Confinement and Dynamical Symmetry Breaking in non-SUSY Gauge Theory from S-duality in String Theory

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(Based on arXiv: 1207.2203)
1 Introduction

Confinement problem:

\[
\begin{align*}
q & \quad \text{quark} \\
\bar{q} & \quad \text{anti-quark}
\end{align*}
\]

\[L\]

squeezed “electric” flux

\[V(L) \propto L\]

linear potential

But, why?

\[\text{cf) Superconductor (Meissner effect)}\]

squeezed magnetic flux

In superconductor, U(1) gauge sym is Higgsed

\[\Rightarrow \text{magnetic flux is squeezed}\]
To apply this idea to the confinement problem,

1. find a magnetic dual description
2. show that the magnetic theory is Higgsed

Confinement via “dual Meissner effect” [Nambu, ’t Hooft, Mandelstam in the ’70s]

We know this scenario really works in many examples in SUSY gauge theory.

Ex) $N=2$ SYM (mass deformed) [Seiberg-Witten ’94]
    $N=1$ SQCD [Seiberg ’95]

How about non-SUSY cases?

* Long history in non-SUSY QCD, but not settled yet.
* It is usually difficult to find convincing evidence of duality.

Can we use string theory?
In this talk, we consider the following duality

\[ O3^+ - D3 \quad \longleftrightarrow \quad \bar{O}3^- - \bar{D}3 \]

[Uranga’ 99]

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<tr>
<th>Electric theory</th>
<th>Magnetic theory</th>
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<td>( SO(2n - 1) )</td>
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<td>( \Phi^I )</td>
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\( 1 \)

\( 4_+ \)

\( 6 \)

\( 1 \)

\( 4_+ \)

\( 6 \)
Electric theory

Conjectured properties:

1. Confinement
2. Dynamical Sym Breaking

\[ \langle Q^i Q^j \rangle \propto \delta^{ij} \]

\[ SO(6) \simeq SU(4) \rightarrow SO(4) \]

Q: Why are they dual?
Q: Can we understand 1, 2 using the duality?

Magnetic theory

Decoupling limit is ambiguous.

Physics at IR should be easier than that in electric theory.
CAUTION

- I am NOT going to “prove” or “derive” confinement and dynamical symmetry breaking,
  but I will just “try to understand” what is going on under duality.

- Some of the arguments are based on a speculative model.

Please be generous!
Plan of Talk

1. Introduction
2. Brief review of D-branes and S-duality
3. Brief review of O3-planes
4. O3-D3 system
5. Confinement and DSB
6. Summary
Type IIB superstring theory

Space-time is 10 dimensional

Massless bosonic fields:

\[
\begin{align*}
\phi & \quad g_{\mu\nu} & \quad B_2 & \quad C_0 & \quad C_2 & \quad C_4 \\
\text{NS-NS field} & & & & & \text{R-R field}
\end{align*}
\]

\[C_n : n\text{-form field}\]
\[C_2 = \frac{1}{2} C_{\mu\nu} dx^\mu \wedge dx^\nu, \text{ etc.}\]

S-duality

This theory is believed to be invariant under “S-duality”

\[
\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ C_2 \end{pmatrix} \quad \tau \to -1/\tau \quad (\tau \equiv C_0 + ie^{-\phi})
\]

Strong coupling \((e^\phi \gg 1)\) ↔ weak coupling \((e^\phi \ll 1)\)
D-brane and Gauge theory

D_p-brane

\(\sim (p+1)\) dim. plane, on which open strings can end.

open string

massless mode

\[ (A_\mu)^a_b \]
eq etc.

\(U(N)\) gauge field

\(a, b = 1 \sim N\)

(p+1) dim \(U(N)\) gauge theory is realized on the Dp-brane
D3-branes and $N=4$ SYM

$n$ D3-branes

massless modes

gauge

fermion

scalar

D3-branes are S-duality invariant

Consistent with S-duality in $N=4$ SYM

$$\tau \to -1/\tau$$

$$\left( \tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \right)$$

Strong coupling ($g^2 \gg 1$) $\leftrightarrow$ weak coupling ($g^2 \ll 1$)

Generalization of electric-magnetic duality in Maxwell theory

$\mathcal{N} = 4$ $U(n)$ Super Yang-Mills

### Table

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<td>adjoint</td>
<td>6</td>
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3 Brief review of O3-planes

- **O3-plane**: fixed plane of $\mathbb{Z}_2$ \(x^{4\sim 9} \rightarrow -x^{4\sim 9}\) and flip orientation of strings

