Calculable Higgs Mass in Gauge-Higgs Unification

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PLAN

- Introduction
- Gauge-Higgs Unification
- 1-loop Corrections to $M_H = 2M_W$ in 6D GHU
- Summary

Introduction

A Higgs boson was discovered, but its origin is still unclear…

Higgs boson is light

H^0 MASS

A combination of the results from ATLAS and CMS, where a recent unpublished result from CMS is used, yields an average value of 125.6 ± 0.3 GeV, see the review on "Status of Higgs Boson Physics."

VALUE (GeV)		*DOCUMENT ID*		*TECN*	*COMMENT*
125.7±0.4 OUR AVERAGE					
$125.5 \pm 0.2 \,^{+0.5}_{-0.6}$	[1,2]	AAD	13AK	ATLS	pp, 7 and 8 TeV
$125.8 \pm 0.4 \pm 0.4$	[1,3]	CHATRCHYAN	13J	CMS	pp, 7 and 8 TeV

Following models beyond the SM might be favored

⇒ Higgs self-coupling is governed by gauge principle

Ex. MSSM "D-term contribution"

$$V = \frac{g^2}{2}\left[H_u^\dagger \tau^a H_u + H_d^\dagger \tau^a H_d \right]^2 + \cdots$$

In MSSM,

$$M_H^2 < M_Z^2 (\cos 2\beta)^2$$ @tree level

$$M_H^2 \simeq M_Z^2 \cos^2 2\beta + \underbrace{\frac{3}{4\pi^2}\frac{m_t^4}{v^2}\sin^2\beta \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)}_{\text{Top-stop loop}}$$ @1-loop

Large but calculable quantum correction can explain the observed Higgs mass

$M_H = 126\ \text{GeV} = M_Z + \Delta M_H$

$\sim\ 90\ \text{GeV} + 35\ \text{GeV}$

GHU in D(>5) dim

$$V = g^2 \left[A_i, A_j \right]^2 \subset F_{MN} F^{MN}$$ @tree level

A_i: extra components of A_M ⇒ Higgs

As will be explained later, in a 6D model of T^2/Z_3

$$M_H = 2M_W \sim 160 \text{ GeV@tree}$$

GHU in D(>5) dim

$$V = g^2 \left[A_i, A_j \right]^2 \subset F_{MN} F^{MN}$$ @tree level

A_i: extra components of A_M ⇒ Higgs

As will be explained later, in a 6D model of T^2/Z_3

$$M_H = 2M_W \sim 160 \text{ GeV@tree}$$

Interesting to realize

$$M_H = 126 \text{ GeV} = 2M_W + \Delta M_H$$
$$\sim 160 \text{ GeV} - 35 \text{ GeV}$$

KK top loop

An issue to be addressed

Quantum corrections to $M_H = 2M_W$ is calculable?

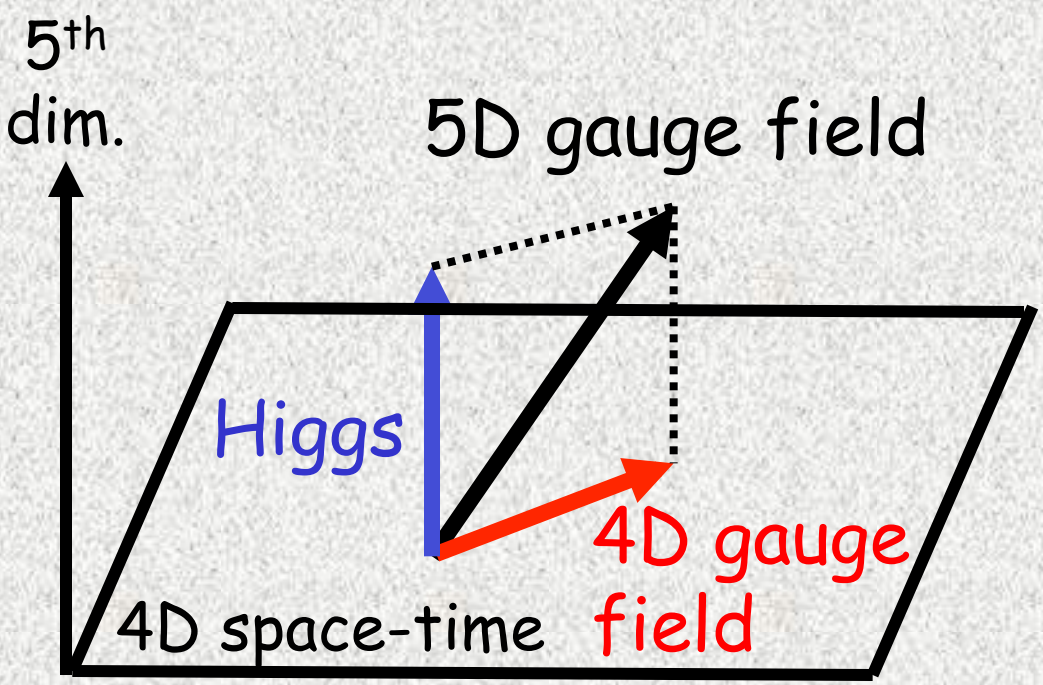
↓

The corrections are explicitly shown to be finite by using a toy model

Gauge-Higgs Unification

In gauge-Higgs unification,
0 mode of extra component of higher dim. gauge field
is identified with the SM Higgs

A_μ, A_i = SM Higgs



(Local) Higgs mass term forbidden by gauge sym.

⇒ Quantum correction to Higgs mass is finite

In 5D,

No Higgs potential@tree & generated@loops

⇒ Higgs mass is likely to be very small

i.e. $M_H^2 \sim g^2/(16\pi^2)\,(1/R)^2$

$\sim 1/(16\pi^2 a^2)\, M_W^2$

$\ll M_W^2$ (if $a \sim O(1)$)

R: compactification radius

$a/R = \langle H \rangle$ (a: dimensionless const.)

In D > 5, Higgs quartic coupling

$g^2[A_i, A_j]^2$ (i,j=5,...D) in $Tr(F_{MN}F^{MN})$

exists at tree level unless $A_i \propto A_j$

➡ Higgs mass can be made heavy??

