Bootstrapping Controversies

@阪大、2015/1/27

Tomoki Ohtsuki (Kavli IPMU)

based on arXiv:1404.0489, 1407.6195

with Yu Nakayama (IPMU/Caltech)

1. Introduction

Role of 3d CFTs in real physics

Why are conformal field theories (CFTs) so important?

=> In the IR limit RG flows, we expect every QFT to approach its scale invariant (≅ conformally invariant) fixed point (FP).

=> Finite temperature continuous phase transitions of 3+1 dim system (e.g. in early universe, condensed matter systems) are expected to be described by d=3 CFTs.

=> "Whether an IR-stable CFT with desired global symmetry exists or not" is crucial. If not, the transition has to be of first order.

How can we actually know the (non)existence of CFTs?

Known theoretical techniques

- Perturbative expansions
  => Only asymptotic and need resummation.
  However, the procedure relies on artisans technique!

- Functional renormalization group
  => must involve uncontrollable truncation.

- Lattice Monte-Carlo methods
  => Becomes heavy in taking the long-wavelength limit (i.e., large # of sites) to judge scaling behavior, especially when the interaction is complicated.

Often these give conflicting conclusions

Huge controversies over $O(n)\times O(m)$ LGW models

- Three dimensional $O(n)\times O(m)$-symmetric Landau-Ginzburg-Wilson models have rich physical realizations. The most famous ones are certain frustrated magnets.

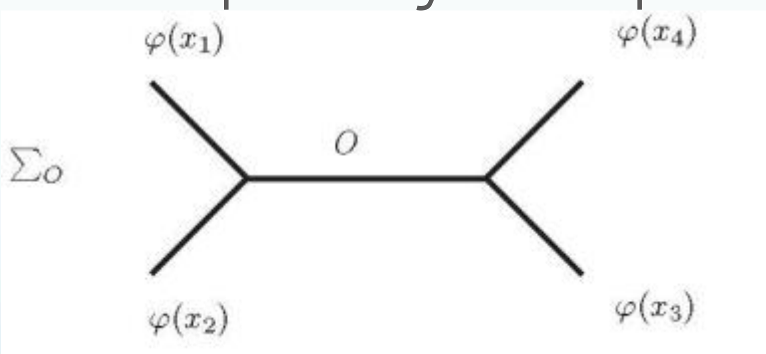
- $U(1)\downarrow A$ - restored 2-flavor QCD chiral phase transitions are also included (when $n=4,\ m=2$) in this family.

- "What kinds of FPs are present for what values of $n,\ m$" has been controversial over decades! Depending on the methods we employ we obtain different answer.

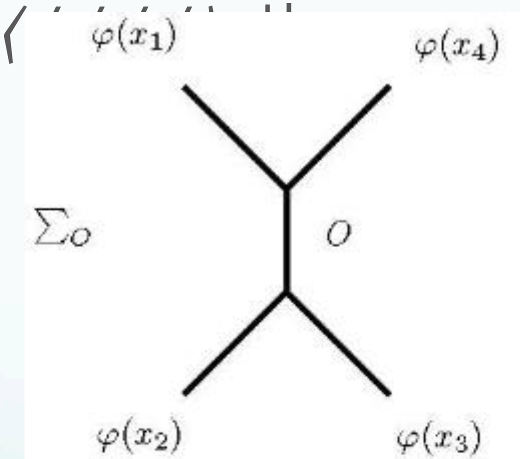
Conformal bootstrap program

(Ferrara, Gatto, Grillo, `73), (Polyakov, `74)

- An attempt to extract non-trivial information for CFTs (in arbitrary dimensions) solely from its basic requirements.
- Especially for 4-pt function ⟨ ing



$=$



are required along with conformal invariance & unitarity.

Hidden power of the program

<u>Remarkably, some non-trivial higher dim CFTs have been "solved" by the program!</u>

- The d=3 Ising model and its Wilson-Fisher family (El-Showk, *et. al.* `12, `13, and `14). They reported the critical exponents

  $$\nu=0.62999(5),\ \eta=0.03631(3),\ c=0.94653(1).$$

- d=3 $O(N)$ vector models at criticality (Kos, Poland, Simmons-Duffin, `13)
- And more… (including SUSY extentions, d=5 $O(N)$ models)

Aim of this talk

- Discuss what the conformal bootstrap can tell you about the (non)existence of the $O(n)\times O(m)$ LGW model fixed points.

Outline

1. Introduction
2. Some aspects of $O(n)\times O(m)$ symmetric Landau-Ginzburg-Wilson models
3. Bootstrap appetizer
4. $O(n)\times O(3)$: Less controversial regime
5. $O(n)\times O(2)$: Controversial regime & discussions

2. Some Aspects of $O(n)\times O(m)$-symmetric Landau-Ginzburg-Wilson models

Lagrangian description

- Consider a Lagrangian formed from scalar field,

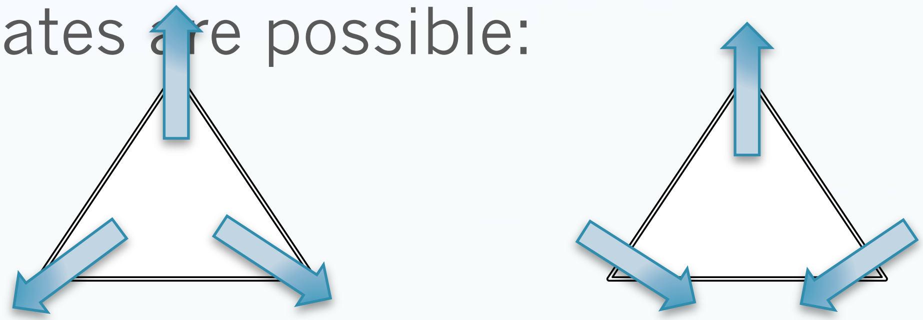
$$\mathcal{L} = \sum_{i,a} \partial_\mu \phi_{ia} \partial_\mu \phi_{ia} + u \left(\sum_{i,a} \phi_{ia}\phi_{ia} \right)^2$$
$$+ v \sum_{i,j,a,b} \left(\phi_{ia}\phi_{ja}\phi_{jb}\phi_{ib} - \phi_{ia}\phi_{ia}\phi_{jb}\phi_{jb} \right)$$

  Here the indices run over $i,j=1,\dots,n$, $a,b=1,\dots,m$, i.e., ϕ transforms as a bifundamental of $O(n)\times O(m)$.

