Anomaly polynomial of general 6d SCFTs

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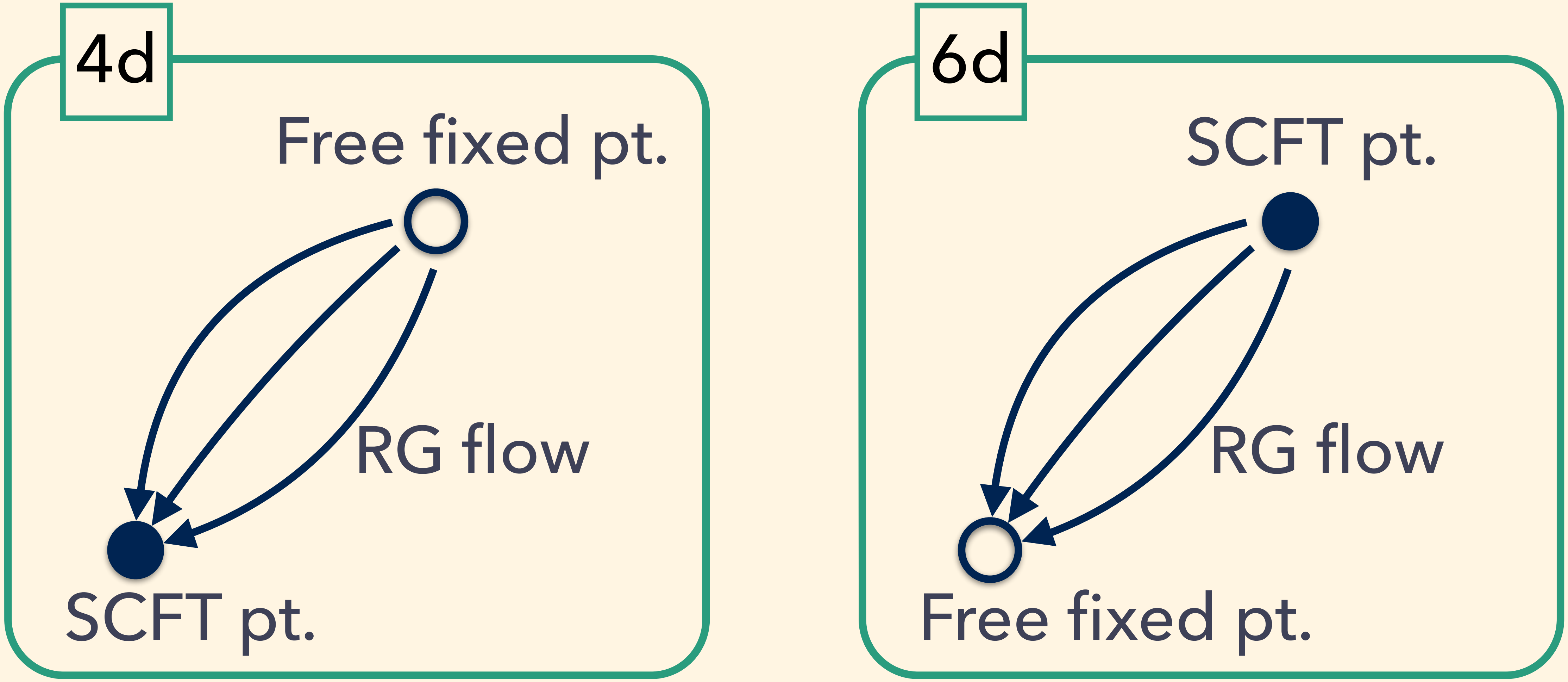
arXiv:1408.5572, 1???.????

Introduction

Interacting 6d SCFTs

- $\mathcal{N}$=(2,0) or $\mathcal{N}$=(1,0) (chiral theories)

- UV fixed point



Why 6d SCFTs?

- Strongly coupled system
  : no known Lagrangian

- Compactification ⇒ low dimensional systems

- Might control low dimensional dualities
  e.g. $\mathcal{N}=(2,0)$ theories of ADE type

What's done?

- String (M,F-) theoretical constructions
  [Witten '96],[Ganor,Hanany '96],[Seiberg '97],[Intrilligator Blum '97]
  [Brunner,Karch '97],[Hanany,Zaffaroni '97],[Aspinwall,Morrison '97] etc.
  [Heckman,Morrison,Vafa '13][Gaiotto Tomasielo '14],
  [Del-Zotto,Heckman,Tomasiello,Vafa '14]
- Enormous works for $\mathcal{N}$=(2,0) theories
  [Gaiotto '09]…
- Anomaly polynomials
  for above $\mathcal{N}$=(1,0) theories
  - Inflow:[Freed, Harvey, Minasian, Moore, '98],[KO,Shimizu,Tachikawa '14]
  - Tensor branch:[Intrilligator '14],[KO,Shimizu,Tachikawa,Yonekura '14]

Plan

- Brane constructions of 6d SCFTs
- Anomaly polynomial
- Tensor branch anomaly matching
- Torus compactifications (if time permits)