6D GHU on T^2/Z_3

Scrucca, Serone, Silvestrini, Wulzer (2004)

Interesting prediction @leading order

$M_H = 2M_W$

Relevant terms for Higgs & W boson masses

$$\mathcal{L} = -\left(-\mu^2 |h_0|^2 + \lambda |h_0|^4\right) + \kappa |h_0|^2 W^{\mu+} W^-_\mu$$

h_0: neutral component of Higgs doublet

Consider two observables

$$M_H^2 = 2\mu^2, \Delta \equiv \frac{\lambda}{\kappa} - 1 = \left(\frac{M_H}{2M_W}\right)^2 - 1$$

Tree level

$$\mu^2_{tree} = 0, \lambda_{tree} = \kappa_{tree} = \frac{g^2}{2}$$

$$M_H^2 = 0$$
$$\Delta_{tree} = 0$$

$$-\frac{1}{2} Tr\left(F_{MN} F^{MN}\right) \supset -g^2 Tr\left(\left[A_\mu, A_5\right]^2 + \left[A_\mu, A_6\right]^2 - \left[A_5, A_6\right]^2\right)$$

All coefficients can be read from the gauge kinetic term

Relevant for Higgs & W boson masses

$$\mathcal{L} \supset -\left(-\mu^2 |h_0|^2 + \lambda |h_0|^4\right) + \kappa |h_0|^2 W^{\mu+} W^-_\mu$$

h_0: neutral component of Higgs doublet

Consider two observables

$$M_H^2 = 2\mu^2, \Delta \equiv \frac{\lambda}{\kappa} - 1 = \left(\frac{M_H}{2M_W}\right)^2 - 1$$

Tree level

$$\mu^2_{tree} = 0, \lambda_{tree} = \kappa_{tree} = \frac{g^2}{2} \Rightarrow \begin{array}{l} M_H^2 = 0 \\ \Delta_{tree} = 0 \end{array}$$

Once EW symmetry is broken

$$\lambda_{tree} = \frac{g^2}{2}, M_H^2 = 2\lambda_{tree} v^2 = g^2 v^2, M_W = \frac{1}{2} g v \Rightarrow M_H = 2M_W$$

An issue to be addressed

Quantum corrections to $M_H = 2M_W$ is calculable?

↓

The corrections are explicitly shown to be finite by using a toy model

1-loop Corrections to $M_H = 2M_W$ in 6D GHU

6D SU(3) model on T^2/Z_3 with a triplet scalar

$$\mathcal{L} = \left(D_M \Phi\right)^\dagger \left(D^M \Phi\right) - M^2 \Phi^\dagger \Phi - \frac{1}{2}\mathrm{Tr}\left(F_{MN} F^{MN}\right)$$

Boundary conditions:

T^2:

$$A_M\left(x, z + e_{1,2}\right) = A_M\left(x,z\right), \Phi\left(x, z + e_{1,2}\right) = \Phi\left(x,z\right)$$

$$\left(e_1 = 2\pi R, e_2 = 2\pi\omega R, z = x_5 + i x_6, \omega = e^{2\pi i/3}\right)$$

Z_3:

$$A_\mu\left(x,\omega z\right) = \Theta A_\mu\left(x,z\right)\Theta^\dagger, A_z\left(x,\omega z\right) = \bar{\omega}\Theta A_z\left(x,z\right)\Theta^\dagger,$$

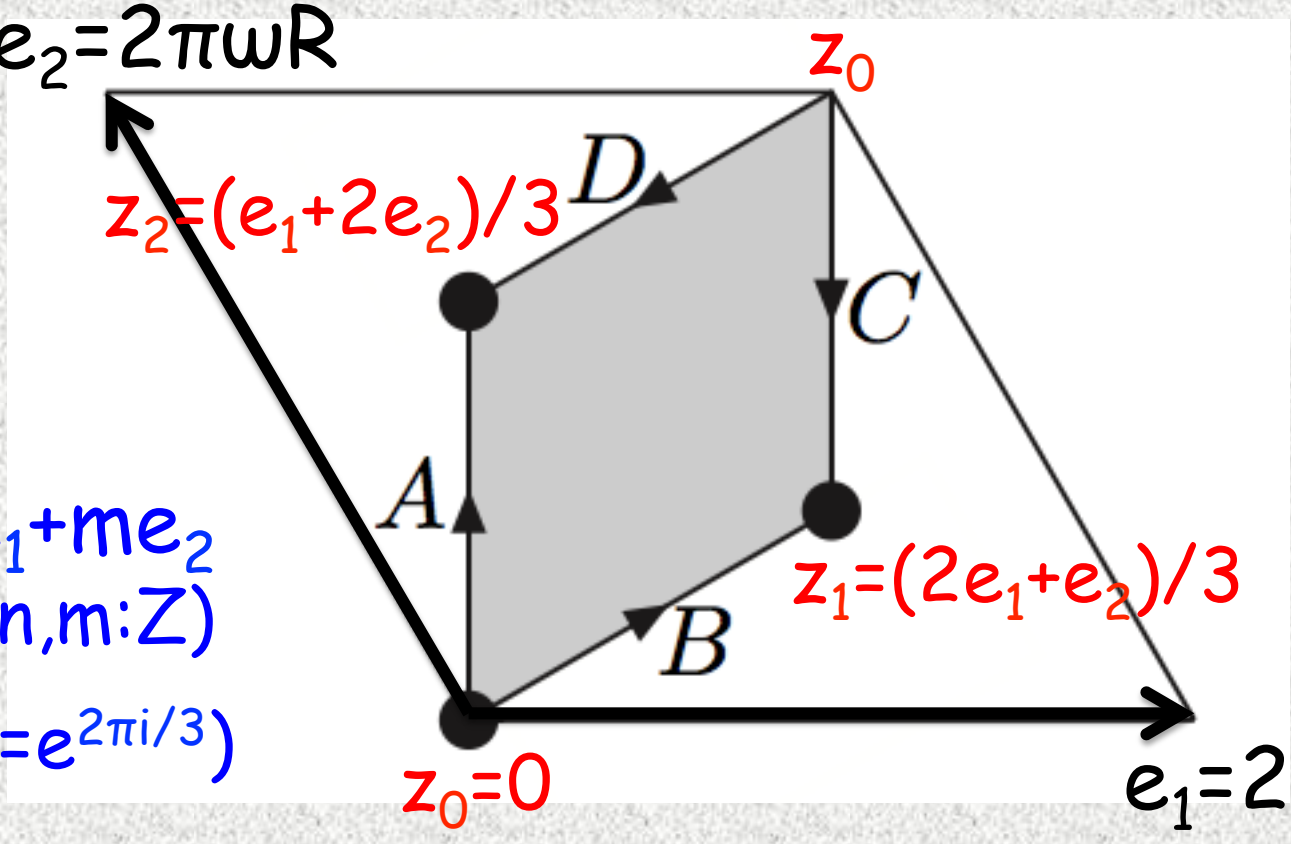
$$A_{\bar{z}}\left(x,\omega z\right) = \omega\Theta A_{\bar{z}}\left(x,z\right)\Theta^\dagger, \Phi\left(x,\omega z\right) = \Theta\Phi\left(x,z\right),$$

$$\Theta = diag\left(1,1,\omega\right) \qquad A_z \equiv \frac{1}{2}\left(A_5 - iA_6\right),\ A_{\bar{z}} \equiv A_z^\dagger$$

