Large-Nc gauge theory and chiral random matrix theory

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M.H., J.-W. Lee (KEK→CCNY) and N. Yamada (KEK),
1302.3532[hep-lat]

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introduction

a ‘puzzle’

- QCD and its SO and USp analogues are ‘equivalent’ at large-Nc. (similar to Eguchi-Kawai)

Lovelace, Nucl. Phys. B 201(1982)
Cherman-M.H.-Robles, Phys. Rev. Lett.106 (arXiv:1009.1623)
M.H.-Yamamoto, JHEP 1202 (arXiv: 1103.5480)

- They are different, according to the effective theory calculations (chiral random matrix theory)

(P. H. Damgaard and K. Splittorff pointed it out to me)

<u>Contradiction?</u>

various large-Nc limits

- ‘t Hooft limit ($\lambda = g^2 N_c$ fixed, $N_c \to \infty$)

’t Hooft, Nucl. Phys. B 72 (1974)

- “M-theory limit” (g^2 fixed, $N_c \to \infty$)

Fujita-M.H.-Hoyos, Phys. Rev. D 86 (arXiv:1205.0853[hep-th])
Azeyanagi-Fujita-M.H., Phys. Rev. Lett. 110 (arXiv:1210.3601[hep-th])

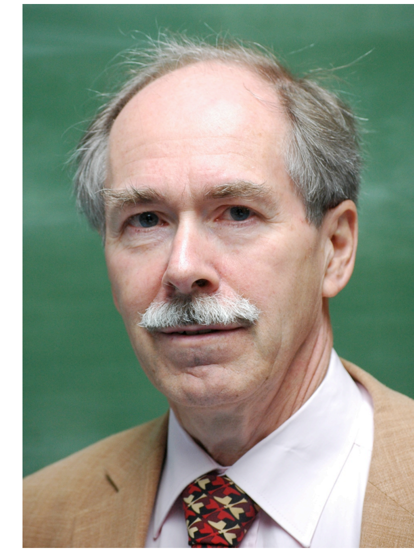
- chi-RMT limit ($m_q V \times (N_c)^\alpha$ fixed, $\alpha > 0$)

M.H.-Lee-Yamada, arXiv:302.3532[hep-lat]

We must understand the similarity and the difference.

't Hooft large-N limit

$\lambda = g^2 N$ *fixed,* $N \to \infty$



why?

- 1/N expansion = genus expansion (string theory)

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

- perturbative series may have a finite radius of convergence at large-N → analytic continuation to strong coupling ?
- Various nice properties (factorization, integrability, gravity dual, ...)

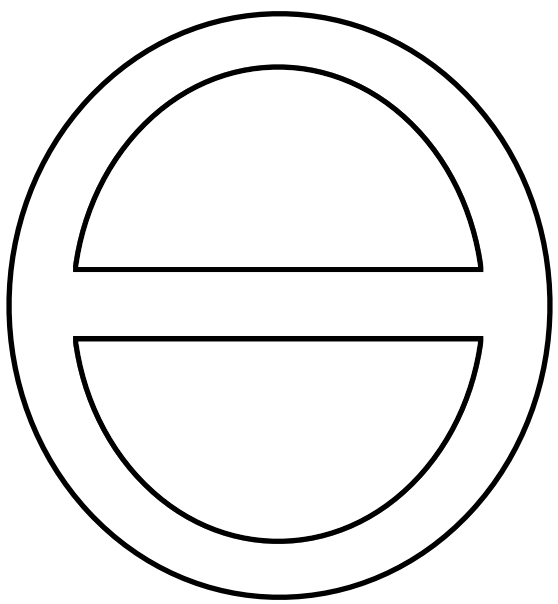
$$S = \frac{N}{4\lambda} \int d^4x Tr F_{\mu\nu}^2$$

vertex $\sim$ N

index loop $\sim$ N

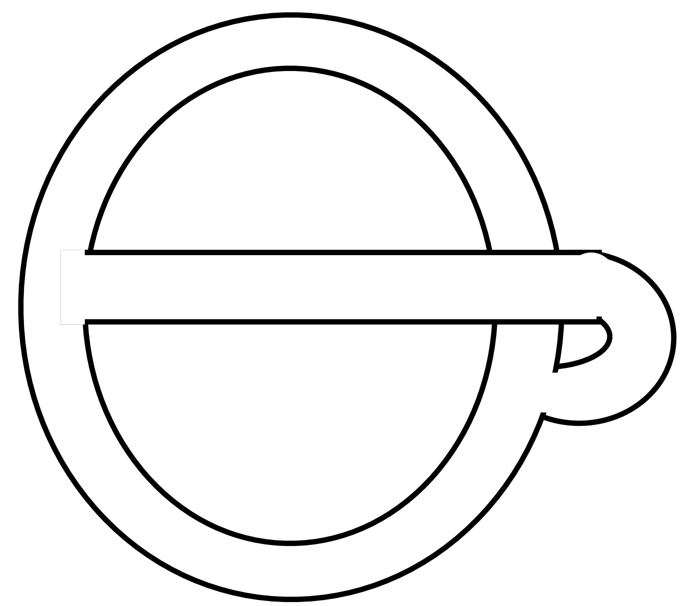
propagator $\sim$ 1/N

planar diagram



$N^2 \times N^{-3} \times N^3 = N^2$

nonplanar diagram
(genus one)



$N^2 \times N^{-3} \times N^1 = N^0$

Assumption

N-dependence appears only through combinatorics.

