Multiverse and Maximum Entropy Principle

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based on

- Y.Hamada, H. Kawai and K.Kawana, IJMP. A 29, arXiv:1405.1310
- Y.Hamada, H. Kawai and K.Kawana, arXiv:1409.6508

Introduction

Fundamental theory of the Particle Physics

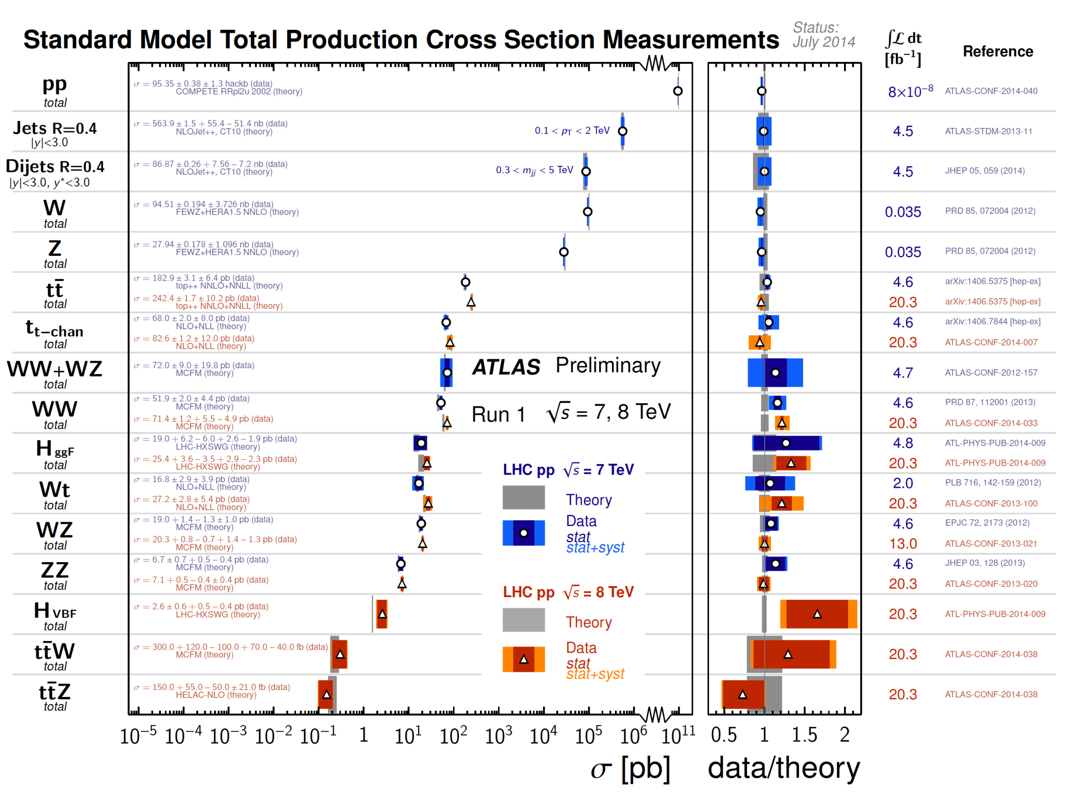
= The Standard Model (SM).

It explains the experimental results very nicely.

Recent ATLAS

Results →

- Gray bands =Theory
- Color bands=data (1 σ)



- However, there are many problems which are difficult to answer within the SM:

i) Why the parameters of the SM are fixed at the observed values ? (theme of today's talk)

ii) Dark Matter (DM)

$$\Omega_{DM}h^2=0.119\pm 0.0031$$ (68%CL, Planck2013)

iii) Dark Energy

$$\Omega_{DE}=0.686\pm 0.020$$ (68%CL, Planck2013)

iv) Baryon Asymmetry

- In this talk, we focus on the first problem:

  Why the parameters of the SM are fixed

  at the observed values ?

- Especially, why the weak scale is O(100)GeV ?

To solve this problem, we want to propose the following idea:

Parameters of the SM are fixed in such a way that

the radiation of the universe S at the late stage becomes

maximum !

⇨ Maximum Entropy Principle (MEP) !

Here, S is defined as

$$S := \rho_{\rm rad} \times a^4$$

- The main part of today’s talk is to show how the MEP can be derived from the quantum theory of Multiverse.
- After that, I show one example :

Higgs Expectation value v_h.

Flow of Story

1) We review the quantum mechanics of the Friedman Universe.

2) Assuming the existence of many universes, we define the wave function of Multiverse and the probability distribution P(λ) of the parameters of universes.

3) We show that P(λ) has a strong peak where the Cosmological Constant (CC) becomes very small, which is given by

$$\Lambda \sim M_{pl}^2/S.$$

This is the Maximum Entropy Principle !

4) Finally, we give an example of the MEP : the Higgs expectation value v_h. We show that S actually becomes maximum around the observed value

$$v_{hob}=246\text{GeV}.$$

1. Path Integral of Friedman Universe

H.Kawai, T.Okada (2011)
H.Kawai, Y.Hamada and
K.Kawana (2013)

- Before discussing Multiverse, we consider the quantum mechanics of a single universe.
- Assumptions in the following discussion:

① We assume the isotropic and homogeneous universe with the S^3 topology:

$$d^2s = -N(t)d^2t + a^2(t)\left(d\mathbf{x}^2 + \frac{(\mathbf{x}\cdot d\mathbf{x})^2}{1-\mathbf{x}^2}\right)$$

② Matter and radiation are effectively included as the energy density. Namely, the Hamiltonian is

$$\hat{\mathcal{H}}(\lambda) = \frac{\hat{p}_a^2}{2} - \frac{a^2\rho(a)}{6M_{pl}^2}$$

where

Potential of a universe

$$\hat{p} = \dot{a}$$

★$\mathcal{H}$=0 is nothing but the Friedman equation !