6D SU(3) model on T^2/Z_3 with a triplet scalar

$$\mathcal{L} = \left(D_M \Phi\right)^{\dagger} \left(D^M \Phi\right) - M^2 \Phi^{\dagger} \Phi - \frac{1}{2} \mathrm{Tr}\left(F_{MN} F^{MN}\right)$$

T^2/Z_3



$T^2: z \sim z + ne_1 + me_2$
$(z=x_5+ix_6,\ n,m:Z)$

$Z_3: z \sim \omega z$ $(\omega=e^{2\pi i/3})$

Boundary conditions:

T^2: $A_M\left(x,z+e_{1,2}\right)=A_M\left(x,z\right),\Phi\left(x,z+e_{1,2}\right)=\Phi\left(x,z\right)$

$$A_\mu\left(x,\omega z\right)=\Theta A_\mu\left(x,z\right)\Theta^\dagger, A_z\left(x,\omega z\right)=\bar{\omega}\Theta A_z\left(x,z\right)\Theta^\dagger,$$

Z_3: $A_{\bar{z}}\left(x,\omega z\right)=\omega\Theta A_{\bar{z}}\left(x,z\right)\Theta^\dagger,\Phi\left(x,\omega z\right)=\Theta\Phi\left(x,z\right),$

$\Theta=diag\left(1,1,\omega\right)$ $\qquad A_z\equiv\frac{1}{2}\left(A_5-iA_6\right),\ A_{\bar{z}}\equiv A_z^\dagger$

$$A_\mu=\begin{pmatrix}1&1&\bar{\omega}\\1&1&\bar{\omega}\\\omega&\omega&1\end{pmatrix},A_z=\begin{pmatrix}\bar{\omega}&\bar{\omega}&\omega\\\bar{\omega}&\bar{\omega}&\omega\\1&1&\bar{\omega}\end{pmatrix},A_{\bar{z}}=\begin{pmatrix}\omega&\omega&1\\\omega&\omega&1\\\bar{\omega}&\bar{\omega}&\omega\end{pmatrix},\Phi=\begin{pmatrix}1\\1\\\omega\end{pmatrix}$$

SU(3)→SU(2)xU(1) One Higgs doublet

First, calculate 1-loop corrections to

Higgs & W masses

Higgs quartic coupling

1-loop effective potential for background fields

Higgs & $W^{\pm}$ due to Φ loop

$$A_\mu^{cl} = \begin{pmatrix} 0 & W_\mu^+/\sqrt{2} & 0 \\ W_\mu^-/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_z^{cl} = \frac{a}{\sqrt{2}gR}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, A_{\bar{z}}^{cl} = \frac{a}{\sqrt{2}gR}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Dimensionless const: $a \equiv gR\left|h^0\right|/\sqrt{2}$

1-loop effective potential

$$V_{eff}(a,W)=\int\frac{d^4p}{(2\pi)^4}\sum_{n,m}\log\left(p^2-M^2-\mathcal{M}^2_{n,m}\right)$$

where

$$\mathcal{M}^2_{n,m}\equiv\begin{pmatrix}-\frac{g^2}{2}W^{+\mu}W^-_\mu+M^2_{n,m} & -\sqrt{2}gp^\mu W^+_\mu & 0\\ -\sqrt{2}gp^\mu W^-_\mu & -\frac{g^2}{2}W^{+\mu}W^-_\mu+M^2_{n,m}+\frac{a^2}{R^2} & \frac{\sqrt{2}a}{R}M_{n,m}\\ 0 & \frac{\sqrt{2}a}{R}M_{n,m} & M^2_{n,m}+\frac{a^2}{R^2}\end{pmatrix}$$

$$M^2_{n,m}\equiv\frac{4}{3R^2}\left(n^2+nm+m^2\right)$$

Eigenvalues of $\mathcal{M}^2_{n,m}$ up to O(W²)

$$\lambda_1 = M^2_{n,m} + \frac{g^2 \left| p^\mu W^+_\mu \right|^2}{M^2_{n,m} - \frac{a^2}{2R^2}} - \frac{g^2}{2} W^{+\mu} W^-_\mu$$

$$\lambda_2 = M^2_{n,m} + \frac{a^2}{R^2} + \frac{\sqrt{2}a}{R} M_{n,m} + \frac{g^2 \left| p^\mu W^+_\mu \right|^2}{\frac{a^2}{R^2} + \frac{\sqrt{2}a}{R} M_{n,m}} - \frac{g^2}{4} W^{+\mu} W^-_\mu$$

$$\lambda_3 = M^2_{n,m} + \frac{a^2}{R^2} - \frac{\sqrt{2}a}{R} M_{n,m} + \frac{g^2 \left| p^\mu W^+_\mu \right|^2}{\frac{a^2}{R^2} - \frac{\sqrt{2}a}{R} M_{n,m}} - \frac{g^2}{4} W^{+\mu} W^-_\mu$$

Our interests are coefficients of the following operators

1: a^2 (Higgs mass)

2: a^4 (Higgs quartic)

3: $a^2 W^{\mu +} W_{\mu}^{-}$ (W-boson mass)

can be read by expanding the potential in "a" and "$W^{\pm}$"

$$V_{eff}(a,W) = \int \frac{d^4 p_E}{(2\pi)^4} \sum_{n,m} \left[\ln\left(p_E^2 + M^2 + \lambda_1\right) + \ln\left(p_E^2 + M^2 + \lambda_2\right) + \ln\left(p_E^2 + M^2 + \lambda_3\right) \right]$$

a^2 term (Higgs mass)

can be read by $W^{\pm}=0$ and expanding in "a"

$$V_{eff}(a,W)=\int\frac{d^4p_E}{(2\pi)^4}\sum_{n,m}\left[\ln\left(p_E^2+M^2+\lambda_1\right)+\ln\left(p_E^2+M^2+\lambda_2\right)+\ln\left(p_E^2+M^2+\lambda_3\right)\right]$$

$$\lambda_1=M_{n,m}^2+\frac{g^2\left|p^\mu W_\mu^+\right|^2}{M_{n,m}^2-\frac{a^2}{2R^2}}-\frac{g^2}{2}W^{+\mu}W_\mu^-,\ \lambda_2=M_{n,m}^2+\frac{a^2}{R^2}+\frac{\sqrt{2}a}{R}M_{n,m}+\frac{g^2\left|p^\mu W_\mu^+\right|^2}{\frac{a^2}{R^2}+\frac{\sqrt{2}a}{R}M_{n,m}}-\frac{g^2}{4}W^{+\mu}W_\mu^-$$

$$\lambda_3=M_{n,m}^2+\frac{a^2}{R^2}-\frac{\sqrt{2}a}{R}M_{n,m}+\frac{g^2\left|p^\mu W_\mu^+\right|^2}{\frac{a^2}{R^2}-\frac{\sqrt{2}a}{R}M_{n,m}}-\frac{g^2}{4}W^{+\mu}W_\mu^-$$

$$2\frac{a^2}{R^2}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n,m}\frac{p_E^2+M^2}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}$$

a^2 term (Higgs mass)

can be read by $W^{\pm}=0$ and expanding in "a" as

$$2\frac{a^2}{R^2}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n,m}\frac{p_E^2+M^2}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}$$

