Split Seesaw Mechanism and Flavor Symmetry

Seminar @ Osaka University

17 April 2012

Ryo Takahashi （NTHU, Taiwan → Osaka University）

Reference

A. Adulpravitchai, RT, JHEP 1109 (2011) 127

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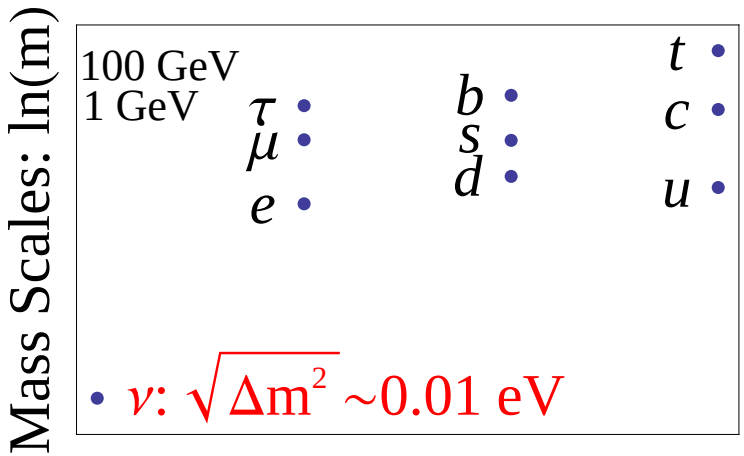

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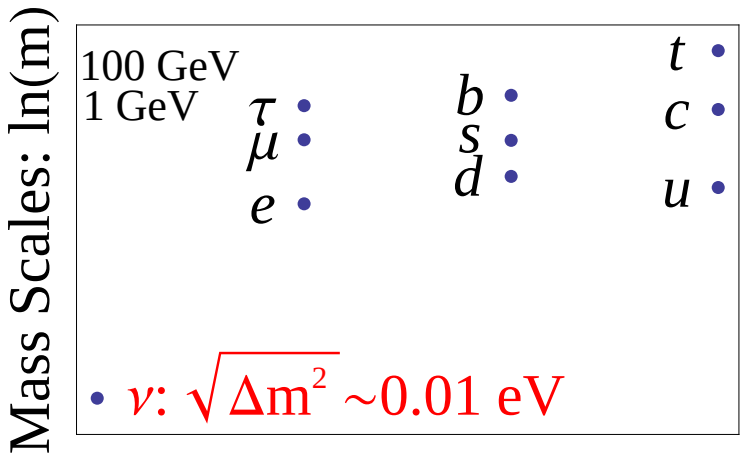

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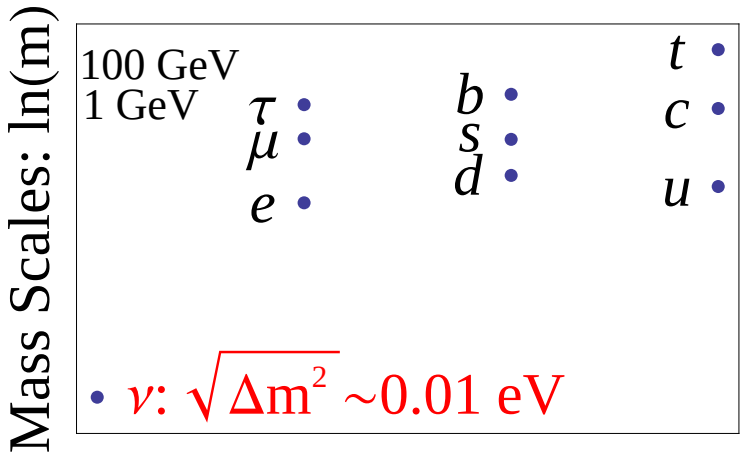

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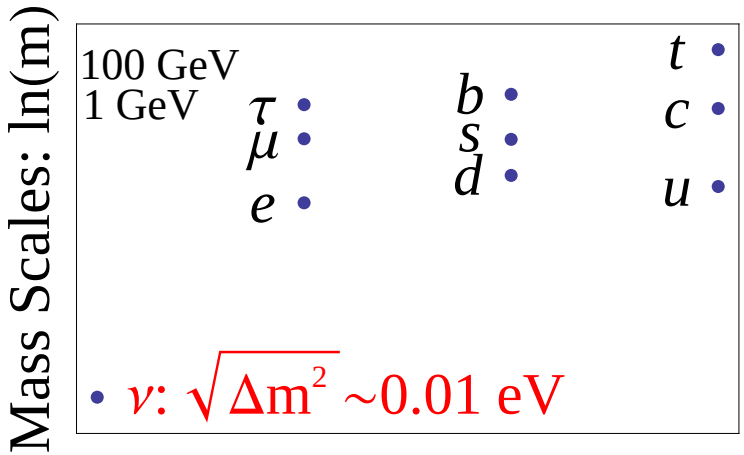

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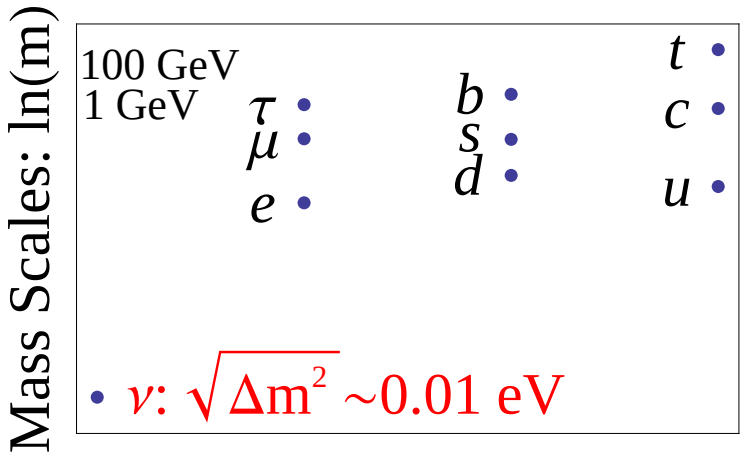

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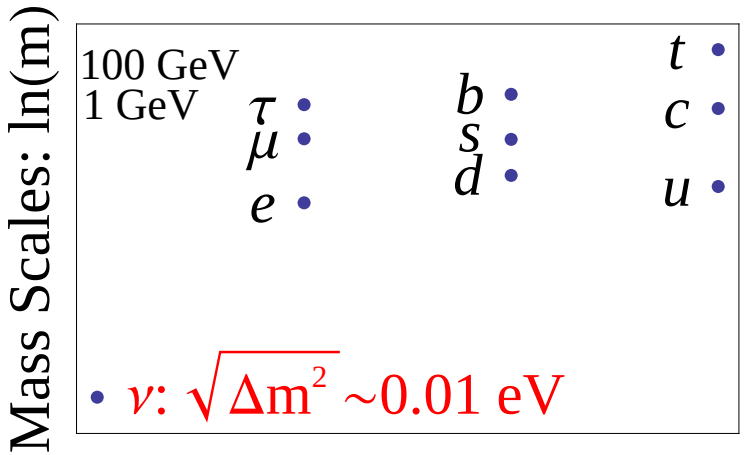

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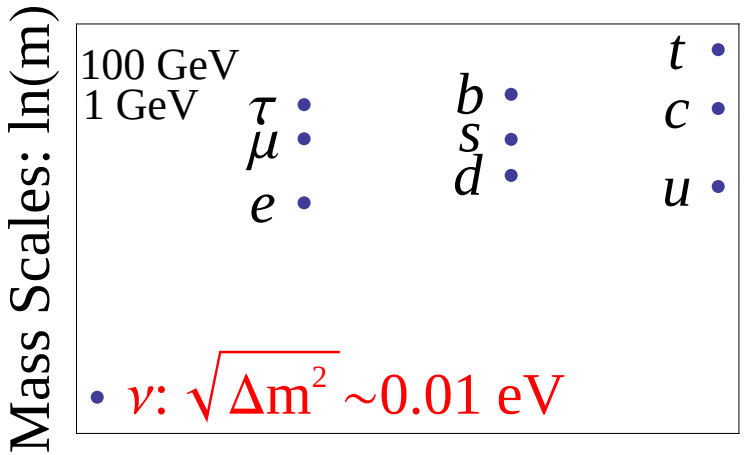

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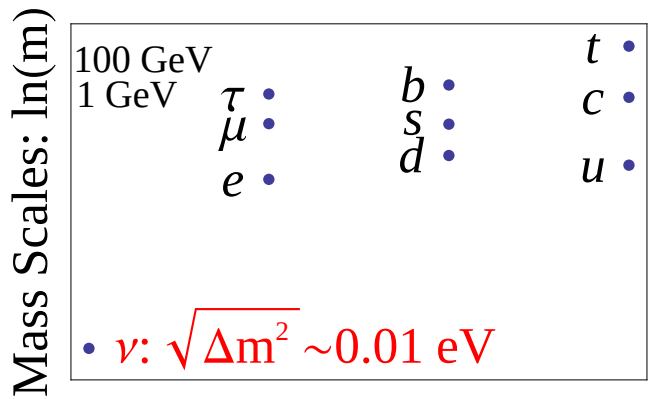