- The term proportional to u is actually $O(nm)$ - invariant. When $v\neq 0$, $O(n\,m)\to O(n)\times O(m)$ explicitly.

Realization I: Frustrated spin systems

- Imagine n–component anti-ferromagnetic spin systems on triangular lattice. When $n=2$, two ground states are possible:



- The effective field theory at criticality is described by $O(n)\times O(2)$ LGW model (Kawamura `85).
- For most important cases of $O(3)\times O(2)$, the transition order has been controversial over decades both in lattice-MC and experiments. RG situations are even worse...

restored
Chiral Phase Transition

- In 2-flavor QCD chiral phase transition, the relevant DOF comprises of mesons, the dynamics of which at criticality is described by some $d=3$ $SU(2)_L \times SU(2)_R \cong O(4)$-symmetric LGW model (Pisarski-Wilczek `84).

- In finite temperature anomalous $U(1)_A$ tends to be restored and symmetry might enhance into

$$SU(2)_L \times SU(2)_R \times U(1)_A \cong O(4) \times O(2)..$$

- Above T_c, the exact restoration of $U(1)_A$ for effective LGW theory is pointed out! (Aoki-Fukaya-Taniguchi `12)

Scaling behaviors vs $U(1)\downarrow A$ restoration

- In (Pisarski - Wilczek `84), they concluded that IR-stable FPs are absent in $O(4)\times O(2)$ LGW model based on 1-loop computation. The conclusion had remained unchanged up to 3-loop order.
- => If $U(1)\downarrow A$ is restored, the chiral transition should be first order without fine tuning!
- Most lattice QCD results support 2nd order transition...

5-loop RG and criticism

- (Pellissetto *et. al.*) claims resummed 5-loop series reveals stable FPs both for $O(3)\times O(2)$ and $O(4)\times O(2)$, but the results are criticized in several contexts :

1. In FRG no such FPs are found. (However recently in 1410.0985, $O(4)\times O(2)$ FPs might be found by going to higher order truncation, but with the different critical exponents.)
2. The parameter in the resummation must be chosen by artisans.
3. Zeroes of β function must be derived without expanding in ε. In other words, there's no such theories in the neighborhood of $d=4$ and they arise only near $d=3$.
4. Even higher loop series might modify the conclusions. As an example, in the Heisenberg model, the known fixed point disappears at 3-loop and resurrects at 4-loop order.

To get out of the "swamp"...

- Non-perturbative evidence from controllable computation is desired.
- This is what we give using the conformal bootstrap method.

3. Bootstrap Appetizer

Conformal four-point function

- Consider (identical) scalar 4pt function in general, radially quantized Euclidean CFT. It can be computed if one knows the OPE $\phi\times\phi$. This is known as “conformal block decomposition.”

- For example in the configuration $|x_1|>|x_2|>|x_3|>|x_4|$, its conformal block decomposition takes the form
$$0\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)0 = \sum_{O\in\phi\times\phi} \lambda_{\phi\phi O}^2 \, g_{\Delta_O, l_O}(z,z) / x_{12}^{2\delta} x_{34}^{2\delta}$$

- λ is the OPE coefficients, δ the operator dimension of ϕ and $g_{\Delta,l}(z,z)$ is the conformal block function.

The bootstrap equation

- In another configuration like $|x_3|>|x_2|>|x_1|>|x_4|$, you have another decomposition of the same correlator.

$$\langle 0|\phi(x_3)\phi(x_2)\phi(x_1)\phi(x_4)|0\rangle = \sum_{O\in\phi\times\phi} \lambda_{\phi\phi O}^2 \frac{g_{\Delta_O,l_O}(1-z,1-\bar{z})}{x_{23}^{2\delta} x_{14}^{2\delta}}$$

- Requirement for the analyticity of the correlator leads to **the bootstrap equation** which holds for every CFT.

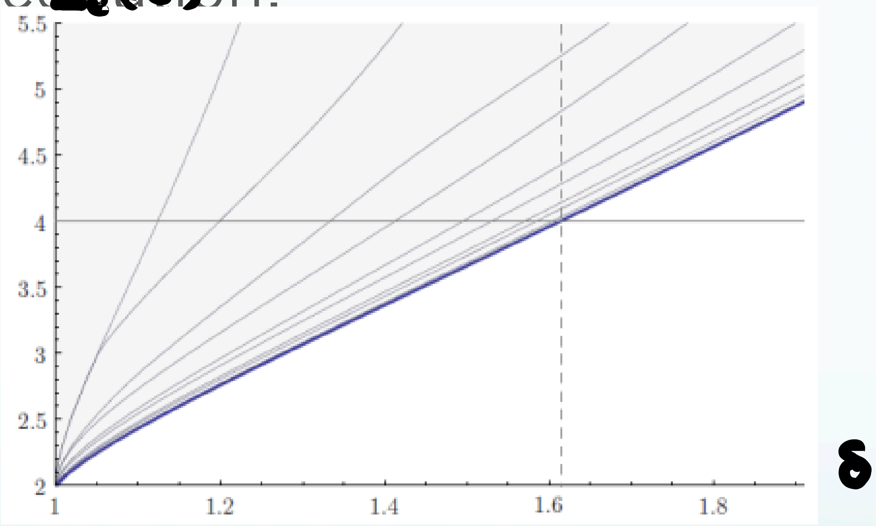
$$0=\sum_{O\in\phi\times\phi} \lambda_{\phi\phi O}^2 \left\{\frac{g_{\Delta_O,l_O}(z,\bar{z})}{x_{12}^{2\delta} x_{34}^{2\delta}} - \frac{g_{\Delta_O,l_O}(1-z,1-\bar{z})}{x_{13}^{2\delta} x_{24}^{2\delta}}\right\}$$

- This equation alone (i.e., without any reference to specific Lagrangian) imposes nontrivial constraint on the spectrum!

