# Non-renormalization Theorem and Cyclic Leibniz Rule in Lattice Supersymmetry

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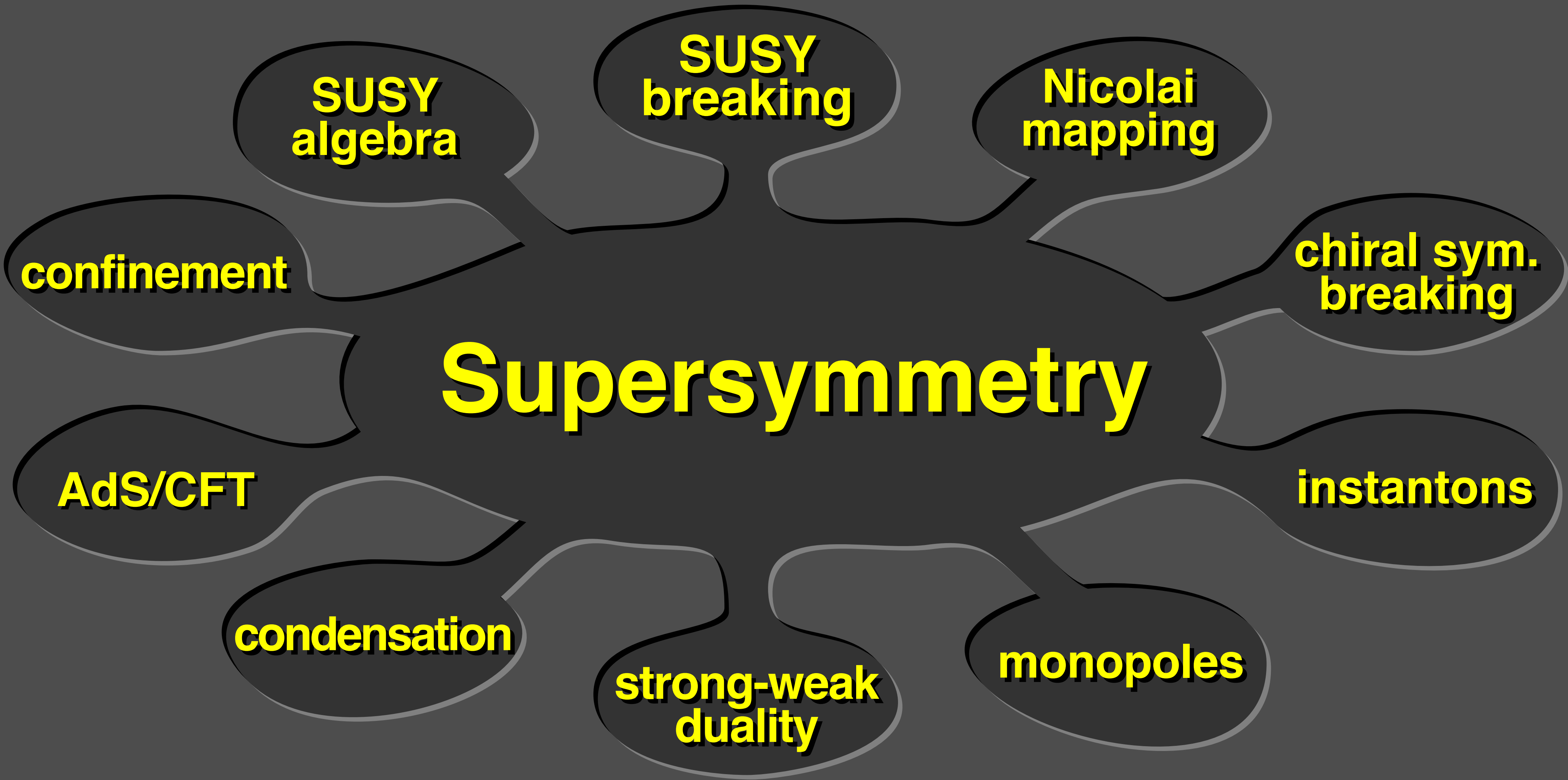
in collaboration with Mitsuhiro Kato and Hiroto So
based on JHEP 1305(2013)089; arXiv:1311.4962;
and in progress

Talk at Osaka University, January 6, 2015

# Non-perturbative dynamics of supersymmetry





# Supersymmetry breaking



S. Weinberg, Phys. Rev. Lett. 80 (1998) 3702

## A theorem for SUSY breaking

Supersymmetry cannot be broken at any finite order of perturbation theory unless it is broken at "tree" level.

- O'Raifeataigh mechanism
- Fayet-Illiopoulous U(1) D term
- SUSY breaking via extra dimensions

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**This theorem may imply that *non-perturbative* analysis is important for SUSY breaking!**

# Lattice Theory

is undoubtedly one of powerful tools to reveal non-perturbative dynamics of field theories!

# History



## Lattice Theory

is undoubtedly one of powerful tools to reveal non-perturbative dynamics of field theories!

However, for more than

**30 years**

no one has succeeded to construct satisfactory lattice supersymmetric models!

# Purpose of my talk



- I explain what is an obstacle to realize super-symmetry on lattice.
  - ► ***No-Go Theorem***

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- I explain what is an obstacle to realize super-symmetry on lattice.
  - ► ***No-Go Theorem***

- I propose a new idea to construct lattice super-symmetric models.
  - ► ***Cyclic Leibniz Rule***
  - ► ***Non-renormalization Theorem***

# Plan of my talk



1. Obstacle to realize SUSY on lattice
2. Attempts to construct lattice SUSY models
3. No-Go theorem
4. Complex SUSY QM on lattice
5. Non-renormalization theorem on lattice
6. Q-exact form and cohomology
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# An obstacle to realize SUSY on lattice



We want to find lattice SUSY transf. $\delta_Q , \delta_{Q'}$ such that

lattice SUSY transf. lattice action

$$\delta_Q S[\phi, \chi, F] = \delta_{Q'} S[\phi, \chi, F] = 0$$

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One might replace $\delta_P$ by a difference operator $\nabla$ on lattice.

Since $\delta_Q, \delta_{Q'}, \delta_P$ satisfy the Leibniz rule

$$\delta(AB) = (\delta A)B + A(\delta B)$$

then, we need to find $\nabla$ which satisfies the ***Leibniz rule***:

$$\boxed{\nabla(AB) = (\nabla A)B + A(\nabla B)}$$ Leibniz rule

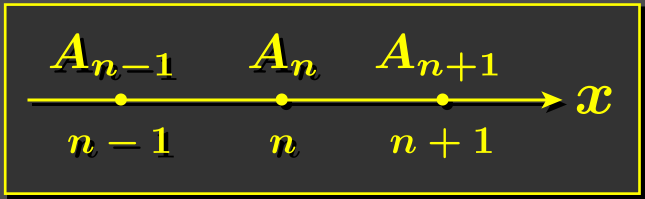
# An obstacle to realize SUSY on lattice



However, simple difference operators do ***not*** satisfy the Leibniz rule on lattice. For example,

*forward difference operator*

$$\nabla^{(+)}(AB)_n \equiv A_{n+1}B_{n+1} - A_n B_n$$



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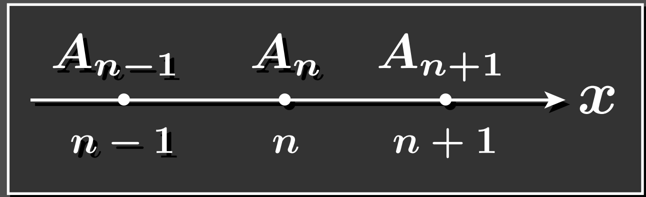


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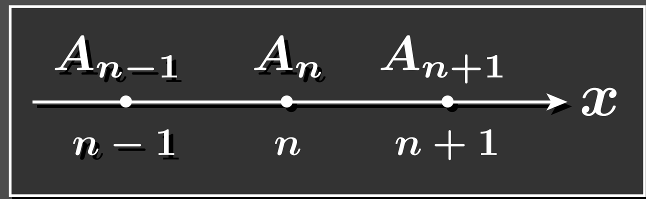


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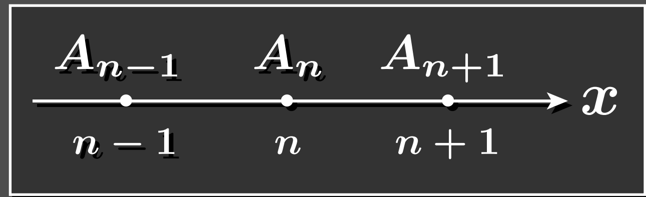


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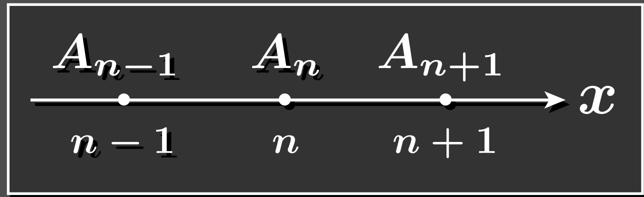


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Indeed, all known (local) difference operators do ***not*** satisfy the Leibniz rule!

