Nonequilibrium Phase Transitions and Nonequilibrium Critical Point from AdS/CFT

Shin Nakamura

(Dept. of Phys. Kyoto Univ.)

S.N. arXiv:1204.1971 [hep-th]

The natural units will be used: $\hbar = c = k_B = 1$

Prologue and Introduction

Physics in the next generation

We should always keep the following question in our mind:

What is coming next in physics?

It is an important question, but can be difficult to have a right answer.

However, I had a chance to get a suggestion for it.

“Baryons’10” at Osaka Univ. 2010

Prof. Y. Nambu's slide @ Baryons'10

Nontrivial problems of SSB
(Jona-Lasinio, 2010)

SSB in non-equilibrium processes

Superfluid *He* flow : Bose condensation
of NG phonon modes

1-D phase transition not possible, but
1-D highway traffic model (Popkov et al 2008)

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Role of SSB in cosmology?

I was inspired very much by his talk, and I decided to study non-equilibrium phase transitions.

What I am going to talk

What I am going to talk today is a result of two-years struggling after Baryons'10 Conference.

The bottom line of the present talk:

A new non-equilibrium phase transition and
a new non-equilibrium critical point are discovered
by using the AdS/CFT correspondence.

The details will be given in the talk.

Non-equilibrium Steady States

Non-equilibrium physics

A challenge in modern physics

Two categories of non-equilibrium states:

- Time-dependent systems
- Time-independent systems

Good place to attack:

Systems that are out of equilibrium, but
(the macroscopic variables are) time independent.

Non-equilibrium steady states (NESS)

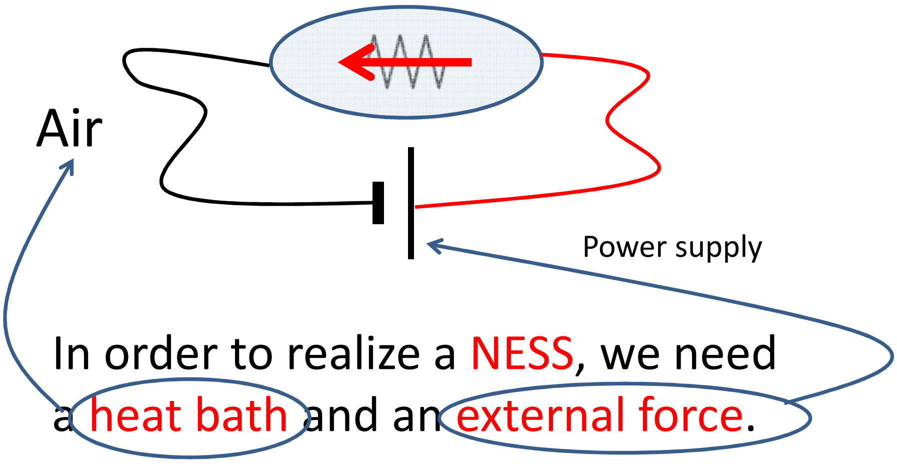
Non-equilibrium steady state (NESS)

Non-equilibrium, but time-independent.

A typical example:

A system with a constant current along the electric field.

- It is non-equilibrium, because heat and entropy are produced.
- The macroscopic variables can be time independent.



In order to realize a NESS, we need a heat bath and an external force.

Systems with constant current

We can again categorize them into two groups:

- Systems within the linear-response regime (Near equilibrium)

  Well-understood

  $J = \sigma E$ σ is a constant.

  The conductivity is given by the Kubo formula: $\sigma = \lim_{\omega \to 0} \mathrm{Im}\left[\frac{G_{\mathrm{R}}(\omega)}{\omega}\right]$

- Systems outside the linear-response regime (Far from equilibrium)

  Still a frontier

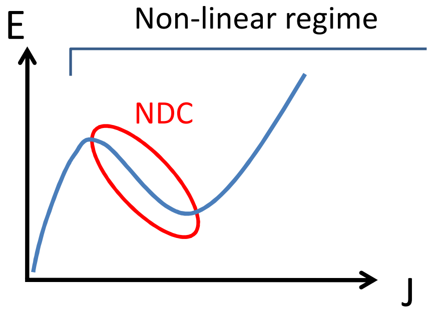
We should attack here!

Non-linear conductivity

A typical behavior of non-linear conductivity of strongly-correlated systems of electrons:

NDC: Negative Differential Conductivity（負性微分伝導度）

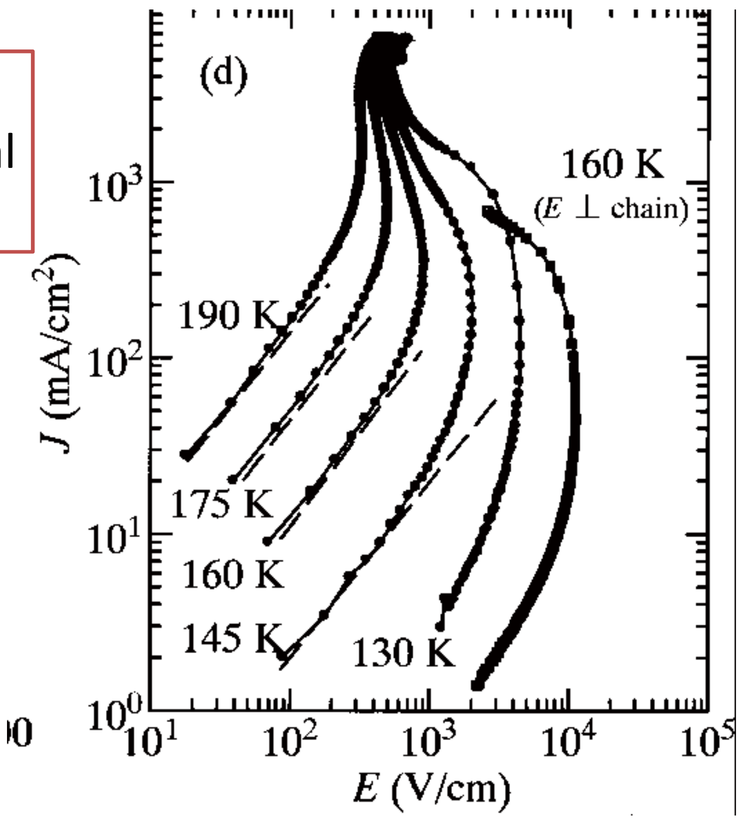
NDC: the voltage goes down when the current increases.



NDC is widely observed in strongly-correlated insulators.

Example of experimental data:

$SrCuO_2$
(1d Mott)



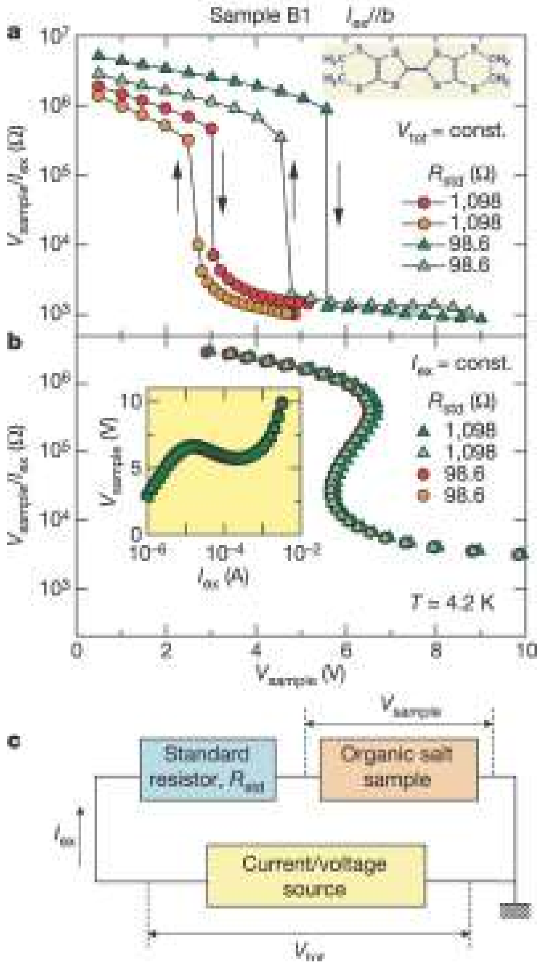
Y. Taguchi T. Matsumoto and Y. Tokura. *Phys. Rev. B, 62:7015, 2000.*

Example of experimental data:

θ-$(BEDT\text{-}TTF)_2CsCo(SCN)_4$ crystal at 4.2 K.