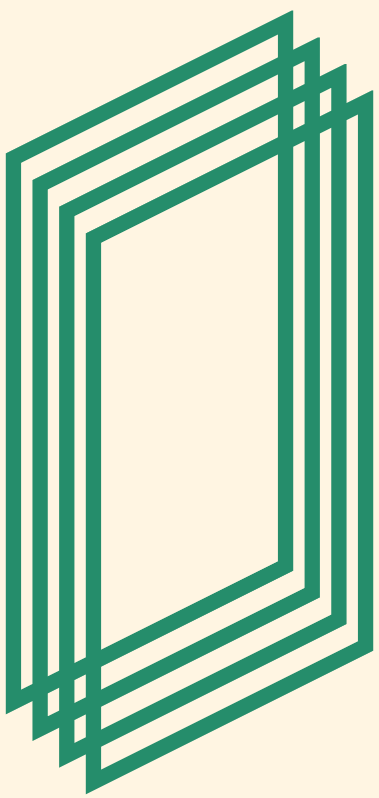
Brane constructions of 6d SCFTs

6d $\mathcal{N}$=(1,0) Supermultiplets

- 8 supercharges, $SU(2)$ R-symmetry
- Tensor multiplet: $(B^{+}_{\mu\nu}, \psi^{+}, a)$
  $a \in \mathbb{R}$: ``tensor branch" vev (preserves R-sym)
- Vector multiplet: (A_{μ}, λ^{-})
  No scalar
- Hyper multiplet: (ϕ_i, ψ^{+}) $i = 1, 2, 3, 4$
  $\phi \in \mathbb{R}^4$: ``Higgs branch" vev (breaks R-sym)

$\mathcal{N}=(2,0)$ theory of A-type

Q coincident M5-branes



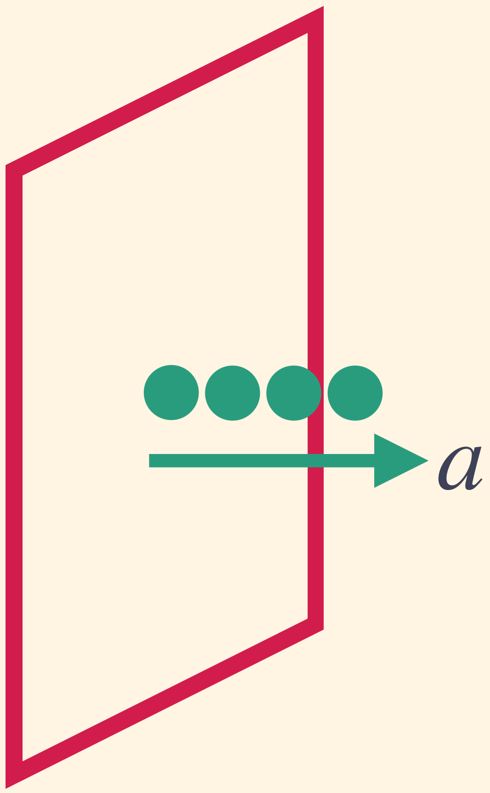
$\Rightarrow$ A_{Q-1}-type $(2,0)$ theory

\+ center of mass mode

E-string theory

Q M5-branes on

``End-of-the-world” brane (10d)



⇒ E-string theory of rank Q

with E_8 flavor symmetry

+ free hyper

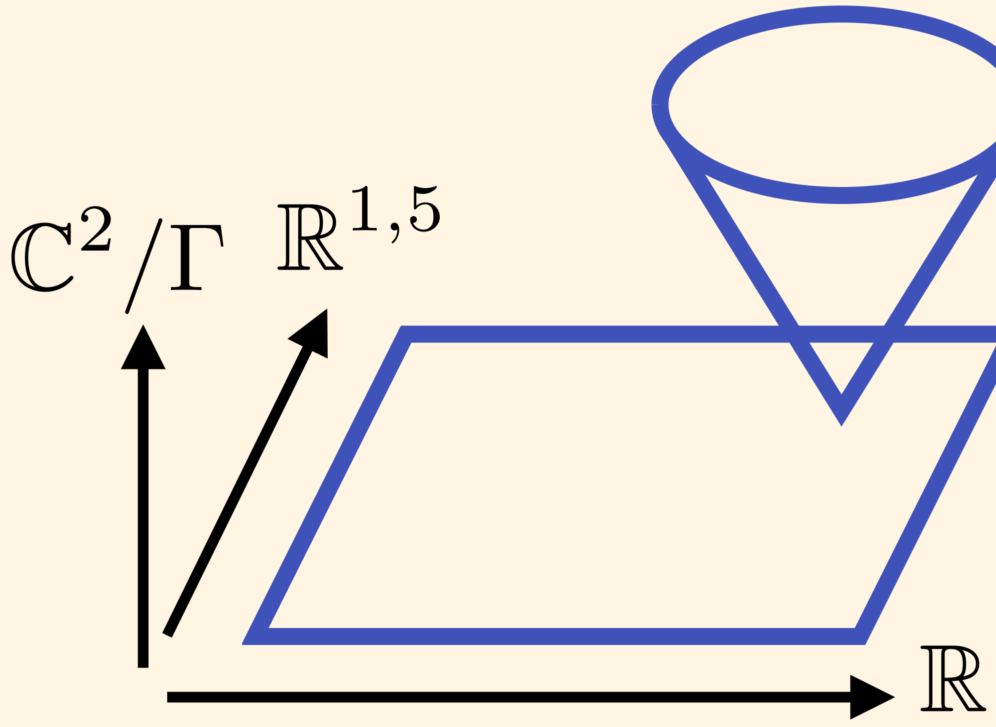
(center of mass mode)