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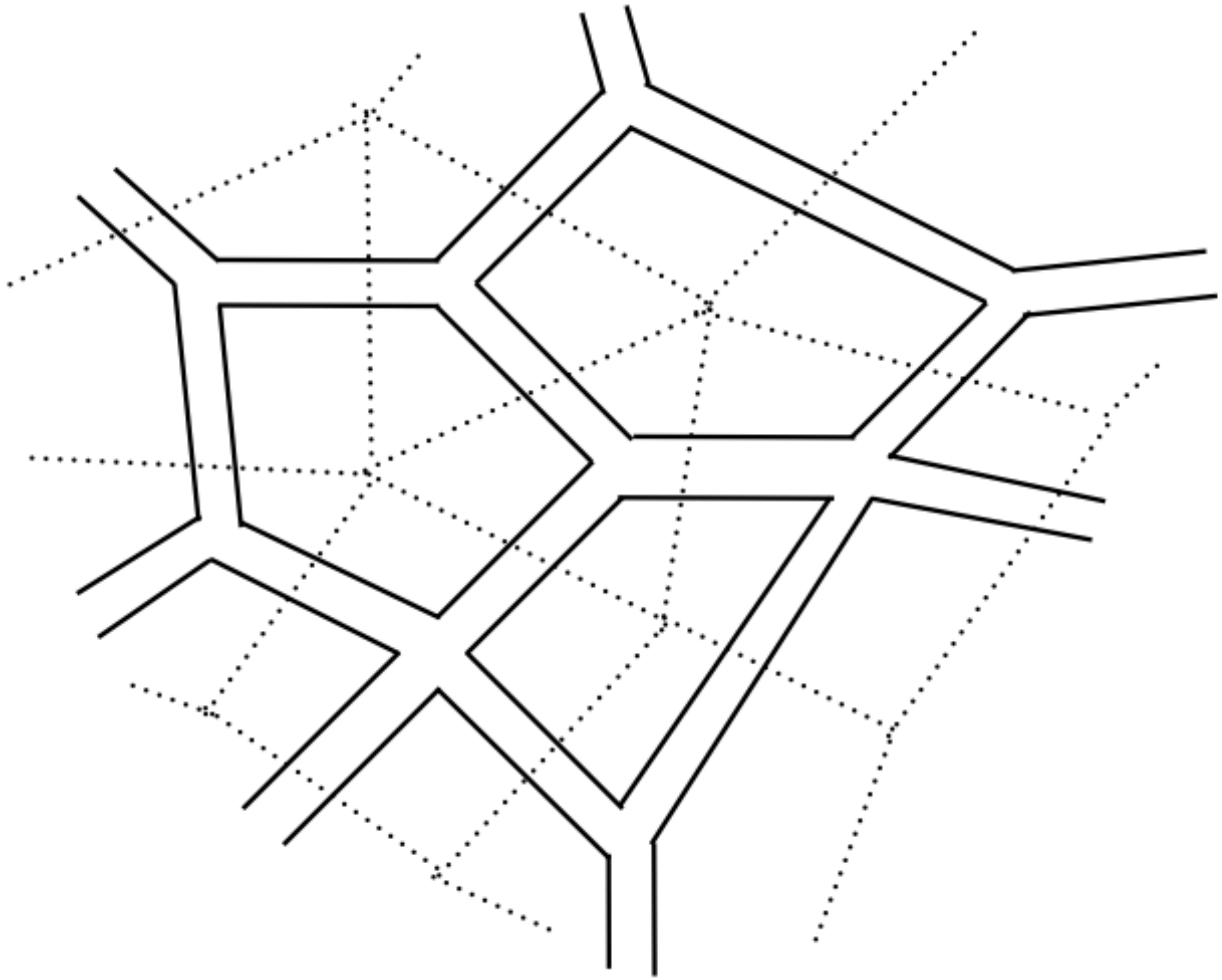
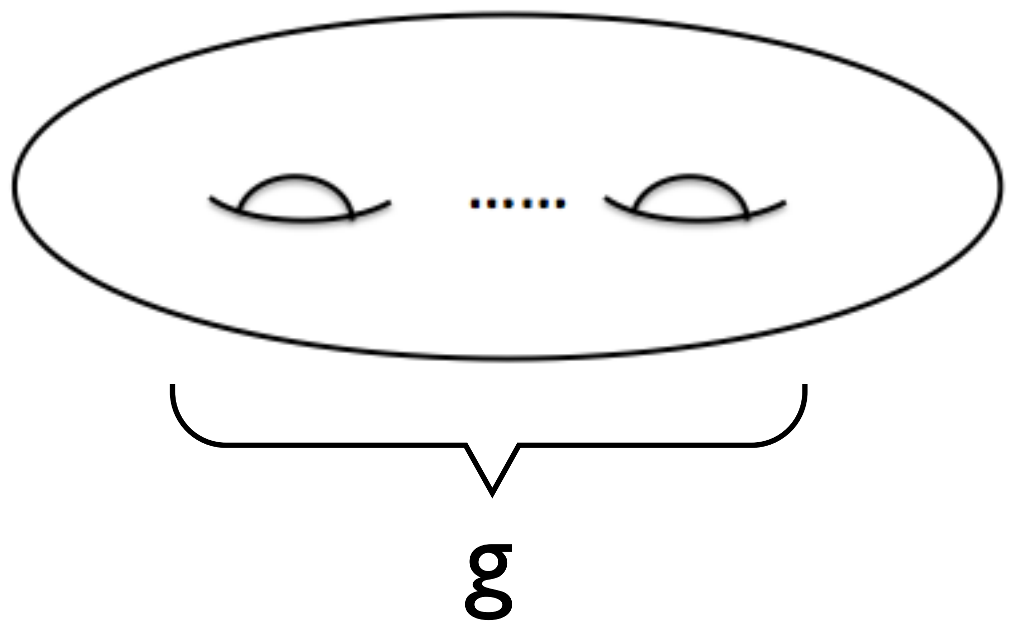
$$f(m, V, N_c) = \sum_{g=0}^{\infty} \frac{f_g(m, V)}{N_c^{2g}}$$

'‘t Hooft counting holds when this coefficient is Nc-independent



ribbon diagrams → two-dimensional surface genus-g diagram = diagram which can be drawn on genus-g surface

→ g closed string loops



1/N correction = g_s correction
't Hooft large-N limit = classical string

‘t Hooft limit
vs
chi-RMT limit

SU(∞), V=∞ gauge theory
with Nf=2 adjoint fermions
(※ other repsensations are also possible)

conformal? confining?
chiral symmetry breaking?

