Black Brane Thermodynamics from Supersymmetric Field Theories

16 July 2013 Seminar at Osaka

Takeshi Morita

KEK (→ Kentucky, from Oct.)

Ref) arXiv:1305.0789 (to appear in JHEP)
with Shotaro Shiba

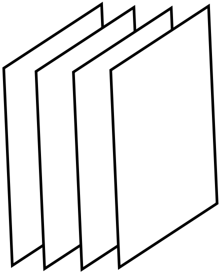
Work in progress, with S. Shiba, T. Wiseman, B. Withers

Introduction

Introduction

◆ What is string theory?

Not only string, there are many extended objects:



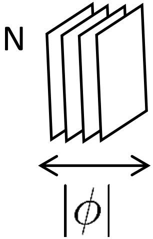
Superstring: F1, Dp, NS5, ･･･

M-theory: M2, M5

Understanding the dynamics of these branes are important to reveal non-perturbative aspects of string theory

Introduction

◆ Thermodynamics of D & M branes from AdS/CFT



(typical spreading of the branes)

Black brane solutions (SUGRA analysis) predict the free energy *F* and the spreading of the branes ϕ as:

N Dp-branes: $F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}}$

$$|\phi| \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

(λ: 't Hooft coupling of Dp, SUGRA is valid when $\underline{\lambda\beta^{3-p} \gg 1}$.)

M-theory N M2-branes: $F \sim N^{3/2}\sqrt{k}T^3$

$$|\phi| \sim N^{\frac{1}{4}} k^{-\frac{1}{4}} T^{\frac{1}{2}}$$

N M5-branes: $F \sim N^3 T^6$

$$|\phi| \sim N T^2$$

Introduction

What is the microscopic origin of
the exotic N dependences of the M-branes?

cf) QGP: $F \sim N^2$, confinement gas: $F \sim 1$, N particles: $F \sim N$

N Dp-branes: $F \sim \boxed{N^2} T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}}$

$$|\phi| \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

(λ: 't Hooft coupling of Dp, SUGRA is valid when $\underline{\lambda\beta^{3-p} \gg 1}$.)

M-theory N M2-branes: $F \sim \boxed{N^{3/2}} \sqrt{k} T^3$

$$|\phi| \sim N^{\frac{1}{4}} k^{-\frac{1}{4}} T^{\frac{1}{2}}$$

N M5-branes: $F \sim \boxed{N^3} T^6$

$$|\phi| \sim N T^2$$

Introduction

What is the microscopic origin of
the exotic N dependences of the M-branes?

AdS/CFT (gauge/gravity) correspondence

SUGRA ⟷ gauge theory
(macro) (micro)

We will estimate these results from dual gauge theories.

(cf. Localization calculations of the gauge theories on spheres at T=0)

M-theory

N M2-branes: $F \sim \boxed{N^{3/2}}\sqrt{k}T^3$

$$|\phi| \sim N^{\frac{1}{4}}k^{-\frac{1}{4}}T^{\frac{1}{2}}$$

N M5-branes: $F \sim \boxed{N^3}T^6$

$$|\phi| \sim NT^2$$

Introduction

◆ Motivations

- Understand the dynamics of M-branes, which might be the most fundamental objects in superstring/quantum gravity.
- Understand the black hole micro states. (cf. D1-D5-P system)
  → Hawking radiation, Information paradox, Fire wall?
- Understand the SYM/SCFT at strong coupling.
- Temporal circle ⟷ spatial circle:
  Witten's Holographic QCD/Sakai-Sugimoto model, SUSY GHU

M-theory N M2-branes: $F \sim N^{3/2}\sqrt{k}T^3$

$$|\phi| \sim N^{\frac{1}{4}}k^{-\frac{1}{4}}T^{\frac{1}{2}}$$

N M5-branes: $F \sim N^3 T^6$

$$|\phi| \sim NT^2$$

Plan of today's talk

1. Introduction
2. Black Dp-brane from SYM
3. Black M2-brane from ABJM
4. Black M5-brane from 6d SCFT
5. Summary

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Effective theory of N Dp-branes = p+1 dim U(N) SYM

$$S_{\mathrm{D}p} = \frac{N}{\lambda}\int d\tau d^p x \,\mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu \Phi^I)^2 - \frac{1}{4}[\Phi^I,\Phi^J]^2 + \cdots\right]$$

-(3-p) -(p+1) 4

◆ mass dimensions of the fields (We will consider dimensional analysis later.)

