Seminar at Osaka University (June 25, 2013)

# Gauged Linear Sigma Model for Exotic Five-brane


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based on arXiv:1304.4061, arXiv:1305.4439

in collaboration with Shin SASAKI (佐々木 伸)

well-known objects
D-branes, NS-branes

string dualities → in $D$-dimensions

| well-known objects | string dualities → in $D$-dimensions | less known objects |
| --- | --- | --- |
| D-branes, NS-branes | | $b_n^c$-branes |

Feature:

✓ deformation of SUGRA in $D$-dimensions (gauging)

✓ some fields around it are not single-valued

(← non-trivial monodromy)

✓ mass and asymptotic behavior as a single object are ill-defined

(← co-dimension 2 or less)

caused by $B_2$, $\Phi$, $C_p$ in string theory

monodrofolds

$$b^{c}_{n}$$

$b$ : spatial dimensions

$c$ : # of isometry directions

$n$ : mass of brane $\sim g_{\mathsf{s}}^{-n}$

NS5-brane : $5_2$-brane

KK-monopole : $5^1_2$-brane

more : $5^2_2$-brane , etc

an instructive discussion : J. de Boer and M. Shigemori `arXiv:1209.6056`

# $5^2_2$-brane

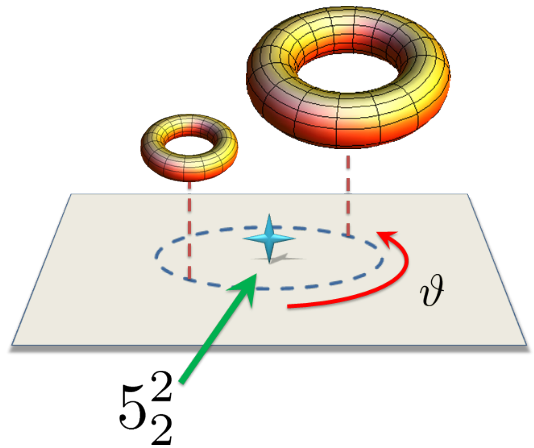
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| NS5-brane | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

↓ T-dual along $x^9$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| KK-monopole | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

↓ T-dual along $x^8$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $5^2_2$-brane | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



This has been analyzed in spacetime (SUGRA) picture.

Ready to study string worldsheet picture!

**string worldsheet picture**

- ☞ nonlinear sigma model (NLSM)
- ☞ conformal field theory (CFT)

- ☞ gauged linear sigma model (GLSM)

GLSM has a rich structure, and involves NLSM and/or CFT in IR

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# $5^2_2$-brane

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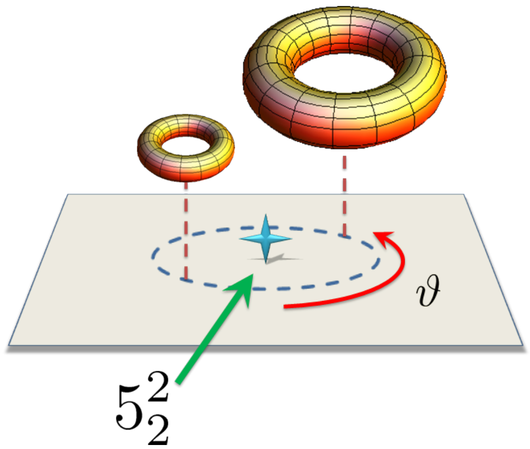
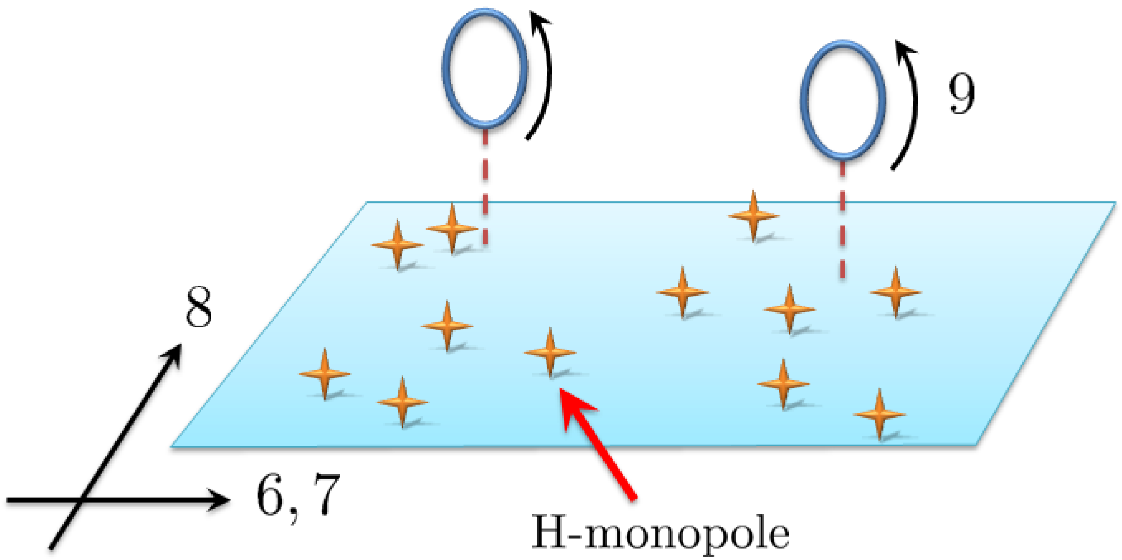


## NS5-branes (smeared), or H-monopoles

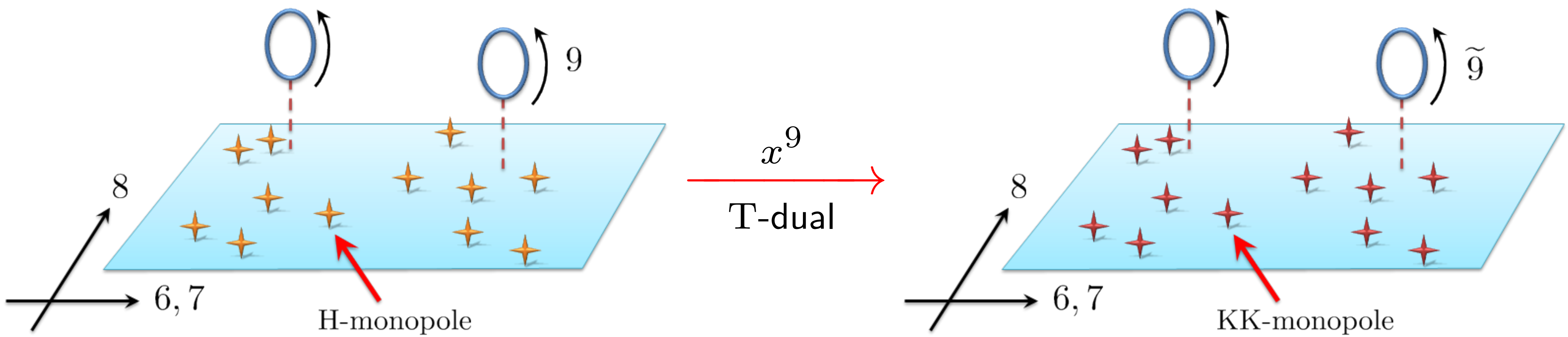
- co-dim. 3 ($\mathbb{R}^3 \times S^1\,,\quad \vec{r} \in \mathbb{R}^3$)
- $\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + (\mathrm{d}x^9)^2\Big]$ NLSM
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q\, \partial_q \log H(x)\,, \qquad \Phi = \frac{1}{2} \log H(x)$

$\mathrm{d}x^2_{012345} = -\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + \ldots + (\mathrm{d}x^5)^2$



