Membranes in the bulk from monopole operators of boundary ABJM theory

or

Large angular momentum in M-theory regime of AdS_4/CFT_3

Hidehiko Shimada
(Okayama Institute for Quantum Physics)

work in progress with
Stefano Kovacs (Dublin IAS)
Yuki Sato (KEK)

1. Introduction

M-theory, matrix model

AdS_4/CFT_3

large angular momentum.

- **M-theory** ('94 Hull-Townsend '95 Witten)

membrane (M2)

$D=11$ theory — no coupling const., single scale ℓ_p

$\downarrow$ on S^1 — $R_{11} = \ell_p g_s^{\frac{2}{3}}$, KK mom. = D0-charge

$D=10$ IIA String

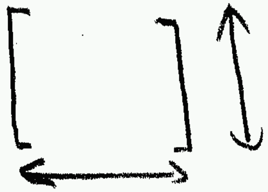
- crucial in non-perturbative String theory
- no established formulation (microscopic DOF? action?)
- best candidate

$\longrightarrow$ Matrix model.

Matrix model

'88 de Wit, Hoppe, Nicolai
'96 Banks, Fischler, Shenker, Susskind

- DOF X^α $(\alpha = 1, \cdots, 9)$



#D0-branes

- action $S = \mathrm{Tr} \int (D_t X^\alpha)^2 + [X^\alpha, X^\beta]^2 \, dt$ + Fermions
- regularised theory of M2 (D=11 Supermembrane) flat space
- matrix model of M-th. on pp-wave BKG ('02 BMN)

$$S = \mathrm{Tr} \int (D_t X^\alpha)^2 - X^2 + i X[X, X] + [X, X]^2 \, dt$$

- best candidate, though with unsolved issues ((matrix size) $\to \infty$? D=11 Lorentz inv.?)
- questions

How much of M-th is captured by M.M.?

How to extract observables using M.M?

AdS_4/CFT_3

$R \sim R_{S^7} = 2R_{AdS_4}$

'06 ABJM

- M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ $=$ D=3 ABJM theory $U(N) \times U(N)$

  ϕ^A, $A = 1, \cdots, 4$ — complex scalar

  $A \quad \hat{A}$

  CS action $\frac{1}{k}$ ← level

$$\frac{R^6}{l_p^6} = Nk$$

- $S^7/\mathbb{Z}_k \rightarrow$ M-th. circle $R_{11} = \frac{R}{k}$
- IIA regime : $N \gg 1$, $\lambda = \frac{N}{k}$: fixed

$\boxed{\text{M-th. regime : } N \gg 1, \; k \sim O(1) \text{ (fixed.)}}$ ← this talk

- learn about
  M-theory
  AdS/CFT

  e.g. matrix model $\longleftrightarrow$ ABJM?

  non-stringy

  all non-planar contributes

- To study M-theory regime, we consider states with large angular mom. $J \gg 1$
- use $\frac{1}{J} \ll 1$ to introduce approximation schemes on both sides, then verify AdS/CFT
- WKB approx ~ large quantum number.
- BMN '02 Berenstein, Maldacena, Nastase in AdS_5/CFT_4

String on $AdS_5 \times S^5$	$D=4$, $\mathcal{N}=4$ SYM $g_{YM}^2 N \gg 1$
$\downarrow J \gg 1$	$\downarrow J \gg 1$, near BPS BMN op.
String on pp-wave BKG	eff. coupling $\frac{g_{YM}^2 N}{J^2} \ll 1$

$\longleftrightarrow$

successful check.

Our work

☆ $J \gg 1$ seems to provide good approx. on both sides

AdS side : pp-wave approx & loop exp.

CFT side : Born-Oppenheimer

☆ AdS/CFT Dictionary for $J \gg 1$

analog of BMN op. (based on monopoleoperator)

~ operator in ABJM

corresponding to near BPS fluctuation of membranes.

☆ Successful test at the leading order

of the approx. (near-BPS)

Outline

1. Introduction
2. AdS side
   - pp-wave approx.
   - BPS states
   - near BPS
3. CFT side
   - radial qtz. & monopole op.
   - BPS
   - near BPS
   - gauge fixing
   - Born-Oppenheimer approx.
4. Summary & Discussion

☆ $J \gg 1$ $\longrightarrow$ approx. on both sides

☆ AdS/CFT dictionary

☆ test at the leading order (for near BPS op.)