't Hooft coupling: $[\lambda] = 3-p$, adjoint scalar: $[\Phi^I] = 1 \quad (I = 1, \cdots, 9-p)$

◆ Moduli at T=0

We are interested in the dynamics of the strong coupling regime $\lambda\beta^{3-p} \gg 1$, in which the gravity analysis is valid. This regime is effectively low temperature.

→ dynamics at T=0 may be important.

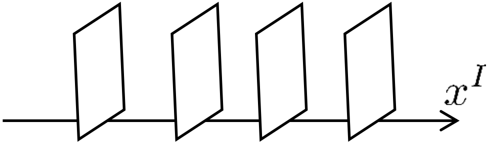
$$\text{Moduli}\begin{cases} A_{\mu,ab} = a_{\mu,a}\delta_{ab} \\ \Phi^I_{ab} = \phi^I_a \delta_{ab} \\ \Psi_{ab} = 0 \end{cases} \quad \text{(constants)}$$

$(a, b = 1, \cdots, N)$

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Physical meaning of the moduli



	τ	1	··	p	p+1	··	9
Dp	-	-	-	-			

ϕ_a^I : position of the a-th Dp-brane

Moduli→ No interactions between the parallel branes at T=0 (BPS).

◆ Moduli at T=0

We are interested in the dynamics of the strong coupling regime $\lambda\beta^{3-p} \gg 1$, in which the gravity analysis is valid. This regime is effectively low temperature.

→ dynamics at T=0 may be important.

$$\text{Moduli} \left\{ \begin{array}{l} A_{\mu,ab} = a_{\mu,a}\delta_{ab} \\ \Phi^I_{ab} = \phi^I_a \delta_{ab} \\ \Psi_{ab} = 0 \end{array} \right. \quad \text{(constants)}$$

$$(a, b = 1, \cdots, N)$$

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Low temperature effective theory of the moduli

☆ assumption: $\beta\phi^I_a \gg 1$ (scale of the moduli) >> (temperature)

The off diagonal modes become massive and we integrate out them.
cf. Higgs $U(N) \to U(1)^N$ (We ignore $a_{\mu,a}$ hereafter.)

➡ $$S^{\text{effective}}_{\text{D}p} = S^{\text{classical}}_{\text{D}p} + S^{\text{one-loop}}_{\text{D}p} + \text{ higher loops}$$

Here,

$$S^{\text{classical}}_{\text{D}p} = \frac{N}{\lambda}\int d\tau d^p x \sum_{a=1}^{N}\left(\frac{1}{2}\partial^\mu\phi^I_a\partial_\mu\phi^I_a\right),$$

$S^{\text{one-loop}}_{\text{D}p}$ {

- Thermal corrections

  $S^{\text{one-loop}}_{\text{D}p,T\neq 0} \propto e^{-\beta|\phi_a-\phi_b|} \to 0,$ → We can ignore them.

- Non-thermal corrections (independent of T)

  $$S^{\text{one-loop}}_{\text{D}p,T=0} \sim -\int d\tau d^p x \sum_{a,b=1}^{N}\frac{(\partial\phi_a-\partial\phi_b)^4}{|\phi_a-\phi_b|^{7-p}} + \cdots,$$

  $\uparrow$ $\beta\phi^I_a \gg 1$

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Comments on the non-thermal one-loop potential

+8 [numerator]

$$S^{\text{one-loop}}_{\mathrm{D}p,T=0} \sim -\int d\tau d^p x \sum_{a,b=1}^{N} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^{7-p}} + \cdots ,$$

-(p+1) -(7-p) [denominator]

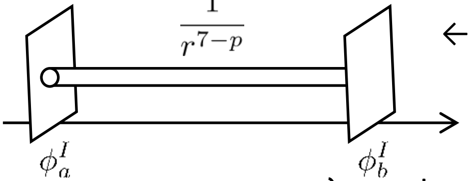
The shape of the potential is roughly fixed by SUSY.