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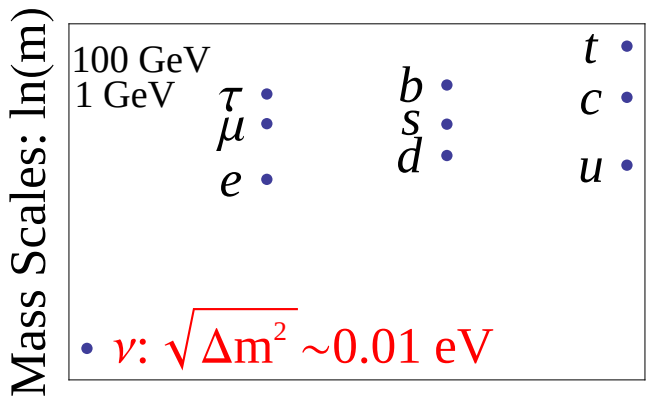

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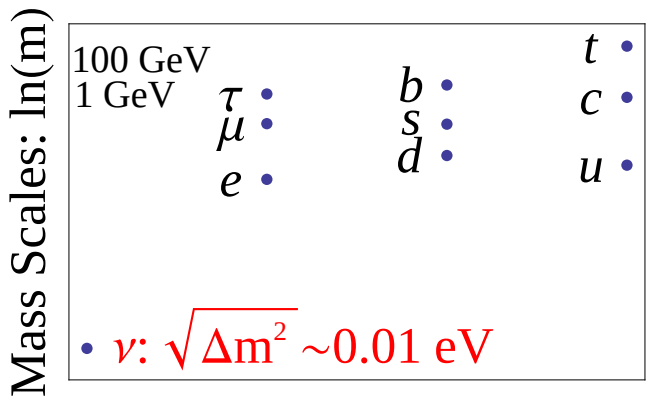

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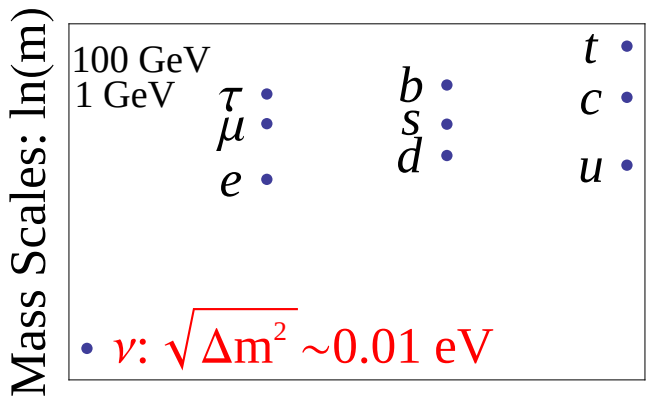

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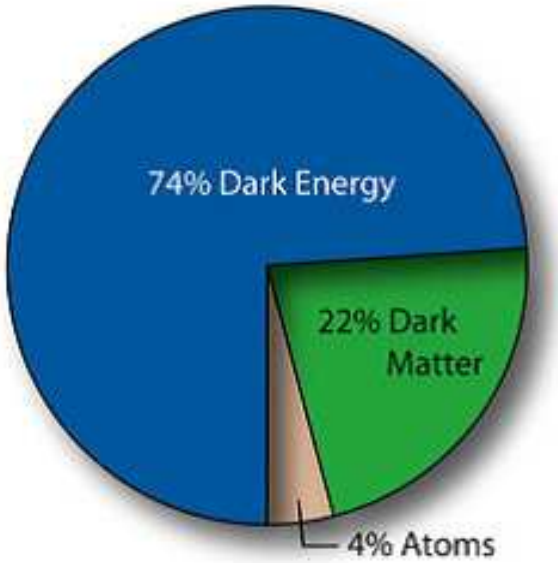

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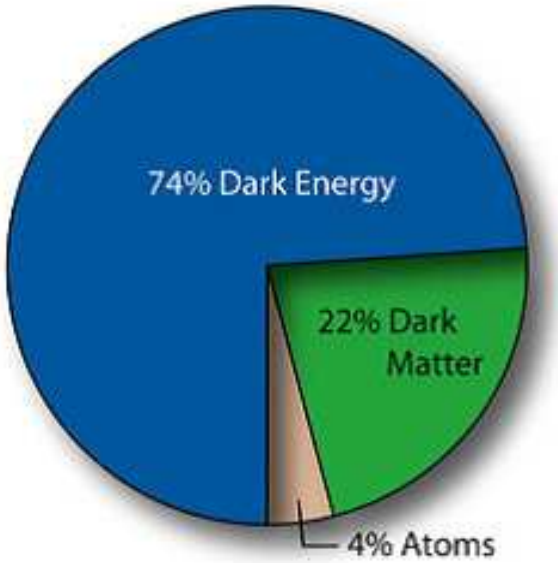

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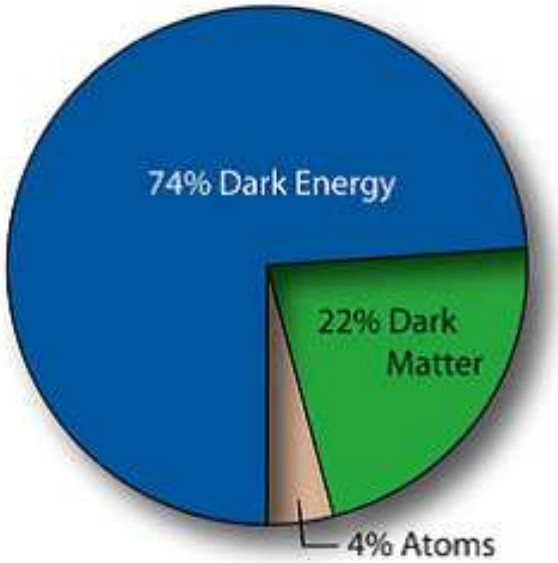

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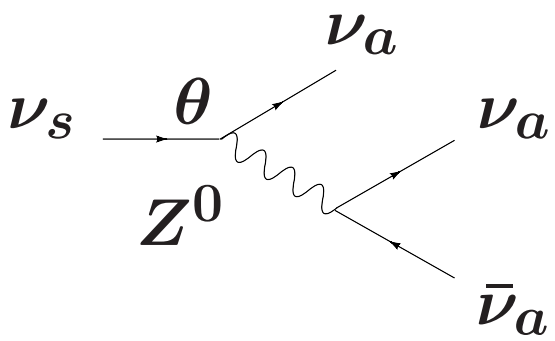
  - In a simple extention of the SM, 3 sterile (right-handed) neutrinos, which are singlets under the SM gauge group, are added: (0906.2968 [hep-ph], good review!)

$$\begin{array}{c c} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix} \end{array} \Longrightarrow \begin{array}{l} |\nu_a\rangle = \cos\theta|\nu_L\rangle + \sin\theta|\nu_R\rangle \\ |\nu_s\rangle = -\sin\theta|\nu_L\rangle + \cos\theta|\nu_R\rangle \end{array}, \quad \theta \sim \frac{M_D}{M_R}$$



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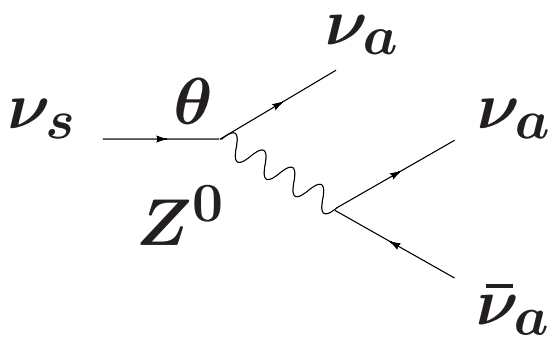

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  - Dodelson and Widrow pointed out that a life-time of the sterile neutrino can be longer than the age of the Universe and a cosmological density $\Omega_s = \Omega_{\rm DM} \sim 0.2$ can be realized if the sterile neutrino mass is in the $\mathcal{O}(\text{keV})$ scale.

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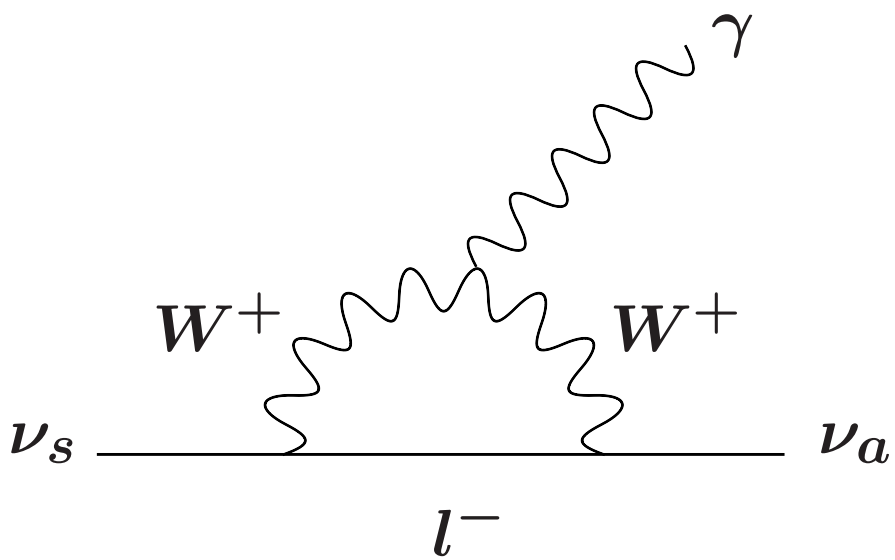