- Based these assumptions, the path integral of an universe is given by

$$Z^{(\lambda)}_{universe}(a_f, a_i) = \int \mathcal{D}p_a \int_{t=0,a(0)=a_i}^{t=1,a(1)=a_f} \mathcal{D}a\mathcal{D}N \exp\{i \int_0^1 dt(p_a\dot{a} - N\mathcal{H}(\lambda))\}$$

- λ represents the parameters of a universe.

  e.g. the Cosmological Constant (CC) Λ

- In the following discussion, we regard these parameters as variables.

$$Z^{(\lambda)}_{universe}(a_f, a_i) = \int \mathcal{D}p_a \int_{t=0,a(0)=a_i}^{t=1,a(1)=a_f} \mathcal{D}a\mathcal{D}N \exp\{i \int_0^1 dt(p_a\dot{a} - N\mathcal{H}(\lambda))\}$$

- As usual, we can make the gauge fixing of N(t).

  But, it is not necessary in the following discussion.

- If an initial state $|\phi_{\text{universe}}>$ is given, the wave function of a single universe is given by

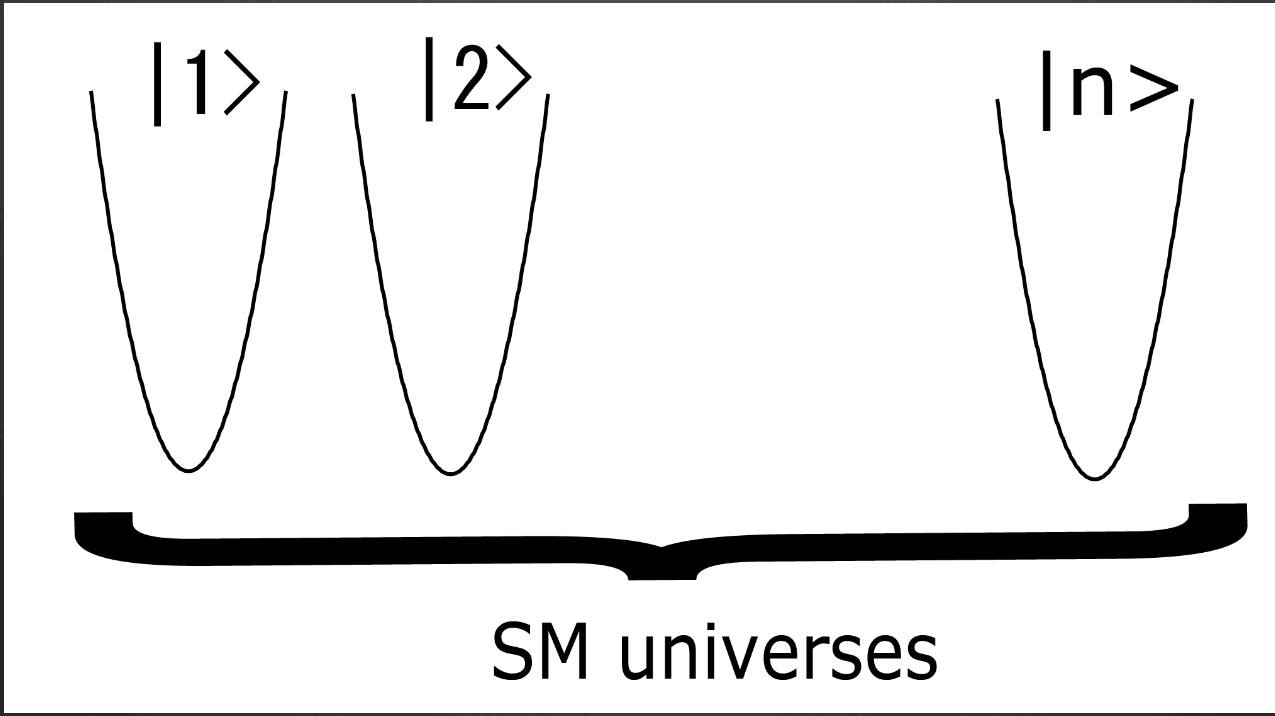
$$\phi_{\text{universe}}(a, \{\lambda_i\}) := \int da' \; Z^{(\lambda)}_{\text{universe}}(a, a')\langle a'|\phi_{\text{universe}}\rangle$$

2. Wave Function of Multiverse and Probability Distribution

- We assume that there are many universes.
- In principle, particle contents and effective theories can be different each other.

→ For simplicity, we consider the situation such that all universes follow the SM.

- Even though all universes follow the SM, their quantum states can be different each other.



Image

- In the following discussion, we consider the simplest situation:

$$|\Psi_n, \{\lambda_i\}\rangle = \frac{\mu_{+1}^n}{\sqrt{n!}} |\phi_{\text{universe}}\rangle \otimes \cdots \otimes |\phi_{\text{universe}}\rangle,$$

$$\leftrightarrow \Psi_n\left(a_1, \cdots a_n, \{\lambda_i\}\right) = \frac{\mu_{+1}^n}{\sqrt{n!}} \prod_{k=1}^{n} \phi_{\text{universe}}(a_k, \{\lambda_i\})$$

where

$$\phi_{\text{universe}}(a, \{\lambda_i\}) := \int da' \; Z_{\text{universe}}^{(\lambda)}(a, a')\langle a' | \phi_{\text{universe}}\rangle$$

- μ_{+1} is the probability amplitude of a universe emerging from nothing.

