Quark Confinement via Magnetic Color-Flavor Locking

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My dream

a new type of unification

quark confinement

chiral symmetry breaking

in QCD

unify as a Higgs mechanism
in the magnetic picture

electroweak symmetry breaking

unify as a higher dimensional QCD

(Higgs mechanism as UV QCD dynamics)

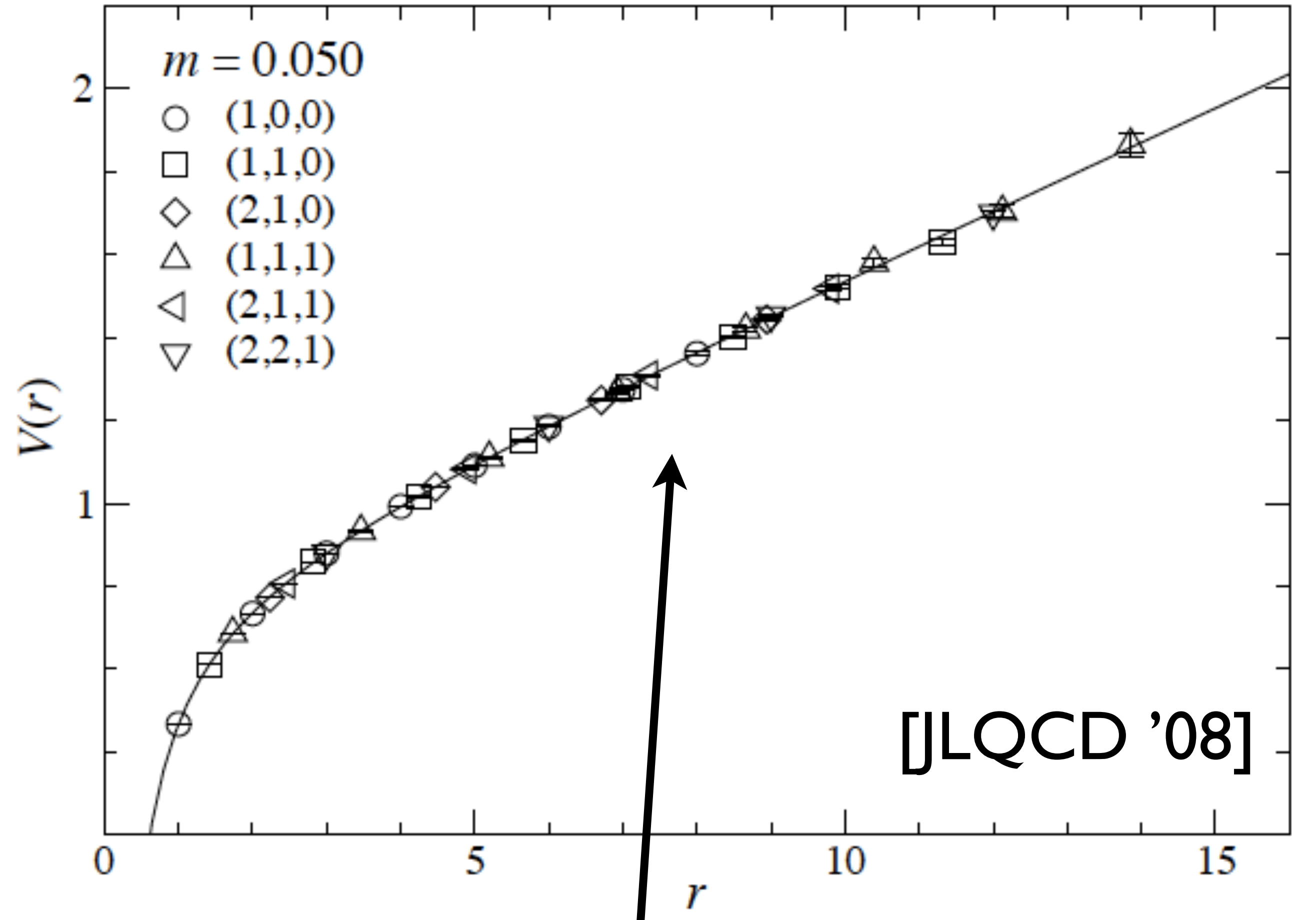
quark confinement

In QCD, quarks are confined.

Lattice QCD

$$V(R) = -\frac{A}{R} + \sigma R.$$

$$A \sim 0.25 - 0.5, \quad \sqrt{\sigma} \sim 430 \text{ MeV}.$$



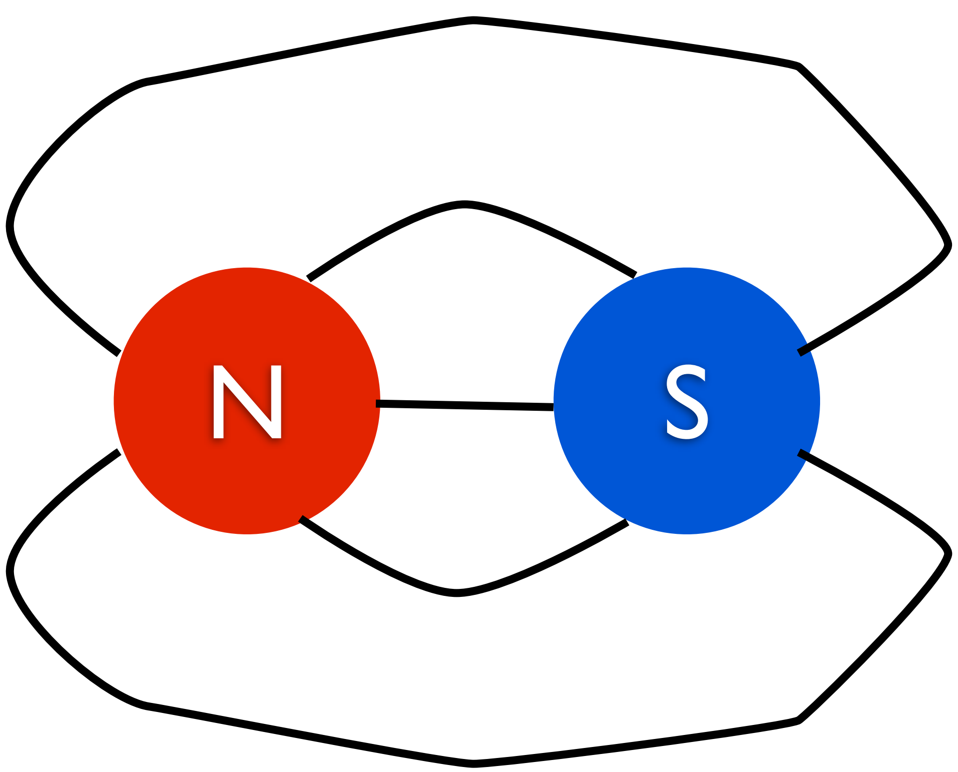
linear potential

Why?

There is a pretty simple picture.

Confinement is dual to Higgs mechanism, and in the dual picture, the quarks are magnetic monopoles.