Winding # sum

$$=\sqrt{3}\pi a^2\int_0^\infty dt\int\frac{d^4p_E}{(2\pi)^4}\left(p_E^2+M^2\right)e^{-t\left(p_E^2+M^2\right)}\sum_{k,l}e^{-(\pi R)^2\left(k^2+kl+l^2\right)/t}$$

$$\frac{1}{A^2}=\int_0^\infty dt\ te^{-At},\quad\sum_{n,m}e^{-tM_{n,m}^2}=\frac{\sqrt{3}\pi R^2}{2}\sum_{k,l}\frac{1}{t}e^{-(\pi R)^2\left(k^2+kl+l^2\right)/t}$$

(Poisson resummation)

a^2 term (Higgs mass)

can be read by $W^{\pm}=0$ and expanding in "a" as

$$2\frac{a^2}{R^2}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n,m}\frac{p_E^2+M^2}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}$$

$$=\sqrt{3}\pi a^2\int_0^\infty dt\int\frac{d^4p_E}{(2\pi)^4}\left(p_E^2+M^2\right)e^{-t\left(p_E^2+M^2\right)}\sum_{k,l}e^{-(\pi R)^2\left(k^2+kl+l^2\right)/t}$$

Winding # sum

"0 winding" part (k=l=0)⇒ UV divergent

$$\int_0^\infty dt\int\frac{d^4p_E}{(2\pi)^4}\left(p_E^2+M^2\right)e^{-t\left(p_E^2+M^2\right)}=\int\frac{d^4p_E}{(2\pi)^4}1=\infty$$

Divergent??
→ Cutoff reg.

$$\xrightarrow{\dim.reg.}\int\frac{d^dp_E}{(2\pi)^4}\frac{p_E^2+M^2}{p_E^2+M^2}=\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{M^d}{\Gamma(1)}\left[\frac{d}{2}\Gamma\left(-\frac{d}{2}\right)+\Gamma\left(1-\frac{d}{2}\right)\right]=0$$

a^4 term (Higgs quartic coupling)

Calculate in a similar way

$$\frac{a^4}{R^4}\int\frac{d^4p_E}{(2\pi)^4}\sum_{n,m}\left[\frac{1}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}-\frac{2\left(p_E^2+M^2\right)^2}{\left(p_E^2+M^2+M_{n,m}^2\right)^4}\right]$$

$$=\frac{\sqrt{3}}{2}\pi\frac{a^4}{R^2}\int_0^\infty dt\int\frac{d^4p_E}{(2\pi)^4}e^{-t\left(p_E^2+M^2\right)}\left[1-\frac{t^2}{3}\left(p_E^2+M^2\right)^2\right]\sum_{k,l}e^{-(\pi R)^2\left(k^2+kl+l^2\right)/t}$$

"0 winding" part (k=l=0)⇒ UV divergent

$$\frac{\sqrt{3}}{6}\pi\frac{a^4}{R^2}\int\frac{d^4p_E}{(2\pi)^4}\frac{1}{p_E^2+M^2}\Rightarrow\frac{\sqrt{3}}{6}\pi\frac{a^4}{R^2}\frac{\Gamma\left(1-d/2\right)}{(4\pi)^{d/2}}M^{d-2}\ (d\to 4)$$

(and t-integration)

$a^2\, W^{\mu+} W_\mu^-$ term (W boson mass)

Similar calculations tell us

$$g^2 \frac{a^2}{R^2} W^{+\mu} W_\mu^- \int \frac{d^4 p_E}{(2\pi)^d} \sum_{n,m} \left[-\frac{1}{2} \frac{1}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^2} + \frac{(1+2/d)\, p_E^2 + M^2}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^3} - \frac{4}{d} \frac{p_E^2 \left(p_E^2 + M^2\right)}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^4} \right]$$

$$= \frac{\sqrt{3}}{2} \pi g^2 \frac{a^2}{R^2} W^{+\mu} W_\mu^- \int_0^\infty dt \int \frac{d^4 p_E}{(2\pi)^d} e^{-t\left(p_E^2 + M^2\right)} \times$$

$$\left[-\frac{1}{2} + \frac{t}{2} \left(\left(1 + \frac{2}{d}\right) p_E^2 + M^2 \right) - \frac{2t^2}{3d} p_E^2 \left(p_E^2 + M^2\right) \right] \sum_{k,l} e^{-(\pi R)^2 \left(k^2 + kl + l^2\right)/t}$$

"0 winding" part (k=l=0) & t-integration

$$\frac{\sqrt{3}}{2} \pi g^2 a^2 W^{+\mu} W_\mu^- \int \frac{d^d p_E}{(2\pi)^d} \left(-\frac{1}{3d} \right) \frac{p_E^2}{\left(p_E^2 + M^2\right)^2}$$

$$= -\frac{\sqrt{3}}{12} \pi g^2 a^2 W^{+\mu} W_\mu^- \frac{\Gamma(1 - d/2)}{(4\pi)^{d/2}} M^{d-2} \quad (d \to 4)$$

Divergent parts of quantum corrections

Zero winding sector

$$\frac{\sqrt{3}}{6}\pi\frac{a^4}{R^2}\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}=\frac{\sqrt{3}}{6}\pi\frac{1}{R^2}\left(\frac{g}{\sqrt{2}}|h_0|R\right)^4\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}$$

$$-\frac{\sqrt{3}}{12}\pi g^2a^2W^{+\mu}W^-_\mu\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}=-\frac{\sqrt{3}}{12}\pi g^2\left(\frac{g}{\sqrt{2}}|h_0|R\right)^2W^{+\mu}W^-_\mu\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}$$

$$a\equiv gR|h^0|/\sqrt{2}$$

$$\mathcal{L}=-\left(-\mu^2|h_0|^2+\lambda|h_0|^4\right)+\kappa|h_0|^2W^{\mu+}W^-_\mu$$

$$\delta\lambda^{div}=\frac{\sqrt{3}}{24}\pi g^4R^2\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}$$

$$\delta\kappa^{div}=\frac{\sqrt{3}}{24}\pi g^4R^2\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}}M^{d-2}$$