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The answer is ***negative!***

**No-Go Theorem**

*M.Kato, M.S. & H.So, JHEP 05(2008)057*

There is no difference operator $\nabla$ satisfying the following three properties:

i) translation invariance
ii) locality
iii) Leibniz rule $\nabla(AB) = (\nabla A)B + A(\nabla B)$

*This prevents us from the realization of SUSY algebra on lattice!*
*I will prove this theorem later.*

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10

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***Give up*** **manifest SUSY on lattice!**

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**It turns out to be hard to control it!**

# Fine tuning

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***Fine tuning*** is necessary to restore it
in the continuum limit.

$\Downarrow$ *In general, there exist various relevant SUSY breaking terms.*

It turns out to be hard to control it!

$\Downarrow$

***Exact SUSY*** on lattice is necessary!

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- N=1 SYM (gluon+an adjoint Majorana fermion)

$$\mathcal{L} = -\frac{1}{4}\underbrace{F^a_{\mu\nu}F^{a\mu\nu}}_{\text{gluon}} + i\underbrace{\bar{\lambda}\gamma_\mu D^\mu\lambda}_{\text{gluino}} + \frac{\theta_{\mathrm{CP}}}{32\pi^2}F^a_{\mu\nu}\widetilde{F}^{a\mu\nu}$$

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| chiral symmetry $m_\lambda = 0$ | $\longleftrightarrow$ | SUSY |
|---|---|---|

# Fine tuning



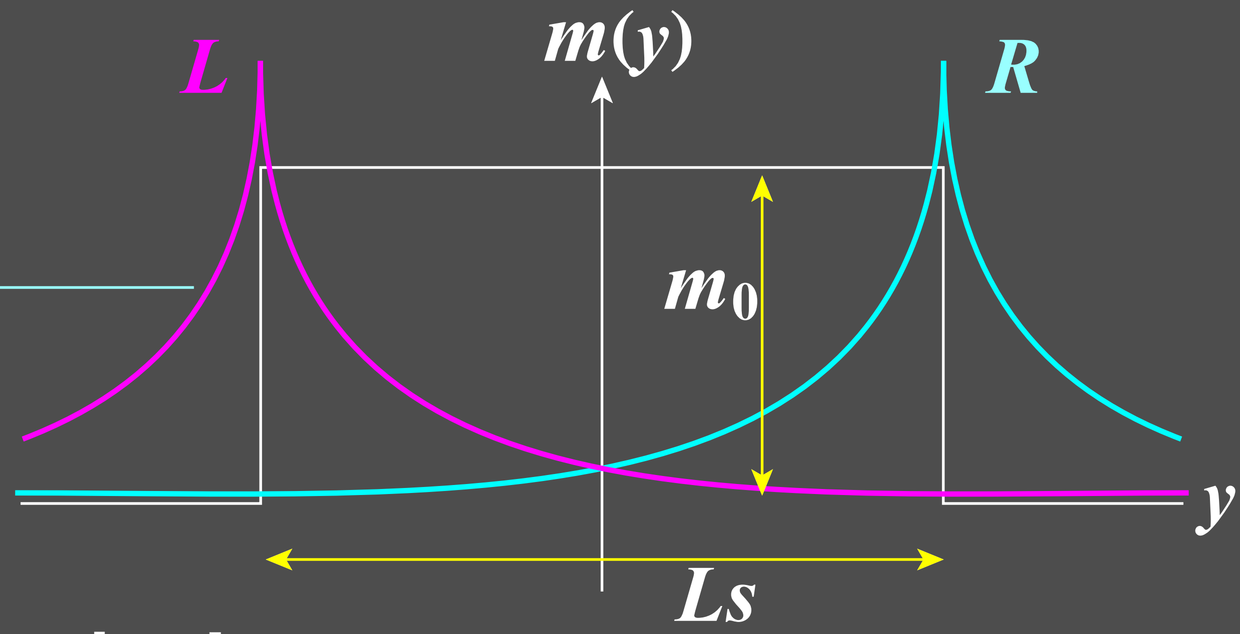
## Domain wall fermions

Nishimura, PLB406(1997)215;
Neuberger, PRD57(1998)5417.

*kink mass*

*5th dimension*

$$+\, m(y)\,\bar{\psi}(x,y)\psi(x,y)$$




*localized fermion zero mode*

↓

*massless gluino!*

- ▶ no need for a fine tuning!
- ▶ two extra parameters: $m_0$, $Ls$
- ▶ The other chirality appears on the opposite wall for finite $Ls$.

# Fine tuning

## Overlap fermions

Maru-Nishimura, IJMPA13(1998)2841;
Neuberger, PRD57(1998)5417.

### Ginsparg-Wilson relation

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

*lattice spacing*

### chiral symmetry

$$\begin{cases} \delta\psi = \tilde{\gamma}_5\psi \\ \delta\bar{\psi} = \bar{\psi}\gamma_5 \end{cases}$$

$$\tilde{\gamma}_5 \equiv \gamma_5(1-2aD)$$

→ massless gluino

↓

**SUSY will appear in the continuum limit!**

# Nicolai map



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*Nicolai, PLB89 (1980) 341; NPB176 (1980) 419*

$\Longleftarrow$

**Nicolai map**

$$\xi = \xi(\phi)$$

**with the properties:**

$$\det \frac{\delta\phi}{\delta\xi} \cdot \det M(\phi) = 1$$

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$\Longleftarrow$ **Gaussian integral!**

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$$Z_{\mathrm{SUSY}}^{\mathrm{lattice}} =$$

$$=$$

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**Start with the Gaussian integral on lattice!**

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*Sakai-Sakamoto, NPB229 (1983) 173*

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Question

Are nilpotent SUSYs enough to restore the full SUSYs in the continuum limit?

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**Question**

Are nilpotent SUSYs enough to restore the full SUSYs in the continuum limit?

Our results suggest that the answer is ***negative!***

# Nilpotent SUSY



N=2 extended SUSY algebra in 2-dim.

$$\{Q_{\alpha i}, Q_{\beta j}\} = \delta_{ij}(\gamma_\mu \gamma_0)_{\alpha\beta} P^\mu \qquad \begin{array}{l} i,j = 1,2 \\ \alpha,\beta = 1,2 \end{array}$$

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$$\Big\downarrow$$

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**Q-exact SUSY**

$$S = \kappa\left(\int d^2x\, O\right)$$

with $\kappa^2 = 0$

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Q-exact SUSY on lattice

# Hamiltonian formulation



Sakai-Sakamoto, NPB229(1983)173;
Elitzur-Schwimmer, NPB226(1983)109.

Time is continuous.

↓

An exact SUSY can be realized in the time-direction.

↓

This guarantees the pairing between bosonic and fermionic states.

# Deconstruction SUSY



Cohen-Kaplan-Katz-Unsal, JHEP12(2003)031.

$U(kN^d)$ **SYM in 3+1 dim. (continuous)**

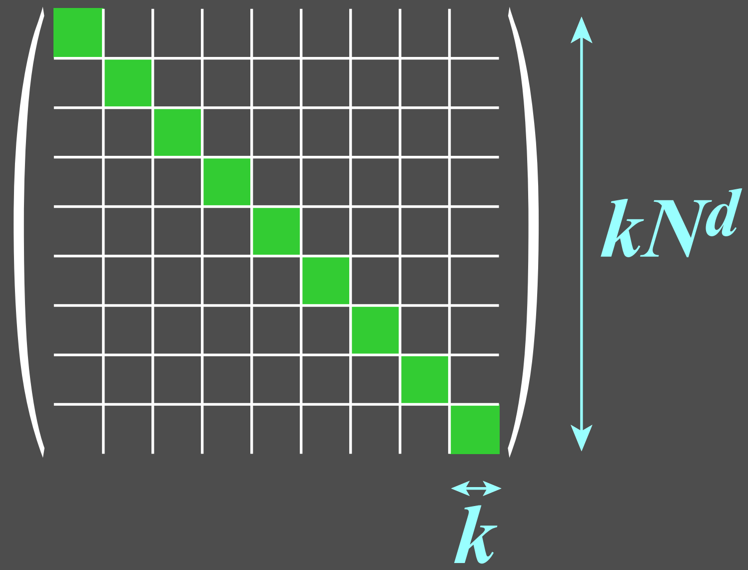
↓ *dimensional reduction:* $\partial_\mu = 0$

$U(kN^d)$ **extended SYM matrix model in 0+0 dim.**

↓ $(Z_N)^d$*-orbifolding*

$U(k)$ **SYM in d-dim. (lattice)**

*Some of SUSY are broken*
*by orbifolding.*



# Perfect action



So-Ukita, PLB457(1999)314;
Bietenholz, MPLA14(1999)51.