[Mandelstam '75, 't Hooft '75]



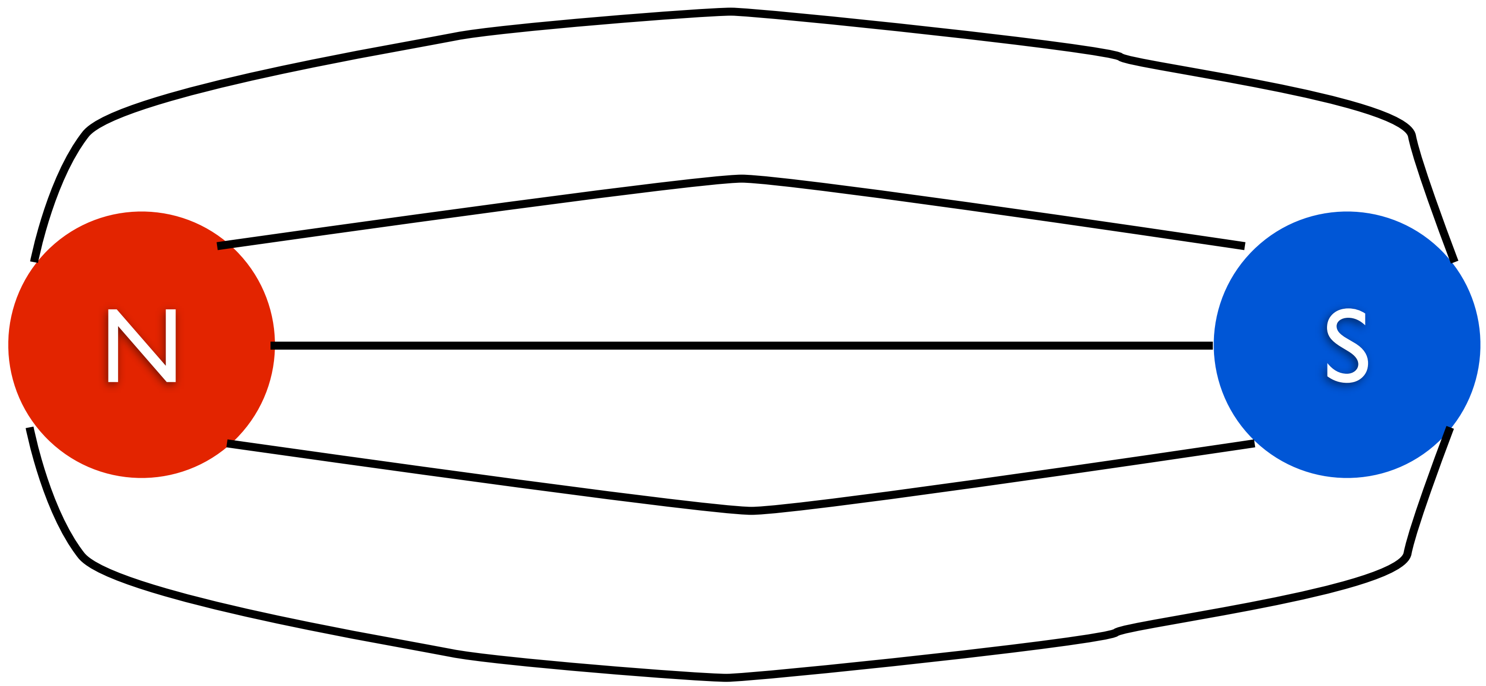
Coulomb like

Why?

There is a pretty simple picture.

Confinement is dual to Higgs mechanism, and in the dual picture, the quarks are magnetic monopoles.

[Mandelstam ’75, ’t Hooft ’75]



Why?

There is a pretty simple picture.

Confinement is dual to Higgs mechanism, and in the dual picture, the quarks are magnetic monopoles.

[Mandelstam ’75, ’t Hooft ’75]

Linear potential

chiral symmetry breaking

$$m_{\pi^\pm} = 139.57 \text{ MeV}$$
$$\ll 1 \text{ GeV}$$

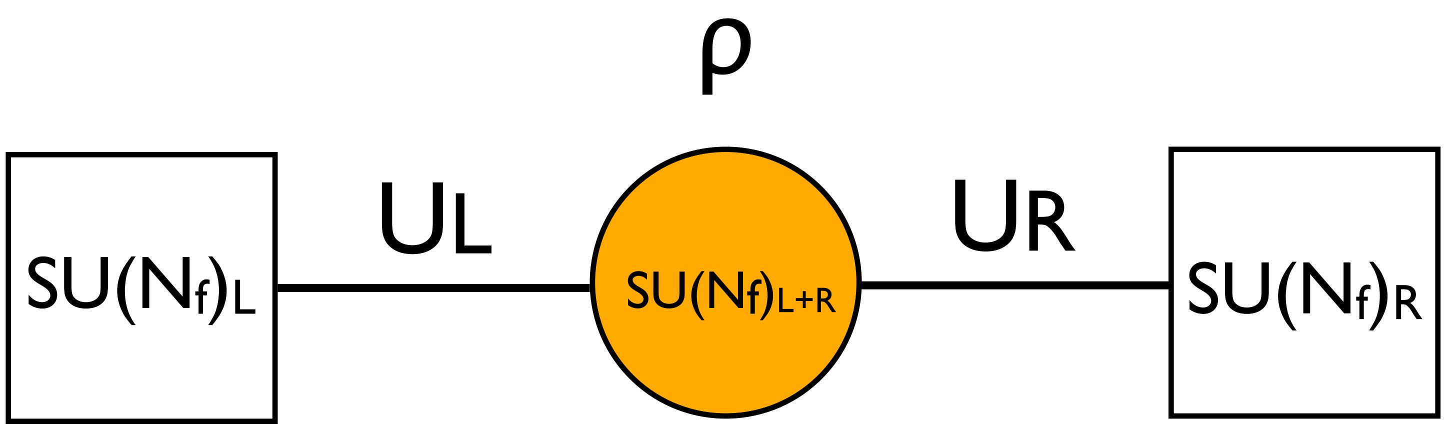
This can be beautifully understood by

$$\langle \bar{q}q \rangle \neq 0$$

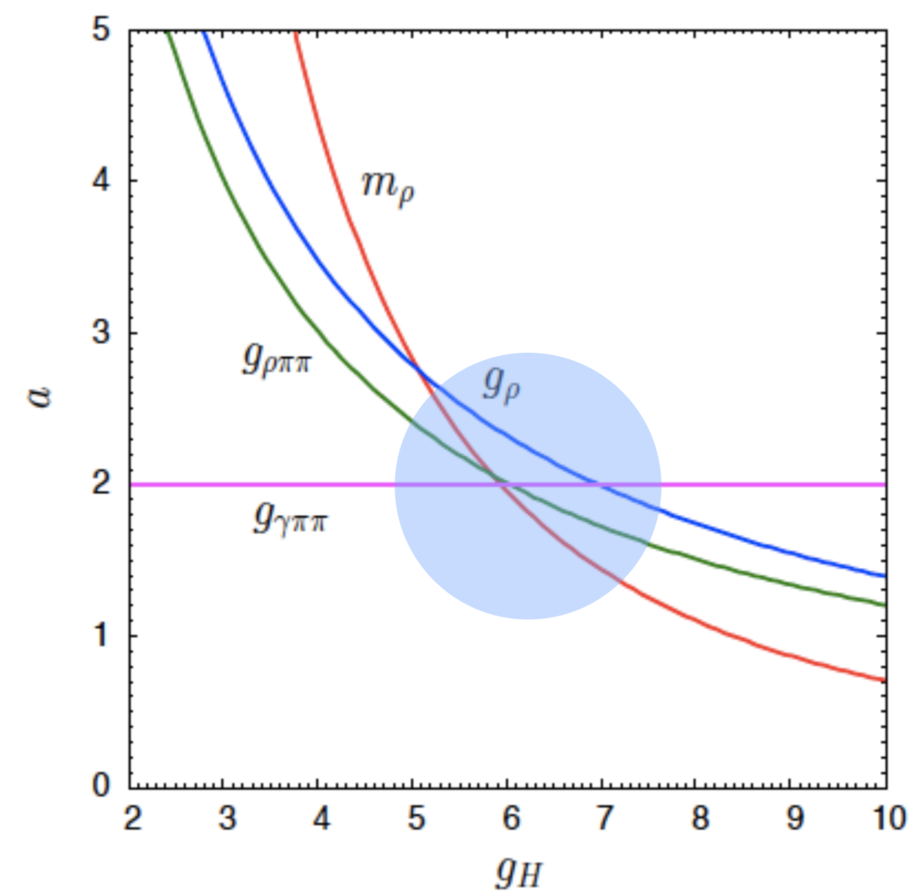
spontaneously broken chiral symmetry

Hidden Local Symmetry

[Bando, Kugo, Uehara, Yamawaki, Yanagida ’85]