- No $(\phi)^n$ potentials without derivatives $(\partial\phi)^m$ (moduli at T=0)
- No $(\partial\phi)^2$ potentials (non-renormalization)

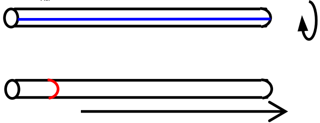
→ corrections start from $(\partial\phi)^4$.

- Dimensional analysis, $[\phi^I_a] = 1$

→ The factor of the denominator is fixed as 7-p.



← Correct power of the classical gravitational potential between the Dp-branes in 10 dim spacetime.



one-loop in gauge theory (open sting)

↕

tree level in SUGRA (closed string)

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Low temperature dynamics of the moduli

$$S_{\mathrm{D}p}^{\mathrm{effective}} = S_{\mathrm{D}p}^{\mathrm{classical}} + S_{\mathrm{D}p}^{\mathrm{one\text{-}loop}} + \text{ higher loops}$$

$$\begin{cases} S_{\mathrm{D}p}^{\mathrm{classical}} = \dfrac{N}{\lambda} \displaystyle\int d\tau d^p x \sum_{a=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_a^I \partial_\mu \phi_a^I \right), \\ S_{\mathrm{D}p}^{\mathrm{one\text{-}loop}} \sim - \displaystyle\int d\tau d^p x \sum_{a,b=1}^{N} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^{7-p}} + \cdots, \end{cases}$$

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial\phi_a^I \sim T\phi$ (derivative ∼ temperature)
3. Strong coupling $S_{\mathrm{D}p}^{\mathrm{classical}} \sim S_{\mathrm{D}p}^{\mathrm{one\text{-}loop}} \sim S_{\mathrm{D}p}^{\mathrm{higher\text{-}loop}}$

1. & 2. →

$$\begin{cases} L_{\mathrm{D}p}^{\mathrm{classical}} \sim \dfrac{N^2}{\lambda} T^2 \phi^2 \\ L_{\mathrm{D}p}^{\mathrm{one\text{-}loop}} \sim \dfrac{N^2 T^4}{\phi^{3-p}} \end{cases}$$

3. → Equating $L_{\mathrm{D}p}^{\mathrm{classical}} \sim L_{\mathrm{D}p}^{\mathrm{one\text{-}loop}}$

$$\phi \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

($\beta\phi_a^I \gg 1$ is OK, if $\lambda\beta^{3-p} \gg 1$)

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Low temperature dynamics of the moduli

◆ Free energy

By substituting ϕ to $S_{\mathrm{D}p}$, we obtain free energy:

$$F_{\mathrm{D}p} \sim S_{\mathrm{D}p}/\beta V_p \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}}$$

$$\phi \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

→ Agrees with SUGRA.

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial\phi_a^I \sim T\phi$ (derivative ~ temperature)
3. Strong coupling $S_{\mathrm{D}p}^{\text{classical}} \sim S_{\mathrm{D}p}^{\text{one-loop}} \sim S_{\mathrm{D}p}^{\text{higher-loop}}$

1. & 2. ➡

$$\begin{cases} L_{\mathrm{D}p}^{\text{classical}} \sim \dfrac{N^2}{\lambda} T^2 \phi^2 \\ L_{\mathrm{D}p}^{\text{one-loop}} \sim \dfrac{N^2 T^4}{\phi^{3-p}} \end{cases}$$

3. ➡ Equating $L_{\mathrm{D}p}^{\text{classical}} \sim L_{\mathrm{D}p}^{\text{one-loop}}$

$$\phi \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

($\beta\phi_a^I \gg 1$ is OK, if $\lambda\beta^{3-p} \gg 1$)

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ Short Summary of this section

☆ We study the dynamics of the moduli ϕ_a^I in p+1 dim SYM.