- **block-spin transformation**

$$\Phi_n \equiv \int dx\, f_n(x)\,\Phi(x)$$

- **perfect action**

*continuum action*

$$e^{-S_L(\varphi_n)} \equiv \int [d\Phi]\; e^{-\sum_n \alpha_n |\varphi_n - \Phi_n|^2 - S_C(\Phi)}$$

- **action invariance**

$$\boxed{S_C(\Phi(x)+\delta\Phi(x)) = S_C(\Phi(x))} \longrightarrow \boxed{\begin{array}{c} S_L(\varphi_n+\delta\varphi_n) = S_L(\varphi_n) \\ \delta\varphi_n \equiv \int dx\, f_n(x)\,\delta\Phi(x) \end{array}}$$

*Success only for the free Wess-Zumino model!*

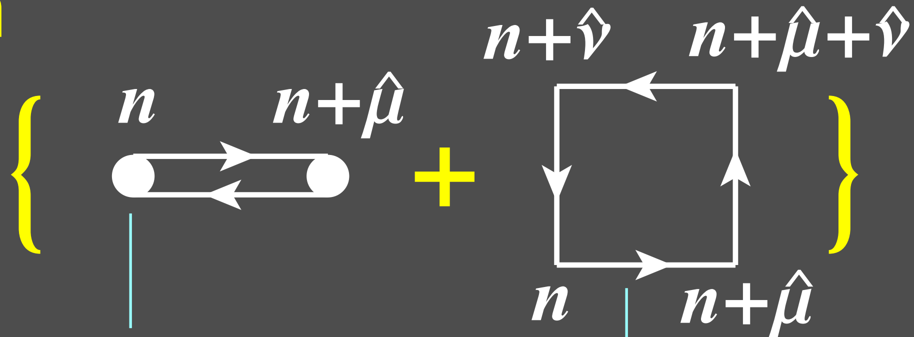
# Ichimatsu lattice



Itoh-Kato-Sawanaka-So-Ukita, NP(Suppl)106(2002)947.

## ▶ lattice action

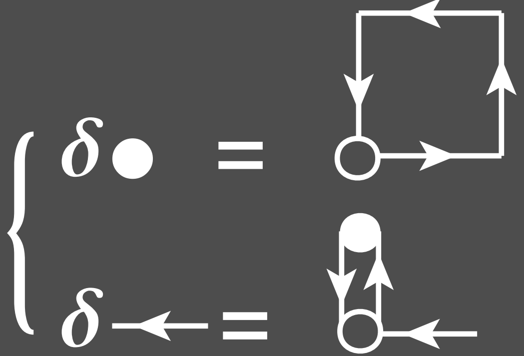
$$S_{\mathrm{lat}} \equiv \sum_{\mathrm{Ichimatsu}}$$




$\xi(n)$
real Staggered fermions

$U_\mu(n)$: link variables

## ▶ fermionic invariance




**It is unclear that in the continuum limit the lattice model reduces to a continuum SUSY model*!***

# Non-commutative approach



D'Adda-Kanamori-Kawamoto-Nagata, NPB707(2005)100.

- difference operator

$$\nabla_\mu^{(-)}(f(n)g(n)) = (\nabla_\mu^{(-)}f(n))\,g(n) + f(n-\hat{\mu})\,(\nabla_\mu^{(-)}g(n))$$

- shift operator

$$T_\mu f(n) = f(n-\hat{\mu})T_\mu$$

- new "difference" operator

$$\hat{\nabla}_\mu^{(-)} \equiv T_\mu \nabla_\mu^{(-)} \longrightarrow$$ satisfies the Leibniz rule!

+ Dirac-Kähler fermions

↓

lattice models with full SUSY!

# Infinite flavors



*M. Kato, M.S. & H.So, JHEP 05(2008)057*

**We find a solution satisfying the Leibniz rule.**

$$D^{ab}(m;n) = d(a-b)\left(\delta_{m-n,a-b} - \delta_{m-n,-(a-b)}\right)$$

$$C^{abc}(l,m;n) = \delta_{l-n,b}\,\delta_{n-m,a}\,\delta_{a+b,c}$$

**characteristic features**

- ★ translationally invariant
- ★ local (= holomorphic)
- ★ non-trivial connection between lattice sites and flavor indices fi need for infinite flavors!
- ★ local in the space direction but "non-local" in the flavor direction!

# Plan of my talk

1. Obstacle to realize SUSY on lattice
2. Attempts to construct lattice SUSY models
3. No-Go theorem
4. Complex SUSY QM on lattice
5. Non-renormalization theorem on lattice
6. Q-exact form and cohomology
7. Summary

# No-Go theorem



**No-Go Theorem**

*M.Kato, M.S. & H.So, JHEP 05(2008)057*

There is no difference operator $\nabla$ satisfying the following three properties:

i) translation invariance

ii) locality

iii) Leibniz rule $\nabla(AB) = (\nabla A)B + A(\nabla B)$

# A proof of No-Go theorem



To prove the theorem, we generalize the difference operator and the field product as follows:

▶ extension of difference operator

$$(\nabla A)_n \longrightarrow \boxed{(\nabla A)_n \equiv \sum_m \nabla_{nm} A_m}$$

▶ extension of field product

$$A_n B_n \longrightarrow \boxed{(A * B)_n \equiv \sum_{lm} M_{nlm} A_l B_m}$$

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$$(\nabla A)_n \quad \longrightarrow \quad \boxed{(\nabla A)_n \equiv \sum_m \boldsymbol{\nabla}_{nm} A_m}$$

forward difference operator

$$(\nabla^{(+)} A)_n = A_{n+1} - A_n \quad \Leftrightarrow \quad \nabla^{(+)}_{nm} = \delta_{n+1,m} - \delta_{n,m}$$

▶ extension of field product

$$A_n B_n \quad \longrightarrow \quad \boxed{(A * B)_n \equiv \sum_{lm} M_{nlm} A_l B_m}$$

normal product

$$A_n B_n \quad \Leftrightarrow \quad M_{nlm} = \delta_{n,l}\delta_{n,m}$$

# A proof of No-Go theorem



**Q** Is it possible to construct a difference operator $\nabla$ satisfying

$$\nabla(A * B) = (\nabla A) * B + A * (\nabla B)$$

- extension of difference operator

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forward difference operator

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normal product

$$A_n B_n \quad \Leftrightarrow \quad M_{nlm} = \delta_{n,l}\delta_{n,m}$$

# A proof of No-Go theorem



- translation invariance

$$\nabla_{nm} = \nabla(n-m)$$

$$M_{nlm} = M(n-l, n-m)$$

# A proof of No-Go theorem



- translation invariance

$$\nabla_{nm} = \nabla(n-m)$$

$$M_{nlm} = M(n-l, n-m)$$

- locality (exponential damping)

$$\nabla(m) \xrightarrow{|m|\to\infty} e^{-\alpha|m|} \qquad (\alpha > 0)$$

$$M(l,m) \xrightarrow{|l|,|m|\to\infty} e^{-\beta|l|-\gamma|m|} \qquad (\beta, \gamma > 0)$$

# A proof of No-Go theorem



- translation invariance

$$\nabla_{nm} = \nabla(n-m)$$
$$M_{nlm} = M(n-l, n-m)$$

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$$\nabla(m) \xrightarrow{|m|\to\infty} e^{-\alpha|m|} \qquad (\alpha > 0)$$
$$M(l,m) \xrightarrow{|l|,|m|\to\infty} e^{-\beta|l|-\gamma|m|} \qquad (\beta,\gamma > 0)$$

$\Longrightarrow$ **holomorphic representation** $(z, w \in \mathbb{C})$

$$\nabla(m) \longrightarrow \tilde{\nabla}(z) \equiv \sum_m \nabla(m)\, z^m$$
$$M(l,m) \longrightarrow \widetilde{M}(z,w) \equiv \sum_{lm} M(l,m)\, z^l w^m$$