M5-branes on $\mathbb{C}^2/\Gamma_{A,D,E}$

$\Gamma_G \subset SU(2)$:Finite Subgroup

$\Rightarrow$ 7d Vecter mult. with gauge group G

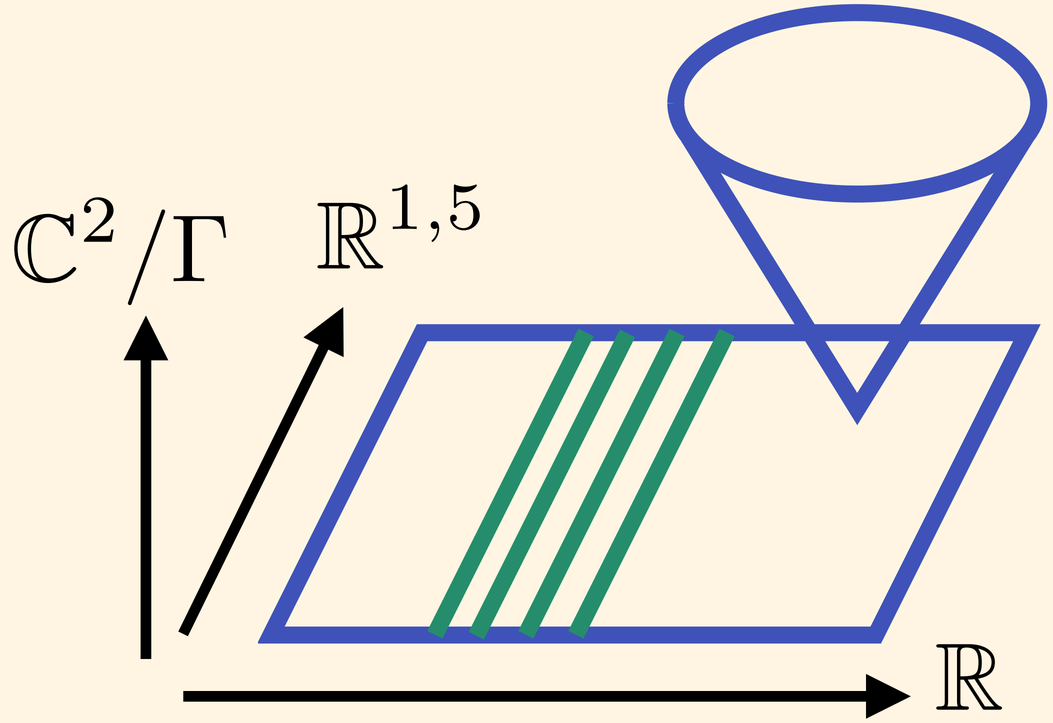
On singular locus of $\mathbb{C}^2/\Gamma \times \mathbb{R}^{1,6}$



M5-branes on $\mathbb{C}^2/\Gamma_{A,D,E}$

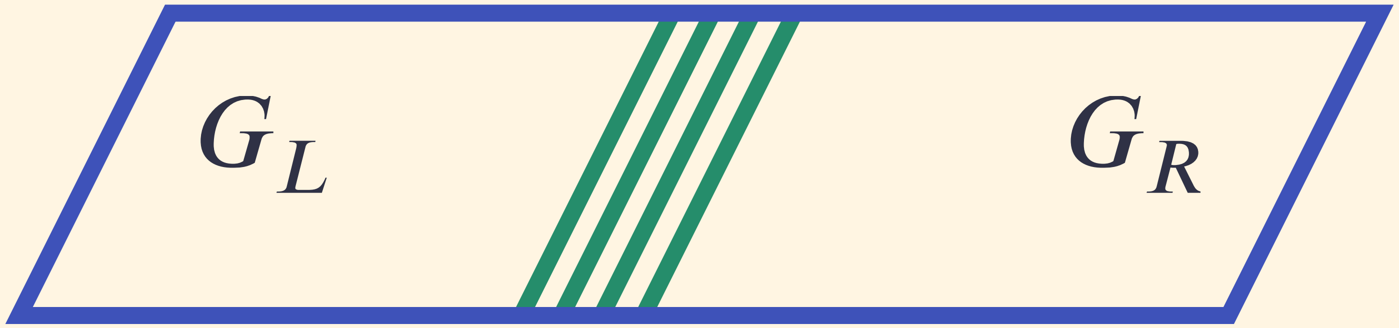
Q M5-branes on

Singular locus of $\mathbb{C}^2/\Gamma \times \mathbb{R}^{1,6}$

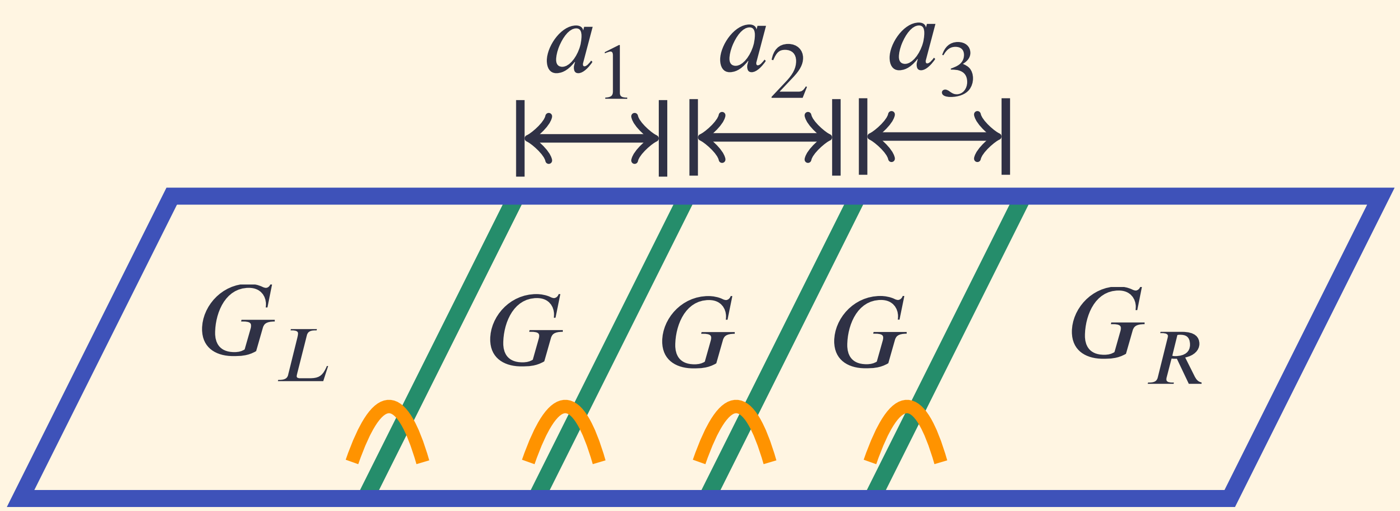


$G \times G$ flavor symmetry

Tensor branch theory
A_k-type singularity case



Tensor branch theory
A_k-type singularity case



$G = SU(k+1)$ +bifundamentals

Q-1 dynamical vector mult.s of $SU(k+1)$

+ bifundamental hypers of neighboring $SU(k+1)$'s

+*Q-1* Tensor multiplets with tensor vev.s a_i (dynamical)