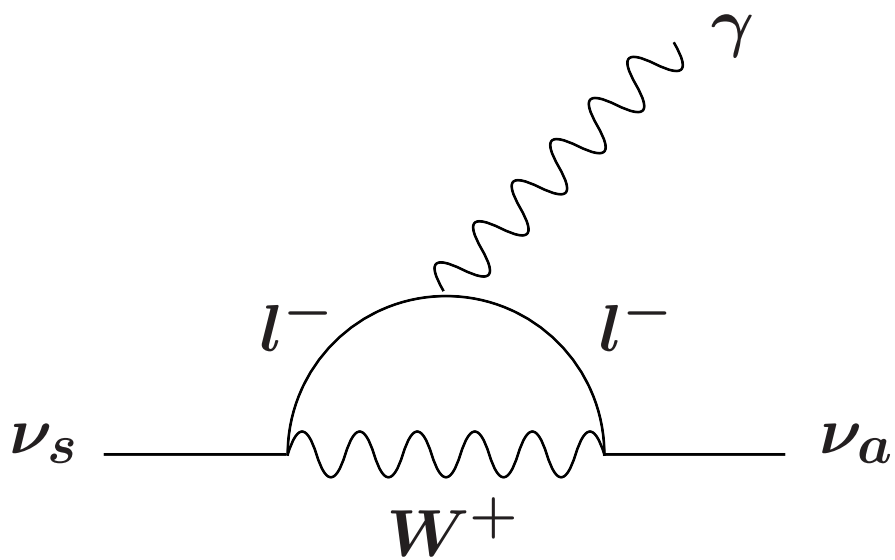

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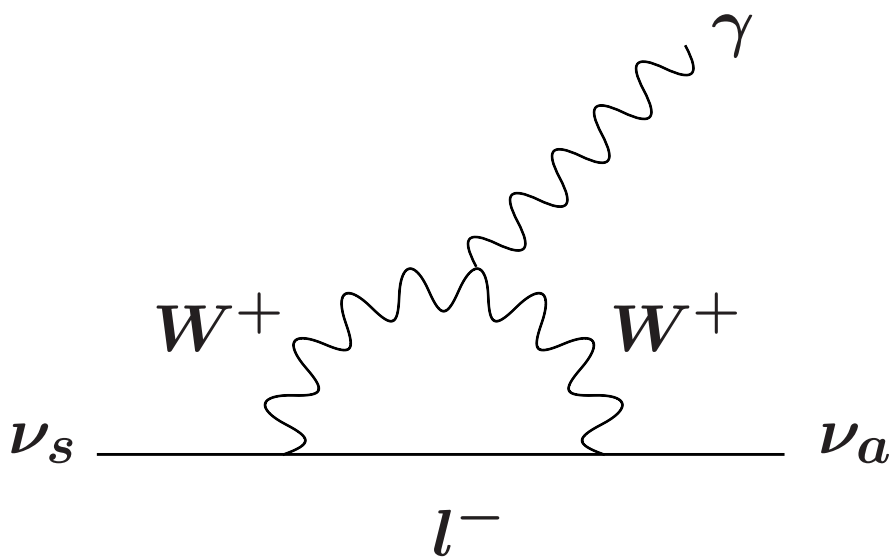

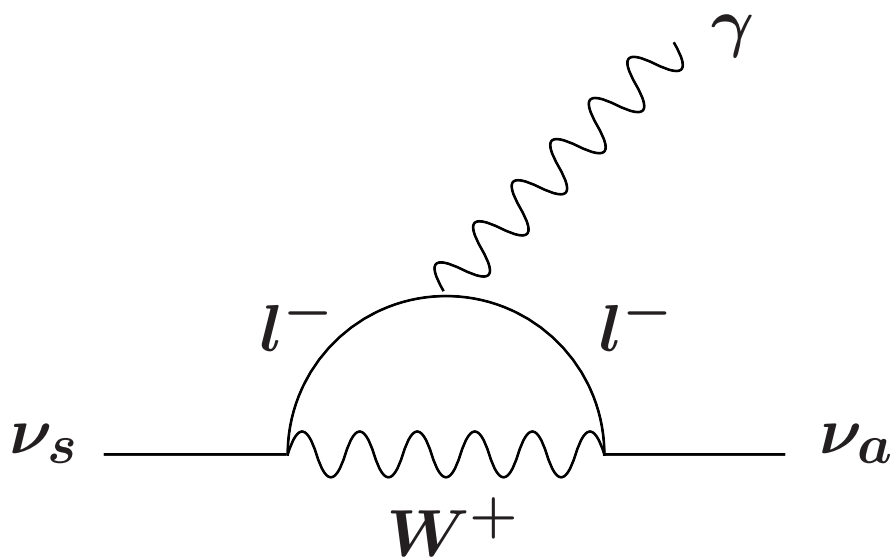

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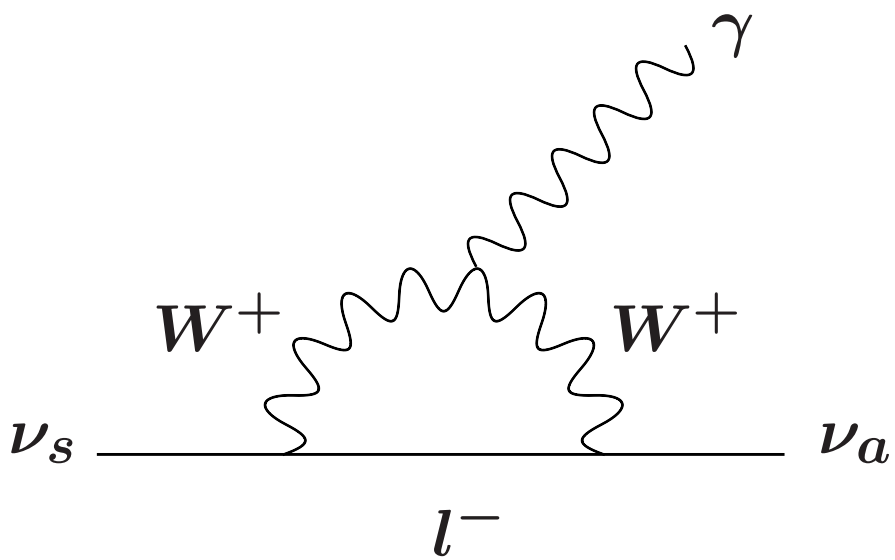

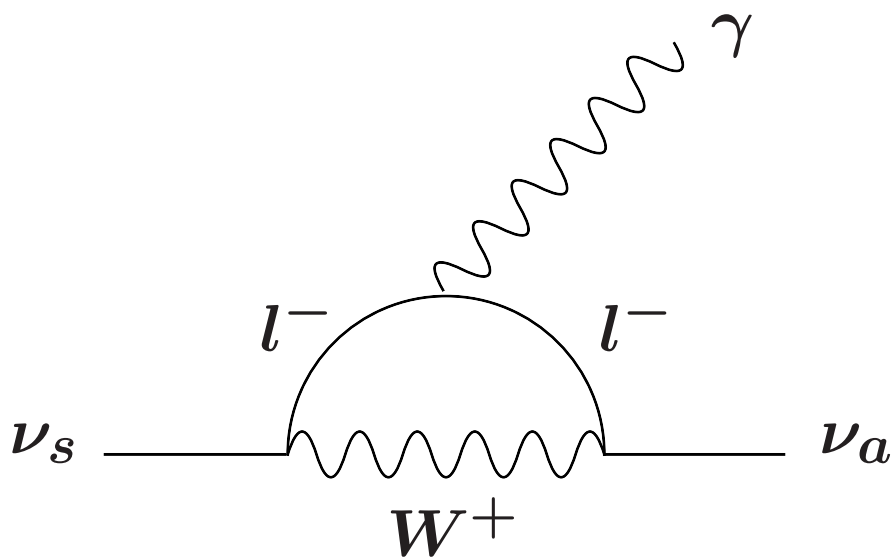

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  - The mass range (1-20)keV of sterile neutrino is also favored by the pulsar kicks as well as DM.

