Propagators in nonstationary spacetimes and a nonequilibrium-thermodynamics character of de Sitter space

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[Refs]

MF-Sakatani-Sugishita

[1] "Propagators in de Sitter space"
[PRD88 (2013) 024041, arXiv:1301.7352]

[2] "Master equation for the Unruh-DeWitt detector and the universal relaxation time in de Sitter space"
[PRD89 (2014) 064024, arXiv:1305.0256]

0. Introduction

Unification of gravity and the Standard Model is one of the most important problems in contemporary physics.

-- Standard Model

⇨ $\phi(x), \psi(x), A_\mu(x)$: **effective fields at low energy**

-- What is special for gravity:

$g_{\mu\nu}(x)$ **is not simply an effective field at low energy.**

It has a thermodynamic character.

[Bekenstein, Hawking, Gibbons-Hawking, …]

Examples:

-- Stationary BH has entropy prop to its horizon area.

[Bekenstein, Hawking]

-- Einstein equation can be regarded as an EoS. [Jacobson]

-- Entropy density naturally depends on $g_{\mu\nu}(x)$. [MF-Sakatani]

Today, I will talk about

- definition of "vacuum" for QFT in curved spacetime
- nonequilibrium-thermodynamics character of de Sitter space

PART 1 : Defining a vacuum in a curved spacetime

[1] MF-Sakatani-Sugishita [PRD88 (2013) 024041, 1301.7352]

1. Basics of QFT in curved spacetimes (esp. in de Sitter space)
2. Propagators for the instantaneous vacuum in curved spacetime (general theory)
3. Propagators in de Sitter space

PART 2 : Unruh-DeWitt detector in curved spacetime

[2] MF-Sakatani-Sugishita [PRD89 (2014) 064024, 1305.0256]

4. Motivation and the result
5. Master equation for Unruh-DeWitt detector (general theory)
6. Nonequilibrium-thermodynamics character of de Sitter space

PART 1 : Defining a vacuum in a curved spacetime

[1] MF-Sakatani-Sugishita [PRD88 (2013) 024041, 1301.7352]

0. Basics of QFT in curved spacetimes

[cf. Birrel-Davies]

[also 福間-酒谷『重力とエントロピー』(サイエンス社2014)]

■ Klein-Gordon inner product

metric: $ds^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu$

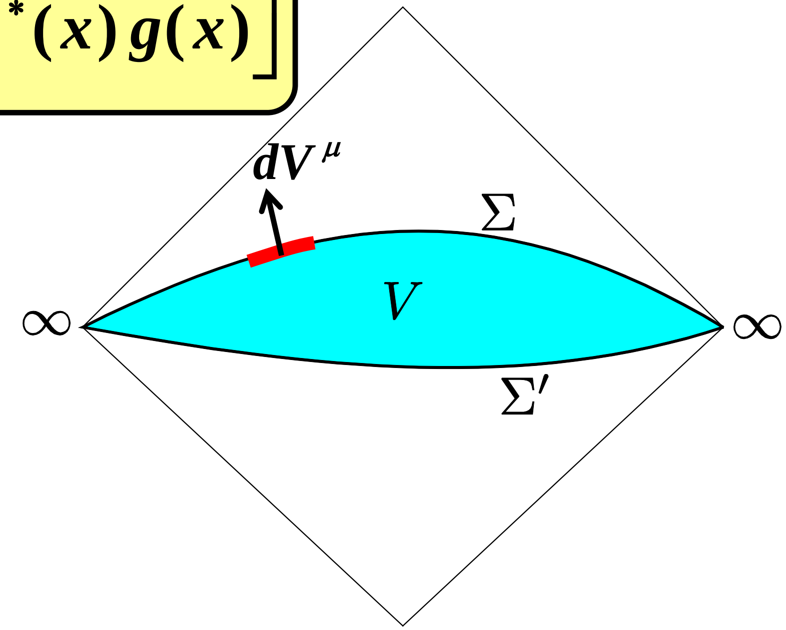
action: $S[\phi(x)] = \int d^d x \left[-\frac{1}{2} g^{\mu\nu}(x)\,\partial_\mu\phi(x)\,\partial_\nu\phi(x) - \frac{m^2}{2}\phi^2(x) \right]$

Sol $\equiv \left\{ \text{c-\# solns of KG eqn } \left(\Box - m^2\right) f(x) = 0 \right\}$