2. AdS side

pp-wave approx.

membrane Hamiltonian

(matrix size) = J

BPS states ~ Concentric Spherical membranes

fluctuation ; near BPS states

tree vs 1-loop

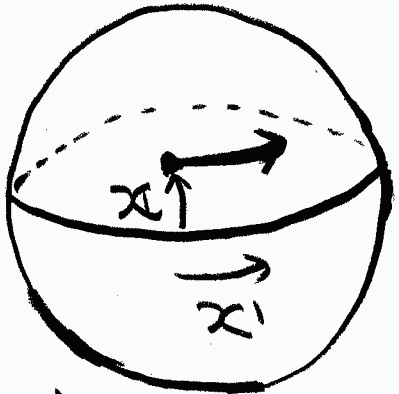
$$N^{\frac{1}{3}} \ll J \ll J^{\frac{1}{2}}$$

- <u>pp-wave approx.</u> (ultrarelativistic (infinite mom.) limit on curved space)

- <u>Simple ex.</u>

particle on $\mathbb{R} \times S^n$

$|x| \ll R$



radius R.

- $ds^2 \approx -dt^2 + (dx^1)^2 \times \left(1 - \frac{x^2}{R^2}\right) + dx^2$

- $P_0 = -E, \quad P_1 = P = \frac{J}{R} \gg 1$

- $0 = G^{\mu\nu} P_\mu P_\nu$ dynamics of a particle.

$= -E^2 + P^2 \times \left(1 + \frac{x^2}{R^2}\right) + \mathbb{p}^2$

$\rightsquigarrow E - P = \frac{\mathbb{p}^2}{2P} + \frac{P}{2} \frac{x^2}{R^2}$

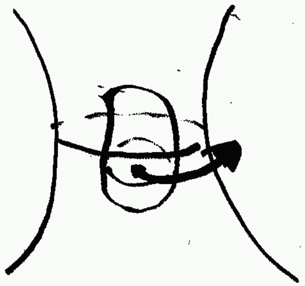
non-rel particle

(using $E + P \approx 2P$)

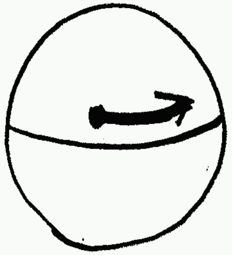
harm. osc. potential

- J : large $\to$ strong harm. osc. potential.

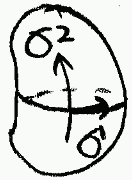
Hamiltonian of membrane theory $\hbar = 1$. $\ell_p = 1$.

- 

$\times$



AdS$_4$ (x^1, x^2, x^3) S^7 $(x^4, \cdots, x^9)$



$X^\alpha(\sigma^1, \sigma^2)$
$\underset{\text{conj.}}{\longleftrightarrow}$ $p^\alpha(\sigma^1, \sigma^2)$

- $$H = \int \underbrace{\frac{(p^\alpha)^2}{2P}}_{\text{inertia}} + \underbrace{\frac{1}{2P}\{X^\alpha, X^\beta\}^2}_{\text{tension}} + \underbrace{\frac{P}{R^2} X^2}_{\substack{\text{pp-wave}\\ \text{approx.}}} + \underbrace{\frac{1}{R}\epsilon_{\alpha\beta\gamma} X^\alpha \{X^\beta, X^\gamma\}}_{\text{flux in AdS}} \, d\sigma^1 d\sigma^2$$

($\epsilon_{123} = 1$, perm. otherwise $= 0$)

$$\{f, g\} = \frac{\partial f}{\partial \sigma^1}\frac{\partial g}{\partial \sigma^2} - \frac{\partial f}{\partial \sigma^2}\frac{\partial g}{\partial \sigma^1}$$

- can use old results on membrane, matrix model on pp-wave BKG
  ('02 Dasgupta, Sheikh-Jabbari, van Raamsdonk
  Kim, Plefka
  Sugiyama, Yoshida)