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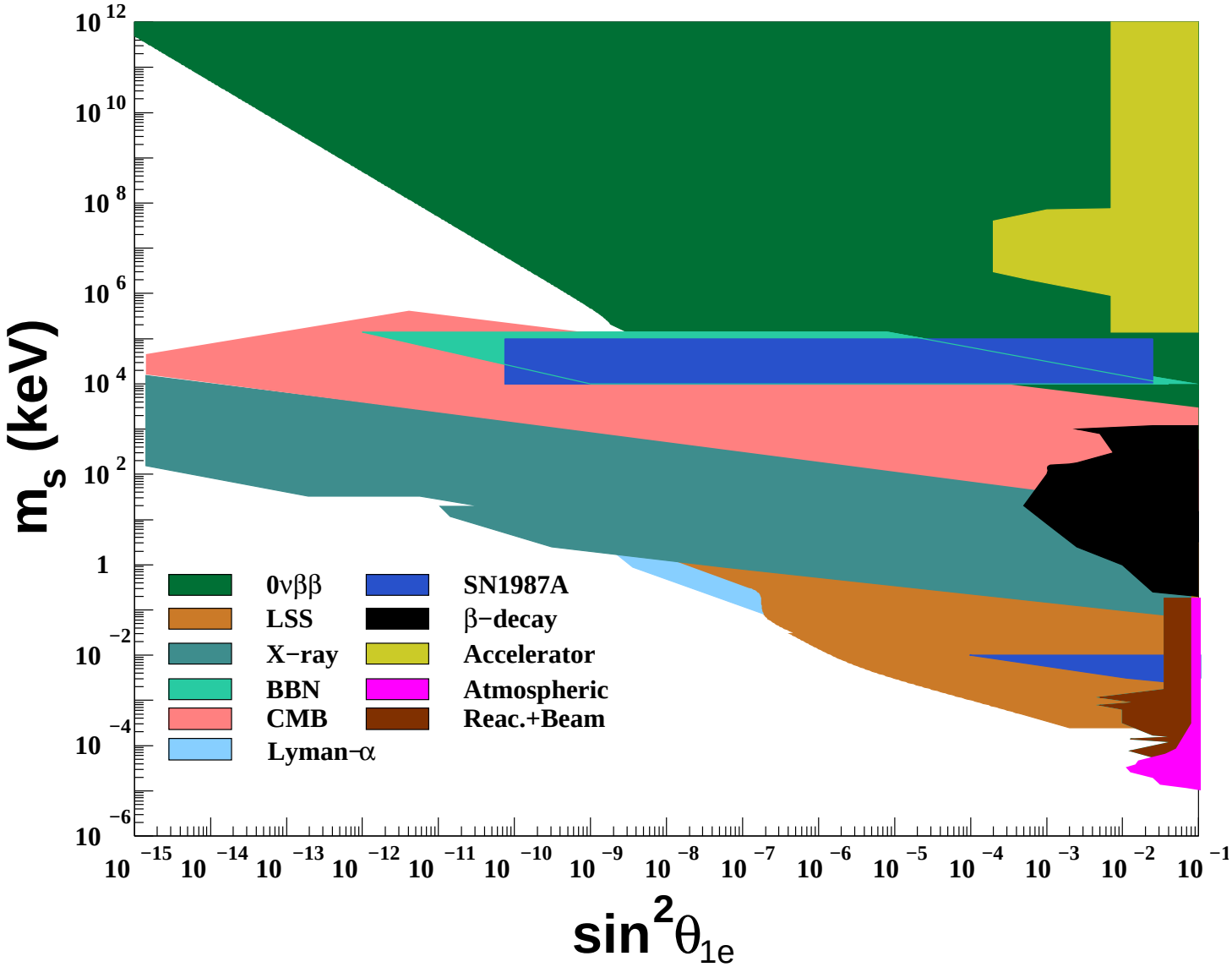

A. Kusenko, 0906.2968 [hep-ph]

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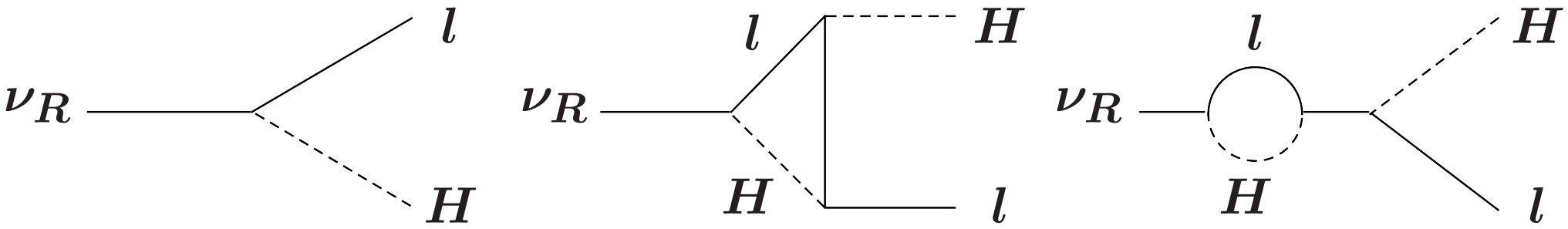

$$M_R \sim 10^{9-10}\ \text{GeV} \quad \Longrightarrow \quad \eta_B \sim 6 \times 10^{-10}$$

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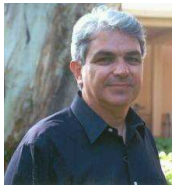
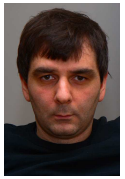

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  - As physics beyond the SM, Arkani-Hamed, Dimopoulos, Dvali proposed a solution to the hierarchy problem in the context of large extra-dimension.

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· As physics beyond the SM, Arkani-Hamed, Dimopoulos, Dvali proposed a solution to the hierarchy problem in the context of large extra-dimension. Randall and Sundrum also proposed a warped five-dimensional model to naturally realize the EW scale from the Plack scale.

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· As physics beyond the SM, Arkani-Hamed, Dimopoulos, Dvali proposed a solution to the hierarchy problem in the context of large extra-dimension. Randall and Sundrum also proposed a warped five-dimensional model to naturally realize the EW scale from the Plack scale. Appelquist, Cheng, Dobrescu considered a possibility of universal extra-dimension (UED) model, which gives a DM candidate (LKP).

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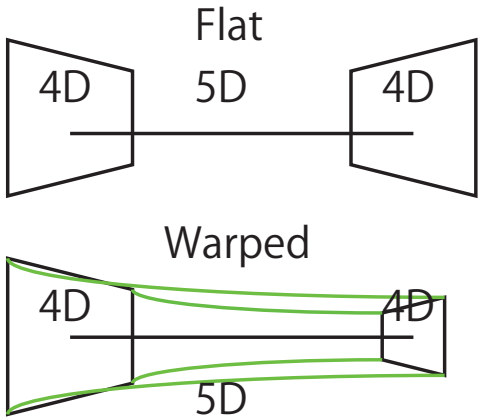

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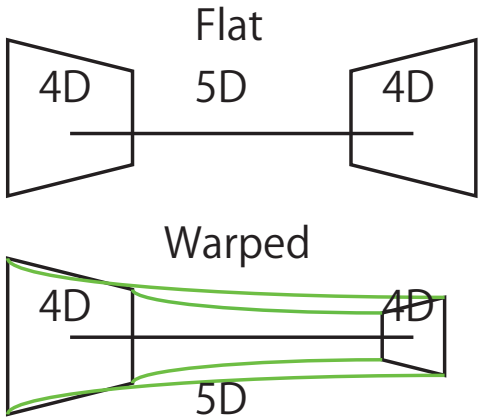

- **Split seesaw mechanism considers a flat five-dimension to obtain *splitting* mass spectra of sterile neutrinos without a large hierarchy among model parameters.**

1. Introduction

Outline of my talk

DM candidate

sterile ν of M~O(keV)

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BAU
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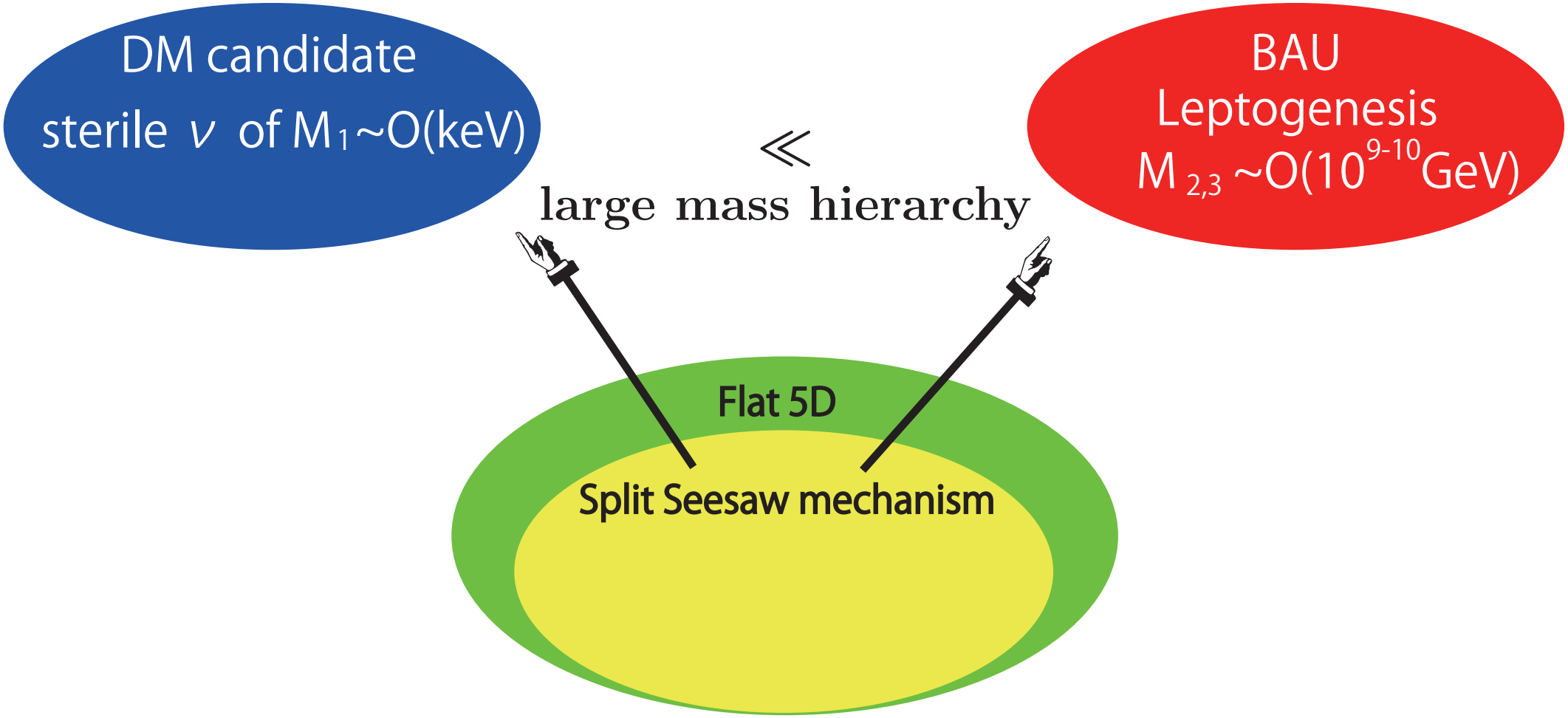