As expected, the coefficients of div. agree

Another possible contributions to $\Delta = (M_H/2M_W)^2 - 1$

Quantum corrections to kinetic terms of $W^{\pm}$ & h_0

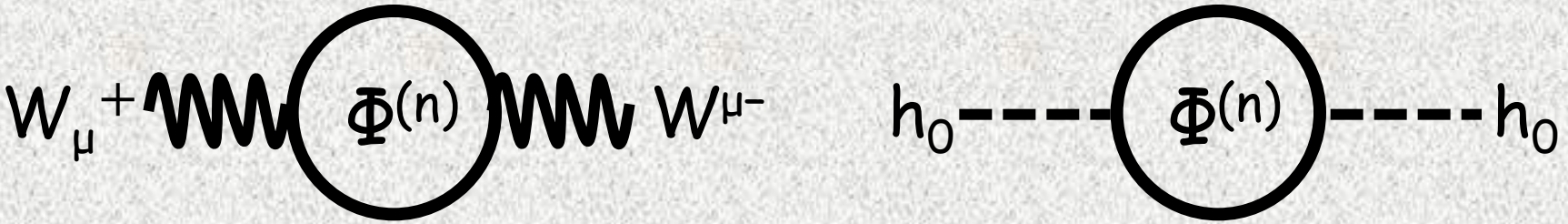
Wave function renormalization:

$$W_\mu^\pm = \sqrt{Z_W}\, W_{r\mu}^\pm, \quad h_0 = \sqrt{Z_h}\, h_{r0}$$

$$\Delta = \left(\frac{M_H}{2M_W}\right)^2 - 1 = \frac{\lambda}{\kappa} - 1 = \frac{\left(g^2/2 + \delta\lambda\right)\left(1 + Z_h - 1\right)}{\left(g^2/2 + \delta\kappa\right)\left(1 + Z_W - 1\right)} - 1$$

$$\simeq \frac{2}{g^2}(\delta\lambda - \delta\kappa) + \left(Z_h - 1\right) - \left(Z_W - 1\right)$$

Relevant to wave fcn. ren. are the self-energy diagrams

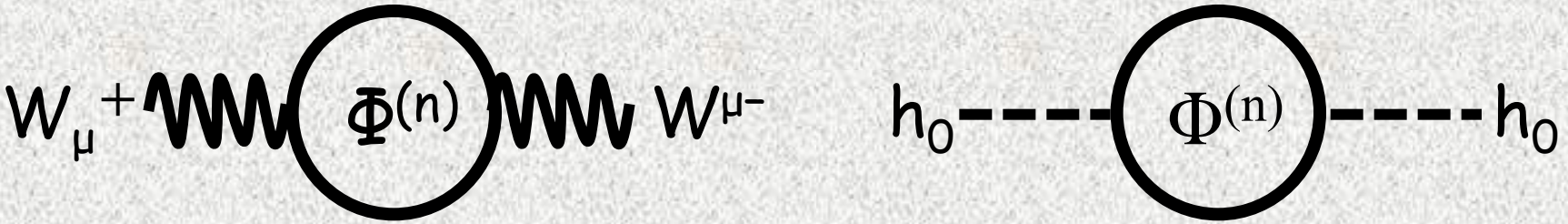


Expectation: div. from these diagrams are the same
because they come from $Z\ F_{MN}F^{MN}$

$$Z_W - 1 = -\frac{\sqrt{3}\pi}{12} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} e^{-t\left(k_E^2 + M^2\right)} \sum_{k,l} e^{-(\pi R)^2 \left(k^2 + kl + l^2\right)/t}$$

$$Z_h - 1 = -\frac{\sqrt{3}\pi}{6} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} \left[1 - \frac{t}{2}\left(k_E^2 + M^2\right)\right] e^{-t\left(k_E^2 + M^2\right)} \sum_{k,l} e^{-(\pi R)^2 \left(k^2 + kl + l^2\right)/t}$$

Relevant to wave fcn. ren. are the self-energy diagrams



Expectation: div. from these diagrams are the same
because they come from $Z\, F_{MN}F^{MN}$

Divergent part

$$\left(Z_W-1\right)_{div}=-\frac{\sqrt{3}\pi}{12}g^2R^2\int_0^\infty dt\int\frac{d^dk_E}{\left(2\pi\right)^d}e^{-t\left(k_E^2+M^2\right)}=-\frac{\sqrt{3}\pi}{12}g^2R^2\int\frac{d^dk_E}{\left(2\pi\right)^d}\frac{1}{k_E^2+M^2}$$

$$\left(Z_h-1\right)_{div}=-\frac{\sqrt{3}\pi}{6}g^2R^2\int_0^\infty dt\int\frac{d^dk_E}{\left(2\pi\right)^d}\left[1-\frac{t}{2}\left(k_E^2+M^2\right)\right]e^{-t\left(k_E^2+M^2\right)}$$

Same!!

$$=-\frac{\sqrt{3}\pi}{6}g^2R^2\int\frac{d^dk_E}{\left(2\pi\right)^d}\left[\frac{1}{k_E^2+M^2}-\frac{1}{2}\frac{k_E^2+M^2}{\left(k_E^2+M^2\right)^2}\right]=-\frac{\sqrt{3}\pi}{12}g^2R^2\int\frac{d^dk_E}{\left(2\pi\right)^d}\frac{1}{k_E^2+M^2}$$

Final expressions

M_H^2 & Δ are calculable

$$M_H^2 = 2\delta\mu^2 = -\frac{\sqrt{3}}{16\pi}\frac{g^2}{R^2}\sum_{(k,l)\neq(0,0)}\int_0^\infty du\left(2u+\hat{M}^2\right)e^{-\hat{M}^2/u}e^{-\pi^2\left(k^2+kl+l^2\right)u}$$

$$\hat{M} \equiv RM$$

$$\Delta \simeq \frac{2}{g^2}\left(\delta\lambda-\delta\kappa\right)+\left(Z_h-1\right)-\left(Z_W-1\right)$$

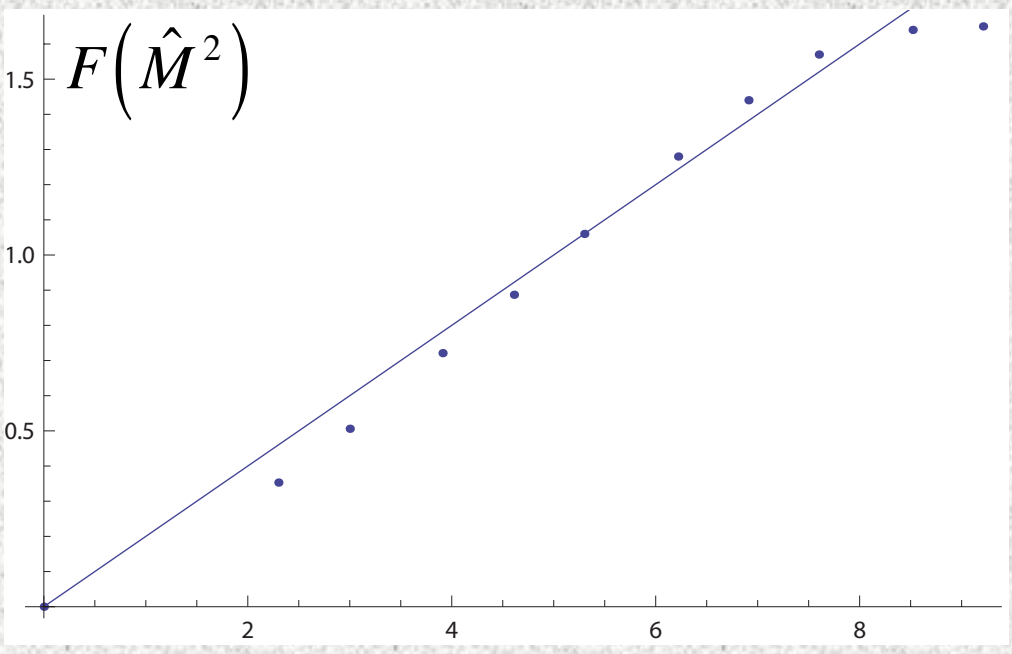
$$= -\frac{\sqrt{3}}{64\pi}g^2\sum_{(k,l)\neq(0,0)}\int_0^\infty du\left(\frac{2}{3}+\frac{2}{3}\frac{\hat{M}^2}{u}+\frac{\hat{M}^4}{3u^2}\right)e^{-\hat{M}^2/u}e^{-\pi^2\left(k^2+kl+l^2\right)u}$$