- **Discrete torsion**
  \[ H_2(S^5/\mathbb{Z}_2, \mathbb{Z}) \cong \mathbb{Z}_2 \]  
  \[ \tau_{\text{NS}} = \exp\left(i \int_{S^2} B_2\right) = \pm 1 \quad \tau_{\text{RR}} = \exp\left(i \int_{S^2} C_2\right) = \pm 1 \]

4 types:

<table>
<thead>
<tr>
<th>(\tau_{\text{NS}}, \tau_{\text{RR}})</th>
<th>$O3^-$</th>
<th>$O3^+$</th>
<th>$\overline{O3}^-$</th>
<th>$\overline{O3}^+$</th>
</tr>
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<tbody>
<tr>
<td>$(+, +)$</td>
<td>$(-, +)$</td>
<td>$(+, -)$</td>
<td>$(-, -)$</td>
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**S-duality of O3-planes**

\[ \tau_{\text{NS}} \leftrightarrow \tau_{\text{RR}} \]

\[ \text{O}_3^- \leftrightarrow \text{O}_3^+ \leftrightarrow \widetilde{\text{O}}_3^- \]

[Witten ’98]

**O3 + n D3 system**

\[ \text{O}_3^- + n \text{D3} \rightarrow \mathcal{N} = 4 \ SO(2n) \ \text{SYM} \]

\[ \text{O}_3^+ + n \text{D3} \rightarrow \mathcal{N} = 4 \ USp(2n) \ \text{SYM} \]

\[ \widetilde{\text{O}}_3^- + n \text{D3} \rightarrow \mathcal{N} = 4 \ SO(2n + 1) \ \text{SYM} \]

Consistent with the S-duality in N=4 SYM.

\[ \widetilde{\text{O}}_3^- \sim \text{O}_3^- + \frac{1}{2} \text{D3} \]

However, this picture is misleading to understand non-perturbative properties, such as S-duality.
**Better picture:**

\[ \widetilde{O}3^- \sim O3^- + \text{spherical D5} \]  (topologically)

\[ \frac{1}{2\pi} \int_{S^2/Z_2} F = \frac{1}{2} \]

\text{RR charge of } \frac{1}{2} \text{ D3-brane}

Similarly, \[ O3^+ \sim O3^- + \text{spherical NS5} \]

**Strings ending on O3-planes:**

Cf) F1 can end on D5, but not on NS5

D1 can end on NS5, but not on D5
**O3–D3 system**

\[ \mathcal{N} = 4 \text{ } USp(2n) \text{ SYM} \]

[S.S. ’99, Uranga ’99]
\( \mathcal{O}^3^+ + n \overline{D3} \rightarrow \mathcal{O}^3^- + n D3 \)

\( S\text{-dual} \)

\( \mathcal{O}^3^- + \frac{1}{2}D3 + n \overline{D3} \)

\( \langle t \rangle \neq 0 \)

Tachyon condensation

\( \mathcal{O}^3^- + \frac{1}{2}(2n - 1) \overline{D3} \)

\[\text{ele} \]

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\[\text{mag A} \]

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\[\text{D3-D3 string} \]

\[\text{mag B} \]

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‘t Hooft anomaly matching condition for $SO(6)^3$ is satisfied

\[ \alpha \frac{1}{2} 2n(2n - 1) = 2n^2 - n \]

\[ \alpha \frac{1}{2} 2n(2n + 1) - 2n = 2n^2 - n \]
**Confinement and DSB**

- **n=1 case**

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$n=1$:

$USp(2) \simeq SU(2)$

$Q^i$ : 1 is gauge singlet

$\Phi^I$ become massive via quantum effect

$SU(2)$ pure YM!

Believed to be confining
Magnetic description of pure YM theory

\[
\begin{array}{c|cc}
\text{gauge} & SO(2n) & \text{global} \\ 
\hline
a_\mu & 1 \\ 
q^i & 4_+ \\ 
\phi^I & 6 \\ 
\Psi_i & 1 \\ 
\psi_i & 4_- \\
\end{array}
\]

\[
\begin{array}{c|cc}
\text{gauge} & SO(2n - 1) & \text{global} \\ 
\hline
a_\mu & 1 \\ 
q^i & 4_+ \\ 
\phi^I & 6 \\
\end{array}
\]

\(\langle t \rangle \neq 0\)

\(n=1\)

\(SO(2) \simeq U(1)\)

t is magnetic monopole

“The monopole condensation”

Gauge symmetry is completely Higgsed

Confinement in electric theory

Manifestation of “dual Meissner effect”!
**Q-\bar{Q} potential**

Quark in fund rep.