Charge order insulator

F. Sawano et. al., Nature 437 (2005) 522.



The system we consider

- Non-equilibrium steady state with a constant flow of current.
- The non-linear region going beyond the linear response theory.
- Strongly-correlated system where NDC typically shows up. (Non-perturbative)

The three "nons" can be overcome, at least in some cases, by using AdS/CFT.

Non-equilibrium Steady States
and
AdS/CFT

How can we employ AdS/CFT?

The AdS/CFT correspondence is a correspondence between a gauge theory and a gravity theory.

(The details will be given later.)

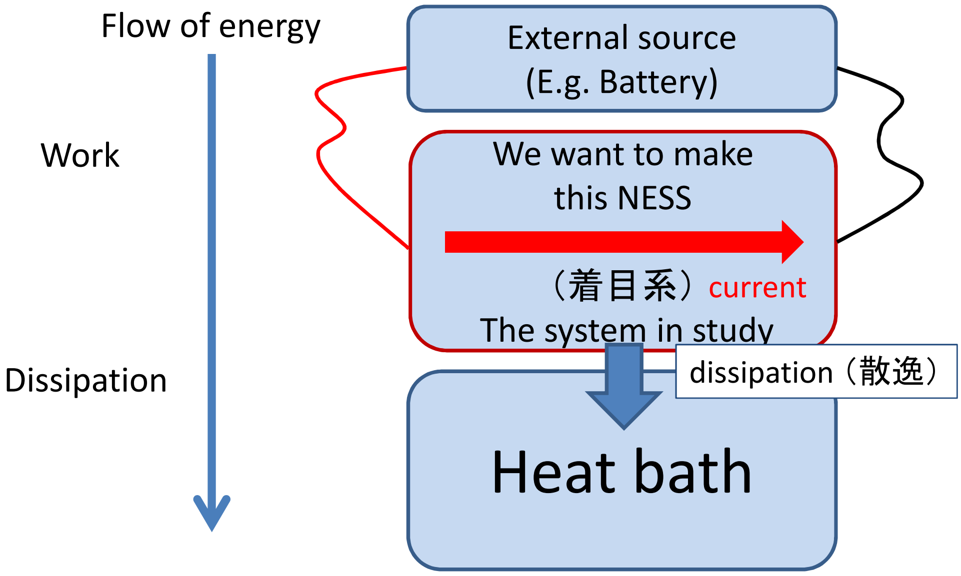
A typical (and the most standard) example of the gauge-theory side is N=4 supersymmetric Yang-Mills (SYM) theory at large-Nc.

- This is quite "different" from what we have in our real world. Does it make sense to employ AdS/CFT?
- If yes, how can we prepare NESS?

Setup for NESS

External force and heat bath are necessary.

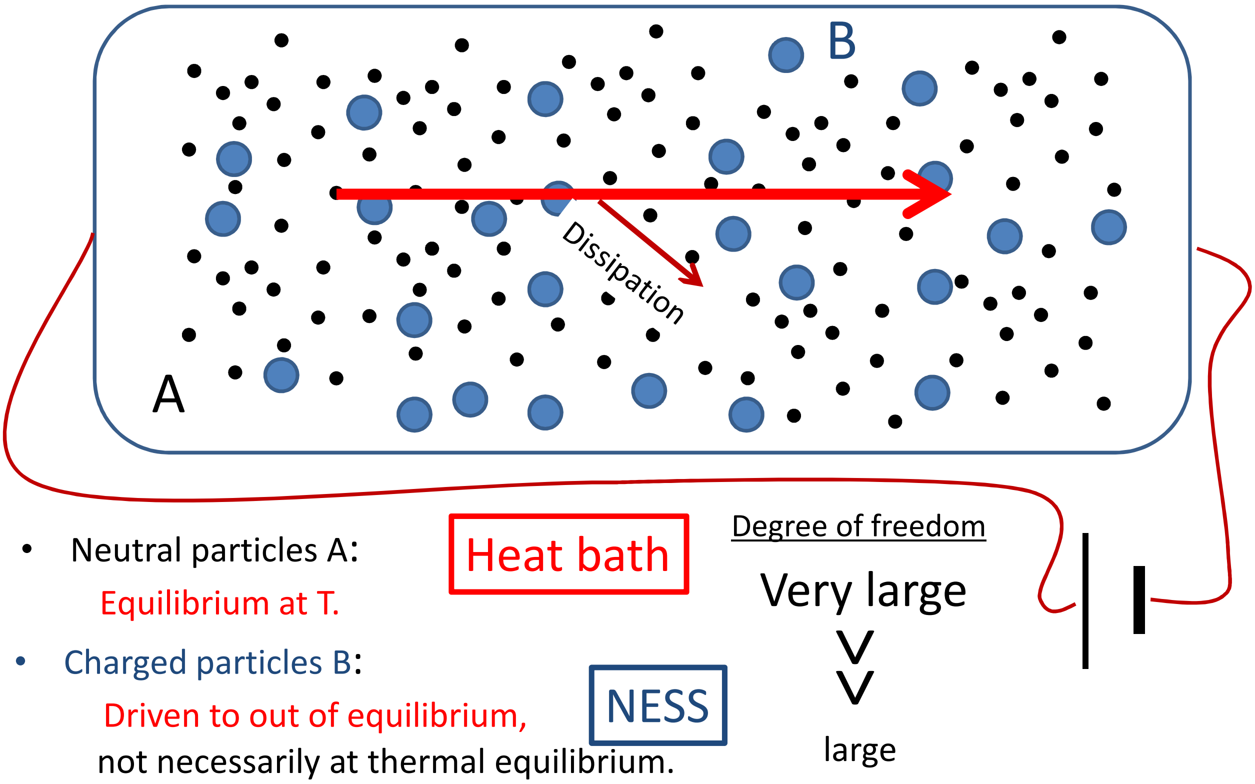
Power supply drives the system our of equilibrium.



The subsystem can be NESS if the work of the source and the energy dissipated into the heat bath are in balance.

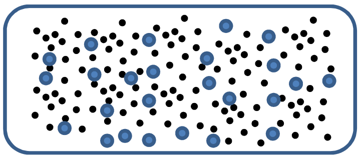
How to prepare the heat bath?

Infinitely large volume



- Neutral particles A:
  Equilibrium at T.
- Charged particles B:
  Driven to out of equilibrium,
  not necessarily at thermal equilibrium.

We can realize this situation.



SU(Nc) gauge theory

Degree of freedom

● particle A → gluon ~N_c^2-1 — Heat bath

● particle B → quark/antiquark ~N_c — NESS

($N_c^2-1 \gg N_c$)

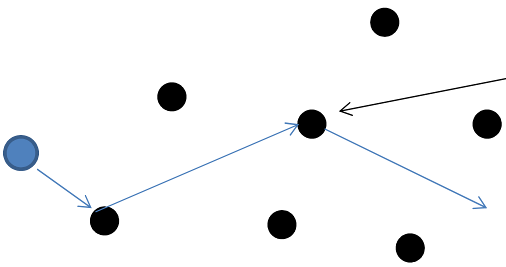
- If we take the large-Nc (Nc>>1) limit:
  The gluon subsystem can be a heat bath.
- We can apply an external electric field acting on the quark charge ($U(1)_B$ charge). — External force

Now, the interaction among the charged particles are given by the large-Nc gauge theory.

The large-Nc gauge theory is not realized in Nature.
Does it make sense to use the large-Nc gauge theory? **Yes.**

Conventional models in condensed matter physics

Drude model:



We do not ask the origin of the interaction between the ion and the electron.

These successful models do not even ask the origin of the interaction.