Final expressions

$$M_H^2 = -\frac{\sqrt{3}}{16\pi}\frac{g^2}{R^2}\sum_{(k,l)\neq(0,0)}\int_0^\infty du\left(2u+\hat{M}^2\right)e^{-\hat{M}^2/u}e^{-\pi^2\left(k^2+kl+l^2\right)u}$$

$$\hat{M} \equiv RM$$

$$\Delta = -\frac{\sqrt{3}}{64\pi}g^2\sum_{(k,l)\neq(0,0)}\int_0^\infty du\left(\frac{2}{3}+\frac{2}{3}\frac{\hat{M}^2}{u}+\frac{\hat{M}^4}{3u^2}\right)e^{-\hat{M}^2/u}e^{-\pi^2\left(k^2+kl+l^2\right)u}$$



$F\left(\hat{M}^2\right)$

Numerical fit

$$F\left(\hat{M}^2\right) \approx 0.2\left(-\ln \hat{M}^2\right)$$

$-\ln \hat{M}$

Rough numerical study

Want to see if the toy model is roughly possible to realize the observed values for suitable choices of parameters

$$\left|M_H^2\right| \simeq \frac{\sqrt{3}}{8\pi^5}\frac{g^2}{R^2}\sum_{(k,l)\neq(0,0)}\frac{1}{\left(k^2+kl+l^2\right)^2} \quad \leftarrow \text{M=0 case is good approx.}$$

$$\approx 0.0685\frac{\alpha}{R^2\sin^2\theta_W} = 2.2\times10^{-3}\frac{1}{R^2} = \left(126GeV\right)^2$$

$$\left|\Delta\right| = \frac{\sqrt{3}}{64\pi}g^2F\left(\hat{M}^2\right) \simeq 0.0217\frac{\alpha}{\sin^2\theta_W}\left(-\ln\hat{M}^2\right) \approx 1.38\times10^{-3}\left(-\ln\hat{M}\right)$$

$$= \left|\left(\frac{M_H}{2M_W}\right)^2 - 1\right| = \left|\left(\frac{126}{160}\right)^2 - 1\right| = 0.380$$

$$\Rightarrow \frac{1}{R} \approx 2.7TeV, -\ln\hat{M} \approx 275$$

Problem?

$-\ln(MR) \sim 275$

$\Rightarrow$ very large log or very small M

In a realistic model,
top quark loop contributions
lead to enhance as

$(m_{top}/m_W)^4 \sim 23$

Too large log or too small M
is expected to be reasonably avoided

Summary

In a certain class of D > 5 GHU models,
$M_H = 2M_W$ is predicted at the leading order

In order to get an observed 126 GeV Higgs mass, important to study the quantum corrections to $M_H = 2M_W$ and its calculability

We have shown that the corrections are indeed calculable in a toy model of 6D GHU

Obviously, getting 160-35~126 GeV in a realistic model is a next challenge and work in progress

Backup

a^2 $W^{\mu+}W_\mu^-$ term (W boson mass)

Coming from $W^+ \cdot W^-$ (1st & 2nd) and
$|p_E \cdot W^+|2 \rightarrow p_E^2\, W^+ \cdot W^-/d$ (3rd & last) terms in the eigenvalues

$$\lambda_1 = M_{n,m}^2 + \frac{g^2\left|p^\mu W_\mu^+\right|^2}{M_{n,m}^2 - \frac{a^2}{2R^2}} - \frac{g^2}{2} W^{+\mu}W_\mu^-,\ \lambda_2 = M_{n,m}^2 + \frac{a^2}{R^2} + \frac{\sqrt{2}a}{R} M_{n,m} + \frac{g^2\left|p^\mu W_\mu^+\right|^2}{\frac{a^2}{R^2} + \frac{\sqrt{2}a}{R} M_{n,m}} - \frac{g^2}{4} W^{+\mu}W_\mu^-$$

$$\lambda_3 = M_{n,m}^2 + \frac{a^2}{R^2} - \frac{\sqrt{2}a}{R} M_{n,m} + \frac{g^2\left|p^\mu W_\mu^+\right|^2}{\frac{a^2}{R^2} - \frac{\sqrt{2}a}{R} M_{n,m}} - \frac{g^2}{4} W^{+\mu}W_\mu^-$$

$$g^2 \frac{a^2}{R^2} W^{+\mu}W_\mu^- \int \frac{d^4 p_E}{(2\pi)^4} \sum_{n,m} \left[-\frac{1}{2} \frac{1}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^2} + \frac{(1+2/d)\,p_E^2 + M^2}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^3} - \frac{4}{d} \frac{p_E^2\left(p_E^2 + M^2\right)}{\left(p_E^2 + M^2 + M_{n,m}^2\right)^4} \right]$$

$$= \frac{\sqrt{3}}{2} \pi g^2 a^2 W^{+\mu}W_\mu^- \int_0^\infty dt \int \frac{d^4 p_E}{(2\pi)^d} e^{-t\left(p_E^2 + M^2\right)} \times$$

$$\left[-\frac{1}{2} + \frac{t}{2}\left(\left(1 + \frac{2}{d}\right) p_E^2 + M^2\right) - \frac{2t^2}{3d} p_E^2\left(p_E^2 + M^2\right) \right] \sum_{k,l} e^{-(\pi R)^2\left(k^2 + kl + l^2\right)/t}$$