For $f(x),\ g(x) \in$ Sol,

$$\langle f, g \rangle \equiv i \int_\Sigma dV^\mu \left[f^*(x)\,\partial_\mu g(x) - \partial_\mu f^*(x)\, g(x) \right]$$

"Klein-Gordon inner product"



Properties:

$$\begin{cases} \text{indep of the choice of } \Sigma \\ \langle g, f \rangle = \langle f, g \rangle^* \\ \langle f^*, g^* \rangle = -\langle g, f \rangle = -\langle f, g \rangle^* \end{cases}$$

■ Quantization of $\phi(x)$

"wave functions"

$\Updownarrow$

Step 1

Choose a complete set of positive-energy solns $\{\varphi_n(x)\}$:

maximal set s.t. $\begin{cases} \langle \varphi_n, \varphi_m \rangle = \delta_{nm}, & \langle \varphi_n, \varphi_m^* \rangle = 0, \\ \langle \varphi_n^*, \varphi_m \rangle = 0, & \langle \varphi_n^*, \varphi_m^* \rangle = -\delta_{nm} \end{cases}$ --- (*)

Step 2

Expand $\phi(x)$ as

$$\phi(x) = \sum_n \left[a_n \varphi_n(x) + a_n^\dagger \varphi_n^*(x) \right]$$

$$\left(\begin{array}{l} \sum_n \Leftrightarrow \int d^3\mathrm{k} \\ \varphi_n(x) \Leftrightarrow \varphi_{\mathrm{k}}(\mathrm{x}, t) = \dfrac{e^{i(\mathrm{k}\,\mathrm{x} - \omega_k t)}}{\sqrt{(2\pi)^3 2\omega_{\mathrm{k}}}} \end{array} \right)$$

Step 3

Impose the commuation relations

$$\left[a_n, a_m^\dagger \right] = \delta_{nm}, \quad \left[a_n, a_m \right] = 0 = \left[a_n^\dagger, a_m^\dagger \right] \left(\Leftrightarrow \left[\phi(\mathrm{x}, \mathrm{t}), \dot{\phi}(\mathrm{y}, \mathrm{t}) \right] = i\,\delta^3(\mathrm{x} - \mathrm{y}) \right)$$

with (perturbative) vacuum $|0\rangle$: $a_n |0\rangle = 0 \ (\forall n)$

■ Quantization of $\phi(x)$

"wave functions"

⇕

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Choose a complete set of positive-energy solns $\{\varphi_n(x)\}$:

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EQUIVALENT

Step 3

Impose the commuation relations

$$\left[a_n, a_m^\dagger\right] = \delta_{nm}, \quad \left[a_n, a_m\right] = 0 = \left[a_n^\dagger, a_m^\dagger\right] \left(\Leftrightarrow \left[\phi(\mathrm{x,t}), \dot{\phi}(\mathrm{y,t})\right] = i\delta^3(\mathrm{x} - \mathrm{y}) \right)$$

with (perturbative) vacuum $|0\rangle$: $a_n |0\rangle = 0 \ (\forall n)$

■ Ambiguity in defining a vacuum

Bogoliubov transformations

$$\varphi'_n(x) = \sum_m \left[\alpha_{nm}\, \varphi_m(x) + \beta_{nm}\, \varphi^*_m(x) \right]$$

s.t. $\begin{cases} \alpha \cdot \alpha^\dagger - \beta \cdot \beta^T = 1, \quad \alpha \cdot \beta^T - \beta \cdot \alpha^T = 0, \\ \alpha^\dagger \cdot \alpha - \beta^T \cdot \beta = 1, \quad \alpha^\dagger \cdot \beta - \beta^T \cdot \alpha^* = 0 \end{cases}$

⇒ **Relations (∗) hold also for $\{\varphi'_n(x)\}$.**

$\exists$ ambiguity of $\{\varphi_n(x)\}$ ⇔ ambiguity of $|0\rangle$

⇒ **ambiguity of Feynman propagator:**

$$G(x,y) = \langle 0 | \mathrm{T}\,\phi(x)\phi(y) | 0 \rangle$$
$$= \theta(x^0 - y^0) \sum_n \varphi_n(x)\varphi^*_n(y) + \theta(y^0 - x^0) \sum_n \varphi_n(y)\varphi^*_n(x)$$

■ What is a natural vacuum?

case 1: $\exists$global timelike Killing vector $\xi = \xi^{\mu}(x)\partial_{\mu}$