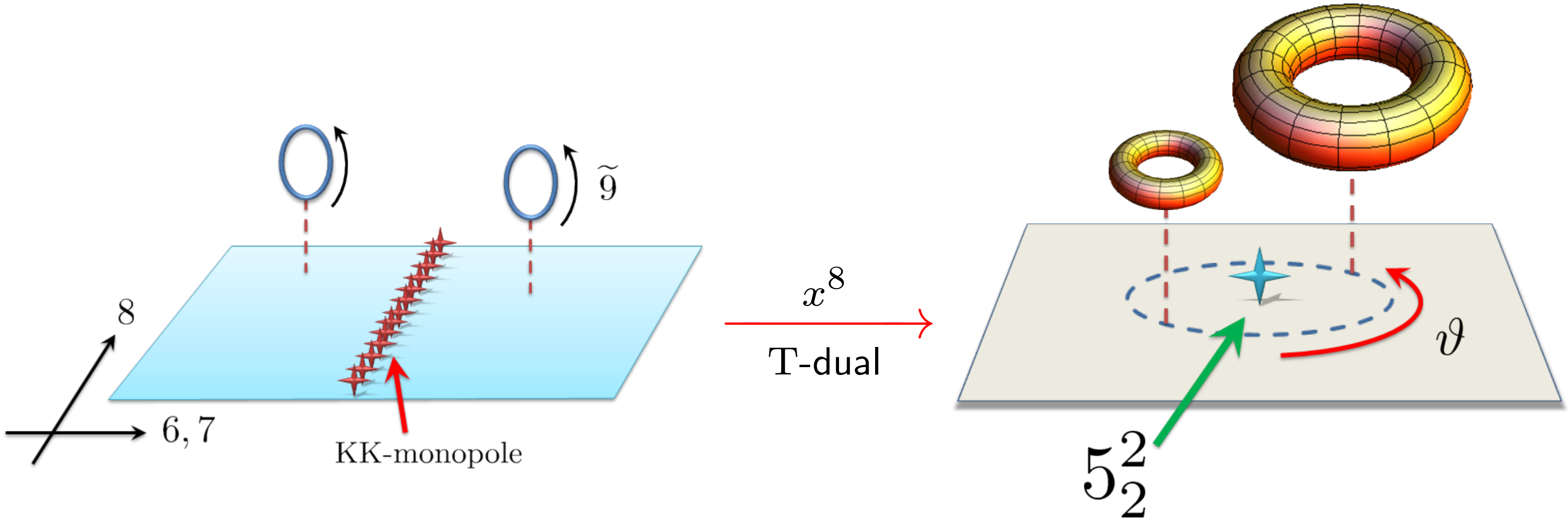
## KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $ds^2 \;=\; dx^2_{012345} + H(x)\Big[(dx^6)^2 + (dx^7)^2 + (dx^8)^2\Big] + \dfrac{1}{H(x)}\big(d\widetilde{x}^9 + \omega\big)^2$ NLSM
- $H(x) = 1 + \sum_p \dfrac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = 0 = \Phi$



## $5^2_2$-brane

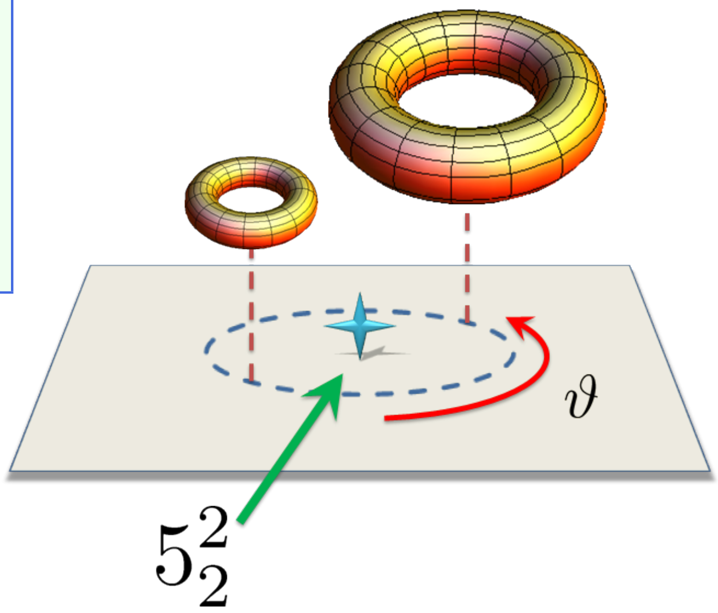
- co-dim. 2 ($\mathbb{R}^2 \times T^2$)
- $H(x) = h + \sigma \log\left(\frac{\mu}{\varrho}\right)$ , $(\varrho, \vartheta) \in \mathbb{R}^2$



$$\mathrm{d}s^2 = \mathrm{d}x^2_{012345} + H\big[\mathrm{d}\varrho^2 + \varrho^2 \mathrm{d}\vartheta^2\big] + \frac{H}{K}\big[(\mathrm{d}\widetilde{x}^8)^2 + (\mathrm{d}\widetilde{x}^9)^2\big]$$

$$B_{89} = -\frac{\sigma\,\vartheta}{K}\,, \quad \mathrm{e}^{2\Phi} = \frac{H}{K}\,, \quad K = H^2 + (\sigma\vartheta)^2$$

$$H = h + \sigma \log\left(\frac{\mu}{\varrho}\right)$$




$$\vartheta = 0 \quad : \quad G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta = 2\pi \quad : \quad G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

Locally geometric, but Globally nongeometric (non–single-valued metric)

T-fold in flux compactifications

J. de Boer and M. Shigemori arXiv:1209.6056

C. Hull hep-th/0406102

## Contents

**Gauged Linear Sigma Model (GLSM) is quite powerful !**

in $\mathcal{N} = (2,2)$ case :

- ✓ CY/LG (geometry/CFT) correspondence [phases]
  - $\leftarrow$ E. Witten hep-th/9301042
- ✓ mirror symmetry [T-duality]
  - $\leftarrow$ K. Hori and C. Vafa hep-th/0002222
- ✓ quantum Kähler moduli space [instantons]
  - $\leftarrow$ N. Doroud et al arXiv:1206.2606; H. Jockers et al arXiv:1208.6244

Gauge theory in UV connects various phases (i.e., NLSM and/or CFT) in IR

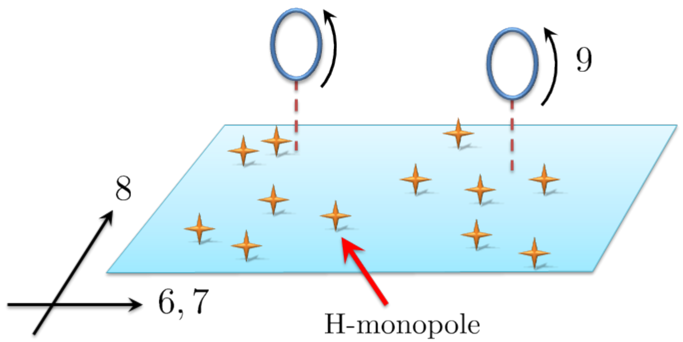
- T-duality transformation is represented as

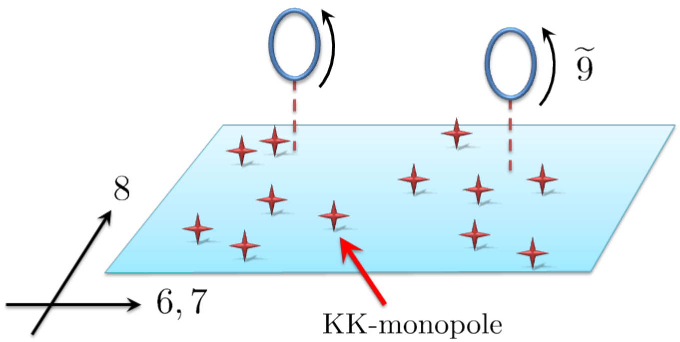
| | | |
|---|---|---|
| (free) string | sign flip (parity) in right-mover | momentum ↔ winding |
| spacetime | Buscher rule | $(G_{mn}, B_{mn}) \to (G'_{mn}, B'_{mn})$ |
| SUSY sigma model | Roček-Verlinde formula | chiral ↔ twisted chiral |

