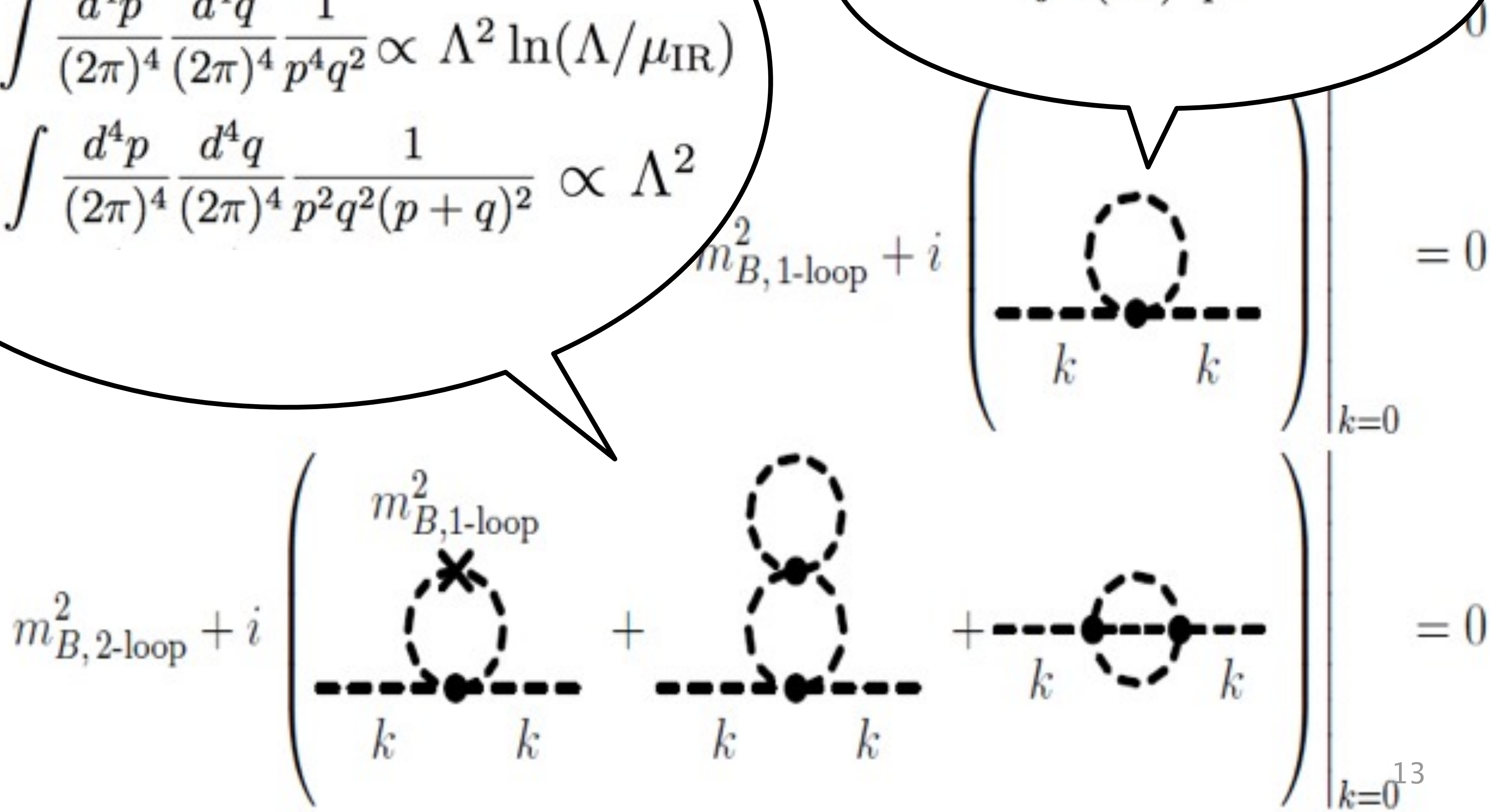


No IR divergences

$$J_2 := \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^4q^2} \propto \Lambda^2 \ln(\Lambda/\mu_{\mathrm{IR}})$$

$$I_2 := \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^2q^2(p+q)^2} \propto \Lambda^2$$

$$I_1 := \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} \propto \Lambda^2$$



Bare Higgs mass result for ϕ^4 theory

- From these conditions, we get

$$m^2_{B,\,\text{1-loop}} = - \frac{\lambda_B}{2} I_1$$

$$m^2_{B,\,\text{2-loop}} = - \frac{5}{72} \lambda_B^2 I_2$$

$$I_1 := \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \propto \Lambda^2$$

$$I_2 := \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{1}{p^2 q^2 (p+q)^2} \propto \Lambda^2$$

SM calculation

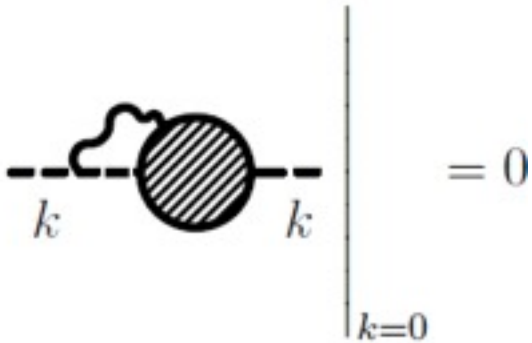
For SM Higgs sector

$$\mathcal{L} = (D_\mu \phi_B)^\dagger (D^\mu \phi_B) - m_B^2 \phi_B^\dagger \phi_B - \lambda_B (\phi_B^\dagger \phi_B)^2$$

Landau gauge and symmetric phase are good

- In **Landau gauge**, gauge field propagator is

$$-\frac{i}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

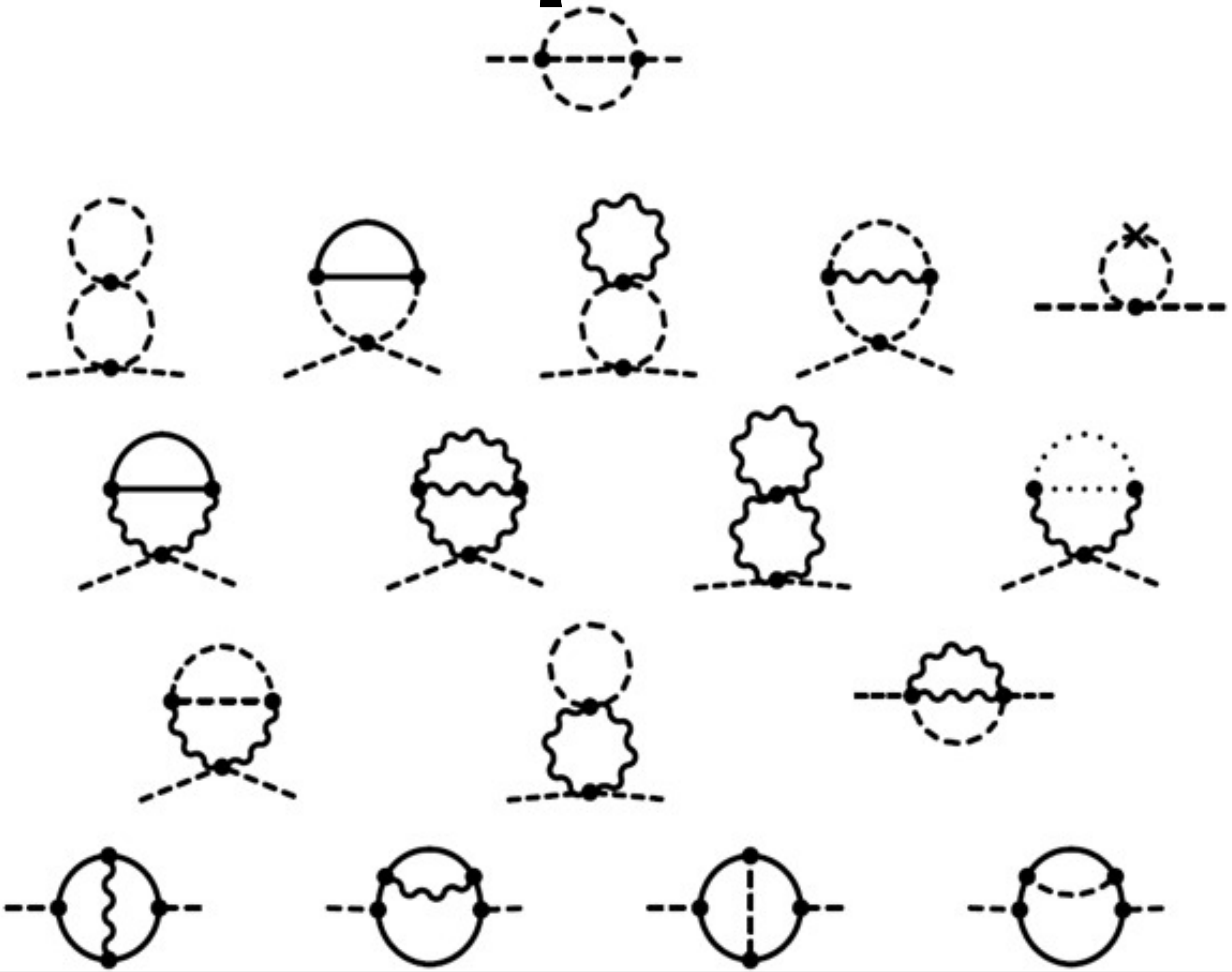


- We work in **symmetric phase** $\langle\phi\rangle = 0$ as we are interested only in **quadratic divergent** terms.