- Because

$$|\Psi_n(a_1, \cdots, a_n, \{\lambda_i\})|^2$$

is the probability density, we can obtain the probability distribution of $\{\lambda_i\}$ by tracing out the number of universes and $\{a_i\}$:

$$P(\{\lambda_i\}) = \sum_{n=0}^{\infty} \int \cdots \int \prod_{k=1}^{n} da_k |\Psi_n(a_1, a_2, \cdots, a_n, \{\lambda_i\})|^2$$

$$= \sum_{n=0}^{\infty} \frac{|\mu_{+1}|^{2n}}{n!} \cdot \prod_{k=1}^{n} \left(\int da_k |\phi_{\text{universe}}(a_k, \{\lambda_i\})|^2 \right)$$

$$= \exp\left(|\mu_{+1}|^2 \cdot \int da |\phi_{\text{universe}}(a, \{\lambda_i\})|^2 \right).$$

- The problem is where $P(\{\lambda_i\})$ has its peak.

→ We can actually check this by the WKB approximation.

Let's understand this intuitively !

$$P(\{\lambda_i\}) = \exp\left(|\mu_{+1}|^2 \cdot \int da |\phi_{\text{universe}}(a, \{\lambda_i\})|^2 \right)$$

We focus on this factor.

The WKB solution is

$$\phi_{\text{WKB}}(a) = \frac{c}{\sqrt{p(a)}} \exp\left(i \cdots\right)$$

where p(a) is the classical momentum.

$$\int da |\phi_{\text{universe}}(a)|^2 \sim \int da \frac{1}{p(a)} \sim \int dt \sim T_{\text{universe}}$$

T_{universe} = Life time of the universe !

- Namely, $P(\{\lambda_i\})$ has its maximum at the point where T_{universe} becomes maximum !

→ We can obtain the solution to the Cosmological Constant Problem (CCP) and the MEP from this result!

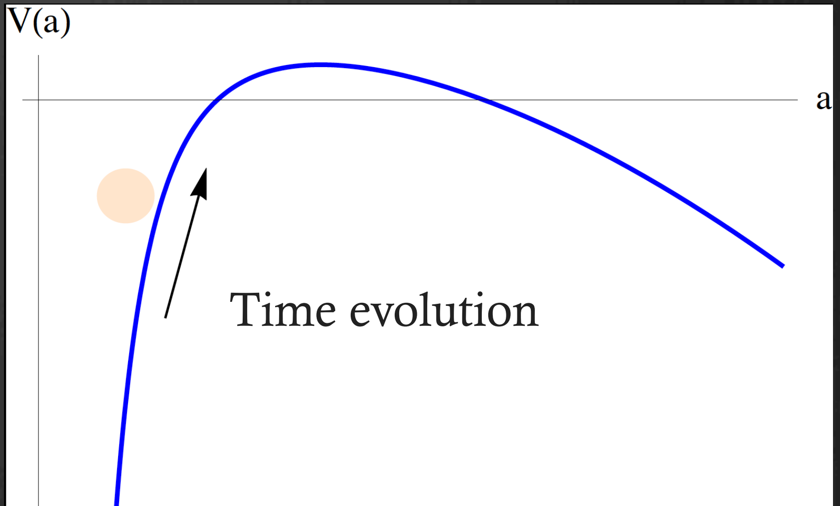
3.Solution to the CCP and Derivation of MEP

Classically, the universe develops following the energy conservation low (Friedman Equation):

$$\hat{\mathcal{H}}(\lambda) = \frac{\hat{p}_a^2}{2} - \frac{a^2\rho(a)}{6M_{pl}^2}$$
$$= \frac{\hat{p}_a^2}{2} - \frac{a^2}{6M_{pl}^2}\left(\frac{M}{a^3} + \frac{S}{a^4} - \frac{M_{pl}^2}{a^2} + M_{pl}^2\Lambda\right) = 0$$

Potential of the universe V(a) !

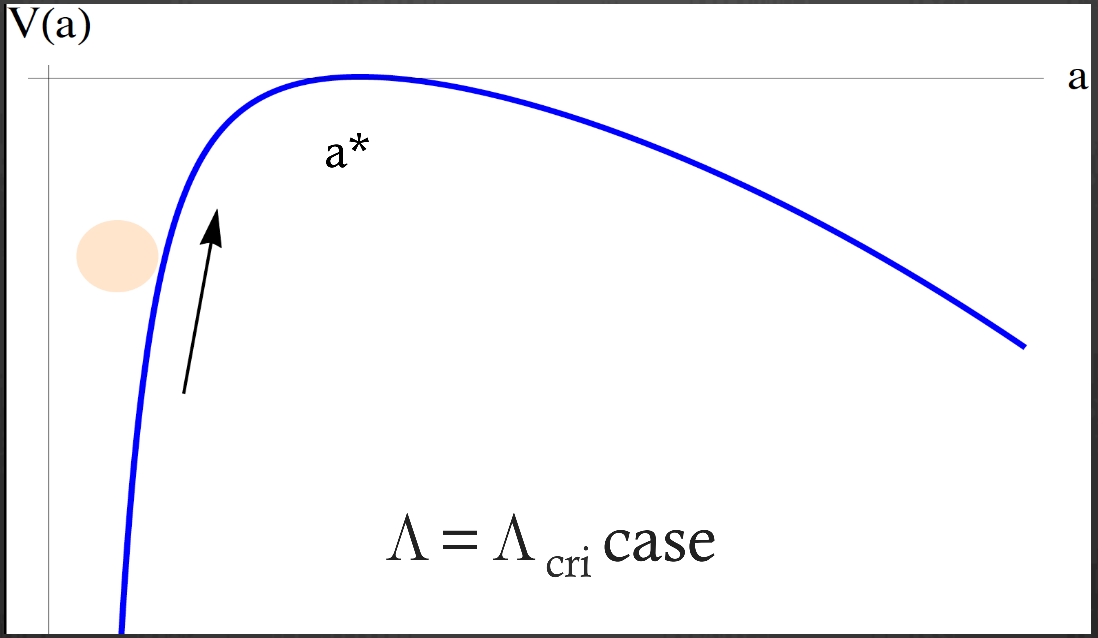
Its typical shape →



If the CC is close to the critical value Λ_{cri} such that

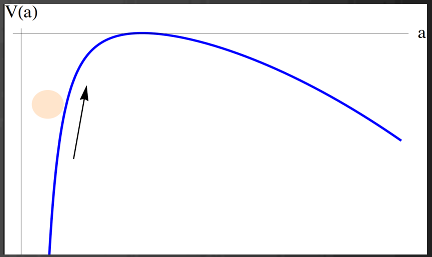
the maximum of V(a) becomes zero,

the universe spends a very long time around a=a* !



