Split Seesaw Mechanism and Flavor Symmetry

Seminar @ Osaka University

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Reference

A. Adulpravitchai, RT, JHEP 1109 (2011) 127

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1. Introduction

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  • Candidate for dark matter (LSP? LKP? sterile neutrino?)

  • Origin of baryon asymmetry of the Universe

  • Stabilization of the Higgs mass (SUSY? extra-dimension?)

  • Inflation

  • Dark energy

Mass Scales: ln(m)

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<thead>
<tr>
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- Sterile neutrino as a candidate for DM;
  - In a simple extension of the SM, 3 sterile (right-handed) neutrinos, which are singlets under the SM gauge group, are added: (0906.2968 [hep-ph], good review!)

\[
\begin{align*}
\begin{pmatrix} \nu_L & \nu_R \\ 0 & M_D \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} & \Rightarrow |\nu_a\rangle = \cos \theta |\nu_L\rangle + \sin \theta |\nu_R\rangle \\
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\Gamma = \frac{G_F^2 M_s^5}{96\pi^3} \theta^2 \Rightarrow \tau_s \sim 10^{20}\text{sec} \left(\frac{\text{keV}}{M_s}\right)^5 \theta^{-2}
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\[
\nu_s \xrightarrow{\theta} \nu_a \\
Z^0 \\
\bar{\nu}_a
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- Dodelson and Widrow pointed out that a life-time of the sterile neutrino can be longer than the age of the Universe and a cosmological density \(\Omega_s = \Omega_{\text{DM}} \sim 0.2\) can be realized if the sterile neutrino mass is in the \(\mathcal{O}(\text{keV})\) scale.
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\[ \gamma \rightarrow W^+ W^- + \nu_s \nu_a \]

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\[ \sin^2 \theta_{1e} \]

\[ m_s \text{ (keV)} \]
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- Sakharov’s conditions;
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  (ii) C and CP invariances are violated,
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![Diagram of extra-dimensional theories]

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- Extra-dimensional theory;

- Split seesaw mechanism considers a flat five-dimension to obtain *splitting* mass spectra of sterile neutrinos without a large hierarchy among model parameters.
1. Introduction

Outline of my talk

DM candidate
sterile $\nu$ of $M \sim O(\text{keV})$
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Flat 5D

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Split Seesaw mechanism

Active neutrino mass $M_\nu \sim O(0.01\,\text{eV})$
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  - sterile $\nu$ of $M_1 \sim O(\text{keV})$

- Large mass hierarchy
  - $M_{2,3} \sim O(10^{9-10} \text{ GeV})$

- Split Seesaw mechanism
  - PMNS
  - A4 flavor models

- Flat 5D

- BAU
  - Leptogenesis
  - Active neutrino mass $M_\nu \sim O(0.01 \text{ eV})$
2. Split seesaw mechanism

- Canonical type I seesaw mechanism;

\[ \mathcal{L} = i \bar{\nu}_R i \gamma^\mu \partial_\mu \nu_R i + \left( \lambda_{i\alpha} \bar{\nu}_R i L_\alpha \phi - \frac{1}{2} M_{R,ij} \bar{\nu}^c R_i \nu_R j + \text{h.c.} \right) \]

\[ \downarrow \text{after integrating out } \nu_R \]

\[ M_\nu = \lambda^T M_R^{-1} \lambda \langle \phi^0 \rangle^2 \quad \text{if} \quad \lambda \langle \phi^0 \rangle \ll M_R \]
2. Split seesaw mechanism

A. Kusenko, F. Takahashi, T.T. Yanagida

\[ S = \int d^4x dy M \left( \overline{\Psi} \Gamma^A \partial_A \Psi + m \overline{\Psi} \Psi \right) \]

Wave function profile of 0-mode (right-handed neutrino):

\[ \Psi(0)^R(y,x) = \sqrt{\frac{2}{m_\ell}} e^{\frac{2m}{m_\ell}y} \Psi(x), \quad \Psi(0)^R(x) = \nu_R(x) \]

