Confinement and Dynamical Symmetry Breaking in non-SUSY Gauge Theory from S-duality in String Theory

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(Based on arXiv: 1207.2203)

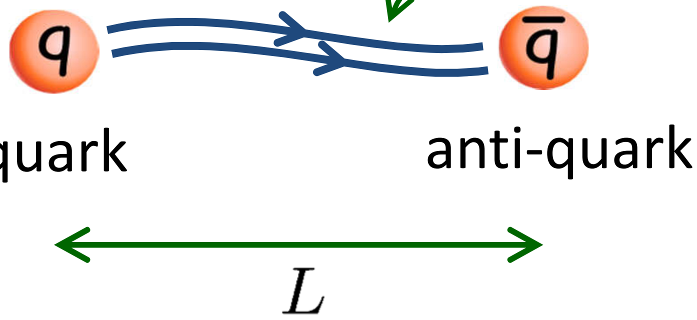
Seminar @ 阪大 2013 July 19

1 Introduction

Confinement problem:

(for SU(3) gauge field)

squeezed "electric" flux



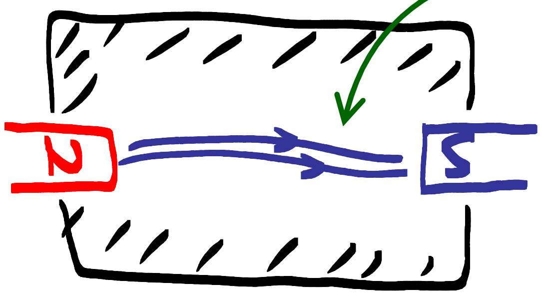
➡ $V(L) \propto L$

linear potential

But, why?

cf) Superconductor (Meissner effect)

squeezed magnetic flux



In superconductor,
U(1) gauge sym is Higgsed

➡ magnetic flux is squeezed

- To apply this idea to the confinement problem,

  (1) find a magnetic dual description

  (2) show that the magnetic theory is Higgsed

➡ Confinement via "dual Meissner effect" [Nambu, 't Hooft, Mandelstam in the '70s]

- We know this scenario really works in many examples in SUSY gauge theory.

  Ex) N=2 SYM (mass deformed) [Seiberg-Witten '94]

  N=1 SQCD [Seiberg '95]

- How about non-SUSY cases?

  * Long history in non-SUSY QCD, but not settled yet.

  * It is usually difficult to find convincing evidence of duality.

  ➡ Can we use string theory?

- In this talk, we consider the following duality

$O3^{+}\text{-}\overline{D3}$ ⟷ S-dual in type IIB ⟷ $\widetilde{O3}^{-}\text{-}\overline{D3}$ [Uranga' 99]

↓ Low energy ↓ Low energy

Electric theory

		gauge	global
	ele	$USp(2n)$	$SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	4_+
scalar	Φ^I	$\square\square$	6

Magnetic theory

		gauge	global
	mag	$SO(2n-1)$	$SO(6)$
gauge	a_μ	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	1
fermion	q^i	$\square\square$	4_+
scalar	ϕ^I	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	6

Electric theory

ele	gauge	global
	$USp(2n)$	$SO(6)$
A_μ	$\square\square$	1
Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\square$	6

Strongly coupled at IR

dual

Magnetic theory

mag	gauge	global
	$SO(2n-1)$	$SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

Weakly coupled at IR

- Conjectured properties:

  ① Confinement

  ② Dynamical Sym Breaking

$$\langle Q^i Q^j \rangle \propto \delta^{ij}$$

$$SO(6) \simeq SU(4) \to SO(4)$$

- Decoupling limit is ambiguous.
- Physics at IR should be easier than that in electric theory

<u>Q</u> : Why are they dual?

<u>Q</u> : Can we understand ① , ② using the duality?

CAUTION

- I am NOT going to "prove" or "derive" confinement and dynamical symmetry breaking,

but I will just "try to understand" what is going on under duality.

- Some of the arguments are based on a speculative model.

Please be generous!