↕

Large-Nc equivalence
(Eguchi-Kawai equivalence)

in the 't Hooft limit

SU(∞), finite-V gauge theory
(Eguchi-Kawai model)

Study this theory
instead of V=∞

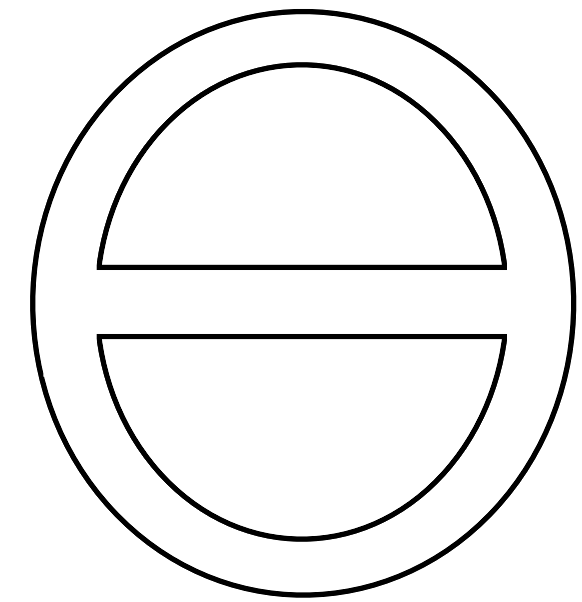
(V=2^4 in our simulation)

Earlier related work:
Narayanan-Neuberger,
Hietanen-Narayanan,
Gonzalez-Arroyo-Okawa, etc

※ To establish the method, we numerically study
Nf=0 case, for which we know the answer.

Perturbative Proof of the Eguchi-Kawai equivalence

- Take the ‘t Hooft large-N limit.
- Then only the panar diagrams remain.



Assumption N-dependence apperas only through combinatorics.

- One can see the one-to-one matching of Feynman diagrams, up to a comon constant factor.
- Perturbative proof is good enough, because there is the convergence raius is finite and can be analytically continued to strong coupling. belief

(If one wants to consider a possible phase transition, one has to take a more sophisticated approach.)

$$f(m, V, N_c) = \sum_{g=0}^{\infty} \frac{f_g(m, V)}{N_c^{2g}}$$

't Hooft counting holds when this coefficient is Nc-independent

Chiral Random Matrix Theory (chi-RMT)

Shuryak-Verbaarschot Nucl. Phys. A 560(1993)

SU(3) QCD

↓ $L \gg 1/\Lambda_{QCD}$

Chiral Perturbation Theory

↓ ε-regime ($L \ll 1/m_\pi$), $m_q V \Sigma$: fixed, $V \to \infty$

chi-RMT

QCD and chi-RMT give the same Dirac spectrum

N×N complex matrix

$V \Leftrightarrow N$ (Nc=3)

$$\mathcal{D}_f = \begin{pmatrix} m_f \mathbf{1} & \Phi \\ -\Phi^\dagger & m_f \mathbf{1} \end{pmatrix}$$

$$Z = \int d\Phi \left(\prod_{f=1}^{N_f} \det \mathcal{D}_f \right) e^{-N \mathrm{tr} \Phi^\dagger \Phi}$$

$m_q N$: fixed, $N \to \infty$

(※ 't Hooft limit of RMT: m_q fixed, $N \to \infty$)

Chiral Random Matrix Theory (chi-RMT)

QCD-like theory (YM + fermion)

↓ $L >> 1/\Lambda_{QCD}$

if the chiral sym. breaking is broken

Chiral Perturbation Theory

↓ ε-regime ($L<<1/m_\pi$), $m_q V\Sigma$: fixed, $V \to \infty$

chi-RMT

(3 classes depending on the chiral symmetry breaking pattern)

<u>The Dirac spectrum coincide</u>

<u>if the chiral symmetry is spontaneously broken.</u>

Large-Nc vs chi-RMT

- In QCD, thermodynamic limit is $V \to \infty$.
- In the SU(Nc) case, it is $V \to \infty$ and/or $Nc \to \infty$.
- N in chi-RMT corresponds to the number of degrees of freedom in QCD.

So, when we compare it with chi-RMT,

$$V \times (Nc)^{\alpha} \Leftrightarrow N$$

$$m_q V \times (Nc)^{\alpha} : \text{fixed.}$$

$$\Sigma \sim (Nc)^{\alpha}$$

$$(\alpha > 0)$$

Let us call it as 'chi-RMT limit.'

Large-Nc vs chi-RMT

The large-Nc 't Hooft limit and chi-RMT limit are different!

't Hooft limit (planar limit) : m_q, V : fix, Nc → ∞

chi-RMT limit : $m_q V \times (Nc)^{\alpha}$ fixed, Nc → ∞

The Eguchi-Kawai equivalence does not hold in the chi-RMT limit!

(※ m_q=0 should be regarded as the chi-RMT limit.)

Large-Nc vs chi-RMT

The large-Nc 't Hooft limit and chi-RMT limit are different!

$$f(m, V, N_c) = \sum_{g=0}^{\infty} \frac{f_g(m, V)}{N_c^{2g}}$$

't Hooft counting holds when this coefficient is Nc-independent

But IR divenrence picks up additional Nc-dependence!

Large-Nc vs chi-RMT

QCD (SU(3))