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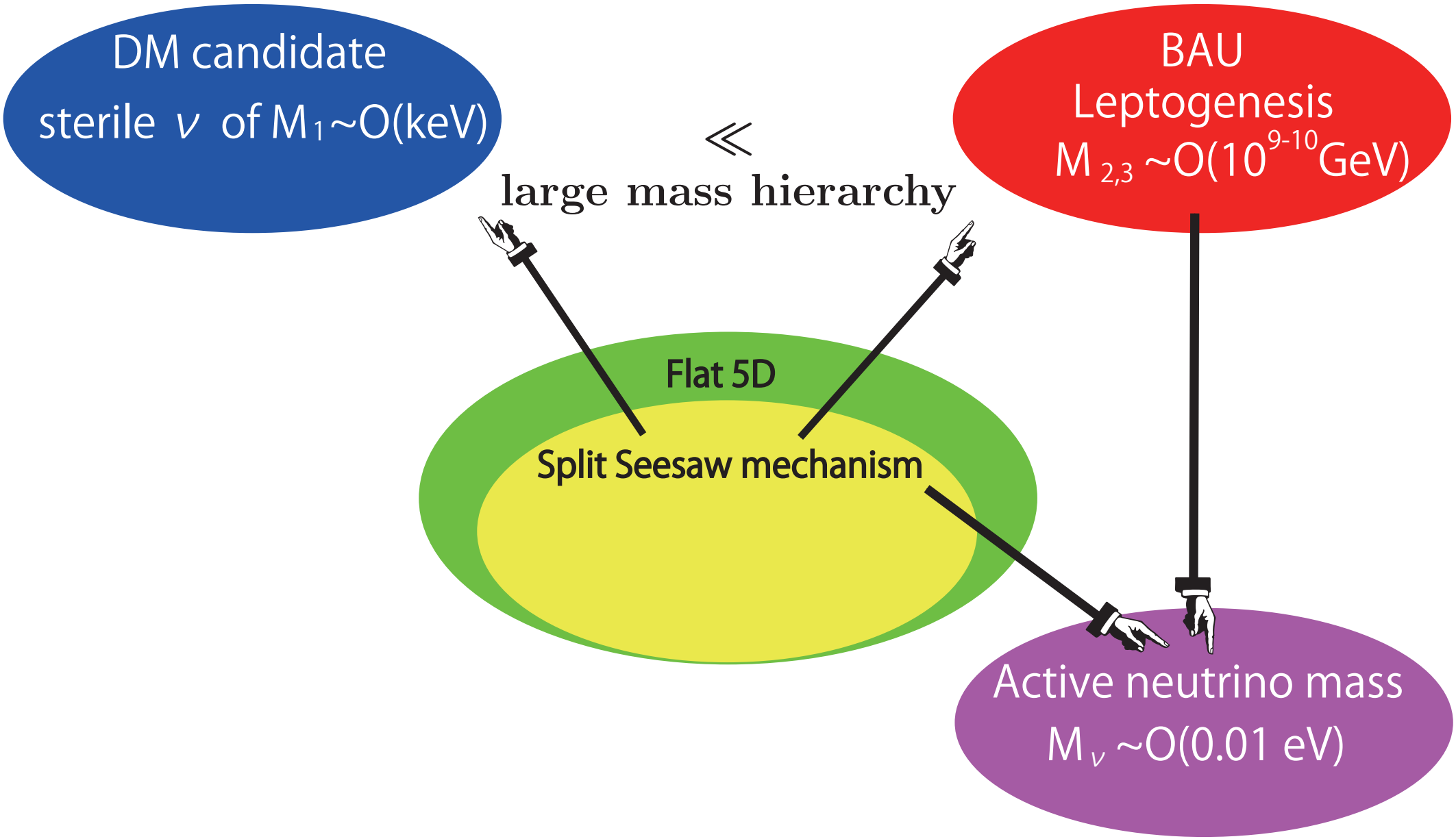

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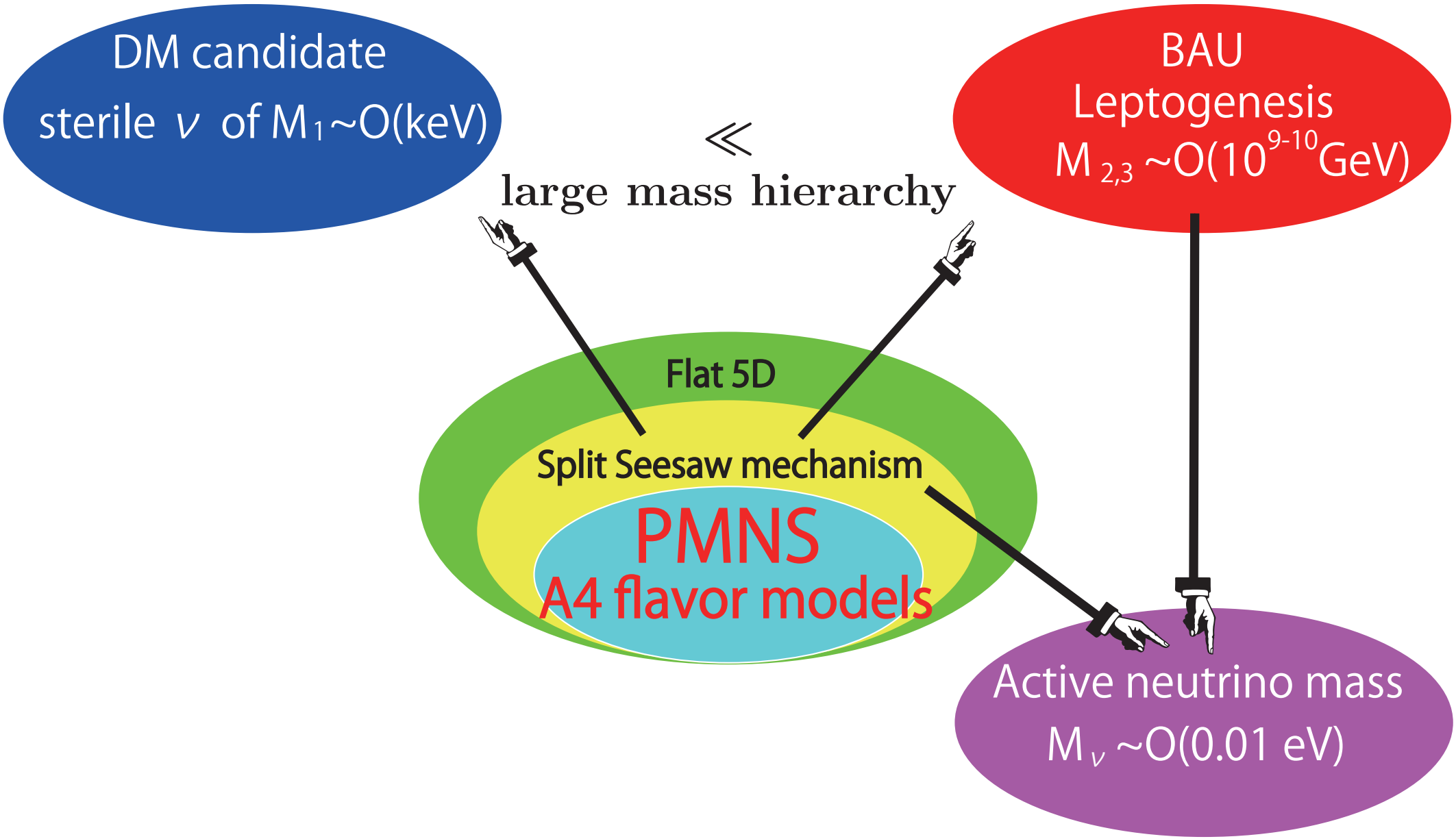