SM brane \((y = 0)\)

Hidden brane \((y = \ell)\)
2. Split seesaw mechanism

A. Kusenko, F. Takahashi, T.T. Yanagida

- Bulk action:
  \[ S = \int d^4x dy M (i \bar{\Psi} \Gamma^A \partial_A \Psi + m \bar{\Psi} \Psi) \]

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  \[ \Psi_R^{(0)}(y, x) = \sqrt{\frac{2m}{e^{2m\ell} - 1}} \frac{1}{\sqrt{M}} e^{my} \psi_R^{(0)}(x), \quad \psi_R^{(0)}(x) = \nu_R(x) \]
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A. Kusenko, F. Takahashi, T.T. Yanagida

- Effective bulk action for the three right-handed neutrinos;

\[ S = \int d^4 x \, dy \left\{ M \left( i \bar{\Psi}_{iR}^{(0)} \Gamma^A \partial_A \Psi_{iR}^{(0)} + m_i \bar{\Psi}_{iR}^{(0)} \Psi_{iR}^{(0)} \right) + \delta(y) \left( \frac{\kappa_i}{2} v_{B-L} \bar{\Psi}_{iR}^{(0) c} \Psi_{iR}^{(0)} + \tilde{\lambda}_i \bar{\Psi}_{iR}^{(0)} L_{\alpha} \phi + h.c. \right) \right\} \]

- Wave function profiles of 0-modes (right-handed neutrinos);

\[ \Psi_{iR}^{(0)}(y, x) = \sqrt{\frac{2m_i}{e^{2m_i \ell} - 1}} \frac{1}{\sqrt{M}} e^{m_i y} \psi_{iR}^{(0)}(x), \quad \psi_{iR}^{(0)}(x) = \nu_{iR}(x) \]
2. Split seesaw mechanism

A. Kusenko, F. Takahashi, T.T. Yanagida

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2. Split seesaw mechanism

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\( (M_{R,1}, M_{R,2}, M_{R,3}) = (1 \text{ keV}, 10^{11} \text{ GeV}, 10^{12} \text{ GeV}) \),

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Outline of my talk

DM candidate
sterile $\nu$ of $M_1 \sim O(\text{keV})$

$\ll$

large mass hierarchy

BAU
Leptogenesis
$M_{2,3} \sim O(10^{9-10}\text{ GeV})$

Flat 5D

Split Seesaw mechanism

Active neutrino mass
$M_\nu \sim O(0.01 \text{ eV})$
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PMNS

A4 flavor models

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3. $A_4$ flavor models in split seesaw mechanism

- The alternating group $(A_N)$ is formed by even permutations among $N$ objects.
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- The alternating group ($A_N$) is formed by even permutations among $N$ objects.
- The $A_4$ group is formed by even permutations among 4 objects:
  \[(x_1, x_2, x_3, x_4) \rightarrow (x_i, x_j, x_k, x_l)\] e.g. \((x_3, x_2, x_1, x_4)\)
3. $A_4$ flavor models in split seesaw mechanism

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3. $A_4$ flavor models in split seesaw mechanism

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- The $A_4$ symmetry can be understood by specific rotations of tetrahedron:

- The $A_4$ group has 3 singlets and one 3 representations.
- The $A_4$ is the smallest non-Abelian discrete group which has the 3 representation.
- An introduction of $A_4$ as a flavor symmetry might be well motivated for the 3 generations of the SM fermions.
3. $A_4$ flavor models in split seesaw mechanism

Barry-Rodejohann (BR) classification for $A_4$ flavor models

<table>
<thead>
<tr>
<th>Type</th>
<th>$L_\alpha$</th>
<th>$E_\alpha$</th>
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- Type A, C, and H models do not have the right-handed neutrinos.
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- Type B, D, F, and J models assign the right-handed neutrinos to the $3$ under the $A_4$ symmetry.
- We found that these models cannot be embedded into the split seesaw mechanism.
3. $A_4$ flavor models in split seesaw mechanism