- H-monopoles and KK-monopoles can also be described by

IR limit of $\mathcal{N} = (4, 4)$ GLSM

D. Tong hep-th/0204186; J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097



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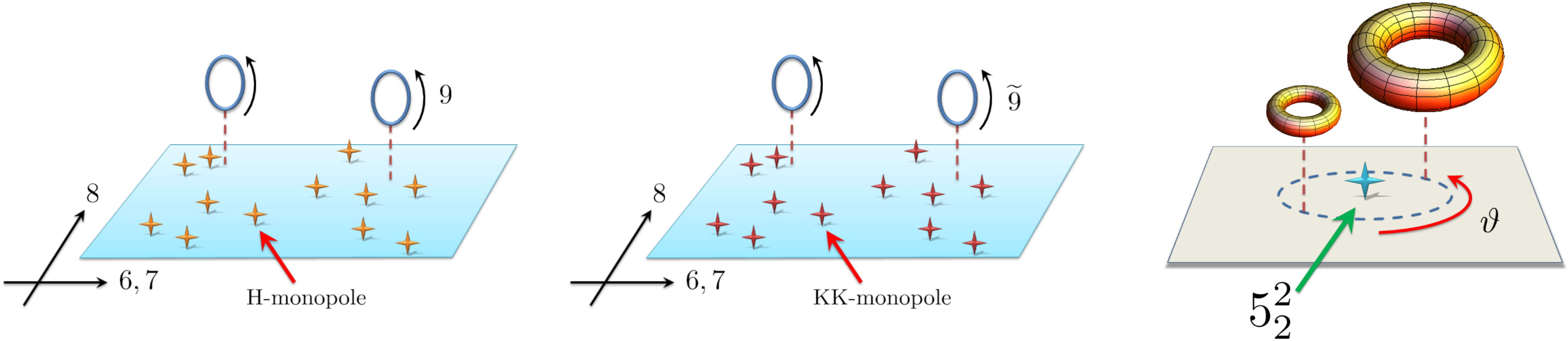
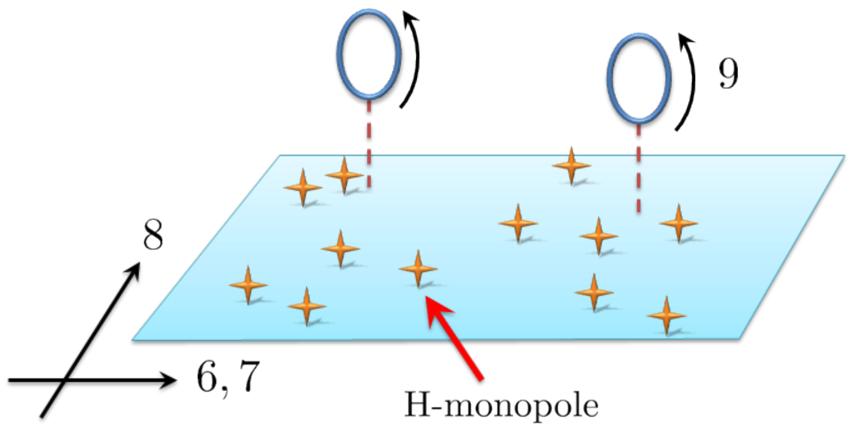



from KK-monopoles to $5^2_2$-brane...

1. logarithmic function (co-dim. 2)
2. isometry, T-duality along $x^8$
3. nongeometric coordinates, non–single-valued metric

# LESSON 1 : GLSM for H-monopoles



- $\mathcal{N} = (4,4)$ for H-monopoles by $\mathcal{N} = (2,2)$ language :

superfields

| $\mathcal{N} = (4,4)$ | $\mathcal{N} = (2,2)$ | | roles in target space |
|---|---|---|---|
| neutral HM | chiral $\Psi = \frac{1}{\sqrt{2}}(r^1 + \mathrm{i}\, r^2) + \ldots$ | twisted chiral $\Theta = \frac{1}{\sqrt{2}}(r^3 + \mathrm{i}\, \theta^4) + \ldots$ | coordinates $(x^6, x^8; x^7, x^9)$ |
| VM | twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}}\overline{D}_+ D_- V_a$ | chiral $\Phi_a$ | gauging isometry |
| charged HMs | chiral $Q_a$ $(+)$ | chiral $\widetilde{Q}_a$ $(-)$ | curving geometry |
| FI parameters | $s_a = s_{1,a} + \mathrm{i}\, s_{2,a}$ | $t_a = t_{1,a} + \mathrm{i}\, t_{2,a}$ | centers of five-branes |

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$$
\begin{aligned}
\mathscr{L}_{\mathrm{H}} \ &= \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big( - \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big( - \overline{\Theta} \Theta + \overline{\Psi} \Psi \Big) \\
&\quad + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big( \widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
&\quad + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, (t_a - \Theta) \Sigma_a + (\text{h.c.}) \Big\}
\end{aligned}
$$

Bosonic Lagrangian after integrating-out auxiliary fields :

$$\begin{aligned}
\mathscr{L}_{\mathsf{H}}^{\mathsf{kin}} &= \sum_a \frac{1}{e_a^2}\Big[\frac{1}{2}(F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2\Big] - \sum_a \Big[|D_m q_a|^2 + |D_m \widetilde{q}_a|^2\Big] \\
&\quad - \frac{1}{2g^2}\Big[(\partial_m \vec{r})^2 + (\partial_m \theta^4)^2\Big] - \sqrt{2}\sum_a (\theta^4 - t_{2,a})\, F_{01,a}
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_{\mathsf{H}}^{\mathsf{pot}} &= -2\sum_a \big(|\sigma_a|^2 + |\phi_a|^2\big)\big(|q_a|^2 + |\widetilde{q}_a|^2 + g^2\big) \\
&\quad - \sum_a \frac{e_a^2}{2}\Big(|q_a|^2 - |\widetilde{q}_a|^2 - \sqrt{2}(r^3 - t_{1,a})\Big)^2 - \sum_a e_a^2 \Big|\sqrt{2}\, q_a \widetilde{q}_a - (r^1 - s_{1,a}) - \mathrm{i}(r^2 - s_{2,a})\Big|^2
\end{aligned}$$

Bosonic Lagrangian after integrating-out auxiliary fields :

$$\mathscr{L}_{\rm H}^{\rm kin} = \sum_a \frac{1}{e_a^2}\Big[\frac{1}{2}(F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2\Big] - \sum_a \Big[|D_m q_a|^2 + |D_m \widetilde{q}_a|^2\Big]$$
$$- \frac{1}{2g^2}\Big[(\partial_m \vec{r})^2 + (\partial_m \theta^4)^2\Big] - \sqrt{2}\sum_a (\theta^4 - t_{2,a})\, F_{01,a}$$

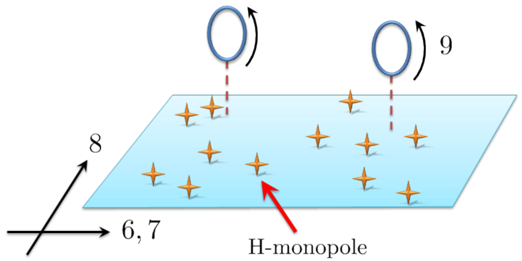
$$\mathscr{L}_{\rm H}^{\rm pot} = -2\sum_a \big(|\sigma_a|^2 + |\phi_a|^2\big)\big(|q_a|^2 + |\widetilde{q}_a|^2 + g^2\big)$$
$$- \sum_a \frac{e_a^2}{2}\Big(|q_a|^2 - |\widetilde{q}_a|^2 - \sqrt{2}(r^3 - t_{1,a})\Big)^2 - \sum_a e_a^2 \Big|\sqrt{2}\, q_a \widetilde{q}_a - (r^1 - s_{1,a}) - {\rm i}(r^2 - s_{2,a})\Big|^2$$

## Steps to NLSM for H-monopoles

geometry details

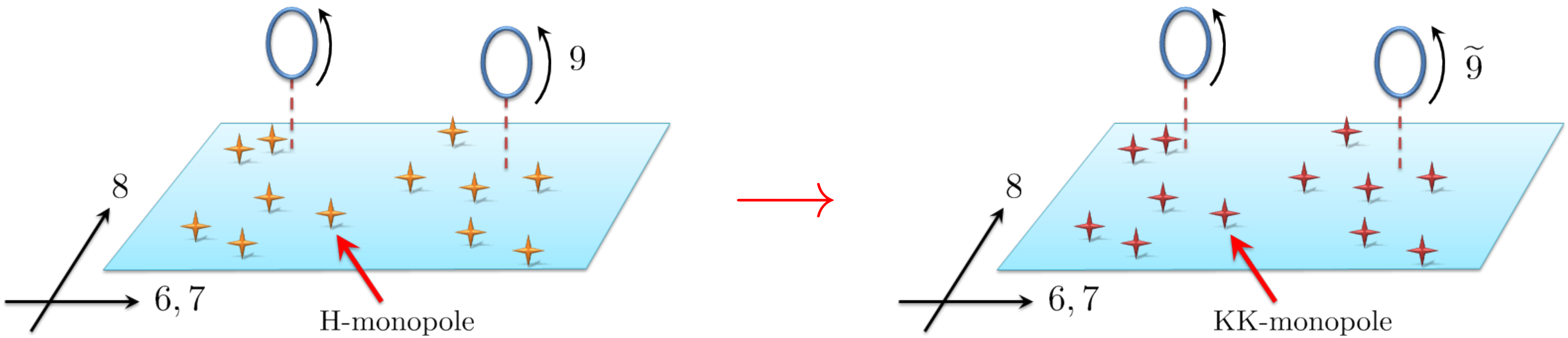
1. SUSY vacua $\mathscr{L}^{\rm pot} = 0$
2. solve constraints on $(q_a, \widetilde{q}_a)$
3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$\Longrightarrow$