Plan of Talk

✓ ① Introduction

② Brief review of D-branes and S-duality

③ Brief review of O3-planes

④ O3-$\overline{\text{D3}}$ system

⑤ Confinement and DSB

⑥ Summary

2 Brief review of D-branes and S-duality

- Type IIB superstring theory

Space-time is 10 dimensional

Massless bosonic fields :

$$\underbrace{\phi \quad g_{\mu\nu} \quad B_2}_{\text{NS-NS field}} \quad \underbrace{C_0 \quad C_2 \quad C_4}_{\text{R-R field}}$$

C_n : n-form field

$$C_2 = \frac{1}{2} C_{\mu\nu}\, dx^\mu \wedge dx^\nu \ , \text{ etc.}$$

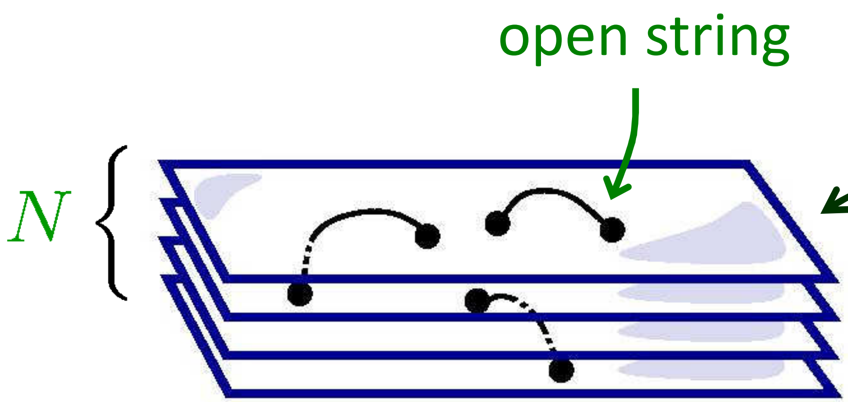
- S-duality

This theory is believed to be invariant under "S-duality"

$$\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ C_2 \end{pmatrix} \qquad \tau \to -1/\tau \qquad (\tau \equiv C_0 + i e^{-\phi})$$

Strong coupling ($e^\phi \gg 1$) $\longleftrightarrow$ weak coupling ($e^\phi \ll 1$)

D-brane and Gauge theory



D*p*-brane

$\simeq$ (p+1) dim. plane, on which open strings can end.

a ⌒ b $\quad\rightarrow\quad$ massless mode $(A_\mu)^a{}_b$ etc.

$a, b = 1 \sim N$ $\qquad\qquad$ $U(N)$ gauge field

⇒ (p+1) dim *U(N)* gauge theory is realized on the Dp-brane

D3-branes and *N*=4 SYM

n D3-branes

massless modes

		gauge	global
		$U(n)$	$SO(6)$
gauge	A_μ	adjoint	1
fermion	Q^i	adjoint	4_+
scalar	Φ^I	adjoint	6

$\mathcal{N}=4$ $U(n)$ Super Yang-Mills

S-duality in *N*=4 SYM

D3-branes are S-duality invariant

Consistent with S-duality in *N*=4 SYM

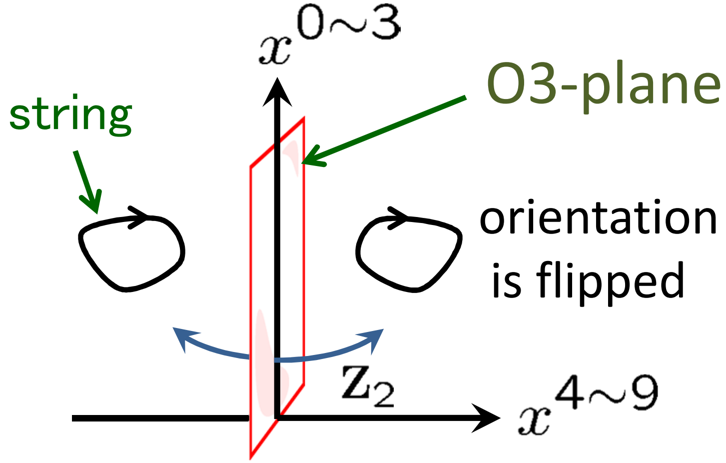
$$\tau \to -1/\tau \qquad \left(\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}\right)$$