$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4g_H^2} F^a_{\mu\nu} F^{a\mu\nu} \\
& + \frac{a f_\pi^2}{2} \mathrm{tr}\left[|D_\mu U_L|^2 + |D_\mu U_R|^2 \right] \\
& + \frac{(1-a) f_\pi^2}{4} \mathrm{tr}\left[|\partial_\mu (U_L U_R)|^2 \right].
\end{aligned}
$$



Two-parameter model for π-ρ-γ system.

hypothesis

$SU(2)_{L+R}$ in HLS as the magnetic gauge group

Higgs mechanism for the ρ meson

confinement
by the dual Meissner effect

chiral symmetry breaking
by the color-flavor locking

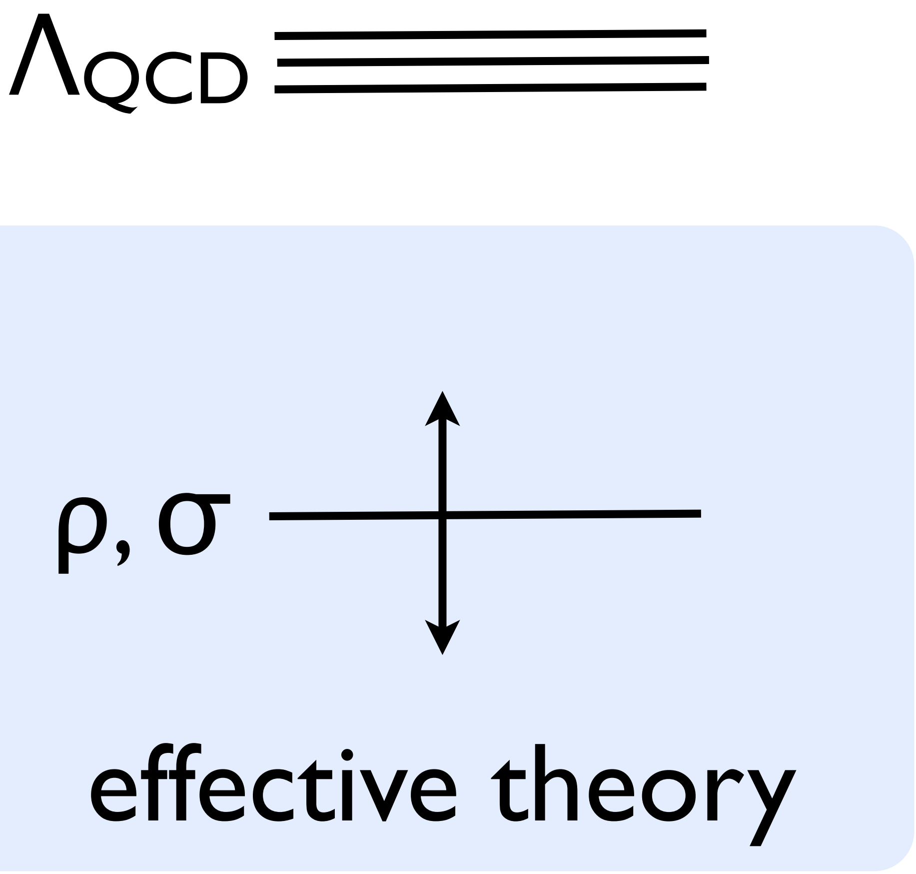
a unified picture

[RK '11][RK, Nakamura, Yokoi '12][RK, Yokoi '13]

Of course,

It is difficult to derive a magnetic theory from QCD.

Our goal today



We demonstrate it
in SUSY QCD.

We introduce
a parameter to the model,
and
show that one can
bring down vector/scalar mesons
as the weakly coupled magnetic
theory in some range of
parameters.

review (Seiberg-Witten Theory)

N=2 SUSY gauge theory

$$(A_\mu^a,\ \lambda^a)$$

SU(2) gauge field   gaugino (adjoint fermion)

$$(\phi^a,\ \psi^a)$$

adjoint scalar   adjoint fermion

Classically, at a generic point

$\phi^3 \neq 0$:   SU(2) $\longrightarrow$ U(1)

review (Seiberg-Witten Theory)

Seiberg and Witten have found a consistent solution.

* $SU(2) \longrightarrow U(1)$ happens everywhere.
* The U(1) gauge coupling at low energy is given by

$$u = \mathrm{Tr}\phi^2/\Lambda^2$$

$$\frac{4\pi i}{g^2} + \frac{\theta}{2\pi} = \frac{da_D}{da}$$

$$a(u) = \frac{\sqrt{2}}{\pi}\int_{-1}^{1} dx \sqrt{\frac{u-x}{1-x^2}}$$

$$a_D(u) = \frac{\sqrt{2}}{\pi}\int_{1}^{u} dx \sqrt{\frac{u-x}{1-x^2}}$$

* monopole/dyon masses are

$$M = \sqrt{2}\,|na + ma_D|$$

n : electric charge
m : magnetic charge
$m = \pm 1$

review (Seiberg-Witten Theory)

$$a(u) = \frac{\sqrt{2}}{\pi} \int_{-1}^{1} dx \sqrt{\frac{u-x}{1-x^2}} \qquad a_D(u) = \frac{\sqrt{2}}{\pi} \int_{1}^{u} dx \sqrt{\frac{u-x}{1-x^2}}$$

$$M = \sqrt{2}\,|na + ma_D| \qquad (m = \pm 1)$$

massless monopole/dyon at $u = \pm 1$

deformation to N=1 ⟶ monopole condensation
(mass term for ϕ^a)

⟶ confinement!

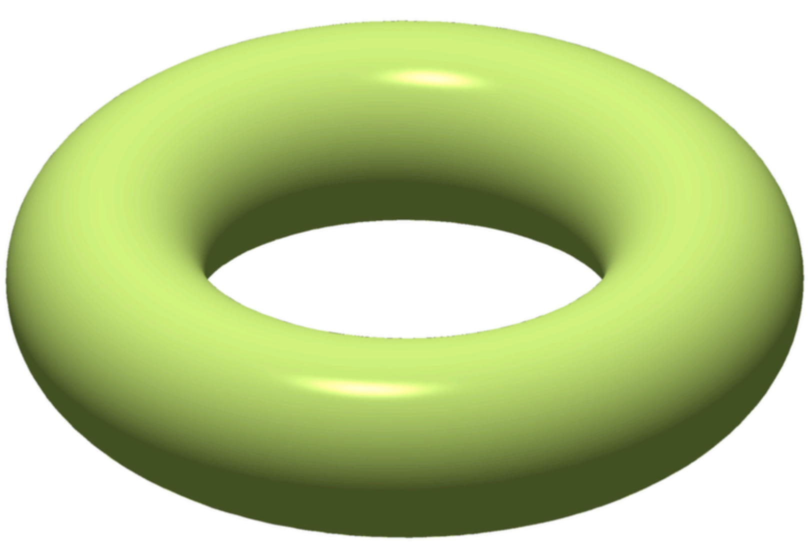
review (Seiberg-Witten Theory)