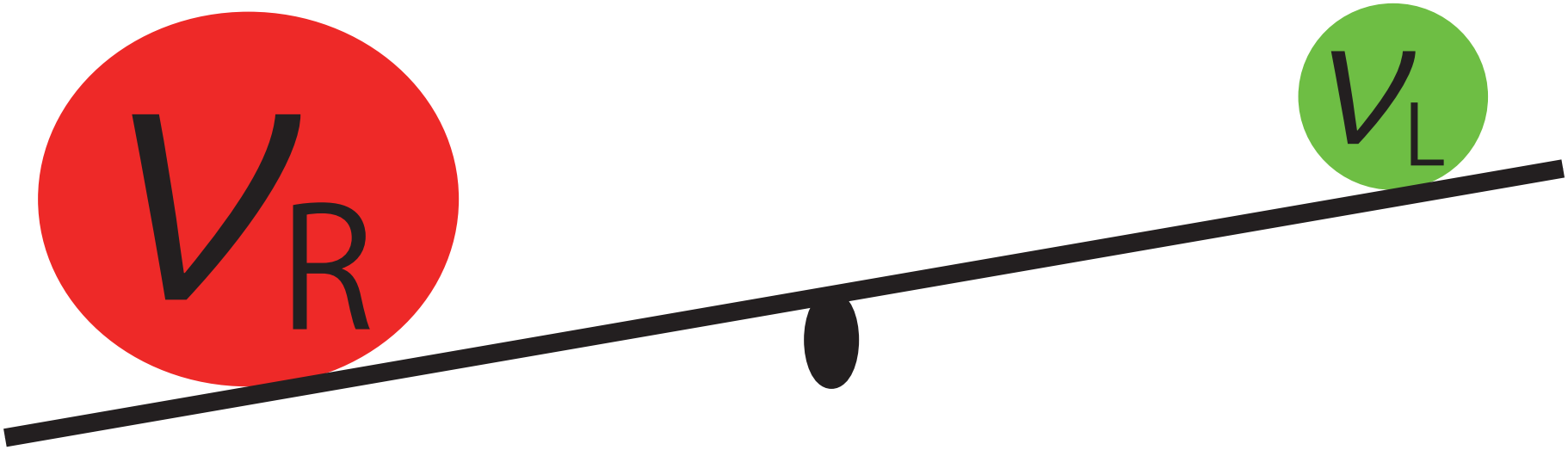
2. Split seesaw mechanism

- Canonical type I seesaw mechanism;

$$\mathcal{L} = i\bar{\nu}_{Ri}\gamma^{\mu}\partial_{\mu}\nu_{Ri} + \left(\lambda_{i\alpha}\bar{\nu}_{Ri}L_{\alpha}\phi - \frac{1}{2}M_{R,ij}\bar{\nu^c}_{Ri}\nu_{Rj} + \text{h.c.}\right)$$

$\Downarrow$ after integrating out ν_R

$$M_{\nu} = \lambda^T M_R^{-1}\lambda\langle\phi^0\rangle^2 \quad \text{if} \quad \lambda\langle\phi^0\rangle \ll M_R$$



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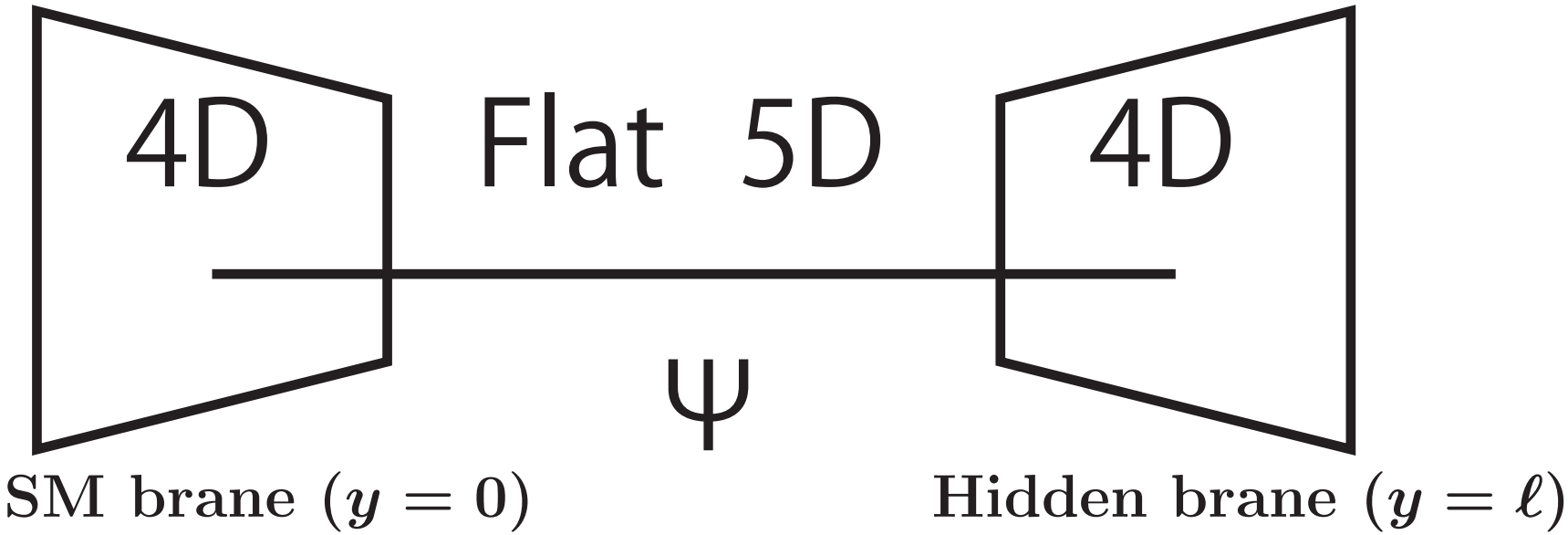
A. Kusenko, F. Takahashi, T.T. Yanagida



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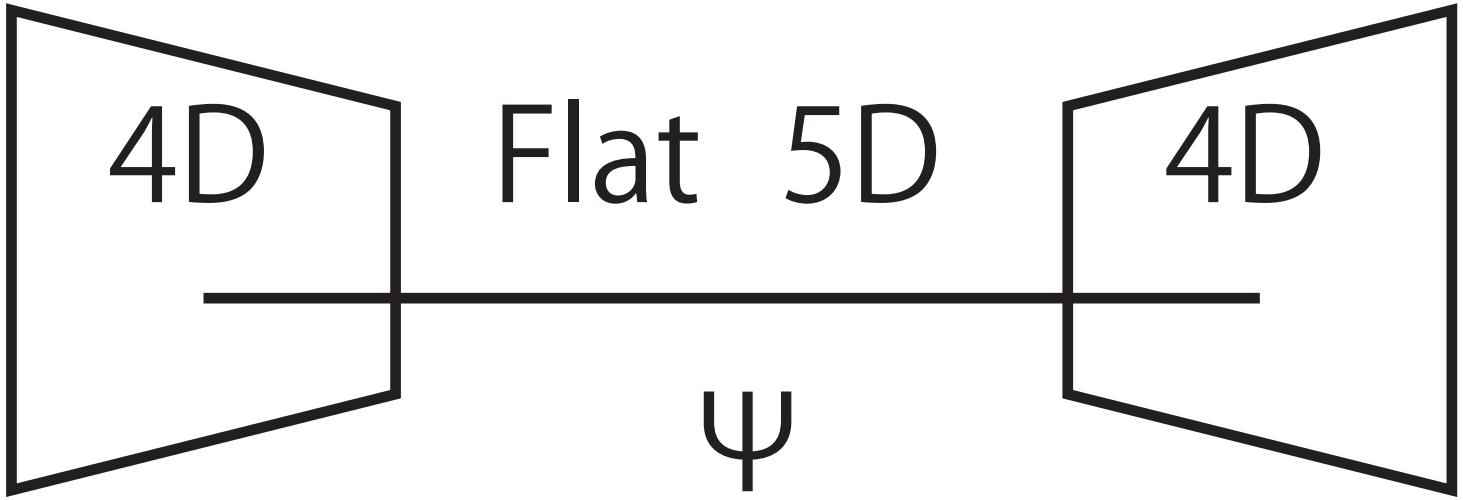

SM brane ($y = 0$) Hidden brane ($y = \ell$)

- Bulk action;

$$S = \int d^4x dy M (i\bar{\Psi}\Gamma^A \partial_A \Psi + m\bar{\Psi}\Psi)$$

- Wave function profile of 0-mode (right-handed neutrino);

$$\Psi_R^{(0)}(y,x) = \sqrt{\frac{2m}{e^{2m\ell}-1}} \frac{1}{\sqrt{M}} e^{my} \psi_R^{(0)}(x), \quad \psi_R^{(0)}(x) = \nu_R(x)$$

