Naturalness, conformal symmetry and duality

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The Content

1．Introduction

Message, Status, Strategy

【Message】

There is a possibility that physics is hidden behind the standard model !

Key word : (Hidden) symmetries

【Present status】

No evidence for superpartners and Kaluza-Klein modes @ LHC

No evidence for proton decay
@ Kamioka

➔ No evidence for physics
beyond the standard model

Supersymmetry, Extra dimensions,
Grand unification !?

Suspicious look

【Present status】

No evidence for superpartners and Kaluza-Klein modes @ LHC

No evidence for proton decay @ Kamioka

➔ No evidence for physics beyond the standard model

➔ Problems relating Higgs mass would be revisited !?

Radiative corrections on Higgs mass

$$\delta m_h^2 = C_h \Lambda^2 + \boxed{C'_h m_h^2 \ln \Lambda^2 / m_h^2} + \boxed{\sum_k C''_{hk} M_k^2 \ln \Lambda^2 / M_k^2} + \cdots$$

Contributions from the SM particles

Contributions from other particles than the SM ones

Λ : a cutoff scale

Problems can be classified into two types.

【Quadratic divergence problem】

$C_h\Lambda^2 >> m_h^2$ ➔ Unnatural because of fine tuning?

$m_{h\,\mathrm{phys}}^2 = m_h^2 + \delta m_h^2$

$$\delta m_h^2 = \boxed{C_h\Lambda^2} + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2$$
$$+ \boxed{\sum_k C_{hk}'' M_k^2 \ln \Lambda^2 \big/ M_k^2} + \cdots$$

Contributions from heavy particles other than SM ones

$C_{hk}'' M_k^2 >> m_h^2$ ➔ Unnatural because of fine tuning?

【Gauge hierarchy problem】

These problems are serious ?

[A possible answer]

Not so serious because they do not threaten the consistency of the theory.

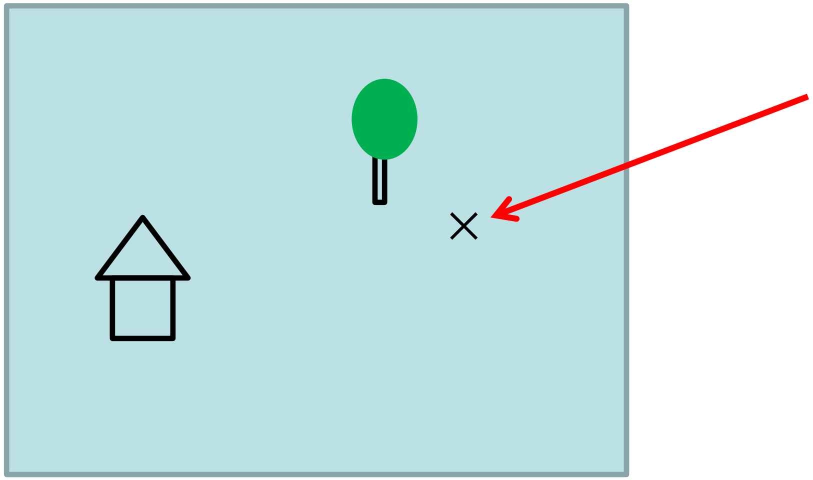
The infinities are renormalized.

$$m_h^2 + \delta m_h^2 \Rightarrow m_{h\mathrm{R}}^2$$

That’s as maybe, but there would be no future in this direction.

【Standpoint】

We are treasure hunters ! Suppose that there is an old treasure map.



Hidden treasure

If the map is genuine and you believe it, you have a chance to get the treasure. ☺

But, if you don’t believe it, you won’t get it. ☹

So let us believe that the problems relating Higgs boson mass are real ones, and we’ ll have a chance to arrive new physics. ☺

【Strategy・Approach】

Let us attack the problems, under the assumption that some exact symmetries are hidden behind the SM and they become key persons !

Q. Who are they ?

【Announcement】 Possible candidates

- Symmetry irrelevant to Action

→ Star in Sec. 2

- Symmetry that the SM particles are singlets.

→ Star in Sec. 3

2. Naturalness

Quadratic divergence, Naturalness, Duality

Y.K., “Naturalness, conformal symmetry and duality”, *Prog. Theor. Exp. Phys.* **11**, 113B04, (2013) arXiv:1308.5069 [hep-ph] .

First of all, let us review the quadratic divergence problem and its related topics, and then we reconsider the essence of the problem

and give a new way out !

【Quadratic divergence problem】

$$\delta m_h^2 = \boxed{C_h \Lambda^2} + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \cdots$$

Λ : a cutoff scale

$$\sqrt{C_h}\,\Lambda >> m_h$$

unnatural because we need a fine tuning ??

Way outs to escape a fine tuning,

$C_h \approx 0$ and/or $\Lambda \leq O(1)\mathrm{TeV}$

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 / m_h^2 + \cdots$$

At the one-loop level,

$$C_h \cong \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$$C_h = 0 \Rightarrow m_h^2 \cong 4m_t^2 - 2M_W^2 - M_Z^2$$

（Veltman condition）

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \cdots$$

At the one-loop level,

$$C_h \cong \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$$C_h = 0\big|_{M_Z} \Rightarrow m_h \cong 320\mathrm{GeV}$$

Unrealistic?

$$C_h \approx 0\big|_{M_{\mathrm{Pl}}}$$

Y. Hamada, H. Kawai & K. Oda,
Phys. Rev. D**87**, 053009 (2013) .

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \cdots$$

$$C_h \cong \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$\Lambda \le O(1)\mathrm{TeV}$ ➔**New Physics**
@ Terascale

Candidates：SUSY, Compositeness,
Extra dimensions, ･･･

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \cdots$$

$$C_h \cong \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$\Lambda \le O(1)\mathrm{TeV}$ → **New Physics @ Terascale**

Problem revisited because of no evidences

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \cdots$$

$$C_h \cong \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$\Lambda \le O(1)\mathrm{TeV}$ → **New Physics @ Terascale**

Reconsider the problem from the viewpoint of symmetry. → **Naturalness**

What is naturalness?

G. ’t Hooft, (1979)

The concept based on the dogma,

“at any energy scale μ, a physical parameter $a(\mu)$ is allowed to be very small, only if the replacement $a(\mu)=0$ would increase the symmetry of the system.”

$$\delta a = a\, h(\Lambda^2) + \cancel{k(\Lambda^2)}$$

$$\xrightarrow[a\to 0]{} 0$$

by some symmetry

Hereafter,
we refer to a parameter with
the feature that the symmetry
of the system enhances when
its value approaches zero as
a natural parameter.

【Example】 Electron mass m_e

$m_e \to 0$ ➔ **chiral symmetry**

$$\psi_L \to e^{i\theta_L}\psi_L, \quad \psi_R \to e^{i\theta_R}\psi_R$$
$$\left(\theta_L, \theta_R : \text{real parameters}\right)$$

For $\theta_L = -\theta_R$,

$$\left\langle \partial_\mu j_A^\mu \right\rangle = 2i\left(m_e + \delta m_e\right)\left(\psi_L^\dagger \psi_R - \psi_R^\dagger \psi_L\right) + \frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$$

$$\delta m_e = \frac{3\alpha}{4\pi} m_e \left(\ln\frac{\Lambda^2}{m_e^2} + \frac{1}{2} \right)$$

For $m_e \to 0$, $\delta m_e \to 0$ and then

$\left\langle \partial_\mu j_A^\mu \right\rangle \to 0$ up to axial anomaly

Quantum corrections respect the chiral symmetry.

【Supplement】

$m_e \to 0$ ➔ Scale invariance

$$\psi_L \to e^{\rho/2}\psi_L, \quad \psi_R \to e^{\rho/2}\psi_R$$

$(\rho : \text{real parameter})$

$$\langle T_\mu^\mu \rangle = 2(m_e + \delta m_e)(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) + \frac{\beta_\alpha}{\alpha} F^{\mu\nu} F_{\mu\nu}$$

$$\delta m_e = \frac{3\alpha}{4\pi} m_e \left(\ln \frac{\Lambda^2}{m_e^2} + \frac{1}{2} \right)$$

For $m_e \to 0$, $\delta m_e \to 0$ and then

$\langle T_\mu^\mu \rangle \to 0$ up to trace anomaly

The conformal symmetry plays the same role as chiral symmetry does.

In the Standard Model, chiral symmetry has a superior quality to conformal symmetry.

The chiral symmetry such as $SU(2)_L \times U(1)_Y$ is a local one and unbroken perturbatively and anomalously.

The conformal symmetry is a global one and broken down explicitely and anomalously.

The chiral gauge symmetry is broken down spontaneously by the VEV of Higgs boson v=246GeV, and fermions acquire masses

$$m_f = y_f \mathrm{v} / \sqrt{2} \,.$$

The smallness of $m_f = y_f \mathrm{v} / \sqrt{2} << M_{\mathrm{Pl}}$ stems from the smallness of $\mathrm{v}(<< M_{\mathrm{Pl}})$.

The (chiral) gauge symmetry enhances in the limit of $\mathrm{v} \rightarrow 0$.

Is a scalar mass m_ϕ a natural parameter or not ?