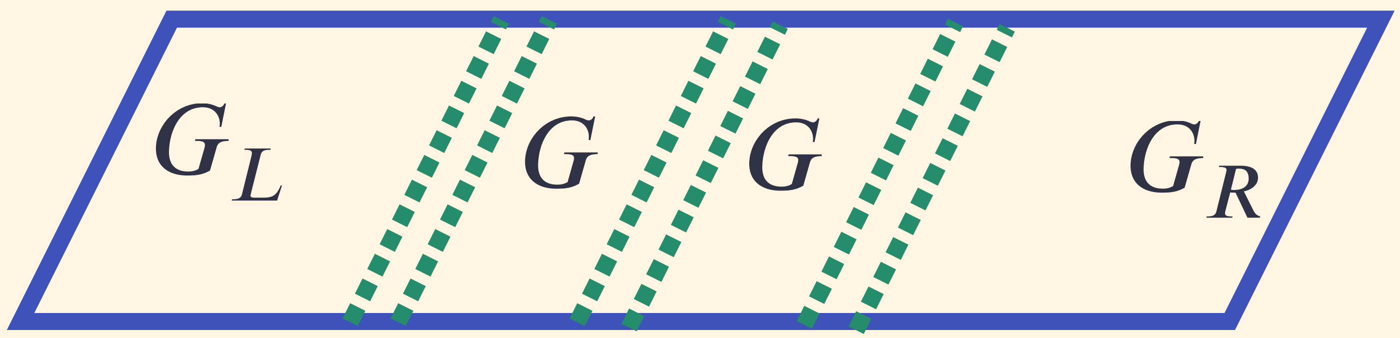
+ massive string (M2 branes)

Tensor branch theory
D_k-type singularity case

$G = SO(2k)$

Tensor branch theory
D_k-type singularity case

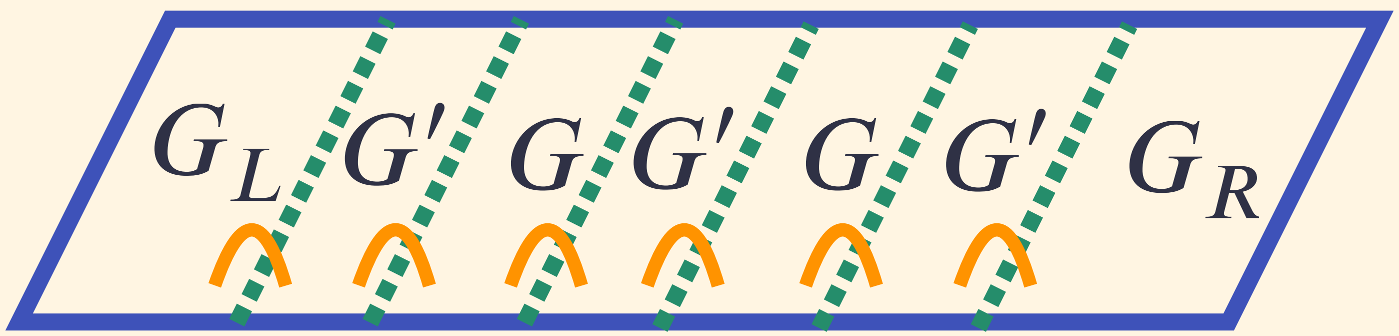
`` Fractional M5"



$G = SO(2k)$

Tensor branch theory
D_k-type singularity case

`` Fractional M5"



$$G = SO(2k)$$
$$G' = USp(2k-8)$$

+half bifundamentals

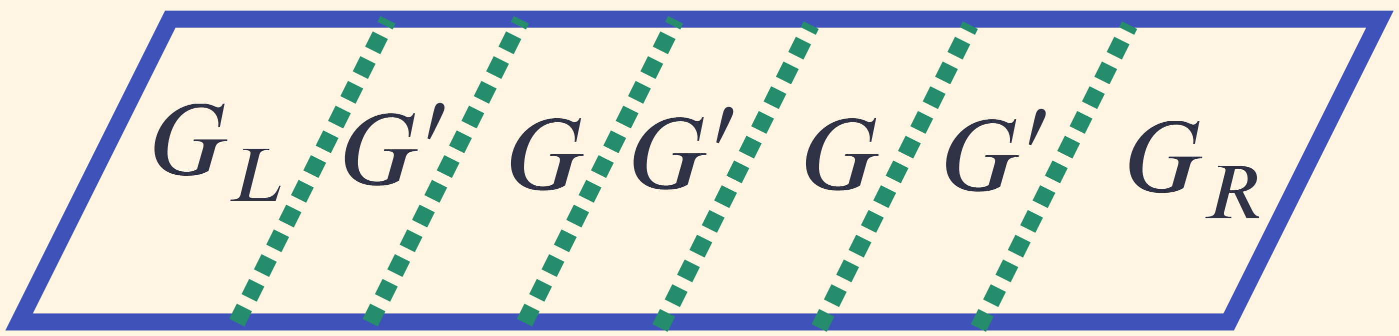
SO(2k)-Usp(2k-8) alternating quiver theory

Gauge couplings are governed by tensor vev.s (dynamical)