agreement with chi-RMT @ mV fixed, $V \to \infty$

↑

nonzero chiral condensate @ $m \to 0$ *after* $V \to \infty$

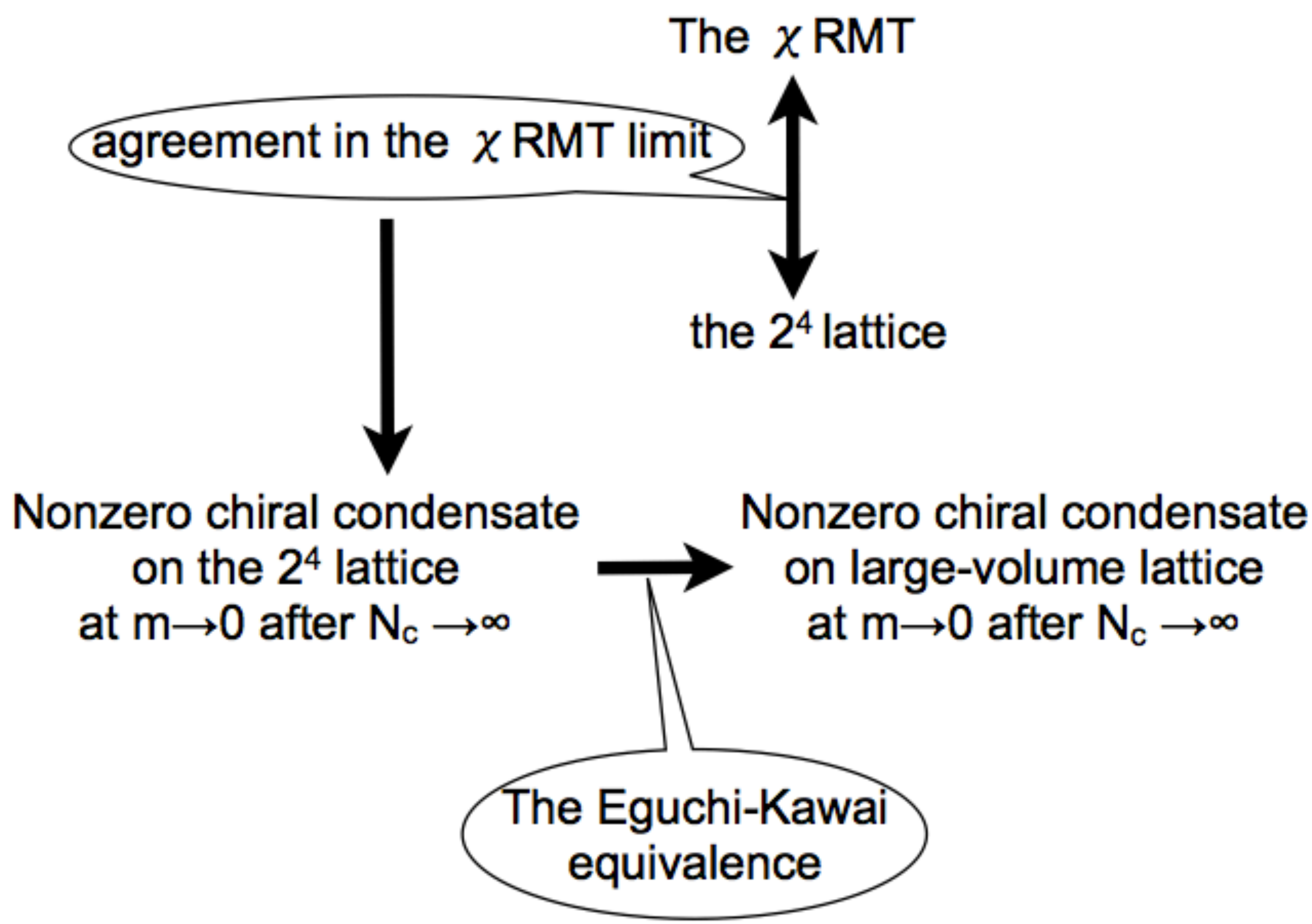
large-Nc YM

agreement with chi-RMT @ $mV \times (Nc)^{\alpha}$ fixed, $Nc \to \infty$

↑

nonzero chiral condensate @ $m \to 0$ *after* $Nc \to \infty$

't Hooft limit

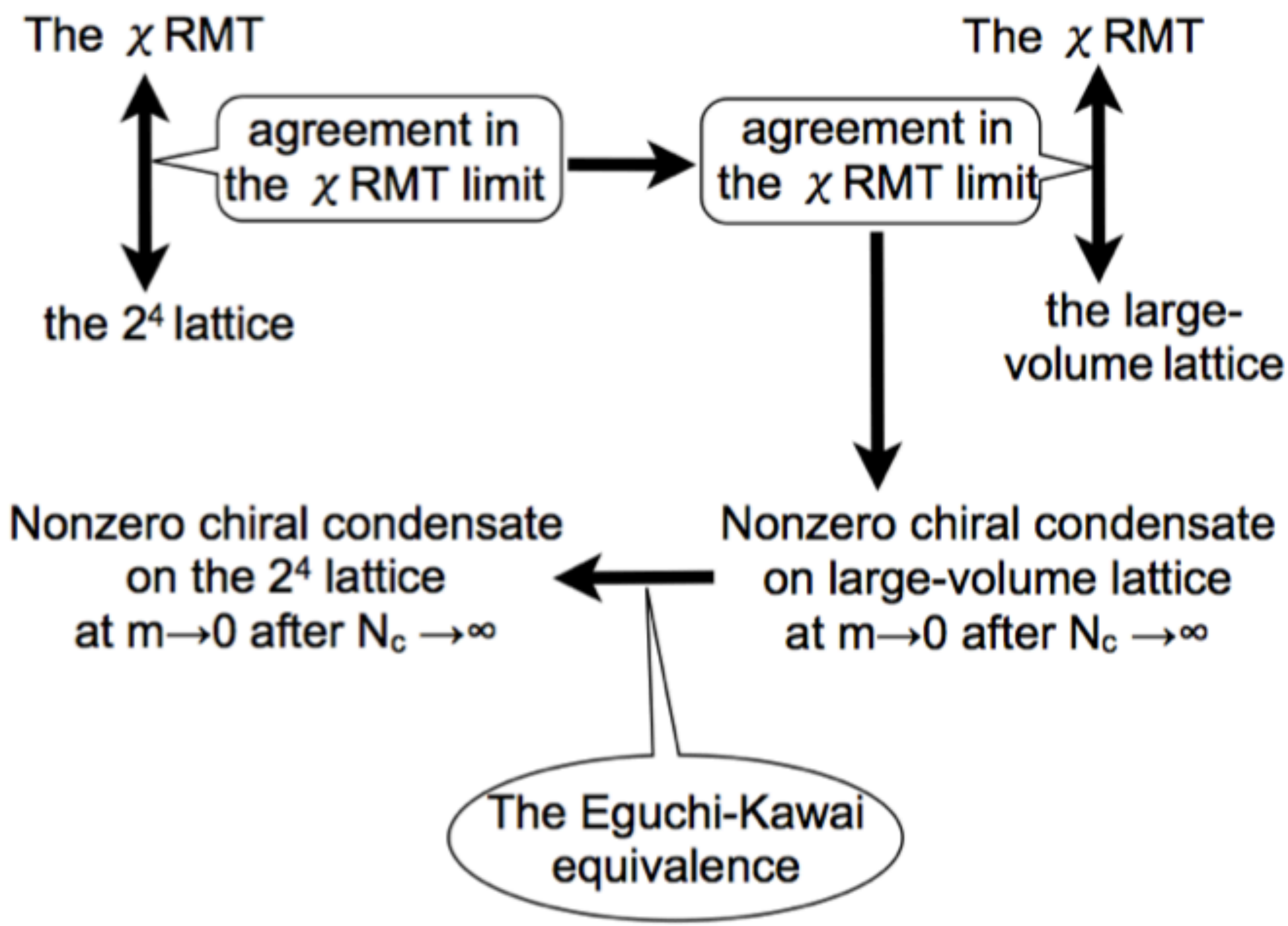

The χ RMT
agreement in the χ RMT limit
the 2⁴ lattice
Nonzero chiral condensate on the 2⁴ lattice at m→0 after Nc →∞
Nonzero chiral condensate on large-volume lattice at m→0 after Nc →∞
The Eguchi-Kawai equivalence

Large-Nc vs chi-RMT

This argument might be too naive for the Eguchi-Kawai model, because the chiral perturbation might not be applicable to 2^4 lattice straightforwardly.