Hubbard model:

$$H = \sum_{i \neq j, \sigma} t_{ij} c^{+}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

We do not ask the origin of the interaction among the electrons.

Statistical physics

The game is to extract the macroscopic physics that is common to wide range of different systems, regardless of the microscopic details.

Example:

Phase transitions and critical phenomena.

Important thing for us is that the charged particles are interacting with each other, with the heat bath and with the external force.

We prepare such a many-body system by using a large-Nc gauge theory.

We focus on

- Non-equilibrium phase transitions and non-equilibrium critical phenomena associated with the non-linear conductivity of strongly-correlated insulators.
- We make qualitative predictions, but not quantitative predictions.

You may still worry:

The interaction of the SU(Nc>1) gauge theory can be qualitatively very different from that of QED (Nc=1). (e.g. confinement,......)

Do not worry. We are going to employ

N=4 Supersymmetric SU(Nc) Yang-Mills theory,
(N=4 SYM)

that produces Coulomb interaction (at T=0).

Why large-Nc?

- Heat bath is naturally prepared within the theory.
- We can employ the AdS/CFT correspondence, easily.

The AdS/CFT correspondence

J. Maldacena 1997

Some strongly-interacting quantum gauge theory = Some gravitational theory (General Relativity + matter)

equivalent

Large-Nc

Classical theory

Advantages in the gravity dual:
the problem becomes much simpler.

“Many-body physics” in the gravity

Particles A: gluons (heat bath) ⇨ single black hole

(Hawking and Bekenstein said that black hole has the notion of temperature and entropy. We still have real time.)

E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505.

Particles B: quark/antiquarks ⇨ single D-brane (a brane-like object)

A. Karch and E. Katz, JHEP 0206 (2002) 043.

The complicated many-body problem of strongly interacting system is reduced to just a “two-body” problem of classical mechanics.

More about AdS/CFT

We have employed large-Nc N=4 SYM

This is good for us: the most standard and the simplest example of AdS/CFT is that for N=4 SYM:

The most standard example of AdS/CFT:

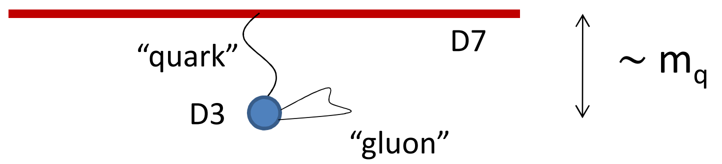
N=4 SYM $\Longleftrightarrow$ $AdS_5 \times S^5$

However, this describes only the gluon sector.

D3-D7 system

We can add the flavor degree of freedom (quarks and anti-quarks) by adding the D7-brane to the system.

(Karch and Katz, JHEP0206(2002)043)



The string between the D3 and the D7 acts as a quark (or antiquark) from the viewpoint of the D3-branes.

The gauge theory realized on the D3-branes is
N=4 SYM + N=2 hyper-multiplet

AdS/CFT based on D3-D7

SU(Nc) N=4 Supersymmetric Yang-Mills (SYM) theory at large-Nc with $\lambda=g_{YM}^2 N_c >> 1$.
（Quantum field theory） Finite T

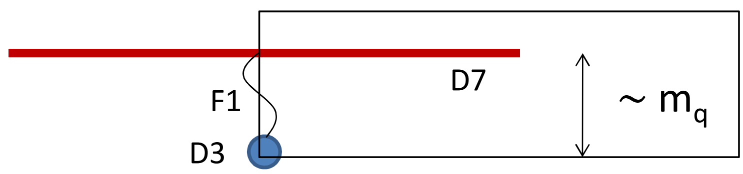
+ quark sector (N=2 hyper-multiplets)

⇕ Equivalent

Type IIBsupergravity at the classical level on weakly curved AdS-BH $\times S^5$

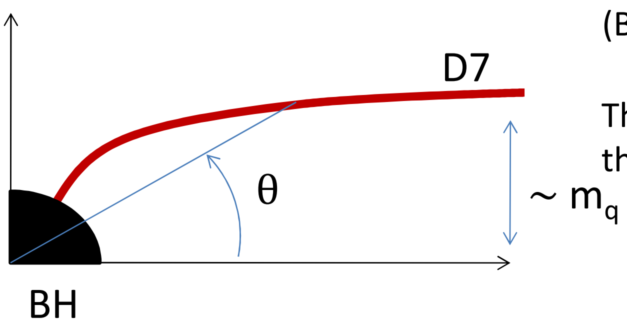
+ D7-brane on this curved spacetime

Gravity Dual



We draw only this part.

- The D3 is replaced with an AdS-BH in the gravity dual.



"z" represents the radial direction.
(But, the boundary is at z=0.)

The shape of the D7 is described by the function θ(z).

$$\theta(z) = m_q z + \text{const.}\, z^3 + \ldots\ldots$$

$$\left.\frac{1}{z}\sin\theta(z)\right|_{z\to 0} = m_q$$

The dual geometry

	$R^{3,1}$	z	S^3	S^2
Horizon	○			
D7	○	○	○	

$\longleftarrow$ AdS_5-BH $\longrightarrow$ $\longleftarrow$ S^5 $\longrightarrow$

$$ds^2{}_{\text{AdS-BH}} = -\frac{1}{z^2}\frac{\left(1-z^4/z_H^4\right)^2}{1+z^4/z_H^4}dt^2 + \frac{1+z^4/z_H^4}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$$

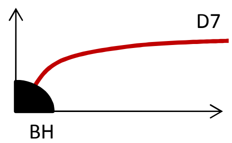
$$ds^2{}_{S^5} = d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2$$

$$\left(0 \le \theta \le \frac{\pi}{2}\right)$$

The D7 is located at $\varphi = 0$ (our choice).

$\boxed{\text{The D7 configuration is given by } \theta(z).}$

Physics of D7-brane



Black hole geometry plays the role of heat bath for the D7-brane.

- The D7-brane is affected by the black hole
- The black hole is not affected by the D7.

$\Rightarrow$ Probe approximation

D7-brane action : Dirac-Born-Infeld action

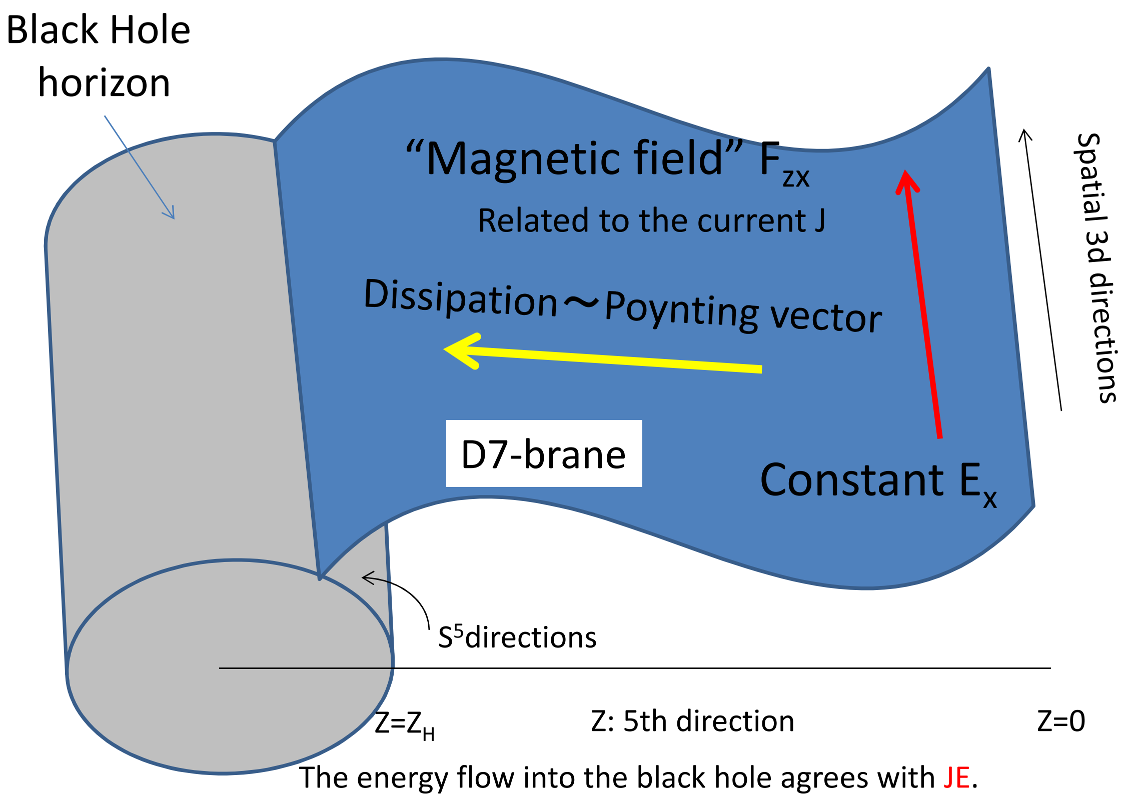
$$S_{D7} = -T_{\text{D7}}\int d^{7+1}x\sqrt{-\det\left(\partial_a x^\mu \partial_b x^\nu g_{\mu\nu} + F_{ab}\right)}$$