☆ SUSY fixes the one-loop potential.

$$S_{\mathrm{D}p}^{\text{one-loop}} \sim -\int d\tau d^p x \sum_{a,b=1}^{N} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^{7-p}} + \cdots ,$$

☆ Estimations:

We estimate the configuration at low temperature as follows

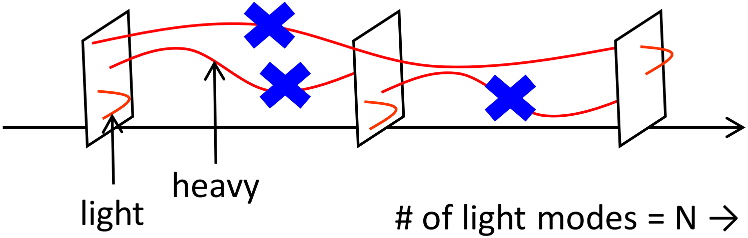
$$\begin{cases} \text{1. } \phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi \ \text{(uniform distribution)} \\ \text{2. } \partial\phi_a^I \sim T\phi \quad \text{(derivative} \sim \text{temperature)} \\ \text{3. Strong coupling } S_{\mathrm{D}p}^{\text{classical}} \sim S_{\mathrm{D}p}^{\text{one-loop}} \sim S_{\mathrm{D}p}^{\text{higher-loop}} \end{cases}$$

➡

$$\begin{cases} F_{\mathrm{D}p} \sim S_{\mathrm{D}p}/\beta V_p \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}} \\ \phi \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}} \end{cases}$$

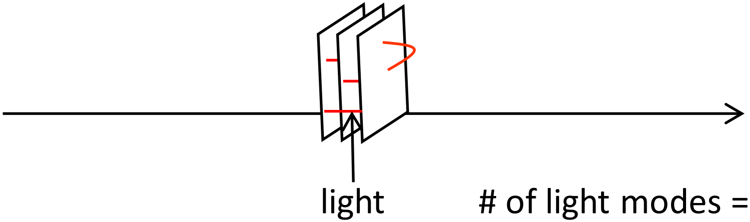
→ Agrees with SUGRA.

- Sufficiently separated D-branes (Higgs)



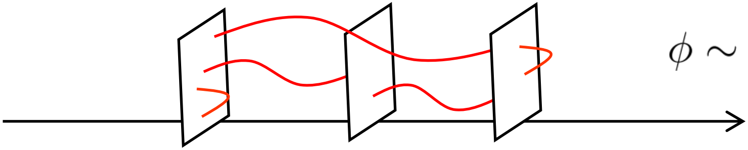
# of light modes = N → $F \sim NT^{p+1}$

- Coincident D-branes (free QGP)



# of light modes = N^2 → $F \sim N^2T^{p+1}$

The black Dp branes are the middle of these two configurations.



$\phi \sim T^{\frac{2}{5-p}}\lambda^{\frac{1}{5-p}}$

$\beta\phi^I_a \gg 1$ but still the interactions are important. → $F_{\mathrm{D}p} \sim N^2 T^{\frac{2(7-p)}{5-p}}\lambda^{-\frac{3-p}{5-p}}$

Plan of today's talk

1. Introduction
2. Black Dp-brane from SYM
3. Black M2-brane from ABJM
4. Black M5-brane from 6d SCFT
5. Summary

Black M2-brane from ABJM [2013 TM-Shiba]

- Effective theory of N M2-branes = ABJM theory (3d SCFT)

$$S_{\mathrm{M2}} = k \int \underset{-3}{d\tau d^2x}\, \mathrm{Tr}\left[\frac{1}{2}\underset{3}{(D_\mu \Phi^I)^2} + \left(\Phi^I\right)^6 + \cdots\right]$$

- mass dimensions of the fields

CS level: $[k] = 0$, bi-fundamental scalar: $[\Phi^I] = 1/2$ $(I = 1, \cdots, 8)$

- Moduli at T=0: $\Phi^I_{ab} = \phi^I_a \delta_{ab}$ $(a, b = 1, \cdots, N)$ positions of M2-branes

- the non-thermal one-loop potential [2008, Baek-Hyun-Jang-Yi]

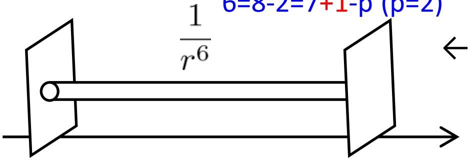
(SUSY & dimensional analysis)

$$S^{\text{one-loop}}_{\mathrm{M2},T=0} \sim -\int \underset{-3}{d\tau d^2x} \sum_{a,b} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^6} + \cdots,$$

4+4 × 1/2=6

-6 × 1/2=-3

6=8-2=7+1-p (p=2)



← Correct power of the classical gravitational potential between the M2-branes in 11 dim spacetime.