Strong coupling ($g^2 \gg 1$) $\longleftrightarrow$ weak coupling ($g^2 \ll 1$)

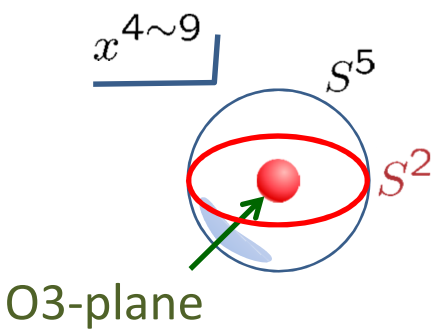
Generalization of electric-magnetic duality in Maxwell theory

③ Brief review of O3-planes

- O3-plane: fixed plane of $\mathbf{Z}_2$ $\begin{cases} x^{4\sim9} \to -x^{4\sim9} \\ \text{and flip orientation of strings} \end{cases}$



- Discrete torsion $\quad H_2(S^5/\mathbf{Z}_2, \tilde{\mathbf{Z}}) \simeq \mathbf{Z}_2$ [Witten '98]



$$\tau_{\mathrm{NS}} = \exp\left(i\int_{S^2} B_2\right) = \pm 1 \qquad \tau_{\mathrm{RR}} = \exp\left(i\int_{S^2} C_2\right) = \pm 1$$

4 types:

	$\mathrm{O3}^-$	$\mathrm{O3}^+$	$\widetilde{\mathrm{O3}}^-$	$\widetilde{\mathrm{O3}}^+$
$(\tau_{\mathrm{NS}}, \tau_{\mathrm{RR}})$	$(+,+)$	$(-,+)$	$(+,-)$	$(-,-)$

S-duality of O3-planes

[Witten '98]

$\tau_{NS} \xleftrightarrow{\text{S-dual}} \tau_{RR}$ $\quad$ $O3^-$ S-dual $\quad$ $O3^+ \xleftrightarrow{\text{S-dual}} \widetilde{O3}^-$

O3 + *n* D3 system

$$O3^- + n\,\mathrm{D3} \Rightarrow \mathcal{N}=4\ SO(2n)\ \mathrm{SYM}$$ S-dual

$$O3^+ + n\,\mathrm{D3} \Rightarrow \mathcal{N}=4\ USp(2n)\ \mathrm{SYM}$$

S-dual

$$\widetilde{O3}^- + n\,\mathrm{D3} \Rightarrow \mathcal{N}=4\ SO(2n+1)\ \mathrm{SYM}$$

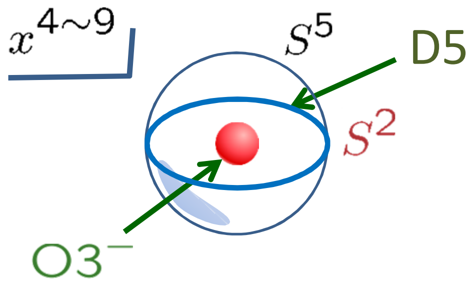
Consistent with the S-duality in N=4 SYM.

- $$\widetilde{O3}^- \sim O3^- + \frac{1}{2}\mathrm{D3}$$

However, this picture is misleading to understand non-perturbative properties, such as S-duality.

- **Better picture:** [Hyakutake-Imamura-S.S. '00]

$$\boxed{\widetilde{\mathrm{O3}}^- \sim \mathrm{O3}^- + \text{spherical D5}}$$ (topologically)