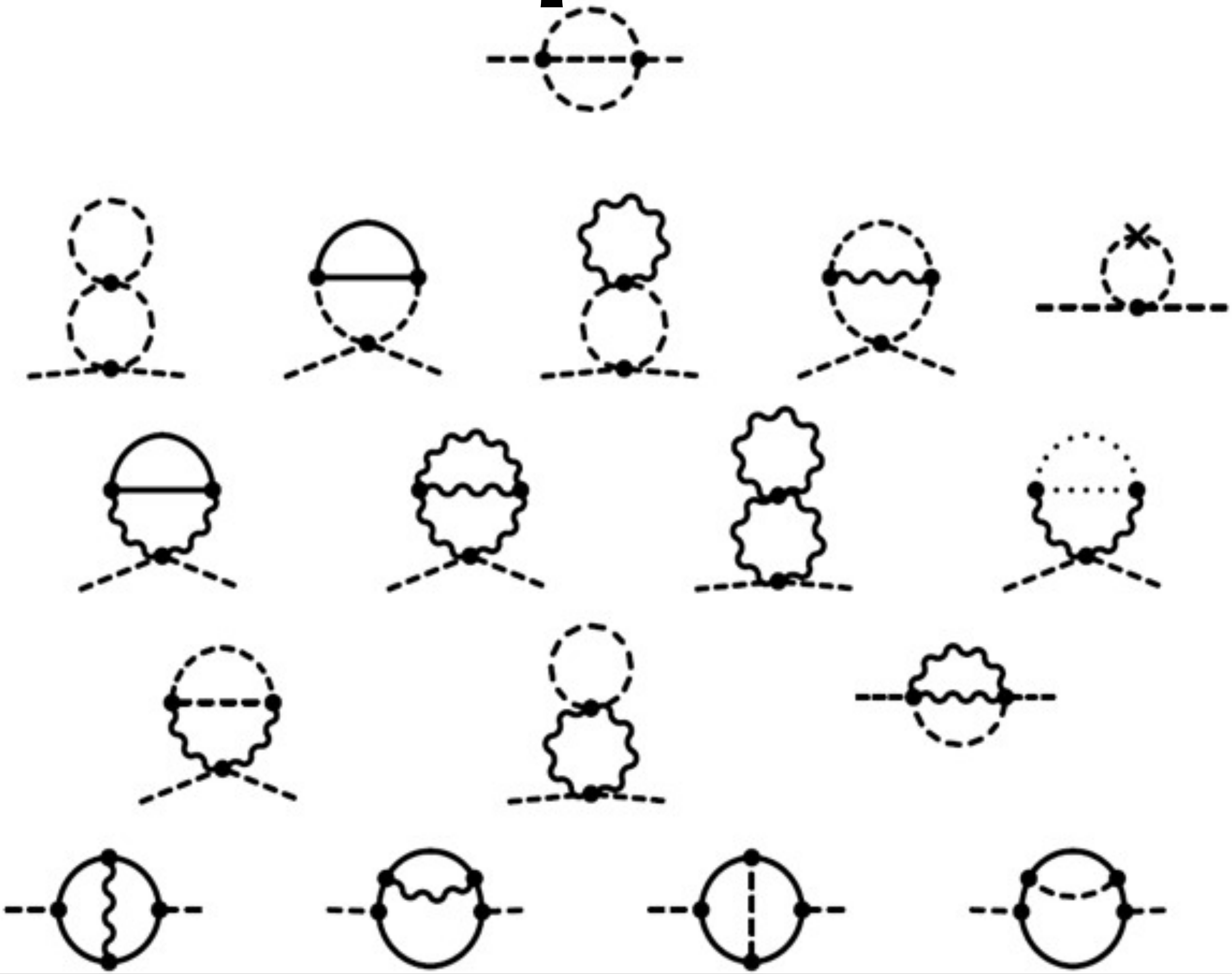
SM 1-loop

$$m^2_{B,\,\text{1-loop}} = -\left(6\lambda_B + \frac{3}{4}g^2_{YB} + \frac{9}{4}g^2_{2B} - 6y^2_{tB}\right) I_1$$

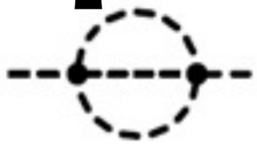
SM 2-loop calculation



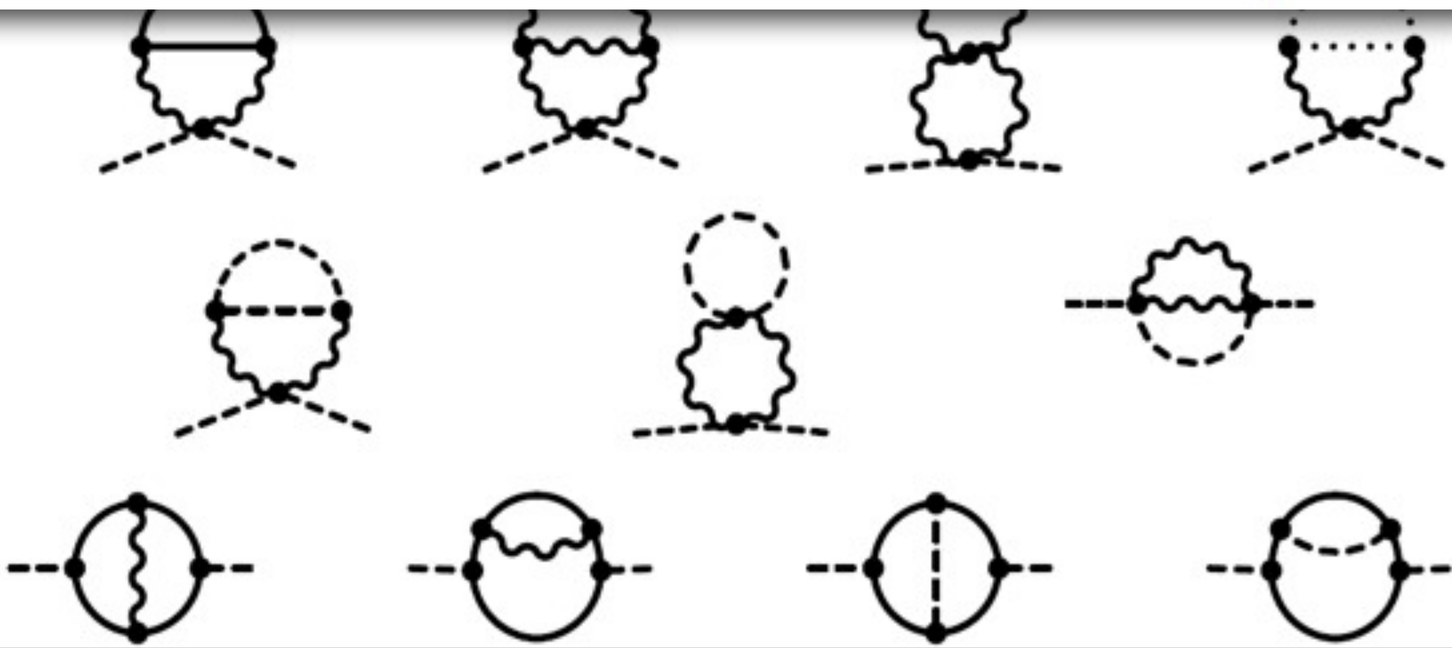
SM 2-loop calculation



SM 2-loop calculation



$$m^2_{B,\,\text{2-loop}} = -\left\{9y_{tB}^4 + y_{tB}^2\left(-\frac{7}{12}g_{YB}^2 + \frac{9}{4}g_{2B}^2 - 16g_{3B}^2\right) + \frac{77}{16}g_{YB}^4 + \frac{243}{16}g_{2B}^4 + \lambda_B\left(-18y_{tB}^2 + 3g_{YB}^2 + 9g_{2B}^2\right) - 10\lambda_B^2\right\} I_2.$$



Relation of 1- and 2-loops

- We need to relate **quadratic divergent integrals** I_1 and I_2.
- We employ following regularization

$$\int d^4k \frac{1}{k^2} = \int_\varepsilon^\infty d\alpha \int d^4k\, e^{-\alpha k^2}$$

to get: $I_1 = \frac{1}{\varepsilon}\frac{1}{16\pi^2}$ $\quad I_2 = \frac{1}{\varepsilon}\frac{1}{(16\pi^2)^2}\ln\frac{2^6}{3^3} \simeq 0.005\, I_1$

- Employing **naive momentum cutoff** by Λ, we get $I_1 = \frac{\Lambda^2}{16\pi^2}$ $\quad 1/\varepsilon = \Lambda^2$

Regularization dependence

$$I_2 = \frac{1}{\varepsilon}\frac{1}{(16\pi^2)^2}\ln\frac{2^6}{3^3} \simeq 0.005\, I_1$$

$$m^2_{B,\text{2-loop}} = -\Big\{9y^4_{tB} + y^2_{tB}\Big(-\frac{7}{12}g^2_{YB} + \frac{9}{4}g^2_{2B} - 16g^2_{3B}\Big) + \frac{77}{16}g^4_{YB} + \frac{243}{16}g^4_{2B} + \lambda_B\left(-18y^2_{tB} + 3g^2_{YB} + 9g^2_{2B}\right) - 10\lambda^2_B\Big\} I_2.$$

- Relation of I_1 and I_2 is regularization dependent.
- If 0.005×(couplings in front of I_2) is large, result suffer from regularization dependence.
- Our two loop computation helps to check it.