$m_\phi \to 0$ ➔ **Scale invariance ?**

$$\langle T_\mu^\mu \rangle = 2\left(m_\phi^2 + \delta m_\phi^2\right)\phi^2 + \sum_k \beta_k O_k$$

O_k : Operators with the mass dimension 4

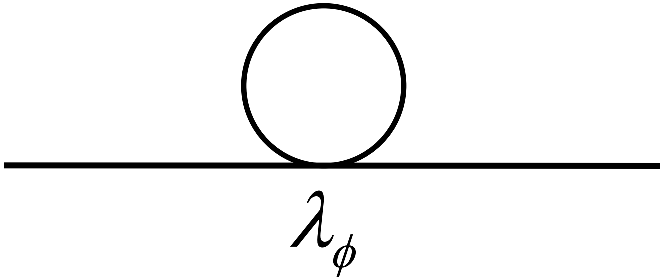
β_k : β functions

For $m_\phi^2 \to 0, \quad \delta m_\phi^2 \to 0$?

$\delta m_\phi^2 \propto m_\phi^2$?

In ϕ^4 theory,

$\delta m_\phi^2 =$



$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m_\phi^2} = \frac{\lambda_\phi}{2}\frac{\pi^2}{(2\pi)^4}\int_0^{\infty}\frac{p^2dp^2}{p^2+m_\phi^2}$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\infty}dp^2 - m_\phi^2\int_0^{\infty}\frac{dp^2}{p^2+m_\phi^2}\right)$$

Regularization

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2}dp^2 - m_\phi^2\int_0^{\Lambda^2-m_\phi^2}\frac{dp^2}{p^2+m_\phi^2}\right)$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\Lambda^2 - m_\phi^2 - m_\phi^2\ln\frac{\Lambda^2}{m_\phi^2}\right)$$

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\left(\Lambda^2 - m_\phi^2 - m_\phi^2 \ln\frac{\Lambda^2}{m_\phi^2}\right)$$

$$\text{For } m_\phi^2 \to 0, \quad \delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\Lambda^2 \neq 0$$

➔ **The scale invariance is not recovered, and hence it is widely thought that m_ϕ is not a natural parameter. This can be the root of quadratic divergence problem.**

Is it true ?

Ambiguities can exist in the regularization procedure.

Such ambiguities, in most case, are resolved by considering symmetries realized manifestly.

Quantities depending on the regularization method should be subtracted, unless the subtraction induces any physical effects.

【Bardeen's argument】

Anomalous relation

W. A. Bardeen,
(1995) SI @ Ontake

$$\left\langle T_{\mu}^{\mu} \right\rangle = m_h^2 + \delta m_h^2 + \sum_k \beta_k O_k$$

For $m_h^2 \rightarrow 0$ and $\beta_k \rightarrow 0$, the classical scale invariance should be restored.

$$\delta m_h^2 = \cancel{C\Lambda^2} + C' m_h^2 \ln \Lambda^2 / m_h^2 + \cdots$$

←Ambiguities can exist in the regularization procedure.

Ambiguities can exist in the regularization procedure.

- In the dimensional regularization,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2} m_\phi^2 \left(-\frac{2}{\varepsilon} + \gamma - 1 + \cdots \right)$$

$\gamma = 0.577\cdots$

- Proposal for subtractive renormalization

K. Fujikawa,
Phys. Rev. D**83**, 105012（2011）

- From the viewpoint of the Wilsonian renormalization group,

H. Aoki & S. Iso,
Phys. Rev. D**86**, 013001（2012）

Quadratic div. might be artifact !?

➔ They can be subtracted, unless it induces any effects

➔ Scale invariance is expected to back up the procedure.

That' s as maybe, but

- Scale invariance in eff. th. might be a secondary concept.
- More direct-connected concept ?

Quadratic div. might be artifact !?

➔ They can be subtracted, unless it induces any effects

➔ Scale invariance is expected to back up the procedure.

That' s as maybe, but

【Conjecture】 The subtraction of quadratic div. is justified by a feature in fundamental theory ?

【Expectation】

- Quadratic div. might be artifact of regularization procedure.
- The calculation scheme can be selected by the physics.
- The subtraction of quadratic div. can be justified by a feature.

➔ As the feature,
let us adopt "duality" !

【Basic idea】

An ultimate theory does not induce any large radiative corrections for low-energy fields owing to a symmetry, and such a symmetry is hidden in the standard model.

Cf. K. Dienes, “Solving the hierarchy problem without supersymmetry or extra dimensions: an alternative approach”
Nucl. Phys. B611, 146（2001）

↓

Misaligned supersymmetry

【Assumptions】

(a) There is an ultimate theory with a fundamental scale Λ .

(b) It has a following duality.

$$\mathrm{The\ physics}@E(\geq \Lambda) \sim \mathrm{The\ physics}@E(\leq \Lambda)$$

(b1) The physics is invariant under the duality.

(b2) The physics is only described by one of the two regions.

(c) A remnant of the duality is hidden in quantities of the low-energy physics involved with Λ.

Ex. Quantum corrections on a

$$\delta a = \int_0^\infty f(p^2)dp^2$$

p^2 : Euclidean momentum squared for a massless virtual particles running in the loop

When δa diverges at $p^2 = \infty$ and $p^2 = 0$,

it is ordinarily regularized as

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$$

μ_0 : a fictitious mass parameter .

【Method based on duality】

$$\delta a = \int_0^\infty f(p^2)dp^2 \rightarrow \delta a = \int_{\mu_0^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

$$\left(\xrightarrow[\mu_0^2 \to 0]{} \delta a = \int_0^\infty f(p^2)dp^2 \right)$$

A tentative one

Then,

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 + \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

If a remnant of duality holds with $\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 \Leftrightarrow \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$, from (b2) we obtain

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 .$$

(b2) The physics is only described by one of the two regions.

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 \Leftrightarrow \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

(b1)

For $\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$ **,**

we take $p^2 \to p'^2 = \Lambda^4/p^2$ **as the remnant of duality transformation**

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 \to \int_{\Lambda^4/\mu_0^2}^{\Lambda^2} f(\Lambda^4/p^2)d\left(\Lambda^4/p^2\right)^2 = \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(\Lambda^4/p^2)\frac{\Lambda^4}{p^4}dp^2 \; .$$

(b1) The physics is invariant under the duality.

From (b1),

Unless $f(p^2)$ contains Λ, $f(p^2) = \frac{c_{-1}}{p^2}$

Then, $\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 = \int_{\mu_0^2}^{\Lambda^2} \frac{c_{-1}}{p^2}dp^2 = c_{-1}\ln\frac{\Lambda^2}{\mu_0^2} \; .$

Our procedure can be not a mere regularization, but a recipe to obtain finite physical values, because Λ is (large but) finite and infinities are taken away by the symmetry relating integration variables, like world-sheet modular invariance in string theory.

From world-sheet modular invariance for the closed string,

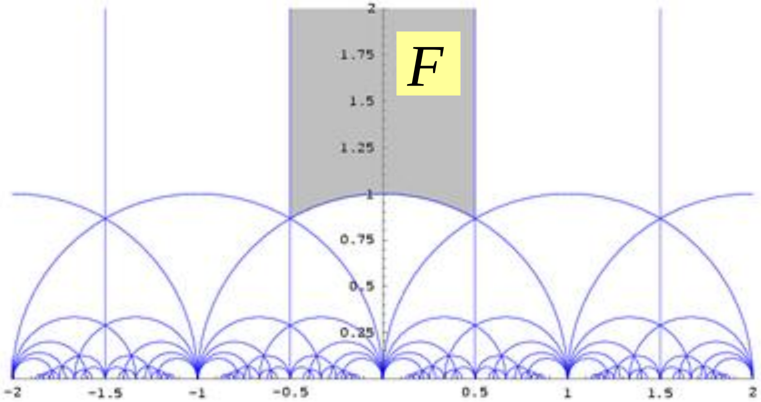
$$\delta a = \int_F \frac{d^2\tau}{\tau_2^2} G(\tau)$$

$$\tau = \tau_1 + i\tau_2$$

$$F = \{\tau : |\mathrm{Re}\,\tau| \le 1/2,\ \ 1 \le |\tau|\}$$

$G(\tau)$: a world-sheet modular invariant function

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad (a, b, c, d \in Z,\ \ ad - bc = 1)$$



$$\left(\tau \to -\frac{1}{\tau}, \quad \tau \to \tau + 1\right)$$

From Wikipedia

In string theory, the world-sheet modular invariance is deeply connected to the consistency of the theory, and radiative corrections should be given in the world-sheet modular invariance form.

In an ultimate theory, the duality is connected to the consistency of the theory.

$$\delta a = \left(\text{Duality invariant terms}\right)$$

In the effective field theory, a remnant of duality is hidden, and it is not connected to the consistency of the theory.

$$\delta a = \int_0^\infty f(p^2)\,dp^2 \Rightarrow \left(\text{Duality invariant terms}\right)$$

Projection is needed.

$$\delta a = \int_0^\infty f(p^2)dp^2 \Rightarrow \left(\text{Duality invariant terms}\right)$$

Projection is needed.

We denote the operation (projection) as $\mathrm{Du}[*]$.

