Strongly coupled gauge theories: In and out of the conformal window

Anna Hasenfratz
University of Colorado Boulder

Osaka University
Feb 18, 2014

In collaboration with A. Cheng, Y. Liu, G. Petropoulos and D. Schaich
July 4th
July 4th 2012: Higgs boson “discovered”

0++ scalar at 126 GeV:
Standard Model like
- no sign of new TeV-scale physics!
For Nobel, They Can Thank the ‘God Particle’
Higgs and Englert Are Awarded Nobel Prize in Physics

Higgs Boson Particle Theory Wins Nobel: Dennis Overbye, a Times reporter, explains the particle’s significance — and insignificance — in understanding our universe.
July 4th 2012: Higgs boson “discovered”

0++ scalar at 126 GeV:
- Standard Model like
- no sign of new TeV-scale physics!

What is this “SMS”?  
- Elementary scalar?  
- SUSY?  
- Composite?
What’s wrong with the SM Higgs?

.... nothing really

The Higgs sector

• Requires enormous fine tuning of the parameters (naturalness)
• (Trivial: mathematically inconsistent: \( \lambda(\mu) \to 0 \) as \( \Lambda \to \infty \))
• Vacuum is metastable due to heavy top quark
• Provides no dynamical explanation for electroweak symmetry breaking or flavor physics

SUSY could solve/explain all this but

• no SUSY particles have been detected
• Higgs is uncomfortably heavy for most SUSY models
Composite Higgs:

Assume a new gauge-fermion system at high energies (techni-)
If it is chirally broken the techni-pions are the Goldstone bosons
of electroweak symmetry breaking, the $0^{++}$ meson is the Higgs

Does it agree with experimental data?

- Scaled-up QCD models are out (were ruled out decades ago)!
  - EW measurements are violated ($g^2$ runs too fast)
- Walking TC models: gauge coupling that evolves slowly with
  energy and a large anomalous dimension could solve most
  these problems;
  - Do they predict Standard Model like scalar?
    - dilaton of spontaneously broken conformal symmetry
    - pseudo-Goldstone of expanded flavor symmetry
Composite Higgs in strongly coupled systems:

SU($N_{\text{color}} \geq 2$) gauge fields + $N_{\text{flavor}}$ fermions in some representation

![Diagram showing the relationship between $N_{\text{flavor}}$ and $N_{\text{color}}$ with regions labeled as IR freedom, Conformal phase, and Chirally broken phase.]
Composite Higgs in strongly coupled systems:

\[ \text{SU}(N_{\text{color}} \geq 2) \text{ gauge fields} + N_{\text{flavor}} \text{ fermions in some representation} \]
Composite Higgs in strongly coupled systems:

SU($N_{\text{color}} \geq 2$) gauge fields + $N_{\text{flavor}}$ fermions in some representation

IR freedom

Conformal phase

Chirally broken phase

$N_{\text{flav}}$

$N_{\text{color}}$
Composite Higgs in strongly coupled systems:

SU($N_{\text{color}} \geq 2$) gauge fields + $N_{\text{flavor}}$ fermions in some representation

IR freedom   Conformal phase   Chirally broken phase

$N_{\text{flav}}$  $N_{\text{color}}$
Composite Higgs in strongly coupled systems:

SU($N_{\text{color}} \geq 2$) gauge fields + $N_{\text{flavor}}$ fermions in some representation

Early lattice results suggest
- light scalar (LatKMI)
- enhanced chiral condensate
- suppressed S parameter
Roadmap for the conformal window

Cartoon

S-D type calculations

Shaded: conformal
Below: confining
Above: IR free
Dotted lines: 2-loop PT

fermion representation:
Fundamental
Adjoint
2Symmetric
2Antisymmm

Needs non-perturbative verification!
Roadmap for the conformal window

Cartoon

S-D type calculations

Shaded: conformal
Below: confining
Above: IR free
Dotted lines: 2-loop PT

F

F

Fundamental
Adjoint
2S
2A
2S

Symmetric
Antisymmm

\[ N_c \rightarrow \text{LATTICE} \]

Needs non-perturbative verification!
Strongly coupled systems

A purely theoretical point of view:

Are there strongly coupled fermion-gauge systems that have a non-trivial, non-perturbative infrared dynamics?