*coefficients of Laurent series*

# A proof of No-Go theorem

## An important observation is that

**locality**

$$\nabla(m) \xrightarrow{|m|\to\infty} e^{-\alpha|m|}$$

$$M(l,m) \xrightarrow{|l|,|m|\to\infty} e^{-\beta|l|-\gamma|m|}$$

$$\Longleftrightarrow$$

# A proof of No-Go theorem

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**locality**

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$\Longleftrightarrow$

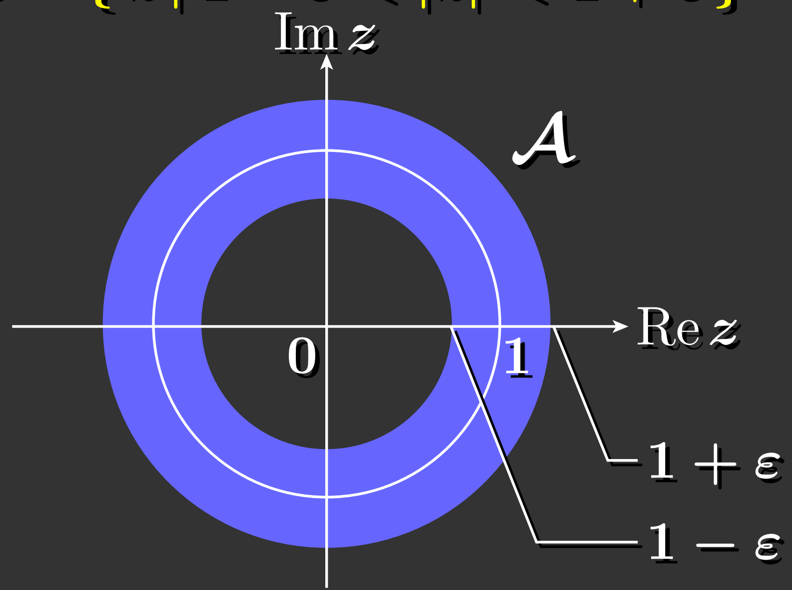
**holomorphy on $\mathcal{A}$**

$$\tilde{\nabla}(z) \equiv \sum_m \nabla(m)\, z^m \qquad (z, w \in \mathcal{A})$$

$$\widetilde{M}(z,w) \equiv \sum_{lm} M(l,m)\, z^l w^m$$

**Annulus domain**

$$\mathcal{A} = \{\, z \,|\, 1-\varepsilon < |z| < 1+\varepsilon \,\}$$



# A proof of No-Go theorem



## Leibniz rule

$$\nabla(A * B) = (\nabla A) * B + A * (\nabla B)$$

# A proof of No-Go theorem



## Leibniz rule

$$\nabla(A * B) = (\nabla A) * B + A * (\nabla B)$$

$$\Longrightarrow \sum_m \left\{ \boldsymbol{\nabla}_{nm} M_{mkl} - M_{nml} \boldsymbol{\nabla}_{mk} - M_{nkm} \boldsymbol{\nabla}_{ml} \right\} = \mathbf{0}$$

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*translation invariance*

$$\Longrightarrow \sum_m \Big\{ \nabla(n-m) M(m-k, m-l) - M(n-m, n-l) \nabla(m-k)$$
$$- M(n-k, n-m) \nabla(m-l) \Big\} = 0$$

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$$\sum_{kl} w^{n-k} z^{n-l} \times$$

$$\Longrightarrow \sum_{klm} \Big\{ \nabla(n-m)(wz)^{n-m} M(m-k, m-l) w^{m-k} z^{m-l}$$
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$$\sum_{kl} w^{n-k} z^{n-l} \times$$

$\widetilde{\nabla}(wz)$ $\quad \widetilde{M}(w,z)$ *in holomorphic rep.*

$$\Longrightarrow \sum_{klm} \Big\{ \boxed{\nabla(n-m)(wz)^{n-m}} \boxed{M(m-k, m-l) w^{m-k} z^{m-l}}$$
$$- \boxed{M(n-m, n-l) w^{n-m} z^{n-l}} \boxed{\nabla(m-k) w^{m-k}}$$
$$- \boxed{M(n-k, n-m) w^{n-k} z^{n-m}} \boxed{\nabla(m-l) z^{m-l}} \Big\} = 0$$

# A proof of No-Go theorem



## Leibniz rule

$$\nabla(A * B) = (\nabla A) * B + A * (\nabla B)$$

$$\Longrightarrow \quad \widetilde{M}(w,z)\left(\tilde{\nabla}(wz) - \tilde{\nabla}(w) - \tilde{\nabla}(z)\right) = 0$$

# A proof of No-Go theorem



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$$\nabla(A * B) = (\nabla A) * B + A * (\nabla B)$$

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$\Downarrow$

By virtue of the identity theorem on holomorphic functions, the domain can be extended to the whole domain $\mathcal{A}$ without

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$$\Longrightarrow \quad \tilde{\nabla}(z) = \beta \log z \qquad \text{on } \mathcal{A}$$

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*non-holomorphic and non-local*

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$\Rightarrow \quad \beta = 0$

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*non-holomorphic*
*and non-local*

$$\Rightarrow \quad \beta = 0$$

$$\Longrightarrow \quad \tilde{\nabla}(z) = 0$$

$\Longrightarrow$ There is ***no*** difference operator which satisfies the Leibniz rule!! $\Longrightarrow$ ***No-Go Theorem!***

# How to escape from No-Go theorem



In order to escape from the No-Go theorem,
we may abandon one of the three requirements.

i) translation inv.

ii) locality

iii) Leibniz rule

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*The continuum limit is highly non-trivial!*

iii) Leibniz rule $\Longrightarrow$ **Modify the Leibniz rule!**

*We will take this approach!*

# Our approach to construct lattice SUSY model



**Nilpotent SUSY algebra**

$$(\delta_Q)^2 = (\delta_{Q'})^2 = \{\delta_Q, \delta_{Q'}\} = 0$$

**+**

**Cyclic Leibniz rule**

Avoid the problem of "translations" on lattice.

A modified version of the Leibniz rule

# Plan of my talk



1. Obstacle to realize SUSY on lattice
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# Complex SUSY quantum mechanics on lattice



## Lattice action

$$
\begin{aligned}
S = & (\nabla\phi_-, \nabla\phi_+) - (F_-, F_+) - i(\chi_-, \nabla\bar{\chi}_+) + i(\nabla\bar{\chi}_-, \chi_+) \\
& -m_+(F_+, \phi_+) + m_+(\chi_+, \bar{\chi}_+) \\
& -m_-(F_-, \phi_-) - m_-(\chi_-, \bar{\chi}_-) \\
& -\lambda_+(F_+, \phi_+ * \phi_+) + 2\lambda_+(\chi_+, \bar{\chi}_+ * \phi_+) \\
& -\lambda_-(F_-, \phi_- * \phi_-) - 2\lambda_-(\chi_-, \bar{\chi}_- * \phi_-)
\end{aligned}
$$

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

inner product: $(\phi, \psi) \equiv \sum_n \phi_n\psi_n$

# N=2 nilpotent SUSYs



N=2 Nilpotent SUSYs: $\boxed{(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0}$

$$\begin{cases} \delta_+\phi_+ = \bar{\chi}_+ \\ \delta_+\chi_+ = F_+ \\ \delta_+\chi_- = -i\nabla\phi_- \\ \delta_+F_- = -i\nabla\bar{\chi}_- \\ \text{others} = 0 \end{cases} \qquad \begin{cases} \delta_-\chi_+ = i\nabla\phi_+ \\ \delta_-F_+ = -i\nabla\bar{\chi}_+ \\ \delta_-\phi_- = -\bar{\chi}_- \\ \delta_-\chi_- = F_- \\ \text{others} = 0 \end{cases}$$

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$$\delta_\pm S = 0$$

$$\Downarrow$$

$$\boxed{(\nabla\bar{\chi}_\pm\,, \phi_\pm * \phi_\pm) + (\nabla\phi_\pm\,, \phi_\pm * \bar{\chi}_\pm) + (\nabla\phi_\pm\,, \bar{\chi}_\pm * \phi_\pm) = 0}$$

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$$\delta_\pm S = 0$$

$$\Downarrow$$

$$\boxed{(\nabla\bar{\chi}_\pm, \phi_\pm * \phi_\pm) + (\nabla\phi_\pm, \phi_\pm * \bar{\chi}_\pm) + (\nabla\phi_\pm, \bar{\chi}_\pm * \phi_\pm) = 0}$$

We call this ***Cyclic Leibniz rule***.

# Cyclic Leibniz rule



We have found that the ***Cyclic Leibniz Rule*** guarantees the N=2 nilpotent SUSYs.

*Cyclic Leibniz Rule* (CLR)

$$(\nabla A,\ B * C) + (\nabla B,\ C * A) + (\nabla C,\ A * B) = 0$$

**vs.**

*Leibniz Rule* (LR)

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No-Go theorem

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$$(\nabla A,\ B * C) + (\nabla B,\ C * A) + (\nabla C,\ A * B) = 0$$

vs.

*Leibniz Rule* (LR)

$$(\nabla A,\ B * C) + (A,\ \nabla B * C) + (A,\ B * \nabla C) \neq 0$$

No-Go theorem

!