In the case that $f(p^2)$ does not contain Λ,

we expand in a series such as $f(p^2) = \sum_n c_n \left(p^2\right)^n$.

$$\delta a = \mathrm{Du}\left[\int_0^\infty f(p^2)dp^2\right] = \mathrm{Du}\left[\int_0^\infty \sum_n c_n \left(p^2\right)^n dp^2\right]$$

$$= \int_{\mu_0^2}^{\Lambda^2} \frac{c_{-1}}{p^2} dp^2 = c_{-1} \ln \frac{\Lambda^2}{\mu_0^2}$$

Radiative corrections on scalar mass

In the massless case,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2} = \frac{\lambda_\phi}{32\pi^2}\int_0^{\infty} dp^2$$

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\int_{\mu_0^2}^{\Lambda^4/\mu_0^2} dp^2 = \frac{\lambda_\phi}{32\pi^2}\int_{\mu_0^2}^{\Lambda^2} dp^2 + \frac{\lambda_\phi}{32\pi^2}\int_{\Lambda^2}^{\Lambda^4/\mu_0^2} dp^2$$

⬇

$$p^2 \to p'^2 = \Lambda^4 / p^2$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^{\infty} dp^2\right] = 0$$

For the massive case,

- Momentum cutoff method
- Proper time method

【Notice】

Our purpose here is not to specify the duality transf. but to impress the idea of our procedure.

Don’t be nervous about the details.

Using momentum cutoff method,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\left(\int_0^\infty dp^2 - m_\phi^2 \int_0^\infty \frac{dp^2}{p^2+m_\phi^2}\right)$$

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda_\phi^2} dp^2 - m_\phi^2 \int_0^{\Lambda_\phi^2} \frac{dp^2}{p^2+m_\phi^2}\right)$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2} dp^2 + \int_{\Lambda^2-m_\phi^2}^{\Lambda_\phi^2} dp^2\right) - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2} \frac{dp^2}{p^2+m_\phi^2} + \int_{\Lambda^2-m_\phi^2}^{\Lambda_\phi^2} \frac{dp^2}{p^2+m_\phi^2}\right)$$

provisional cutoff

$$\Lambda_\phi^2 \equiv \left(\Lambda^4 / m_\phi^2\right) - m_\phi^2$$

$$p^2 + m_\phi^2 \rightarrow \Lambda^4 / \left(p^2 + m_\phi^2\right)$$

⇓

$$\delta m_\phi^2 = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_0^{\Lambda^2-m_\phi^2} \frac{dp^2}{p^2+m_\phi^2} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2} \ln \frac{\Lambda^2}{m_\phi^2}$$

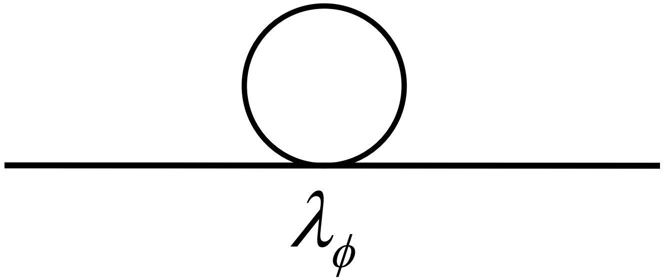
Using the proper time method,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m_\phi^2} = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\int_0^\infty e^{-(p^2+m_\phi^2)t}dt$$

$$= \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt$$

t : proper time

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t^2} - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t} + \frac{\lambda_\phi m_\phi^4}{64\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}dt + \cdots$$



$$\tilde{\Lambda}_\phi^2 \equiv \Lambda^4 / m_\phi^2$$

provisional cutoff

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t^2} - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t} + \frac{\lambda_\phi m_\phi^4}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2} dt + \cdots$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t^2} + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2}\frac{dt}{t^2}\right) - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t} + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2}\frac{dt}{t}\right)$$

$$+ \frac{\lambda_\phi m_\phi^4}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2} dt + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2} dt\right) + \cdots$$

$$\tilde{\Lambda}_\phi^2 \equiv \Lambda^4 \big/ m_\phi^2$$

⇓

$$t \rightarrow 1/(\Lambda^4 t)$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2} dt\right] = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\ln\frac{\Lambda^2}{m_\phi^2}$$

It is important to examine the applicable scope of our method.

Here, we point out that the result depends on the choice of duality transformation.

Different choice

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2}$$

$$\tau_2 \equiv \Lambda^2 t \qquad \tau = \tau_1 + i\tau_2$$

⬇

$$\tau \to -\frac{1}{\tau}, \quad \tau \to \tau + 1$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2}\right]$$

$$= \frac{\lambda_\phi \Lambda^2}{32\pi^2}\int_F \frac{d^2\tau}{\tau_2^2} = \frac{\lambda_\phi}{32\pi^2}\frac{\pi}{2}\Lambda^2$$

$$F = \left\{\tau : |\mathrm{Re}\,\tau| \le 1/2,\ \ 1 \le |\tau|\right\}$$

$$\tau \to -\frac{1}{\tau}, \quad \tau \to \tau + 1$$

$$F = \{\tau : |\mathrm{Re}\,\tau| \le 1/2, \;\; 1 \le |\tau|\}$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2}\right] = \frac{\lambda_\phi \Lambda^2}{32\pi^2}\int_F \boxed{\frac{d^2\tau}{\tau_2^2}} = \frac{\lambda_\phi}{32\pi^2}\frac{\pi}{2}\Lambda^2$$

↕ **Difference of invariant measures**

$$t \to 1/(\Lambda^4 t)$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2} dt\right] = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\Lambda^2}^{1/m_\phi^2} \boxed{\frac{dt}{t}} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\ln\frac{\Lambda^2}{m_\phi^2}$$

We need to specify the duality in order to obtain phys. results.

Different choice

➔ The form of duality could be determined by matching the counterpart in the ultimate theory.

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2}$$

$\tau \rightarrow \frac{a\tau+b}{c\tau+d}$ ← $\tau \rightarrow -\frac{1}{\tau}$ & $\tau \rightarrow \tau+1$

$\tau = \tau_1 + i\tau_2$

Field th. limit

$\tau_1 = 0$ ↓

$\tau_2 \equiv \Lambda^2 t$

$\tau_2 \rightarrow \frac{1}{\tau_2}$ ➔ $t \rightarrow 1/(\Lambda^4 t)$

【A conjecture】

A duality can be hidden behind the standard model.

3. Gauge hierarchy

Gauge hierarchy problem, Fermionic symmetries

Y.K., "Gauge hierarchy problem, supersymmetry and fermionic symmetry", arXiv:1311.2365 [hep-ph] .

First of all, let us review the gauge hierarchy problem and its related topics, and then we reconsider the essence of the problem
and give a new way out !

【Gauge hierarchy problem】

Even if quadratic div. are removed,

$$\delta m_h^2 = C'_h m_h^2 \ln \Lambda^2 / m_h^2 + \sum_k \boxed{C''_{hk} M_k^2} \ln \Lambda^2 / M_k^2 + \cdots$$

If $C''_{hk} M_k^2 >> m_h^2$, unnatural because we need a fine tuning ??

Serious problem for Grand Unified Theory ➔ Supersymmetry ?

$$\delta m_h^2 = C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \sum_k C_{hk}'' M_k^2 \ln \Lambda^2 \big/ M_k^2 + \cdots$$

Way outs to escape a fine tuning,

$$\sum_k C_{hk}'' M_k^2 \ln \Lambda^2 \big/ M_k^2 = 0$$

→ **A miracle (e.g. an excellent symmetry)**

or

$$M_k \le O(1)\mathrm{TeV} \ \text{ unless } \ C_{hk}'' \approx 0.$$

→ **If new particles exist irrelevant to the miracle, they would be around the terascale.**

Candidates of the miracle

- (Softly broken) supersymmetry

$$m_{SUSY} \le O(1)\mathrm{TeV}$$

$$\delta m_h^2 = \tilde{C}_h' \, m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \tilde{C}_h'' \, m_{SUSY}^2 \ln \Lambda^2 \Big/ m_{SUSY}^2 + \cdots$$

$m_{SUSY} \cong$ Masses of Superpartners of SM particles

➔ Supersymmetric GUTs

N. Sakai, *Z. Phys.* C**11**, 153 (1981).
S. Dimopoulos & H. Georgi, *Nucl. Phys.* B**193**, 150 (1981).

- Misaligned supersymmetry

“Cancellation due to infinite numbers of massive modes” K. R. Dienes, *Nucl. Phys.* B**611**, 146 (2001).

Commonly, the problem is recognized as follows.

【Gauge hierarchy problem】

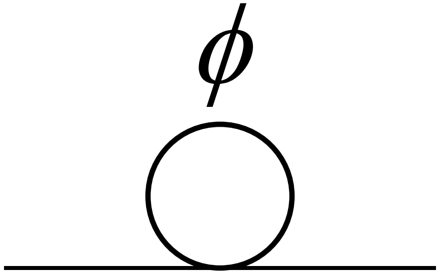
Are values of parameters in the SM stabilized against radiative corrections involving heavy particles?