What are the properties of these systems?
– conformal?
– large anomalous mass dimension?

These systems are interesting even if there is no direct BSM application
In this talk: $N_f = 4, 8$ and $12$ fundamental fermions

Concentrate on
$N_f=12$:
- controversial system near the conformal boundary

$N_f=8$:
- likely chirally broken, probably walking

$N_f=4$:
- QCD like, uncontroversial

Questions to answer:
- Is the system conformal or chirally broken (and walking)?
- Is there a light scalar?
- Is the $S$ parameter small? What is the anomalous mass dim.?
- ....
Simple enough .... cannot be much harder than QCD

It is surprisingly difficult to distinguish conformal, walking, and chirally broken systems on the lattice
Fixed point structure of a **chirally broken system**

**m=0 critical surface:** **one fixed point**

Continuum limit:
Tune bare $g^2 \rightarrow 0$ and $m \rightarrow 0$ : renormalized $g^2$ anywhere on renormalized trajectory

Wilson RG

$g_1$: gauge coupling
$g_2, \ldots$ : irrelevant couplings

Perturbative FP
$g_1=0, m=0$ : 2 relevant directions

$g_{2r}$

$g_1$

$g_2/g_1$

Perturb FP
Fixed point structure of a \textit{conformal system}

\begin{itemize}
  \item Two possible continuum limits:
    \begin{enumerate}
      \item Tune bare $g^2 \to 0$ and $m \to 0$ : renormalized $g^2$ anywhere on renormalized trajectory
      \item Tune only $m \to 0$ : renormalized $g^2 = g^2_{\text{IRFP}}$
    \end{enumerate}
\end{itemize}
Fixed point structure of a conformal system

m=0 critical surface: two fixed points

Two possible continuum limits:
1. Tune bare $g^2 \to 0$ and $m \to 0$: renormalized $g^2$ anywhere on renormalized trajectory
2. Tune only $m \to 0$: renormalized $g^2 = g^2_{\text{IRFP}}$

Perturbative FP
$g_1=0, m=0$: 2 relevant directions

IRFP
$g_1=g_{\text{IRFP}}, m=0$: 1 relevant direction

Wilson RG
Fixed point structure of a conformal system

m=0 critical surface: two fixed points

Two possible continuum limits:
1. Tune bare $g^2 \to 0$ and $m \to 0$: renormalized $g^2$ anywhere on renormalized trajectory
2. Tune only $m \to 0$: renormalized $g^2 = g^2_{\text{IRFP}}$
It is surprisingly difficult to distinguish conformal, walking, and chirally broken systems on the lattice.

- they look very similar along the RT
- if the gauge coupling “walks” : g is nearly marginal!
  (non-QCD like)
SU(3) gauge with \( N_f = 4, 8 \) and 12 fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density \( \rho(\lambda) \)
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
   Shows the effect of the near marginal gauge coupling
SU(3) gauge with $N_f= 4, 8$ and $12$ fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density $\rho(\lambda)$
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
   Shows the effect of the near marginal gauge coupling
SU(3) gauge with $N_f = 4, 8$ and $12$ fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density $\rho(\lambda)$
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
   Shows the effect of the near marginal gauge coupling $m \to 0$, $L \to \infty$
SU(3) gauge with N_f= 4, 8 and 12 fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density ρ(λ)
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
   Shows the effect of the near marginal gauge coupling

Mostly N_f=4 and 12 flavor to test the methods and understand/resolve existing controversies.
Some N_f=8 : preliminary but exciting!
Scaling of the Dirac eigenvalue spectrum - **conformal system**

Eigenvalue density \( \rho(0)=0 \), scales as \( \rho(\lambda) \propto \lambda^{\alpha(\lambda)} \)

RG invariance implies \( \frac{4}{1+\alpha} = \gamma_m = 1 + \gamma_m \)

\( \lambda \) provides an energy scale

- **IR** – small \( \lambda \) region:
  \[ \gamma_m(\lambda \to 0) = \gamma_m^* \]
  predicts the universal anomalous dimension at the IRFP

- **UV** – large \( \lambda = O(1) \) region:
  if governed by the asymptotically free perturbative FP
  \[ \gamma_m(\lambda=O(1)) = \gamma_0 g^2 + ... \]