Still, however:

- For sufficiently large lattice, there is no problem. There, the eigenvalue distribution depends only on $mV \times (Nc)^{\alpha}$. (This α might depend on V.)
- If there is no phase transition (center symmetry breaking), the same expression should hold even at small V.


The χRMT
agreement in the χRMT limit
the 2^4 lattice
agreement in the χRMT limit
The χRMT
the large-volume lattice
Nonzero chiral condensate on the 2^4 lattice at m→0 after N_c →∞
Nonzero chiral condensate on large-volume lattice at m→0 after N_c →∞
The Eguchi-Kawai equivalence

Numerical results (Nf=0)

- 2^4 plaquette action + heavy Dirac adjoint fermion

  → unbroken center symmetry

  (Bringoltz-Sharpe 2009, Azeyanagi-M.H.-Unsal-Yacoby 2010)

- Probe massless overlap fermion
  in the adjoint representation

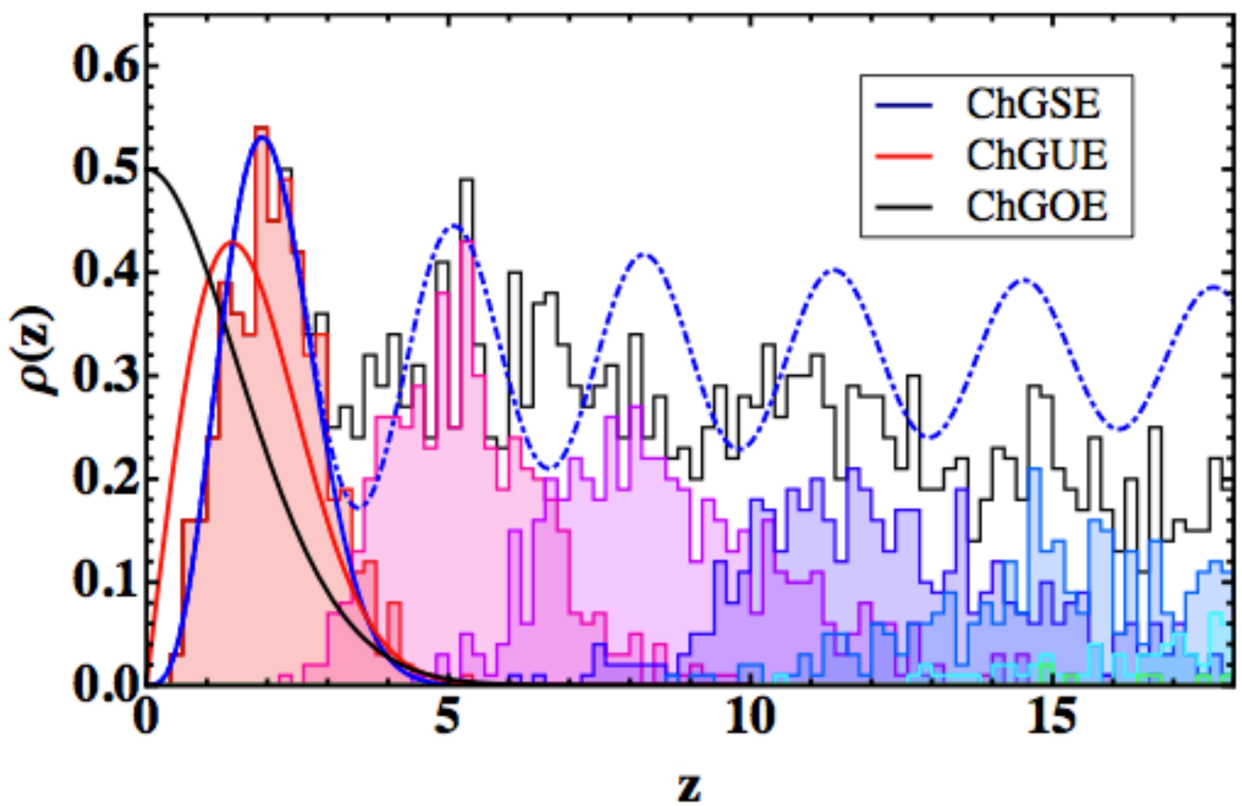
- Low-lying Dirac eigenvalues scales as 1/Nc→ α=1

  (Naive expectation from the 't Hooft counting is α=2)