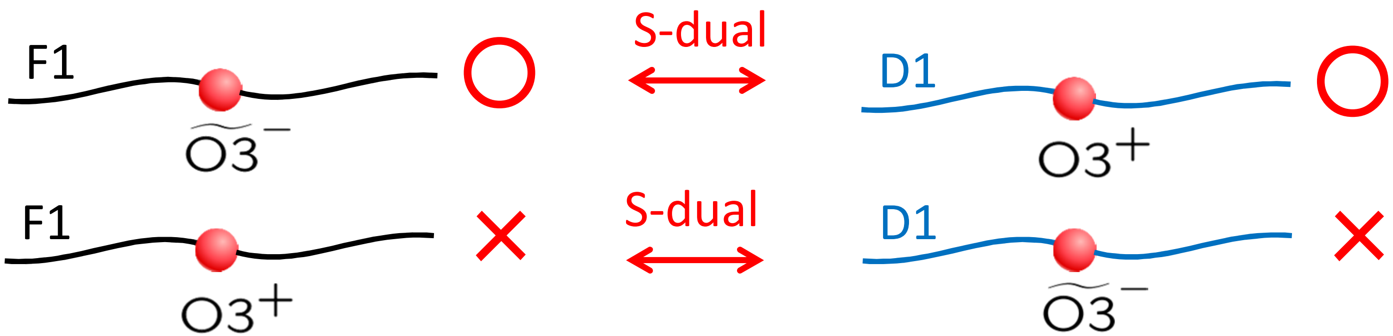


magnetic flux on D5

$$\frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = \frac{1}{2}$$ ➡ RR charge of ½ D3-brane

Similarly, $$\boxed{\mathrm{O3}^+ \sim \mathrm{O3}^- + \text{spherical NS5}}$$

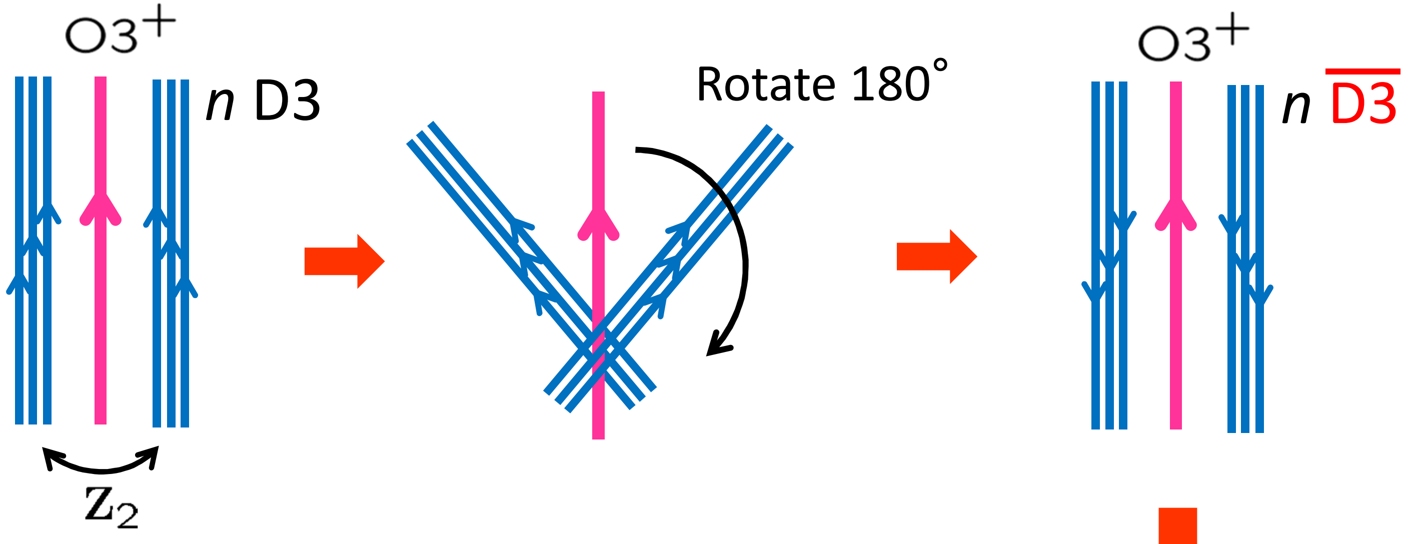
- **Strings ending on O3-planes:**



Cf) F1 can end on D5, but not on NS5
D1 can end on NS5, but not on D5

4 O3-$\overline{\text{D3}}$ system

[S.S. '99, Uranga '99]



		gauge $USp(2n)$	global $SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\square\square$	4_+
scalar	Φ^I	$\square\square$	6