The solutions can be written as

$$a(u) = \frac{1}{\sqrt{2}\pi}\int_\alpha \frac{x^2}{y}dx \qquad a_D(u) = \frac{1}{\sqrt{2}\pi}\int_\beta \frac{x^2}{y}dx$$

$$y^2 = (x^2 - u)^2 - 4$$

(equation for torus)   Seiberg-Witten curve

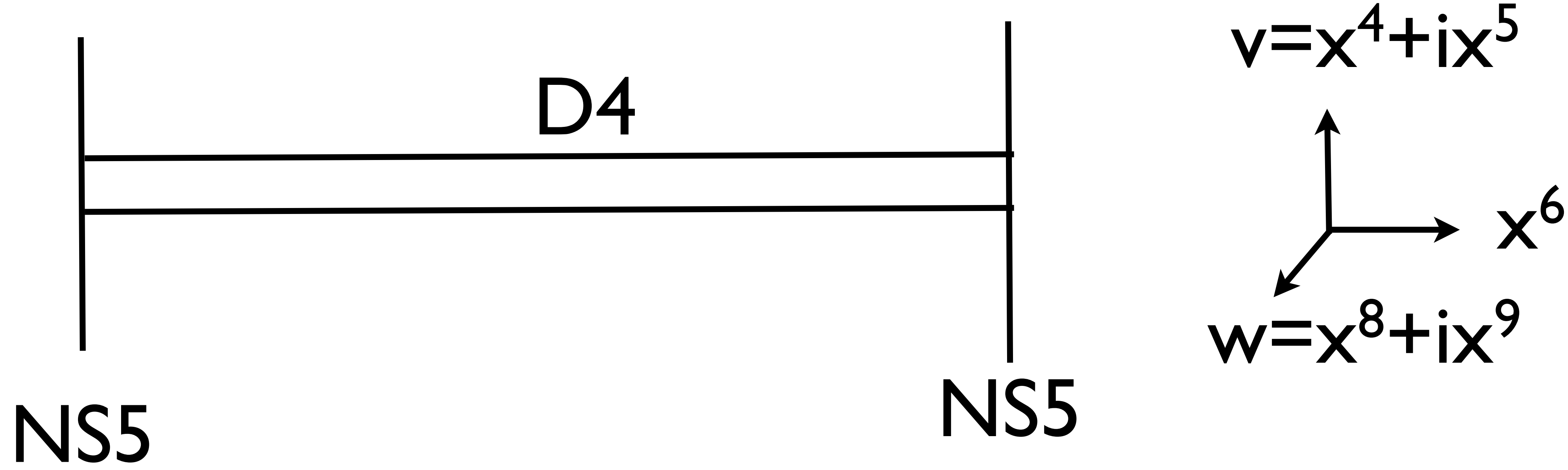


massless monopole

⟶ vanishing cycle

review (Seiberg-Witten Theory)

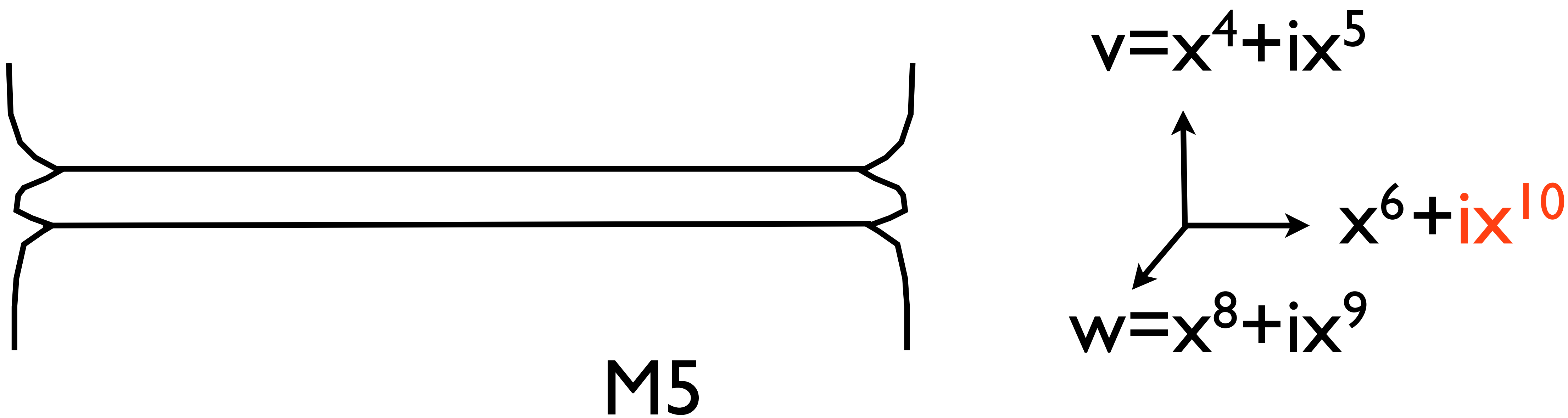
Type IIA string theory



describes SU(2) gauge theory

review (Seiberg-Witten Theory)

M theory



describes SU(2) gauge theory

The configuration is $t^2 - (v^2 - u)t + 1 = 0$

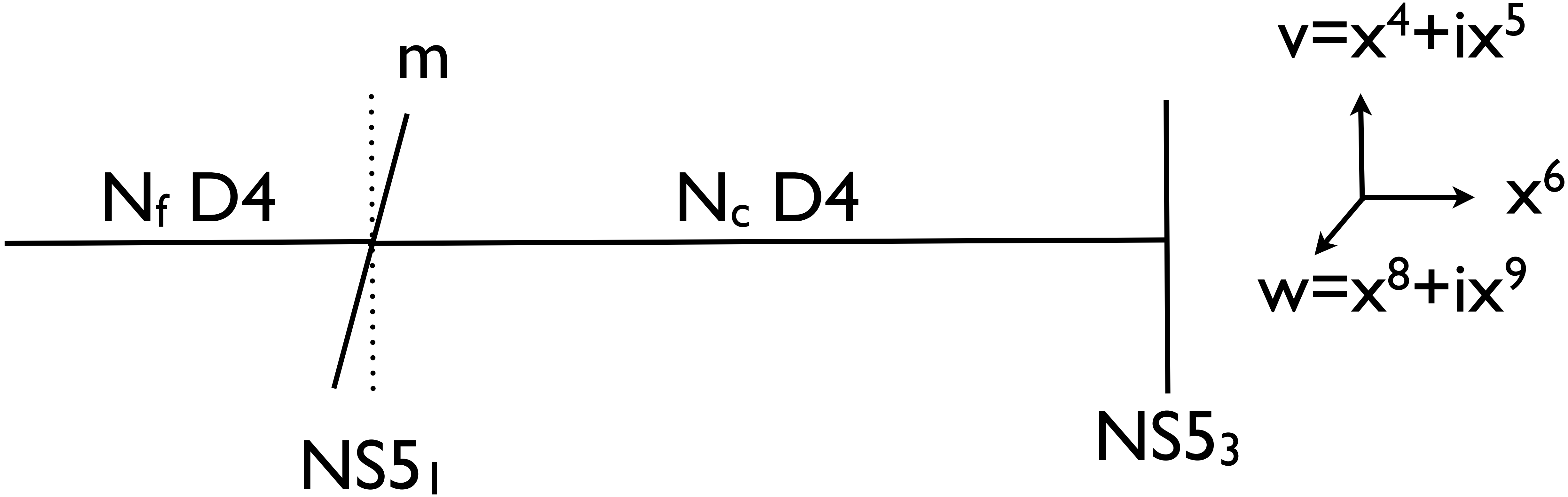
$$t = e^{-x^6 + ix^{10}}$$

$$\longrightarrow \quad y^2 = (x^2 - u)^2 - 4 \quad \left(y = 2t - (v^2 - u), \quad x = v \right)$$

Shape of the M5 brane = Seiberg-Witten curve !!!