$\rightarrow$ What is the value of Λ_{cri} ?

Rough Estimation of Λ_{cri}



Around a=a*, each term of V(a) balances.

For simplicity, we consider the case such that radiation is dominated than matter around a*. In this case,

$$\frac{S}{a^{*4}} \sim \frac{M_{pl}^2}{a^{*2}} \sim M_{pl}^2 \Lambda_{\rm cri}$$

$$\leftrightarrow a^* \sim \frac{S^{1/2}}{M_{pl}} \quad , \quad \Lambda_{\rm cri} \sim \frac{M_{pl}^2}{S}$$

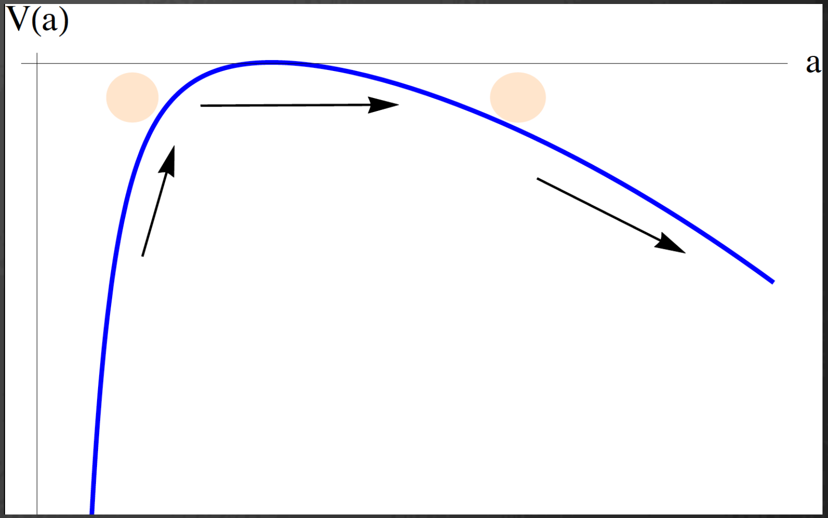
Very small !

If we assume $a(t_{now})=10^{10}$ light year and the photon density $n_\gamma \sim 400/\mathrm{m}^3$ at present, this becomes $\Lambda_{cri} \sim 10^{-51}\mathrm{GeV}^2$.

MEP

The discussion so far is yet classical. By the quantum tunneling, the universe can exist at a>a*.

Then, the universe continues to expand.



This expansion takes a lot of time when Λ_{cri} is small !

Conclusion

$P(\{\lambda_i\})$ has its maximum at the point where

$\Lambda = \Lambda_{cri} \sim M_{pl}^2/S$, and S becomes

as large as possible! (MEP)

5. Example (Higgs Expectation value v_h)

Predictions of MEP depend on what kind of particle

becomes dominated when a~a*.

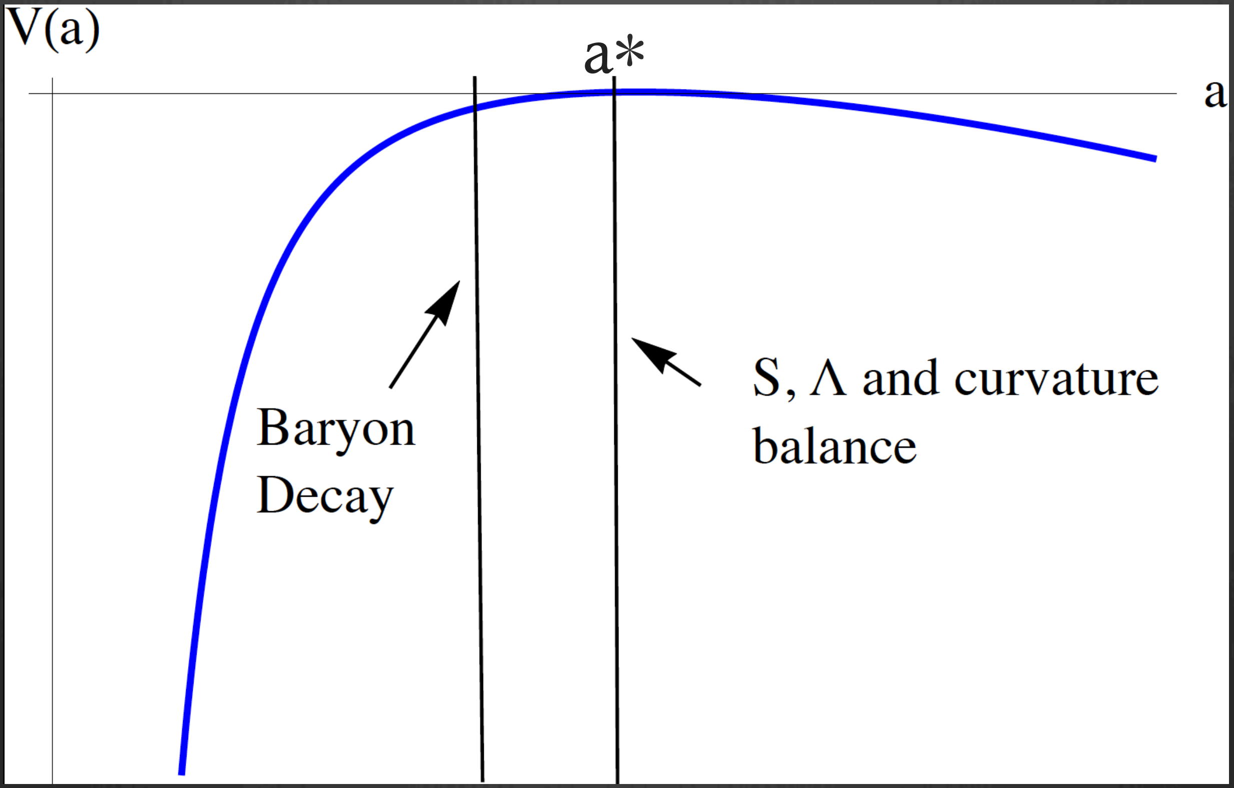
→ We must choose a scenario of the universe.