In between:
  scale dependent effective \( \gamma_m \)
Scaling of the Dirac eigenvalue spectrum - conformal system

Eigenvalue density $\rho(0) = 0$, scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies
$$\frac{4}{1 + \alpha} = \gamma_m = 1 + \gamma_m$$

$\lambda$ provides an energy scale

IR – small $\lambda$ region:
$$\gamma_m(\lambda \to 0) = \gamma_m^*$$
predicts the universal anomalous dimension at the IRFP

UV – large $\lambda = \mathcal{O}(1)$ region:
if governed by the asymptotically free perturbative FP
$$\gamma_m(\lambda = \mathcal{O}(1)) = \gamma_0 g^2 + ...$$

In between:
scale dependent effective $\gamma_m$
Scaling of the Dirac eigenvalue spectrum - **conformal system**

Eigenvalue density $\rho(0)=0$, scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies $\frac{4}{1+\alpha} = y_m = 1 + \gamma_m$

$\lambda$ provides an energy scale

\[\gamma_m \leq 1, \quad \alpha \geq 1\]

\[\gamma_m \to 0, \quad \alpha \to 3\]
Scaling of the Dirac eigenvalue spectrum - conformal system

Eigenvalue density $\rho(0) = 0$, scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies $\frac{4}{1+\alpha} = y_m = 1 + \gamma_m$

$\lambda$ provides an energy scale

![Diagram showing perturbative and IRF regimes with $\lambda$ as an energy scale]
Scaling of the Dirac eigenvalue spectrum - conformal system

Eigenvalue density $\rho(0)=0$, scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies $\frac{4}{1+\alpha} = y_m = 1 + \gamma_m$

$\lambda$ provides an energy scale
Chirally broken systems show only the asymptotically free region.
Results: $N_f = 4$

Broken chiral symmetry in IR, asymptotic freedom in UV

Lattice spacing from Wilson flow:

\[
\frac{a_{6.4}}{a_{7.4}} = 2.84(3) \\
\frac{a_{6.6}}{a_{7.4}} = 2.20(5) \\
\frac{a_{7.0}}{a_{7.4}} = 1.45(3) \\
\frac{a_{8.0}}{a_{7.4}} = 0.60(4)
\]
Results: $N_f = 4$

Broken chiral symmetry in IR, asymptotic freedom in UV

Lattice spacing from Wilson flow:

\[
\frac{a_{6.4}}{a_{7.4}} = 2.84(3) \\
\frac{a_{6.6}}{a_{7.4}} = 2.20(5) \\
\frac{a_{7.0}}{a_{7.4}} = 1.45(3) \\
\frac{a_{8.0}}{a_{7.4}} = 0.60(4)
\]
Results: $N_f = 4$

Broken chiral symmetry in IR, asymptotic freedom in UV

Lattice spacing from Wilson flow:

\[
\begin{align*}
    a_{6,4} / a_{7,4} &= 2.84(3) \\
    a_{6,6} / a_{7,4} &= 2.20(5) \\
    a_{7,0} / a_{7,4} &= 1.45(3) \\
    a_{8,0} / a_{7,4} &= 0.60(4)
\end{align*}
\]
Rescaling: $N_f = 4$

The dimension of $\lambda$ is carried by the lattice spacing: $\lambda_{\text{lat}} = \lambda p a$

Rescale to a common physical scale:

$$\lambda_\beta \rightarrow \lambda_\beta \left( \frac{a_{7.4}}{a_\beta} \right)^{1+\gamma_m(\lambda_\beta)}$$

Lattice spacing from Wilson flow:

$$a_{6.4} / a_{7.4} = 2.84(3)$$
$$a_{6.6} / a_{7.4} = 2.20(5)$$
$$a_{7.0} / a_{7.4} = 1.45(3)$$
$$a_{8.0} / a_{7.4} = 0.60(4)$$
Rescaling: \( N_f = 4 \)

The dimension of \( \lambda \) is carried by the lattice spacing: \( \lambda_{\text{lat}} = \lambda_{p}a \)

Rescale to a common physical scale:

\[
\lambda_{\beta} \rightarrow \lambda_{\beta} \left( \frac{a_{7.4}}{a_{\beta}} \right)^{1+\gamma_{m}(\lambda_{\beta})}
\]

Universal curve covering almost 2 orders of magnitude in energy!