- regularisation. $k=1$

$$x^\alpha(\sigma^1, \sigma^2), \quad p^\alpha(\sigma^1, \sigma^2)$$

$\longrightarrow$ X^α, P^α : matrices

- (matrix size) = J

$S^7/\mathbb{Z}_k \rightarrow$ M-th. circle

$\rightsquigarrow$ J = KK mom. on M-theory circle
= # D0
= matrix size

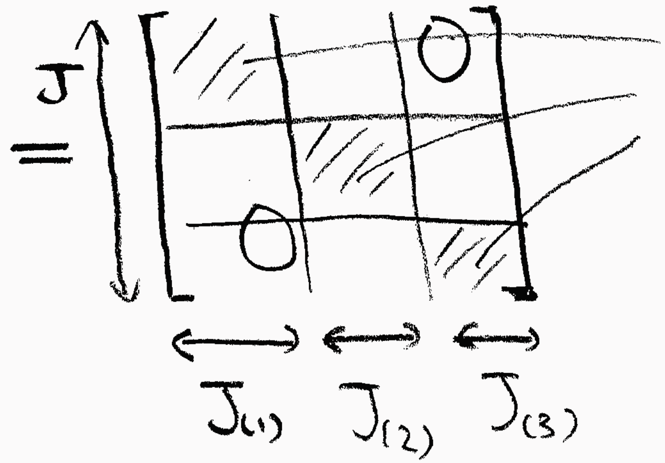
- cf. BMN op. $\sim \mathrm{Tr}\, Z^J \sim$ closed string reg. by lattice with J sites.

- $$H = \mathrm{Tr}\left(R(P^\alpha)^2 - R\,[X^\alpha, X^\beta]^2 + \frac{1}{R^3}(X^\alpha)^2 + i\,\frac{1}{R}\,\epsilon_{\alpha\beta\gamma} X^\alpha [X^\beta, X^\gamma] \right)$$

pp-wave matrix model.

BPS states

• classical ground states $E=0$

$X^\alpha =$ 

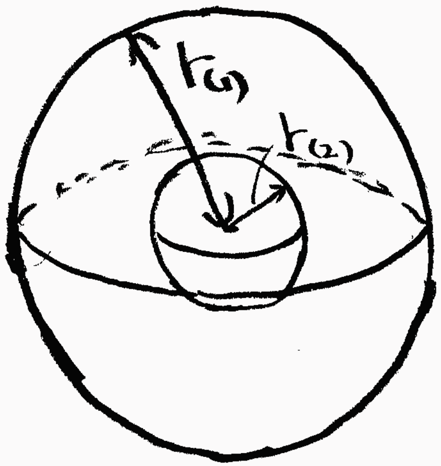
$\alpha = 1,2,3$

$$r_{(i)} \propto \frac{1}{J_{(i)}} L_\alpha^{(i)}$$

$$[L_\alpha^{(i)}, L_\beta^{(i)}] = i\epsilon_{\alpha\beta\gamma} L_\gamma^{(i)}, \quad J_{(i)} \times J_{(i)}$$ matrices

• $\boxed{J = J_{(1)} + J_{(2)} + \cdots}$

• collection of fuzzy spheres



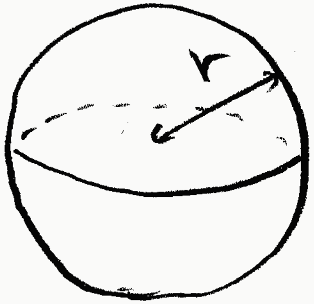
Concentric spherical membranes with radii $r_{(i)}$

$$r_{(i)} = \frac{J_{(i)}}{R^2}$$

· pp-wave approx.

$$\frac{r}{R} = \frac{J}{R^3} = \frac{J}{N^{\frac{1}{2}}} \ll 1$$

- **near BPS fluct.** · single membrane case $J = J_{(1)}$,



spherical membrane

· focus on fluct. in S^7-direction ~ 6-scalars ← polarisation

· oscillation mode corresponds to $Y_{\ell m}(\theta, \varphi)$

$$\omega = \frac{1}{R}\sqrt{\frac{1}{4} + \ell(\ell+1)}$$ ← tree level

($\frac{1}{4}$: harm. osc.; $\ell(\ell+1)$: tension)

- **1-loop correction** ('02 Dasgupta et al.) $H = H_2 + H_3 + H_4$ (H_2: free)

$$\Delta E = \sum_{\psi'} \frac{|\langle\psi|H_3|\psi'\rangle|^2}{E - E'} + \langle\psi|H_4|\psi\rangle.$$