This system is also realized by D6-O6-NS5 system in IIA

Tensor branch theory
D_k-type singularity case

$$k = 4$$



$$G = SO(8)$$
$$G' = USp(0)$$

Tensor branch theory
D_k-type singularity case

$$k = 4$$

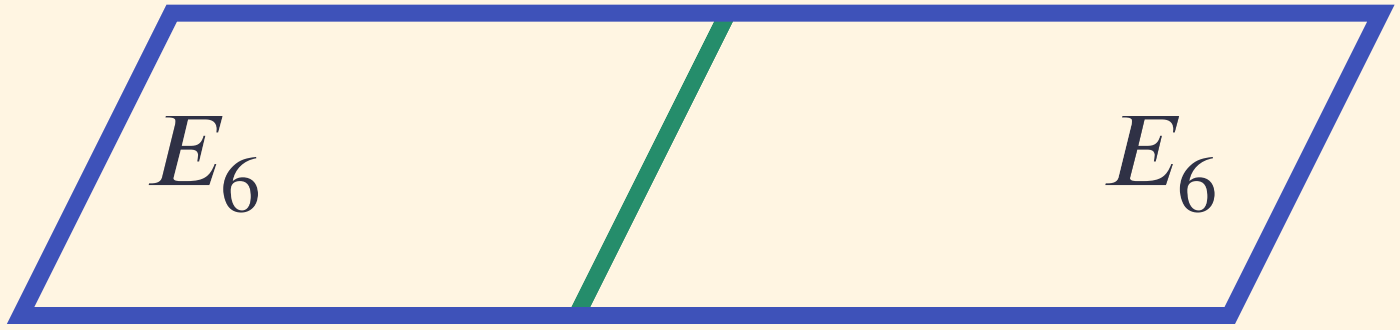
$G = SO(8)$ +rank 1 E-string theories

``$(SO(8),SO(8))$ conformal matter''= E-string

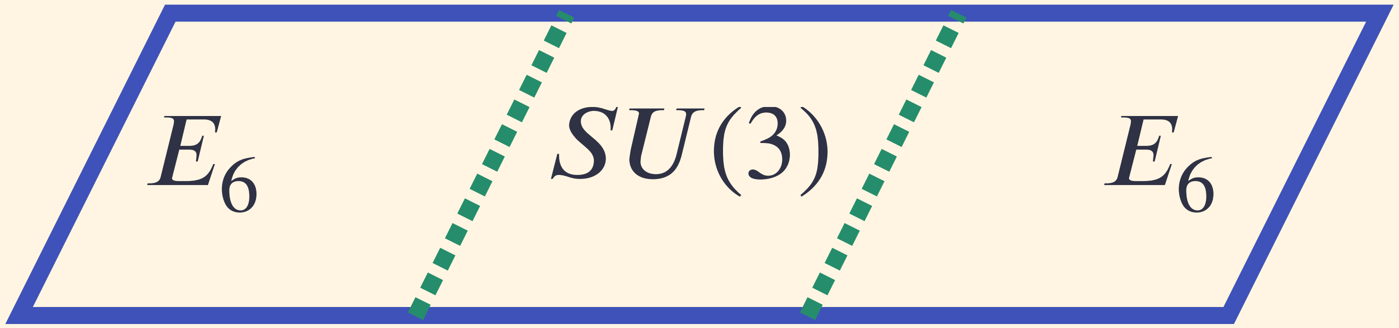
$$SO(8) \times SO(8) \subset E_8$$

[Del-Zotto,Heckman,Tomasiello,Vafa '14]

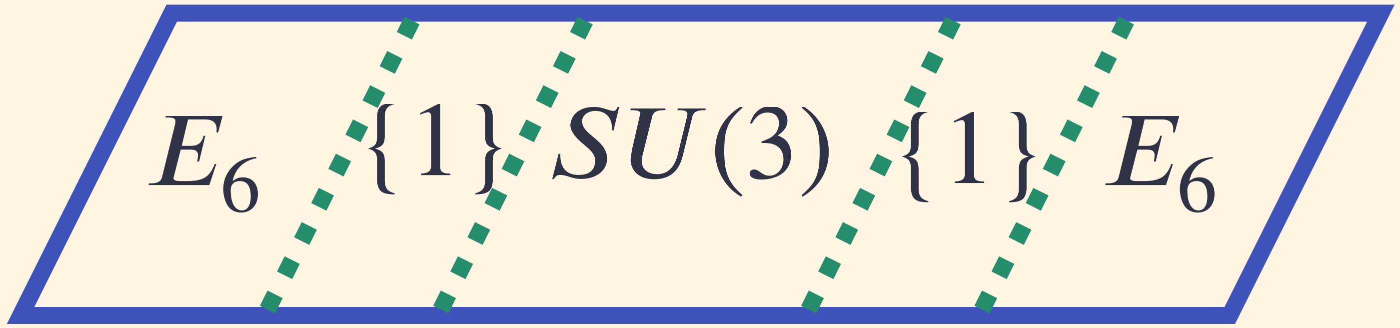
Tensor branch theory E_6-type singularity case