$\mathcal{N}=4$ $USp(2n)$ SYM

	ele	gauge $USp(2n)$	global $SO(6)$
gauge	A_μ	$\square\square$	1
fermion	Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
scalar	Φ^I	$\square\square$	6

non-SUSY

- $\mathrm{O3}^{+} + n\,\overline{\mathrm{D3}}$

ele

		gauge $USp(2n)$	global $SO(6)$
gauge	A_μ	$\square\!\square$	1
fermion	Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
scalar	Φ^I	$\square\!\square$	6

S-dual

- $\underbrace{\widetilde{\mathrm{O3}}^{-}}_{\mathrm{O3}^{-} + \frac{1}{2}\mathrm{D3}} + n\,\overline{\mathrm{D3}}$

$$\overbrace{\mathrm{O3}^{-} + \frac{1}{2}\mathrm{D3} + n\,\overline{\mathrm{D3}}}$$

Tachyon condensation [Sen '98]

$$\xrightarrow[\langle t\rangle \neq 0]{} \quad \mathrm{O3}^{-} + \frac{1}{2}(2n-1)\,\overline{\mathrm{D3}}$$

[Uranga '99]

mag A

		gauge $SO(2n)$	global $SO(6)$
gauge	a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion	q^i	$\square\!\square$	4_+
scalar	ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
tachyon	t	$\square$	1
fermion	ψ_i	$\square$	4_-

D3-$\overline{\mathrm{D3}}$ string

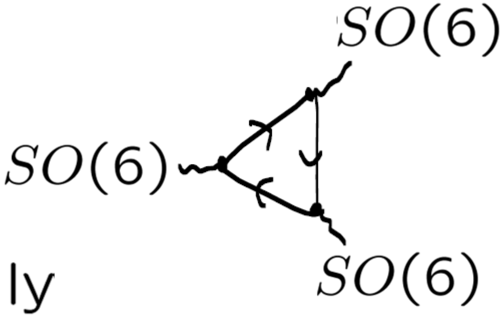
mag B

		gauge $SO(2n-1)$	global $SO(6)$
gauge	a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion	q^i	$\square\!\square$	4_+
scalar	ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

- ’t Hooft anomaly matching condition for SO(6)[3] is satisfied

ele

	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\!\square$	1
fermion Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\!\square$	6



$SO(6)^3$ anomaly

$$\propto \frac{1}{2}\underset{\substack{\uparrow\\ Q^i}}{2n(2n-1)} = \underline{2n^2 - n}$$

mag A

	gauge $SO(2n)$	global $SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
fermion q^i	$\square\!\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
t	$\square$	1
fermion ψ_i	$\square$	4_-

$$\propto \frac{1}{2}\underset{\substack{\uparrow\\ q^i}}{2n(2n+1)} - \underset{\substack{\uparrow\\ \psi_i}}{2n} = \underline{2n^2 - n}$$

5 Confinement and DSB

- *n*=1 case

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\square$	1
Q^i	$\begin{array}{c}\square\\\square\end{array}$	4_+
Φ^I	$\square\square$	6

n=1

$USp(2) \simeq SU(2)$

Q^i : $\begin{array}{c}\square\\\square\end{array}$ is gauge singlet

Φ^I become massive via quantum effect

$\Rightarrow$ SU(2) pure YM !