Spacetime metric.

$$F_{ab} = \partial_a A_b - \partial_b A_a \quad (2\pi\alpha' = 1)$$

Field strength of the U(1) gauge field on the D7-brane.

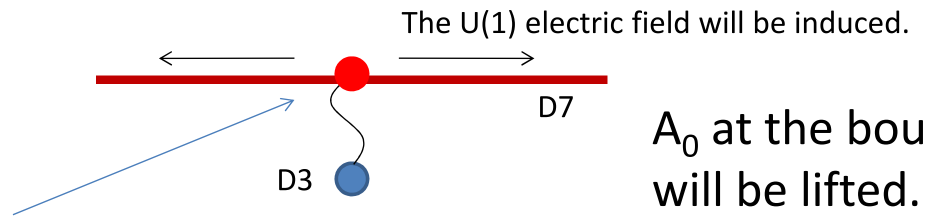
Cartoon of D-brane configuration



The energy flow into the black hole agrees with JE.
(Karch, O'Bannon, Thompson, 2008 and 2009)

The U(1) on the D7

The U(1) gauge field on the D7 is linked to the $U(1)_B$ charge ($U(1)_B$ current) in the YM side.



This endpoint acts as a unit charge of the U(1) gauge field on the D7.

A_0 at the boundary will be lifted.

If the quark moves (in x-direction), the "magnetic" field will be induced on the D7 and A_x at the boundary will be lifted as well.

AdS/CFT dictionary: GKP-Witten relation

z=0: boundary

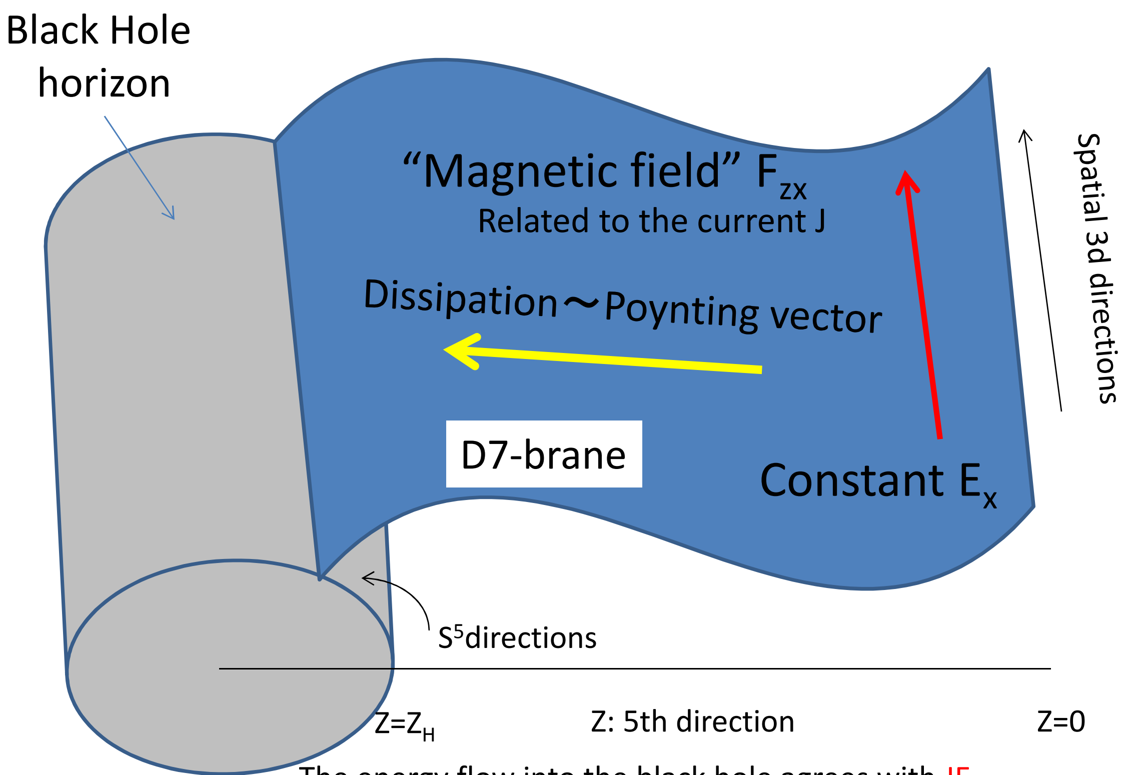
$$A_0(z) = \mu - \frac{(2\pi)^2}{2N_c}\left\langle J^0 \right\rangle z^2 + O(z^4)$$

$$A_x(z) = -Et + \frac{(2\pi)^2}{2N_c}\left\langle J^x \right\rangle z^2 + O(z^4)$$

If the configuration of $A_x(z)$ on the D7-brane is specified as a function of z, the relationship between E and J^x can be read from it.

⇨ We obtain the (non-linear) conductivity.

Cartoon of D-brane configuration



The energy flow into the black hole agrees with JE.
(Karch, O'Bannon, Thompson, 2008 and 2009)

However,

$$A_x(z) = -Et + \tfrac{(2\pi)^2}{2N_c}\left\langle J^x \right\rangle z^2 + O(z^4)$$

A_x obeys a second-order differential equation.

➡ We need two boundary conditions to fix the solution.

The first and the second terms are the input conditions we need to specify by hand!

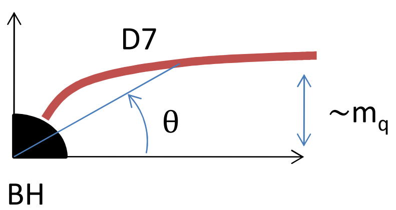
However, if we specify them as we like,
the on-shell D7-brane action will be complex in general.

The reality of the D7-brane action (the stability of the system) constrains the relationship between the first and the second terms.

[Karch, O'Bannon JHEP0709(2007)024]

The on-shell D7-brane action

$$S_{\mathrm{D7}} = -N\int dzdt \cos^6\theta\, g_{xx}^{5/2}\left|g_{tt}\right|^{1/2}\sqrt{W}$$



$$W = \frac{g_{zz}\left(\left|g_{tt}\right|g_{xx} - E^2\right)}{\left|g_{tt}\right|g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}}$$

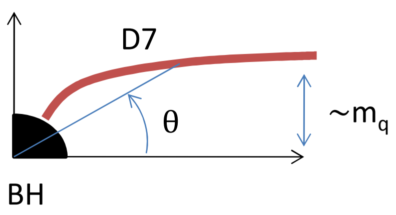
The metric of the AdS-BH

$$ds^2{}_{\text{AdS-BH}} = -\frac{1}{z^2}\frac{\left(1-z^4/z_H^4\right)^2}{1+z^4/z_H^4}dt^2 + \frac{1+z^4/z_H^4}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$$

- The horizon is located at $z=z_H$.
- The boundary is at $z=0$.