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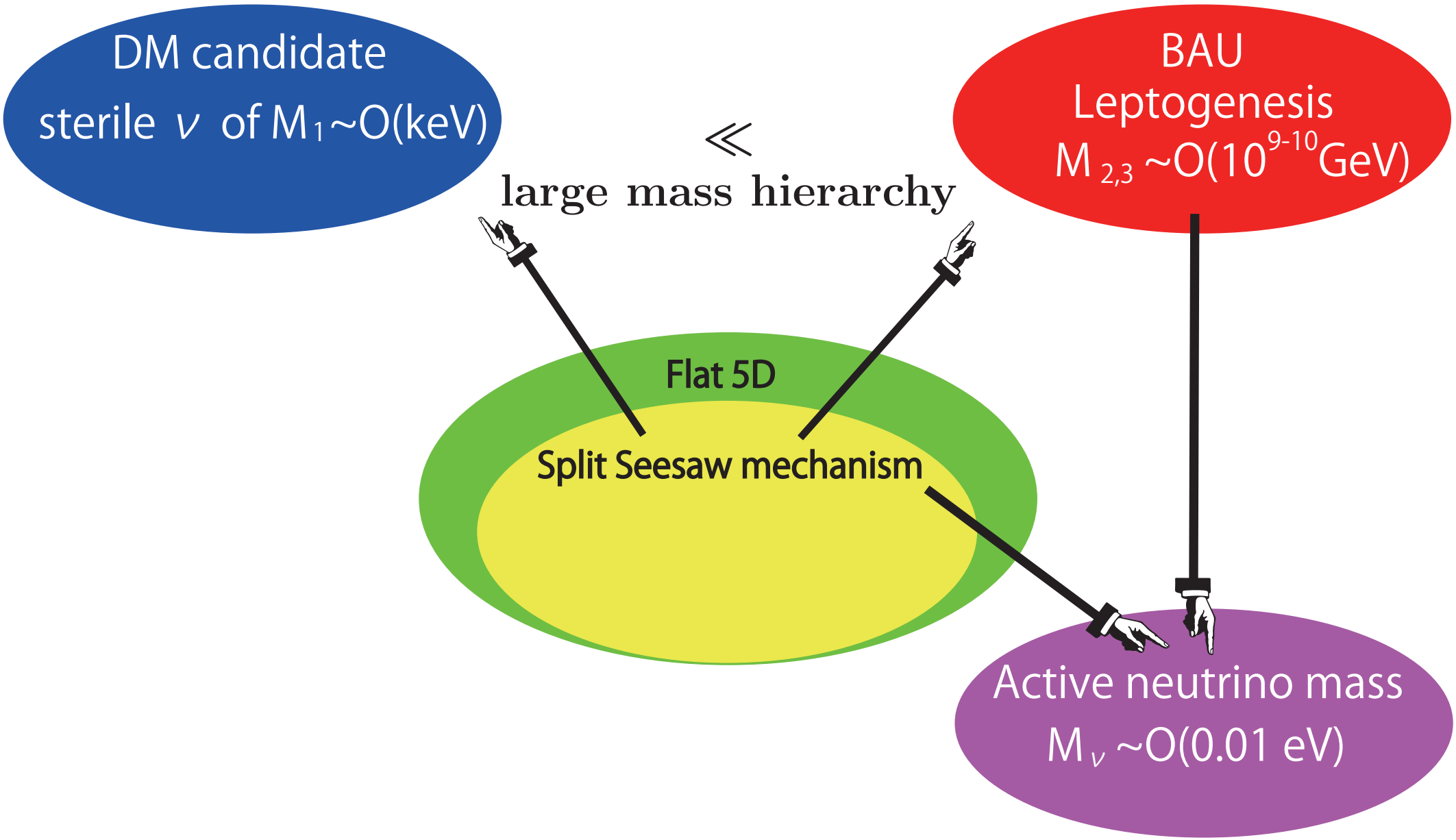

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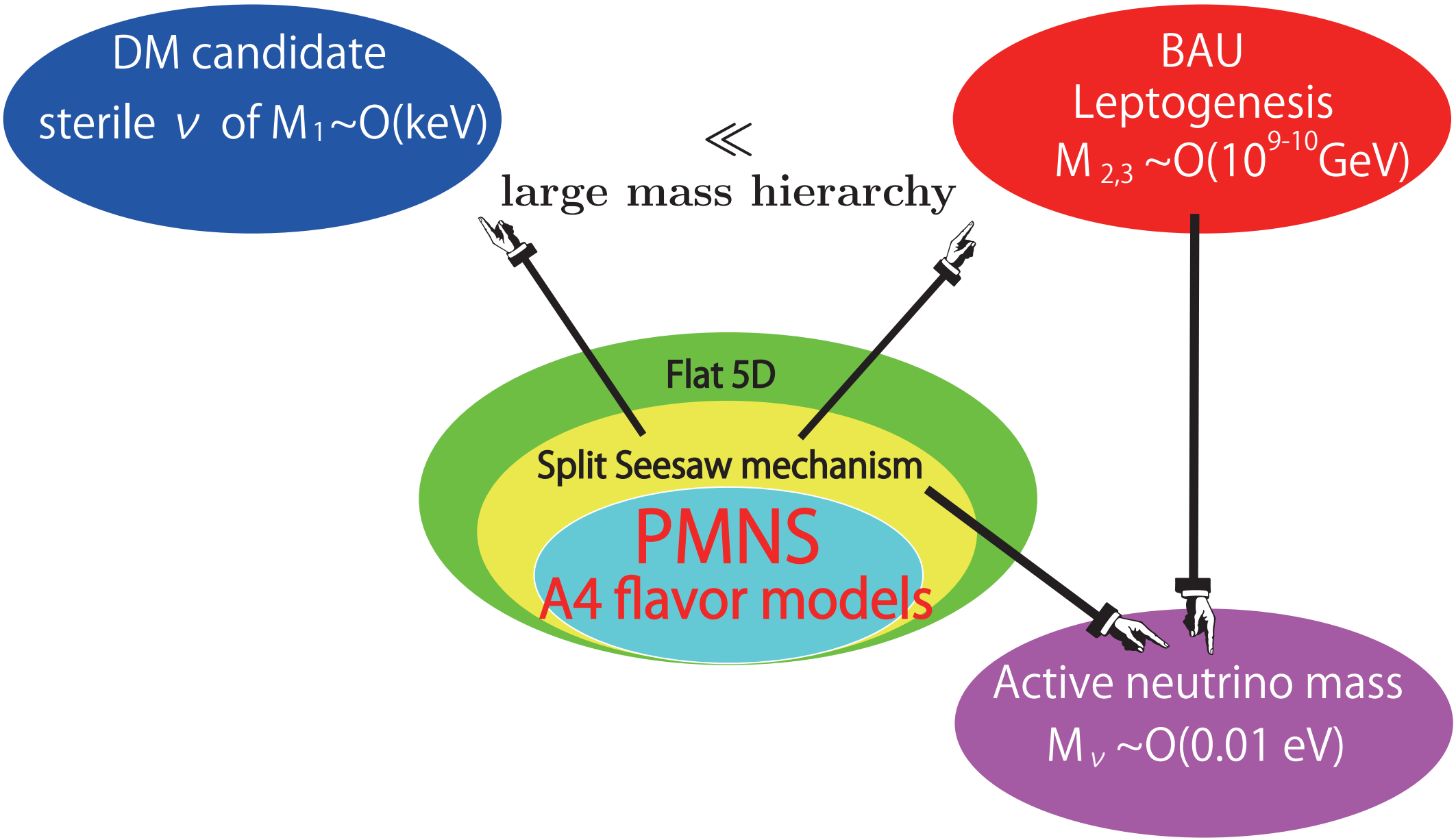

3. A_4 flavor models in split seesaw mechanism

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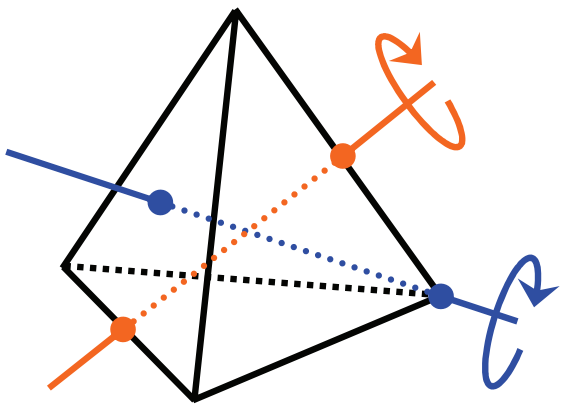

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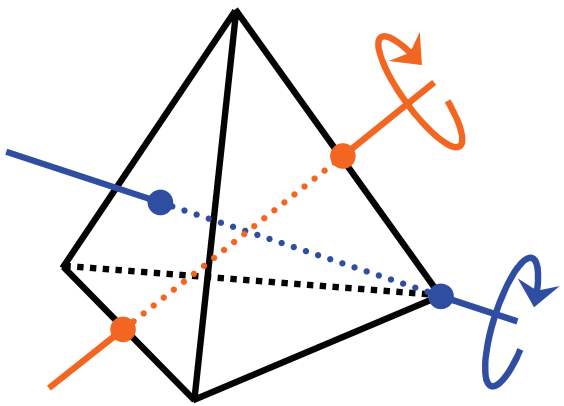

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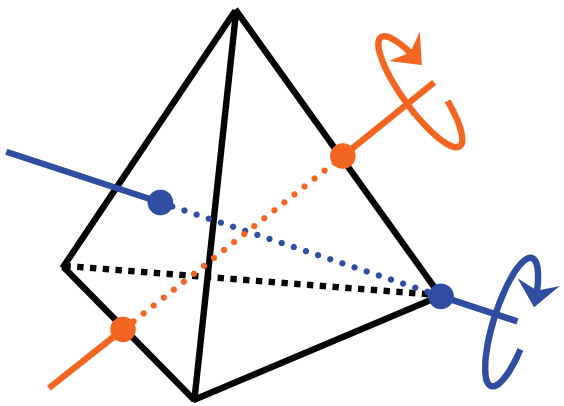

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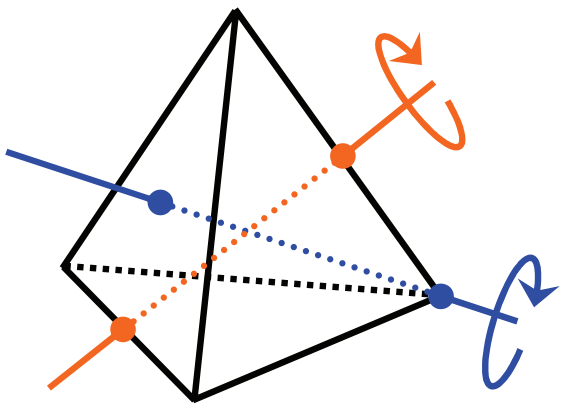

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- The A_4 is the smallest non-Abelian discrete group which has the $\underline{\mathbf{3}}$ representation.
- An introduction of A_4 as a flavor symmetry might be well motivated for the $\mathbf{3}$ generations of the SM fermions.