Plan

1. Now we can evaluate bare mass
2. Quartic coupling can take zero at Planck scale
3. Bare Higgs mass can take zero at Planck scale

Quartic coupling can take zero at Planck scale

"Quartic coupling vanishes at Planck scale if m_t=171GeV"

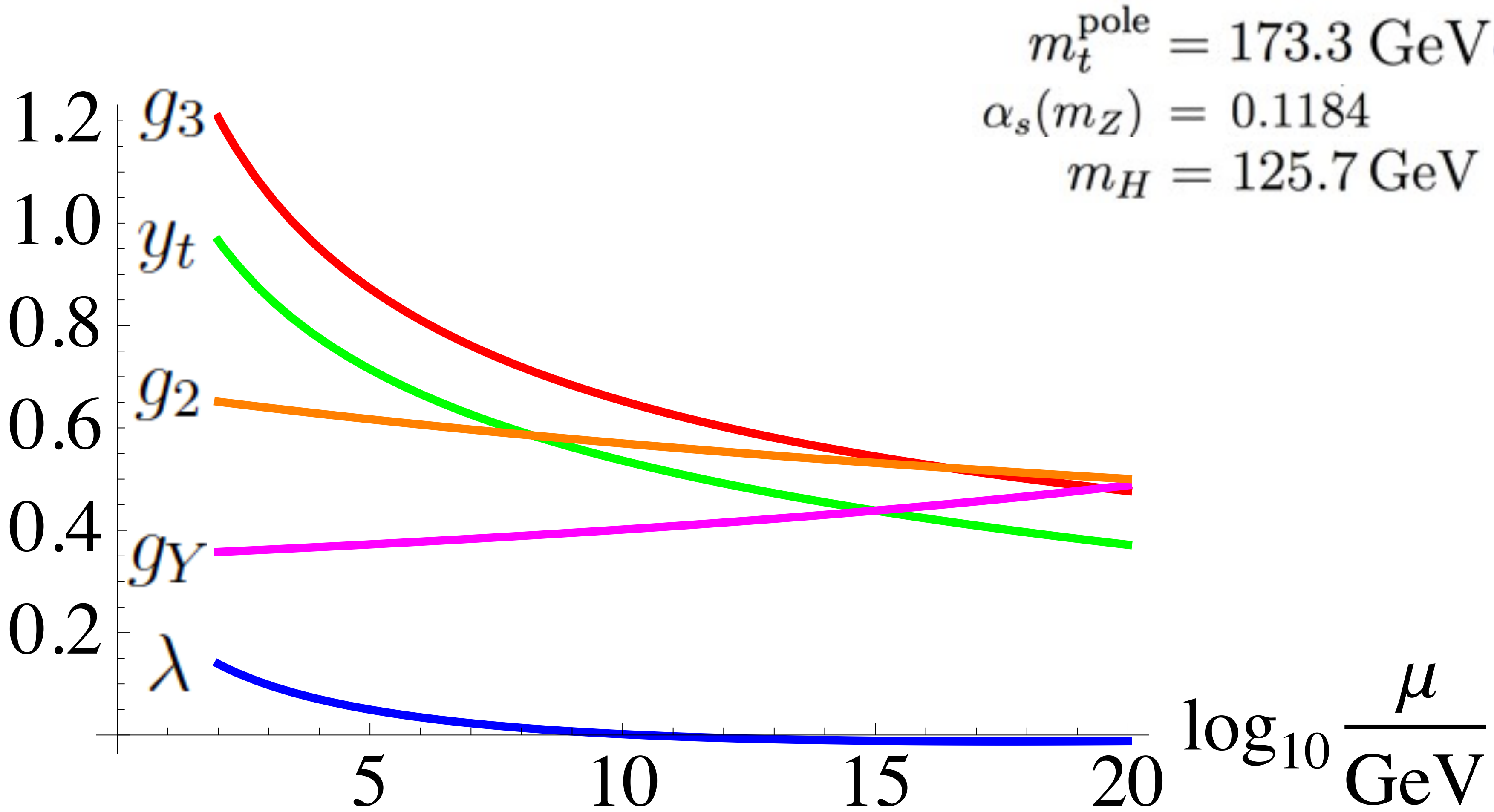
Approximating bare parameters by MS-bar

- In bare mass formula, there are dimensionless bare parameters
- We approximate **dimensionless bare parameters** by MS-bar ones at UV cutoff scale Λ.
- We apply two-loop RGE to get MS-bar couplings.

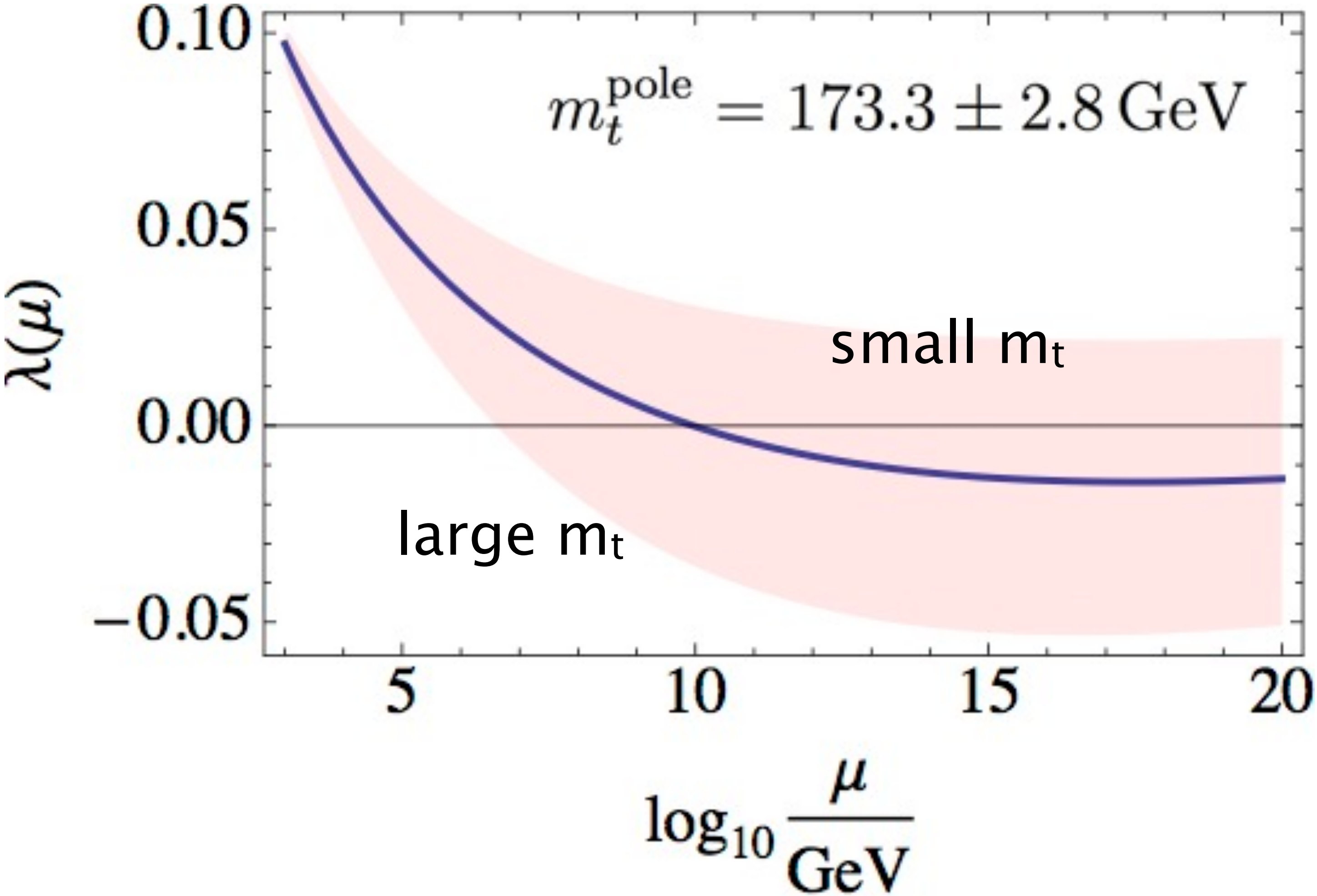
$$\lambda_B^i \simeq \lambda_{\overline{\rm MS}}^i(\mu = \Lambda)$$