On-shell D7-brane action

$$S_{\text{D7}} = -N\int dzdt \cos^6\theta\, g_{xx}^{5/2}\left|g_{tt}\right|^{1/2}\sqrt{W}$$



$$W = \frac{g_{zz}\left(\left|g_{tt}\right|g_{xx} - E^2\right)}{\left|g_{tt}\right|g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}}$$

Both the numerator and the denominator go across zero somewhere between the boundary and the horizon.

Only the way to make the action real is to make them go across zero at the same point.
(We define this point as $z=z_*$.)

The conditions for reality

$$\left(-g_{tt}\right)g_{xx}\Big|_{z=z_*} - E^2 = 0 \Rightarrow$$ z_* in terms of E

$$\left(-g_{tt}\right)g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}\Bigg|_{z=z_*} = 0$$

⇒ J_x is given in terms of E and $\theta(z_*)$.

$$\frac{1}{z}\sin\theta(z)\Big|_{z\to 0} = m_q$$

⇒ J_x is given by using E and m_q.

The conductivity is given as a function of m_q (at given T, λ).

The non-linear conductivity

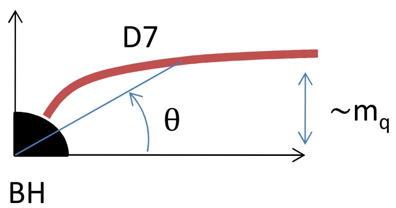
The charge density is also taken into account.
[Karch, O'Bannon JHEP0709(2007)024]

Normal conduction

$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Pair-creation

$$d \equiv \frac{\langle J^t\rangle}{\frac{\pi}{2}\sqrt{2\lambda}T^2}, \quad e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2},$$



⇒
- $\cos\theta(z_*)$ goes to 1 at $m_q \to 0$.
- $\cos\theta(z_*)$ goes to zero at $m_q \to$ infinity.

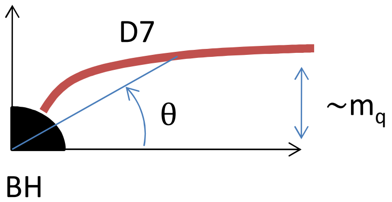
We have no way to reproduce NDC from the normal-conduction part.

$$J_x\big|_{m_q\to\infty} = \frac{d}{\sqrt{1+e^2}}E \to \begin{cases} \approx & d\cdot E & (e << 1) \\ \approx & \text{saturate} & (e >> 1) \\ \approx & 0 & (T >> 1) \\ & (e \leftrightarrow -e \text{ symmetric}) & \end{cases}$$

We consider the neutral case: the contribution of the pair-creation

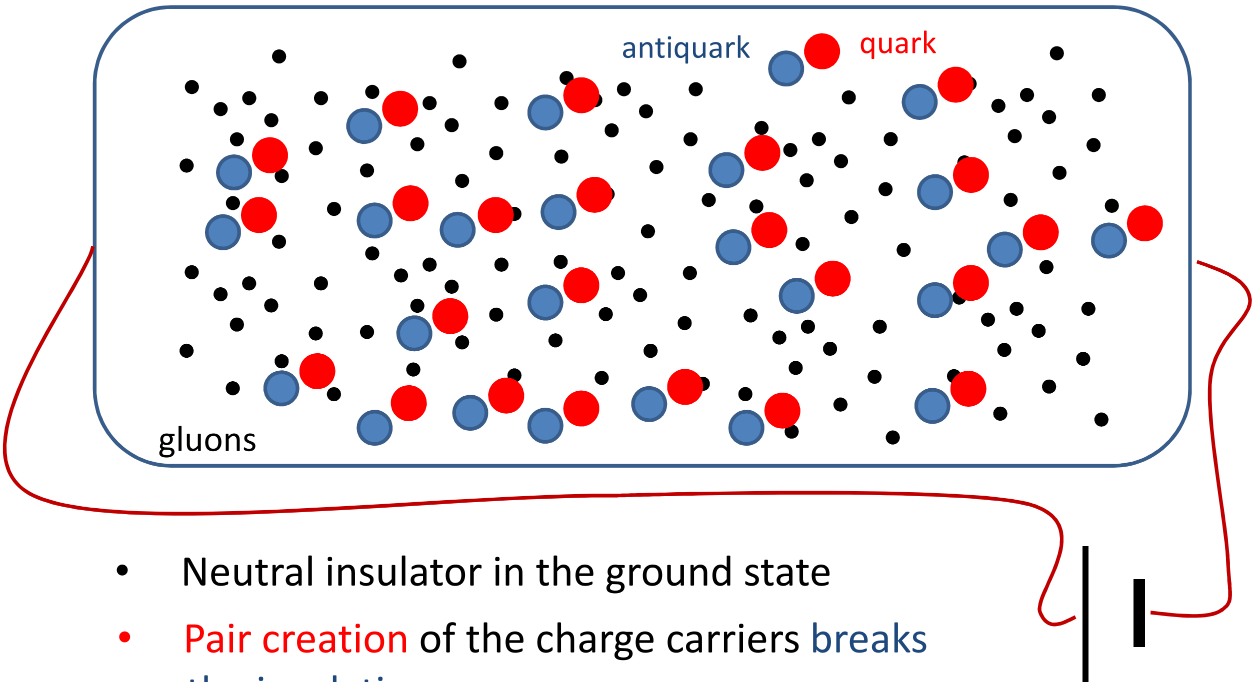
$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*)}$$

$$e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2}$$



This function can be given by solving a non-linear differential equation, numerically.

Insulation breaking



- Neutral insulator in the ground state
- Pair creation of the charge carriers breaks the insulation.

⇒ Insulation breaking.

Our setup

- We consider a neutral system.
- The volume of the system is infinite.
- Strongly interacting system.
- Our current is non-ballistic.
- Insulator in the ground state.
- Strong-enough electric field breaks the insulation.

Interaction among the charged particles, and the pair creation of the charges are taken into account.

Results of analysis

S.N. arXiv:1204.1971

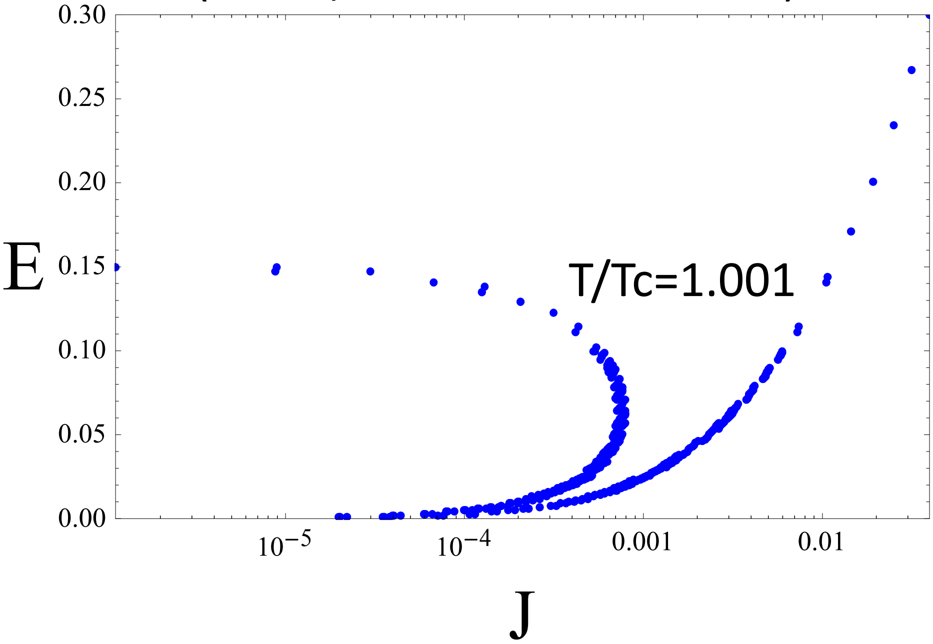
（See also, S.N. PTP124(2010)1105.）

$$\lambda = (2\pi)^2/2, \quad N_C = 40,$$

We solve a non-linear differential equation numerically to obtain the conductivity.