3d SCFT knows the 11 dimension!

Black M2-brane from ABJM [2013 TM-Shiba]

◆ Dynamics of the moduli (we assume $\beta^{1/2}\phi_a^I \gg 1$.)

$$S_{\text{M2}}^{\text{effective}} = S_{\text{M2}}^{\text{classical}} + S_{\text{M2}}^{\text{one-loop}} + \text{ higher loops}$$

$$\begin{cases} S_{\text{M2}}^{\text{classical}} = k \int d\tau d^2x \sum_{a=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_a^I \partial_\mu \phi_a^I \right), \\ S_{\text{M2},T=0}^{\text{one-loop}} \sim - \int d\tau d^2x \sum_{a,b} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^6} + \cdots , \end{cases}$$

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial\phi_a^I \sim T\phi$ (derivative ∼ temperature)
3. Strong coupling $S_{\text{M2}}^{\text{classical}} \sim S_{\text{M2}}^{\text{one-loop}} \sim S_{\text{M2}}^{\text{higher-loop}}$

1. & 2. →

$$\begin{cases} L_{\text{M2}}^{\text{classical}} \sim kNT^2\phi^2 \\ L_{\text{M2}}^{\text{one-loop}} \sim \frac{N^2T^4}{\phi^2} \end{cases}$$

3. →

Equating $L_{\text{M2}}^{\text{classical}} \sim L_{\text{M2}}^{\text{one-loop}}$

$$\phi \sim N^{\frac{1}{4}} k^{-\frac{1}{4}} T^{\frac{1}{2}}$$

($\beta^{1/2}\phi_a^I \gg 1$ is OK, if $N/k \gg 1$.)

Black M2-brane from ABJM [2013 TM-Shiba]

◆ Free energy

By substituting ϕ to S_{M2}, we obtain free energy:

$$F_{\mathrm{M2}} \sim N^{\frac{3}{2}}\sqrt{k}T^3$$

$$\phi \sim N^{\frac{1}{4}}k^{-\frac{1}{4}}T^{\frac{1}{2}}$$

→ Agrees with 11dim SUGRA.

(ϕ scales with N, akin to the localization matrix model.)

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial\phi_a^I \sim T\phi$ (derivative ~ temperature)
3. Strong coupling $S_{\mathrm{M2}}^{\text{classical}} \sim S_{\mathrm{M2}}^{\text{one-loop}} \sim S_{\mathrm{M2}}^{\text{higher-loop}}$

1. & 2. ⇒

$$\begin{cases} L_{\mathrm{M2}}^{\text{classical}} \sim kNT^2\phi^2 \\ L_{\mathrm{M2}}^{\text{one-loop}} \sim \dfrac{N^2T^4}{\phi^2} \end{cases}$$

3. ⇒

Equating $L_{\mathrm{M2}}^{\text{classical}} \sim L_{\mathrm{M2}}^{\text{one-loop}}$

$$\phi \sim N^{\frac{1}{4}}k^{-\frac{1}{4}}T^{\frac{1}{2}}$$

($\beta^{1/2}\phi_a^I \gg 1$ is OK, if $N/k \gg 1$.)

Plan of today's talk

1. Introduction
2. Black Dp-brane from SYM
3. Black M2-brane from ABJM
4. Black M5-brane from 6d SCFT
5. Summary

Black M5-brane from 6d SCFT [2013 TM-Shiba]

- Effective theory of N M5-branes = 6d (2,0) SCFT

$$S_{\rm M5} = \text{UNKNOWN !!}$$

But the moduli ϕ_a^I, which represents the positions of the M5 must exit.