⬇

Using t s.t. $\xi = \partial_t$,

we choose $\{\varphi_n(x) = \varphi_n(\mathbf{x}, t)\}$ s.t. they take the form

$$\varphi_n(x) = \varphi_n(\mathbf{x}, t) = \varphi_n(\mathbf{x})\, e^{-i\omega_n t} \quad (\omega_n > 0)$$

case 2: $\exists$vector $\xi = \xi^{\mu}(x)\partial_{\mu}$ that is asympt. timelike Killing

⬇

$\xi \to \partial_{t_{\mathrm{in}}}$ (far past) ⇒ $|0;\mathrm{in}\rangle$

$\xi \to \partial_{t_{\mathrm{out}}}$ (far future) ⇒ $|0;\mathrm{out}\rangle$

(Generally, $|0;\mathrm{in}\rangle \not\propto |0;\mathrm{out}\rangle$)

de Sitter space does not belong to case 1 or 2.

1. Basics of QFT in de Sitter space

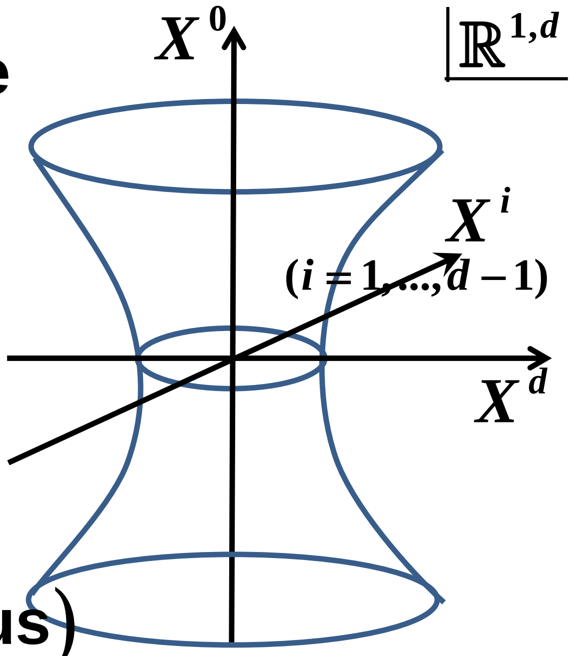
[1] MF-Sakatani-Sugishita [PRD88 (2013) 024041, 1301.7352]

■ d-dimensional de Sitter space

hyperboloid in $(d+1)$-dim Minkowski space

$$-(X^0)^2+(X^1)^2+\cdots+(X^{d-1})^2+(X^d)^2=\ell^2$$

$$\left(\text{topology: } \mathbb{R}\times S^{d-1}\right)$$



This is a solution to Einstein's equation with positive cosmological constant

$$\Lambda=\frac{(d-2)(d-1)}{2\ell^2}>0 \quad (\ell:\text{ curvature radius})$$

■ QFT in de Sitter space

Motivation

- one of the simplest but nontrivial spacetimes
- application to { inflation / cosmological const problem
- helpful in understanding the thermal character of curved spacetime

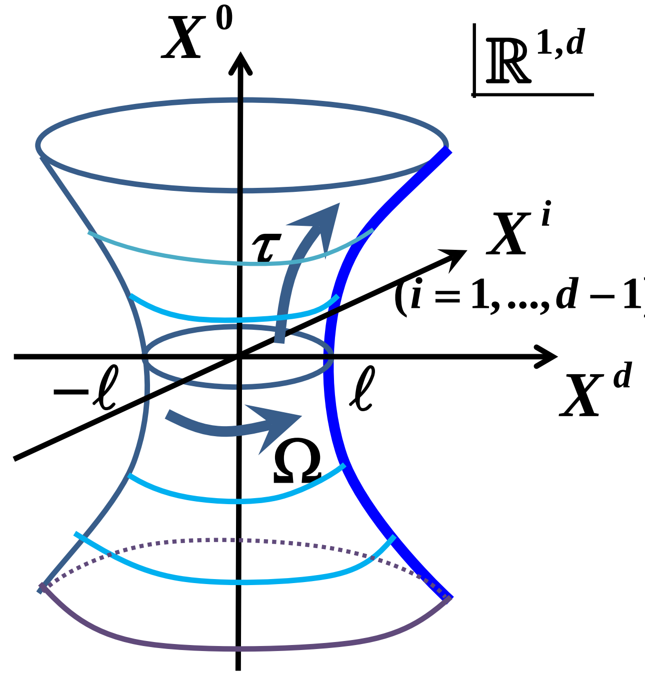
■ various coordinate patches

(1) global patch

$$\begin{cases} X^0 = \ell \sinh \dfrac{\tau}{\ell} \\ X^I = \ell \cosh \dfrac{\tau}{\ell}\, \Omega^I \ (I = 1, \ldots, d) \end{cases}$$

$$\left(\Omega \in S^{d-1}\ ; \quad \sum_{i=1}^{d} (\Omega^I)^2 = 1 \right)$$

$$ds^2 = -d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell}\, d\Omega_{d-1}^2$$