$$\{\lambda^i\}_{i=1,\ldots,5} = \{g_Y^2, g_2^2, g_3^2, y_t^2, \lambda\}$$

SM running couplings

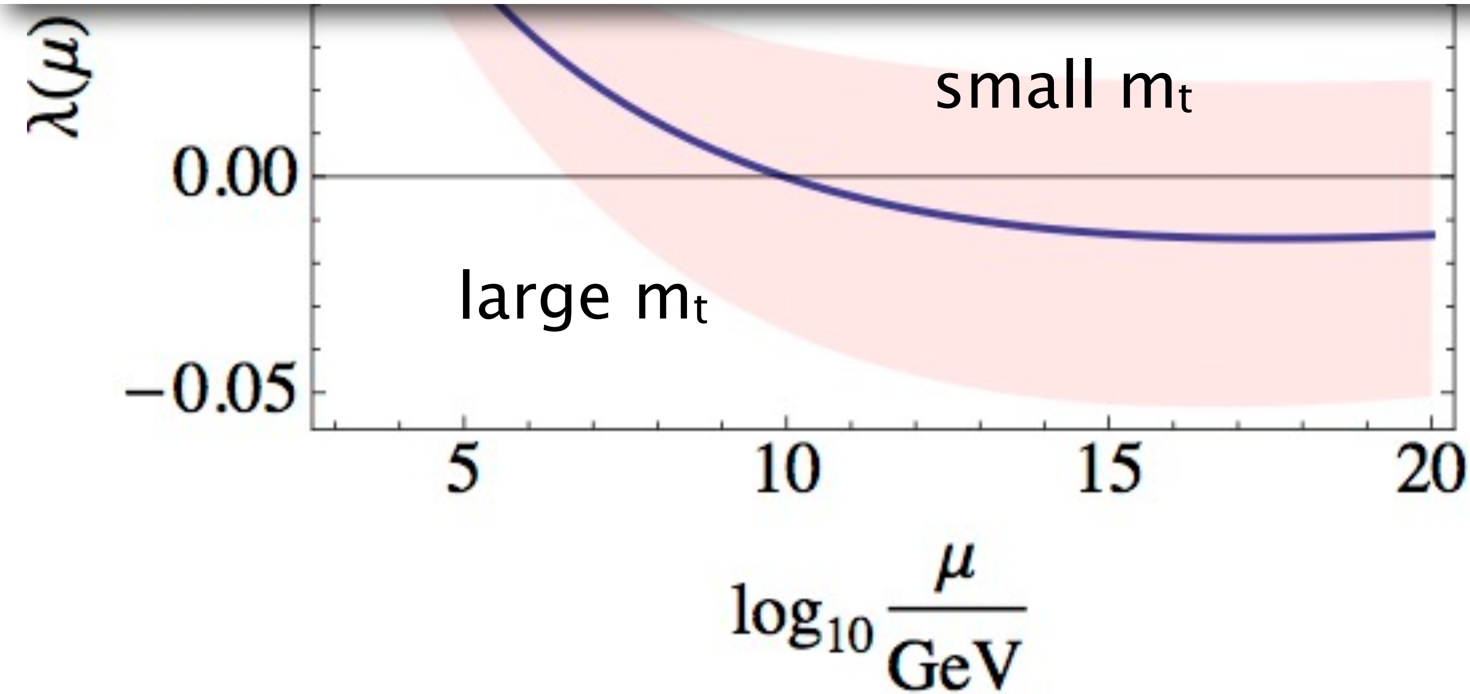


$\lambda \simeq$ 0 at high energy

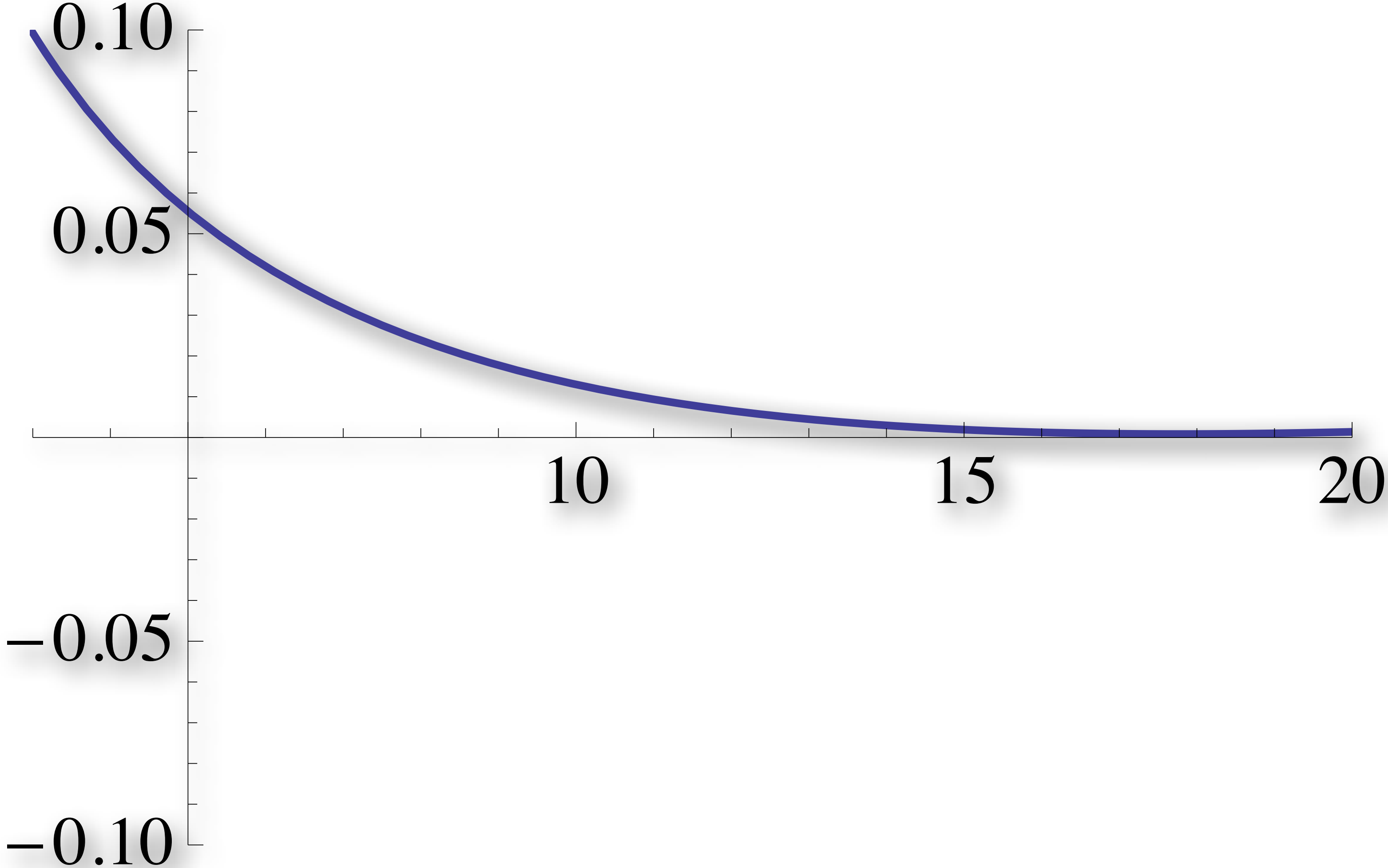


$\lambda \simeq$ 0 at high energy

$$\lambda(M_{\rm Pl}) = -0.014 - 0.018\left(\frac{m_t^{\rm pole} - 173.3\,{\rm GeV}}{2.8\,{\rm GeV}}\right) + 0.002\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) + 0.002\left(\frac{m_H - 125.7\,{\rm GeV}}{0.6\,{\rm GeV}}\right) \pm 0.004_{\rm th}.$$



Quartic coupling vanishes at M_P for $m_t = 171\text{GeV}$



Plan

1. Now we can evaluate bare mass
2. Quartic coupling can take zero at Planck scale
3. Bare Higgs mass can take zero at Planck scale

Bare Higgs mass can take zero at Planck scale

"Bare Higgs mass vanishes at Planck scale cutoff if m_t=170GeV."