$(r^1, r^2, r^3; \theta^4) = (x^6, x^8, x^7; x^9)$

# LESSON 2 : T-duality



$\Theta \to \Gamma$ :

$$\mathscr{L}_{\mathsf{H}} \ni \mathscr{L}_{\Theta} = \int d^4\theta \Big( -\frac{1}{g^2}\overline{\Theta}\Theta \Big) + \sum_a \Big\{ \sqrt{2} \int d^2\widetilde{\theta}\, (-\Theta)\Sigma_a + (\text{h.c.}) \Big\}$$

$$= \int d^4\theta \Big\{ -\frac{1}{2g^2}(\Theta + \overline{\Theta})^2 - 2(\Theta + \overline{\Theta}) \sum_a V_a \Big\} - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m(\theta^4 A_{n,a})$$

$$\downarrow$$

$$\mathscr{L}_{B\Gamma} = \int d^4\theta \Big\{ -\frac{1}{2g^2}B^2 - 2B \sum_a V_a - B(\Gamma + \overline{\Gamma}) \Big\} - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m(\theta^4 A_{n,a})$$

real $\overline{B} = B$ ; chiral $\overline{D}_{\pm}\Gamma = 0$

$\Theta \to \Gamma$ :

$$\mathscr{L}_{B\Gamma} \;=\; \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2} B^2 - 2B \sum_a V_a - (\Gamma + \overline{\Gamma}) B \Big\} - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m (\theta^4 A_{n,a})$$

Integrating out $\Gamma$, $\overline{\Gamma}$ : $\to$ GLSM for H-monopoles $\qquad \overline{D}_+ \overline{D}_- B = 0 = D_+ D_- B \quad \to \quad B = \Theta + \overline{\Theta}$

or, Integrating out $B$ : $\to$ GLSM for KK-monopoles $\qquad \frac{1}{g^2} B = -(\Gamma + \overline{\Gamma}) - 2 \sum_a V_a$

duality relation : $\qquad \Theta = \frac{1}{\sqrt{2}}(r^3 + \mathrm{i}\theta^4) + \ldots, \quad \Gamma = \frac{1}{\sqrt{2}}(\gamma^3 + \mathrm{i}\gamma^4) + \ldots$

$$\Theta + \overline{\Theta} \;=\; -g^2 (\Gamma + \overline{\Gamma}) - 2\, g^2 \sum_a V_a \quad \to \quad \boxed{\begin{aligned} r^3 &= -g^2 \gamma^3 \\ \pm(\partial_0 \pm \partial_1)\theta^4 &= -g^2 (D_0 \pm D_1) \gamma^4 \\ D_m \gamma^4 &= \partial_m \gamma^4 + \sqrt{2} \sum_a A_{m,a} \end{aligned}}$$

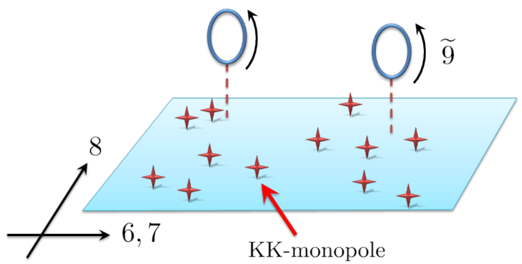
$$
\begin{aligned}
\mathscr{L}_{\text{KK}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big( - \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big( \Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \, \overline{\Psi} \Psi \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big( \widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (\theta^4 A_{n,a})
\end{aligned}
$$

## Steps to NLSM for KK-monopoles

geometry | details | superfields

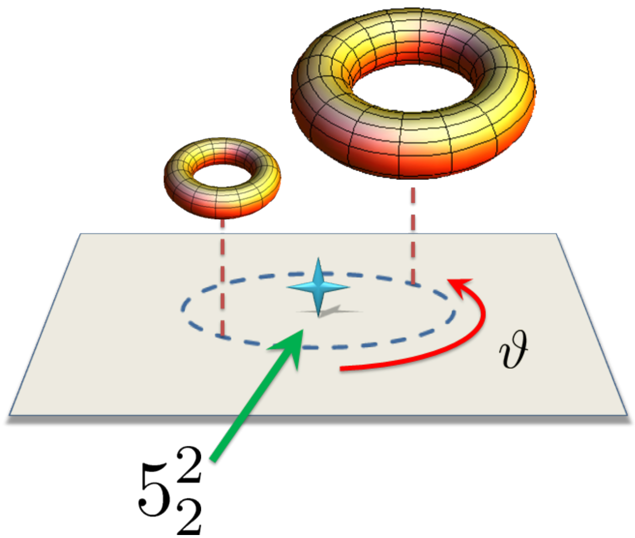
1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q_a, \widetilde{q}_a)$
3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$\Longrightarrow$




$(r^1, r^2, r^3; \gamma^4) = (x^6, x^8, x^7; \widetilde{x}^9)$

## MAIN : $5^2_2$-brane



T-duality : KKM $\to 5_2^2$

**technique 1**

Roček-Verlinde formula with F-terms

$\downarrow$

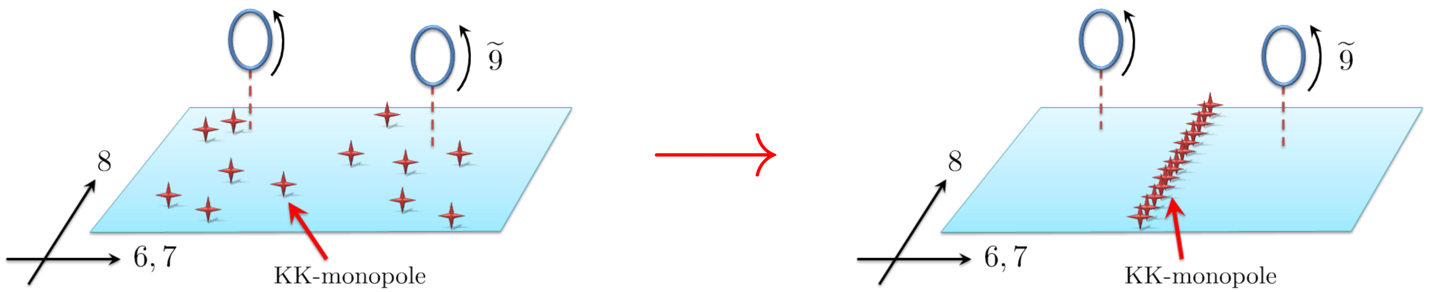
F-terms $\to$ D-terms

**technique 2**

Isometry along $r^2$ $(\sim x^8)$

$\downarrow$

$$s_{1,a} = 0 = t_{1,a} = t_{2,a}\,, \qquad s_{2,a} \neq 0\,, \qquad k \to \infty$$



$\Psi \to \Xi$ :

$$
\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \ni \mathscr{L}_{\Psi} &= \int \mathrm{d}^4\theta \, \frac{1}{g^2} \overline{\Psi} \Psi + \sum_a \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, (-\Psi) \Phi_a + (\mathsf{h.c.}) \Big\} \\
&= \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - \sqrt{2} (\Psi + \overline{\Psi}) \sum_a (C_a + \overline{C}_a) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\Psi - \overline{\Psi})^2 - \sqrt{2} (\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a) \Big\}
\end{aligned}
$$

$\downarrow$

$\Psi \to \Xi$ :

$$\mathscr{L}_{\mathsf{KK}} \ni \mathscr{L}_{\Psi} = \int d^4\theta \, \frac{1}{g^2} \overline{\Psi}\Psi + \sum_a \Big\{ \sqrt{2} \int d^2\theta \, (-\Psi) \Phi_a + (\text{h.c.}) \Big\}$$

$$= \int d^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - \sqrt{2} (\Psi + \overline{\Psi}) \sum_a (C_a + \overline{C}_a) \Big\}$$

$$+ \int d^4\theta \, \Big\{ \frac{2a - 1}{2g^2} (\Psi - \overline{\Psi})^2 - \sqrt{2} (\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a) \Big\}$$

$$\downarrow$$

$$\mathscr{L}_{RS\Xi X} = \int d^4\theta \, \Big\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \overline{C}_a) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\}$$

$$+ \int d^4\theta \, \Big\{ \frac{2a - 1}{2g^2} (\mathrm{i}S)^2 - \sqrt{2} (\mathrm{i}S) \sum_a (C_a - \overline{C}_a) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}$$

$\overline{R} = R, \ \overline{S} = S, \ \overline{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \ \overline{D}_\pm X = 0, \ \Phi_a = \overline{D}_+ \overline{D}_- C_a$

$\Psi \to \Xi$ :

$$\begin{aligned}\mathscr{L}_{RS\Xi X} &= \int \mathrm{d}^4\theta \Big\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \overline{C}_a) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\ &\quad + \int \mathrm{d}^4\theta \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - \sqrt{2}(\mathrm{i}S) \sum_a (C_a - \overline{C}_a) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}\end{aligned}$$