Penrose

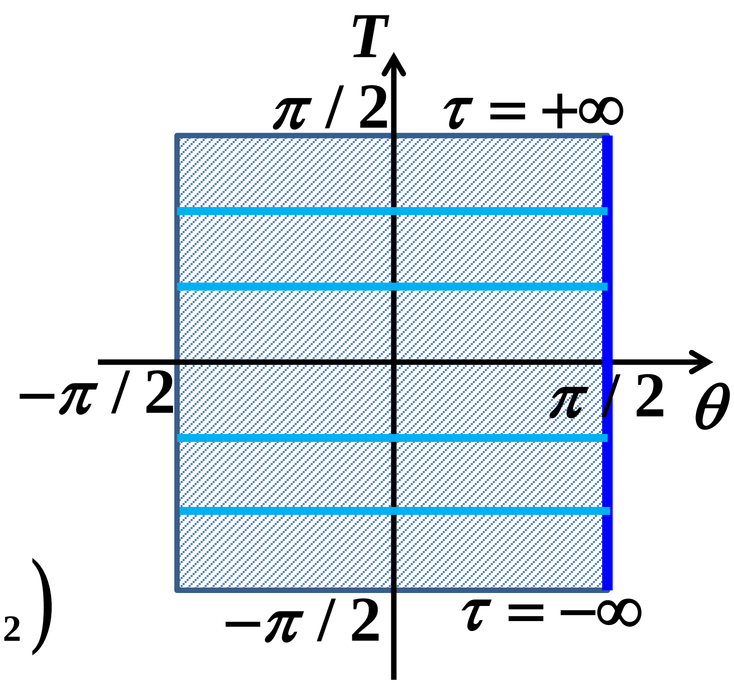
$$\sin T = \tanh \frac{\tau}{\ell},$$

$$\Omega^I = (\Omega^i, \theta)$$

$$\left(-\pi/2 < \tau, \theta < \pi/2 \right)$$

$$\begin{pmatrix} X^0 = \ell \sinh \dfrac{\tau}{\ell} \\ X^d = \ell \sin \theta \\ X^i = \ell \cos \theta\, \Omega^i \\ (i = 1, \ldots, d-1) \end{pmatrix}$$

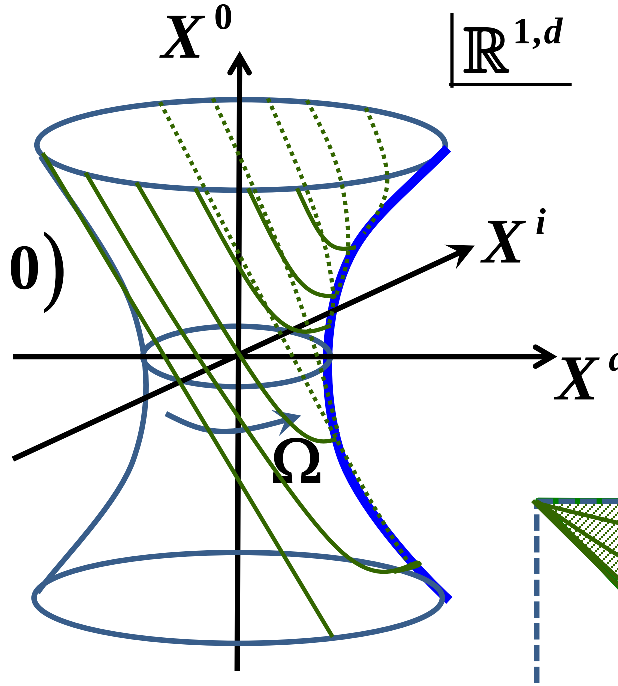
$$\Rightarrow ds^2 = \frac{\ell^2}{\cos^2 T} \left(-dT^2 + d\theta^2 + \cos^2 \theta\, d\Omega_{d-2}^2 \right)$$