⭐Our scenario⭐

i) The Dark Matter (DM) decays much earlier than baryons. This guarantees that the radiation produced by the DM is negligible.

ii) If the Dark Energy (DE) is the CC, it closes $\Lambda_{cri.}$ If not, the DE becomes negligible before the decay of baryons. Λ is fixed to Λ_{cri}.

iii) Baryons decay, and S is produced. Finally, the CC and curvature balance with S.



Potential of our scenario

Qualitative Understanding of S

First, we consider the situation such that baryons N_B are all protons, and decay simultaneously at $t = \tau_p$.

From the energy conservation low,

$$N_B m_p = a(\tau_p)^3 \rho_{rad} = \frac{S}{a(\tau_p)} \rightarrow S = a(\tau_p) N_B m_p$$

We can eliminate $a(\tau_p)$ by the Friedman equation at τ_p:

$$\frac{1}{\tau_p^2} \sim \frac{1}{M_{pl}^2} \frac{m_p N_B}{a(\tau_p)^3} \leftrightarrow a(\tau_p) \sim \left(\frac{\tau_p^2 m_p N_B}{M_{pl}^2} \right)^{\frac{1}{3}}$$

$$S \sim \left(\frac{\tau_p m_p^2 N_B^2}{M_{pl}} \right)^{\frac{2}{3}}$$

This is the qualitative expression when there is no atomic nucleus.

⭐ Effects of Atomic (Helium) Nuclei

i) A helium nucleus has the binding energy

$$\Delta = -28\text{MeV}.$$

→ This decreases S !

ii) However, because of Δ, each nucleon in ^{4}He has

a longer life time than τ_p.

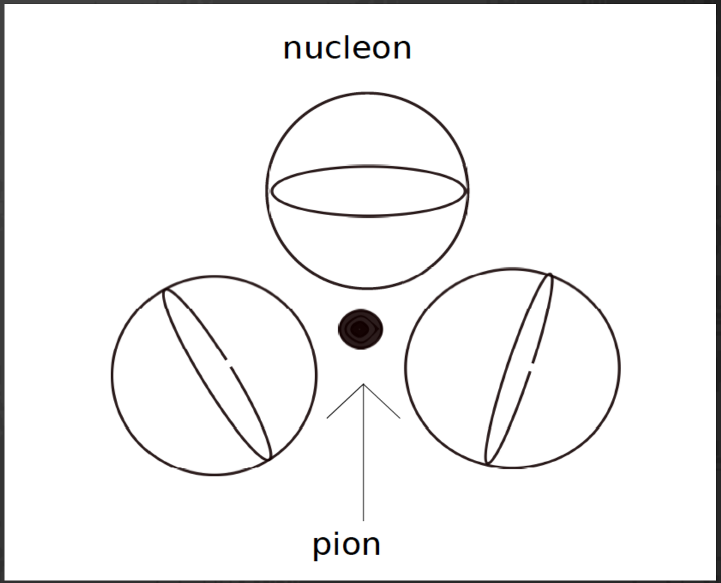
→ This increases S !

iii) A pion produced by the nucleon decay in ^{4}He can

be scattered by the remaining nucleons, and

lose its energy.

$$\epsilon := \frac{E_{\mathrm{after}}}{E_{\mathrm{before}}}$$



→ This decreases S !

- In principle, how these effects change S can be calculated by solving the Friedman equation and evolution equations.

$$\frac{dN_p(t)}{dt} = -\tau_p^{-1} \cdot N_p(t) + 3\tau_{He}^{-1} \cdot N_{He}(t),$$

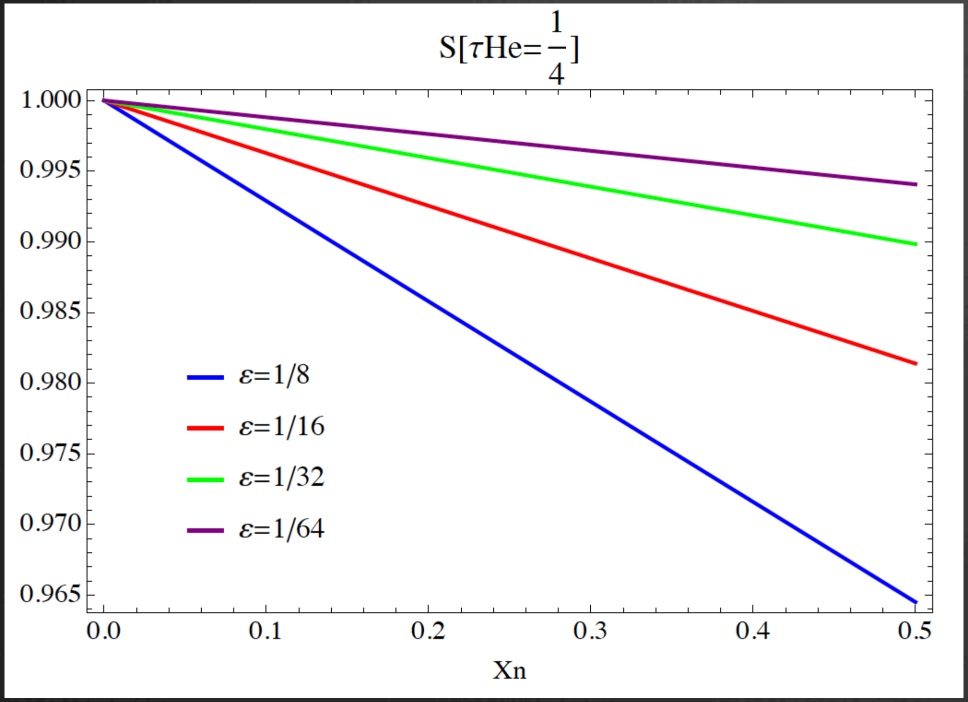
$$\frac{dN_{He}(t)}{dt} = -\tau_{He}^{-1} \cdot N_{He}(t),$$