- Classical action of the moduli (= N single-M5-brane action)

$$S_{\rm M5}^{\rm classical} = \int \underbrace{d\tau d^5x}_{-6} \sum_{a=1}^{N} \left(\frac{1}{2} \underbrace{\partial^\mu \phi_a^I \partial_\mu \phi_a^I}_{+6} \right),$$

- mass dimension $[\phi_a^I] = 2 \quad (I = 1, \cdots, 5)$

- the non-thermal "one-loop" potential

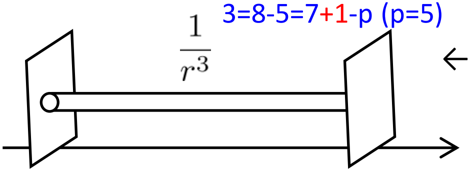
(SUSY & dimensional analysis)

$$S_{{\rm M5},T=0}^{\text{one-loop}} \sim - \int \underbrace{d\tau d^5x}_{-6} \sum_{a,b} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^3} + \cdots,$$

4+4 × 2=12

-3 × 2=-6

3=8-5=7+1-p (p=5)



$\frac{1}{r^3}$

← Correct power of the classical gravitational potential between the M5-branes in 11 dim spacetime.

6d SCFT knows the 11 dimension!

Black M5-brane from 6d SCFT [2013 TM-Shiba]

◆ Dynamics of the moduli (we assume $\beta^2\phi_a^I \gg 1$.)

$$S_{\rm M5}^{\rm effective} = S_{\rm M5}^{\rm classical} + S_{\rm M5}^{\rm one\text{-}loop} + \text{ higher loops}$$

$$\begin{cases} S_{\rm M5}^{\rm classical} = \displaystyle\int d\tau d^5x \sum_{a=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_a^I \partial_\mu \phi_a^I \right) , \\ S_{{\rm M5},T=0}^{\rm one\text{-}loop} \sim - \displaystyle\int d\tau d^5x \sum_{a,b} \frac{(\partial\phi_a - \partial\phi_b)^4}{|\phi_a - \phi_b|^3} + \cdots , \end{cases}$$

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial\phi_a^I \sim T\phi$ (derivative ∼ temperature)
3. Strong coupling $S_{\rm M5}^{\rm classical} \sim S_{\rm M5}^{\rm one\text{-}loop} \sim S_{\rm M5}^{\rm higher\text{-}loop}$

1. & 2. →

$$\begin{cases} L_{\rm M5}^{\rm classical} \sim NT^2\phi^2 \\ L_{\rm M5}^{\rm one\text{-}loop} \sim N^2T^4\phi \end{cases}$$

3. →

Equating $L_{\rm M5}^{\rm classical} \sim L_{\rm M5}^{\rm one\text{-}loop}$

$$\phi \sim NT^2$$

($\beta^2\phi_a^I \gg 1$ is OK, if $N \gg 1$.)

Black M5-brane from 6d SCFT [2013 TM-Shiba]

◆ Free energy

By substituting ϕ to $S_{\rm M5}$, we obtain free energy:

$$F_{\rm M5} \sim N^3 T^6$$

$$\phi \sim NT^2$$

→ Agrees with 11dim SUGRA.

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi_a^I \sim \phi_a^I - \phi_b^I \equiv \phi$ (uniform distribution)
2. $\partial \phi_a^I \sim T\phi$ (derivative ~ temperature)
3. Strong coupling $S_{\rm M5}^{\rm classical} \sim S_{\rm M5}^{\text{one-loop}} \sim S_{\rm M5}^{\text{higher-loop}}$

1. & 2. →

$$\begin{cases} L_{\rm M5}^{\rm classical} \sim NT^2\phi^2 \\ L_{\rm M5}^{\text{one-loop}} \sim N^2T^4\phi \end{cases}$$