Numerical Results

(Rattazzi, Rychkov, Tonni, Vichi, `08)

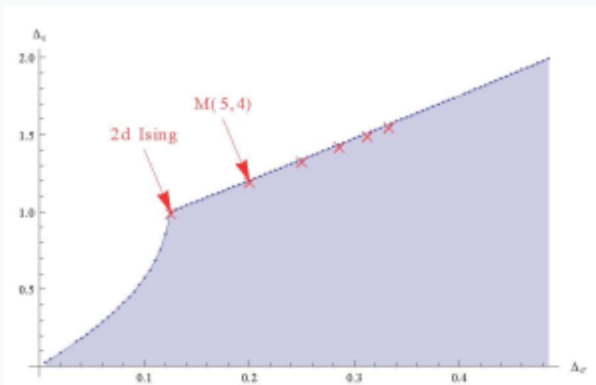
- By numerical methods the following curve $\Delta_c(\delta)$ can be drawn for <u>general</u> d=4 CFTs from the bootstrap equation.

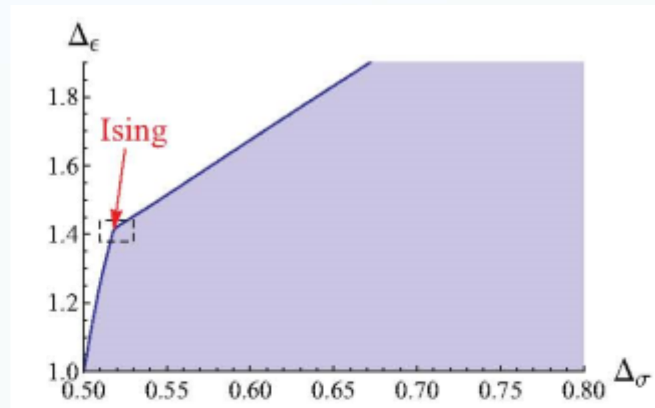


- The precise meaning: There must ∃ a scalar operator with dimension below $\Delta_c(\delta)$ in $\phi\times\phi$ OPE. (Recall δ=dim ϕ.)

“solution” to the d > 2 Ising

- For $d<4$, the behavior of $\Delta_c(\delta)$ is singular!





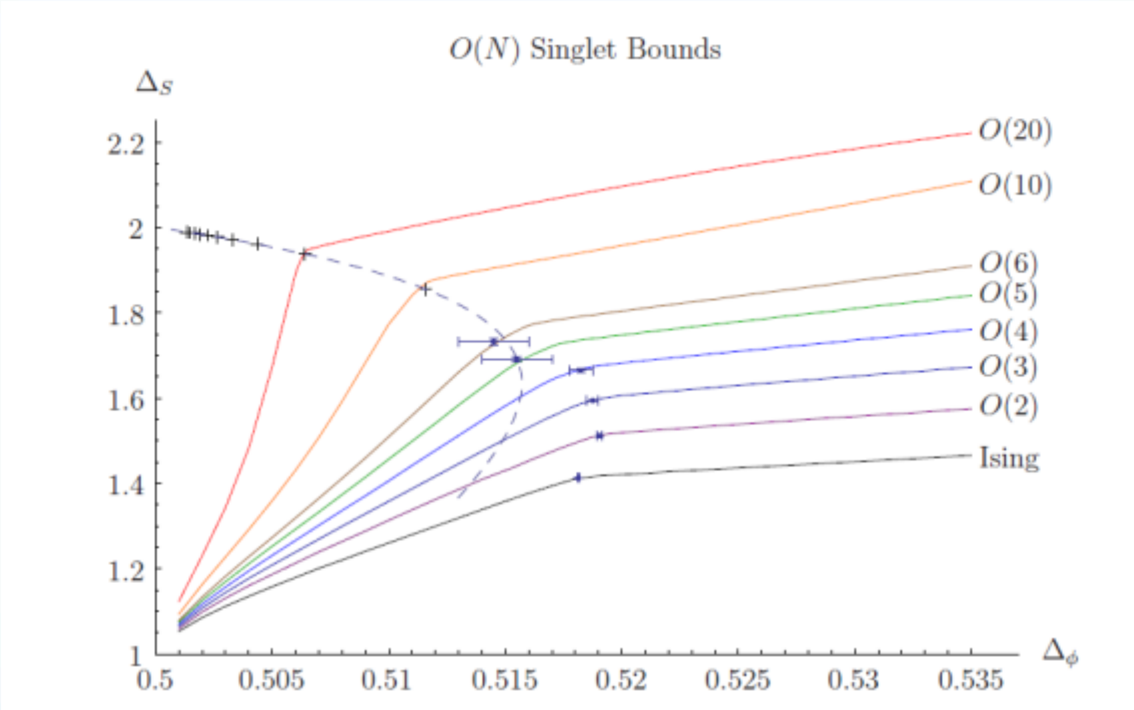
Conversely assume the 3d Ising model actually sits at the “kink”. Then you will obtain critical exponents and various OPE coefficients with tremendous precision. => “Solution”