Integrating out $\Xi_1$, $\Xi_2$, $X$ : $\to$ GLSM for KK-monopoles

or, Integrating out $R$, $\Xi_2$ : $\to$ new GLSM

$$\frac{2a}{g^2} R = -(\Xi_1 + \overline{\Xi}_1) + \sqrt{2} \sum_a (C_a + \overline{C}_a)$$

duality relation at $a = \frac{1}{2}$ :

$$\Psi = \frac{1}{\sqrt{2}}(r^1 + \mathrm{i}\, r^2) + \ldots$$

$$\Psi + \overline{\Psi} = -g^2 (\Xi_1 + \overline{\Xi}_1) + \sqrt{2}\, g^2 \sum_a (C_a + \overline{C}_a)$$

$r^1 \sim$ real part of $\Xi$

$\partial r^2 \sim \partial$(imaginary part of $\Xi$) + "gauge" fields in $C_a$

$$\begin{aligned}
\mathscr{L}_{\text{new}} \;=\; & \sum_{a=1}^{k} \int d^4\theta \left\{ \frac{1}{e_a^2} \Big( - \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, e^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, e^{-2V_a} \widetilde{Q}_a \right\} \\
& + \int d^4\theta \, \frac{g^2}{2} \Big( \Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int d^4\theta \left\{ - \frac{g^2}{2} \Big( \Xi + \overline{\Xi} - \sqrt{2} \sum_{a=1}^{k} (C_a + \overline{C}_a) \Big)^2 \right\} \\
& - \sqrt{2} \int d^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a=1}^{k} (C_a - \overline{C}_a) \\
& + \sum_{a=1}^{k} \left\{ \sqrt{2} \int d^2\theta \, \Big( \widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \sqrt{2} \int d^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \right\} - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (\theta^4 A_{n,a})
\end{aligned}$$

$\int d^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_a (C_a - \overline{C}_a)$ plays a crucial role in T-duality !

$\int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_a (C_a - \overline{C}_a)$ is essential. If absent,

- ☹ 2D theory in IR is reduced to a chiral model [conflict w/ $\mathcal{N} = (4,4)$ SUSY]
- ☹ Metric is single-valued [conflict w/ non-trivial monodromy]
- ☹ Target space B-field does not appear [conflict w/ original NS5-brane]

$\int d^4\theta\,(\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_a (C_a - \overline{C}_a)$ is essential. If absent,

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Since this is present,

dual (no longer geometric) coordinate $r^2$ still contributes to the system

↓

Integrating-out ! (possible!)

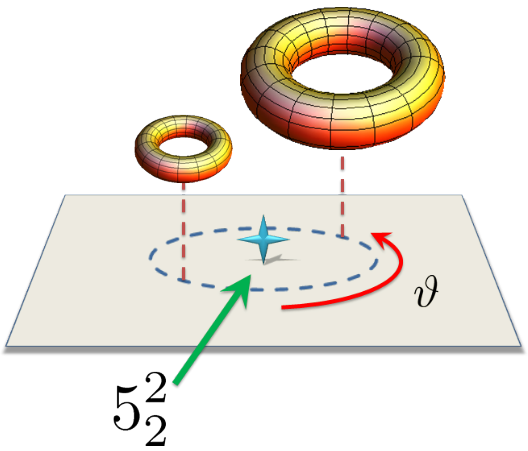
correct non–single-valued metric and B-field emerge!

## Steps to NLSM for $5^2_2$-brane geometry details superfields

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q_a, \widetilde{q}_a)$
3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

★4. integrate out $r^2$

($r^2$: dual of physical coordinate $y^2$)




$$(r^1, y^2; r^3, \gamma^4) = (x^6, \widetilde{x}^8; x^7, \widetilde{x}^9)$$

# Contents

GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by soliton (vortex) solutions in gauge theory

Take the configuration : $\phi_a = 0 = \sigma_a$ with $g^2 \to 0$ and finite $e_a^2$

$$\mathscr{L}_{\mathsf{E}} = \sum_{a=1}^{k} \Big[ \frac{1}{2e_a^2} (F_{12,a})^2 + |D_m q_a|^2 + \frac{e_a^2}{2} \big( |q_a|^2 - \sqrt{2}\, \zeta_a \big)^2 + \mathrm{i} \sqrt{2}\, \theta^4\, F_{12,a} \Big]$$

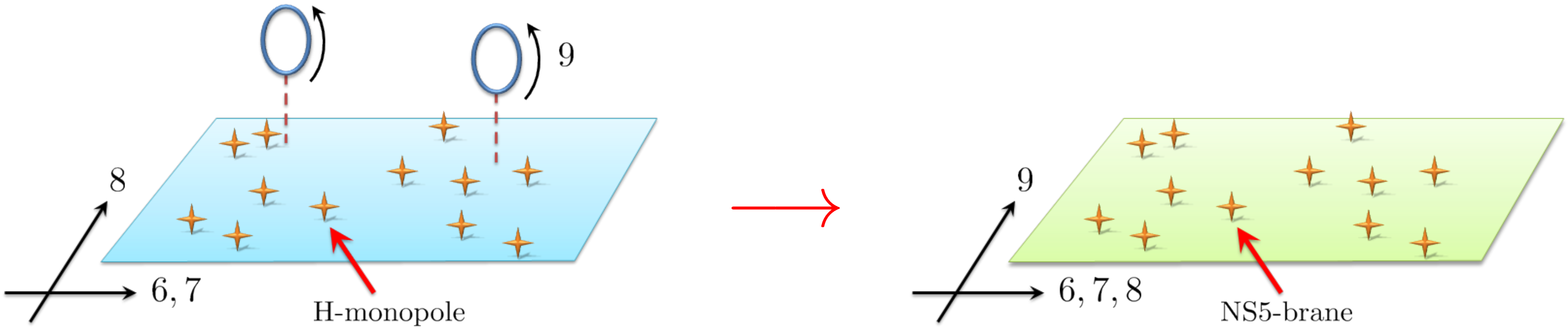
$$F_{12,a} = \mp e_a^2 \big( |q_a|^2 - \sqrt{2}\, \zeta_a \big) , \qquad 0 = (D_1 \pm \mathrm{i} D_2) q_a$$

Abrikosov-Nielsen-Olesen vortex eq.

then, $S_{\mathsf{E}} = \frac{1}{2\pi} \int \mathrm{d}^2 x\, \mathscr{L}_{\mathsf{E}} = \sqrt{2} \sum_{a=1}^{k} \Big( \zeta_a |n_a| - \mathrm{i}\, \theta^4\, n_a \Big)$ $\qquad n_a = -\frac{1}{2\pi} \int \mathrm{d}^2 x\, F_{12,a}$

Worldsheet instanton corrections to GLSM for H-monopoles :

unfolding effect on compactified circle $\theta^4 \sim x^9$



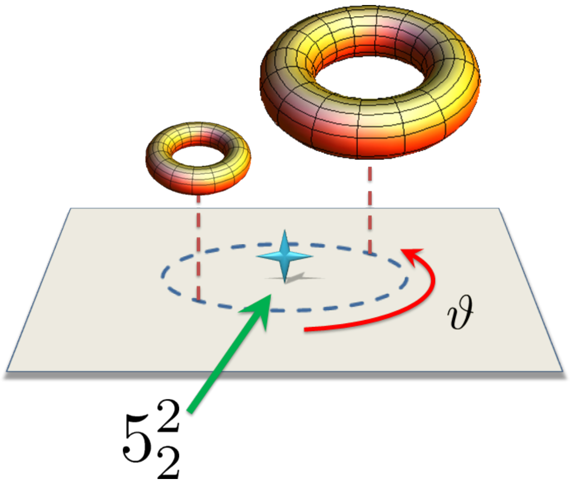

$$H = \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2}R_a} \to \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2}R_a} \sum_{n_a=1}^{\infty} \mathrm{e}^{-n_a R_a} \Big[ \mathrm{e}^{+\mathrm{i}\, n_a \theta^4} + \mathrm{e}^{-\mathrm{i}\, n_a \theta^4} \Big]$$

$$= \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2}R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(\theta^4)}$$

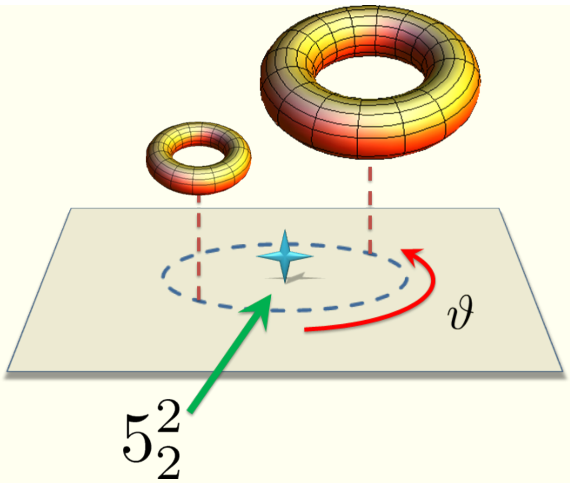
D. Tong hep-th/0204186

Worldsheet instanton corrections to GLSM for $5^2_2$-brane :

string winding modes along $\theta^4 \sim x^9$, rather than $\gamma^4 \sim \widetilde{x}^9$