【Gauge hierarchy and SUSY】

Boson ←→ **Fermion**

ϕ **SUSY** ψ

$\delta m_\phi^2 =$



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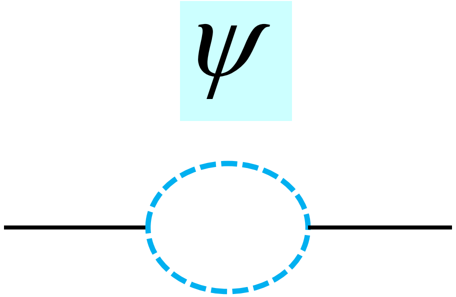


$= \ 0$

Boson ↔ Fermion

SUSY

massless	ϕ_0	ψ_0
heavy Mass: M	ϕ_1 ϕ_2	$\left.\begin{matrix}\psi_1\\\psi_2\end{matrix}\right\}\Psi$

$$\delta m^2_{\phi_i} = 0,\ \ \delta m_{\psi_0} = 0,\ \ \delta m_{\Psi} = 0$$

Boson ←→ Fermion

SUSY

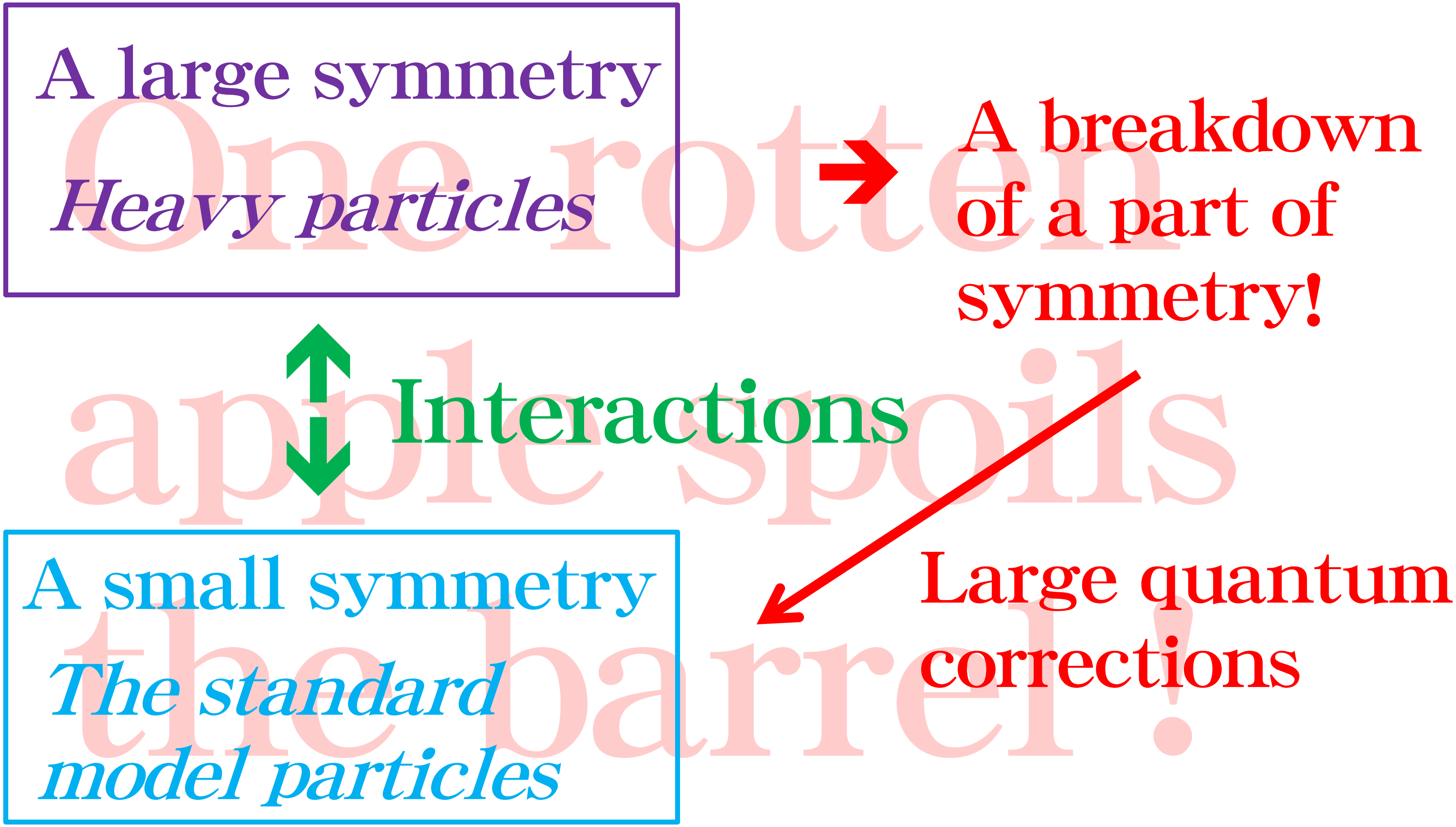
massless	ϕ_0	$\cancel{\psi_0}$ Omit it by hand
heavy Mass: M	ϕ_1 ϕ_2	$\left.\begin{matrix}\psi_1\\ \psi_2\end{matrix}\right\}\Psi$

$$\delta m^2_{\phi_0}\,|_{1\,\mathrm{loop}} = 0$$

← SUSY invariant interaction between ϕ_0 and heavy fields

$$\delta m^2_{\phi_1} - \delta m^2_{\Psi}\,|_{1\,\mathrm{loop}} = O(M^2) \rightarrow \delta m^2_{\phi_0}\,|_{2\,\mathrm{loop}} = O(M^2)$$

A root of the problem



A large symmetry

Heavy particles

Interactions

A small symmetry

The standard model particles

A breakdown of a part of symmetry!

Large quantum corrections

One rotten apple spoils the barrel!

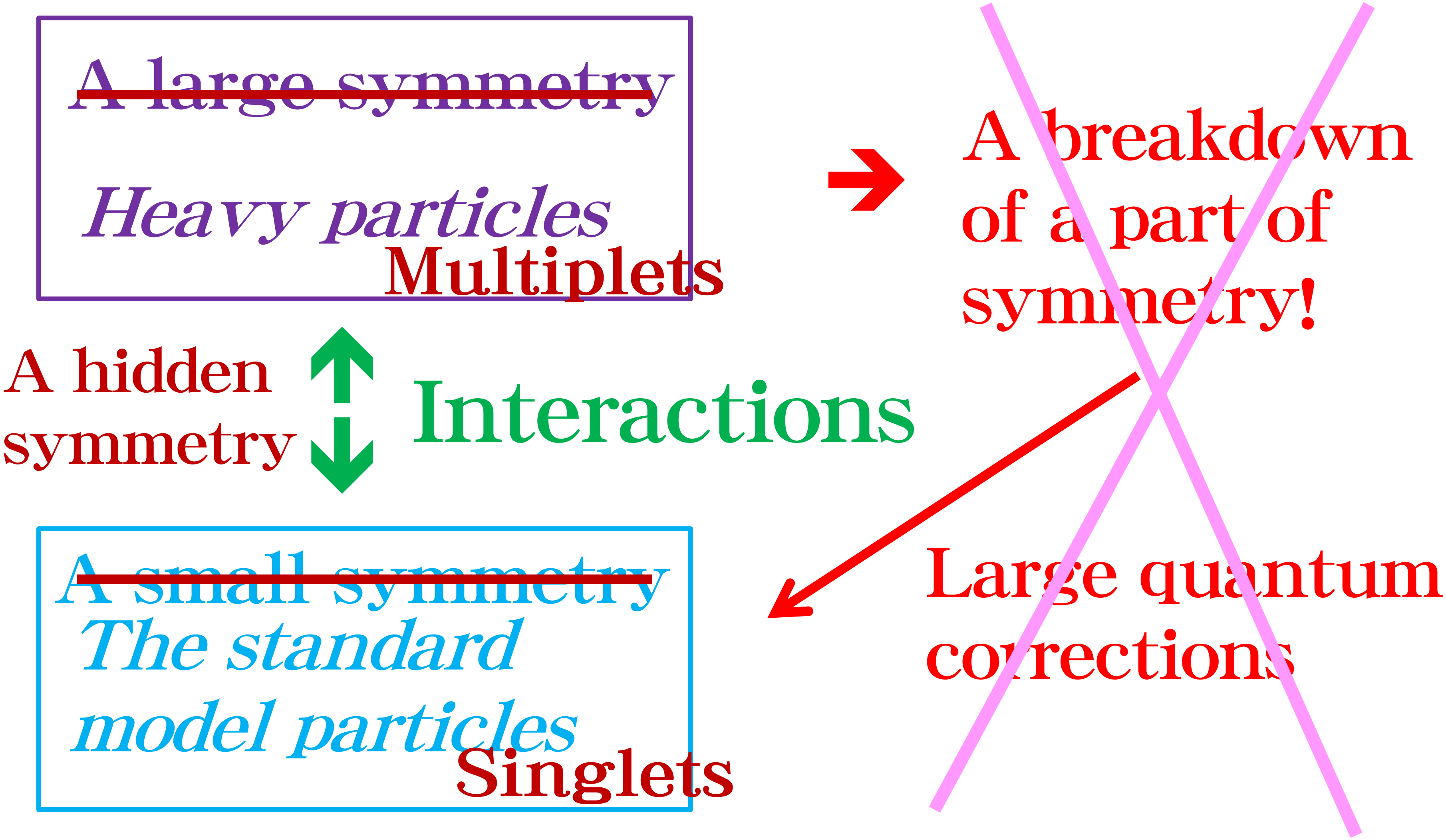
A root of the problem

Is it possible to construct an effective theory, without spoiling the structure of a high-energy physics?