D-brane construction

Following Witten, one can construct 4D SQCD by branes and solve it by lifting it to M-theory.

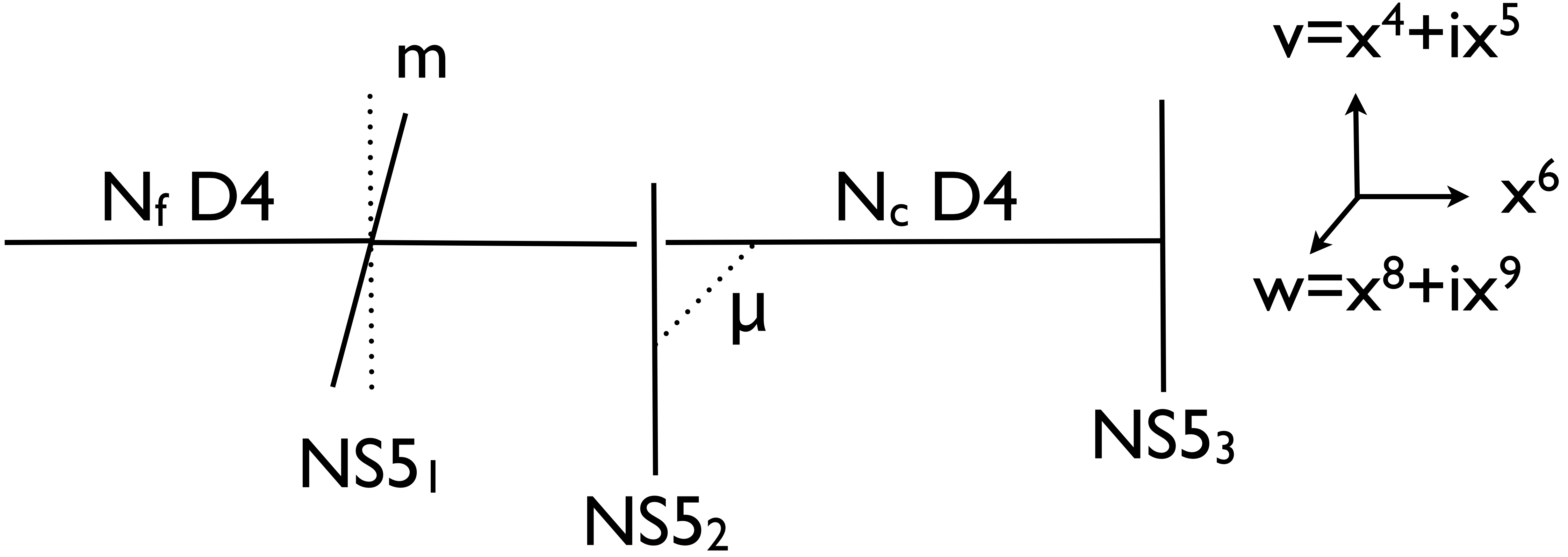


construction of SU(Nc) Nf flavor theory

[Witten '97]

D-brane construction

Following Witten, one can construct 4D SQCD by branes and solve it by lifting it to M-theory.



We introduce a parameter μ as a deformation.
This parameter is going to control the mass of ρ.

SW curve

For Nc=3, Nf=2 (μ=0, m=0), we get

$$\Lambda_2^3 t^3 - (v-\phi_1')(v-\phi_2')(v-\phi_3')t^2 + \left(v-\hat{\phi}_1\right)\left(v-\hat{\phi}_2\right)\left(v-\hat{\phi}_3\right)t + \Lambda_1 v^2 = 0,$$

$$\hat{\phi}_1 + \hat{\phi}_2 + \hat{\phi}_3 = -\Lambda_1, \quad \phi_1' + \phi_2' + \phi_3' = 0.$$

[Giveon, Pelc '97][Erlich, Naqvi, Randall '98]

from this, one can identify the vacua which remain after turning on μ and m:

$$\rho = \frac{\Lambda_1^2}{2} + c_1\Lambda'^2, \quad \sigma = c_2\Lambda'^3, \quad \rho' = c_1\Lambda'^2 \quad \sigma' = -\Lambda_2^3 + c_2\Lambda'^3,$$

$$\rho = \frac{1}{2}(\hat{\phi}_1^2 + \hat{\phi}_2^2 + \hat{\phi}_3^2), \quad \rho' = \frac{1}{2}(\phi_1'^2 + \phi_2'^2 + \phi_3'^2),$$

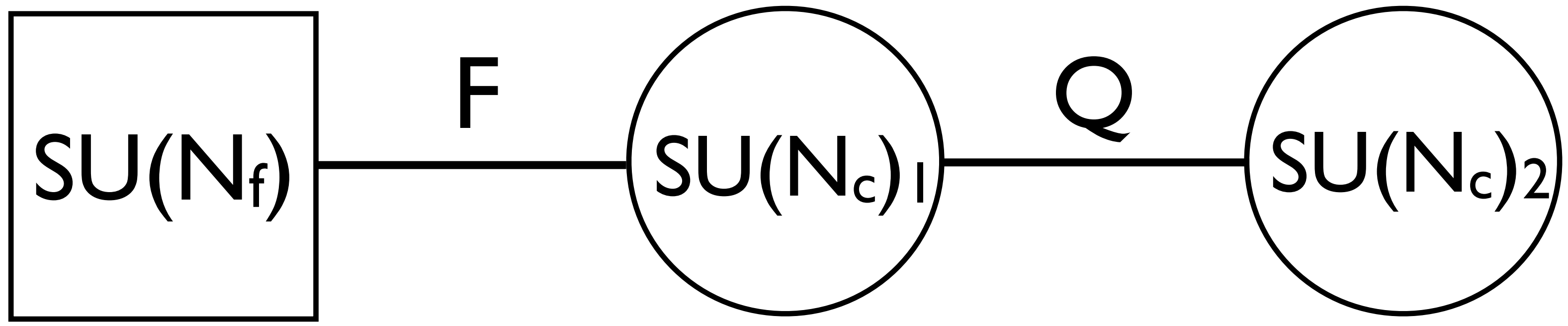
$$\sigma = \hat{\phi}_1\hat{\phi}_2\hat{\phi}_3, \quad \sigma' = \phi_1'\phi_2'\phi_3',$$

$$\Lambda'^4 = \Lambda_1\Lambda_2^3.$$

$$(c_1, c_2) = \left(\frac{2i}{\sqrt{3}}, \pm\frac{8\sqrt{2}}{9\cdot 3^{1/4}}(1+i)\right), \left(-\frac{2i}{\sqrt{3}}, \pm\frac{8\sqrt{2}}{9\cdot 3^{1/4}}(1-i)\right)$$

[Carlino, Konishi, Murayama '00]

Field theoretic analysis



	$SU(N_c)_1$	$SU(N_c)_2$	$SU(N_f)$	$U(1)_B$	$U(1)_{B'}$
Q	N_c	$\bar{N}_c$	1	0	1
F	N_c	1	N_f	1	0

gauged but frozen

$$\rho = \frac{\Lambda_1^2}{2} + c_1 \Lambda'^2, \quad \sigma = c_2 \Lambda'^3, \quad \rho' = c_1 \Lambda'^2 \quad \sigma' = -\Lambda_2^3 + c_2 \Lambda'^3, \qquad \Lambda' = (\Lambda_1 \Lambda_2^3)^{1/4}$$