Perturbative: functional form from 1-loop PT, relative scale is fitted

Most of these data were obtained on deconfined (small) volumes at \( m = 0 \)!
Spectral density results: $N_f = 12$

$\beta = 3.0, 4.0, 5.0, 6.0$

- There is no sign of asymptotic freedom behavior for $\beta < 6.0$,
- $\gamma_m$ grows towards UV
- Not possible to rescale different $\beta$'s to a single universal curve

Looks as if there was an IRFP between $\beta = 5.0 - 6.0$
Spectral density results: \( N_f = 12 \)

\( \beta = 3.0, 4.0, 5.0, 6.0 \)

- There is no sign of asymptotic freedom behavior for \( \beta < 6.0 \),
- \( \gamma_m \) grows towards UV
- Not possible to rescale different \( \beta \)'s to a single universal curve

Looks as if there was an IRFP between \( \beta = 5.0 - 6.0 \)
Spectral density results: $N_f = 12$

- $\beta = 3.0, 4.0, 5.0, 6.0$
  - There is no sign of asymptotic freedom behavior for $\beta < 6.0$,
  - $\gamma_m$ grows towards UV
  - Not possible to rescale different $\beta$’s to a single universal curve

Looks as if there was an IRFP between $\beta = 5.0 - 6.0$
Rescaling $N_f=4$ vs $N_f=12$

$N_f=4$ : smaller $\beta$ matches to the left (forward flow)

$N_f=12$ : no consistent rescaling but even an approximate one matches to the right of $\beta<6.0$
Anomalous dimension, $N_f = 8$

Expected to be chirally broken - looks like walking!

- No asymptotic free scaling
- No rescale of different couplings
- When $\gamma_m \sim 1$ in the UV, the $S^4b$ phase develops
Anomalous dimension, $N_f = 8$

Expected to be chirally broken - looks like walking!

- No asymptotic free scaling
- No rescale of different couplings
- When $\gamma_m \sim 1$ in the UV, the $S^4b$ phase develops
Anomalous dimension, $N_f = 8$

Expected to be chirally broken - looks like walking!

- No asymptotic free scaling
- No rescale of different couplings

- When $\gamma_m \sim 1$ in the UV, the $S^4b$ phase develops
Dirac operator eigenvalue spectrum and spectral density

Unique & promising method!
- Can distinguish strong and weak coupling region of conformal / chirally broken systems

Predictions:
\( N_f=4 \) : scaling & anomalous dimension
\( N_f=12 \) : looks conformal
\( N_f=8 \) : could be walking with large anomalous dimension!
II : Finite size scaling

Well understood method in systems governed by one relevant operator
\( \rightarrow \) in conformal systems it could predict the mass anomalous dimension

Is this prediction internally consistent? Is it consistent with results of spectral density?

Results for Nf=12 system only
Finite size scaling - textbook case

Consider a FP with one relevant operator 
\( m \approx 0 \) with scaling dimension \( y_m > 0 \)
and irrelevant operators 
\( g_i \) with scaling dimensions \( y_i < 0 \).

Renormalization group arguments in volume \( L^3 \) predict scaling of physical masses as

\[
M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0
\]

as \( m \to 0, \ L \to \infty: \ g_i m^{-y_i/y_0} \to 0 \)

\[
M_H L = f(x), \quad x = Lm^{1/y_m}
\]

--tune \( y_m \) until different volumes “collapse”
Scaling exponents

Result of “curve collapse” for pseudo-scalar, vector and $f_{\pi}$:

$y_m$ depends strongly on $\beta$ and the operator considered

→ Internally inconsistent !!!
Finite size scaling with a near-marginal operator

Consider a FP with one relevant operator
\[ m \approx 0 \text{ with scaling dimension } y_m > 0 \]
and irrelevant operators
\[ g_i \text{ with scaling dimensions } y_i < 0 \]
g\(_0\) (near) marginal, \( y_0 \ll 0 \)