Tensor branch theory
E_6-type singularity case

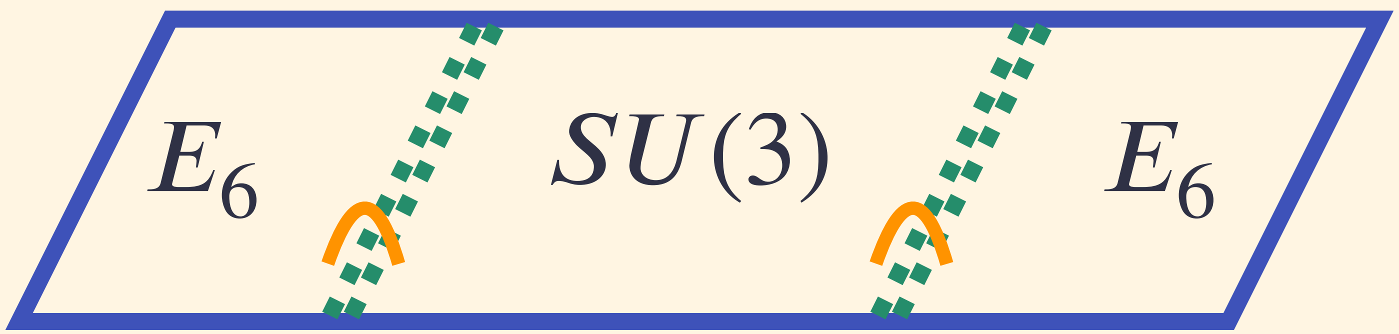


Tensor branch theory
E_6-type singularity case



Tensor branch theory
E_6-type singularity case

[Del-Zotto,Heckman,Tomasiello,Vafa '14]



rank 1 E-string theories

"$(SU(3),E_6)$ conformal matter"= E-string

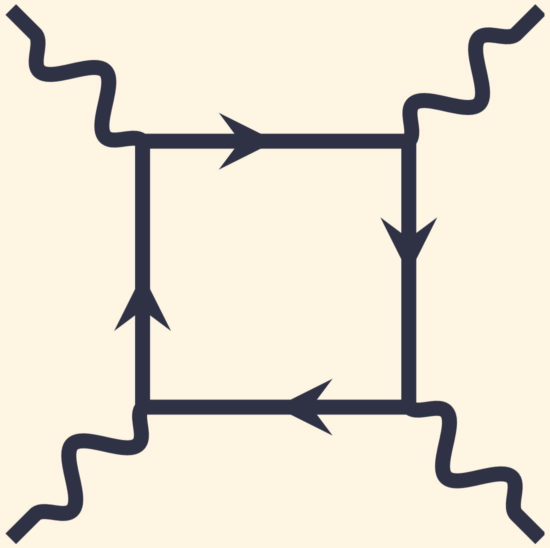
$$SU(3) \times E_6 \subset E_8$$

Other 6d SCFTs

- End-of-the-world brane + ALE singularity + (multiple) M5 branes
  [Del-Zotto,Heckman,Tomasiello,Vafa '14]
- F-theory construction
  [Heckman,Morrison,Vafa '13]
- Lagrangian constructions
  [Smilga '07],[Samtleben, E. Sezgin, and R. Wimmer '11,'12]
  [Ho, Matsuo '14] for (2,0) theories

Anomaly polynomials

Anomaly in 6d SCFTs



- Anomaly polynomial 8-from I_8

$$
\begin{aligned}
I_8 \supset & (\mathrm{Tr} F_R^2)^2, && SU(2)_R^4 \\
& (\mathrm{Tr} R^2)^2, (\mathrm{Tr} R^4), && \text{grav.}^4 \\
& (\mathrm{Tr} F_G^2)^2, && G^4 \\
& \mathrm{Tr} F_R^2 \mathrm{Tr} R^2, \mathrm{Tr} F_R^2 \mathrm{Tr} F_G^2, \cdots && \text{mixed}
\end{aligned}
$$

- Anomaly polynomial should exist even for non-perturbative SCFTs.

6d Green-Schwarz term

- $\int B \wedge X_4\,,\ X_4 \supset \mathrm{Tr}\, F_R^2, \mathrm{Tr}\, F_G^2, \mathrm{Tr}\, R^2$

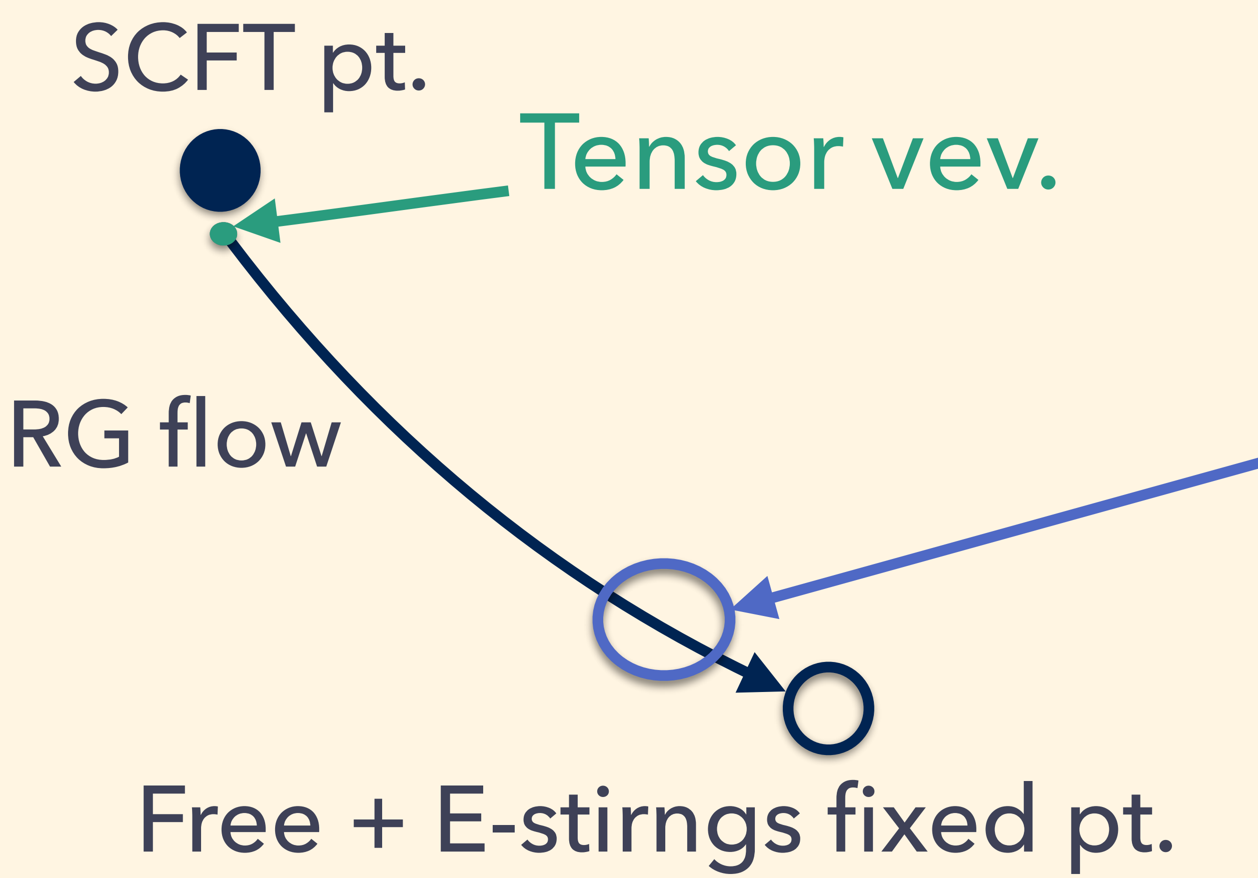
  induces modified Bianchi identity $\mathrm{d}H \propto X_4$ and additional anomaly $\delta I_8 = \frac{1}{2} X_4^2$ [Monnier '13]