Bare mass as function of cutoff

- Now we can evaluate bare mass in units of I_1 as function of cutoff Λ

$$\frac{m_B^2}{\Lambda^2/16\pi^2} = \frac{m^2_{B,\,\text{1-loop}}}{I_1} + \frac{m^2_{B,\,\text{2-loop}}}{I_2}\frac{I_2}{I_1}$$

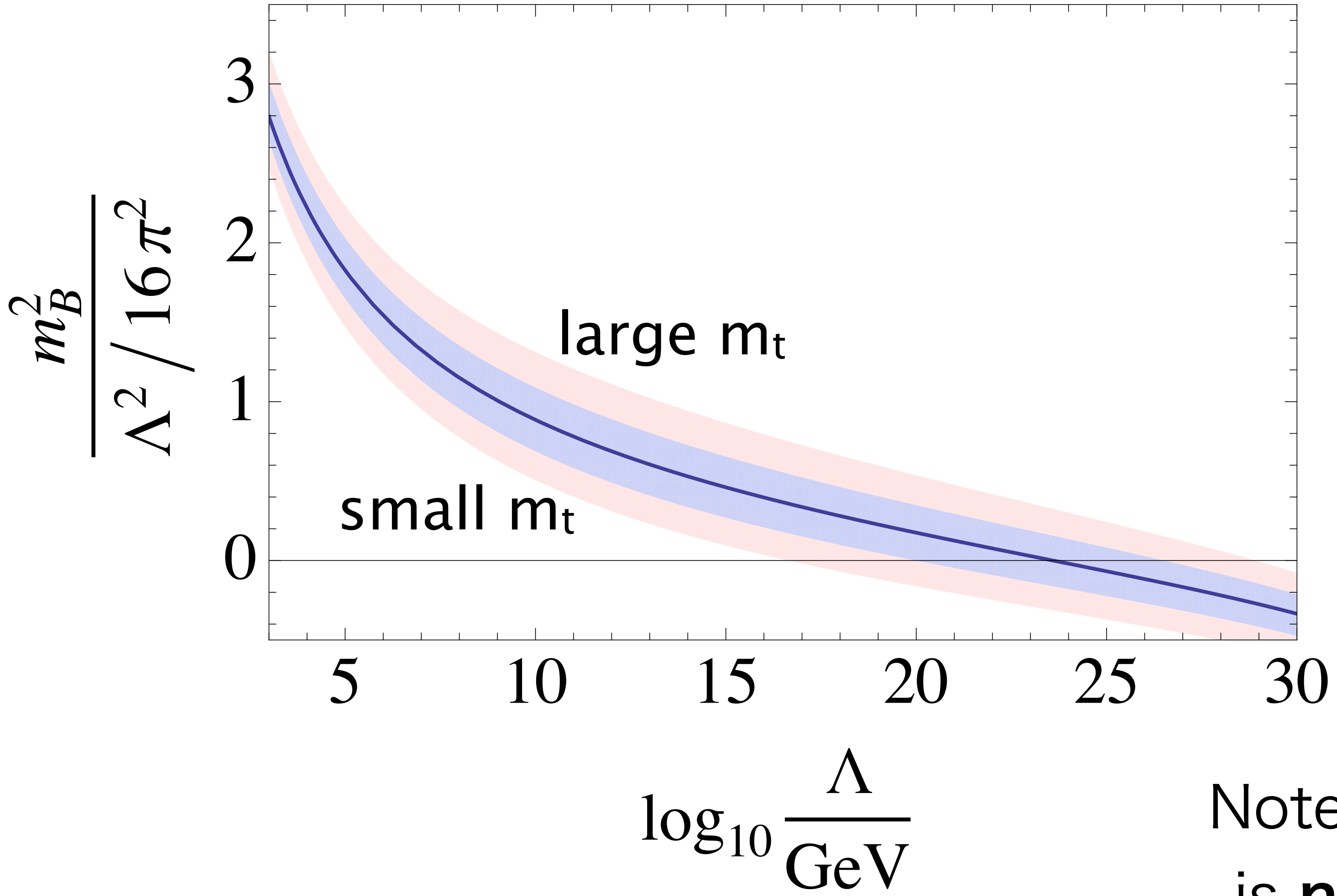
$$m^2_{B,\,\text{1-loop}} = -\left(6\lambda_B + \frac{3}{4}g^2_{YB} + \frac{9}{4}g^2_{2B} - 6y^2_{tB}\right) I_1$$

$$m^2_{B,\,\text{2-loop}} = -\left\{9y^4_{tB} + y^2_{tB}\left(-\frac{7}{12}g^2_{YB} + \frac{9}{4}g^2_{2B} - 16g^2_{3B}\right) + \frac{77}{16}g^4_{YB} + \frac{243}{16}g^4_{2B} + \lambda_B\left(-18y^2_{tB} + 3g^2_{YB} + 9g^2_{2B}\right) - 10\lambda^2_B\right\} I_2.$$

$$\lambda^i_B \simeq \lambda^i_{\overline{\text{MS}}}(\mu = \Lambda)$$

m_B^2 vanishes for $\Lambda = 10^{17} \sim 10^{28}$ GeV

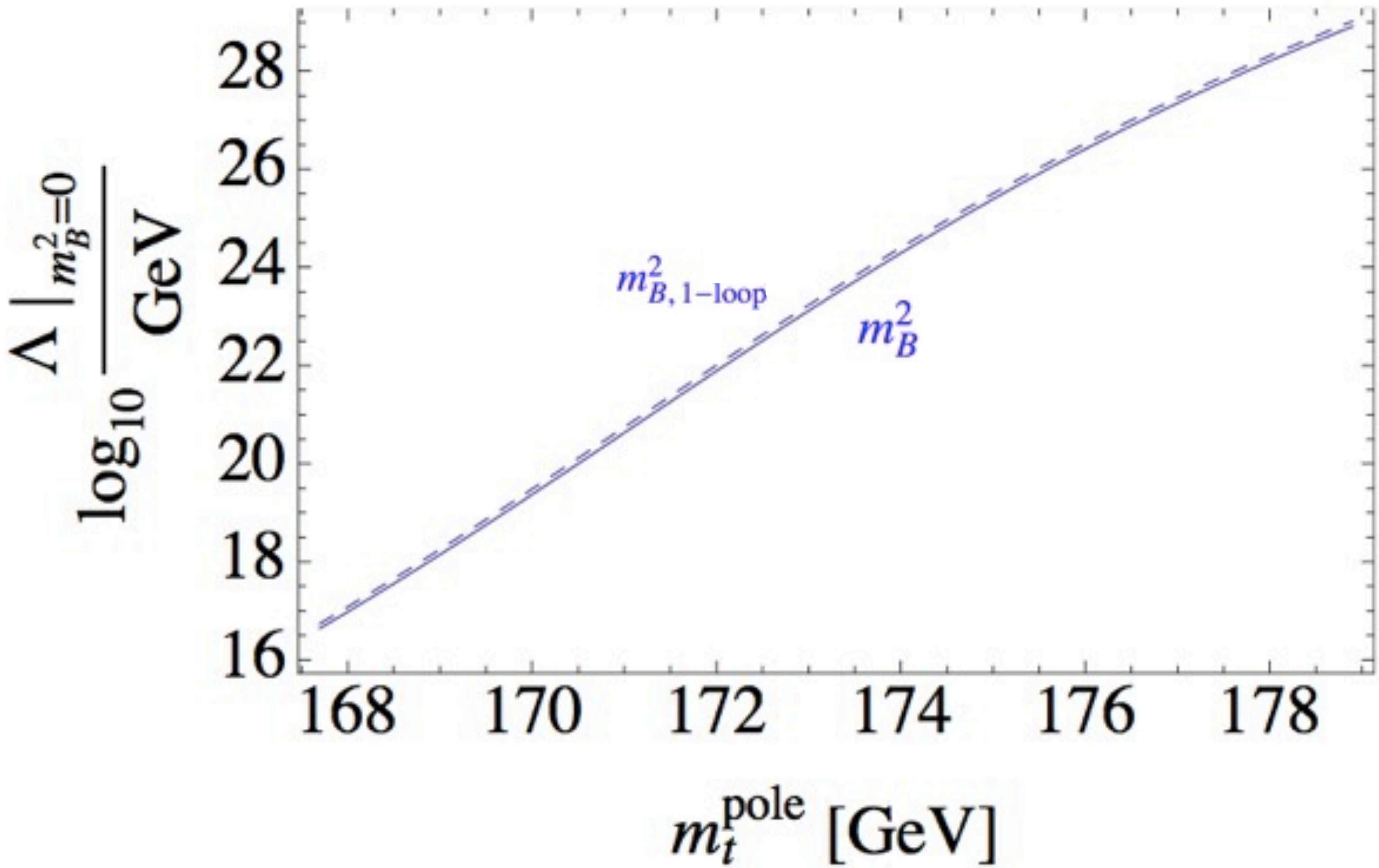
$m_t^{\text{pole}} = 173.3 \pm 2.8\,\text{GeV}$ Alekhin, Djouadi, Moch



Note: Bare mass is **not** running.