The cyclic Leibniz rule ensures ***a lattice analog of vanishing surface terms!***

$$(\nabla \phi,\ \phi * \phi) = 0 \longleftarrow \int dx\, \partial_x \left(\phi(x)\right)^3 = 0$$

on lattice  *CLR*  in continuum

# An example of CLR



The important fact is that the cyclic Leibniz rule can be realized on lattice, though the Leibniz rule cannot !

# An example of CLR



The important fact is that the cyclic Leibniz rule can be realized on lattice, though the Leibniz rule cannot !

An explicit example of the ***Cyclic Leibniz Rule*** :

$$(\nabla\phi)_n = \frac{1}{2}\left(\phi_{n+1} - \phi_{n-1}\right)$$

$$(\phi * \psi)_n = \frac{1}{6}\left(2\phi_{n+1}\psi_{n+1} + 2\phi_{n-1}\psi_{n-1} + \phi_{n+1}\psi_{n-1} + \phi_{n-1}\psi_{n+1}\right)$$

lattice spacing
$a=1$

*M.Kato, M.S. & H.So, JHEP 05(2013)089*

which satisfy i) *translation invariance*, ii) *locality* and iii) *Cyclic Leibniz Rule.*

# An example of CLR



The important fact is that the cyclic Leibniz rule can be realized on lattice, though the Leibniz rule cannot !

An explicit example of the ***Cyclic Leibniz Rule*** :

$$(\nabla\phi)_n = \frac{1}{2}\left(\phi_{n+1} - \phi_{n-1}\right)$$

$$(\phi * \psi)_n = \frac{1}{6}\big(2\phi_{n+1}\psi_{n+1} + 2\phi_{n-1}\psi_{n-1} + \phi_{n+1}\psi_{n-1} + \phi_{n-1}\psi_{n+1}\big)$$

lattice spacing
$a=1$

*M.Kato, M.S. & H.So, JHEP 05(2013)089*

which satisfy i) *translation invariance*, ii) *locality* and iii) *Cyclic Leibniz Rule.*

! *The field product* $(\phi * \psi)_n$ *should be non-trivial!*

# Advantages of CLR



Advantages of our lattice model with ***CLR*** are given by

| | CLR | no CLR |
|---|---|---|
| nilpotent SUSYs | | |
| Nicolai maps | | |
| “surface” terms | | |
| non-renormalization theorem | | |
| cohomology | | |

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# Plan of my talk



1. Obstacle to realize SUSY on lattice
2. Attempts to construct lattice SUSY models
3. No-Go theorem
4. Complex SUSY QM on lattice
5. Non-renormalization theorem on lattice
6. Q-exact form and cohomology
7. Summary

# Non-renormalization theorem in continuum



One of the striking features of SUSY theories is the ***non-renormalization theorem***.

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❑ 4d N=1 Wess-Zumino model in continuum

*chiral superfield* *superpotential*

$$S = \int d^4x \left\{ \int d^2\theta d^2\bar{\theta}\; \Phi^\dagger(\bar{\theta})\Phi(\theta) + \int d^2\theta\, W(\Phi) + c.c. \right\}$$

D term (kinetic terms)

F term (potential terms)

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**Non-renormalization Theorem**

There is ***no quantum correction to the F-terms*** in any order of perturbation theory.

*Grisaru, Seigel, Rocek, NPB159 (1979) 429*

# Essence of non-renormalization theorem



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***Holomorphy*** plays an important role in the non-renormalization theorem.

*tree superpotential*

*chiral superfield*

*coupling constant*

*anti-chiral superfield*

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quantum corrections $\Downarrow$

*effective superpotential*

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*holomorphy* $\Downarrow$ *anti-holomorphy*

*No quantum correction!!* $\quad W_{\rm eff} = W_{\rm tree}$

*N.Seiberg, Phys. Lett. B318 (1993) 469*

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$$\Downarrow$$

*anti-chiral superfield*

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*holomorphy* *anti-holomorphy*

$$\Downarrow$$

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# Difficulty in defining chiral superfield on lattice



The holomorphy requires that the F term $W(\Phi)$ depends only on the ***chiral*** superfield $\Phi(x,\theta)$, which is defined by

$$\bar{D}\Phi(x,\theta) \equiv \left(\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma_\mu\partial_\mu\right)\Phi(x,\theta) = 0 \quad \text{in continuum}$$

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However, the above definition of the chiral superfield is ***ill-defined*** because any products of chiral superfields are not chiral due to the ***breakdown of LR on lattice!***

$$\bar{D}\Phi_1 = \bar{D}\Phi_2 = 0 \implies \bar{D}(\Phi_1\Phi_2) \neq 0$$

*the breakdown of the Leibniz rule on lattice*

# Superfield formulation in our lattice model



❑ Lattice superfields

$$\Psi_\pm(\theta_+, \theta_-) \equiv \chi_\pm + \theta_\pm F_\pm + \theta_\mp i\nabla\phi_\pm + \theta_\pm\theta_\mp i\nabla\bar{\chi}_\pm$$

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$$\delta_\pm\mathcal{O}(\theta) = \frac{\partial}{\partial\theta_\pm}\mathcal{O}(\theta) \qquad \text{for any superfield } \mathcal{O}(\theta)$$

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- Lattice action in superspace $S = S_{\text{type I}} + S_{\text{type II}}$

$$S_{\text{type I}} = \int d\theta_+ d\theta_- \,(\Psi_-, \Psi_+)$$
$$S_{\text{type II}} = \int d\theta_+ d\theta_- \,\Big\{\theta_-\big(m_+(\Psi_+,\Phi_+) + \lambda_+(\Psi_+,\Phi_+ * \Phi_+)\big) - \theta_+\big(m_-(\Psi_-,\Phi_-) + \lambda_-(\Psi_-,\Phi_- * \Phi_-)\big)\Big\}$$

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# Superfield formulation in our lattice model



- general Lattice action in superspace $S = S_{\mathbf{type\,I}} + S_{\mathbf{type\,II}}$

$$S_{\mathbf{type\,I}} = \int d\theta_+ d\theta_-\ K(\Psi_+, \Phi_+; \Psi_-, \Phi_-)$$

$$S_{\mathbf{type\,II}} = \int d\theta_+ d\theta_-\ \left\{\theta_- W_+(\Psi_+, \Phi_+) - \theta_+ W_-(\Psi_-, \Phi_-)\right\}$$

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!

$S_{\mathrm{type\,II}}$ is SUSY-invariant ***if and only if*** $W_+(\Psi_+, \Phi_+)$ depends only on $\Psi_+, \Phi_+$ and is written into the form

$$W_+(\Psi_+, \Phi_+) = \sum_n \lambda_+^{(n)} (\Psi_+, \overbrace{\Phi_+ * \Phi_+ * \cdots * \Phi_+}^{n-1})$$

and $(\Psi_+, \Phi_+ * \Phi_+ * \cdots * \Phi_+)$ has to obey ***CLR***.

*M.Kato, M.S., H.So, in preparation*

# Non-renormalization theorem on lattice



$$\int d\theta_+ d\theta_- \, \theta_- W_+^{\mathrm{tree}}(\Psi_+, \Phi_+; m_+, \lambda_+)$$

$$W_+^{\mathrm{tree}} = m_+(\Psi_+, \Phi_+) + \lambda_+(\Psi_+, \Phi_+ * \Phi_+)$$

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$$\Big\Downarrow \text{ quantum corrections}$$

$$\int d\theta_+ d\theta_-\ \theta_- W_+^{\mathrm{eff}}(\Psi_+, \Phi_+; m_+, \lambda_+; \Psi_-, \Phi_-; m_-, \lambda_-)$$

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SUSY-invariance with CLR forbids these!

To prove it, we extend $m_\pm, \lambda_\pm$ to superfields!

$$m_\pm \to\ m_\pm(\theta) \equiv m_\pm + \theta_\pm \zeta_\pm^m$$

$$\lambda_\pm \to\ \lambda_\pm(\theta) \equiv \lambda_\pm + \theta_\pm \zeta_\pm^\lambda$$

This extension preserves SUSY-invariace!

*N.Seiberg, Phys. Lett. B318 (1993) 469*

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$\Downarrow$

$$\int d\theta_+ d\theta_-\ \theta_- W_+^{\rm eff}(\Psi_+, \Phi_+; m_+, \lambda_+)$$

The holomorphic property is realized in our lattice model!