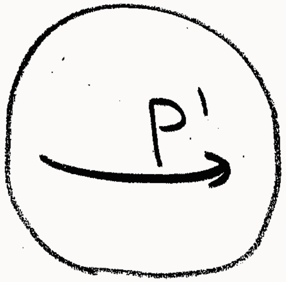
· $J \gg 1$ → H_2: large; → #(intermediate state): large

SUSY

- $$\frac{\text{1-loop}}{\text{tree}} = \frac{N}{J^3} \leftarrow = \left(\frac{\ell_p}{r}\right)^3$$

- $$N^{\frac{1}{3}} \underset{\text{loop exp.}}{\ll} J \underset{\text{pp-wave}}{\ll} N^{\frac{1}{2}}$$

Summary of 2. AdS side

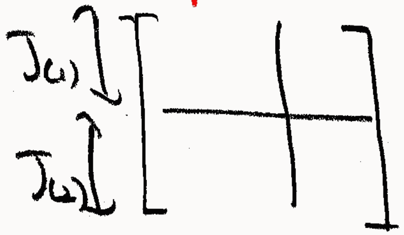
- 

  large (angular) mom. in curved space

  $\longrightarrow$ H contains $\frac{P}{R^2}(X^{\alpha})^2$

- matrix size $= J$
- BPS states   collection of fuzzy spheres

  $$J = J_{(1)} + J_{(2)} + \cdots$$

- near BPS states

  $$Y_{\ell m} \longrightarrow \omega = \sqrt{\frac{1}{4} + \ell(\ell+1)} = \frac{1}{2} + \ell$$

- validity of approx.

  loop exp.   pp-wave approx.

  $$N^{\frac{1}{3}} \ll J \ll N^{\frac{1}{2}}$$

3. CFT side

monopole op.

BPS states

near BPS fluctuation

gauge fixing & "Higgs mechanism"

Born - Oppenheimer approx.

large J in ABJM

- J : R-charge in ABJM

  4 complex scalars

  $\phi^1, \phi^2, \phi^3, \boxed{\phi^4}$: bi-fundamental of $U(N)\times U(N)$

- ϕ^4 carries unit charge

- $\cancel{\mathrm{Tr}(\phi^4)^J}$ ← wrong, not gauge inv

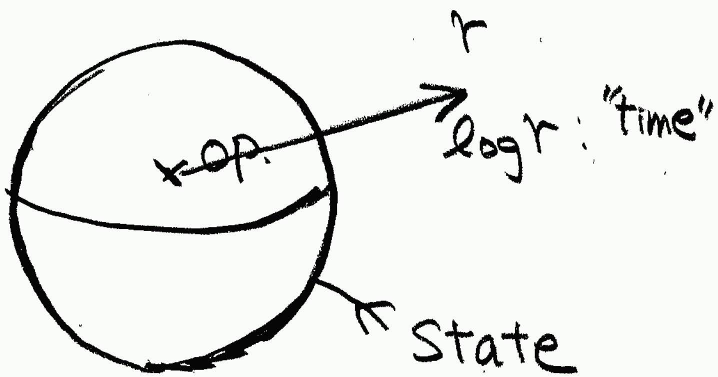
  needs monopole op. ('06 ABJM '02 Kapustin et al.)

- <u>State op. mapping</u>

  ABJM on $S^2 \times \mathbb{R}^1$

  ϕ^A : mass $= \frac{1}{2}$

  Energy = conformal dimension



- ## ABJM on $S^2 \times \mathbb{R}^1$

$\begin{cases} A = 1, 2, 3, 4 & SU(4) \text{ flavour.} \\ i = 1, \cdots, N & U(N) \text{ colour} \\ \hat{i} = 1, \cdots, N & \text{the other } U(N) \text{ colour.} \end{cases}$

- $\begin{pmatrix} \phi^{A\,i}{}_{\hat{j}} & \overset{\text{conj.}}{\longleftrightarrow} & \Pi_A{}^{\hat{i}}{}_j \\ \phi^\dagger{}_A{}^{\hat{i}}{}_j & \longleftrightarrow & \Pi^{\dagger A\,i}{}_{\hat{j}} \end{pmatrix}$ $\begin{pmatrix} A_r{}^i{}_j \\ \hat{A}_r{}^{\hat{i}}{}_{\hat{j}} \end{pmatrix}$ $r = 1, 2$ sphere.