$\longrightarrow$




$+$

string winding along
dual circle $\theta^4 \sim x^9$

$$H \;\to\; h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n \neq 0} \exp(\mathrm{i}\, n\, \theta^4) K_0(|n|\varrho)$$

S. Sasaki and TK arXiv:1305.4439

# Contents

# We established GLSM for exotic five-brane!

- ✓ co-dim. 2 ; isometry ; non–single-valued metric
- ✓ First application : worldsheet instanton corrections along $x^9$-direction

# We established GLSM for exotic five-brane!

- ✓ co-dim. 2 ; isometry ; non–single-valued metric
- ✓ First application : worldsheet instanton corrections along $x^9$-direction

We hope this $\mathcal{N} = (4,4)$ GLSM tells us more and more ! as $\mathcal{N} = (2,2)$ GLSM GLSM

- ✓ Another instantons : string winding modes along $x^8$-direction ?
- ✓ modular invariance, dipole description ? T. Kikuchi, T. Okada and Y. Sakatani
- ✓ more exotic as "bound states of NS5 + $5^2_2$" de Boer and Shigemori

This work contains my experiences/knowledge of

- GLSM (2D gauge theory; SUSY NLSM)
- flux compactifications with B-field
- $\mathbb{T}$-fold and non-geometry
- gauging SUGRA in $D$-dimensions

I hope this work will show us a deeper insight in stringy geometry.

I want to develop this model more and more!

Thanks

# Appendix

10D string theory $=$ $D$-dim spacetime $\otimes$ compact space $\mathcal{M}_d$ main

| | | | |
|---|---|---|---|
| $\mathcal{M}_d$ | geometry associated with $G_{mn}$ | Conventional geometry (manifold)<br>$O(d)$ global symmetry<br>[Calabi-Yau, etc] | ordinary compactifications |
| | geometry associated with $G_{mn}$, $B_{mn}$ | Generalized geometry<br>$O(d, d; \mathbb{Z})$ T-duality symmetry<br>[T-fold] | flux compactifications |
| | geometry associated with $G_{mn}$, $\tau = C_{(0)} + \mathrm{i}\, \mathrm{e}^{-\Phi}$ | Generalized geometry<br>$SL(2, \mathbb{Z})$ S-duality symmetry<br>[S-fold] | F-theory |
| | geometry associated with $G_{mn}$, $B_{mn}$, $\Phi$, $C_{(p)}$ | Generalized geometry<br>$E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry<br>[U-fold] | compactifications with non-abelian gauge |

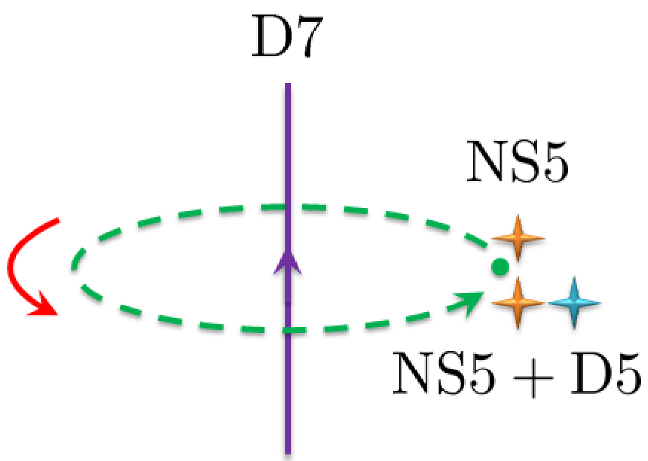
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge $q$

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0,\ \text{NS5}) \to (\text{D5},\ \text{NS5})$$

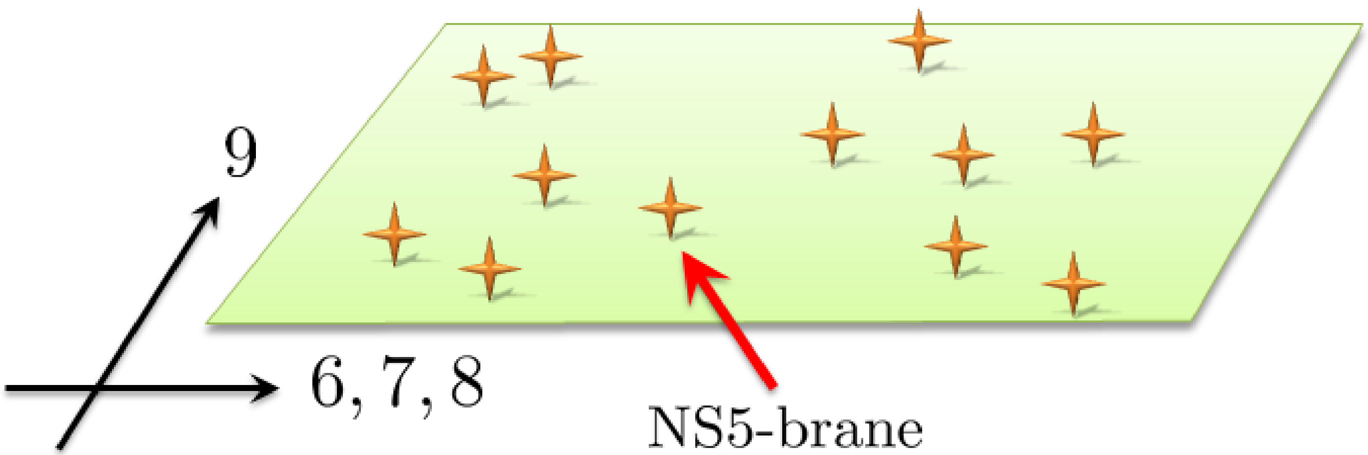



an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

## NS5-branes

- co-dim. 4 ($\mathbb{R}^4$, $\vec{x} \in \mathbb{R}^4$)
- $ds^2 = dx^2_{012345} + H(x)\Big[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2\Big]$
- $H(x) = 1 + \sum_p \frac{Q}{|\vec{x} - \vec{x}_p|^2}$, $\quad H_{mnp} = \varepsilon_{mnp}{}^q \, \partial_q \log H(x)$, $\quad \Phi = \frac{1}{2} \log H(x)$

$dx^2_{012345} = -dt^2 + (dx^1)^2 + \ldots + (dx^5)^2$



## 1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a \ , \qquad |q_a|^2 - |\widetilde{q}_a|^2 = \sqrt{2}(r^3 - t_{1,a}) \ , \qquad \sqrt{2}\, q_a \widetilde{q}_a = (r^1 - s_{1,a}) + \mathrm{i}(r^2 - s_{2,a})$$

## 2. solve constraints on $(q_a, \widetilde{q}_a)$

$$q_a = -\frac{\mathrm{i}}{2^{1/4}}\,\mathrm{e}^{-\mathrm{i}\alpha_a}\sqrt{R_a + (r^3 - t_{1,a})} \ , \qquad \widetilde{q}_a = \frac{\mathrm{i}}{2^{1/4}}\,\mathrm{e}^{+\mathrm{i}\alpha_a}\frac{(r^1 - s_{1,a}) + \mathrm{i}(r^2 - s_{2,a})}{\sqrt{R_a + (r^3 - t_{1,a})}}$$

$$|D_m q_a|^2 + |D_m \widetilde{q}_a|^2 = \frac{1}{2\sqrt{2}R_a}\Big[(\partial_m r^1)^2 + (\partial_m r^2)^2 + (\partial_m r^3)^2\Big] + \sqrt{2}\,R_a\Big(\partial_m \alpha_a + A_{m,a} - \frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m r^i\Big)^2$$

$$R_a = \sqrt{(r^1 - s_{1,a})^2 + (r^2 - s_{2,a})^2 + (r^3 - t_{1,a})^2}$$

$$\Omega_{i,a}\,\partial_m r^i = \frac{-(r^1 - s_{1,a})\partial_m r^2 + (r^2 - s_{2,a})\partial_m r^1}{\sqrt{2}\,R_a(R_a + (r^3 - t_{1,a}))}$$