# Non-renormalization theorem on lattice



To restrict the form of $W_+^{\text{eff}}(\Psi_+, \Phi_+; m_+, \lambda_+)$, we use the "conserved" charges:

| | $\Psi_\pm$ | $\Phi_\pm$ | $m_\pm$ | $\lambda_\pm$ | $\theta_\pm$ | $d\theta_\pm$ | $S_{\text{type II}}$ | $\frac{\lambda_\pm}{m_\pm}\Phi_+$ |
|---|---|---|---|---|---|---|---|---|
| $N_F$ | $+1$ | $0$ | $0$ | $0$ | $+1$ | $-1$ | $0$ | $0$ |
| $U(1)$ | $\pm 1$ | $\pm 1$ | $\mp 2$ | $\mp 3$ | $0$ | $0$ | $0$ | $0$ |
| $U(1)_{\text{R}}$ | $0$ | $\pm 1$ | $0$ | $\mp 1$ | $\pm 1$ | $\mp 1$ | $0$ | $0$ |

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$$\int d\theta_+ d\theta_-\, \theta_- W_+^{\text{eff}}(\Psi_+, \Phi_+; m_+, \lambda_+)$$

$$= \int d\theta_+ d\theta_-\, \theta_- \frac{(m_+)^2}{\lambda_+} \left(\Psi_+, f\left(\frac{\lambda_+}{m_+}\Phi_+\right)\right)$$

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*zero charge combination*

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$$= \int d\theta_+ d\theta_-\ \theta_-\ \frac{(m_+)^2}{\lambda_+}\,\Big(\Psi_+, f\big(\boxed{\tfrac{\lambda_+}{m_+}\Phi_+}\big)\Big)$$

$$\Downarrow \qquad f(z)=\sum_n a_n z^n$$

$$= \int d\theta_+ d\theta_-\ \theta_-\ \sum_n a_n \frac{(\lambda_+)^{n-1}}{(m_+)^{n-2}}\Big(\Psi_+, \underbrace{\Phi_+ * \Phi_+ * \cdots * \Phi_+}_{n}\Big)$$

# Non-renormalization theorem on lattice



$$W_+^{\mathrm{eff}} \sim (\lambda_+)^{n-1}\Psi_+(\Phi_+)^n = \Psi_+(\lambda_+\Phi_+)^{n-1}\Phi_+$$

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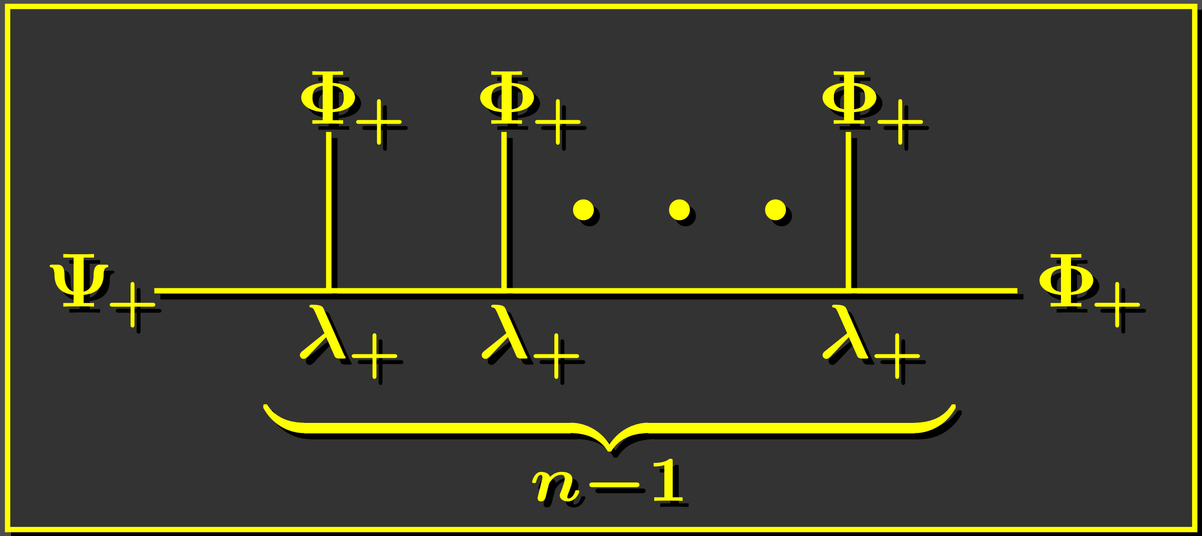

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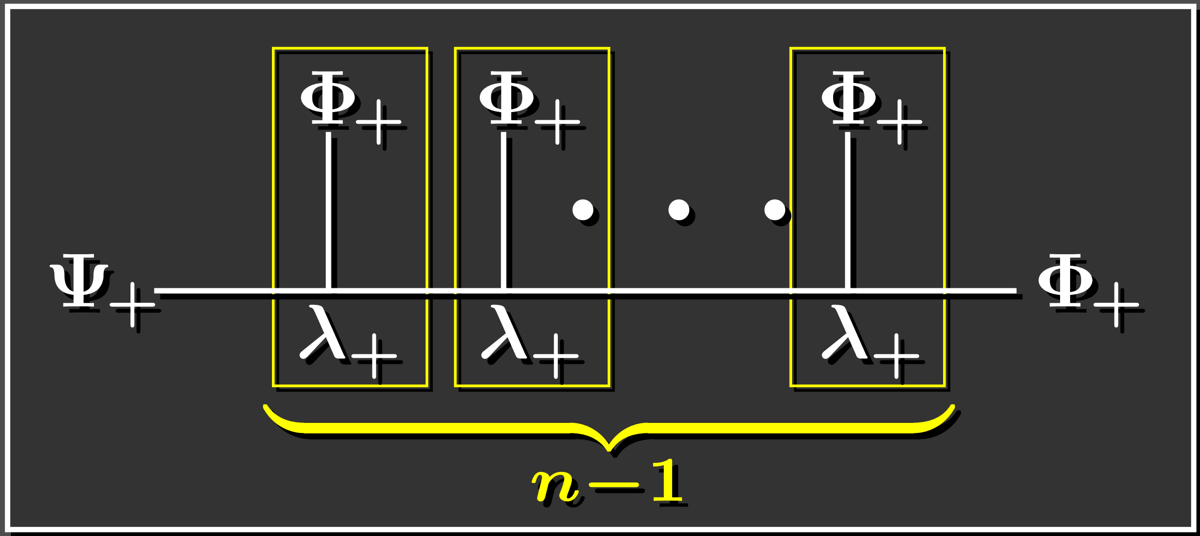

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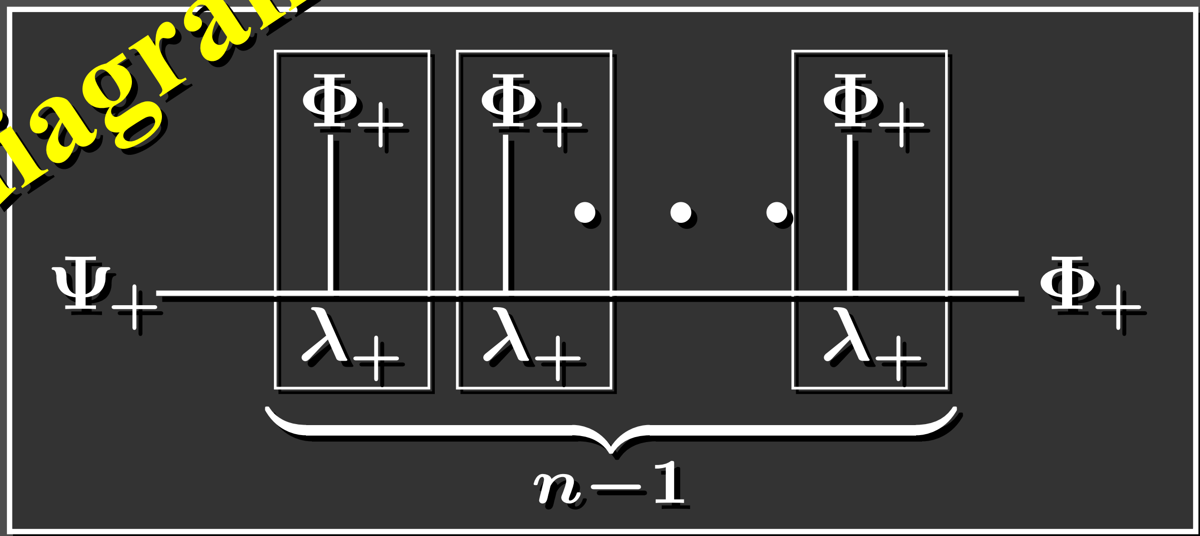
$$\Downarrow$$

**Tree diagrams!**




These diagrams should be excluded from $W_+^{\mathrm{eff}}$
because it consists of ***1PI diagrams***!