- This contribution is important for calculating anomaly polynomial of 6d SCFTs.

Tensor branch anomaly matching

Basic idea

[Intrilligator '14],[KO,Shimizu,Tachikawa,Yonekura '14]



vector multiplets,
``bifundamentals'',
(= hyper or E-string)
tensor multiplets
+ corrections
from massive string

↓

GS terms $\int B \wedge X$

↓

$$I^{UV} = I^{\mathrm{Naive}} + I^{\mathrm{GS}} \longleftarrow I^{\mathrm{GS}} = \frac{1}{2} X^2$$

Tensor branch anomaly matching

- Vector, tensor, hyper mult. and E-strings generate $I^{\mathbf{Naive}} \supset (\mathrm{Tr}\, F^2_{\mathbf{gauge}})^2, \mathrm{Tr}\, F^2_{\mathbf{gauge}} \mathrm{Tr}\, R^2,$
  $\mathrm{Tr}\, F^2_{\mathbf{gauge}} \mathrm{Tr}\, F^2_R, \mathrm{Tr}\, F^2_{\mathbf{gauge}} \mathrm{Tr}\, F^2_{\mathbf{flavor}}$
- Anomaly for E-strings are calculated from anomaly inflow: [KO,Shimizu,Tachikawa '14]
- Because the UV theory are superconformal, $I^{UV} = I^{\mathbf{Naive}} + I^{\mathbf{GS}}$ should not contain $F_{\mathbf{gauge}}$

Tensor branch anomaly matching

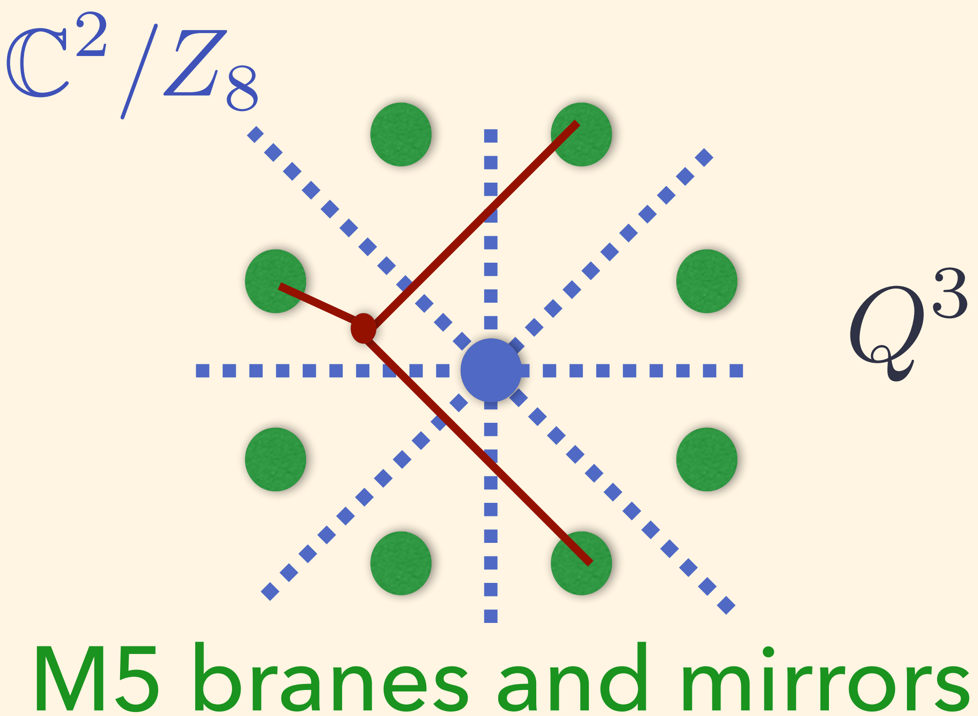
- Example: $I^{\textbf{Naive}} \supset -\alpha(\mathrm{Tr}\,F^2_{\textbf{gauge}})^2 - \beta\mathrm{Tr}\,F^2_{\textbf{gauge}}\mathrm{Tr}\,R^2$
- To cancel this by $I^{\textbf{GS}} = \frac{1}{2}X^2$
  X should contain
  $X \supset \sqrt{2\alpha}\mathrm{Tr}\,F_{\textbf{gauge}} + \frac{\beta}{\sqrt{2\alpha}}\mathrm{Tr}\,R^2$
- We can fix all of the GS terms $\int B_i \wedge X^i$
  in this manner. (If # of X^i = # of gauge fields)
- $I^{UV} = I^{\textbf{Naive}} + I^{\textbf{GS}}$: Do square completion!