For $\Lambda_1 \gg \Lambda_2$, $\mu \ll \Lambda_1 \longrightarrow \rho \simeq \frac{\Lambda_1^2}{2}, \quad \sigma \simeq 0, \longrightarrow (\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3) \simeq (0, 0, -\Lambda_1).$

baryonic root

magnetic IR free

	$SU(2)_1$	$U(1)_1$	$SU(3)_2$	$SU(2)_f$	$U(1)_B$	$U(1)_{B'}$
q	2	1/2	1	2	0	−1
q'	2	1/2	$\bar{3}$	1	−1	0
e	1	−1	1	1	0	−1

effective theory below Λ_1

[Argyres, Plesser, Seiberg '96]

$SU(3)_2$ factor gets strong at a scale

$$\Lambda' = (\Lambda_1 \Lambda_2^3)^{1/4}$$

$\Lambda' \gg \Lambda_2$ $\quad \rho = \frac{\Lambda_1^2}{2} + c_1 \Lambda'^2, \quad \sigma = c_2 \Lambda'^3, \quad \rho' = c_1 \Lambda'^2 \quad \sigma' = -\Lambda_2^3 + c_2 \Lambda'^3,$

$$\longrightarrow \quad \rho' = c_1 \Lambda'^2, \quad \sigma' \simeq c_2 \Lambda'^3$$

↖ Point at which massless monopoles appear.

[Carlino, Konishi, Murayama '00]

	$SU(2)_1$	$U(1)_{1'}$	$U(1)_2$	$U(1)_{2'}$	$SU(2)_f$	$U(1)_B$	$U(1)_{B'}$
q	2	1/2	0	0	2	0	-1
e	1	-1	0	0	1	0	-1
e_1	1	0	1	0	1	0	0
e_2	1	0	0	1	1	0	0

effective theory below Λ'

turn on m (N=2 to N=1)

$$W \ni \frac{m}{2}\mathrm{Tr}\Phi_1^2.$$

↓

[Argyres, Plesser, Seiberg '96]

$$W \ni e\Phi_{D1}\bar{e} - e_1\Phi_{D2}\bar{e}_1 - e_2\Phi_{D2'}\bar{e}_2 + m\Lambda_1 x_1 \Phi_{D1} + m\Lambda' x_2 \Phi_{D2} + m\Lambda' x_{2'}\Phi_{D2'},$$

	$SU(2)_1$	$U(1)_{1'}$	$U(1)_2$	$U(1)_{2'}$	$SU(2)_f$	$U(1)_B$	$U(1)_{B'}$
q	2	1/2	0	0	2	0	−1
e	1	−1	0	0	1	0	−1
e_1	1	0	1	0	1	0	0
e_2	1	0	0	1	1	0	0

condensations of e, e_1, e_2

↓

<e> would not cause confinement since $U(1)_{B'}$ is not dynamical

magnetic gauge group →

	$SU(2)_1$	$U(1)_X$	$SU(2)_f$	$U(1)_B$
q	2	3/2	2	0

effective theory below $(m\Lambda')^{1/2}$

turn on μ

$$W \ni -\frac{3}{2}\mathrm{Tr}(q\Phi_X\bar{q}) + \mu^2\Phi_X, \quad \longrightarrow \quad q = \bar{q} = \frac{\mu}{\sqrt{3}} \cdot 1,$$

$SU(2)_1 \times SU(2)_f \longrightarrow SU(2)_{1+f}$

color-flavor locking

magnetic gauge boson ⟶ vector meson (ρ)

$U(1)_X$ breaking ⟶ string formation

⟶ quark confinement

This is what we wanted.

We have seen that

Quiver deformation provides us with
an understanding of HLS as
the magnetic gauge theory.

large μ ⟷ small μ

QCD

HLS as
magnetic theory

[RK ’11]

N=0?

Seiberg duality + soft SUSY breaking terms

electric

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
Q	N_c	N_f	1	1	1	0	$(N_f - N_c)/N_f$
$\overline{Q}$	$\overline{N_c}$	1	$\overline{N_f}$	-1	1	0	$(N_f - N_c)/N_f$
Q'	N_c	1	1	0	$\overline{N_c}$	1	1
$\overline{Q}'$	$\overline{N_c}$	1	1	0	N_c	-1	1

Table 1: Quantum numbers in the electric picture.

magnetic

	$SU(N_f)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
q	N_f	$\overline{N_f}$	1	0	1	N_c/N_f	N_c/N_f
$\overline{q}$	$\overline{N_f}$	1	N_f	0	1	$-N_c/N_f$	N_c/N_f
Φ	1	N_f	$\overline{N_f}$	0	1	0	$2(N_f - N_c)/N_f$
q'	N_f	1	1	1	N_c	$-1 + N_c/N_f$	0
$\overline{q}'$	$\overline{N_f}$	1	1	-1	$\overline{N_c}$	$1 - N_c/N_f$	0
Y	1	1	1	0	$1 +$ Adj.	0	2
Z	1	1	$\overline{N_f}$	-1	$\overline{N_c}$	1	$(2N_f - N_c)/N_f$
$\overline{Z}$	1	N_f	1	1	N_c	-1	$(2N_f - N_c)/N_f$