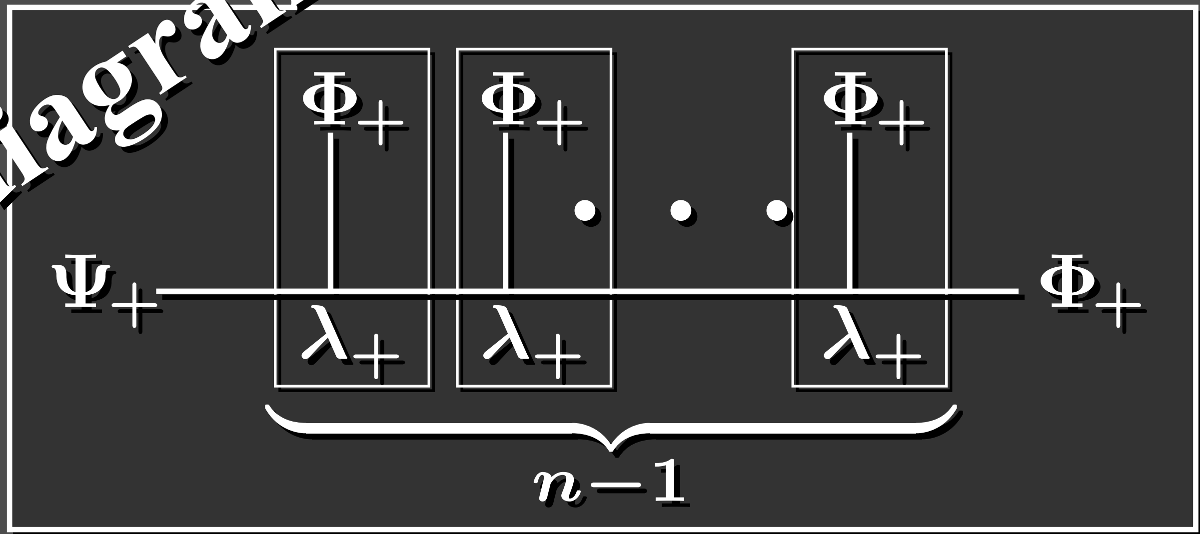
# Non-renormalization theorem on lattice



$$W_+^{\mathrm{eff}} \sim (\lambda_+)^{n-1}\Psi_+(\Phi_+)^n = \Psi_+(\lambda_+\Phi_+)^{n-1}\Phi_+$$

$$\Downarrow$$

**Tree diagrams!**




These diagrams should be excluded from $W_+^{\mathrm{eff}}$ because it consists of ***1PI diagrams***!

$$\Downarrow$$

non-renormalization $\boxed{W_+^{\mathrm{eff}} = W_+^{\mathrm{tree}}}$ ***No quantum correction!***

# Plan of my talk

1. Obstacle to realize SUSY on lattice
2. Attempts to construct lattice SUSY models
3. No-Go theorem
4. Complex SUSY QM on lattice
5. Non-renormalization theorem on lattice
6. Q-exact form and cohomology
7. Summary

# Q-exact form and cohomology



$$S = \overbrace{\delta_+\delta_- K(\phi_\pm, F_\pm, \chi_\pm, \bar{\chi}_\pm)}^{S_{\text{type I}}} + \overbrace{\delta_+ W(\phi_+, \chi_+) + \delta_- \bar{W}(\phi_-, \chi_-)}^{S_{\text{type II}}}$$

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$$(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0$$

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but not exact

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$W \sim (\chi_+, \phi_+ * \phi_+ * \cdots * \phi_+)$
which has the properties:
$W \neq \delta_- K'$
$\delta_- W \sim (\nabla\phi_+, \phi_+ * \phi_+ * \cdots * \phi_+) \overset{\text{CLR}}{=} 0$

*M.Kato, M.S., H.So in preparation*

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*M.Kato, M.S., H.So in preparation*

The type II terms are ***cohomologically non-trivial with CLR!***

# Summary



- ❑ We have proved the ***No-Go theorem*** that the Leibniz rule cannot be realized on lattice under reasonable assumptions.
- ❑ We proposed a lattice SUSY model equipped with the ***cyclic Leibniz rule*** as a modified Leibniz rule.
- ❑ A striking feature of our lattice SUSY model is that the ***non-renormalization theorem*** holds for a finite lattice spacing.
- ❑ Our results suggest that ***the cyclic Leibniz rule grasps important properties of SUSY.***

# Remaining tasks



❑ Extension to higher dimensions

*We have to extend our analysis to higher dimensions. Especially, we need to find solutions to CLR in more than one dimensions.*

❑ inclusion of gauge fields

❑ Nilpotent SUSYs with CLR $\overset{?}{\longleftrightarrow}$ full SUSYs

*Are nilpotent SUSYs extended by CLR enough to guarantee full SUSYs ?*

# History repeats itself!

# History repeats itself!

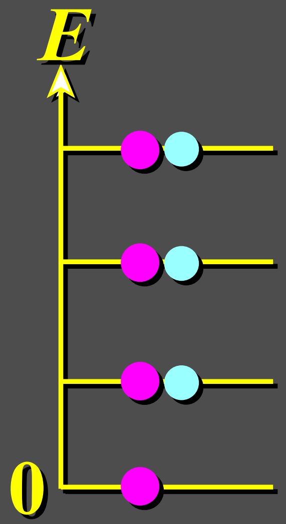


**chiral gauge theory**

# History repeats itself!



## chiral gauge theory



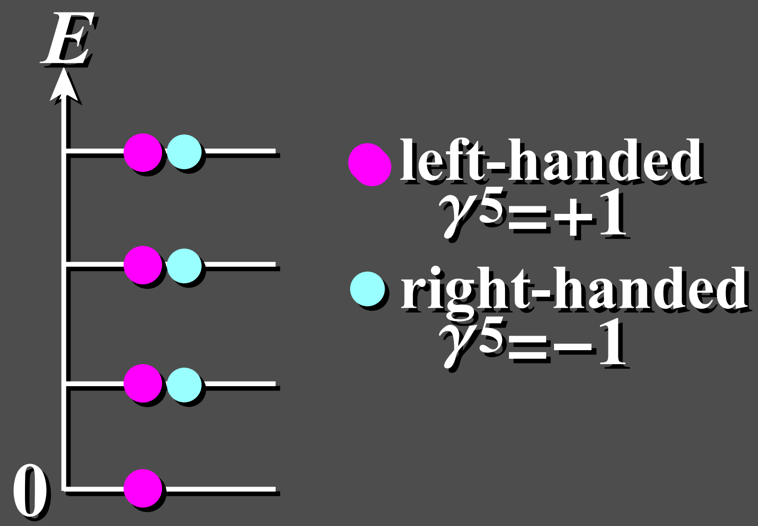

left-handed $\gamma^5 = +1$

right-handed $\gamma^5 = -1$

# History repeats itself!



## chiral gauge theory



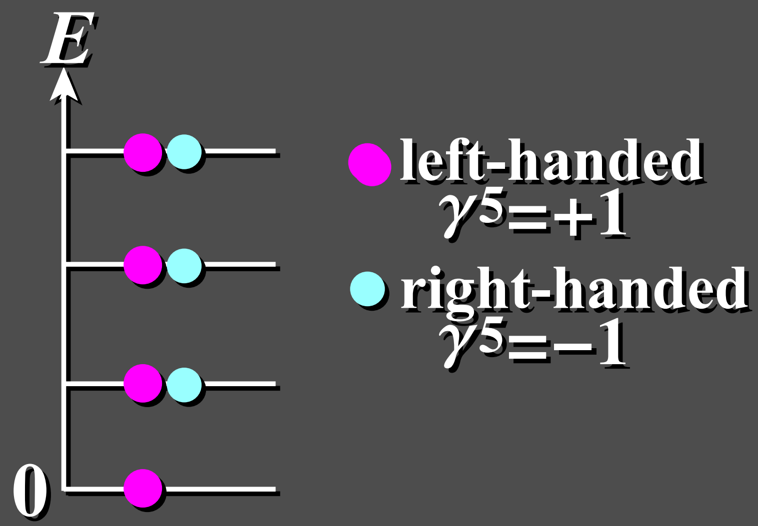

**continuous theory**

There is no renormalization scheme to preserve the chiral structure non-perturbatively!

# History repeats itself!



## chiral gauge theory

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The lattice theory can make it possible!!