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- Bulk mass term for $\Psi_R = (\Psi_{1R}, \bar{\Psi}_{2R}, \bar{\Psi}_{3R})^T$:

$$m \bar{\Psi}_R \Psi_R = m(\bar{\Psi}_{1R} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{2R} + \bar{\Psi}_{3R} \Psi_{3R})$$

⇒ Degeracy problem: $m_1 = m_2 = m_3$

⇒ cannot realize mass *splitting* in split seesaw mechanism.
2. Split seesaw mechanism

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- The type E model was proposed by Ma in 2005.
- Detailed numerical analyses were given by Lavoura and Kuhbock in 2006.
- This model is a simple extension of the SM. ⇒ later on
Barry-Rodejohann (BR) classification for $A_4$ flavor models

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<td>$1', 1', 1'$</td>
</tr>
<tr>
<td>J</td>
<td>$\bar{3}$</td>
<td>$1', 1', 1'$</td>
<td>$3$</td>
</tr>
</tbody>
</table>

• Frampton and Matsuzaki pointed out that a simple model in a class of type $G$ cannot give a realistic active neutrino mass spectrum.
3. $A_4$ flavor models in split seesaw mechanism

Barry-Rodejohann (BR) classification for $A_4$ flavor models

<table>
<thead>
<tr>
<th>Type</th>
<th>$L_\alpha$</th>
<th>$E_\alpha$</th>
<th>$\bar{\Psi}_{iR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>1, 1 ′, 1 ″</td>
<td>...</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1, 1 ′, 1 ″</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>...</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
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<td>3</td>
<td>1, 1 ′, 1 ″</td>
</tr>
<tr>
<td>F</td>
<td>1, 1 ′, 1 ″</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>3</td>
<td>1, 1 ′, 1 ″</td>
<td>1, 1 ′, 1 ″</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>1, 1, 1</td>
<td>...</td>
</tr>
<tr>
<td>I</td>
<td>3</td>
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<td>1, 1, 1</td>
</tr>
<tr>
<td>J</td>
<td>3</td>
<td>1, 1, 1</td>
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</tr>
</tbody>
</table>

- Type I model is the first complete supersymmetric model of flavor based on $A_4$ symmetry together with the $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ Pati-Salam gauge symmetry.
- This model does not suffer from the degeneracy problem.
3. $A_4$ flavor models in split seesaw mechanism

- Ma model (2005):

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<tr>
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<th>$\Psi_{2R}$</th>
<th>$\Psi_{3R}$</th>
<th>$\phi$</th>
<th>$\varphi_{\nu,t}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>1'</td>
<td>1''</td>
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<td>1''</td>
<td>1</td>
<td>3</td>
</tr>
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</table>

- Neutrino Dirac Mass matrix:

$$M_D = \begin{pmatrix} rf_1y_1' & f_1y_1' & f_1y_1' \\ rf_2y_2' & \omega f_2y_2' & \omega^2 f_2y_2' \\ rf_3y_3' & \omega^2 f_3y_3' & \omega f_3y_3' \end{pmatrix} \frac{u}{\Lambda} v_{ew}, \quad \langle \varphi_{\nu,t} \rangle \equiv \begin{pmatrix} ru \\ u \\ u \end{pmatrix}$$

- Right-handed Majorana mass matrix:

$$M_R = \begin{pmatrix} af_1^2 & 0 & 0 \\ 0 & 0 & bf_2f_3 \\ 0 & bf_2f_3 & 0 \end{pmatrix} v_{B-L}, \quad f_i \equiv \frac{1}{\sqrt{M}} \sqrt{\frac{2m_i}{e^{2m_i\ell} - 1}}$$

- Light neutrino mass matrix after the seesaw mechanism:

$$M_{\nu} = \begin{pmatrix} r^2x & ry & ry \\ ry & x & y \\ ry & y & x \end{pmatrix} m_{\nu}, \quad \begin{cases} x \equiv \frac{(y_1')^2}{a} + \frac{2y_2'y_3'}{b}, \\ y \equiv \frac{(y_1')^2}{a} - \frac{y_2'y_3'}{b}, \end{cases} \quad m_{\nu} \equiv \frac{v_{ew}^2}{v_{B-L}} \left( \frac{u}{\Lambda} \right)^2$$
3. $A_4$ flavor models in split seesaw mechanism