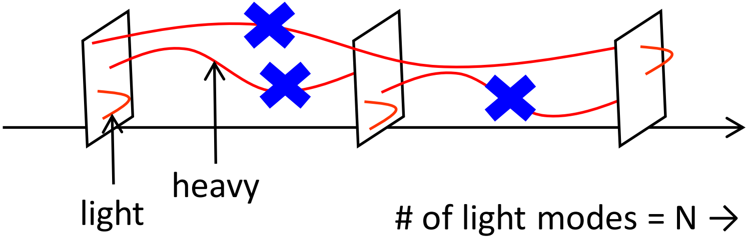
3. →

Equating $L_{\rm M5}^{\rm classical} \sim L_{\rm M5}^{\text{one-loop}}$

$$\phi \sim NT^2$$

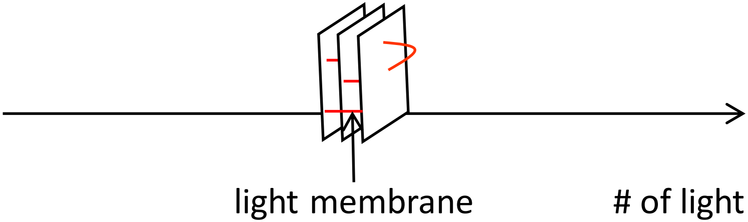
($\beta^2\phi_a^I \gg 1$ is OK, if $N \gg 1$.)

- Sufficiently separated M-branes (Higgs)



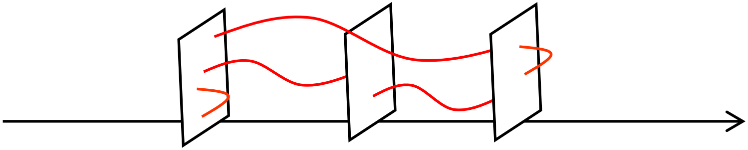
# of light modes = N → $F \sim NT^{p+1}$

- Coincident M-branes



# of light modes = ?? → $F \sim$??

The black M-branes are also the middle of these two configurations.



$\beta^{1/2}\phi_a^I \gg 1$ $(\beta^2\phi_a^I \gg 1)$ but still the interactions are important.

Summary

We can reproduce the black brane thermodynamics form SYM and SCFT. These results suggest the following points:

- Black hole micro states = Dynamics of the moduli fields ϕ_a^I (≠ QGP?)
- SUSY is crucial to reproduce SUGRA from field theories.
- The dynamics of M2 and M5 are similar to Dp.
  → M2 and M5 brane are not so exotic objects.

Future directions

- Exact computation, like the localization technique.
- Study ABJ theory to see the ABJ triality.
  → Connection between SUGRA and HS theory.
- Understand the universal viscosity ratio.
- Understand Witten’s Holographic QCD and Sakai-Sugimoto model.
  → Confinement/chiral symmetry breaking in 4d QCD

Appendix

Black Dp-brane from SYM

[For D0, 2008, Smilga]
[For Dp, 2013, Wiseman]

◆ p=5 case [Work in progress, TM-Shiba-Wiseman-Withers]

◆ Free energy

By substituting ϕ to $S_{\mathrm{D}p}$, we obtain free energy:

$$E_{\mathrm{D5}} \sim \frac{N^2}{\lambda^2}\phi^2$$

ϕ : undetermined

→ Agrees with Hagedron nature of black D5 in SUGRA.
(The position of the horizon is not fixed.)

☆ Estimations:

We estimate the configuration at low temperature as follows

1. $\phi^I_a \sim \phi^I_a - \phi^I_b \equiv \phi$ (uniform distribution)
2. $\partial\phi^I_a \sim T\phi$ (derivative ∼ temperature)
3. Strong coupling $S^{\text{classical}}_{\mathrm{D}p} \sim S^{\text{one-loop}}_{\mathrm{D}p} \sim S^{\text{higher-loop}}_{\mathrm{D}p}$

1. & 2. →

$$\begin{cases} L^{\text{classical}}_{\mathrm{D}p} \sim \dfrac{N^2}{\lambda}T^2\phi^2 \\ L^{\text{one-loop}}_{\mathrm{D}p} \sim N^2T^4\underline{\phi^2} \end{cases}$$

3. → Equating $L^{\text{classical}}_{\mathrm{D}p} \sim L^{\text{one-loop}}_{\mathrm{D}p}$

$$T \sim 1/\sqrt{\lambda}$$

ϕ : undetermined