Cf. Are values of parameters in the SM stabilized against radiative corrections involving heavy particles?

A root of the problem

【Expectation】



~~A large symmetry~~

Heavy particles

Multiplets

A hidden symmetry

Interactions

~~A small symmetry~~

The standard model particles

Singlets

~~A breakdown of a part of symmetry!~~

~~Large quantum corrections~~

【Assumptions】

(a) A fundamental theory at M_U .

- Heavy particles with masses of $O(M_U)$
- Physical massless particles → SM+ α

(b) Unknown property X behind SM+ α . Effective th. with X.

(c) Full effective th. $= L_{heavy} + L_{light} + L_{mix}$

Free of gauge hierarchy problem

(c1) Parameters in SM+ α are stabilized against rad. corrections.

(c2) X is preserved, independent of the SM+ α physics. **What is X?**

SUSY provides a hint !

Boson ⟷ Fermion

SUSY

Particles with different statistics

→ Cancellations of contributions

(Feature) $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$

No SUSY singlets !

Q_α - singlet, $\bar{Q}_{\dot{\alpha}}$ - singlet

$Q_\alpha \Psi(\mathrm{x}) = 0, \quad \bar{Q}_{\dot{\alpha}} \Psi(x) = 0$

$\because P_\mu \Psi(x) = i\partial_\mu \Psi(x) \neq 0$

Pairs !

X as new symmetries $\left(Q_F\,,\ Q_F^{\dagger}\right)$

SM+α particles Q_F - singlets

Heavy particles φ Q_F - doublets

$$\varphi \quad \xleftrightarrow[\left(Q_F\,,\ Q_F^{\dagger}\right)]{} \quad ???$$

Cancellation of contributions

➔ particles with different statistics ?

X as new symmetries $\left(Q_F\,,\ Q_F^\dagger\right)$

SM+α particles Q_F - singlets

Heavy particles φ Q_F - doublets

$$\varphi \quad \xleftrightarrow[\left(Q_F\,,\ Q_F^\dagger\right)]{} \quad c_\varphi$$

Ghosts ?

Cancellation of contributions

➔ particles with different statistics ?

Toy model

Light particle ϕ Q_F - singlet

Heavy particles (φ, c_φ) Q_F - doublet

$$L_T = L_\phi + L_{\varphi,c} + L_{mix}$$

$$L_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^2$$

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - M_\varphi^2 (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$- \lambda_\varphi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi) \bigstar (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

Non-local interaction

$$L_{mix} = -\lambda' \phi^\dagger \phi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

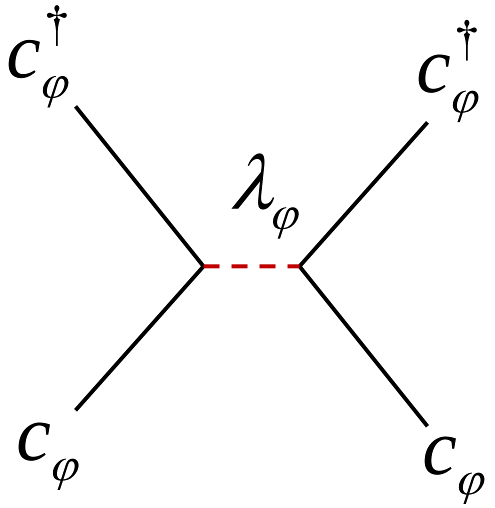
$$L_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^2$$

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - M_\varphi^2 (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$- \lambda_\varphi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi) \bigstar (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

Non-local interaction

$$L_{mix} = -\lambda' \phi^\dagger \phi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$



Schematic diagram for non-local interaction

Self-interactions of ghosts are induced radiatively.

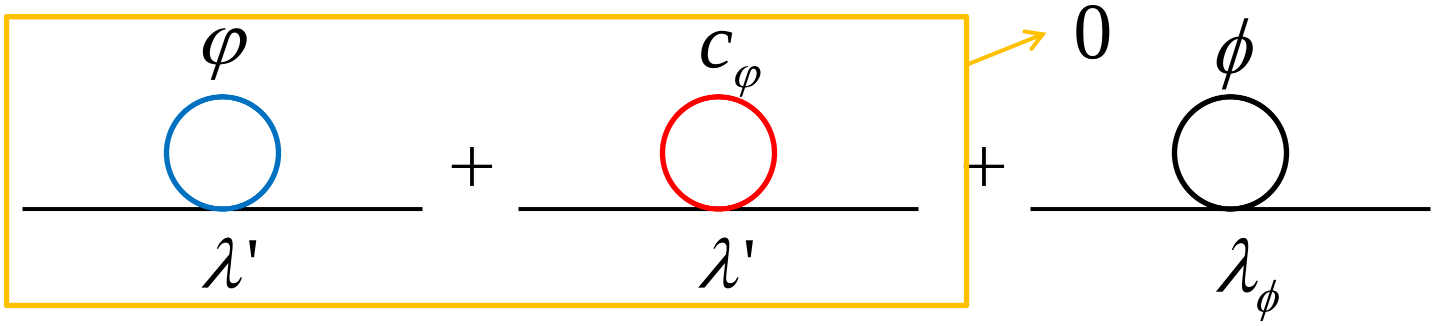
In case with local interaction,

$$-\lambda_\varphi : c_\varphi^\dagger c_\varphi c_\varphi^\dagger c_\varphi : = 0$$

$$(\because c_\varphi{}^2 = 0)$$

Corrections for light one ϕ Q_F - singlet

$\delta m_\phi^2 =$

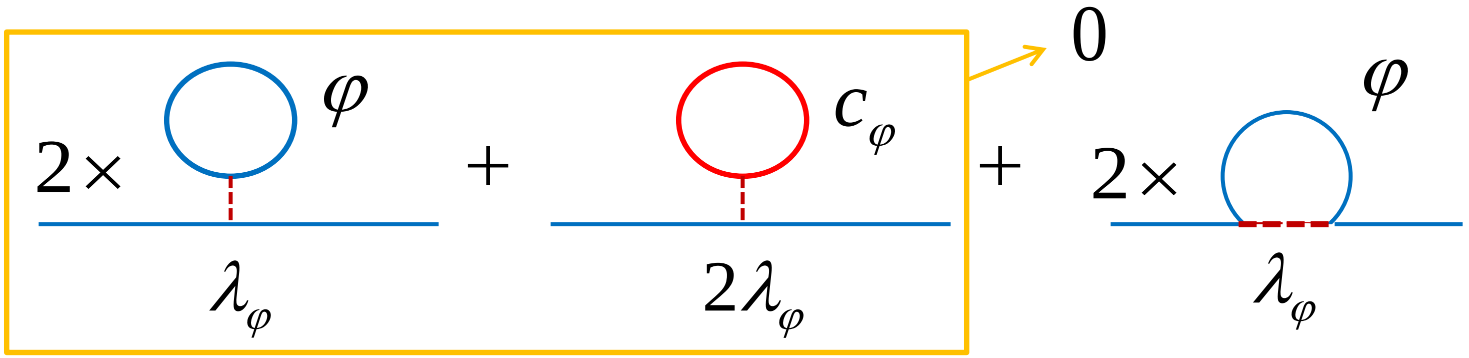


$$= -\frac{\lambda_\phi}{4\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}$$

← Quad. div. has been removed.

Corrections for heavy ones (φ, c_φ) Q_F - doublet

$\delta M_\varphi^2 =$



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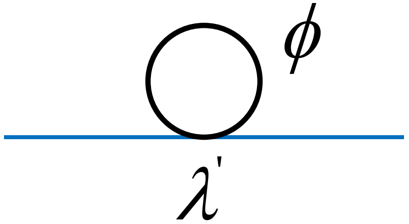


$$= (\varphi \leftrightarrow c_\varphi) = 2\lambda_\varphi J_\varphi - \frac{\lambda'}{4\pi^2} m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}$$

$$L_T = L_\phi + L_{\varphi,c} + L_{mix}$$

$$L_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^2$$

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - M_\varphi^2 (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$- \lambda_\varphi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi) \bigstar (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$L_{mix} = -\lambda' \phi^\dagger \phi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

The mass hierarchy can be stabilized by symmetries X !?

What is X ?

$$I = \varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi$$ **is a key!**

Transf. group with the invariant

$$I=\varphi^{\dagger}\varphi+c_{\varphi}^{\dagger}c_{\varphi}$$

$OSp(2|2)$

(1) $\delta_o\varphi=i\varepsilon_o\varphi,\ \delta_o\varphi^{\dagger}=-i\varepsilon_o\varphi^{\dagger},\ \delta_o c_{\varphi}=0,\ \delta_o c_{\varphi}^{\dagger}=0$ Q_o

(2) $\delta_g\varphi=0,\ \delta_g\varphi^{\dagger}=0,\ \delta_g c_{\varphi}=i\varepsilon_g c_{\varphi},\ \delta_g c_{\varphi}^{\dagger}=-i\varepsilon_g c_{\varphi}^{\dagger}$ Q_g

(3) $\delta_F\varphi=-\zeta c_{\varphi},\ \delta_F c_{\varphi}=0,\ \delta_F\varphi^{\dagger}=0,\ \delta_F c_{\varphi}^{\dagger}=\zeta\varphi^{\dagger}$ Q_F

$\delta_F^{\dagger}\varphi=0,\ \delta_F^{\dagger}c_{\varphi}=\zeta^{\dagger}\varphi,\ \delta_F^{\dagger}\varphi^{\dagger}=\zeta^{\dagger}c_{\varphi}^{\dagger},\ \delta_F^{\dagger}c_{\varphi}^{\dagger}=0$ $Q_F^{\dagger}$

Fermionic symmetries

$$Q_F{}^{2}=0,\ Q_F^{\dagger\,2}=0,\ \left\{Q_F,Q_F^{\dagger}\right\}=Q_o+Q_g\equiv N_D$$

$$I=\varphi^{\dagger}\varphi+c_{\varphi}^{\dagger}c_{\varphi}=\tilde{\delta}_F\left(c_{\varphi}^{\dagger}\varphi\right)=\tilde{\delta}_F^{\dagger}\left(\varphi^{\dagger}c_{\varphi}\right)=\tilde{\delta}_F\tilde{\delta}_F^{\dagger}\left(\varphi^{\dagger}\varphi\right)$$

Here, $\tilde{\delta}_F$ and $\tilde{\delta}_F^{\dagger}$ represent transformations omitting Grassmann parameters.