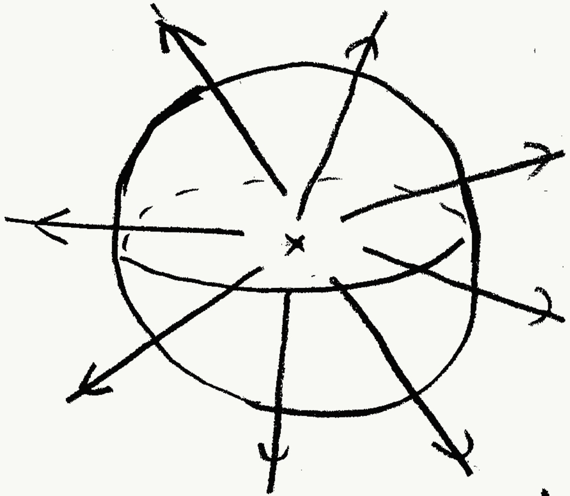
- $$H = \mathrm{tr} \int \frac{1}{\sin\theta} \Pi^A \Pi_A + g^{rs} D_r \phi^A D_s \phi_A \sin\theta + \underbrace{\frac{1}{4} \phi^A \phi_A \sin\theta}_{\text{mass from radial qtz.}} + \left(\frac{2\pi}{k}\right)^2 \underbrace{V_6(\phi)}_{\phi^6\text{-potential}} + \cdots \, d\theta \, d\varphi$$

- Gauß' law constr.

$$\frac{k}{2\pi} F_{\theta\varphi} = \rho \,, \quad -\frac{k}{2\pi} \hat{F}_{\theta\varphi} = \hat{\rho}$$

- $$\rho = i \phi^A \Pi_A - i \Pi^{\dagger A} \phi^\dagger{}_A + \cdots \,, \quad \hat{\rho} = i \phi^\dagger{}_A \Pi^{\dagger A} - i \Pi_A \phi^A + \cdots$$

- <u>monopole op. & BPS states</u> ('06 ABJM, '09 Seok Kim)



- excite 0-mode of ϕ^4, J times
- Gauß' law constr.

$$\frac{k}{2\pi} F_{\theta\varphi} = \rho \sim J$$

- needs to introduce const. BKG $F_{\theta\varphi} \sim J$

- <u>GNO charge</u> (Goddard, Nuyts, Olive '77)

$$A_\mu = N \begin{bmatrix} \square & & & & 0 \\ & \square & & & \\ & & \square & & \\ & & & \ddots & \\ 0 & & & & \square \end{bmatrix} \begin{array}{l} \text{Dirac monopole } g_{(1)} \\ g_{(2)} \\ \vdots \\ g_{(N)} \end{array}$$

N

$$\boxed{J = \sum_{i=1}^{N} 2 g_{(i)}}$$

→ matches with AdS side for large $J_{(i)}$ (cf. '09 Simon, Sheikh-Jabbari)

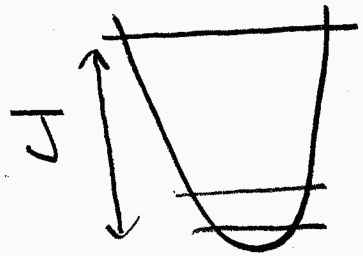
- <u>near BPS fluct.</u> S^7-direction ~ 6 polarisation.

excite ϕ^1, ϕ^2, ϕ^3 modes corresponding to $Y_{\ell m}$

$$\boxed{\Delta \sim \sqrt{\left(\frac{1}{2}\right)^2 + \ell(\ell+1)}}$$

→ matches with AdS side.