Believed to be confining

Magnetic description of pure YM theory

mag A

	gauge $SO(2n)$	global $SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
tachyon t	$\square$	1
ψ_i	$\square$	4_-

$$\xrightarrow{\langle t\rangle \neq 0}$$

mag B

	gauge $SO(2n-1)$	global $SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

n=1 (mag A)

$$SO(2) \simeq U(1)$$

t is magnetic monopole

$$\xrightarrow{\langle t\rangle \neq 0}$$

"monopole condensation"

n=1 (mag B)

Gauge symmetry is completely Higgsed

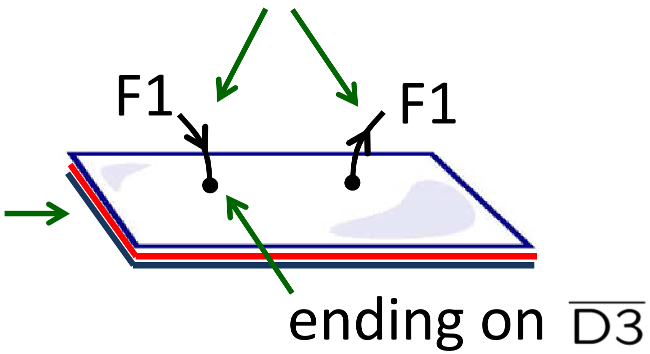
→ Confinement in electric theory

Manifestation of "dual Meissner effect" !

- Q-$\bar{\text{Q}}$ potential

ele

$O3^+ + \overline{D3}$

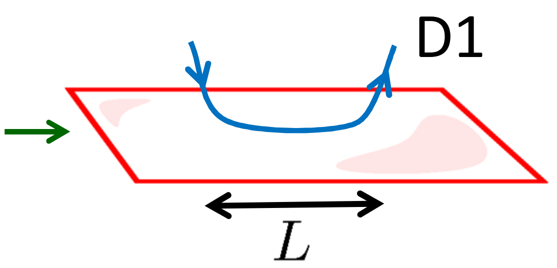


This picture is valid only when the coupling is small

mag B

$$O3^- + \frac{1}{2}\overline{D3} \sim \left(O3^- + \text{spherical D5 with } \frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = -\frac{1}{2}\right) \equiv \widehat{O3}^-$$

$\widehat{O3}^-$



D1 cannot end on $\widehat{O3}^-$

- Should be connected
- linear potential $V(L) \propto L$
- Confinement !
  (in electric theory)

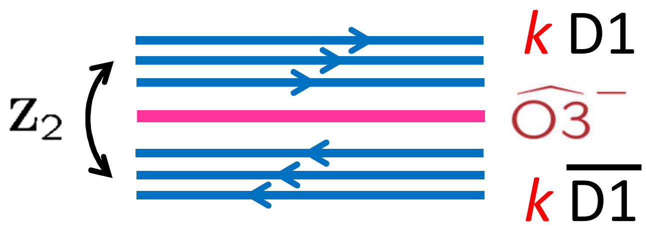
2-ality

ele All the fields are rank two tensors of USp(2n)

Flux tube associated with fundamental rep. is stable,
but rank 2 tensor can be screened.

➡ The fluxes are classified by $\mathbf{Z}_2$

mag B Consider k flux tubes,



D1 world-volume gauge theory is $U(k)$ theory with tachyon in

[Bergman '00]

➡ k=1 is stable (no tachyon), while two D1-$\overline{\text{D1}}$ pairs can be annihilated via tachyon condensation.

➡ Consistent with the above $\mathbf{Z}_2$ property !

Cf) non-BPS D7 in type I is a $\mathbf{Z}_2$ charged object. [Witten '98]

n > 1 case

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\!\square$	1
Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\!\square$	6

mag B	gauge $SO(2n-1)$	global $SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\!\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

Weakly coupled at UV

$m_\Phi^2 > 0 \quad$ (1-loop)