Top mass dependece

$m_t^{\rm pole} = 173.3 \pm 2.8\,{\rm GeV}$ Alekhin, Djouadi, Moch



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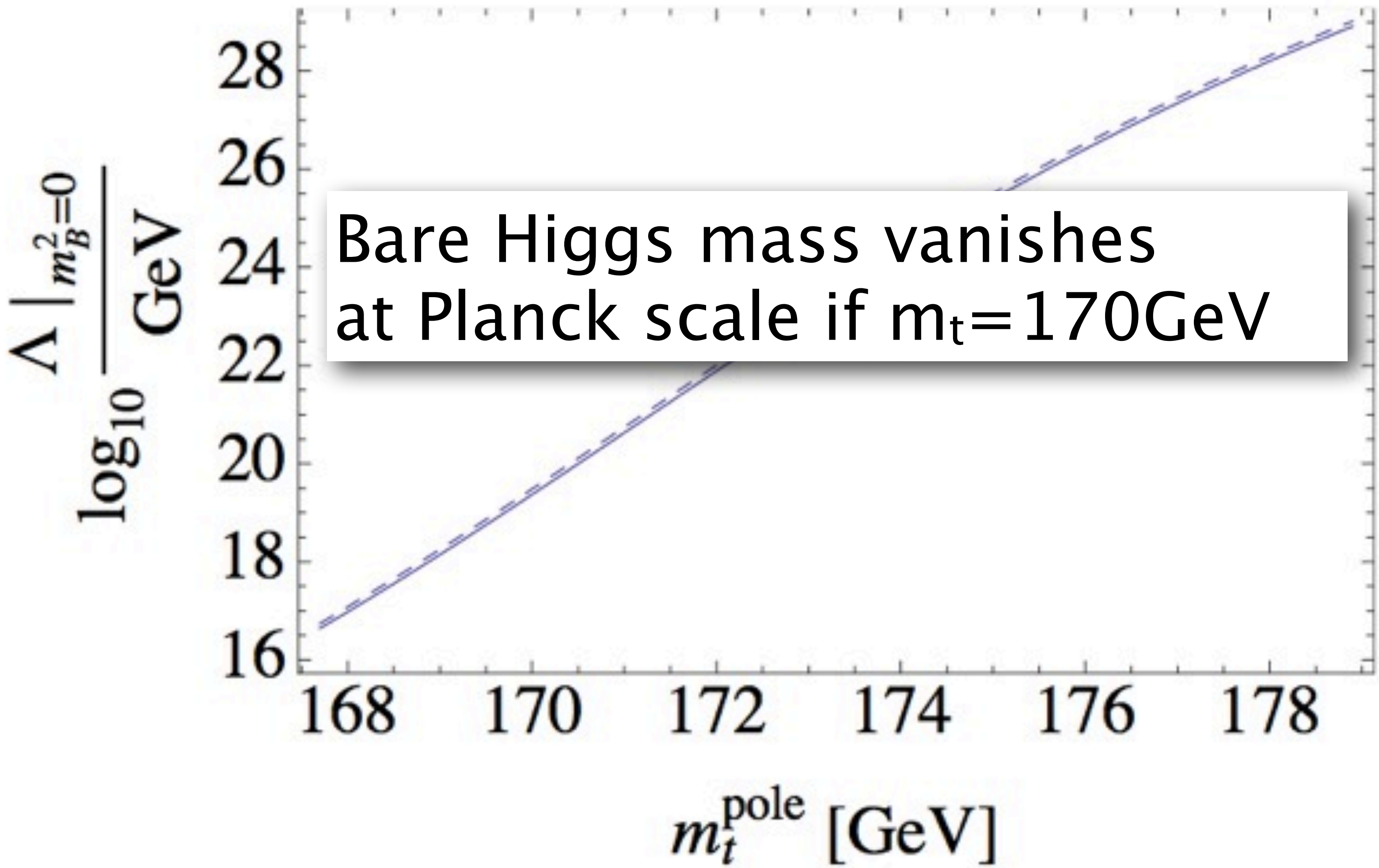


Regularization dependence is small

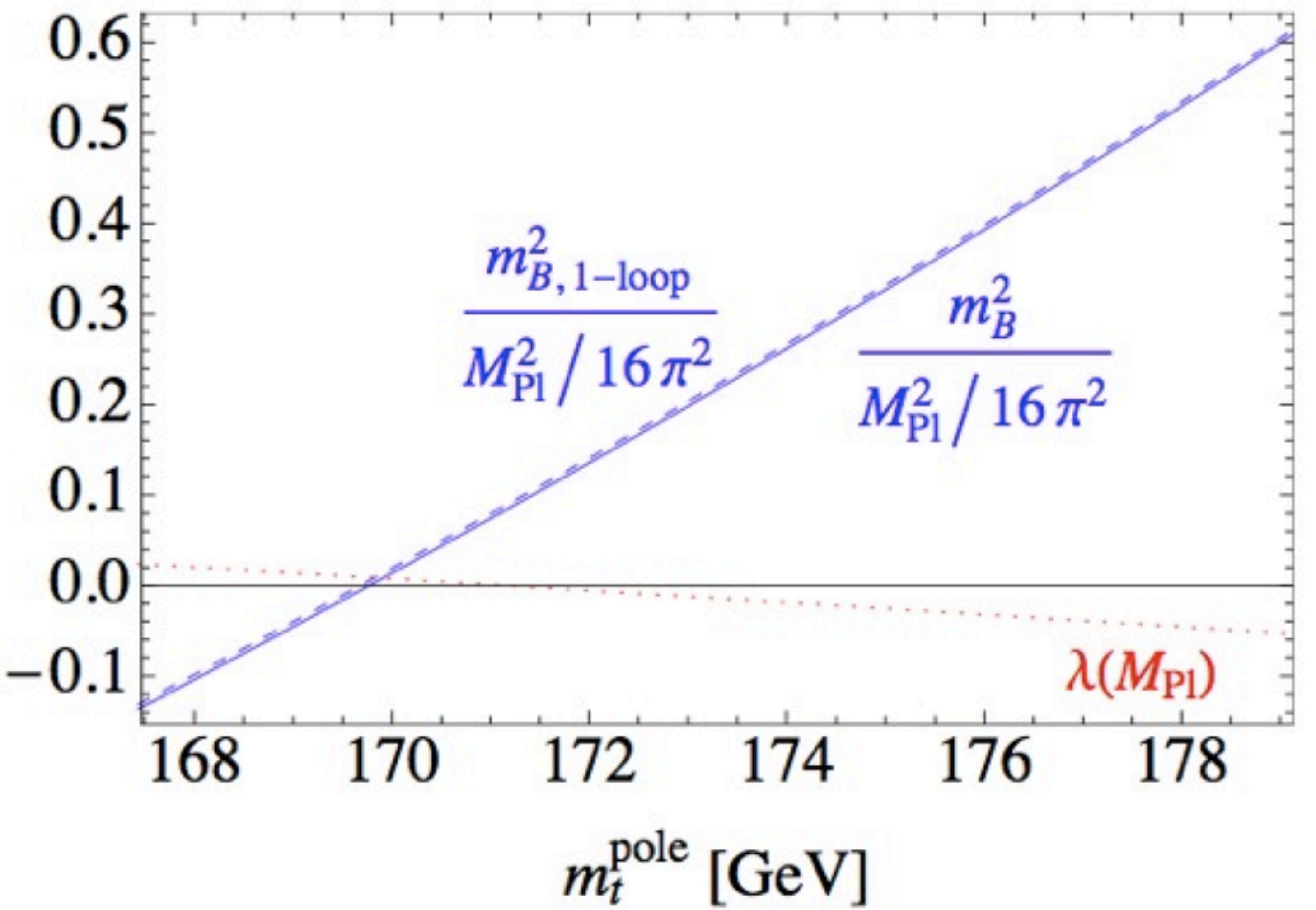
$$m_B^2 = \Big[0.22 + 0.18\left(\frac{m_t^{\rm pole} - 173.3\,{\rm GeV}}{2.8\,{\rm GeV}}\right) - 0.02\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right)$$
$$- 0.01\left(\frac{m_H - 125.7\,{\rm GeV}}{0.6\,{\rm GeV}}\right) \pm 0.02_{\rm th}\Big]\frac{M_{\rm Pl}^2}{16\pi^2}.$$

$$m^2_{B,\,\text{2-loop}} \simeq -0.005\, M_{\rm Pl}^2/16\pi^2$$

- As advertised, we can see that two loop correction can be safely neglected.