Non-linear conductivity at various T

(T: temperature of the heat bath)

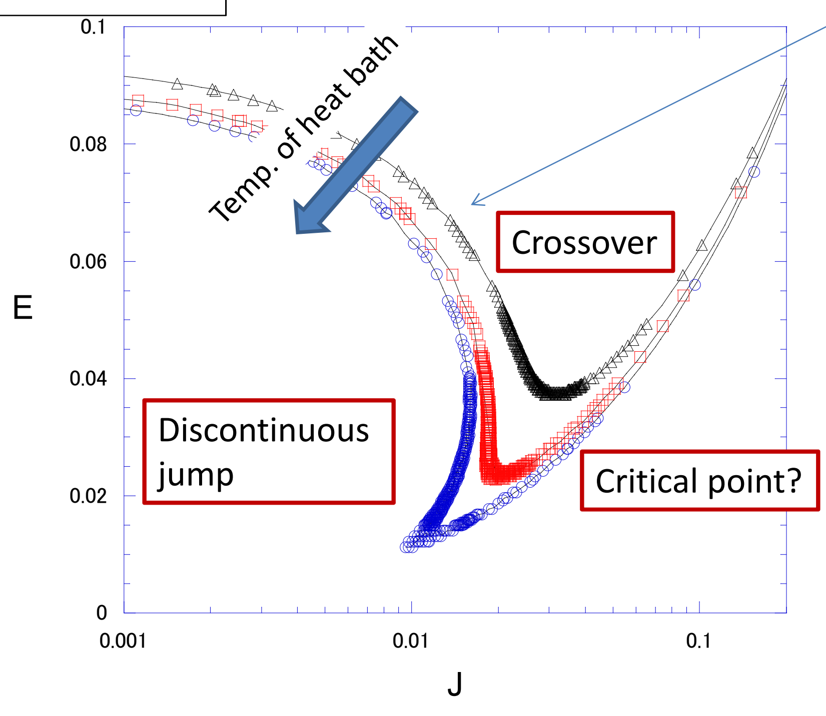


J-E characteristics

S.N. PTP124(2010)1105.

Temp. of heat bath

△ : T=0.34337
□ : T=0.34365=Tc
○ : T=0.34379



Negative Differential conductivity (NDC)

It is widely observed in strongly-correlated insulators

(See, e.g. [Oka, Aoki, arXiv:0803.0422])

How to determine the transition point?

(S.N. arXiv:1204.1971)

Equilibrium cases:

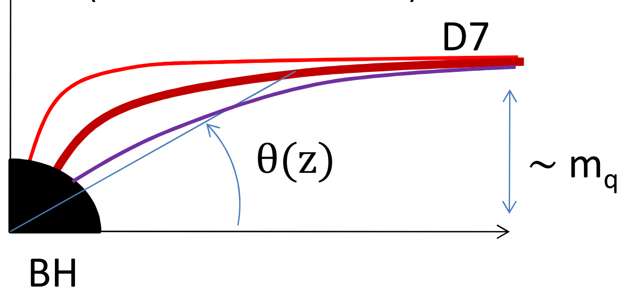
We compute the free energy and compare which branch (phase) is most economic.

In the non-equilibrium systems:

- Are there a non-equilibrium generalization of free energy?
- If yes, how to compute it?

The things are classical mechanics

(S.N. arXiv:1204.1971)



The question is which D-brane configuration is most stable.
This is a problem of classical mechanics of membrane with electro-magnetic flux on a curved geometry.

The most natural way is to compare the Hamiltonian.

Hamiltonian of D7

Renormalized Hamiltonian

"Bare Hamiltonian"

$$H = V \lim_{\varepsilon \to 0} \int_{\varepsilon}^{z_H} dz \left[\dot{A}_x \frac{\partial L_{D7}}{\partial \dot{A}_x} - \left(L_{D7} - A'_x \frac{\partial L_{D7}}{\partial A'_x} \right) - L_{count}(\varepsilon) \right]$$

"Routhian"

E J

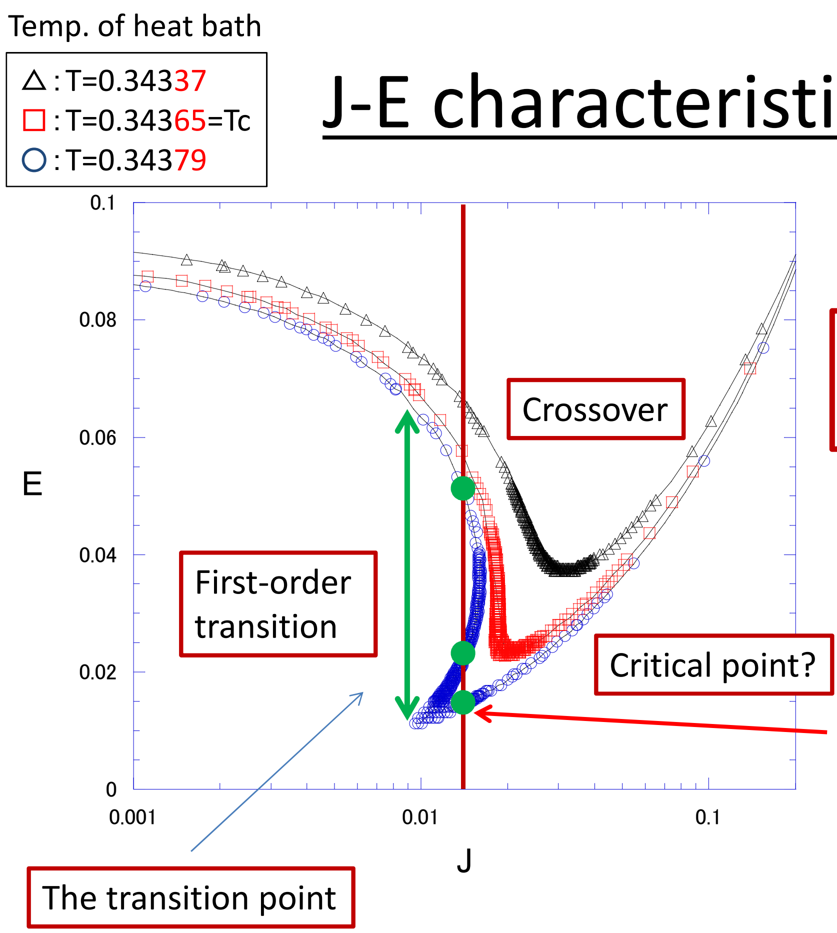
- The UV divergence is renormalized by the counter terms.
- The IR divergence is canceled within the Legendre transformation.

We propose to define this Hamiltonian as a NESS generalization of the free energy.

Any relationship to the steady state thermodynamics?

(Y. Oono and M. Paniconi, PTP.Suppl. **130** (1998), 29.)

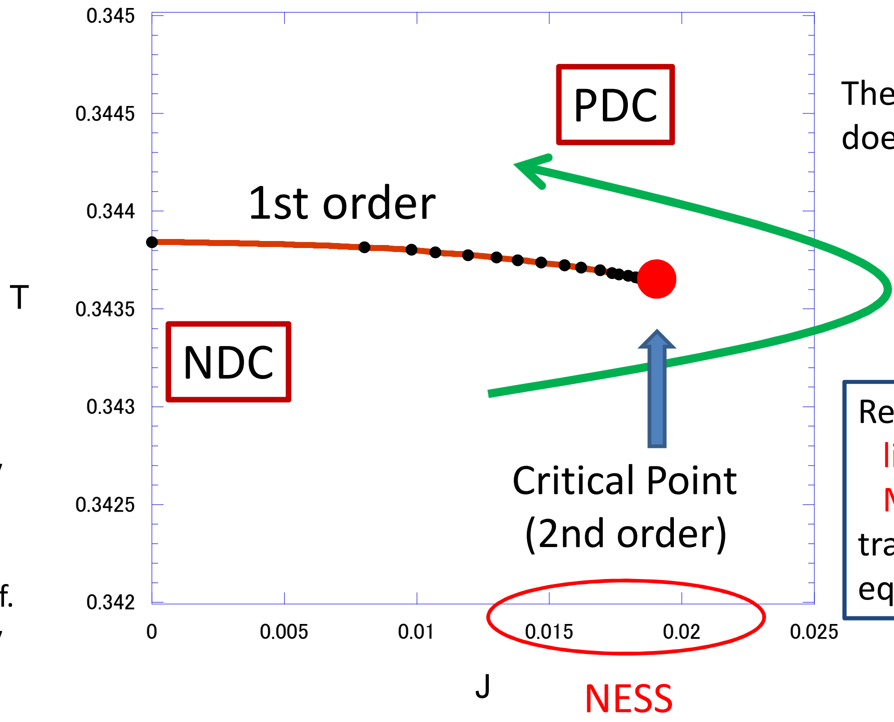
J-E characteristics



The system prefers smaller dissipation.