- Fitting the experimentally observed values:

$$
M_\nu = \begin{pmatrix}
    r^2x & ry & ry \\
    ry & x & y \\
    ry & y & x
\end{pmatrix}
\Rightarrow
\begin{cases}
\theta_{23} = \frac{\pi}{4}, \theta_{13} = 0 @ LO, \\
\Delta m^2_{21}(x, y, r), \Delta m^2_{31}(x, y, r), \\
\sin^2 \theta_{12}(x, y, r)
\end{cases}
$$

Active neutrino mass scale requires

$$
x m_\nu, y m_\nu \simeq \sqrt{\Delta m^2_{i1}} \sim \mathcal{O}(10^{-2}) \text{ eV} \quad (i = 2, 3)
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3. $A_4$ flavor models in split seesaw mechanism

- Fitting the experimentally observed values:

$$M_{\nu} = \begin{pmatrix} r^2x & ry & ry \\ ry & x & y \\ ry & y & x \end{pmatrix} m_{\nu} \Rightarrow \begin{cases} \theta_{23} = \frac{\pi}{4}, \theta_{13} = 0 @ LO, \\ \Delta m_{21}^2(x, y, r), \Delta m_{31}^2(x, y, r), \\ \sin^2 \theta_{12}(x, y, r) \end{cases}$$

Active neutrino mass scale requires

$$xm_{\nu}, ym_{\nu} \simeq \sqrt{\Delta m_{i1}^2} \sim \mathcal{O}(10^{-2}) \text{ eV} \quad (i = 2, 3)$$

- Cosmological constraints on the left-right mixing angle in keV sterile neutrino DM scenario:

$$\theta^2 = \frac{\sum_\alpha |(M_D)_{1\alpha}|^2}{M_{R,1}^2} < 5.8 \times 10^{-9} \left(\frac{5 \text{ keV}}{M_{R,1}}\right)^5 \Leftarrow \text{X-ray bound}$$

$$M_D = \begin{pmatrix} rf_1y_1'' & f_1y_1'' & f_1y_1'' \\ rf_2y_2'' & \omega f_2y_2'' & \omega^2 f_2y_2'' \\ rf_3y_3'' & \omega^2 f_3y_3'' & \omega f_3y_3'' \end{pmatrix} \frac{u}{\Lambda} v_{ew}, \quad M_R = \begin{pmatrix} af_1^2 & 0 & 0 \\ 0 & 0 & bf_2f_3 \\ 0 & bf_2f_3 & 0 \end{pmatrix} v_{B-L}$$

$$\Rightarrow \frac{|y_{1\nu}|^2}{a} m_{\nu} \simeq (x + 2y)m_{\nu} < 1.4 \times 10^{-5} \text{ eV} \neq \mathcal{O}(10^{-2}) \text{ eV}$$
3. $A_4$ flavor models in split seesaw mechanism

- What’s happened?

$$\theta^2(<10^{-8}) = \sum_{\alpha} |(M_D)_{1\alpha}|^2/M_{R,1}^2 \approx m_ \nu / M_{R,1} \sim 10^{-6}$$