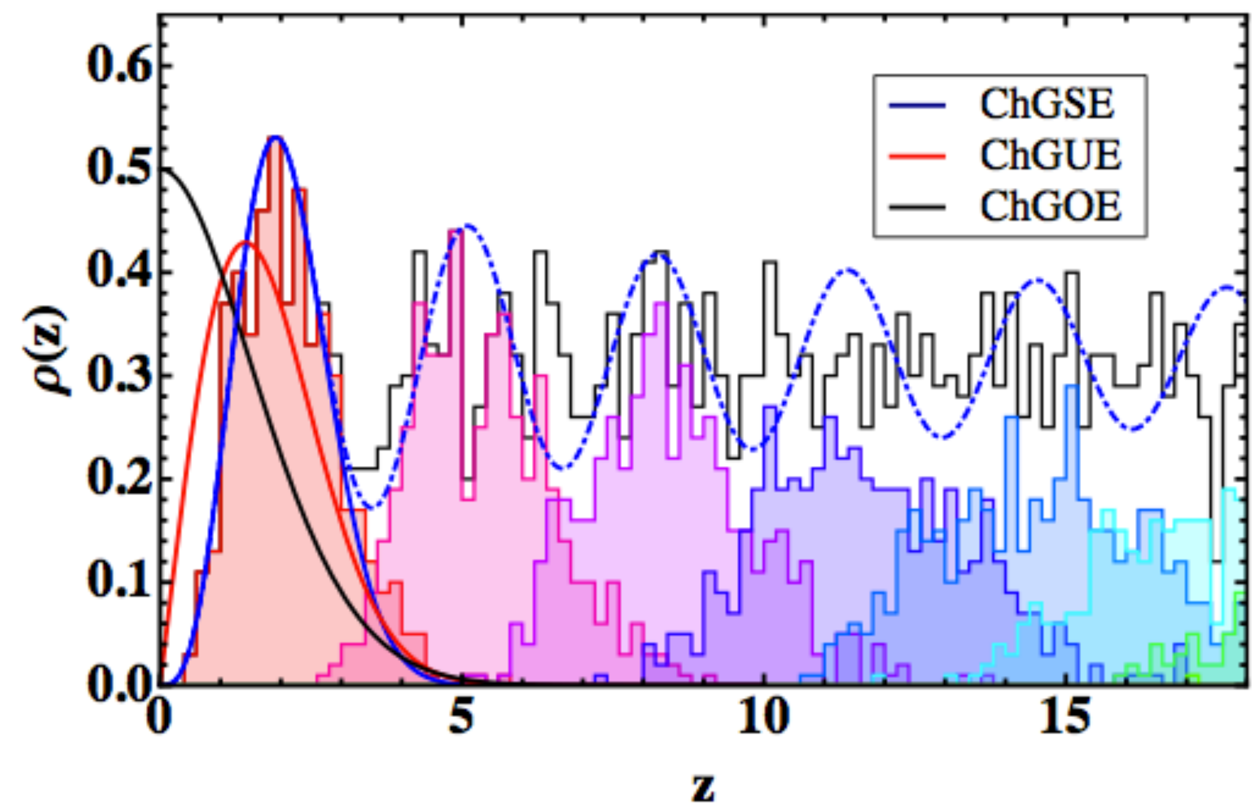
- Chiral symmetry must be broken.
  Can we detect it by comparing the simulation data with the chi-RMT prediction?

Numerical results (Nf=0)