$$H^2(t) := \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{pl}^2} \cdot \left(\frac{M(t)}{a^3} + \frac{S_{rad}(t)}{a^4} - \frac{M_{pl}^2}{a^2} + M_{pl}^2 \Lambda\right),$$

• Numerically, we found the following result:

Previous qualitative result

$$S = const \times \left(N_B^2 m_p^2 \tau_p\right)^{\frac{2}{3}} \times \left(1 - c\left(\epsilon, \frac{\tau_{\rm He}}{\tau_p}, \frac{m_{\rm He}}{m_p}\right) X_n\right)$$



Effect from atomic nuclei. Here, X_n is the ratio of neutrons to all nucleons.

$$S = const \times \left(N_B^2 m_p^2 \tau_p\right)^{\frac{2}{3}} \times \left(1 - c\left(\epsilon, \frac{\tau_{\mathrm{He}}}{\tau_p}, \frac{m_{\mathrm{He}}}{m_p}\right) X_n\right)$$

What we have to do is to calculate the parameter dependences of this !

→ We focus on v_h ! Namely, we regard S as a function of v_h only. All the other parameters are fixed at the observed value.

→ But, there are a few possibilities how we fix them.

Fixing the Current Quark Masses

$$m_i = \frac{y_i v_h}{\sqrt{2}} = \text{fixed}$$

In this case, quantities like $m_p(\tau_p)$, $m_{He}(\tau_{He})$ and c are all fixed. As a result,

$$S = const \times \left(N_B^2 m_p^2 \tau_p\right)^{\frac{2}{3}} \times \left(1 - c\left(\epsilon, \frac{\tau_{He}}{\tau_p}, \frac{m_{He}}{m_p}\right) X_n\right)$$

Only N_B and X_n depend on v_h.

$$S = const \times N_B^{\frac{4}{3}}(v_h)\left(1 - cX_n(v_h)\right)$$

- Because the detail calculations are not important, we understand how X_n and N_B depend on v_h intuitively.

1) X_n

- At a high temperature, protons and neutrons are in thermal equilibrium through the weak interaction. X_n at that time is given by

$$X_n = \frac{1}{1+\exp(\frac{Q}{T})}$$

$Q := m_n - m_p$
$= 1.29\,\mathrm{MeV}$

- However, if H becomes comparable with the reaction rate, the weak interaction is frozen out. We denote this temperature as $T_{dec.}$

- Below T_{dec}, neutrons decreases through the beta decay until the Big Bang Nucleosynthesis.

$$n \to p + e^- + \bar{\nu}$$

Life time: τ_n

- As a result, X_n is fixed at

$$X_n = \frac{e^{-\tau_n^{-1}(t_{\rm BBN} - t_{dec})}}{1 + e^{Q/T_{dec}}}$$

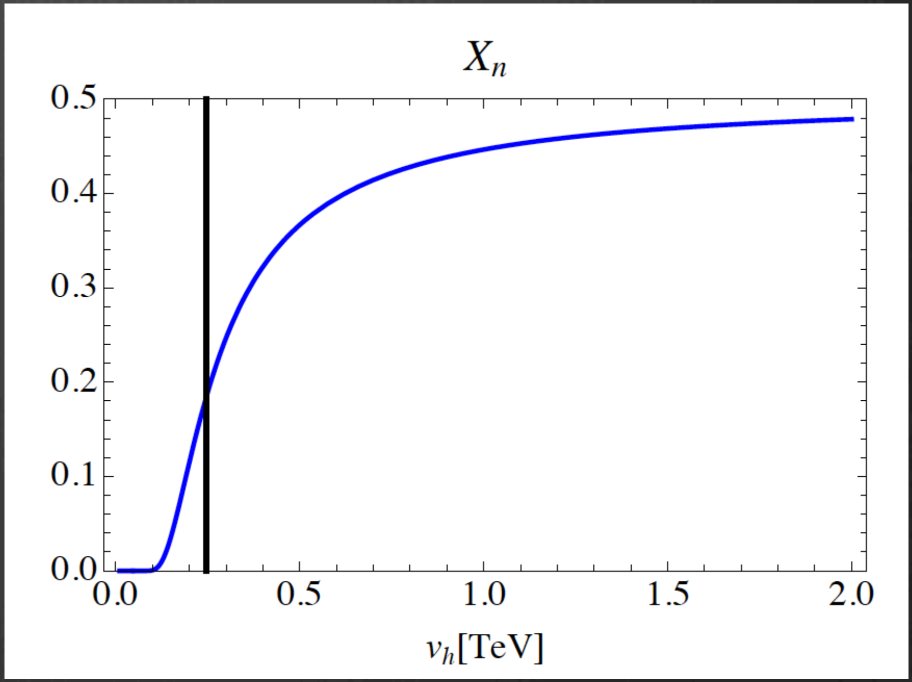
← Beta decay

← Equilibrium formula

When v_h ↗

⇕

T_{dec} ↗, τ_n^{-1} ↘



2) N_B

- If $N_{(B)-L}$ is given, we can produce N_B from the Sphaleron Process.