Renormalization group arguments in volume \( L^3 \) predict
\[ M_H L = f(L m^{1/y_m}, g_i m^{-y_i/y_m}) \text{ as } m \approx 0 \]
as \( m \to 0, \ L \to \infty : g_i m^{-y_i/y_0} \to 0 \)
\[ g_0 \to g_0 m^{\omega}, \ \omega = -y_0 / y_m \gtrsim 0 \]
\[ M_H L = f(x, g_0 m^{\omega}), \ x = L m^{1/y_m} \]

The scaling function depends on two variables now!
Corrections to finite size scaling

Physical masses scale as

\[ M_H = L^{-1} f(x, g_0 m^\omega), \quad \omega = -y_0 / y_m \]

If the \( g_0 m^\omega \) corrections are small, expand

\[ L M_H = F(x)(1 + g_0 m^\omega G(x)) \]

Approximate \( G(x) = c \) (should be checked) \( \rightarrow \)

\[ \frac{L M_H}{1 + c g_0 m^\omega} = F(x) \]

Fit needs minimization in \( y_m, \omega, \) and \( c_0 = c g_0 \)
Scaling exponent \textbf{with} corrections

Include all data $M_{\Pi} L$, $M_{\Psi} L$, $f_{\Pi} L$ points

Leading operator only \hspace{2cm} With correction

Fits show
- good curve collapse
- consistent scaling exponent $y_m = 1.22(2)$
- can we constrain the fit parameters better?
Scaling exponent with corrections

Include all data $M_{\pi L}$, $M_{V L}$, $f_{\pi L}$ points

Leading operator only

With correction

Fits show
- good curve collapse
- consistent scaling exponent $y_m=1.22(2)$
- can we constrain the fit parameters better?
Combining data sets:

If the gauge coupling is irrelevant, the scaling function $F(x)$

\[
\frac{LM}{1+cg_0m^\omega} = F(x)
\]

is unique, independent of

- gauge coupling $\beta$
- lattice action (nHYP or stout or HISQ)

Combine different data sets

- we need to rescale the bare fermion mass $m(\beta) \rightarrow s \, m(\beta)$
- remnant scaling violations could be different for different sets
  $\rightarrow$ most noticeable at small $x$ (or $L$)
Combining gauge couplings:

\textbf{pion} at $\beta=4.0,4.5$ (all available volumes):

$y_m=1.23[2]$, $y_0=-0.47[6]$; \hspace{1cm} $\chi^2$/dof $=1.2$ [60]
Combining gauge couplings AND operators

pion and vector at $\beta=4.0, 4.5$ (new fit!)

$y_m=1.22[2]$, $y_0=-0.50[5]$; $\chi^2/$dof $=1.4$ [108]
Combining gauge couplings AND actions

**pion** at $\beta=4.0, 4.5$, LH, KMI:

$y_m=1.24[1], \quad y_0=-0.51[5] \quad ; \quad \chi^2 / \text{dof} = 1.4 \,[ 95]$

**Diagram:**

- $\beta = 4.0$
- $\beta = 4.5$
- $\beta = 2.2; \ LH$
- $\beta = 3.7; \ LatKMI$
- $\beta = 4.0; \ LatKMI$
Combining gauge couplings AND actions AND operators

pion and vector at $\beta=4.0,4.5$, LHC, KMI:
$y_m=1.27[1]$, $y_0=-0.51[5]$; $\chi^2$/dof $=2.7$ [188]
Consistency:

Fit 30-300 points with 10 - 20 parameters ...
Consistency:

Fit 30-300 points with 10 - 20 parameters ...

“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.”

John von Neumann
Consistency:

Fit 30-300 points with 10 - 20 parameters ... yet $y_m, y_0$, are consistent

“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.”

John von Neumann

Fits combining different data sets, operators, predict $y_m = 1 + \gamma_m = 1.235[15]$ with $\chi^2$/dof $\approx 1 - 3$
The gauge coupling of strongly coupled conformal systems are expected to run slowly ("walking") → scaling is strongly influenced by this near-marginal coupling

This is universal in every walking system!

- In finite size scaling analysis the marginal coupling can be accounted for
- Its effect should be considered in every other approach
Summary

Strongly coupled gauge-fermion systems are exciting
- show non-perturbative dynamics with unusual properties
- can offer BSM description with composite Higgs

Near the conformal window they (could)
- walk: slowly changing gauge coupling
- large anomalous dimension
- dilaton: light scalar?

Lattice studies are only starting to understand these systems