- The fundamental reason for the phenomena is mysterious…

symmetry: $O(N)$ as an example

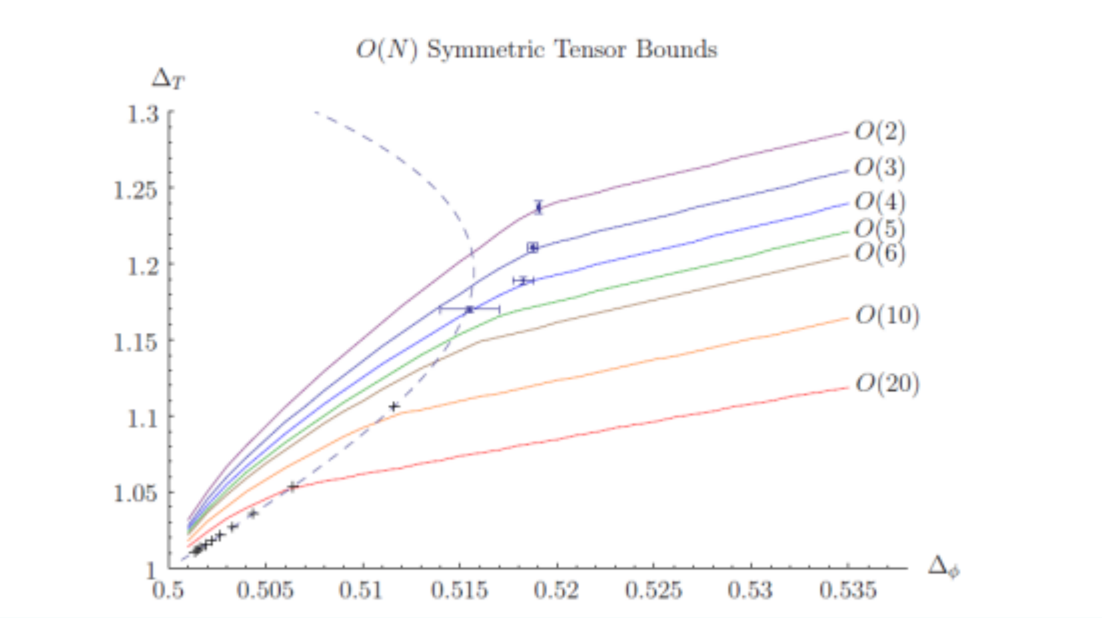
- Assume the presence of global symmetry and let $\phi_I(x)$ transforms according to an irreducible representation R.
- In $\phi_I \times \phi_J$ OPE, additional structures appear depending on the irreducible components in $R \otimes R$.
- Important example: Let ϕ_i be an $O(N)$ fundamental. $\phi_i \times \phi_j$ OPE includes singlet(S), anti-symmetric tensor (A), symmetric-traceless tensor (T) representations.
- Now the Bootstrap eq. looks vectorial and encodes the informations about group structure.

d=3 $O(N)$ results (Kos, Poland, Simons-Duffin, `13)



- The precise meaning: ∃ a scalar operator in the S-rep in $\phi_i \times \phi_j$ OPE with dimension below $\Delta_{c,S,N}(\delta)$.

d=3 $O(N)$ results for symmetric tensors



<u>Lesson:</u> We can separately write down the curve for each
global symmetry channel in the intermediate states.

4. $O(n)\times O(3)$: Less Controversial Regime

Based on arXiv:1404.0489

$O(n)\times O(3)$ with $n \gg 3$ as a laboratory

- We chose to begin with well-established models, avoiding controversial (though physically important) cases.
- We start from $O(n)\times O(3)$ models with $n \gg 3$ as a laboratory: due to large n solvability they are well-known.
- In these cases there are two more FPs in addition to Gaussian and $O(3n)$-Heisenberg FPs.

Can we observe (or "solve") these additional FPs as we could for the 3d Ising model??

Huge bootstrap equation for $O(n)\times O(m)$ ...

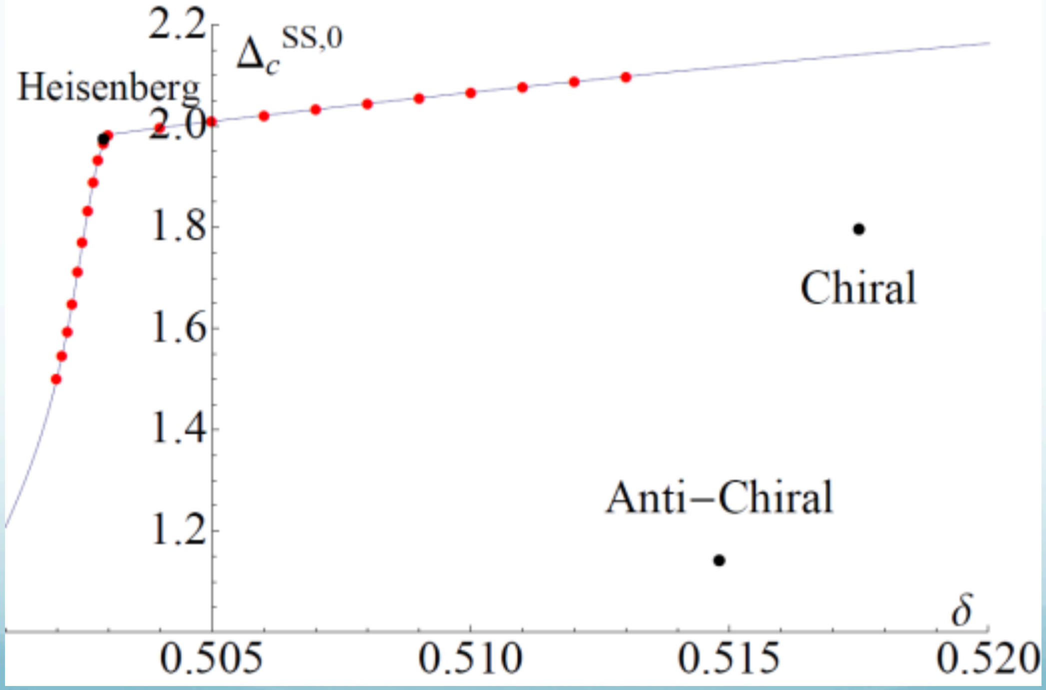
- Assume the precence of $O(n)\times O(m)$- bifundamental scalar $\phi\downarrow ia$. OPE $\phi\downarrow ia \times \phi\downarrow jb$ contains 9 irreducible channels. According to $O(N)$ terminology, they are

  SS, ST, SA, TS, TT, ...