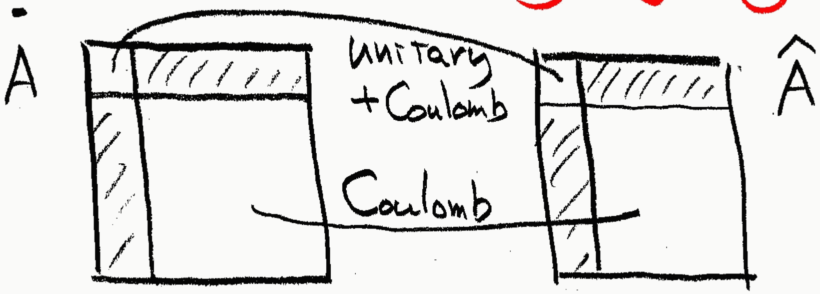
- <u>gauge fixing</u> BKG gauge + unitary gauge + Coulomb gauge.
- Gauß' law constr. $\frac{k}{2\pi} F_{\theta\varphi} = \rho \leftarrow \underbrace{\phi^{4\,\prime}{}_{\uparrow}}_{\text{complex scalar}}$ excited J-times.
- <u>BKG gauge</u> (choose sector) A_r = monopole BKG + fluct. (Wu-Yang)
- RHS "background" seems to be quantum rather than classical.



$\langle \phi \rangle = 0$, $\langle \phi \phi^* \rangle \sim J$.

- ⇝ use "polar coordinate" in ϕ-space

  ~ unitary gauge. $\mathrm{Im}\, \phi'{}_{\uparrow} = 0$.



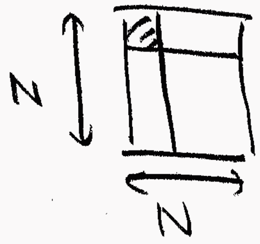
/// : unitary gauge $\quad i', \hat{i}' = 2, \dots, N$

$\phi^{4\,\prime}{}_{\hat{i}'} = 0 \quad \phi^{4\,i'}{}_{\uparrow} = 0$

• next step.

eliminate conj. mom. of gauge fixed variables
by solving the Gauß' law constr (cf '73 GGRT LC gauge string)

• $\Phi^{41} = (\underbrace{\overset{\text{const.}}{f}+u}_{\text{real part}}) + i\underbrace{v}_{\text{imag. part}}$, $v = 0$



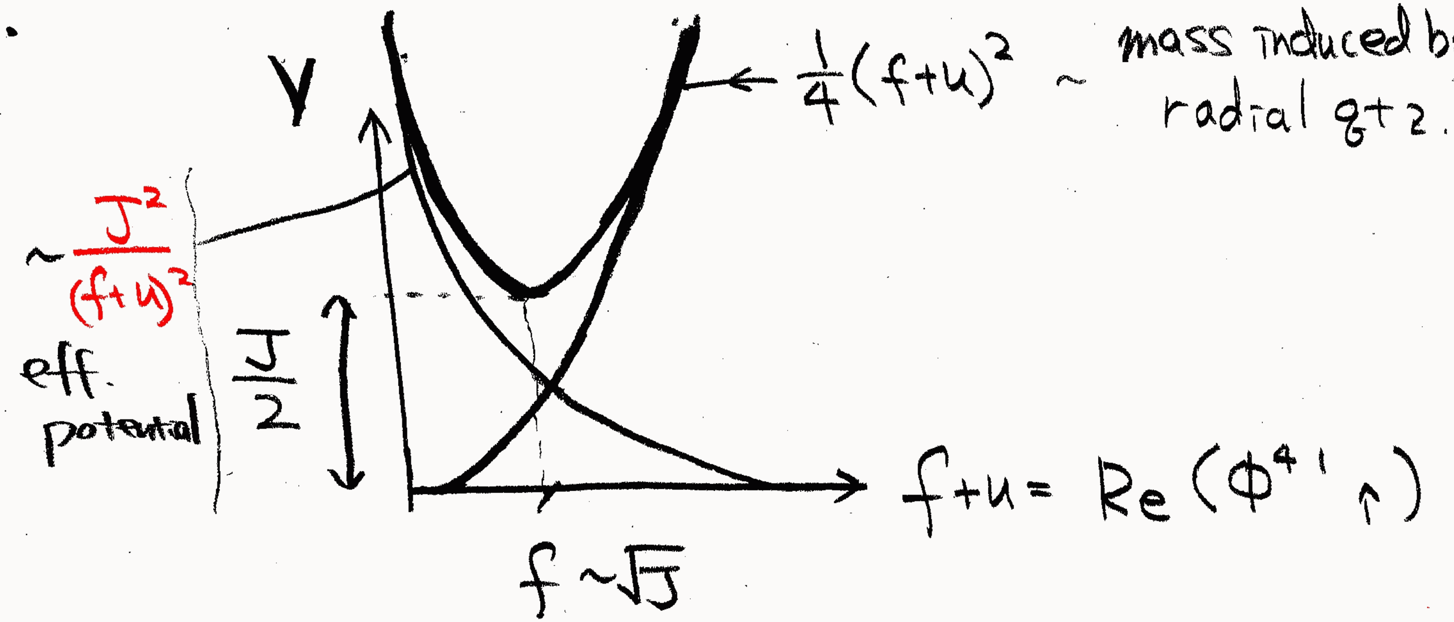
• Gauß' law: $\frac{k}{2\pi} F_{\theta\varphi} = \rho$