(2) Poincaré patch

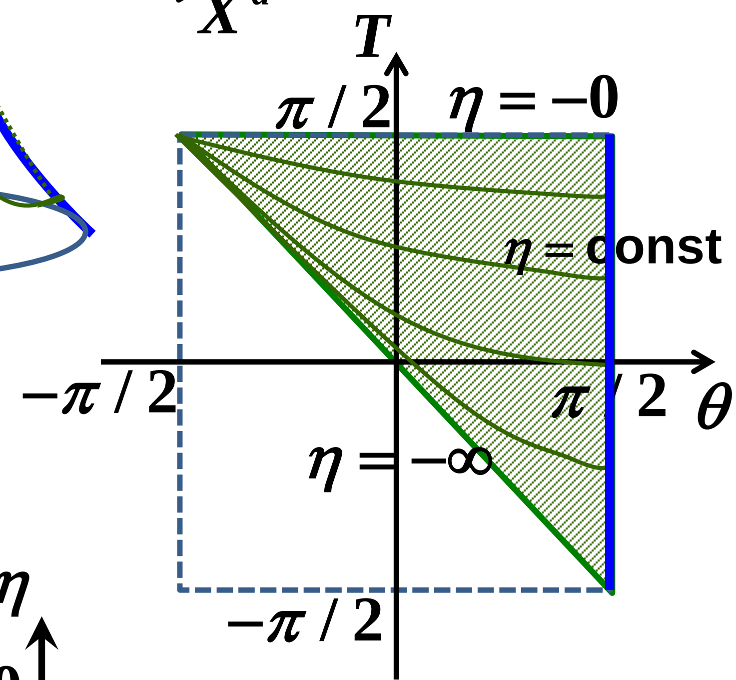
$$
\begin{cases}
X^0 = \dfrac{\ell^2 - \eta^2 + \mathbf{x}^2}{-2\eta} \\[2ex]
X^d = \dfrac{\ell^2 + \eta^2 - \mathbf{x}^2}{-2\eta} \\[2ex]
X^i = \dfrac{\ell x^i}{-\eta} \quad (i = 1, \ldots, d)
\end{cases}
$$

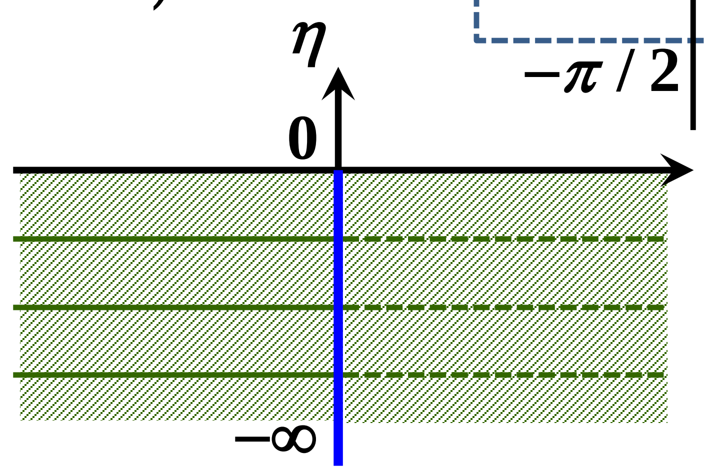
$(\eta < 0)$



$$
ds^2 = \ell^2 \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2} = -dt^2 + e^{2t/\ell} d\mathbf{x}^2
$$

$\left(-\eta = \ell\, e^{-t/\ell}\right)$

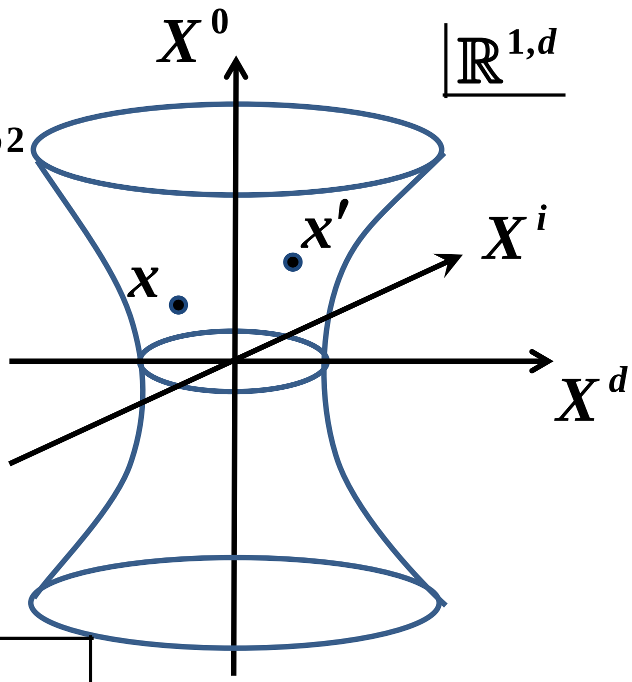




NB:

$$
X^0 + X^d = \frac{\ell^2}{-\eta} > 0
$$

■ de Sitter invariant



$$\eta_{MN} X^M X^N = -(X^0)^2 + (X^1)^2 + \cdots + (X^d)^2 = \ell^2$$

correspondence

$$x \in \mathrm{dS} \Leftrightarrow X(x) \in \mathrm{R}^{1,d}$$

⇓

invariant under $O(1,d)$:

$$Z(x,x') \equiv \frac{1}{\ell^2} \eta_{MN} X^M(x) X^N(x') \equiv \frac{1}{\ell^2} X \cdot X'$$

[global] $\left(ds^2 = -d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell} d\Omega_{d-1}^2 \right)$

$$Z(x,x') = -\sinh\frac{\tau}{\ell}\sinh\frac{\tau'}{\ell} + \cosh\frac{\tau}{\ell}\cosh\frac{\tau'}{\ell}\,\Omega\cdot\Omega'$$

[Poincaré] $\left(ds^2 = \ell^2 \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2} \right)$

$$Z(x,x') = \frac{\eta^2 + \eta'^2 - |\mathbf{x} - \mathbf{x}'|^2}{2\eta\eta'}$$

■ Issue of defining a vacuum in de Sitter space

no global timelike Killing vector

⇓

time-dependent Hamiltonian

⇓

no stationary states

If adiabaticity holds asymptotically (i.e. no level crossing occurs at the far past/future), one can define an asymptotic vacuum.

This cannot be expected for de Sitter space (except for $\eta = -\infty$ in Poincare).