Tensor branch anomaly matching

- Result for Q M5's on Singularities:

$$I^{\mathrm{UV}} \supset \frac{Q^3|\Gamma|^2}{96}(\mathrm{Tr}F_R^2)^2$$

- The leading behavior can be understood from Higgs branch and M2 junction:



Q^3 behavior ←M2 junction.

$Q^3|\Gamma|^3$ ways of suspending junction divided by Γ

Torus compactifications

(on going with the same collaborators)

Torus compactifications

- What is the 4d theory of M5 branes on torus-compactified singularities?

	$\mathbb{R}^{1,3}$	T^2	$\mathbb{R}$	$\mathbb{C}^2/\Gamma$
M5	$\circ$	$\circ$		

- Is that SCFT?

- Can we find candidates in Class S theories?

- How to check?

Known examples and Class S

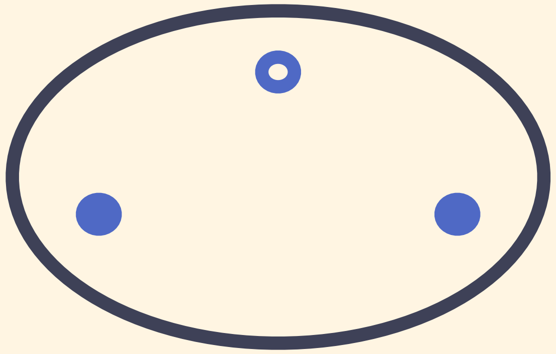
- Single M5 on $\Gamma = A_k$ singularity
  $\Rightarrow$ bifundamental both in 6d and 4d.

- Single M5 on $\Gamma = D_4$ singularity
  $\Rightarrow$ Rank 1 E-sting theory in 6d

  $\Rightarrow$ Minahan-Nemeshansky E_8 theory in 4d

Known examples and Class S

- Single M5 on $\Gamma = A_k$ singularity
  $\Rightarrow$ bifundamental both in 4d.

  = Class S of type A_k def'ed by 

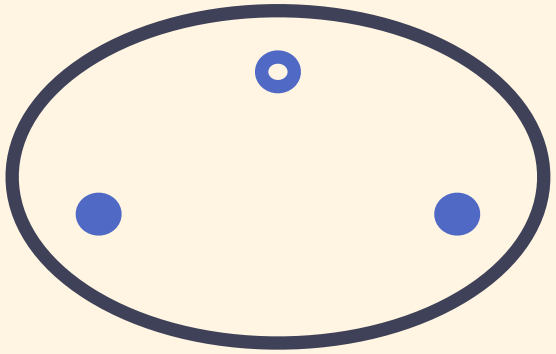
● : ``full" puncture (full symmetry)

○ : ``simple" puncture ($U(1)$ or no sym.)

Known examples and Class S

- Single M5 on $\Gamma = D_4$ singularity
  ⇒ Rank 1 E-sting theory in 6d

  ⇒ Minahan-Nemeshansky E_8 theory in 4d

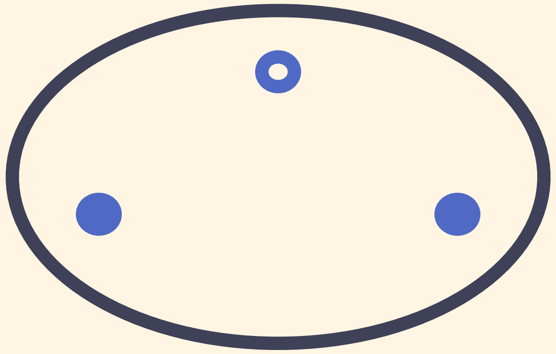
  = Class S of type D_4 def'ed by 

● : ``full" puncture (full symmetry)

○ : ``simple" puncture ($U(1)$ or no sym.)

Guess for Q=1

Single M5 on $\mathfrak{g}$-sing.$\times T^2$ without center of mass
(``(G,G) conformal matter”)

$\stackrel{?}{=}$ Class S of type $\mathfrak{g}$ def’ed by 

- ● : ``full” puncture (full symmetry)
- ○ : ``simple” puncture ($U(1)$ or no sym.)

`` 4d conformal matter”

Checks

- Dim.s of Higgs/Coulomb branches matches
- The geometry of Higgs branch = $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$
- With plausible assumption, we can calculate the conformal anomalies and flavor levels for one M5 on $\mathfrak{g}$-sing. $\times T^2$
  $\Rightarrow$ matches with the class S conjecture.
- M/IIB duality chain? Not so clear.

Conclusion

- The anomaly polynomials of 6d SCFTs can be calculated by adding tensor branch contributions and GS contributions.

- For ``6d (G,G) conformal matter”, the candidate for the torus compactified theories can be found in the Class S theories.

Further directions

- Comactifications of 6d (1,0) SCFTs
  - 4d anomaly polynomials for T^2 compactified theories?
  - What and how the 4d theories are?
- 6d a-theorems?

Appendix

Tensor branch anomaly matching wo/ vector

- N=(2,0) and E-string theories :
  there is no vector in tensor branch theory

- We know 5d theories obtained by circle compactifications for those theories.

- In 5d coulomb branch corresponds to 6d tensor branch, we have U(1) vector A_i
  come from 6d self-dual tensor B_i
  and massive mode because of coulomb vev.

Tensor branch anomaly matching wo/ vector

- In 5d coulomb branch corresponds to 6d tensor branch, we have U(1) vector A_i come from 6d self-dual tensor B_i and massive mode because of coulomb vev.

- Those missive modes generates 5d CS terms $\int A_i \wedge X^i$, which is calculable.
- This should come from 6d GS term $\int B_i \wedge X^i$