$$N_{-L} \longrightarrow N_B$$

Sphaleron Process

- We assume that the initial $N_{(B)-L}$ does not depend on v_h.
- We denote the transition rate as Γ_{sph}, and this is

$$\Gamma_{\mathbf{sph}} = \alpha_W^4 T e^{-\frac{E_{\mathbf{sph}}}{T}} \quad , \quad \alpha_W = \frac{g_2^2}{4\pi}$$

$$v_h \sim \frac{T_{BBN}^2}{M_{pl} y_e^5},$$

- When $H < \Gamma_{sph}$, quarks and leptons are in thermal equilibrium, and N_B is determined by thermodynamics.

$$N_B = N_{-\mathrm{L}} \times f\left(\frac{m_i}{T}\right) \quad , \quad f\left(\frac{m_i}{T}\right) = \frac{n_B(m_i/T)}{n_{-L}(m_i/T)}$$

- When $H \sim \Gamma_{sph}$, the SP decouples. N_B is fixed at

$$N_B = N_{-\mathrm{L}} \times f\left(\frac{m_i}{T_{\mathrm{sph}}}\right)$$

where T_{sph} is the decoupling temperature.

$$N_B = N_{-\mathrm{L}} \times f\left(\frac{m_i}{T_{\mathrm{sph}}}\right)$$

Fixed !

- The order of T_{sph} is T_c which is the critical temperature of the phase transition. By solving $H = \Gamma_{sph}$, we obtain

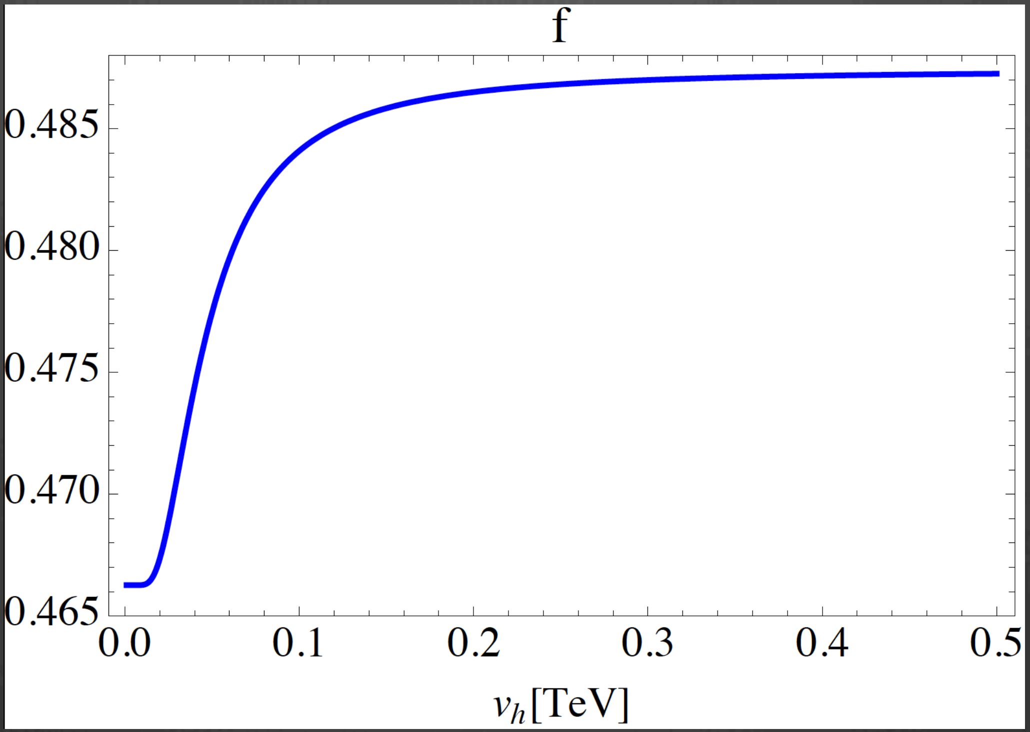
M. D'Onofrio et all ,PoS LATTICE 2012,[arXiv:1212.3206].

$$T_{\mathrm{sph}} = \frac{140}{246} \times v_h$$

- f (x) is a decreasing function because the heavy particle suffers a Boltzmann suppression factor

$$e^{-\frac{m}{T_{sph}}}$$

Result



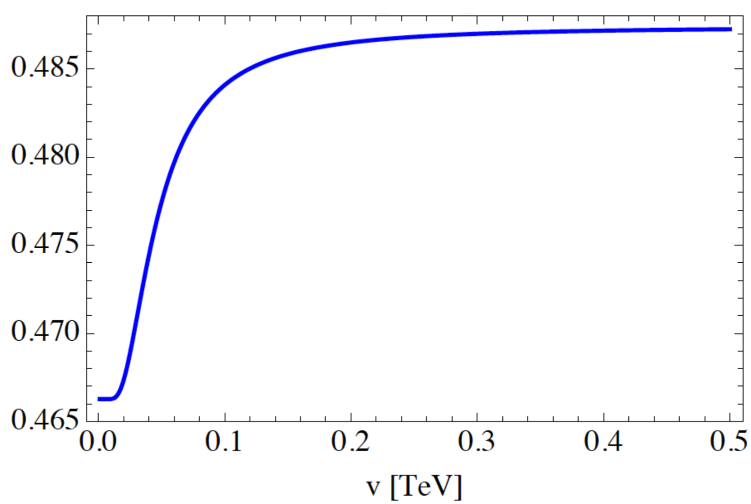
- The decrease around O(170~180) GeV is due to the top mass.

→ We can finally draw a picture of S !