- Now the vectorial bootstrap eq. is quite lengthy and the computational cost is ~100 times heavier then the Ising case!

operator in $O(15)\times O(3)$ model

- As the first sample we took $O(15)\times O(3)$ model, where the presence of non-Heisenberg FPs (called "chiral" and "anti-chiral") is undoubtable.
- The bound for SS, spin 0 operator is shown by red dots:



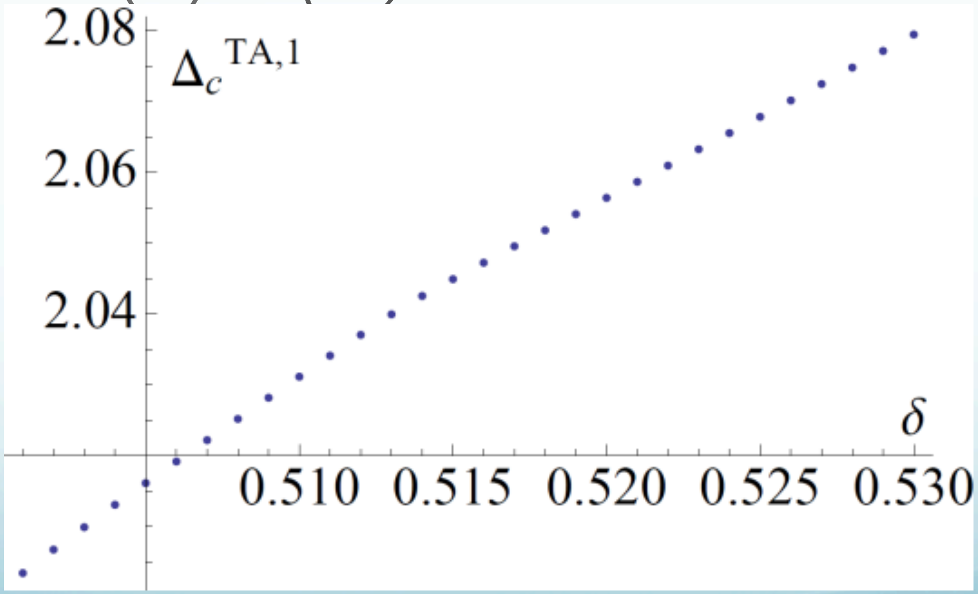
Symmetry enhancement

- Within the precision the bound is identical to that of O(45).
- Such "symmetry enhancement" has been reported for the 4d SU(N)/SO(2N). Is it a general mathematical statement?
- large N prediction for the additional FPs are well-below the bounds. There are two aspects:

1. 😊 The upper bounds are satisfied and consistent!
2. 😢 We cannot observe any symptom of these fixed points from this computation. Can't we "solve" them??

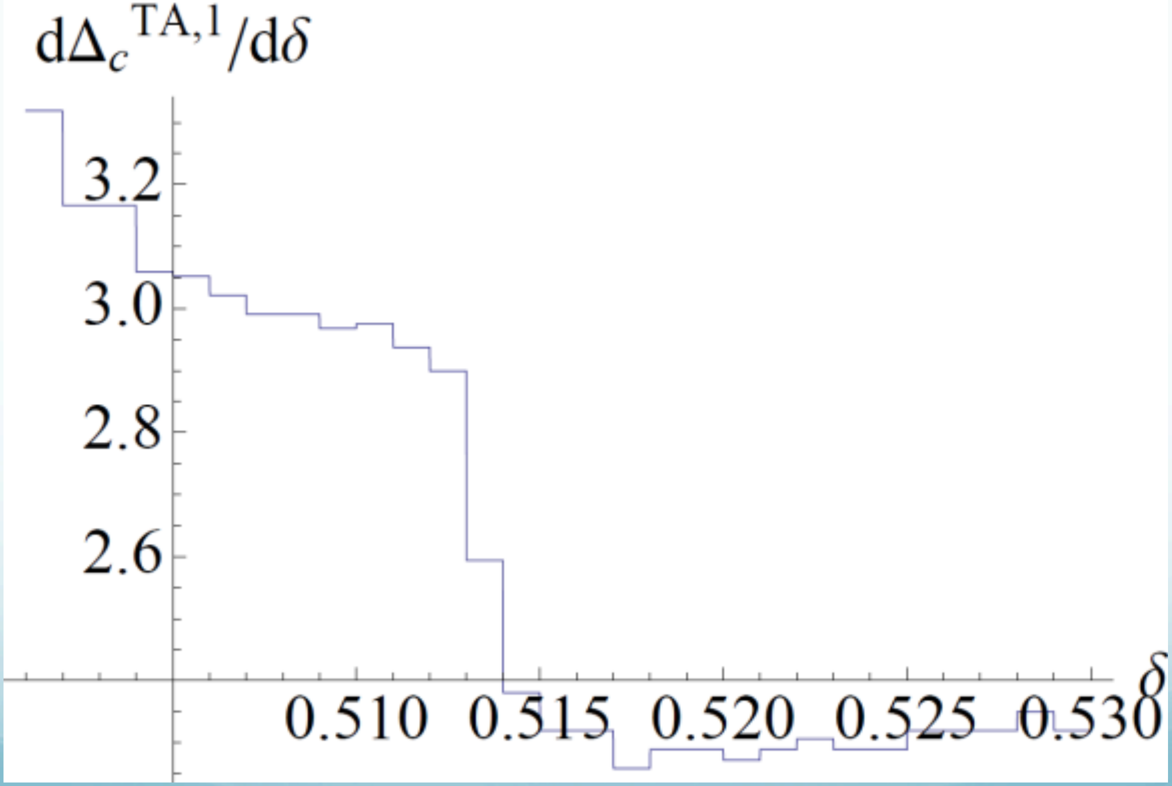
Salvation : Bounds for spin 1 operator in TA sector

- Then we computed the dimension bounds for spin 1 operator in TA representation. Note that such operator has dimension exactly 2 at $O(nm)$ Heisenberg fixed points but not when $O(nm)$ is broken to $O(n)\times O(m)$.