For quantization of coexisting systems with ordinary complex scalar fields and their ghost partners and with ordinary Dirac spinors and their ghost partners,

$Q_F|\mathrm{phys}\rangle = 0 \quad Q_F^\dagger|\mathrm{phys}\rangle = 0 \quad N_D|\mathrm{phys}\rangle = 0$

Cf. Kugo-Ojima subsidiary condition

➔ Quartet mechanism

$(\varphi, c_\varphi\,;\ \varphi^\dagger, c_\varphi^\dagger)$ (➔ unphysical)

Y.K., “Fermionic scalar field”, arXiv:1406.6155 [hep-th] .

$$L_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^2$$

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi - M_\varphi^2 (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$
$$- \lambda_\varphi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi) \bigstar (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$L_{mix} = -\lambda' \phi^\dagger \phi (\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi)$$

$$\varphi^\dagger \varphi + c_\varphi^\dagger c_\varphi = \tilde{\delta}_F \left(c_\varphi^\dagger \varphi \right) = \tilde{\delta}_F^\dagger \left(\varphi^\dagger c_\varphi \right) = \tilde{\delta}_F \tilde{\delta}_F^\dagger \left(\varphi^\dagger \varphi \right)$$

$$L_T = L_\phi + L_{\varphi,c} + L_{mix} = L_\phi + \tilde{\delta}_F \tilde{\delta}_F^\dagger \left(\Delta L \right)$$

This is the secret of the stabilization of mass hierarchy !

【Expectation】

SM+α particles Q_F -singlets

Others including heavy particles

Q_F -doublets

$$L_{BSM} = L_{light} + L_{heavy} + L_{mix}$$

$$L_{light} = L_{SM+\alpha} + \tilde{\delta}_F \tilde{\delta}_F^{\dagger} \left(\Delta L_{light} \right)$$

$$L_{heavy} + L_{mix} = \tilde{\delta}_F \tilde{\delta}_F^{\dagger} \left(\Delta L_{heavy} \right)$$

Under $Q_F \left| \mathrm{phys} \right\rangle = 0 \quad Q_F^{\dagger} \left| \mathrm{phys} \right\rangle = 0 \quad N_D \left| \mathrm{phys} \right\rangle = 0$,

system is a same as $L_{SM+\alpha}$?!

【Expectation】

SM+α particles Q_F - singlets

Others including heavy particles Q_F - doublets

$$L_{BSM} = L_{light} + L_{heavy} + L_{mix}$$

$$L_{light} = L_{SM+\alpha} + \tilde{\delta}_F \tilde{\delta}_F^\dagger \left(\Delta L_{light} \right)$$

$$L_{heavy} + L_{mix} = \tilde{\delta}_F \tilde{\delta}_F^\dagger \left(\Delta L_{heavy} \right)$$

Q. Is there a proof of Q_F - doublets ?

Q. Is there a proof of Q_F -doublets ?

Or, is the existence of unphysical fields verified ?

It is almost impossible because of no dynamical effects.

All we can say is that

- no almighty proof.
- an indirect proof only in a very special case.

What is a very special case ?

(a) Effective th. has multiplets of G $\{\Phi_s\}$ and parameters f_i , and it is invariant under G .
f_i are measured precisely.

(b) Using the obs. values $f_i(E_l)$ and the RG eqs., we obtain specific relations among parameters.

Ex. $f_1 = f_2 = \cdots = f_k \Big|_{M_U}$

(c) The relations hold exactly without any threshold corrections around M_U.

Ex. $f_1 = f_2 = \cdots = f_k \Big|_{M_U}$

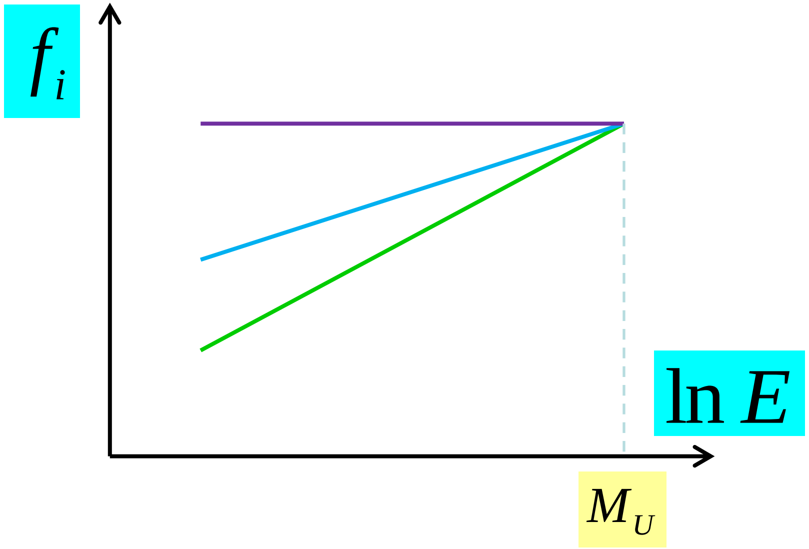
Or, they hold in the effective th. without heavy particles.

(d) The relations suggest a large symmetry (G_U) at M_U.

Particles in effective th. are parts of G_U, or incomplete multiplets !?

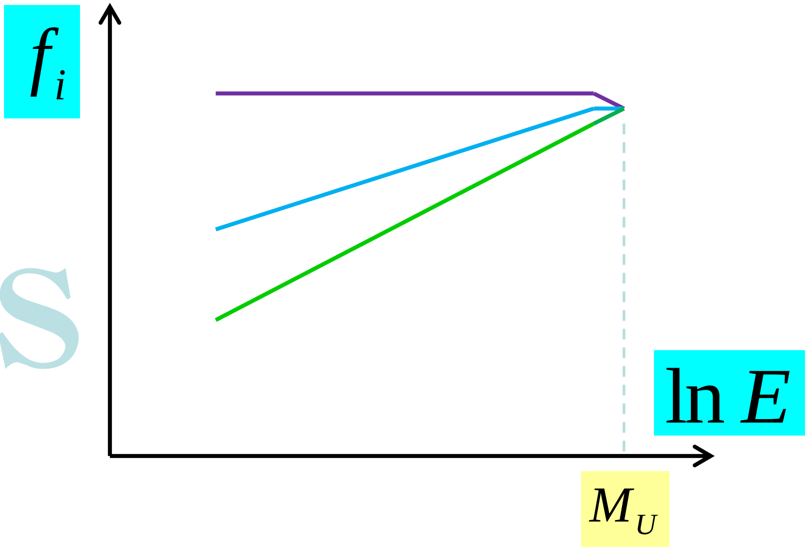
(Ex.) No threshold corrections at M_U

Our case



(Ex.) Threshold corrections at M_U

Most cases



It is hard to explain them using ordinary SSB. Because

The theory at M_U

$=$ The theory invariant under G_U containing multiplets of G_U $\{\Phi_U\}$.

➔ Decomposition of G-multiplets

$\{\Phi_S\}+\{\Phi_D\}$

$$\begin{cases} \{\Phi_D\} \text{ acquire masses of } O(M_U). \\ \{\Phi_S\} \rightarrow G\text{-inv. effective th.} \end{cases}$$

Threshold corrections from $\{\Phi_D\}$ appear (inevitably) !?

【Expectation】Novel Sym. Br.

Ex. $f_1 = f_2 = \cdots = f_k\big|_{M_U}$ → G_U $\{\Phi_U\}$

G_U -multiplets

The sector with $\{\Phi_U\}$ alone has the invariance of G_U at M_U.

Below M_U, it is reduced into G.

The decomposition: $\{\Phi_S\} + \{\Phi_D\}$

$\left\{\begin{array}{l}\text{Below } M_U\text{, only } \{\Phi_S\} \text{ are observed.}\\ \text{No quantum corrections from } \{\Phi_D\}\end{array}\right.$

→ $\{\Phi_D\}$ are unphysical ! → Ghosts $\{C_D\}$

【Expectation】 Novel Sym. Br.

Ex. $f_1 = f_2 = \cdots = f_k \big|_{M_U}$ → G_U $\{\Phi_U\}$ G_U -multiplets

The sector with $\{\Phi_U\}$ alone has the invariance of G_U at M_U.