$$\frac{k}{2\pi}\left(\xi + \partial_\theta a_\varphi - \partial_\varphi a_\theta \right) = (f+u) P_v - \cancel{v P_u} + \cdots$$

$$P_v \sim \frac{J}{f+u} + \cdots$$

"effective potential"

• H contains P_v^2 - term. $\rightarrow H_{u\cdots} + \frac{J^2}{(f+u)^2} + \cdots$



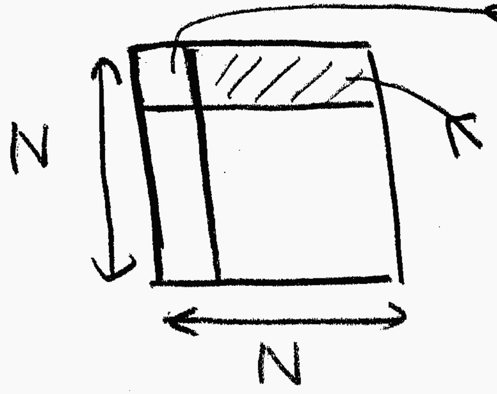
- $f^2 \sim J$: vacuum expectation value of $|\Phi^{41}{}_{\uparrow}|^2$

$\longrightarrow$ contributes to spectrum & vertices via Φ^6 potential, …
(resembles standard Higgs mechanism)

- # Born-Oppenheimer approx.

"slow" or "light" modes

- fields in ABJM



do not feel flux, "free" $\rightarrow Y_{\ell m}$ $\omega=\sqrt{\left(\frac{1}{2}\right)^2+\ell(\ell+1)}$ $\ell=0,1,\cdots$

feel flux $\rightarrow Y_{8\ell m}$ $\ell=8,8+1,\cdots$

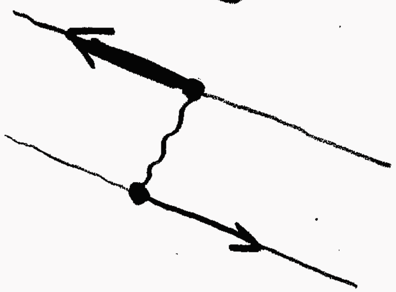
$\omega=\sqrt{\left(\frac{1}{2}\right)^2+\ell(\ell+1)}$ "fast" or "heavy" modes

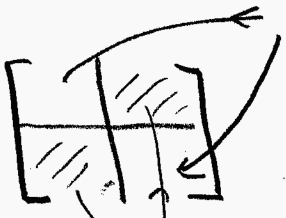
- order J gap in the frequency or energy

$\rightarrow$ integrate out fast modes ~ B-O approx.

should introduce exp. param $\left(\frac{1}{J}\right)^{\odot}$

- SUSY important ~ cancellation of large contribution to the potential from zero pt. osc. of fast modes
- analogous to D-brane scattering