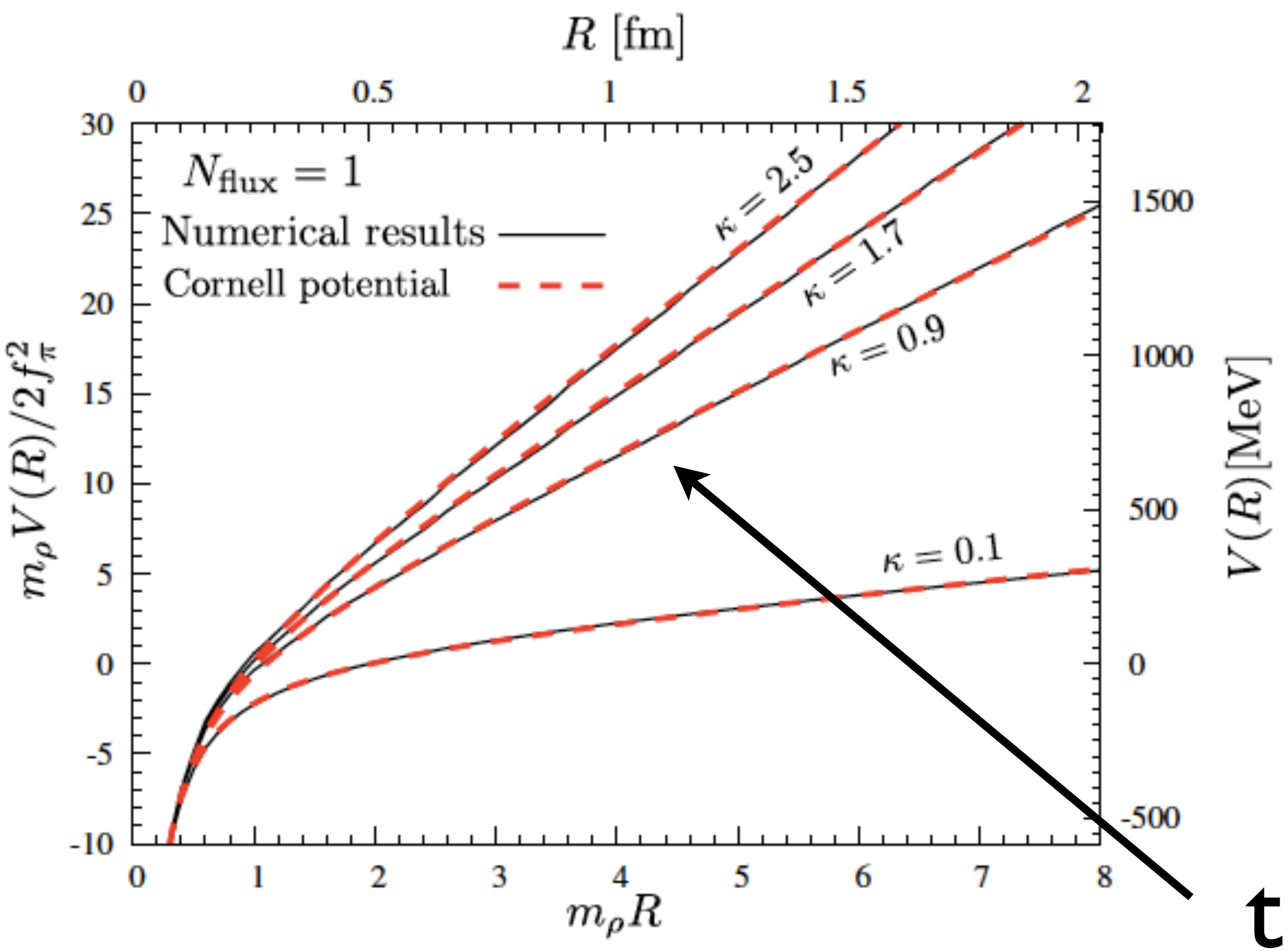
Tachyonic

One can also see HLS.

[RK, Nakamura, Yokoi '12]

QCD may not be so far

We have calculated the monopole-antimonopole potential by using hadron data.



this line

$$g_\rho = (340\ \text{MeV})^2,$$

$$m_\rho = 770\ \text{MeV},$$

~ m_ω

$$m_S = m_A = 980\ \text{MeV},$$

(scalar meson masses)

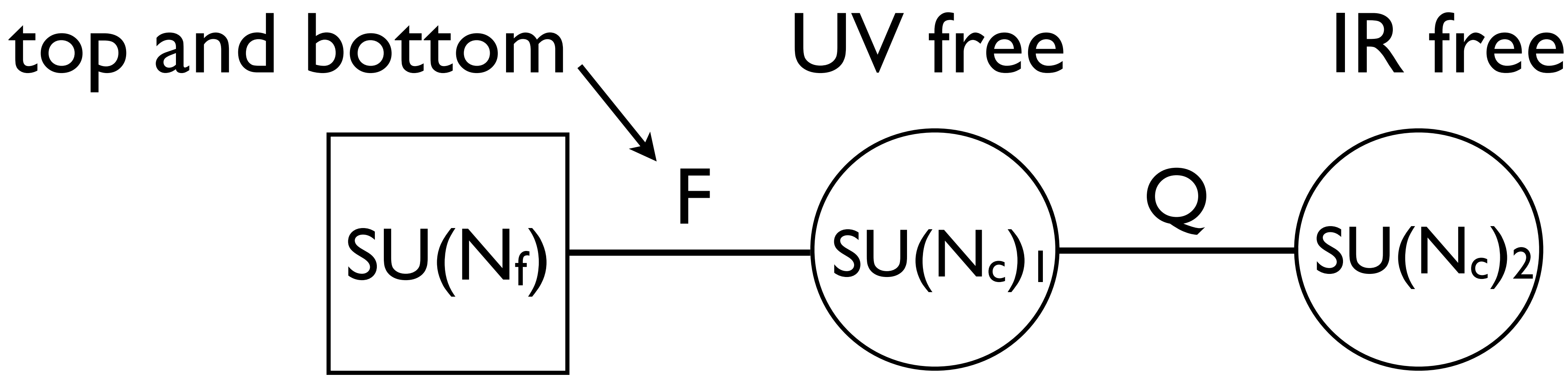
$$V(R) = -\frac{A}{R} + \sigma R.$$

$A = 0.25 \quad \sqrt{\sigma} = 400$ MeV

consistent with lattice QCD

Application to EWSB??

[RK, Nakai '12]



	$SU(3)_1$	$SU(3)_2$	$U(1)_B$	$SU(2)_L \times U(1)_Y$
Q	3	$\overline{3}$	1	1_0
$\bar{Q}$	$\overline{3}$	3	-1	1_0
Φ	1	$1+8$	0	1_0
q_1	3	1	1	$2_{1/6}$
t_1^c	$\overline{3}$	1	-1	$1_{-2/3}$
b_1^c	$\overline{3}$	1	-1	$1_{1/3}$
q_2	1	3	0	$2_{1/6}$
t_2^c	1	$\overline{3}$	0	$1_{-2/3}$
b_2^c	1	$\overline{3}$	0	$1_{1/3}$
$\bar{q}_2$	1	$\overline{3}$	0	$\bar{2}_{-1/6}$
$\bar{t}_2^c$	1	3	0	$1_{2/3}$
$\bar{b}_2^c$	1	3	0	$1_{-1/3}$

other quarks
+ top/bottom

gauged

no Higgs field

$$W = \sqrt{2}g\left(q_1\bar{Q}\bar{q}_2 + t_1^c Q\bar{t}_2^c + b_1^c Q\bar{b}_2^c + \bar{Q}\Phi Q - v^2\mathrm{Tr}\Phi + v_q\bar{q}_2 q_2 + v_t\bar{t}_2^c t_2^c + v_b\bar{b}_2^c b_2^c\right).$$

For $\Lambda \ll 4\pi v,$

$$SU(3)_1 \times SU(3)_2 \longrightarrow SU(3)_{1+2}$$

We get MSSM without Higgs as low energy theory.

not interesting.

Below, we study the case with

$\Lambda \gg 4\pi v,$ (strongly coupled region)

$\longrightarrow$ magnetic description gets better.

Seiberg duality

SU(3)$_1$ factor gets strong

$\longrightarrow$ weakly coupled magnetic picture (CFT)

	$SU(3)_1$	$SU(3)_2$	$U(1)_B$	$SU(2)_L \times U(1)_Y$
Q	3	$\overline{3}$	1	1_0
$\bar{Q}$	$\overline{3}$	3	-1	1_0
Φ	1	$1+8$	0	1_0
q_1	3	1	1	$2_{1/6}$
t_1^c	$\overline{3}$	1	-1	$1_{-2/3}$
b_1^c	$\overline{3}$	1	-1	$1_{1/3}$
q_2	1	3	0	$2_{1/6}$
t_2^c	1	$\overline{3}$	0	$1_{-2/3}$
b_2^c	1	$\overline{3}$	0	$1_{1/3}$
$\bar{q}_2$	1	$\overline{3}$	0	$\bar{2}_{-1/6}$
$\bar{t}_2^c$	1	3	0	$1_{2/3}$
$\bar{b}_2^c$	1	3	0	$1_{-1/3}$

$\longrightarrow$

Higgs appeared.