Below M_U, it is reduced into G.

The decomposition: $\{\Phi_S\} + \{\Phi_D\}$

$$\begin{cases} \text{Below } M_U\text{, only } \{\Phi_S\} \text{ are observed.} \\ \text{No quantum corrections from } \{\Phi_D\} \end{cases}$$

$\{\Phi_D, C_D\}$ → Unphysical by quartet mechanism.

Features of fund. theory (?)

- It is defined just at M_U.

Ex. $\text{The physics}@E(\geq\Lambda) \sim \text{The physics}@E(\leq\Lambda)$

Duality

- The sector with ordinary particles alone has G_U symmetry. → Fundamental objects (?)

Ex. $f_1 = f_2 = \cdots = f_k\big|_{M_U}$

- Ordinary particles $\{\Phi_U\} \Rightarrow \{\Phi_S\} + \{\Phi_D\}$

$\{\Phi_D\}$ become unphys. with the advent of ghosts $\{C_D\}$. → The reduction into G!

Features of fund. theory (?)

- It is defined just at M_U.

Ex. The physics@$E(\geq \Lambda)$ ~ The physics@$E(\leq \Lambda)$

Duality

- The sector with ordinary particles alone has G_U symmetry. → Fundamental objects (?)

Ex. $f_1 = f_2 = \cdots = f_k \big|_{M_U}$

- Ordinary particles $\{\Phi_U\} \Rightarrow \{\Phi_S\} + \{\Phi_D\}$

$\{\Phi_D\}$ become unphys. with the advent of ghosts $\{C_D\}$. → $\{C_D\}$ might be solitons (?)

Based on these features (guesses), let us explore physics behind the SM !

【Notice】

Our purpose here is not to present a complete model but to impress the idea of our mechanism.

Don’t be nervous about the details.

【Assumptions】

☆ Fundamental object O possesses a large gauge symmetry.

Gauge boson : $A_\mu^\alpha(x)$

Cf. D - brane

Gauge symmetry

☆ "Matters" originate from O as solutions of ultimate theory.

"Matters" $\phi,\ \psi,\ \underline{(\varphi,\ c_\varphi)}$

Fermionic symmetries

Construct effective th. by sym. !

(Ex.) Grand unification, SU(5)

Case (a) Massless particles

Gauge boson : $A_\mu^\alpha(x)$ $(\alpha = 1 \sim 24)$

Higgs boson : $H(x) = \left(H_C, H_W\right)$

H_C : Colored Higgs , H_W : Weak Higgs

From the gauge symmetry,

$$L^{(a)} = -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu}$$

$$+ \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - \lambda \left(H^\dagger H\right)\left(H^\dagger H\right)$$

Case (b)

Gauge boson : $A_\mu^\alpha(x) \quad (\alpha = 1 \sim 24)$

Higgs boson : $H(x) = (H_C, H_W)$

Higgs ghost : $C_H(x) = \left(C_{H_C}, C_{H_W}\right)$

$$\begin{aligned} L^{(b)} &= -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} + \left(D_\mu H\right)^\dagger \left(D^\mu H\right) + \left(D_\mu C_H\right)^\dagger \left(D^\mu C_H\right) \\ &\quad - \lambda\left(H^\dagger H + C_H^\dagger C_H\right)\bigstar\left(H^\dagger H + C_H^\dagger C_H\right) \\ &= -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} + \tilde{\delta}_F \tilde{\delta}_F^\dagger \Big[\left(D_\mu H\right)^\dagger \left(D^\mu H\right) \\ &\qquad\qquad - \frac{\lambda}{2}\left(H^\dagger H \bigstar H^\dagger H - C_H^\dagger C_H \bigstar C_H^\dagger C_H\right)\Big] \end{aligned}$$

Pure SU(5) Yang-Mills theory

Case (c)

Gauge boson : $A^a_\mu(x)\quad (a = 1 \sim 8, 21 \sim 24)$

→ The SM gauge bosons

X boson : $X_\mu(x)\ \ (A^i_\mu(x)\quad i = 9 \sim 20)$

X ghost : $C_\mu(x)$

→ The SM Higgs doublet

Higgs boson : $H(x) = \left(H_C, H_W\right)$

Colored Higgs ghost : C_{H_C}

$$L^{(c)} = L^{\bigstar}_{GUT} + L_{gh} + L_{\text{int}} = L^{\bigstar}_{SM} + \tilde{\delta}_F \tilde{\delta}^{\dagger}_F \left(\Delta L\right)\Big|_{M_U}$$

$$L^{\bigstar}_{SM} = -\frac{1}{4} F'^{a}_{\mu\nu} F'^{a\mu\nu} + \left(D'_\mu H_W\right)^{\dagger}\left(D'^{\mu} H_W\right) - \lambda\left(H^{\dagger}_W H_W\right)\bigstar\left(H^{\dagger}_W H_W\right)$$

$$L^{\star}_{GUT} + L_{gh} + L_{\mathrm{int}} = L^{\star}_{SM} + \tilde{\delta}_F \tilde{\delta}_F^{\dagger}(\Delta L)\Big|_{M_U}$$

【Features】

Gauge coupling unif. $g_3 = g_2 = g_1 = g_U\Big|_{M_U}$

Triplet-doublet splitting

$$H_5 = \begin{pmatrix} H_C \\ H_W \end{pmatrix} \qquad c_{H_C}$$

Q_F - doublet

Proton stability $X_\mu : (3,2) \qquad C_\mu : (3,2)$

Q_F - doublet

Proton acquires an eternal life as a result that extra colored particles sell their souls to the ghosts.

【Almost ultimate scenario】

Our world comes from "nothing" !

"Nothing" means not an empty but a world with unphysical objects or only gauge bosons.

"Beings" are generated after the change of fundamental objects !

"Beings" means a world with physical particles or SM + α.

Ref. Y.K., "Creation of physical modes from unphysical fields", arXiv:1409.0276 [hep-th].

【A conjecture】

Fermionic symmetries can be hidden behind the standard model.

4. Summary

【Message】

There is a possibility that physics (duality & fermionic symmetries) is hidden behind the standard model !

【Quadratic divergence problem】

➔ Duality relating scale

$$\delta m_h^2 = \boxed{C_h \Lambda^2} + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \boxed{\sum_k C_{hk}'' M_k^2 \ln \Lambda^2 \big/ M_k^2} + \cdots$$

【Gauge hierarchy problem】

➔ Fermionic symmetries relating ghosts

If problems were solved, then the following scenario can be realistic !

The SM + new particles at O(1) TeV

No physical superpartners !

↓ Almost big desert

Fundamental theory @M_U

☺

【Assumption】

Grand unification at M_U

$$\left. L^{\bigstar}_{GUT} + L_{gh} + L_{\text{int}} = L^{\bigstar}_{SM+\alpha} + \tilde{\delta}_F \tilde{\delta}_F^{\dagger} (\Delta L) \right|_{M_U}$$

「$+ \alpha$」 stands for new particles around O(1) TeV.

【Subject】 To construct a realistic model that realizes the unification of gauge couplings exactly, without threshold corrections at M_U, in corporation with「$+ \alpha$」.

【Assumption】
Grand unification at M_U

$$L_{GUT}^{\star} + L_{gh} + L_{\text{int}} = L_{SM+\alpha}^{\star} + \tilde{\delta}_F \tilde{\delta}_F^{\dagger} (\Delta L) \Big|_{M_U}$$

「+ α」 stands for new particles around O(1) TeV.

Y.K., “Terascale remnants of unification and supersymmetry at the Planck scale”,
Prog. Theor. Exp. Phys. 8, 081B01, (2013)
arXiv:1304.7885 [hep-ph] .

【Subjects】

SM+α particles would be massless at M_U!

Origin of masses？
Or origin of EW breaking？

Derivation of $\boxed{m_h \simeq 126\,\mathrm{GeV}}$

New particles around the terascale ？

Fundamental theory？

Corrections on vacuum energy

$$\delta\Lambda_{\mathrm{V}} = \frac{(-1)^F}{2}\sum_k \int_{-\infty}^{\infty} \frac{d^4 p}{(2\pi)^4} \ln(p^2 + m_k^2)$$

Duality

$$= -\sum_k \frac{(-1)^F}{64\pi^2} m_k^4 \ln \frac{\Lambda^2}{m_k^2}$$

$$= -\sum_{SM+\alpha} \frac{(-1)^F}{64\pi^2} m_a^4 \ln \frac{\Lambda^2}{m_a^2}$$

Fermionic symmetries

The cosmological constant problem would be a problem in the SM+α.

【Present status】

No evidence for superpartners
and Kaluza-Klein modes @ LHC

No evidence for proton decay
@ Kamioka

➔ No evidence for physics
beyond the standard model

Supersymmetry, Extra dimensions,
Grand unification !?

Suspicious look

【Present status】

No evidence for superpartners and Kaluza-Klein modes @ LHC

No evidence for proton decay @ Kamioka

➔ No evidence for physics beyond the standard model

Evidence for unphysical particles!

Supersymmetry, Extra dimensions, Grand unification !?

【Wish】

Even if our duality and fermionic symmetries are products of fantasy,
I hope the expectation would survive.

- The calculation scheme is selected by the physics.
- Radiative corrections are constrained by symmetries in an ultimate theory.
- The gauge hierarchy is stabilized by the symmetry that the SM particles are singlets.