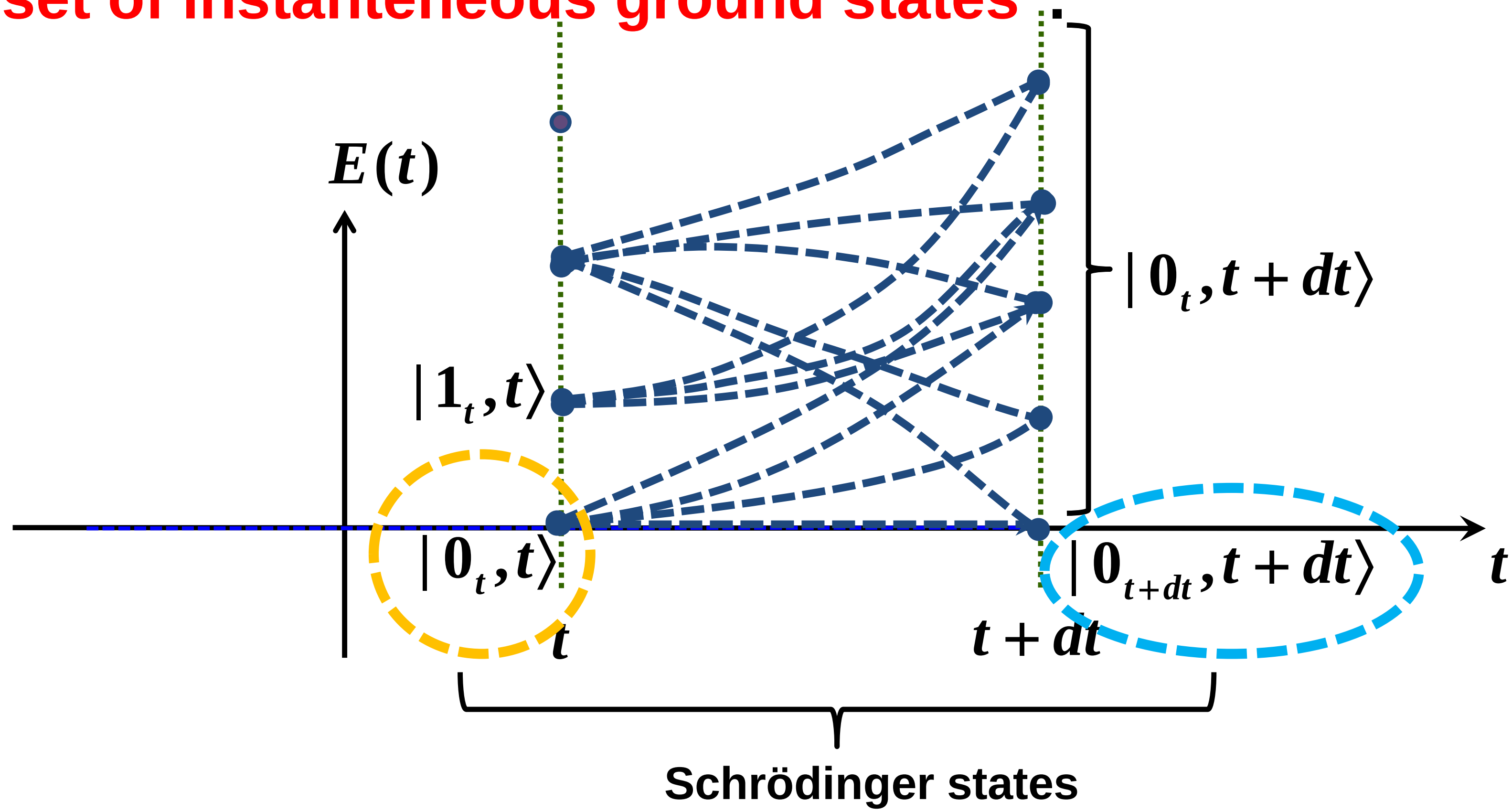
⇓

There exists an ambiguity in defining propagators.

■ Our strategy

[MF-Sakatani-Sugishita [1] 1301.7352]

- Instead of defining a global vacuum, we consider the ground state at each instant, and define the vacuum to be "the set of instanteneous ground states".



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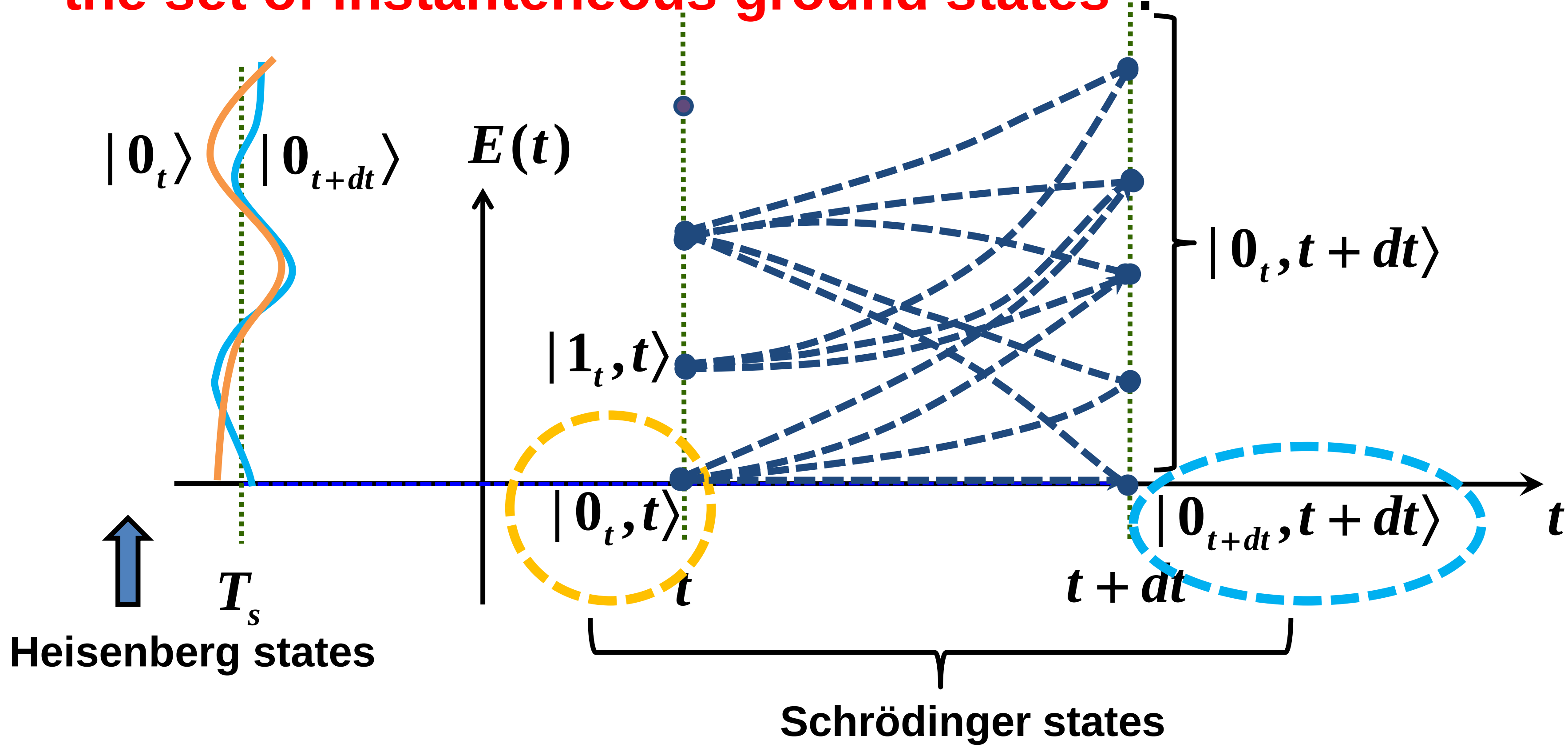
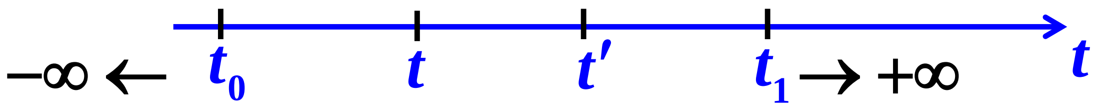