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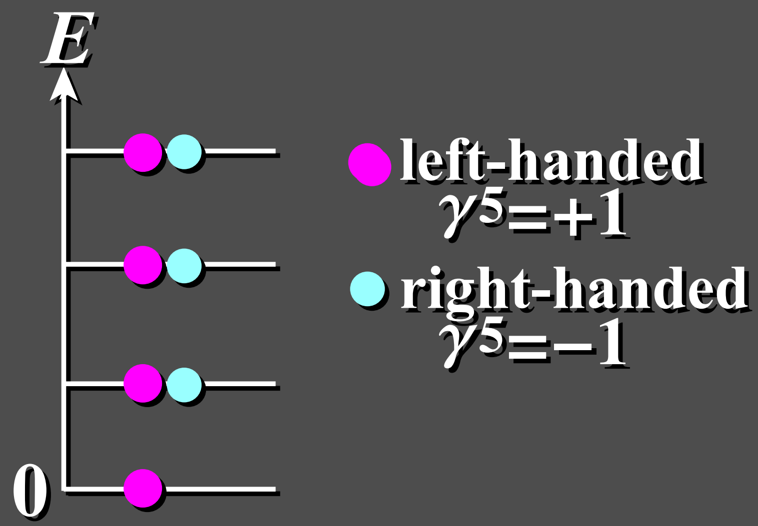


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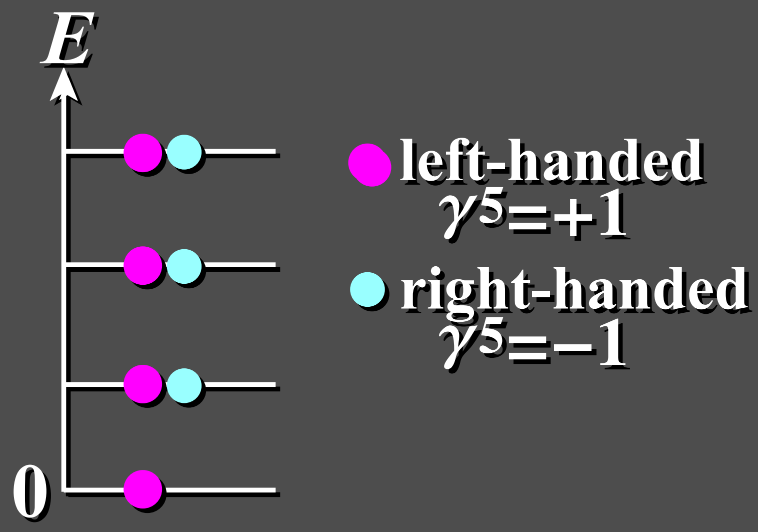
## SUSY theory

# History repeats itself!



## chiral gauge theory

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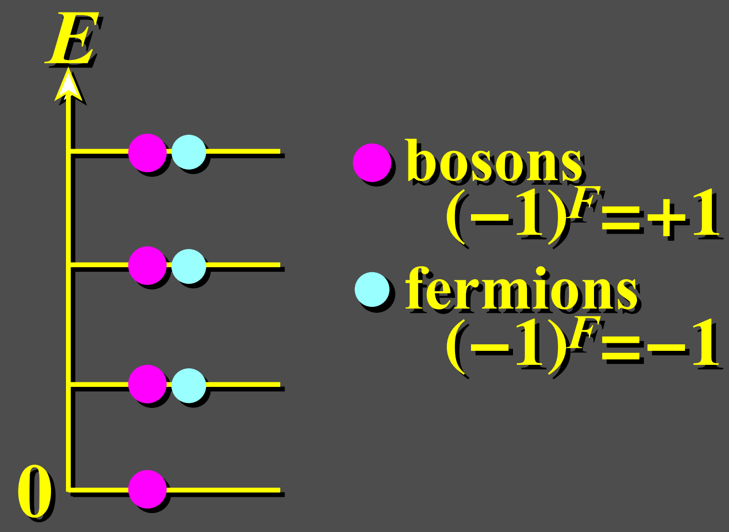
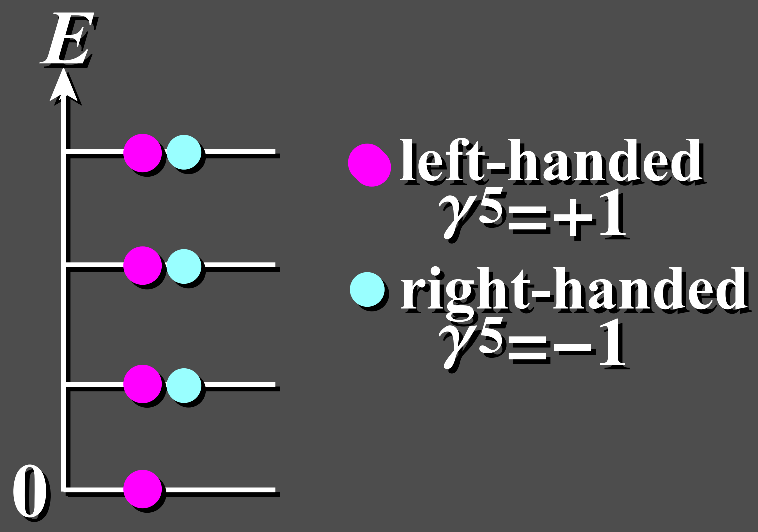


# History repeats itself!



## chiral gauge theory

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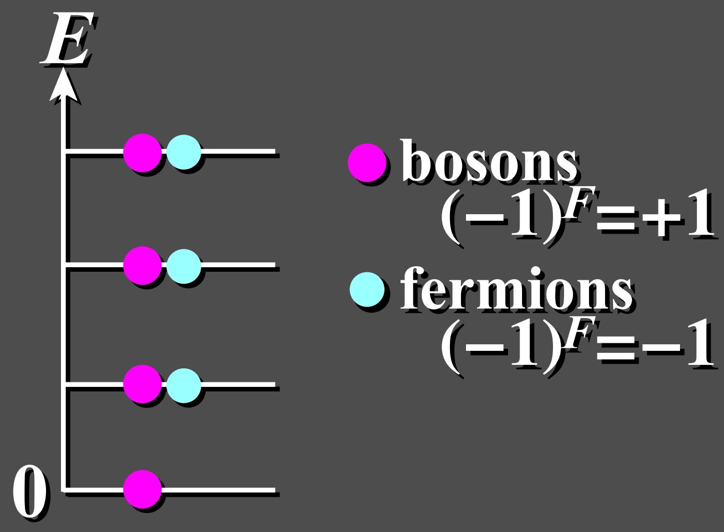


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# History repeats itself!

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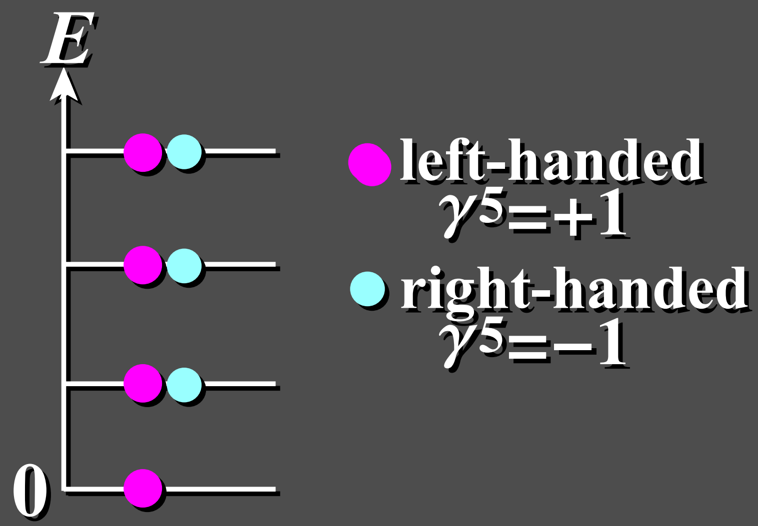


**continuous theory**

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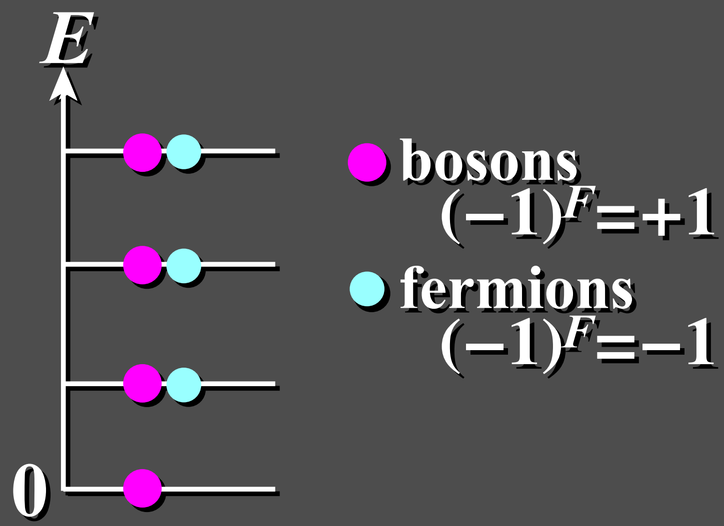
**lattice theory**

The lattice theory can make it possible!!

## SUSY theory




**continuous theory**

There is no renormalization scheme to preserve the SUSY structure non-perturbatively!

**lattice theory**

The lattice theory will make it possible again!!