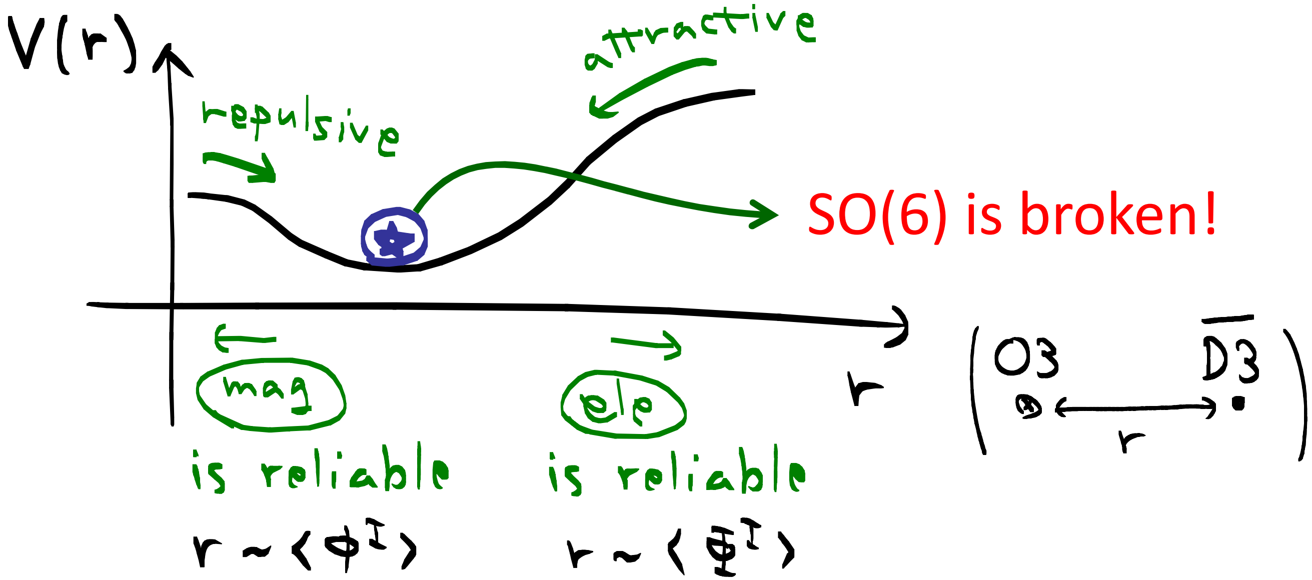
O3$^+$ and $\overline{\text{D3}}$ are attractive

Weakly coupled at IR

$m_\phi^2 < 0 \quad$ (1-loop)

$\widehat{\text{O3}}^-$ and $\overline{\text{D3}}$ are repulsive

➡ We expect:



- Unfortunately, we do not know the precise form of the potential.
- To proceed, we consider the following *speculative model* that seems to capture some of the qualitative features in the magnetic description.

$$V(\phi^I) = -\frac{\mu^2}{2}\,\mathrm{tr}(\phi^I\phi^I) - \frac{g}{4}\,\mathrm{tr}\left([\phi^I,\phi^J]^2\right) + \frac{\lambda}{2}\,\mathrm{tr}\left((\phi^I\phi^I)^2\right)$$

One-loop tachyonic mass term

Tree level potential

Added to stabilize the potential

- This model has a fuzzy sphere solution:

(2*n*-1) dim representation

$$\phi^{1\sim3} = aJ^{1\sim3}$$
$$\phi^{4\sim6} = 0$$

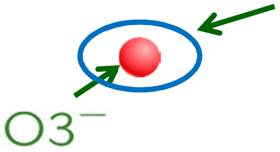
J^i : spin (*n*-1) representation of SU(2)

$$a = \sqrt{\frac{\mu^2}{2g + 2\lambda n(n-1)}}$$

- This fuzzy sphere solution corresponds to the following picture

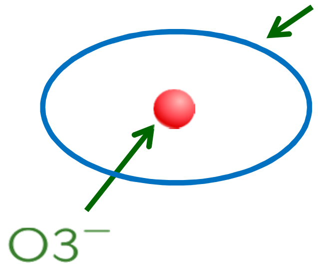
mag B : $\widehat{\mathrm{O3}}^{-} + (n-1)\,\overline{\mathrm{D3}}$

$\widehat{\mathrm{O3}}^{-} \sim$



spherical D5 with $\frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = -\frac{1}{2}$