Thank you for your attention.

Back up

The spin-statistics theorem

In relativistic QFT,

- Positivity of energy
- Causality
- No negative norm states

➔

☆ Integer spin particles obey Bose-Einstein statistics.

➔ Quantization using commutation relations

☆ Half odd integer spin particles obey Fermi-Dirac statistics.

➔ Quantization using anti-commutation relations

Q. What happens if integer spin particles are quantized using anti-commutation relations?

Q. What happens if half odd integer spin particles are quantized using commutation relations?

→ Negative norm states

The causality is not violated !

Refs. N. Ohta, "Causal fields and spin-statistics connection for massless particles in higher dimensions", *Phys. Rev.* D**31**, 442 (1985).

K. Fujikawa, "Spin-Statistics Theorem in Path Integral Formulation", *Int. J. Mod. Phys.* **A16**, 4025 (2001).

Y.K., "Fermionic scalar field", arXiv:1406.6155 [hep-th].

Scalar fields with $OSp(2|2)$

Transf. group of the invariant $x^2 + y^2 + 2i\theta_1\theta_2$

$$\left(x, y \in R,\ \theta_1^\dagger = \theta_1,\ \theta_2^\dagger = \theta_2,\ \theta_1{}^2 = \theta_2{}^2 = 0\right)$$

$$L_{OSp(2|2)} = \frac{1}{2}\left(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2\right) + i\partial_\mu c_1 \partial^\mu c_2$$

⬇

$$\phi_1 = \frac{\varphi + \varphi^\dagger}{\sqrt{2}},\ \phi_2 = \frac{\varphi - \varphi^\dagger}{i\sqrt{2}},$$

$$c_1 = \frac{c_\varphi + c_\varphi^\dagger}{\sqrt{2}},\ c_2 = \frac{c_\varphi - c_\varphi^\dagger}{i\sqrt{2}}$$

Quantization using anti-commutation relations

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi$$

Scalar fields with $OSp(2|2)$

$$L_{\varphi,c}=\partial_\mu\varphi^\dagger\partial^\mu\varphi+\partial_\mu c_\varphi^\dagger\partial^\mu c_\varphi$$

$$\delta_F\varphi=-\zeta c_\varphi,\delta_F c_\varphi=0,\delta_F\varphi^\dagger=0,\ \delta_F c_\varphi^\dagger=\zeta\varphi^\dagger \qquad Q_F$$

$$\delta_F^\dagger\varphi=0,\delta_F^\dagger c_\varphi=\zeta^\dagger\varphi,\delta_F^\dagger\varphi^\dagger=\zeta^\dagger c_\varphi^\dagger,\delta_F^\dagger c_\varphi^\dagger=0 \qquad Q_F^\dagger$$

$$\delta_D\varphi=-i\zeta\zeta^\dagger\varphi,\delta_D c_\varphi=-i\zeta\zeta^\dagger c_\varphi,\delta_D\varphi^\dagger=i\zeta\zeta^\dagger\varphi^\dagger,\ \delta_D c_\varphi^\dagger=i\zeta\zeta^\dagger c_\varphi^\dagger \qquad N_D$$

$$Q_F{}^2=0,\ Q_F^{\dagger 2}=0,\ \left\{Q_F,Q_F^\dagger\right\}=N_D$$

$$Q_1\equiv Q_F+Q_F^\dagger,\quad Q_2\equiv i\left(Q_F-Q_F^\dagger\right)$$

Hermitian fermionic charges

$$Q_1{}^2=N_D,\ Q_2{}^2=N_D,\ \left\{Q_1,Q_2\right\}=0$$

cf. $N=2$ QM SUSY : $Q_1{}^2=Q_2{}^2=H,\ \left\{Q_1,Q_2\right\}=0$

Scalar fields with $OSp(2|2)$

$$L_{\varphi,c} = \partial_\mu \varphi^\dagger \partial^\mu \varphi + \partial_\mu c_\varphi^\dagger \partial^\mu c_\varphi$$

$(c_\varphi, c_\varphi^\dagger)$ ➔ **Negative norm states**

$$Q_F{}^2 = 0,\ Q_F^{\dagger\,2} = 0,\ \{Q_F, Q_F^\dagger\} = N_D$$

$$Q_F|\mathrm{phys}\rangle = 0 \quad Q_F^\dagger|\mathrm{phys}\rangle = 0 \quad N_D|\mathrm{phys}\rangle = 0$$

$$Q_1{}^2 = N_D,\ Q_2{}^2 = N_D,\ \{Q_1, Q_2\} = 0$$

$$Q_1|\mathrm{phys}\rangle = 0 \quad Q_2|\mathrm{phys}\rangle = 0 \quad N_D|\mathrm{phys}\rangle = 0$$

Negative norm states are eliminated !

Only the vacuum state $|0\rangle$ survives .

Systems with $OSp(2|2)$

$$Q_F{}^2 = 0,\ Q_F^{\dagger\,2} = 0,\ \left\{Q_F, Q_F^\dagger\right\} = N_D$$

or

$$Q_1{}^2 = N_D,\ Q_2{}^2 = N_D\,,\ \left\{Q_1, Q_2\right\} = 0$$

➔ Different from BRST sym.

Q. Systems with BRST sym. ?

Systems with $OSp(1,1|2)$

Scalar fields with $OSp(1,1|2)$

Transf. group of the invariant $x^2 - y^2 + 2i\theta_1\theta_2$

$\left(x, y \in R,\ \theta_1^\dagger = \theta_1,\ \theta_2^\dagger = \theta_2,\ \theta_1{}^2 = \theta_2{}^2 = 0\right)$

$$L_{OSp(1,1|2)} = \frac{1}{2}\left(\partial_\mu\phi_3\partial^\mu\phi_3 - \partial_\mu\phi_0\partial^\mu\phi_0\right) + i\partial_\mu c_1\partial^\mu c_2$$

$$\phi_3 = \frac{B+\phi}{\sqrt{2}},\ \phi_0 = \frac{B-\phi}{\sqrt{2}},$$

$$\boxed{c_1 = \bar{c},\ c_2 = c}$$

Quantization using anti-commutation relations

$$L_{\phi,c} = \partial_\mu B\partial^\mu\phi + i\partial_\mu\bar{c}\partial^\mu c$$

Cf. K. Fujikawa, *Prog. Theor. Phys.* **63**, 1364 (1980)

$B,\ \phi,\ \bar{c},\ c$: hermitian scalar fields

Scalar fields with $OSp(1,1|2)$

$$L_{\phi,c} = \partial_\mu B \partial^\mu \phi + i\partial_\mu \bar{c} \partial^\mu c$$

$\tilde{\delta}_B \phi = c, \tilde{\delta}_B c = 0, \ \tilde{\delta}_B \bar{c} = iB, \ \tilde{\delta}_B B = 0$ **BRST transf.**

$\tilde{\bar{\delta}}_B \phi = \bar{c}, \tilde{\bar{\delta}}_B c = -iB, \tilde{\bar{\delta}}_B \bar{c} = 0, \ \tilde{\bar{\delta}}_B B = 0$ **Anti-BRST transf.**

$$Q_B{}^2 = 0, \ \bar{Q}_B{}^2 = 0, \ \{Q_B, \bar{Q}_B\} = 0$$

$$L_{\phi,c} = \tilde{\delta}_B\left(-i\partial_\mu \bar{c} \partial^\mu \phi\right) \Rightarrow \tilde{\delta}_B\left(i\bar{c}\partial_\mu \partial^\mu \phi\right) = -B\partial_\mu \partial^\mu \phi - i\bar{c}\partial_\mu \partial^\mu c$$

$L(\phi) = 0$ ➔ **Local symmetry**

$$\phi(x) \rightarrow \phi_\Lambda(x) = \phi(x) + \Lambda(x)$$

➔ **Gauge fixing & FP ghosts**

$F(\phi) = \partial_\mu \partial^\mu \phi(x) = 0$ $\bar{c}(x), \ c(x)$

Scalar fields with $OSp(1,1|2)$

$$L_{OSp(1,1|2)} = \frac{1}{2}\left(\partial_\mu \phi_3 \partial^\mu \phi_3 - \partial_\mu \phi_0 \partial^\mu \phi_0\right) + i\partial_\mu c_1 \partial^\mu c_2$$

(ϕ_0, c_1, c_2) ➔ **Negative norm states**

$$Q_B{}^2 = 0,\ \overline{Q}_B{}^2 = 0,\ \left\{Q_B, \overline{Q}_B\right\} = 0$$

➔ $Q_B|\mathrm{phys}\rangle = 0$ **Kugo-Ojima subsidiary condition**

Negative norm states are eliminated !

Physics is independent of gauge fixing condition.

$$L_{\phi,c} = \tilde{\delta}_B\left(-i\partial_\mu \bar{c} \partial^\mu \phi\right) \Rightarrow \tilde{\delta}_B\left(i\bar{c}\partial_\mu \partial^\mu \phi\right) \Rightarrow \tilde{\delta}_B\left(i\bar{c}F(\phi)\right)$$

For systems with $OSp(2|2)$

in the same as those with $OSp(1,1|2)$

under suitable subsidiary conditions, doublets of fermionic symmetries become unphysical by the quartet mechanism, and the theory can be unitary.