## 3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = \frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m r^i - \frac{1}{2R_a}\,\varepsilon_{mn}\,\partial^n \theta^4 \,, \qquad \Omega_i = \sum_a \Omega_{i,a}$$

$$\Longrightarrow \quad \mathscr{L}_{\mathrm{H}}^{\mathrm{NLSM}} = -\frac{1}{2}\Big(\frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}\Big)\Big[(\partial_m \vec{r})^2 + (\partial_m \theta^4)^2\Big] - \varepsilon^{mn}\,\Omega_i\,\partial_m r^i\,\partial_n \theta^4$$

1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a \,, \qquad |q_a|^2 - |\widetilde{q}_a|^2 = \sqrt{2}(r^3 - t_{1,a}) \,, \qquad \sqrt{2}\, q_a \widetilde{q}_a = (r^1 - s_{1,a}) + \mathrm{i}(r^2 - s_{2,a})$$

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$$q_a = -\frac{\mathrm{i}}{2^{1/4}}\, \mathrm{e}^{-\mathrm{i}\alpha_a} \sqrt{R_a + (r^3 - t_{1,a})} \,, \qquad \widetilde{q}_a = \frac{\mathrm{i}}{2^{1/4}}\, \mathrm{e}^{+\mathrm{i}\alpha_a} \frac{(r^1 - s_{1,a}) + \mathrm{i}(r^2 - s_{2,a})}{\sqrt{R_a + (r^3 - t_{1,a})}}$$

$$|D_m q_a|^2 + |D_m \widetilde{q}_a|^2 = \frac{1}{2\sqrt{2}R_a}\Big[(\partial_m r^1)^2 + (\partial_m r^2)^2 + (\partial_m r^3)^2\Big] + \sqrt{2}\, R_a \Big(\partial_m \alpha_a + A_{m,a} - \frac{1}{\sqrt{2}}\Omega_{i,a}\, \partial_m r^i\Big)^2$$

$$R_a = \sqrt{(r^1 - s_{1,a})^2 + (r^2 - s_{2,a})^2 + (r^3 - t_{1,a})^2}$$

$$\Omega_{i,a}\, \partial_m r^i = \frac{-(r^1 - s_{1,a})\partial_m r^2 + (r^2 - s_{2,a})\partial_m r^1}{\sqrt{2}\, R_a (R_a + (r^3 - t_{1,a}))}$$

3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = -\frac{1}{2R_a H}\big(\partial_m \gamma^4 + \Omega_i\, \partial_m r^i\big) + \frac{1}{\sqrt{2}}\Omega_{i,a}\, \partial_m r^i \,, \qquad \Omega_i = \sum_a \Omega_{i,a} \,, \qquad H = \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}$$

$$\Longrightarrow \quad \mathscr{L}^{\mathsf{NLSM}}_{\mathsf{KK}} = -\frac{1}{2}H(\partial_m \vec{r})^2 - \frac{1}{2}H^{-1}(\partial_m \gamma^4 + \Omega_i\, \partial_m r^i)^2 - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m((\theta^4 - t_{2,a})\mathring{A}_{n,a})$$

$$\mathscr{L}^{\text{kin}}_{\text{new}} = \sum_a \frac{1}{e_a^2}\Big[\frac{1}{2}(F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2\Big] - \sum_a \Big[|D_m q_a|^2 + |D_m \widetilde{q}_a|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m r^1)^2 + (\partial_m r^3)^2\Big] - \frac{g^2}{2}\Big[(\partial_m y^2)^2 + (D_m \gamma^4)^2\Big] - \sqrt{2}\sum_a (\theta^4 - t_{2,a})\, F_{01,a}$$

$$\mathscr{L}^{\text{pot}}_{\text{new}} = -2g^2 \sum_{a,b}(\sigma_a \overline{\sigma}_b + 4 M_{c,a}\overline{M}_{c,b}) - 2\sum_a \big(|\sigma_a|^2 + 4|M_{c,a}|^2\big)\big(|q_a|^2 + |\widetilde{q}_a|^2\big)$$
$$-\sum_a \frac{e_a^2}{2}\Big(|q_a|^2 - |\widetilde{q}_a|^2 - \sqrt{2}(r^3 - t_{1,a})\Big)^2 - \sum_a e_a^2 \Big|\sqrt{2}\, q_a \widetilde{q}_a - (r^1 - s_{1,a}) - \mathrm{i}(r^2 - s_{2,a})\Big|^2$$
$$+ \frac{g^2}{2}\sum_{a,b}(A_{c=,a} + \overline{A}_{c=,a})(B_{c+\!\!+,b} + \overline{B}_{c+\!\!+,b})$$

$$(\partial_0 + \partial_1) r^2 = -g^2(\partial_0 + \partial_1) y^2 + g^2 \sum_a (B_{c+\!\!+,a} + \overline{B}_{c+\!\!+,a})$$

$$(\partial_0 - \partial_1) r^2 = +g^2(\partial_0 - \partial_1) y^2 + g^2 \sum_a (A_{c=,a} + \overline{A}_{c=,a})$$

$$+\frac{g^2}{2}\sum_{a,b}(A_{c=,a} + \overline{A}_{c=,a})(B_{c+\!\!+,b} + \overline{B}_{c+\!\!+,b}) = -\frac{1}{2g^2}(\partial_m r^2)^2 + \frac{g^2}{2}(\partial_m y^2)^2 + \varepsilon^{mn}(\partial_m r^2)(\partial_n y^2)$$

Step 1.,2.,3. :

$$\begin{aligned}\mathscr{L} \;=\; & -\frac{1}{2}H\Big[(\partial_m r^1)^2 + (\partial_m r^2)^2 + (\partial_m r^3)^2\Big] - \frac{1}{2H}(\partial_m \gamma^4)^2 - \frac{(\Omega_2)^2}{2H}(\partial_m r^2)^2 - \frac{\Omega_2}{H}(\partial_m \gamma^4)(\partial^m r^2) \\ & -\frac{(\Omega_1)^2}{2H}(\partial_m r^1)^2 - \frac{\Omega_1 \Omega_2}{H}(\partial_m r^1)(\partial^m r^2) - \frac{\Omega_1}{H}(\partial_m \gamma^4)(\partial^m r^1) \\ & -\sqrt{2}\,\varepsilon^{mn}\sum_a \partial_m((\theta^4 - t_{2,a})\mathring{A}_{n,a}) + \varepsilon^{mn}(\partial_m r^2)(\partial_n y^2)\end{aligned}$$

$$H \;=\; \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}\,, \qquad \Omega_1 \;=\; \sum_a \frac{r^2 - s_{2,a}}{\sqrt{2}\,R_a(R_a + (r^3 - t_{1,a}))}\,, \quad \Omega_2 \;=\; -\sum_a \frac{r^1 - s_{1,a}}{\sqrt{2}\,R_a(R_a + (r^3 - t_{1,a}))}$$

$$A_{m,a} = -\frac{1}{2R_a H}\big(\partial_m \gamma^4 + \Omega_i\,\partial_m r^i\big) + \frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m r^i$$

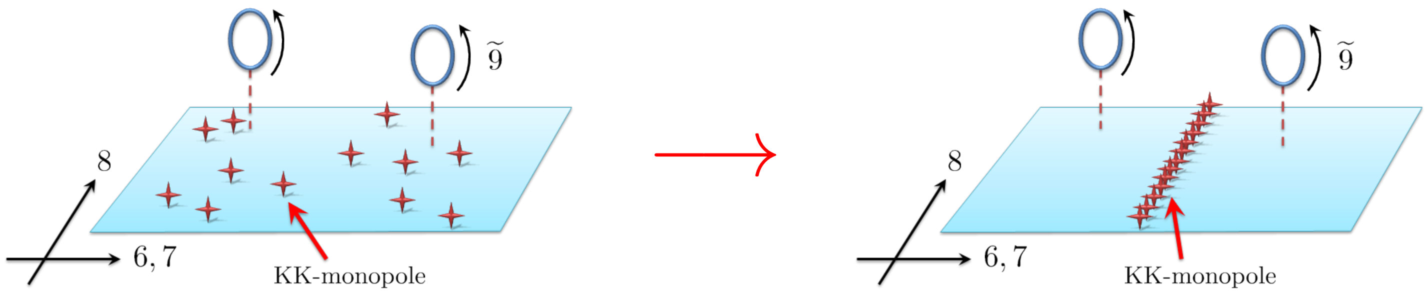
This looks like a sigma model for KK-monopoles, but...

the topological term is essential!