“Kink” in the bound

- When differentiated, it becomes apparent that the slope changes around $\delta \cong 0.515$.



Spectral study

- (El-Showk, Paulos `12) has shown that once a CFT saturate this kind of bounds, spectrum contained in $\phi_I \times \phi_J$ can be uniquely reproduced from the bootstrap output.

- Our result: $(\Delta_\phi, \Delta^{SS}) = (0.515, 1.16)$

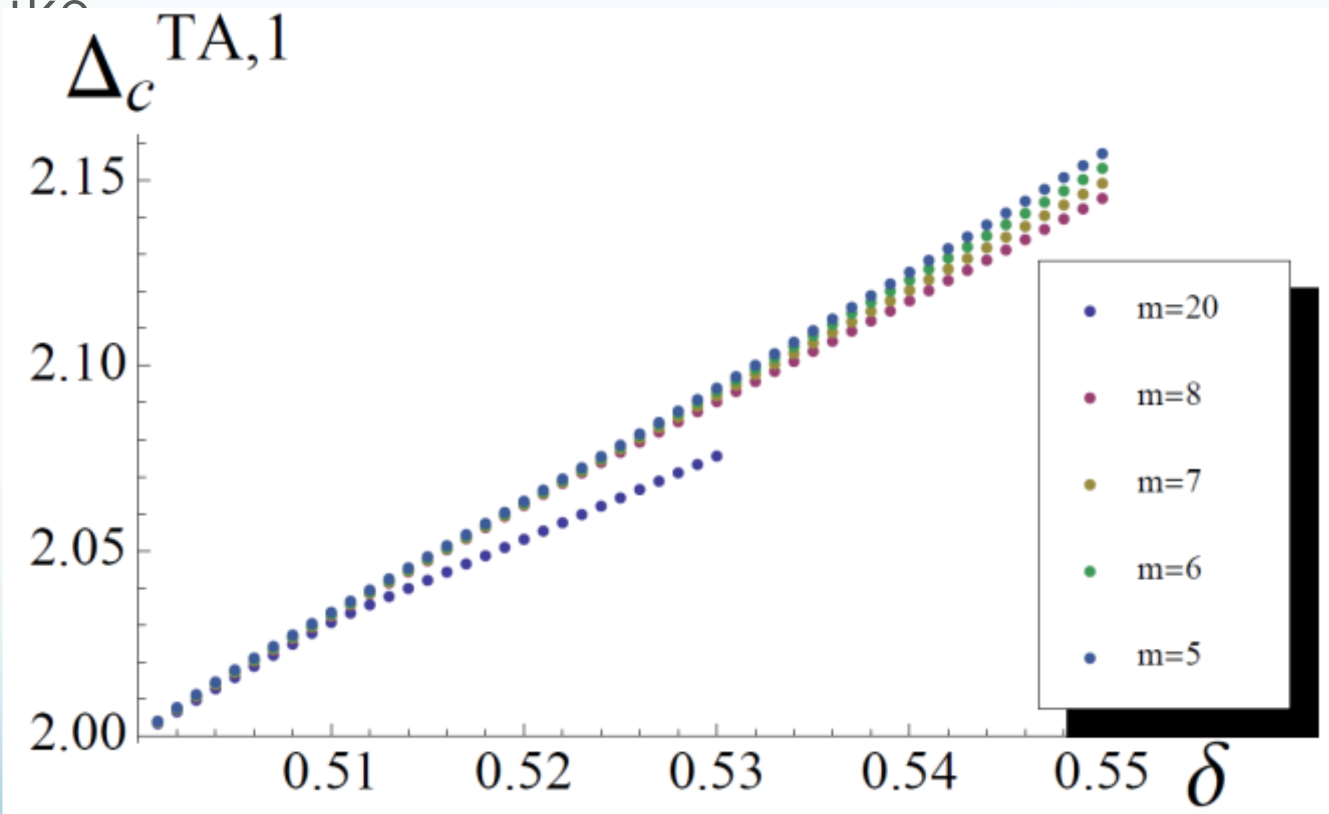
  Note: although this CFT saturate $\Delta_{c,TA}(\delta)$, it may not do so for the bound in the other sector like $\Delta_{c,SS}(\delta)$!

- The $1/n$ – prediction for "anti-chiral" fixed point : $(\Delta_\phi, \Delta^{SS}) = (0.5148, 1.142)$

=>anti-chiral fixed point is observed!