<table>
<thead>
<tr>
<th></th>
<th>w/ flavor sym. (e.g. $A_4$)</th>
<th>w/o flavor sym.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_D$</td>
<td>$\propto \begin{pmatrix} rf_1 y_1^\nu &amp; f_1 y_1^\nu &amp; f_1 y_1^\nu \ r f_2 y_2^\nu &amp; \omega f_2 y_2^\nu &amp; \omega^2 f_2 y_2^\nu \ r f_3 y_3^\nu &amp; \omega^2 f_3 y_3^\nu &amp; \omega f_3 y_3^\nu \end{pmatrix}$</td>
<td>$\propto \begin{pmatrix} y_1^\nu e &amp; y_1^\nu \mu &amp; y_1^\nu \tau \ y_2^\nu e &amp; y_2^\nu \mu &amp; y_2^\nu \tau \ y_3^\nu e &amp; y_3^\nu \mu &amp; y_3^\nu \tau \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
<td>constrained</td>
<td>unconstrained</td>
</tr>
<tr>
<td>$M_R$</td>
<td>$\propto \begin{pmatrix} a f_1^2 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; b f_2 f_3 \ 0 &amp; b f_2 f_3 &amp; 0 \end{pmatrix}$</td>
<td>$\propto \begin{pmatrix} M_{R,1} &amp; 0 &amp; 0 \ 0 &amp; M_{R,2} &amp; 0 \ 0 &amp; 0 &amp; M_{R,3} \end{pmatrix}$</td>
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<tr>
<td></td>
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<td>unconstrained</td>
</tr>
<tr>
<td>$M_\nu$</td>
<td>$\propto \begin{pmatrix} r^2 x &amp; ry &amp; ry \ ry &amp; x &amp; y \ ry &amp; y &amp; x \end{pmatrix}$</td>
<td>$\propto \begin{pmatrix} A &amp; B &amp; C \ B &amp; D &amp; E \ C &amp; E &amp; F \end{pmatrix}$</td>
</tr>
<tr>
<td></td>
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<td>parameters unreduced</td>
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- Ma model can be embedded into *split seesaw mechanism* but cannot satisfy cosmological bounds on the *keV sterile neutrino DM scenario.*
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- This is because we used

$$\theta^2(<10^{-8}) = \sum_{\alpha} |(M_D)_{1\alpha}|^2/M_{R,1}^2 \approx m_\nu/M_{R,1} \sim 10^{-6}.$$  

This equality means the keV sterile neutrino contributes to the active neutrino mass through the seesaw mechanism.
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- We had to use this relation in Ma model because the Yukawa structure was *constrained by* $(A_4)$ flavor symmetry.
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- Flavor models tend to suffer from such kind of problem.

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<th>PMNS</th>
<th>Split seesaw</th>
<th>keV $\nu_s$</th>
<th>DM</th>
</tr>
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<tbody>
<tr>
<td>$\bigcirc$</td>
<td>$\bigtriangleup$</td>
<td>$\bigtriangleup$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. $A_4$ flavor models in split seesaw mechanism

• How can we avoid the problem in the context of flavor symmetry, split seesaw mechanism, and keV $\nu_s$ scenario?
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- How can we avoid the problem in the context of flavor symmetry, split seesaw mechanism, and keV $\nu_s$ scenario?
  - extension of the basic Ma model (but so complicated ...)
    - adding other scalar fields

<table>
<thead>
<tr>
<th>Field</th>
<th>$\Delta L$</th>
<th>$\varphi_{\nu, s}$</th>
<th>$\tilde{\varphi}_{\nu, t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4$</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
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- type II seesaw contribution to neutrino mass

$$S_\nu = S_I + S_{II}$$

type II contribution can realize the two neutrino mass squared differences while satisfying the cosmological bounds on the sterile neutrino coupling and respecting with $A_4$ symmetry.

- Leptogenesis can be realized in type I+II scenario.
4. Summary (motivation)

♠ There are unsolved problems in the context of SM:
  ● Property of neutrino (tiny masses and PMNS structure)
  ● Candidate for DM
  ● Origin of BAU
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  • $A_4$ has 3 singlets and one 3 representations, and is the smallest non-Aberian discrete group having 3 representation.
  • An introduction of $A_4$ as a flavor symmetry might be well motivated for the 3 generations of the SM fermions.
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♠ $A_4$ flavor models with 3 right-handed neutrinos being $A_4$ triplet suffer from a *degeneracy problem* for the bulk mass term, which disturbs the split mechanism for right-handed neutrino mass spectrum.
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♠ We can make the model realistic in the context of split seesaw mechanism with $A_4$ flavor symmetry by extending the scalar sector of the model (but so complicated \ldots).