Step 4. : $s_{1,a} = 0 = t_{1,a} = t_{2,a}$, and $k \to \infty$

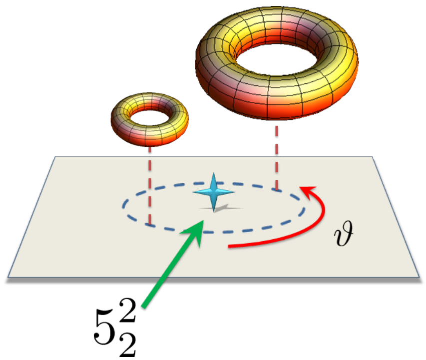
main

This makes
$$\begin{cases} H \to h_0 + \sigma \log(\mu/\varrho) & : \text{co-dim. 2} \quad \varrho^2 = (r^1)^2 + (r^3)^2 \\ \Omega_1 \to 0 & : \text{isometry along } r^2 \\ \Omega_2 \to \sigma \arctan(r^3/r^1) \equiv \sigma\vartheta & : \text{"non–single-valued" metric} \end{cases}$$




Finally, integrate out $r^2$ : $\partial_m r^2 = -\frac{H}{K}\Big[\frac{\sigma\vartheta}{H}(\partial_m \gamma^4) - \varepsilon_{mn}(\partial^n y^2)\Big]$ $\quad K = H^2 + (\sigma\vartheta)^2$

$$\begin{aligned} \mathscr{L} \;=\; & -\frac{1}{2}H\Big[(\partial_m r^1)^2 + (\partial_m r^3)^2\Big] - \frac{1}{2}HK^{-1}\Big[(\partial_m y^2)^2 + (\partial_m \gamma^4)^2\Big] \\ & + (\sigma\vartheta)K^{-1}\,\varepsilon^{mn}(\partial_m y^2)(\partial_n \gamma^4) - \sqrt{2}\,\varepsilon^{mn}\sum_a \partial_m(\theta^4 \mathring{A}_{n,a}) \end{aligned}$$



$$\Psi = \frac{1}{\sqrt{2}}(r^1 + \mathrm{i} r^2) + \mathrm{i}\sqrt{2}\,\theta^+ \chi_+ + \mathrm{i}\sqrt{2}\,\theta^- \chi_- + 2\mathrm{i}\,\theta^+\theta^- G + \ldots$$

$$\Xi = \frac{1}{\sqrt{2}}(y^1 + \mathrm{i}\, y^2) + \mathrm{i}\sqrt{2}\,\theta^+ \overline{\xi}_+ + \mathrm{i}\sqrt{2}\,\overline{\theta}^- \xi_- + 2\mathrm{i}\,\theta^+\overline{\theta}^- G_\Xi + \ldots$$

$$\Theta = \frac{1}{\sqrt{2}}(r^3 + \mathrm{i}\theta^4) + \mathrm{i}\sqrt{2}\,\theta^+ \overline{\widetilde{\chi}}_+ + \mathrm{i}\sqrt{2}\,\overline{\theta}^- \widetilde{\chi}_- + 2\mathrm{i}\,\theta^+\overline{\theta}^- \widetilde{G} + \ldots$$

$$\Gamma = \frac{1}{\sqrt{2}}(\gamma^3 + \mathrm{i}\,\gamma^4) + \mathrm{i}\sqrt{2}\,\theta^+ \zeta_+ + \mathrm{i}\sqrt{2}\,\theta^- \zeta_- + 2\mathrm{i}\,\theta^+\theta^- G_\Gamma + \ldots$$

$$Q_a = q_a + \mathrm{i}\sqrt{2}\,\theta^+ \psi_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \psi_{-,a} + 2\mathrm{i}\,\theta^+\theta^- F_a + \ldots$$

$$\widetilde{Q}_a = \widetilde{q}_a + \mathrm{i}\sqrt{2}\,\theta^+ \widetilde{\psi}_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \widetilde{\psi}_{-,a} + 2\mathrm{i}\,\theta^+\theta^- \widetilde{F}_a + \ldots$$

$$\begin{aligned} V_a = {} & \theta^+\overline{\theta}^+(A_{0,a} + A_{1,a}) + \theta^-\overline{\theta}^-(A_{0,a} - A_{1,a}) - \sqrt{2}\,\theta^-\overline{\theta}^+\sigma_a - \sqrt{2}\,\theta^+\overline{\theta}^-\overline{\sigma}_a \\ & -2\mathrm{i}\,\theta^+\theta^-\big(\overline{\theta}^+\overline{\lambda}_{+,a} + \overline{\theta}^-\overline{\lambda}_{-,a}\big) + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\big(\theta^+\lambda_{+,a} + \theta^-\lambda_{-,a}\big) - 2\,\theta^+\theta^-\overline{\theta}^+\overline{\theta}^- D_{V,a} \end{aligned}$$

$$\Phi_a = \phi_a + \mathrm{i}\sqrt{2}\,\theta^+ \widetilde{\lambda}_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \widetilde{\lambda}_{-,a} + 2\mathrm{i}\,\theta^+\theta^- D_{\Phi,a} + \ldots \qquad = \overline{D}_+\overline{D}_- C_a$$

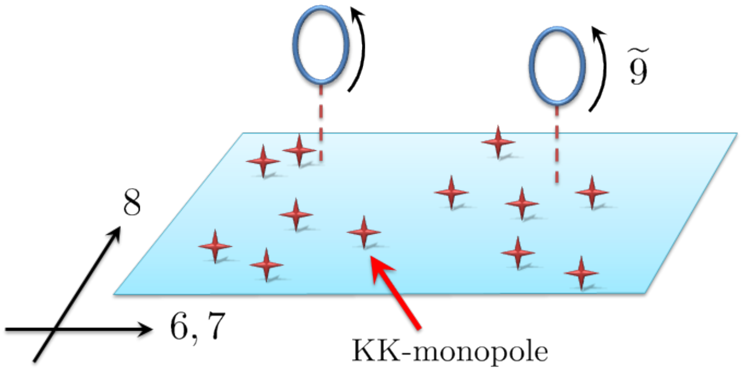
$$\begin{aligned} C_a = {} & \phi_{c,a} + \mathrm{i}\sqrt{2}\,\theta^+\psi_{c+,a} + \mathrm{i}\sqrt{2}\,\theta^-\psi_{c-,a} + \mathrm{i}\sqrt{2}\,\overline{\theta}^+\chi_{c+,a} + \mathrm{i}\sqrt{2}\,\overline{\theta}^-\chi_{c-,a} \\ & +2\mathrm{i}\,\theta^+\theta^- F_{c,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^- M_{c,a} + 2\mathrm{i}\,\theta^+\overline{\theta}^- G_{c,a} + 2\mathrm{i}\,\overline{\theta}^+\theta^- N_{c,a} + \theta^-\overline{\theta}^- A_{c=,a} + \theta^+\overline{\theta}^+ B_{c+\!\!+,a} \\ & -2\mathrm{i}\,\theta^+\theta^-\overline{\theta}^+\zeta_{c+,a} - 2\mathrm{i}\,\theta^+\theta^-\overline{\theta}^-\zeta_{c-,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\theta^+\lambda_{c+,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\theta^-\lambda_{c-,a} - 2\theta^+\theta^-\overline{\theta}^+\overline{\theta}^- D_{c,a} \end{aligned}$$

with $\overline{D}_\pm = -\dfrac{\partial}{\partial\overline{\theta}^\pm} + \mathrm{i}\,\theta^\pm(\partial_0 \pm \partial_1)$, $D_\pm = \dfrac{\partial}{\partial\theta^\pm} - \mathrm{i}\,\overline{\theta}^\pm(\partial_0 \pm \partial_1)$

H KK 522

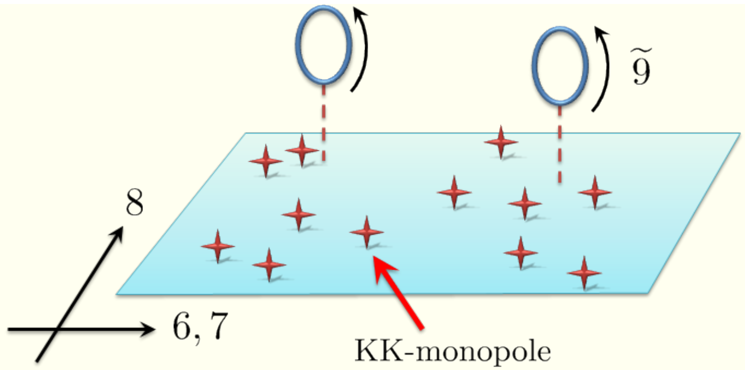
Worldsheet instanton corrections to GLSM for KK-monopoles :

string winding modes along $\theta^4 \sim x^9$, rather than $\gamma^4 \sim \widetilde{x}^9$




$$H \to \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2}R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(\theta^4)}$$

$\longrightarrow$




+

string winding along
dual circle $\theta^4 \sim x^9$

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