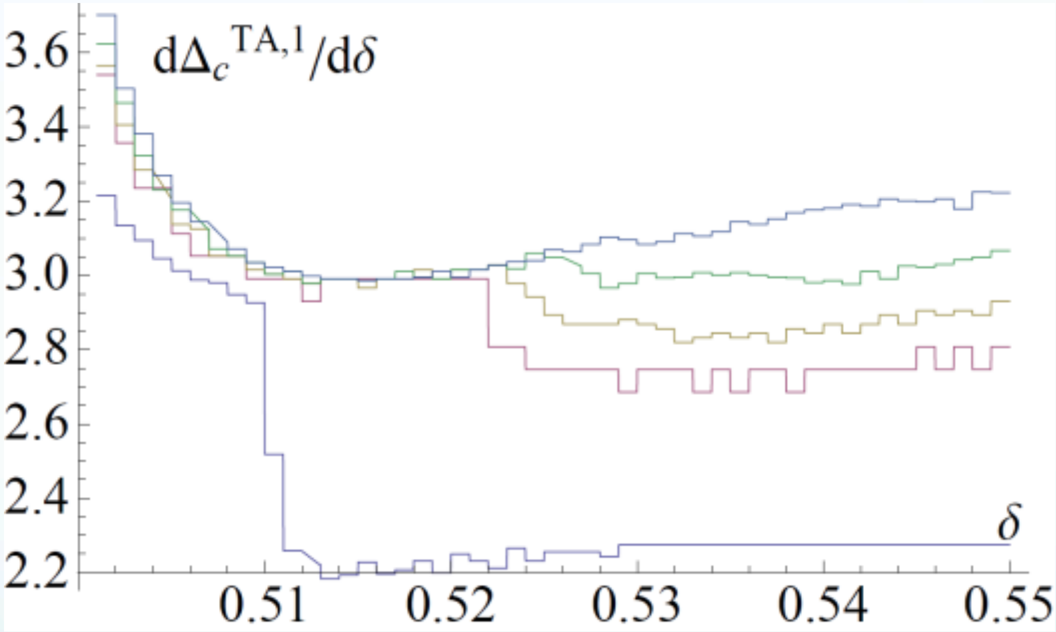
$O(n)\times O(3)$ family

- Varying n, the bounds $\Delta_{c,TA}(\delta)$ changes its form like



Slope change disappearance

- Around n=6~7, the kink in $\Delta\downarrow c,TA\ (\delta)$ disappears.



- According to large n analysis, such a fixed point disappears at n=7.3 .

Summary for $O(n)\times O(3)$

- We examined operator dimension bounds for $O(15)\times O(3)$ model in various global symmetry sector and found that the one in TA sector is saturated by the anti-chiral fixed point. => It is "solvable" as in the $d=3$ Ising!

- For smaller values of n, the kink present in spin 1 TA sector bounds of $O(n)\times O(3)$ model disappears. $=>$Might be a reflection of the conformal window.

- This is the first example where we can observe multiple interacting CFTs in single bootstrap eq.

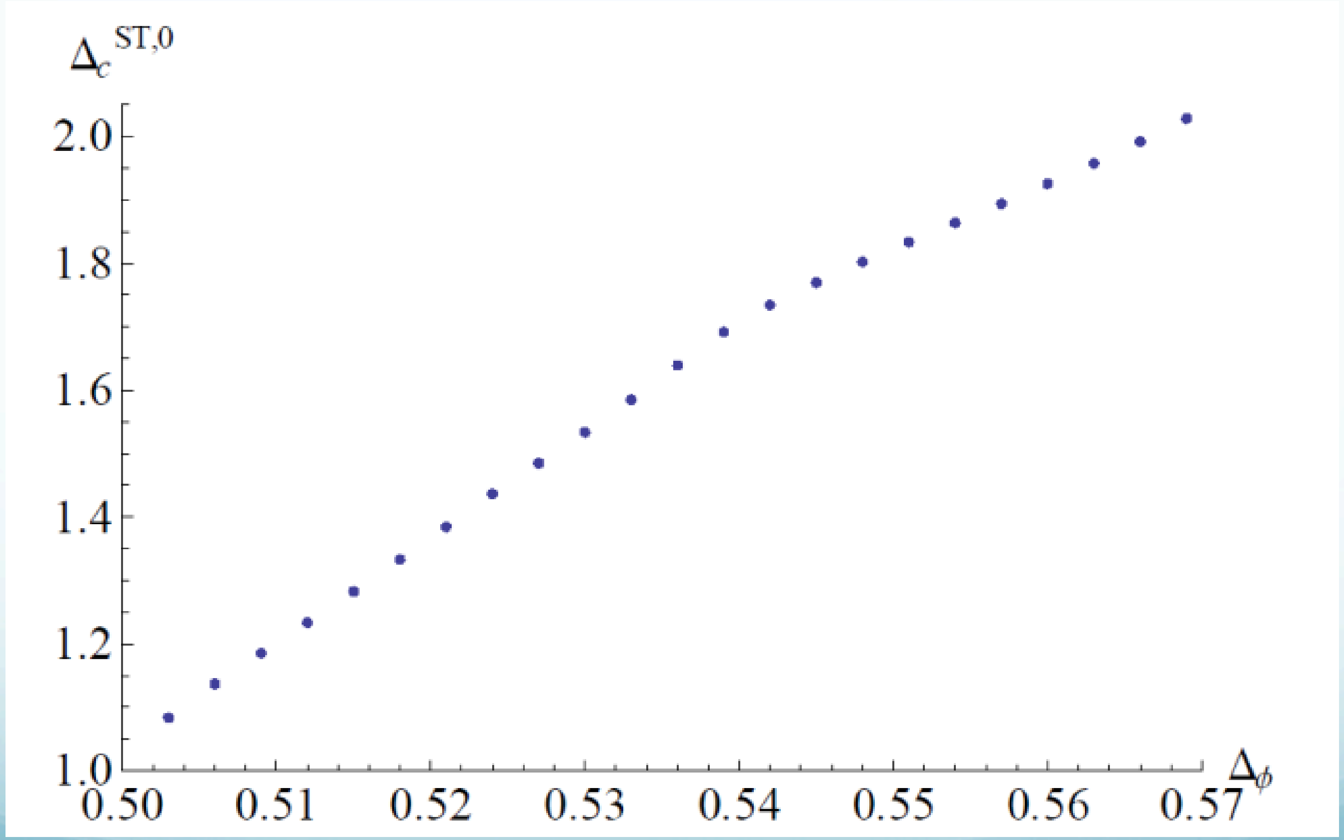
- Conclusion: Everything is consistent with the bootstrap!