3. A_4 flavor models in split seesaw mechanism

Barry-Rodejohann (BR) classification for A_4 flavor models

Type	L_α	$\overline{E}_\alpha$	$\overline{\Psi}_{iR}$
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F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$
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- Type B, D, F, and J models assign the right-handed neutrinos to the $\underline{3}$ under the A_4 symmetry.
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3. A_4 flavor models in split seesaw mechanism

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E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\cdots$
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$

- The type E model was proposed by Ma in 2005.
- Detailed numerical analyses were given by Lavoura and Kuhbock in 2006.
- This model is a simple extension of the SM. $\Rightarrow$ later on

3. A_4 flavor models in split seesaw mechanism

Barry-Rodejohann (BR) classification for A_4 flavor models

Type	L_α	$\overline{E}_\alpha$	$\overline{\Psi}_{iR}$
A	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\cdots$
B	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$
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- Frampton and Matsuzaki pointed out that a simple model in a class of type G cannot give a realistic active neutrino mass spectrum.

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- Type I model is the first complete supersymmetric model of flavor based on A_4 symmetry together with the $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ Pati-Salam gauge symmetry.
- This model does not suffer from the *degeneracy problem.*

3. A_4 flavor models in split seesaw mechanism

- Ma model (2005):

Field	L_α	$\overline{\Psi}_{1R}$	$\overline{\Psi}_{2R}$	$\overline{\Psi}_{3R}$	ϕ	$\varphi_{\nu,t}$
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{1}$	$\underline{3}$

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- Neutrino Dirac Mass matrix:

$$M_D = \begin{pmatrix} r f_1 y_1^\nu & f_1 y_1^\nu & f_1 y_1^\nu \\ r f_2 y_2^\nu & \omega f_2 y_2^\nu & \omega^2 f_2 y_2^\nu \\ r f_3 y_3^\nu & \omega^2 f_3 y_3^\nu & \omega f_3 y_3^\nu \end{pmatrix} \frac{u}{\Lambda} v_{ew}, \quad \langle \varphi_{\nu,t} \rangle \equiv \begin{pmatrix} ru \\ u \\ u \end{pmatrix}$$

- Right-handed Majorana mass matrix:

$$M_R = \begin{pmatrix} a f_1^2 & 0 & 0 \\ 0 & 0 & b f_2 f_3 \\ 0 & b f_2 f_3 & 0 \end{pmatrix} v_{\mathrm{B-L}}, \quad f_i \equiv \frac{1}{\sqrt{M}} \sqrt{\frac{2 m_i}{e^{2 m_i \ell} - 1}}$$

- Light neutrino mass matrix after the seesaw mechanism:

$$M_\nu = \begin{pmatrix} r^2 x & r y & r y \\ r y & x & y \\ r y & y & x \end{pmatrix} m_\nu, \quad \begin{cases} x \equiv \dfrac{(y_1^\nu)^2}{a} + \dfrac{2 y_2^\nu y_3^\nu}{b}, \\ y \equiv \dfrac{(y_1^\nu)^2}{a} - \dfrac{y_2^\nu y_3^\nu}{b}, \end{cases} m_\nu \equiv \frac{v_{ew}^2}{v_{\mathrm{B-L}}} \left(\frac{u}{\Lambda}\right)^2$$

3. A_4 flavor models in split seesaw mechanism

- Fitting the experimentally observed values:

$$M_\nu = \begin{pmatrix} r^2 x & ry & ry \\ ry & x & y \\ ry & y & x \end{pmatrix} m_\nu \Rightarrow \begin{cases} \theta_{23} = \frac{\pi}{4},\ \theta_{13} = 0 \text{ @ LO}, \\ \Delta m^2_{21}(x,y,r),\ \Delta m^2_{31}(x,y,r), \\ \sin^2\theta_{12}(x,y,r) \end{cases}$$

Active neutrino mass scale requires

$$x m_\nu,\ y m_\nu \simeq \sqrt{\Delta m^2_{i1}} \sim \mathcal{O}(10^{-2})\ \text{eV} \quad (i = 2, 3)$$

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- Cosmological constraints on the left-right mixing angle in keV sterile neutrino DM scenario:

$$\theta^2 = \frac{\sum_\alpha |(M_D)_{1\alpha}|^2}{M^2_{R,1}} < 5.8 \times 10^{-9} \left(\frac{5\ \text{keV}}{M_{R,1}}\right)^5 \Leftarrow \text{X-ray bound}$$

$$M_D = \begin{pmatrix} r f_1 y^\nu_1 & f_1 y^\nu_1 & f_1 y^\nu_1 \\ r f_2 y^\nu_2 & \omega f_2 y^\nu_2 & \omega^2 f_2 y^\nu_2 \\ r f_3 y^\nu_3 & \omega^2 f_3 y^\nu_3 & \omega f_3 y^\nu_3 \end{pmatrix} \frac{u}{\Lambda} v_{ew},\ M_R = \begin{pmatrix} a f_1^2 & 0 & 0 \\ 0 & 0 & b f_2 f_3 \\ 0 & b f_2 f_3 & 0 \end{pmatrix} v_{\text{B}-\text{L}}$$

$$\Rightarrow \frac{|y^\nu_1|^2}{a} m_\nu \simeq (x + 2y) m_\nu < 1.4 \times 10^{-5}\ \text{eV} \neq \mathcal{O}(10^{-2})\ \text{eV}$$

3. A_4 flavor models in split seesaw mechanism

- What's happened?

$$\theta^2(<10^{-8}) = \sum_\alpha |(M_D)_{1\alpha}|^2/M_{R,1}^2 \simeq m_\nu/M_{R,1} \sim 10^{-6}$$

	w/ flavor sym. (e.g. A_4)	w/o flavor sym.
M_D	$\propto \begin{pmatrix} r f_1 y_1^\nu & f_1 y_1^\nu & f_1 y_1^\nu \\ r f_2 y_2^\nu & \omega f_2 y_2^\nu & \omega^2 f_2 y_2^\nu \\ r f_3 y_3^\nu & \omega^2 f_3 y_3^\nu & \omega f_3 y_3^\nu \end{pmatrix}$ constrained	$\propto \begin{pmatrix} y_{1e}^\nu & y_{1\mu}^\nu & y_{1\tau}^\nu \\ y_{2e}^\nu & y_{2\mu}^\nu & y_{2\tau}^\nu \\ y_{3e}^\nu & y_{3\mu}^\nu & y_{3\tau}^\nu \end{pmatrix}$ unconstrained
M_R	$\propto \begin{pmatrix} a f_1^2 & 0 & 0 \\ 0 & 0 & b f_2 f_3 \\ 0 & b f_2 f_3 & 0 \end{pmatrix}$ constrained	$\propto \begin{pmatrix} M_{R,1} & 0 & 0 \\ 0 & M_{R,2} & 0 \\ 0 & 0 & M_{R,3} \end{pmatrix}$ unconstrained
M_ν	$\propto \begin{pmatrix} r^2 x & r y & r y \\ r y & x & y \\ r y & y & x \end{pmatrix}$ parameters reduced	$\propto \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}$ parameters unreduced

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3. A_4 flavor models in split seesaw mechanism

- How can we avoid the problem in the context of flavor symmetry, split seesaw mechanism, and keV ν_s scenario?
  - extension of the basic Ma model (but so complicated $\cdots$)
    - adding other scalar fields

Field	Δ_L	$\varphi_{\nu,s}$	$\tilde{\varphi}_{\nu,t}$
A_4	$\underline{1}$	$\underline{1}$	$\underline{3}$

    - type II seesaw contribution to neutrino mass

$$S_\nu = S_I + S_{II}$$

      type II contribution can realize the two neutrino mass squared differences while satisfying the cosmological bounds on the sterile neutrino coupling and respecting with A_4 symmetry.
    - Leptogenesis can be realized in type I+II scenario.

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- **Candidate for DM**
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- A_4 has **3** singlets and one $\underline{\mathbf{3}}$ representations, and is the smallest non-Aberian discrete group having $\underline{\mathbf{3}}$ representation.
- An introduction of A_4 as a flavor symmetry might be well motivated for the **3** generations of the SM fermions.

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♠ **A_4 flavor models with 3 right-handed neutrinos being A_4 triplet suffer from a *degeneracy problem* for the bulk mass term, which disturbs the split mechanism for right-handed neutrino mass spectrum.**

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	PMNS	Split seesaw	keV ν_s DM
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♠ We can make the model realistic in the context of split seesaw mechanism with A_4 flavor symmetry by extending the scalar sector of the model (but so complicated $\cdots$).