Both m_B^2 and λ_B almost vanish

($\Lambda = M_{Pl}$)



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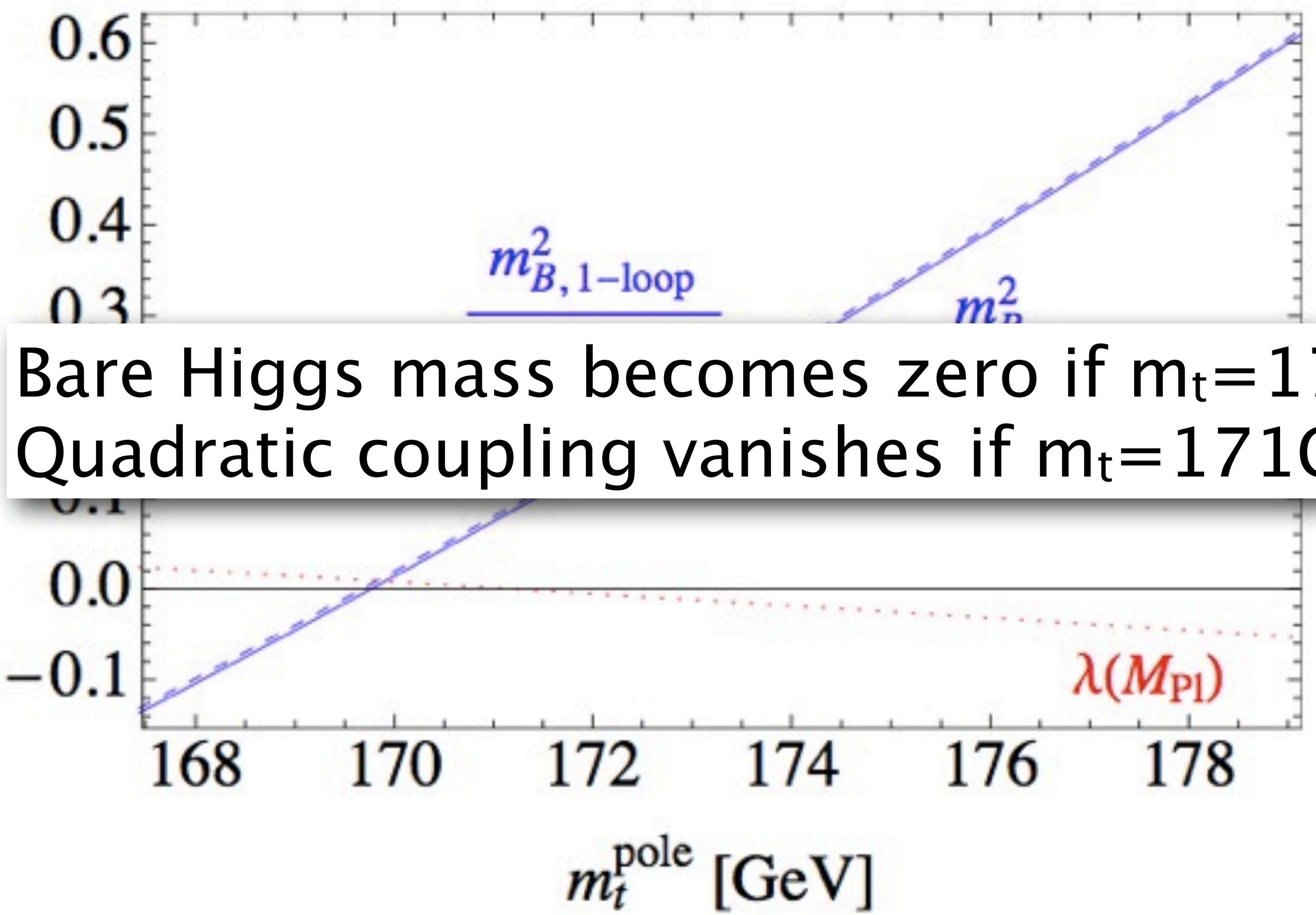
Both m_B^2 and λ_B almost vanish

($\Lambda = M_{Pl}$)



Bare Higgs mass becomes zero if m_t=170GeV.
Quadratic coupling vanishes if m_t=171GeV.

Discussion

Vanishing bare mass?

- fine tuning problem

$$m_B^2 + \delta m^2 = m_H^2$$

Quadratic divergence is canceled.

- One possibility:
  - Both are fine tuned: $m_B{}^2=0$ and $\delta m^2=0$.
  - For this to be true, fine tuning may be achieved in framework beyond ordinary QFT(?)

Or, nonzero bare mass as string threshold correction?

- Interpretation for m_B^2 at Planck scale cutoff as string threshold correction
- Integrating out string massive modes,

$$m_B^2 \sim C \frac{g_s^2}{16\pi^2} m_s^2$$

$$m_s := (\alpha')^{-1/2}$$

C : a model dependent constant

Neutrino mass

- If we assume see–saw mechanism,
- Our analysis corresponding to the case where M_R is small: $m_\nu \sim y_D^2 v^2 / M_R \sim 0.1\,\mathrm{eV}$ $y_D \lesssim 10^{-2}$

$$\downarrow$$

$$M_R \lesssim 10^{10}\,\mathrm{GeV}$$

- The case where M_R is large is also interesting.

Supersymmetry

- When supersymmetry is softly broken,
  - There are no quadratic divergence,
  - Our study cannot apply.

- In the case of split supersymmetry,
  - It is possible to perform a parallel analysis. (work in progress)

Works in progress

- Small bare mass as string threshold corrections?
  - ★ Integrating out string massive modes,

$$m_B^2 \sim C \frac{g_s^2}{16\pi^2} m_s^2$$

C : computable constant

$$m_s := (\alpha')^{-1/2}$$

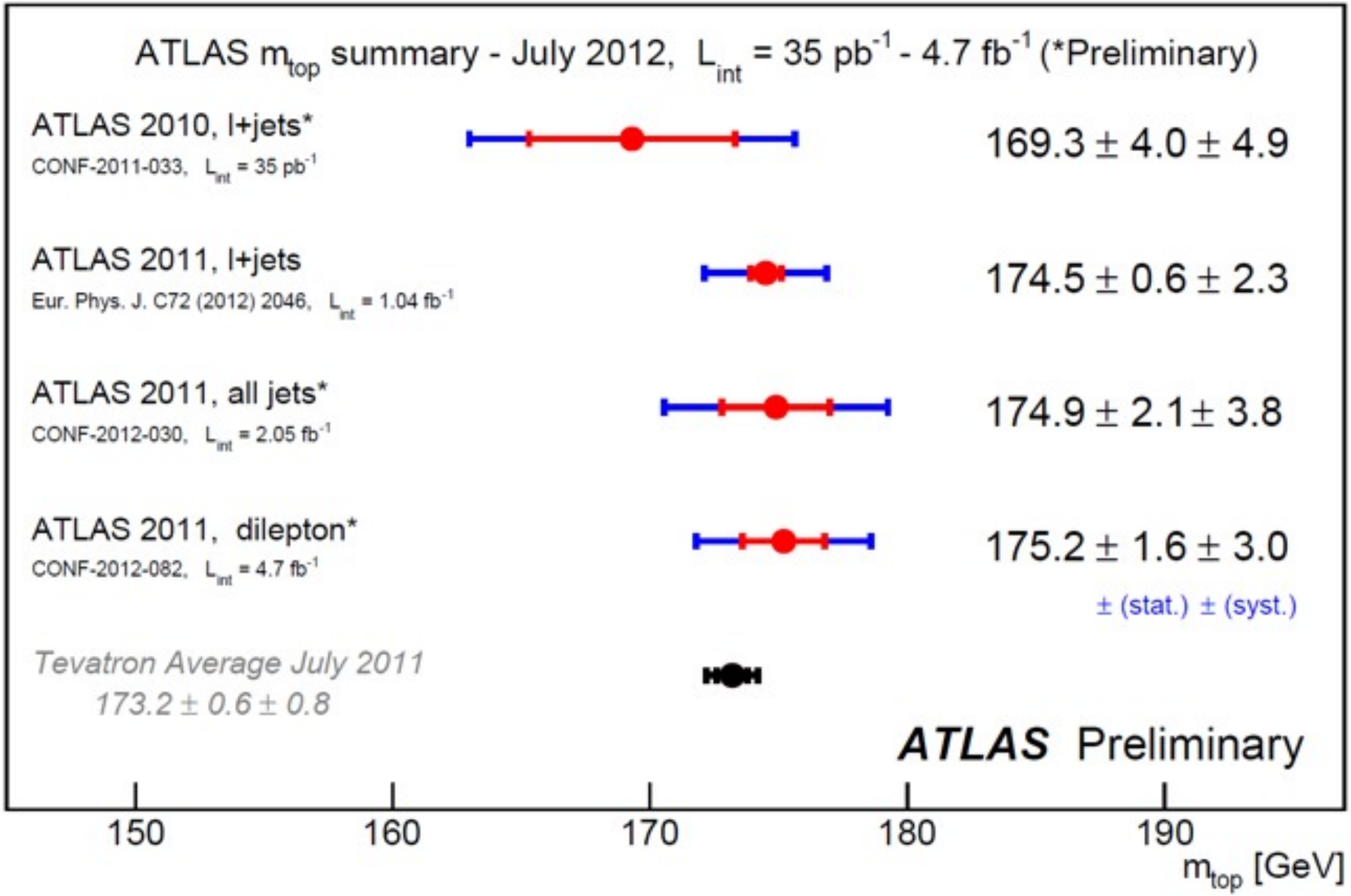
- Neutrino mass?
  - ★ Assuming seesaw and $M_R > 10^{10}$GeV, neutrino Yukawa's contribute too.
- Split SUSY?
  - ★ Similar analysis apply.
- **<u>A lot to do. Join!!</u>**