This picture is valid only when the coupling is small.

\[ \text{mag } B \quad O3^- + \frac{1}{2} D3 \sim \left( O3^- + \text{spherical D5 with } \frac{1}{2\pi} \int_{S^2/Z_2} F = -\frac{1}{2} \right) \equiv \hat{O}3^- \]

D1 cannot end on \( \hat{O}3^- \)

- Should be connected
- linear potential \( V(L) \propto L \)
- Confinement!
  (in electric theory)
2-ality

All the fields are rank two tensors of USp(2n).

Flux tube associated with fundamental rep. is stable, but rank 2 tensor can be screened.

The fluxes are classified by $\mathbb{Z}_2$.

Consider $k$ flux tubes, $\mathbb{Z}_2$ stabilizes $k$ D1 pairs.

D1 world-volume gauge theory is $U(k)$ theory with tachyon in $\mathbb{Z}_2$.

$k=1$ is stable (no tachyon), while two D1-D1 pairs can be annihilated via tachyon condensation.

Consistent with the above $\mathbb{Z}_2$ property!

Cf) non-BPS D7 in type I is a $\mathbb{Z}_2$ charged object. [Witten ’98]
\( n > 1 \) case

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Weakly coupled at UV
\( m_\phi^2 > 0 \) (1-loop)
\( O3^+ \) and \( D3 \) are attractive

Weakly coupled at IR
\( m_\phi^2 < 0 \) (1-loop)
\( \hat{O3}^- \) and \( D3 \) are repulsive

We expect:

\[ V(r) \]

So(6) is broken!
Unfortunately, we do not know the precise form of the potential.

To proceed, we consider the following *speculative model* that seems to capture some of the qualitative features in the magnetic description.

\[
V(\phi^I) = -\frac{\mu^2}{2} \text{tr}(\phi^I \phi^I) - \frac{g}{4} \text{tr} ([\phi^I, \phi^J]^2) + \frac{\lambda}{2} \text{tr} ((\phi^I \phi^I)^2)
\]

- One-loop tachyonic mass term
- Tree level potential
- Added to stabilize the potential

This model has a fuzzy sphere solution:

\[
\phi^{1 \sim 3} = a J^{1 \sim 3} \\
\phi^{4 \sim 6} = 0
\]

\[
a = \sqrt{\frac{\mu^2}{2g + 2\lambda n(n - 1)}}
\]

\[
J^i : \text{spin (n-1) representation of SU(2)}
\]

(2n-1) dim representation
This fuzzy sphere solution corresponds to the following picture:

\[
\text{mag } B : \ O_3^- + (n-1) \overline{D_3} \\
\]

\( O_3^- \sim \) spherical \( D_5 \) with \( \frac{1}{2\pi} \int_{S^2/Z_2} F = \frac{1}{2} \)

\( \overline{O_3^-} + (n-1) D_3 \sim \) spherical \( D_5 \) with \( \frac{1}{2\pi} \int_{S^2/Z_2} F = \frac{1}{2} - (n-1) \)

\( \overline{D_3} \) are absorbed into \( D_5 \)

\( O_3^- \) and \( \overline{D_3} \) are repulsive \( \rightarrow \) makes the sphere bigger.
Symmetry breaking

Suppose that the sphere is embedded in $x^{4 \sim 6}$.

$SO(6)$ is broken to $SO(3) \times SO(3) \simeq SU(2) \times SU(2)$.

This breaking pattern is consistent with the dynamical symmetry breaking expected in electric theory.

$$SO(6) \simeq SU(4) \rightarrow SO(4) \simeq SU(2) \times SU(2)$$

$$\langle Q^i Q^j \rangle \propto \delta^{ij}$$
Confinement

The SO(2n-1) gauge symmetry is completely broken.

\[ \phi^{1\sim3} = a J^{1\sim3} \]
\[ \phi^{4\sim6} = 0 \]

\[ j^i : \text{spin } (n-1) \text{ representation of SU(2)} \]
\[ a = \sqrt{\frac{\mu^2}{2g + 2\lambda n(n - 1)}} \]

- The SO(2n-1) gauge symmetry is completely broken.
- Confinement in electric theory!
Note:

For large enough $\lambda$, we can show the following:

- The fuzzy sphere solution is stable against small fluctuations.
- The fuzzy sphere solution has lower energy than the following configurations:
Summary

S-dual

Electric theory

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Magnetic theory

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Confinement
Dynamical Sym Breaking

Tachyon condensation
Fuzzy sphere configuration
Thank you !