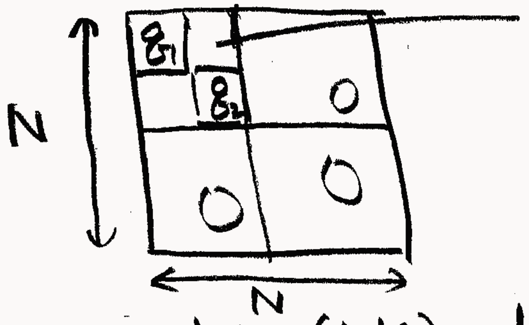


SUSY

$\rightarrow$ $\frac{v^4}{r^7} \ll 1$

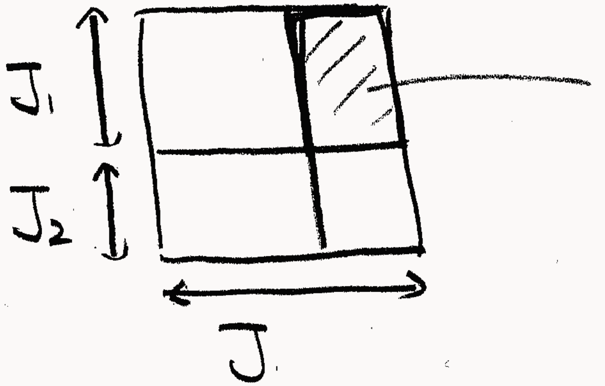
Case with two membranes

- ABJM (CFT) side $\quad q_1 - q_2 \sim O(1)$.



$Y_{q\ell m}$ with $q = q_1 - q_2$
not heavy

- matrix model (AdS) side $\quad J_1 \sim q_1$, $J_2 \sim q_2 \quad J_1 - J_2 \sim O(1)$.



rectangular matrices
can be decomposed by matrices
corresponding to $Y_{q\ell m} \quad q \sim J_1 - J_2$
(Tsuchiya et al. '06)

- off-diag. DOF in ABJM

$\longleftrightarrow$ block off-diag. in matrix model

4. Summary

- $J \gg 1 \rightarrow$ approx. on both sides. $N^{\frac{1}{3}} \ll J \ll N^{\frac{1}{2}}$
-

	AdS	CFT
framework	pp-wave matrix model	radial gtz with large flux
approx.	$J \gg 1 \rightarrow$ large H_2	$J \gg 1 \rightarrow$ off diag heavy $\rightarrow$ Born-Oppenheimer approx.
exp. param.	$\frac{NR}{J^3} \ll 1$	?
BPS	Fuzzy spheres $J = J_{(1)} + J_{(2)} + \cdots$	const. flux $g = g_{(1)} + g_{(2)} + \cdots$
near BPS fluctuation	modes labelled by Tem $\ell = 0, 1, \cdots$ $\omega = \sqrt{\left(\frac{1}{2}\right)^2 + \ell(\ell+1)}$ $S^7: X^4, \cdots, X^9$ $(AdS_4: X^1, X^2, X^3)$	Φ^1, Φ^2, Φ^3 $(\Phi^4, A, \hat{A})$

- **Important next step**

quantum correction agrees ?

Same exp. param ?

(exp. param on AdS sides $\frac{Nk}{J^3} \sim \frac{N}{g^3 k^2}$

Seems to suggest 1-loop in AdS = 2-loop in ABJM

1-loop in ABJM $\equiv 0$?)

- **Other directions**

Symmetry in this gauge? renormalisation?

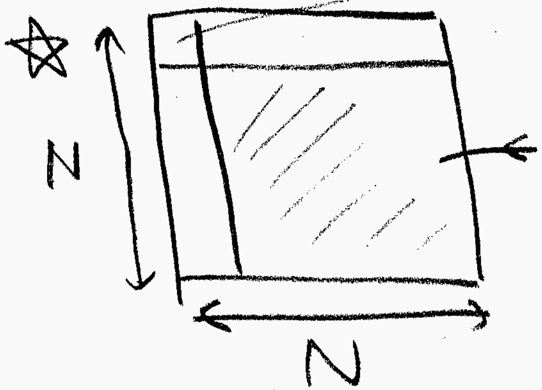
How to see $J \ll N^{\frac{1}{2}}$ in ABJM ?

(neglecting processes GNO charges change
membrane interaction
longi. mom. transfer)

3pt functions ?

matrix model may contain splitting / joining of membranes

- Observation



create
excite → phonon on spherical membrane
1st qtz.

excite → Create small membrane
2nd qtz.

AdS / CFT
bulk / boundary

☆ rather direct correspondence

spherical membrane in bulk ⟷ sphere used in radial qtz.

- We hope that our work leads to
  good approximation
  and then to deeper understanding of
  M-theory, matrix model, AdS/CFT