SU(3) GHU on T^2/Z_N

2 doublets for N=2

1 doublet for N=3

0 doublets for N=4,6

$$Z_W - 1 = -\frac{\sqrt{3}\pi}{12} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} e^{-t\left(k_E^2 + M^2\right)} \sum_{k,l} e^{-(\pi R)^2 \left(k^2 + kl + l^2\right)/t}$$

$$Z_h - 1 = -\frac{\sqrt{3}\pi}{6} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} \left[1 - \frac{t}{2}\left(k_E^2 + M^2\right)\right] e^{-t\left(k_E^2 + M^2\right)} \sum_{k,l} e^{-(\pi R)^2 \left(k^2 + kl + l^2\right)/t}$$

$$\left(Z_W - 1\right)_{div} = -\frac{\sqrt{3}\pi}{12} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} e^{-t\left(k_E^2 + M^2\right)} = -\frac{\sqrt{3}\pi}{12} g^2 R^2 \int \frac{d^d k_E}{(2\pi)^d} \frac{1}{k_E^2 + M^2}$$

$$\left(Z_h - 1\right)_{div} = -\frac{\sqrt{3}\pi}{6} g^2 R^2 \int_0^\infty dt \int \frac{d^d k_E}{(2\pi)^d} \left[1 - \frac{t}{2}\left(k_E^2 + M^2\right)\right] e^{-t\left(k_E^2 + M^2\right)}$$

$$= -\frac{\sqrt{3}\pi}{6} g^2 R^2 \int \frac{d^d k_E}{(2\pi)^d} \left[\frac{1}{k_E^2 + M^2} - \frac{1}{2} \frac{k_E^2 + M^2}{\left(k_E^2 + M^2\right)^2}\right] = -\frac{\sqrt{3}\pi}{12} g^2 R^2 \int \frac{d^d k_E}{(2\pi)^d} \frac{1}{k_E^2 + M^2}$$

a^2 term (Higgs mass)

can be read by $W^{\pm}=0$ and expanding in "a"

log part

$$\ln\left[\left(p_E^2+M^2+M_{n,m}^2\right)^2+2\left(p_E^2+M^2\right)\frac{a^2}{R^2}+\frac{a^4}{R^4}\right]$$

$O(a^2)$

$$\frac{2\left(p_E^2+M^2\right)}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}\frac{a^2}{R^2}$$

a^4 term (Higgs quartic coupling)

log part

$$\ln\left[\left(p_E^2+M^2+M_{n,m}^2\right)^2+2\left(p_E^2+M^2\right)\frac{a^2}{R^2}+\frac{a^4}{R^4}\right]$$

$$O(a^4)\quad\left[\frac{1}{\left(p_E^2+M^2+M_{n,m}^2\right)^2}-\frac{2\left(p_E^2+M^2\right)^2}{\left(p_E^2+M^2+M_{n,m}^2\right)^4}\right]\frac{a^4}{R^4}$$

Comments

1: Divergent localized tadpoles

Gersdorff, Irges & Quiros (2003)
Scrucca, Serone, Sivestrini & Wulzer (2004)

U(1) component of kinetic term linear in F_{56}
is allowed on fixed points
if U(1) is unbroken by BCs ($SU(3) \to SU(2) \times U(1)$)

↓

F_{56} has Higgs mass term $\propto [A_5, A_6]$,
which is divergent by quantum corrections

Cannot be cancelled locally, but can be globally
in T^2/Z_4 with a suitable choice of fermions

Scrucca, Serone, Sivestrini & Wulzer (2004)

Comments

2: Divergent gauge invariant operators
with higher mass dimensions

In non-renormalizable theory,
higher dim. gauge inv. terms are generated @loops
and its coefficients are divergent in general

ex.

$(F_{MN}F^{MN})^n$ $(n \geqq 2) \Rightarrow$ harmless in 6D $(d \geqq 8)$

$(D_L F_{MN})(D^L F^{MN}) \Rightarrow$ log div. $(d=6)$

$\Rightarrow$ 6-point Higgs self-coupling
suppressed by M_W^2/M_C^2

Comments

3: Wave function renormalization

Wave function renormalizations for Higgs & $W^{\pm}$ are divergent, but Δ is still calculable

∵ all kinetic terms are included in $F^{MN}F_{MN}$

⇒ same coefficient of div. parts

Heavy particle effects to finite terms of wave function. ren. are NOT remarkable comparing to Higgs & W mass renormalizations

[Oblique corr.: $T \propto O(m_{heavy}{}^2)$, S, $U \propto O(\log m_{heavy}{}^2)$]

$$A_z \equiv \frac{1}{2}\left(A_5 - iA_6\right) = \frac{|h_0|}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$A_{\bar{z}} \equiv A_z^{\dagger} = \frac{1}{2}\left(A_5 + iA_6\right) = \frac{|h_0|}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_5 = \frac{|h_0|}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, A_6 = i\frac{|h_0|}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$-\frac{1}{2}Tr\left(F_{MN}F^{MN}\right)\supset-\frac{1}{2}Tr\left(2g^2\left[A_\mu,A_5\right]^2+2g^2\left[A_\mu,A_6\right]^2-2g^2\left[A_5,A_6\right]^2\right)$$

$$Tr\left[A_\mu,A_5\right]^2=\left(\frac{|h_0|}{2}\right)^2 Tr\left[\begin{pmatrix}0 & W_\mu^+/\sqrt{2} & 0\\ W_\mu^-/\sqrt{2} & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\right]^2=\left(\frac{|h_0|}{2}\right)^2 Tr\begin{pmatrix}0 & 0 & W_\mu^+/\sqrt{2}\\ 0 & 0 & 0\\ -W_\mu^-/\sqrt{2} & 0 & 0\end{pmatrix}^2$$

$$=\left(\frac{|h_0|}{2}\right)^2 Tr\begin{pmatrix}-W^{\mu+}W_\mu^-/2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -W^{\mu+}W_\mu^-/2\end{pmatrix}=-\left(\frac{|h_0|}{2}\right)^2 W^{\mu+}W_\mu^-$$

$$Tr\left[A_\mu,A_6\right]^2=-\left(\frac{|h_0|}{2}\right)^2 Tr\left[\begin{pmatrix}0 & W_\mu^+/\sqrt{2} & 0\\ W_\mu^-/\sqrt{2} & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}\right]^2=\left(\frac{|h_0|}{2}\right)^2 Tr\begin{pmatrix}0 & 0 & -W_\mu^+/\sqrt{2}\\ 0 & 0 & 0\\ -W_\mu^-/\sqrt{2} & 0 & 0\end{pmatrix}^2$$

$$=-\left(\frac{|h_0|}{2}\right)^2 Tr\begin{pmatrix}W^{\mu+}W_\mu^-/2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & W^{\mu+}W_\mu^-/2\end{pmatrix}=-\left(\frac{|h_0|}{2}\right)^2 W^{\mu+}W_\mu^-$$

$$Tr\left[A_5,A_6\right]^2=-\left(\frac{|h_0|}{2}\right)^4 Tr\left[\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix},\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}\right]^2=-\left(\frac{|h_0|}{2}\right)^4 2Tr\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix}^2=-\left(\frac{|h_0|^2}{2}\right)^2$$

$$\Rightarrow\quad -\frac{1}{2}Tr\left(F_{MN}F^{MN}\right)\supset\frac{g^2}{2}|h_0|^2 W^{\mu+}W_\mu^--\frac{g^2}{2}|h_0|^4$$