$\widehat{\mathrm{O3}}^{-} + (n-1)\,\overline{\mathrm{D3}} \sim$



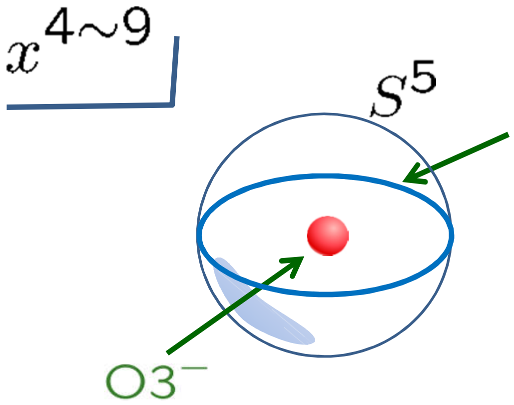
spherical D5

with $\frac{1}{2\pi}\int_{S^2/\mathbf{Z}_2} F = -\frac{1}{2} - (n-1)$

($\overline{\mathrm{D3}}$ are absorbed into D5)

$\mathrm{O3}^{-}$ and $\overline{\mathrm{D3}}$ are repulsive ➡ makes the sphere bigger.

Symmetry breaking



Suppose that the sphere is embedded in $x^{4\sim6}$

$\underset{x^{4\sim9}}{SO(6)}$ is broken to $\underset{x^{4\sim6}}{SO(3)} \times \underset{x^{7\sim9}}{SO(3)} \simeq SU(2) \times SU(2)$!

- This breaking pattern is consistent with the dynamical symmetry breaking expected in electric theory.

$$SO(6) \simeq SU(4) \xrightarrow{\langle Q^i Q^j \rangle \propto \delta^{ij}} SO(4) \simeq SU(2) \times SU(2)$$

Confinement

The fuzzy sphere solution:

(2*n*-1) dim representation

$$\phi^{1\sim3} = aJ^{1\sim3}$$
$$\phi^{4\sim6} = 0$$

$$\begin{cases} J^i & \text{: spin } (n\text{-}1) \text{ representation of SU(2)} \\ a = \sqrt{\dfrac{\mu^2}{2g + 2\lambda n(n-1)}} & \end{cases}$$

➡ The SO(2n-1) gauge symmetry is completely broken.

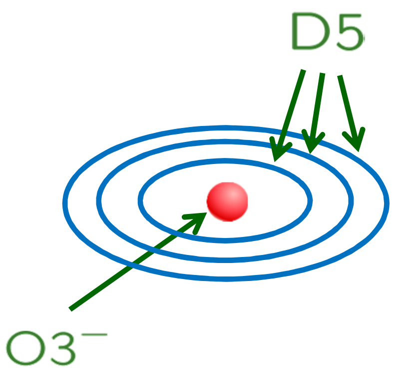
➡ Confinement in electric theory !

Note:

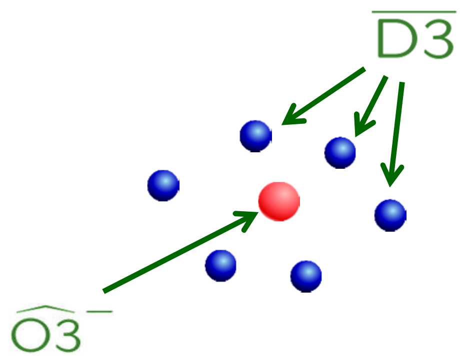
For large enough λ, we can show the following:

- The fuzzy sphere solution is stable against small fluctuations.
- The fuzzy sphere solution has lower energy than the following configurations:

multiple spheres



isolated $\overline{\text{D3}}$'s



6 Summary

$O3^{+}\text{-}\overline{D3}$ $\xleftrightarrow{\text{S-dual}}$ $\widetilde{O3}^{-}\text{-}\overline{D3}$

$\downarrow$ Low energy $\qquad\qquad$ $\downarrow$ Low energy

Electric theory

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\square$	1
Q^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4_+
Φ^I	$\square\square$	6

Magnetic theory

mag	gauge $SO(2n-1)$	global $SO(6)$
a_μ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
q^i	$\square\square$	4_+
ϕ^I	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

Confinement
Dynamical Sym Breaking
$\longleftrightarrow$
Tachyon condensation
Fuzzy sphere configuration

Thank you !