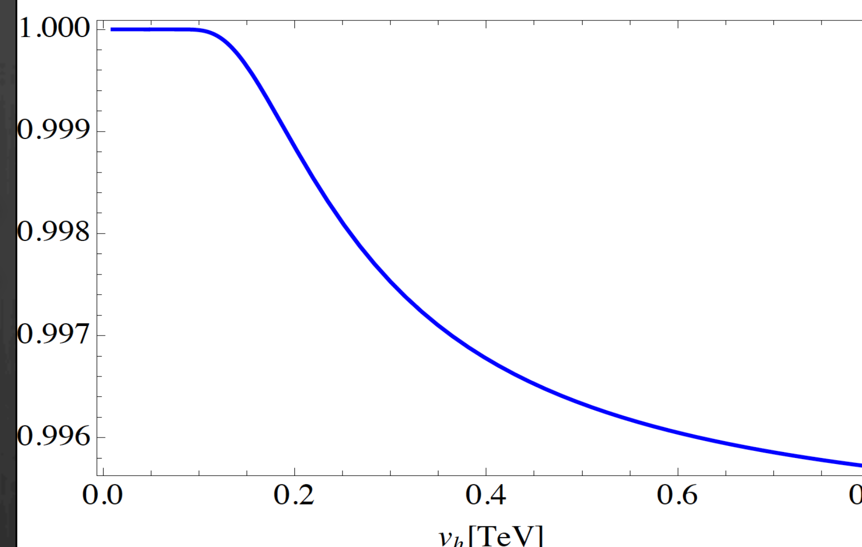
S is

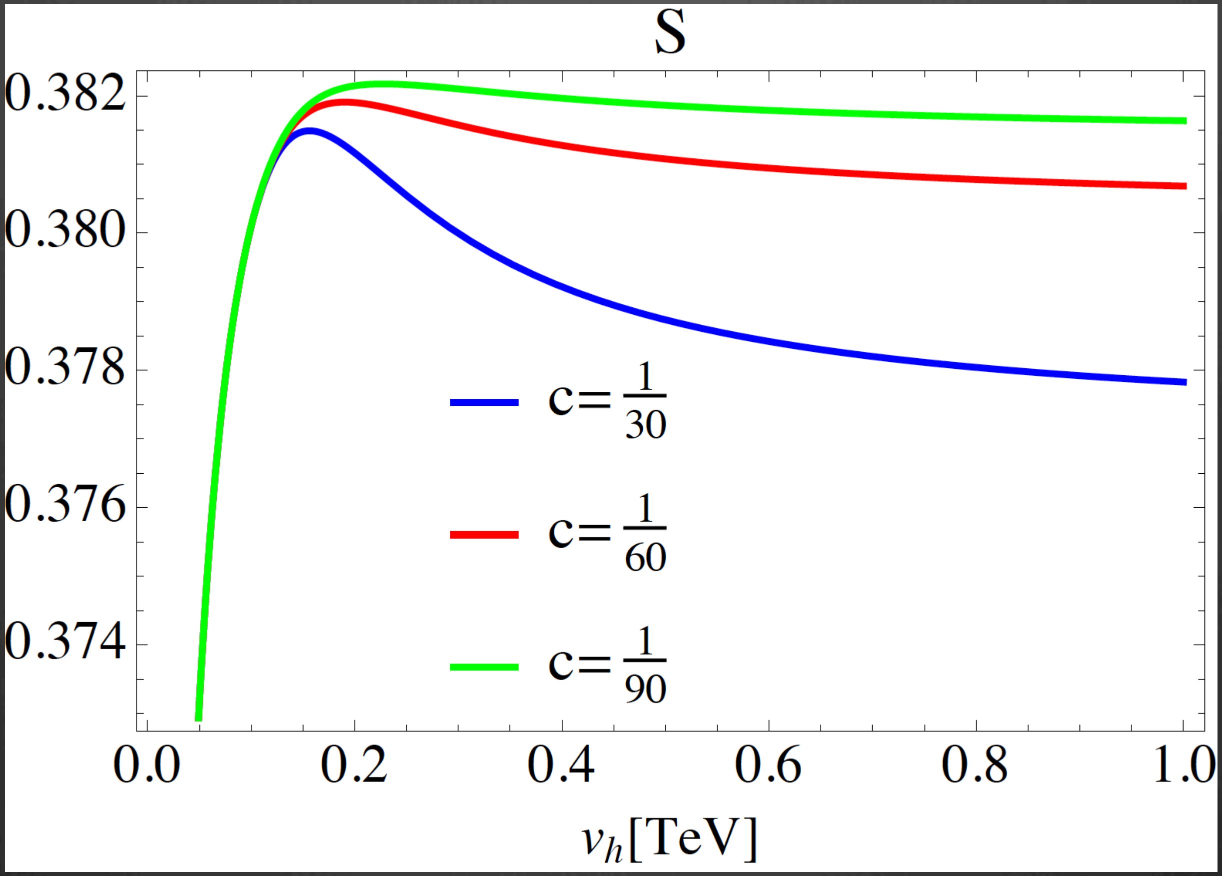
$$S = const \times N_B^{\frac{4}{3}}(v_h)\,(1 - cX_n(v_h))$$

$$N_B^{\frac{4}{3}}$$



$$1 - c \cdot X_n$$





S has a maximum around v_h=200GeV !

- To obtain the peak around 200GeV, the top mass played a very important role.
- Precisely speaking, we have just checked the MEP for one direction of the parameter space.
- Although I do not speak here, we have also checked that even if we fix the Yukawa couplings, S also has a maximum around 200GeV.

Y.Hamada, H. Kawai and K.Kawana, arXiv:1409.6508

4. Summary and Future Work

1) We found that the solution to the CCP and the MEP can be obtained from the quantum theory of Multiverse.

2) We have checked that S actually has the global maximum around O(200)GeV as a function of v_h when we fix the current quark masses.

Future Work

- Confirming the MEP to the remaining parameters of the SM.

  e.g. Gauge couplings, top Yukawa, ・・・ and so on

- It might be interesting to consider how the physics beyond the SM (such as DM) contributes S.

At any late, considering the MEP is very interesting !

Thank you for your listening.