This has the smallest Hamiltonian in the gravity side.

Phase diagram



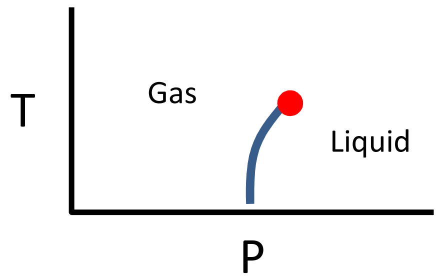
The symmetry does not change.

Resembles of liquid-gas Mott transitions at equilibrium.

PDC:
Positive Diff. Conductivity

NDC:
Negative Diff. Conductivity

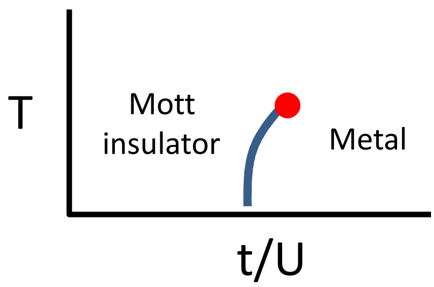
Critical Phenomena（equilibrium）



Liquid-gas transition (equilibrium)

$$n_L - n_g \propto (T_C - T)^\beta$$

Difference of density



Mott transition (equilibrium)

Experimentally detected by using the conductivity σ, instead of the density.

P. Limelette et al., Science 302 (2003) 89.
F. Kagawa et al.,Nature 436 (2005) 534.

- Universality class = Ising universality class
- β=1/2 within the mean-field theory.

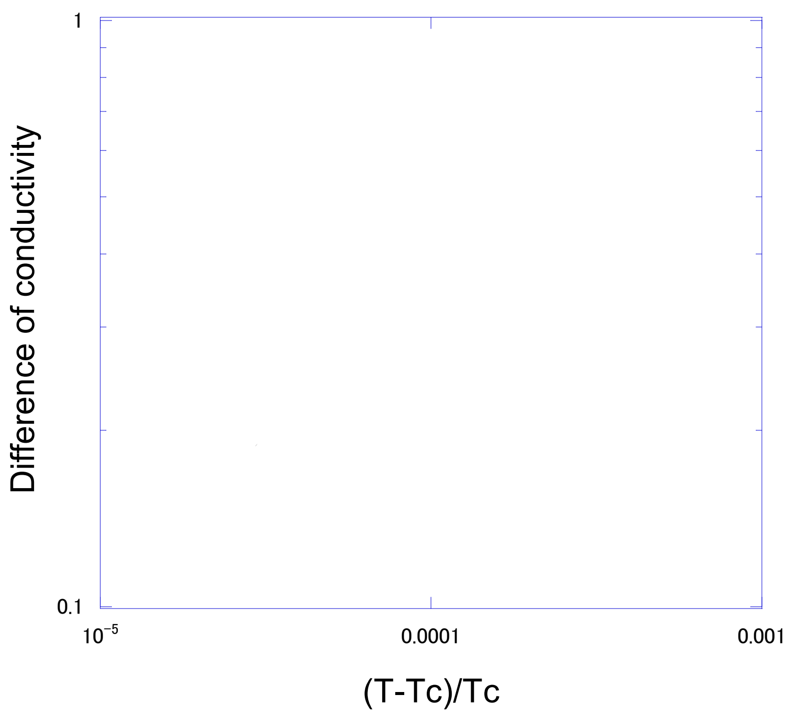
We propose to see

$$\sigma_{\mathrm{PDC}} - \sigma_{\mathrm{NDC}} \propto \left(T - T_C\right)^{\beta}$$

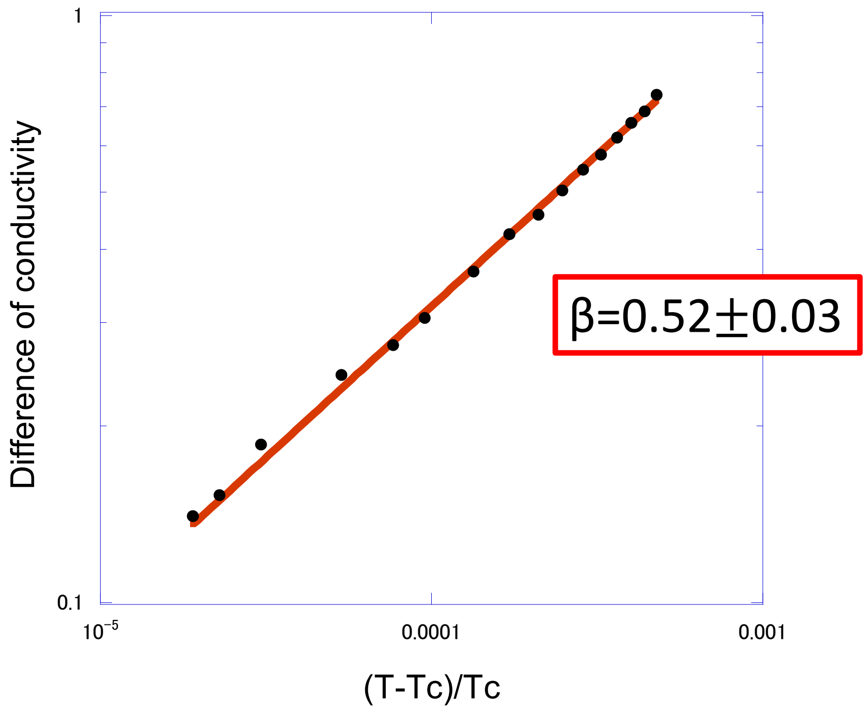
Difference of conductivity Temperature of heat bath

Let us see what is going on.

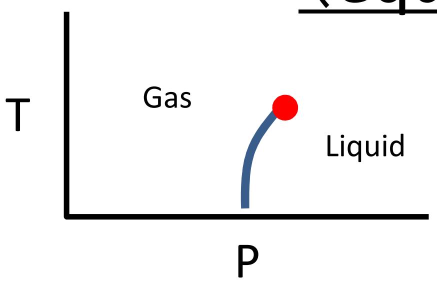
Behavior of diff. of conductivity



Behavior of diff. of conductivity

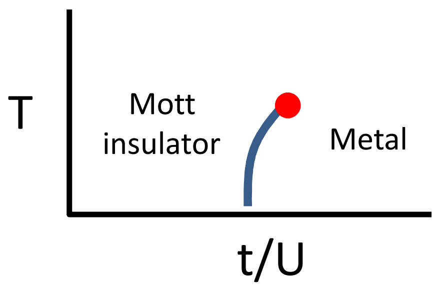


Another critical exponent (equilibrium)



Liquid-gas transition (equilibrium)

$$\left(n-n_c\right)\Big|_{T=Tc} \propto \left|P-P_c\right|^{1/\delta}$$



Mott transition (equilibrium)

$$\left(\sigma-\sigma_c\right)\Big|_{T=Tc} \propto \left|P-P_c\right|^{1/\delta}$$

Pressure

- δ=3 for the mean-field theory.