5. $O(n)\times O(2)$: Controversial Case & Discussions

Based on: arXiv:1407.6195

frustrated magnet transitions

- For $O(3)\times O(2)$, the bound for ST sector look like:



The spectra agree!

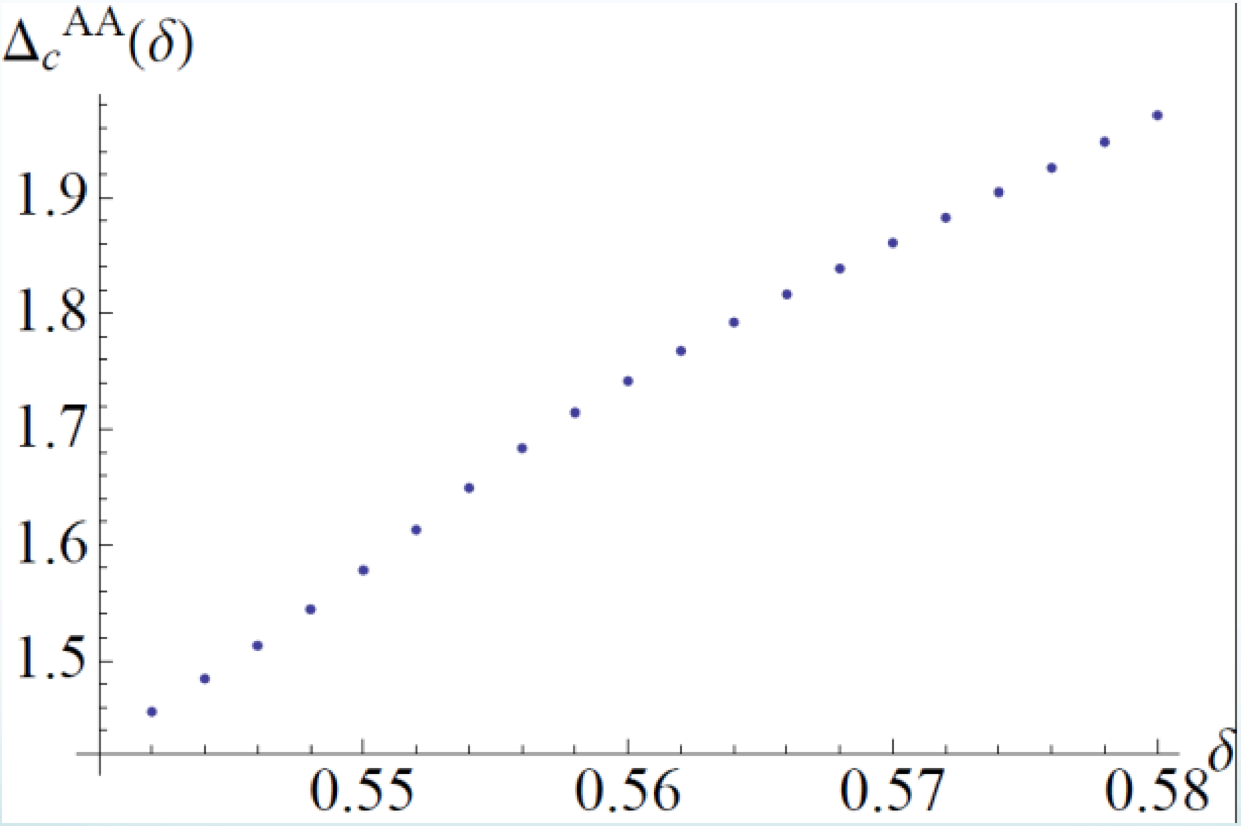
	Δ_ϕ	$\Delta_{\rm SS}$	$\Delta_{\rm ST}$	$\Delta_{\rm TS}$	$\Delta_{\rm TT}$	$\Delta_{\rm AA}$
bootstrap	0.539(3)	1.42(4)	1.69(6)	1.39(3)	1.113(3)	0.89(2)
MS	0.54(2)	1.41(12)	1.79(9)	1.46(8)	1.04(11)	0.75(12)
MZM	0.55(1)	1.18 (10)	1.91(5)	1.49(3)	1.01(4)	0.65(13)

- The spectra read off around the kink and the higher order $\mathcal{MS}$ results agree within systematic errors.

- **Most natural explanation: the fixed point actually exists!**

- According to the perturbative analysis, this is IR-stable.

$O(4)\times O(2)$: Signal of the chiral phase transition CFT



The spectral agreement

	Δ_ϕ	Δ_{SS}	Δ_{ST}	Δ_{TS}	Δ_{TT}	Δ_{AA}
bootstrap	0.556(6)	1.54(6)	0.83(3)	1.044(3)	1.26(2)	1.70(6)
MS	0.56(3)	1.68(17)	1.0(3)	1.10(15)	1.35(10)	1.9(1)
MZM	0.56(1)	1.59(14)	0.95(15)	1.25(10)	1.34(5)	1.90(15)

- Again they agree and we conjecture that the FP exists.
- IR stable according to the perturbative results.

Summary & Discussions

- Despite various criticism, resumed perturbative RG seems to be robust from the comparison with the bootstrap.
- In particular certain frustrated Heisenberg models, i.e., $O(3)\times O(2)$ LGW model can transit continuously.
- Even when $U(1)_A$ is restored, 2-flavor QCD chiral phase transition could be of second order!! We predicted the critical exponents most precisely.

Theoretical backup needed?

- Our working hypothesis “kink => CFT” has not been rigorously proven even for the simplest cases. At this stage our results are phenomenological.
- The deeper understanding of the bootstrap program will revolutionize modern physics.

Thank you for your attention!!

In (near?) future...