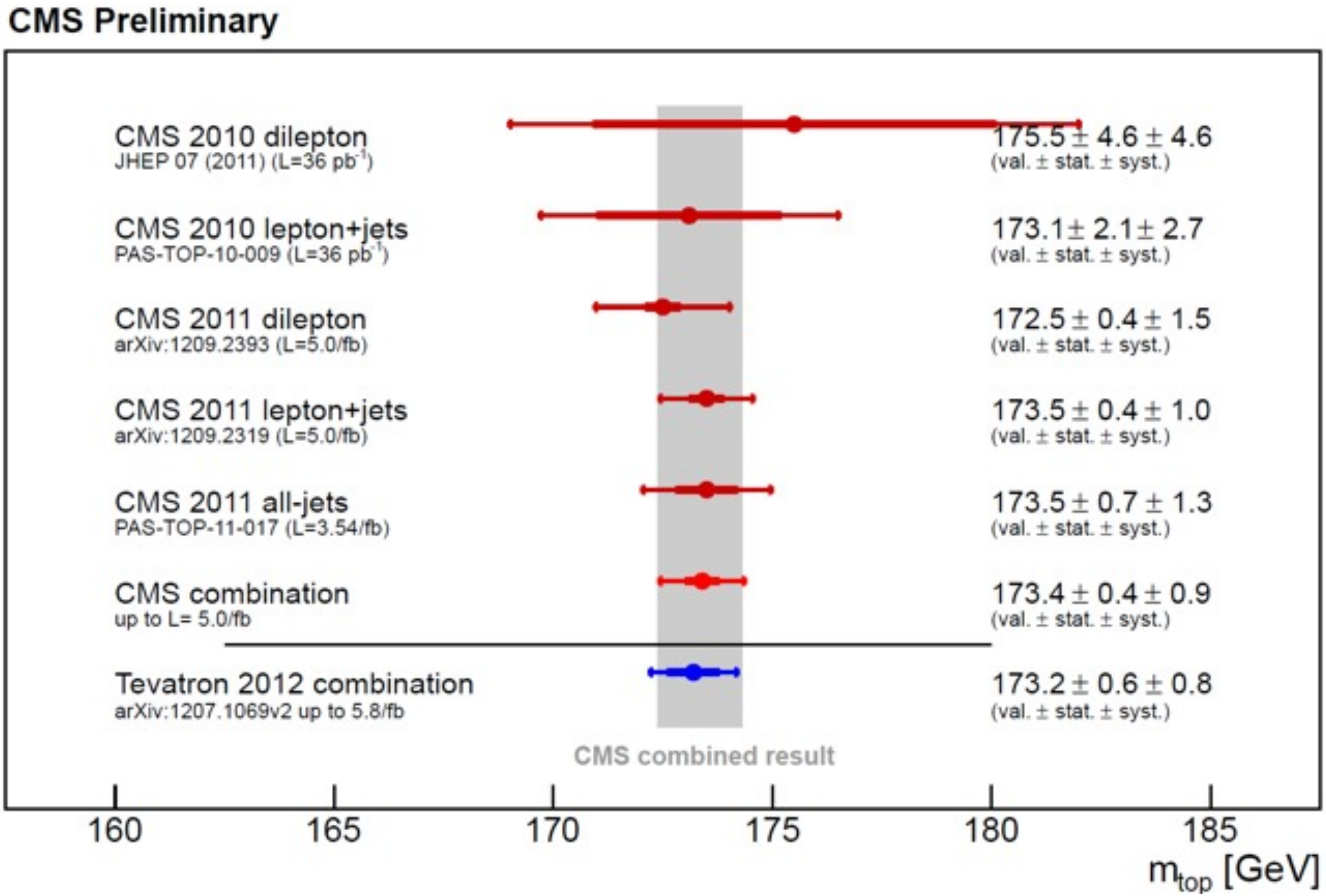
Summary

- We can discuss bare Lagrangian at Planck scale.
- We compute quadratic divergence in bare Higgs mass up to 2-loop orders.
  - We find 2-loop contribution is small.
  - Negligible regularization dependence.
- At Planck scale,
  - Bare Higgs mass vanishes for m_t=170GeV.
  - Quartic coupling vanishes for m_t=171GeV.


Thank you!!

Backup slides


ATLAS m_top summary - July 2012, L_int = 35 pb^-1 - 4.7 fb^-1 (*Preliminary)
ATLAS 2010, l+jets*
CONF-2011-033, L_int = 35 pb^-1
169.3 ± 4.0 ± 4.9
ATLAS 2011, l+jets
Eur. Phys. J. C72 (2012) 2046, L_int = 1.04 fb^-1
174.5 ± 0.6 ± 2.3
ATLAS 2011, all jets*
CONF-2012-030, L_int = 2.05 fb^-1
174.9 ± 2.1 ± 3.8
ATLAS 2011, dilepton*
CONF-2012-082, L_int = 4.7 fb^-1
175.2 ± 1.6 ± 3.0
± (stat.) ± (syst.)
Tevatron Average July 2011
173.2 ± 0.6 ± 0.8
ATLAS Preliminary
150
160
170
180
190
m_top [GeV]


CMS Preliminary
CMS 2010 dilepton
JHEP 07 (2011) (L=36 pb-1)
175.5 ± 4.6 ± 4.6
(val. ± stat. ± syst.)
CMS 2010 lepton+jets
PAS-TOP-10-009 (L=36 pb-1)
173.1 ± 2.1 ± 2.7
(val. ± stat. ± syst.)
CMS 2011 dilepton
arXiv:1209.2393 (L=5.0/fb)
172.5 ± 0.4 ± 1.5
(val. ± stat. ± syst.)
CMS 2011 lepton+jets
arXiv:1209.2319 (L=5.0/fb)
173.5 ± 0.4 ± 1.0
(val. ± stat. ± syst.)
CMS 2011 all-jets
PAS-TOP-11-017 (L=3.54/fb)
173.5 ± 0.7 ± 1.3
(val. ± stat. ± syst.)
CMS combination
up to L= 5.0/fb
173.4 ± 0.4 ± 0.9
(val. ± stat. ± syst.)
Tevatron 2012 combination
arXiv:1207.1069v2 up to 5.8/fb
173.2 ± 0.6 ± 0.8
(val. ± stat. ± syst.)
CMS combined result
160
165
170
175
180
185
m_top [GeV]

Note: It's not running mass!

- $m^2_{phys} = m^2_{bare}$ + (radiative corrections).
- In mass independent renormalization (dim reg):
  1. m^2_{bare} is tuned to cancel Λ^2 and to make $m^2_{phys} = 0$.
  2. A mass parameter is inserted as perturbation.
  3. Running mass obtained as **multiplicative** renormalization of this mass parameter.
- What we compute is **additive** renormalization constant, tuned before above prescription.

Cutoff vs $\overline{\mathbf{MS}}$

We have approximated the bare couplings by the running ones in the $\overline{\mathbf{MS}}$ scheme. The resulting error can be evaluated once the cutoff scheme is explicitly specified.

$$\lambda^i_{\overline{\mathrm{MS}}}(\mu) = \lambda^i_B + \sum_{jk} c^{ijk}(\mu/\Lambda)\ \lambda^j_B \lambda^k_B + O(\lambda^3_B),$$

$$c^{ijk}(x) := f^{ijk} + b^{ijk} \ln x + O(x^2),$$

This expression is valid for

$$\left| \frac{\lambda^i_{\overline{\mathrm{MS}}}}{16\pi^2} \ln(\mu/\Lambda) \right| \ll 1 \qquad \mu \ll \Lambda$$

Thus we have

$$\lambda^i_{\overline{\mathrm{MS}}}(\mu) = \lambda^i_B + \sum_{jk} \left(f^{ijk} + b^{ijk} \ln \frac{\mu}{\Lambda} \right) \lambda^j_B \lambda^k_B$$

On the other hand, from the RGE, we get

$$\lambda^i_{\overline{\mathrm{MS}}}(\Lambda) = \lambda^i_{\overline{\mathrm{MS}}}(\mu) + \sum_{jk} b^{ijk} \lambda^j_{\overline{\mathrm{MS}}}(\mu)\, \lambda^k_{\overline{\mathrm{MS}}}(\mu) \ln \frac{\Lambda}{\mu}$$

From these equations, we obtain

$$\lambda^i_{\overline{\mathrm{MS}}}(\Lambda) = \lambda^i_B + \sum_{jk} f^{ijk} \lambda^j_B \lambda^k_B$$

This gives the relation between the bare and the MS couplings at the same scale.

With the above correction, the formula for the bare Higgs mass is modified by

$$\Delta m_B^2 = -\sum_{ijk} a^i f^{ijk} \lambda^j_{\overline{\mathrm{MS}}}(\Lambda)\, \lambda^k_{\overline{\mathrm{MS}}}(\Lambda)$$

$$\Lambda\big|_{m^2=0} \Rightarrow \Lambda\big|_{m^2=0}\, e^{\delta t}$$

$$\delta t = \frac{\sum_{ijk} a^i f^{ijk} \lambda^j_{\overline{\rm MS}}(\Lambda)\, \lambda^k_{\overline{\rm MS}}(\Lambda)}{\sum_{ijk} a^i b^{ijk} \lambda^j_{\overline{\rm MS}}(\Lambda)\, \lambda^k_{\overline{\rm MS}}(\Lambda)}$$

The ambiguity for the vanishing scale would be at most $e^{\delta t} \lesssim 10$