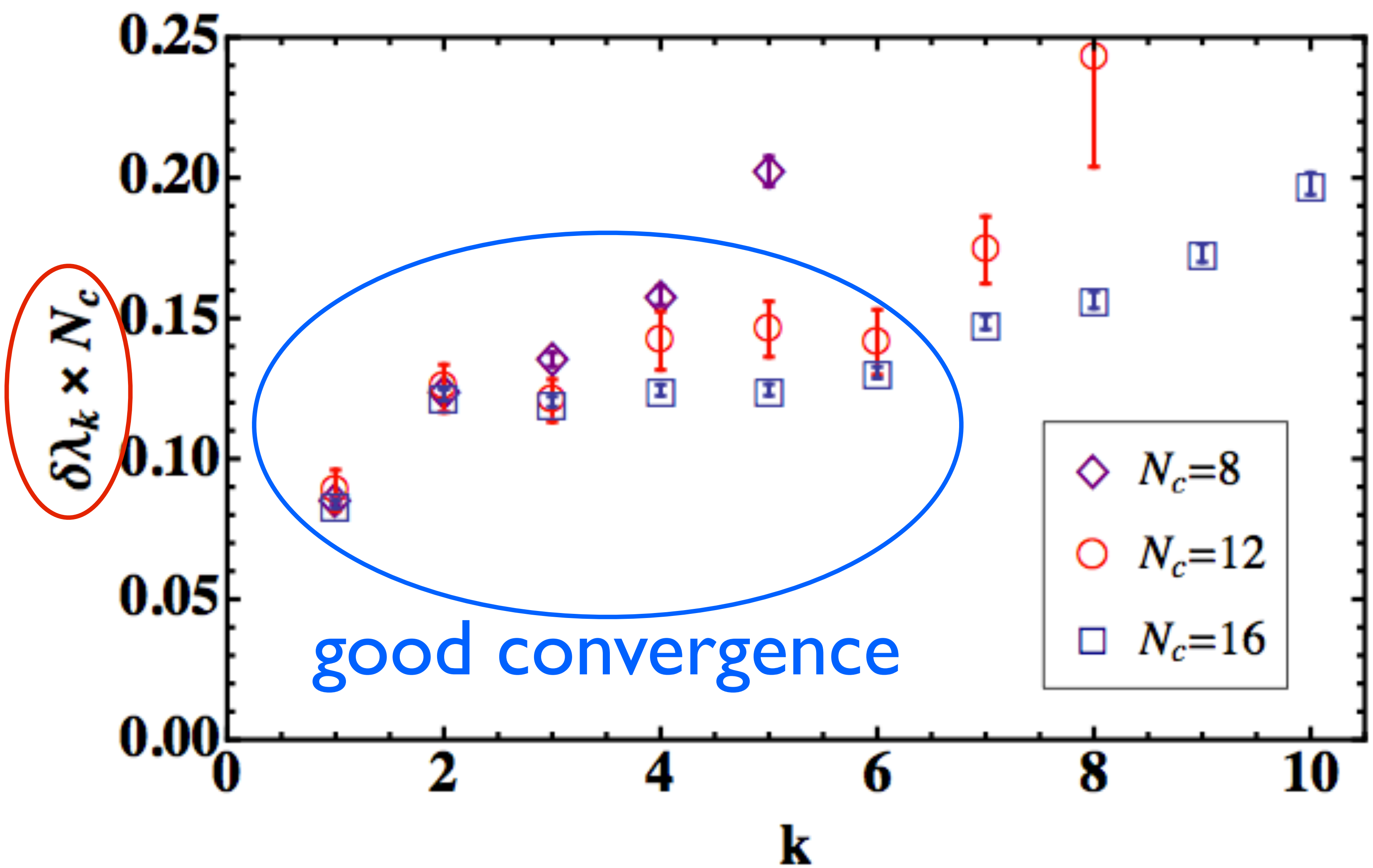
Nc=8



Nc=16



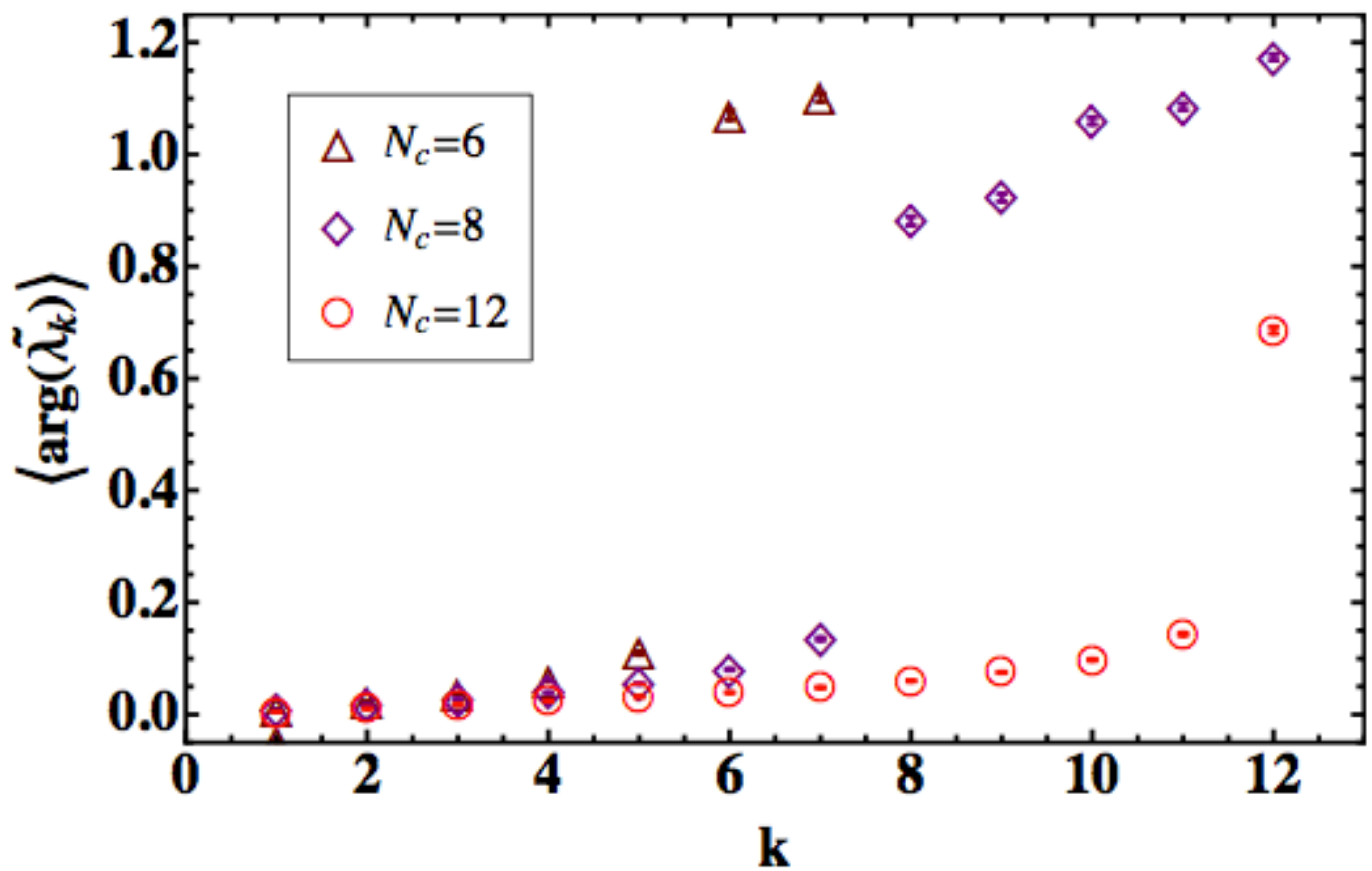
Numerical results (Nf=0)



2^4 lattice

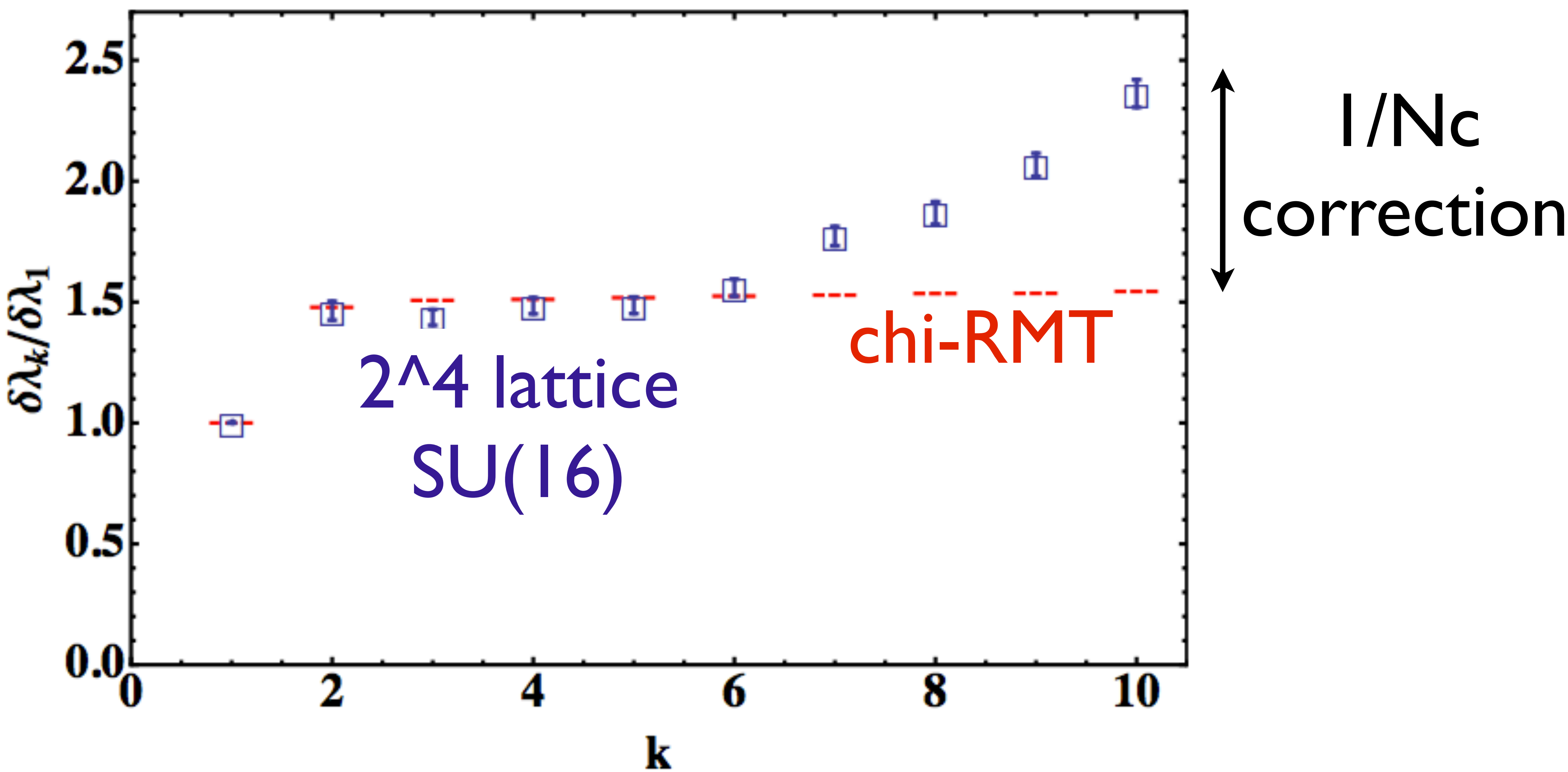
$\delta\lambda_k$ = < Im[λ_k-λ_{k-1}] > , $\delta\lambda_1$ = <λ_1>