	$SU(2)_1$	$SU(3)_2$	$U(1)_B$	$SU(2)_L \times U(1)_Y$
f	2	1	$3/2$	2_0
$\bar{f}_u$	$\overline{2}$	1	$-3/2$	$1_{1/2}$
$\bar{f}_d$	$\overline{2}$	1	$-3/2$	$1_{-1/2}$
H_u	1	1	0	$2_{1/2}$
H_d	1	1	0	$2_{-1/2}$
f'	2	3	$3/2$	$1_{1/6}$
$\bar{f}'$	$\overline{2}$	$\overline{3}$	$-3/2$	$1_{-1/6}$
q	1	3	0	$2_{1/6}$
t^c	1	$\overline{3}$	0	$1_{-2/3}$
b^c	1	$\overline{3}$	0	$1_{1/3}$

below the dynamical scale Λ.

below Λ’

$$W = \lambda_d \bar{f}_u H_d f + \lambda_u \bar{f}_d H_u f - \frac{\lambda_q \lambda_t}{\Lambda'} f \bar{f}_u t^c q - \frac{\lambda_q \lambda_b}{\Lambda'} f \bar{f}_d b^c q.$$

SU(2)$_1$ factor confines

$$\longrightarrow \quad W = \frac{\lambda_u \Lambda'}{4\pi} H_u H'_d + \frac{\lambda_d \Lambda'}{4\pi} H_d H'_u - \frac{\lambda_q \lambda_t}{4\pi} H'_u t^c q - \frac{\lambda_q \lambda_b}{4\pi} H'_d b^c q.$$

	$SU(3)_2$	$U(1)_B$	$SU(2)_L \times U(1)_Y$
H_u	1	0	$2_{1/2}$
H_d	1	0	$2_{-1/2}$
H'_u	1	0	$2_{1/2}$
H'_d	1	0	$2_{-1/2}$
S	1	3	1
$\bar{S}$	1	-3	1
q	3	0	$2_{1/6}$
t^c	$\bar{3}$	0	$1_{-2/3}$
b^c	$\bar{3}$	0	$1_{1/3}$

$$\left(H'_u H'_d - S\bar{S} = \frac{\Lambda'^2}{(4\pi)^2}. \right)$$

gauged

<S> $\longrightarrow$ S is not dynamical

MSSM like model

$$W = \frac{\lambda_u \Lambda'}{4\pi} H_u H'_d + \frac{\lambda_d \Lambda'}{4\pi} H_d H'_u - \frac{\lambda_q \lambda_t}{4\pi} H'_u t^c q - \frac{\lambda_q \lambda_b}{4\pi} H'_d b^c q.$$

$$K \ni \frac{\Lambda'^\dagger}{\Lambda'} H'_u H'_d + \mathrm{h.c.}$$

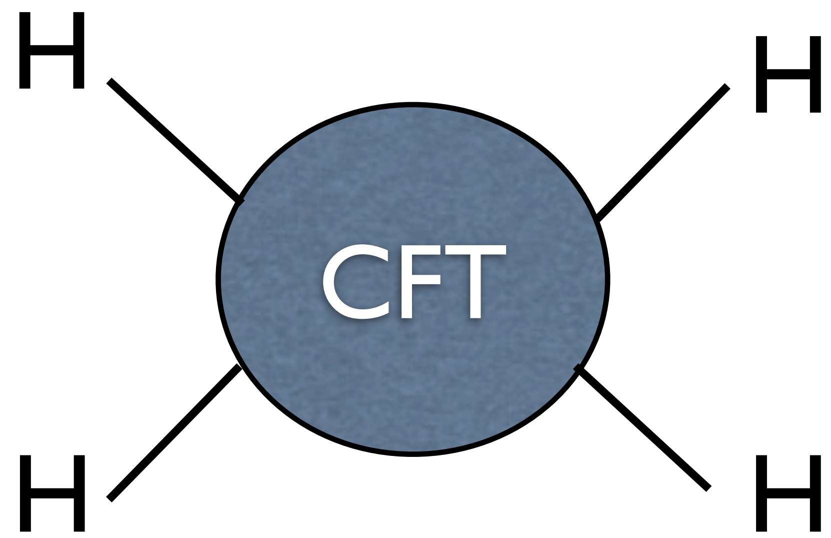
μ-like terms

obtained from kinetic terms for S and $\bar{S}$.

We consider SUSY breaking by turning on

$\Lambda'(1 + m_{\mathrm{SUSY}}\theta^2)$ with $m_{\mathrm{SUSY}} \sim \Lambda' \sim 1\ \mathrm{TeV}$

Higgs potential



$$V \ni \frac{m_{\rm SUSY}^2}{(4\pi)^2}(|\lambda_u H_u|^2 + |\lambda_d H_d|^2) + \frac{1}{(4\pi)^2}(|\lambda_u H_u|^4 + |\lambda_d H_d|^4).$$

new quartic!!

$$V \ni m_{\rm SUSY}^2(|H_u'|^2 + |H_d'|^2) + \cdots$$

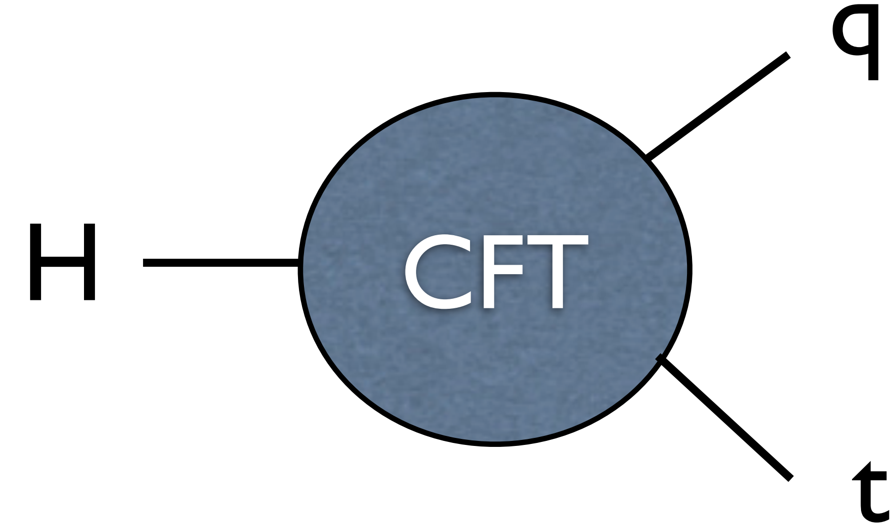
$$V \ni m_{\rm SUSY}\left(\frac{\lambda_u \Lambda'}{4\pi} H_u H_d' + \frac{\lambda_d \Lambda'}{4\pi} H_d H_u' + \text{h.c.}\right),$$

$$W \ni \frac{\Lambda'}{4\pi}(\lambda_u H_u H_d' + \lambda_d H_d H_u') + m_{\rm SUSY} H_u' H_d'.$$

$$V \ni m_{\rm SUSY}^2 H_u' H_d' + \text{h.c.}$$

H’ are heavy

top mass



$$K \ni \frac{\lambda_q \lambda_t \lambda_d}{(4\pi)^2} \frac{1}{\Lambda'^\dagger} H_d^\dagger t^c q, \qquad W \ni -\frac{\lambda_q \lambda_t}{4\pi} H_u' t^c q \,.$$

$$\longrightarrow \quad m_t \sim \frac{\lambda_q \lambda_t \lambda_d}{(4\pi)^2} \langle H_d \rangle \sim 160 \text{ GeV} \cdot \left(\frac{\lambda_d/4\pi}{0.2} \right) \left(\frac{\lambda_q/4\pi}{0.6} \right) \left(\frac{\lambda_t/4\pi}{0.6} \right) .$$

not bad.

note: top obtains mass from H_d

Summary

- We studied a strongly coupled regime of a quiver theory. We see that the quark confinement can be understood as the magnetic color-flavor locking.
- We studied a similar model for EWSB. The Higgs fields emerge as the magnetic degrees of freedom. By adding SUSY breaking terms, EWSB can occur while 125GeV Higgs boson is naturally explained.