However, the pressure is not a control parameter in our system.
(Our system has infinite volume.)

nature Vol 436|28 July 2005|doi:10.1038/nature03806

LETTERS

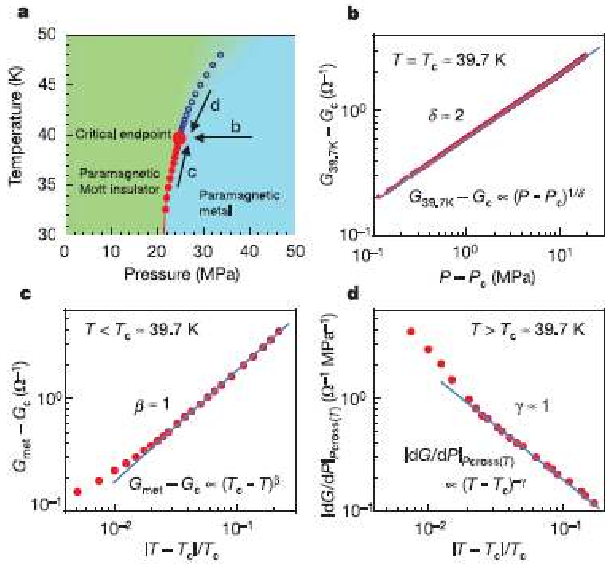
Unconventional critical behaviour in a quasi-two-dimensional organic conductor

F. Kagawa[1], K. Miyagawa[1,2] & K. Kanoda[1,2]

Nature 436 (2005) 534.

Metal-insulator transition at equilibrium (J~0)

Fig. 2



Definition of a new critical exponent

Our pressure is not a control parameter.

The remaining available control parameter :

J

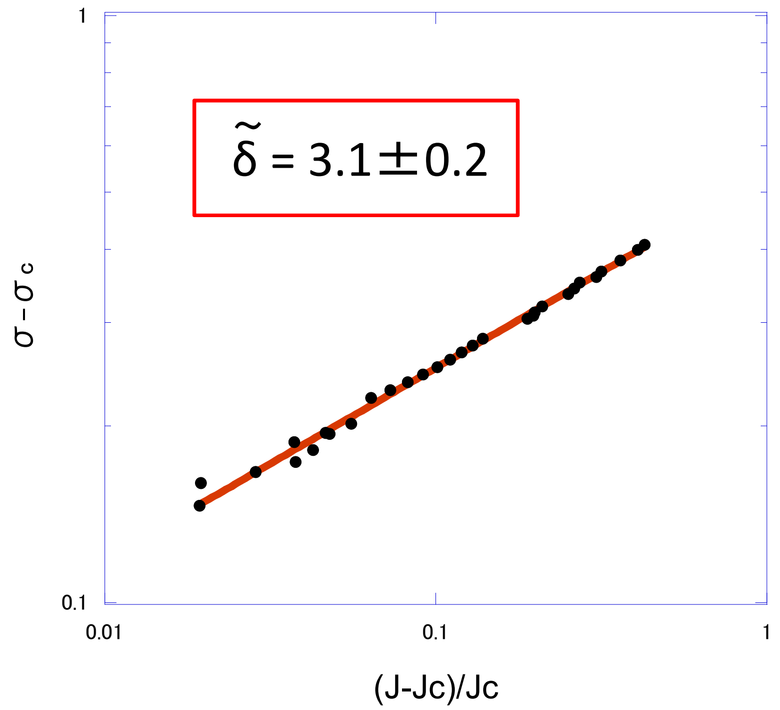
Essentially a non-equilibrium quantity

Proposal:

We define $\tilde{\delta}$ by

$$\left.(\sigma - \sigma_c)\right|_{T=Tc} \propto |J - J_c|^{1/\tilde{\delta}}$$

Does $\tilde{\delta}$ make sense?



Presence of mean-field theory for non-equilibrium phase transitions?

β=0.52±0.03 ～ 0.5

$\tilde{\delta}$ = 3.1±0.2 ～ 3

Suggest mean-field values?

It is interesting to see whether we can construct a non-equilibrium version of Landau-Ginzburg theory.

By the way, it is natural to have mean-field values in large-Nc theories.

Large-Nc as mean-field approximation

SU(Nc)

Internal degree of freedom

Large-Nc:

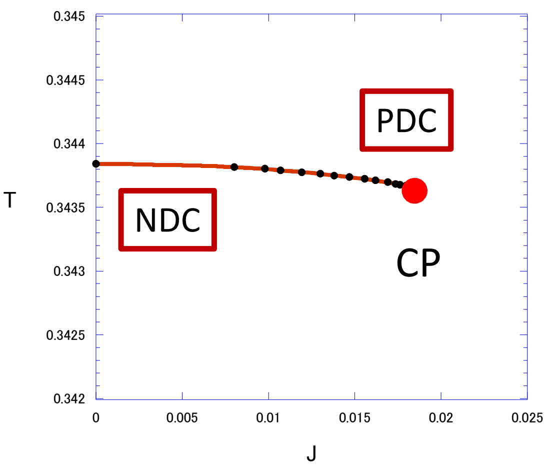
We are taking the internal degree of freedom of the particles very large: the mean-field behavior appears.

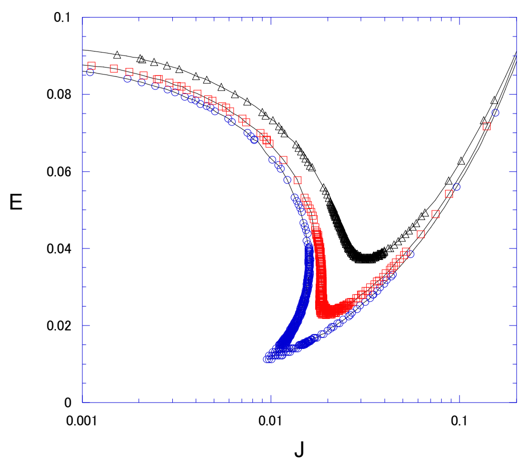
Similar situations can be found in the condensed matter physics.

- O(N) model, at N→∞.
- Mean-field theory: d →∞.
- ....

Summary

An AdS/CFT analysis of non-linear conductivity of a "strongly-correlated insulator" indicates the unknown non-equilibrium phase transitions and non-equilibrium critical point.



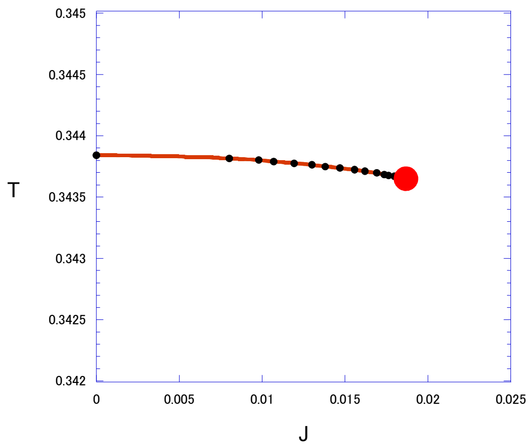


A possible contact with real materials:

Hope:

In light of possible universality, the current-driven non-equilibrium critical point found in the present work may have a chance to be observed even in real materials of strongly-correlated insulators.

The precise values of the critical exponent can be different (from the mean-field values.)



Towards experimental verification

I have visited the experimental physicists at Nagoya University on May 24th.

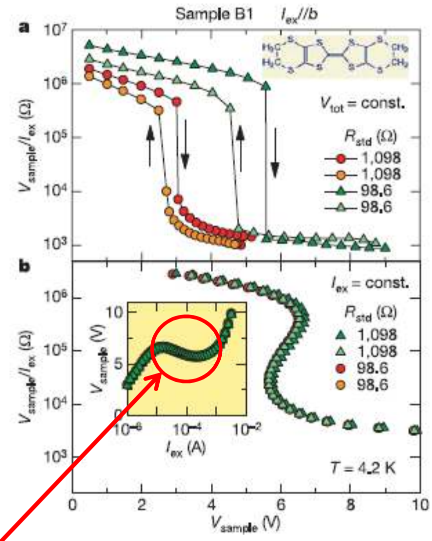
Question to the experimental physicists:

“Organic thyristor”

F. Sawano, I. Terasaki, Y. Nogami et al., Nature 437 (2005) 522.

θ-(BEDT-TTF)$_2$CsCo(SCN)$_4$ crystal at 4.2 K.

Charge order insulator



What happens if we raise the temperature?