$\lambda \sim 1/N_c$

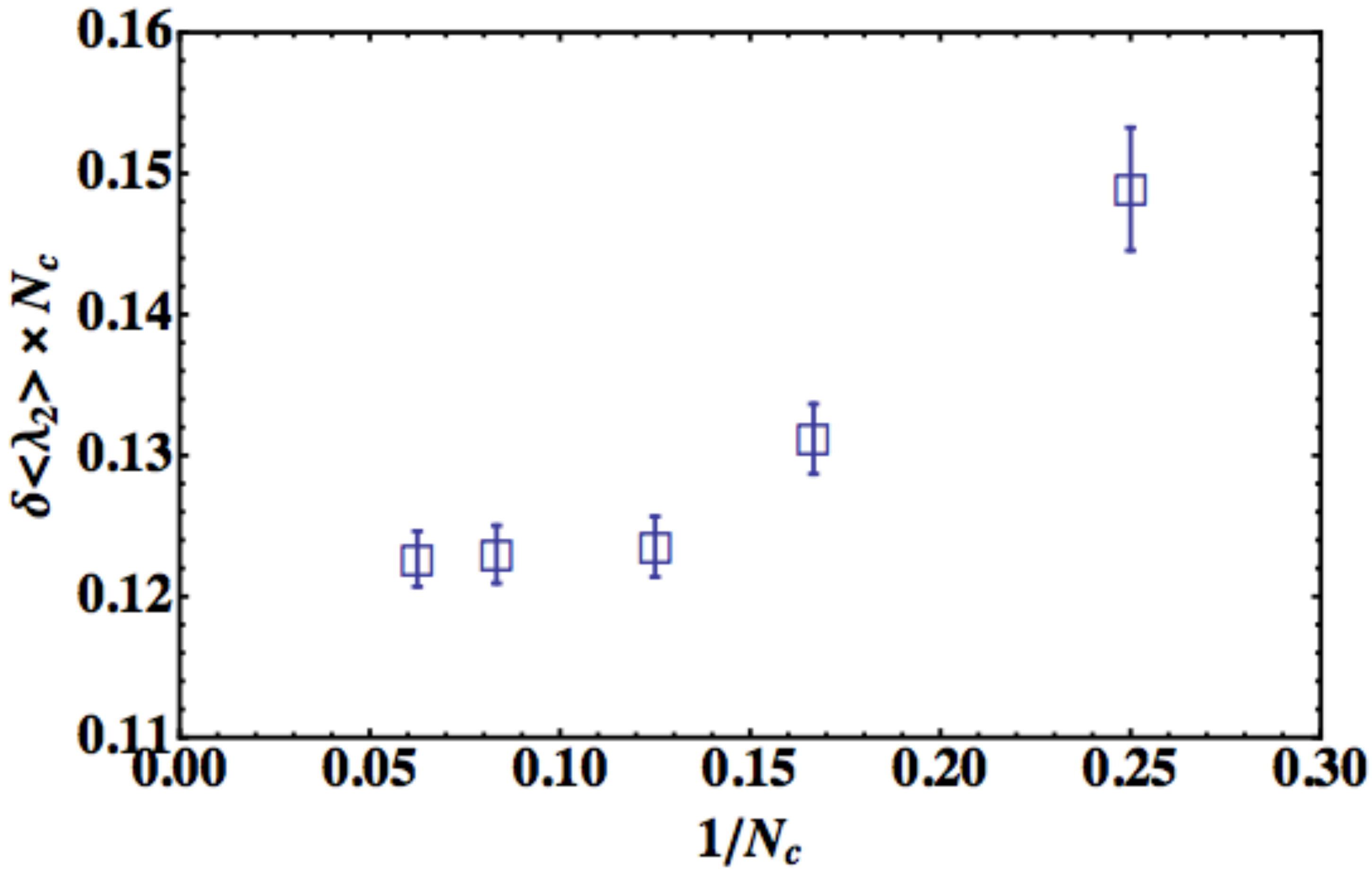


(Nc −1) eigenvalues are small

Numerical results (Nf=0)



perfect agreement with chi-RMT!


0.16
0.15
0.14
0.13
0.12
0.11
$\delta < \lambda_2 > \times N_c$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
$1/N_c$

Conclusion & Outlook

- Chiral symmetry breaking at large-Nc can be detected by comparing small-size lattice and chi-RMT.

- 2^4, SU(8) (or SU(16)) is good enough.

- Simulaton of Nf=2 theory is ongoing.

- Be careful about the difference between the 't Hooft limit and chi-RMT limit when you use them.

- Twisted boundary condition?

(Gonzalez-Arroyo and Okawa 1982, 2010-2013)