Finding fixed points on T^2/Z_3

$$z \to \omega z + n e_1 + m e_2 = z \left(n, m \in \mathbb{Z} \right)$$

$$\Rightarrow z = \frac{n e_1 + m e_2}{1 - \omega} = \frac{1}{3}(2 + \omega)(n e_1 + m e_2)$$

$$= \frac{1}{3}\left[(2n - m) e_1 + (n + m) e_2 \right] \left(\because 1 + \omega + \omega^2 = 0 \right)$$

$$0 \le 2n - m < 3, 0 \le n + m < 3 \Rightarrow (n, m) = (0,0), (1,0), (1,1)$$

Mode functions

$$1: f^{(0)}_{n,m}(z) = f_{n,m}(z) + f_{n,m}(\omega z) + f_{n,m}(\bar{\omega} z)$$

$$\omega: f^{(1)}_{n,m}(z) = f_{n,m}(z) + \bar{\omega} f_{n,m}(\omega z) + \omega f_{n,m}(\bar{\omega} z)$$

$$\omega^2: f^{(2)}_{n,m}(z) = f_{n,m}(z) + \omega f_{n,m}(\omega z) + \bar{\omega} f_{n,m}(\bar{\omega} z)$$

$$f_{n,m}(z) = \exp\left[\frac{i}{2R}\left\{\left(n - \frac{n+2m}{\sqrt{3}} i\right) z + c.c\right\}\right]$$

Naïve Dimensional Analysis

$$\varepsilon = \frac{\pi^{D/2}}{(2\pi)^D \Gamma(D/2)} g_D^2 \Lambda^{D-4} = \frac{\pi^{D/2}}{(2\pi)^D \Gamma(D/2)} g_4^2 V_{D-4} \Lambda^{D-4}$$

Loop expansion parameter in D-dim ($g^2/(16\pi^2)$ in 4D)

$$\int \frac{d^D k}{(2\pi)^D} = \frac{\Omega_{D-1}}{2} \int \frac{dk^2}{(2\pi)^D} \left(k^2\right)^{D-2/2}, \Omega_D \equiv \frac{2\pi^{(D+1)/2}}{\Gamma\left((D+1)/2\right)}$$

Requiring perturbativity up to the cutoff scale, i.e. $\varepsilon < 1$, the upper bound can be obtained

$$\Lambda < \left[\frac{2^{D-2} \pi^{D/2-1}}{\alpha_4 V_{D-4}} \Gamma\left(\frac{D}{2}\right) \right]^{1/(D-4)}$$

$$\alpha_4 \equiv \frac{g_4^2}{4\pi}$$

D=6, T^2/Z_3 case ($V_2 = (2\pi R)^2$)

$$\Lambda_6 < \left[\frac{2^4 \pi^2}{\alpha_4 \left(4\pi^2 R^2\right)} \Gamma(3) \right]^{1/2} = \sqrt{\frac{8}{\alpha_4}} \frac{1}{R} \simeq \frac{3}{R} \left(\alpha_4 \simeq 1\right)$$

$$\Lambda < \left[\frac{2^{D-2} \pi^{D/2-1}}{\alpha_4 V_{D-4}} \Gamma\left(\frac{D}{2}\right) \right]^{1/(D-4)}$$

The most stringent case

Finite parts

M_H^2 & Δ are calculable

$$M_H^2 = 2\delta\mu^2$$

$$\Delta = \frac{\lambda}{\kappa} - 1 = \frac{\left(g^2/2 + \delta\lambda\right)\left(1 + Z_h - 1\right)}{\left(g^2/2 + \delta\kappa\right)\left(1 + Z_W - 1\right)} - 1 \simeq \frac{2}{g^2}\left(\delta\lambda - \delta\kappa\right) + \left(Z_h - 1\right) - \left(Z_W - 1\right)$$

$$\delta\mu^2 = -\frac{\sqrt{3}}{2}\pi g^2 R^2 \int_0^\infty dt \int \frac{d^4 p_E}{(2\pi)^4}\left(p_E^2 + M^2\right) e^{-t\left(p_E^2 + M^2\right)} \sum_{(k,l)\neq(0,0)} e^{-(\pi R)^2\left(k^2 + kl + l^2\right)/t}$$

$$\delta\lambda = \frac{\sqrt{3}}{8}\pi g^4 R^2 \int_0^\infty dt \int \frac{d^d p_E}{(2\pi)^d}\left(1 - \frac{t^2}{3}\left(p_E^2 + M^2\right)^2\right) e^{-t\left(p_E^2 + M^2\right)} \sum_{(k,l)\neq(0,0)} e^{-(\pi R)^2\left(k^2 + kl + l^2\right)/t}$$

$$\delta\kappa = -\frac{\sqrt{3}}{4}\pi g^4 R^2 \int_0^\infty dt \int \frac{d^d p_E}{(2\pi)^d}\left(-\frac{1}{2} + \frac{t}{2}\left(\left(1 + \frac{2}{d}\right)p_E^2 + M^2\right) - \frac{2t^2}{3d} p_E^2\left(p_E^2 + M^2\right)\right)$$

$$\times e^{-t\left(p_E^2 + M^2\right)} \sum_{(k,l)\neq(0,0)} e^{-(\pi R)^2\left(k^2 + kl + l^2\right)/t}$$

T^2/Z_N (N=2,3,4,6)

T^2: $x \sim x + V$, $V = n_a e_a$ (e_a: basis of T^2, n_a: integer)

Z_N: $x \sim e^{2\pi i/N} x$

$$\theta = \begin{pmatrix} \exp\left(\frac{2\pi i}{N}k\right) & 0 \\ 0 & \exp\left(-\frac{2\pi i}{N}k\right) \end{pmatrix} \in Z_N$$

$y \sim \Theta x + V$

$\Rightarrow z \sim \Theta y + V \sim \Theta^2 x + \Theta V + V$

$\Theta V = V' = n'_a e_a$

$$Tr\theta = \exp\left(2\pi ik/N\right) + \exp\left(-2\pi ik/N\right) = 2\cos\left(2\pi k/N\right) \in \mathbb{Z}$$

$$\det\left(1-\theta\right) = \det\begin{pmatrix} 1-\exp\left(2\pi ik/N\right) & 0 \\ 0 & 1-\exp\left(-2\pi ik/N\right) \end{pmatrix}$$

$$= 4\sin^2\left(\pi k/N\right) \in \mathbb{Z}$$

$\Rightarrow$ N=2,3,4,6