- **Introduce 2-pt functions:**

$$G_{00}(x,x';t_0,t_0) \equiv \frac{\langle 0_{t_0} | \mathrm{T}\phi(x)\,\phi(x') | 0_{t_0} \rangle}{\langle 0_{t_0} | 0_{t_0} \rangle}$$

$$G_{10}(x,x';t_1,t_0) \equiv \frac{\langle 0_{t_1} | \mathrm{T}\phi(x)\,\phi(x') | 0_{t_0} \rangle}{\langle 0_{t_1} | 0_{t_0} \rangle}$$



- **Take the limit $t_0 \to -\infty,\ t_1 \to +\infty$:**

$$G^{\text{in/in}}(x,x') \equiv \lim_{t_0\to-\infty} G_{00}(x,x';t_0,t_0) \equiv \lim_{t_0\to-\infty} \frac{\langle 0_{t_0} | T\phi(x)\,\phi(x') | 0_{t_0} \rangle}{\langle 0_{t_0} | 0_{t_0} \rangle}$$

$$G^{\text{out/in}}(x,x') \equiv \lim_{\substack{t_0\to-\infty \\ t_1\to+\infty}} G_{10}(x,x';t_1,t_0) \equiv \lim_{\substack{t_0\to-\infty \\ t_1\to+\infty}} \frac{\langle 0_{t_1} | T\phi(x)\,\phi(x') | 0_{t_0} \rangle}{\langle 0_{t_1} | 0_{t_0} \rangle}$$

- **Result:** **A careful calculation shows that the finite limit exists for all patches.**

(**except for $G^{\text{in/in}}$ in the global patch for $m < (d-1)/2\ell$**)

2. Propagators for the instantaneous vacuum in curved spacetime (general theory)

[1] MF-Sakatani-Sugishita [PRD88 (2013) 024041, 1301.7352]

Mode expansion

Assume that the metric takes the form

$$ds^2 = -N^2(t)\,dt^2 + A^2(t)\,h_{ij}(\mathrm{x})dx^i dx^j$$

Inroduce a complete set $\{Y_n(\mathrm{x}) \in \mathbb{R}\}$

for the spatial Laplacian $\Delta_{d-1} \equiv \frac{1}{\sqrt{h}}\partial_i\left(\sqrt{h}h^{ij}\partial_j\right)$ **s.t.**

$$\Delta_{d-1} Y_n(\mathrm{x}) = -\lambda_n Y_n(\mathrm{x}), \quad \int d^{d-1}\mathrm{x}\sqrt{h}\,Y_n(\mathrm{x})Y_m(\mathrm{x}) = \delta_{n,m}$$

Expand $\phi(x) = \phi(t,\mathrm{x}) = \sum_n \phi_n(t)\,Y_n(\mathrm{x}) \quad (\phi_n(t) \in \mathbb{R})$

⇓

$$S[\phi] = \sum_n \int dt\left[\frac{\rho(t)}{2}\dot{\phi}_n^2(t) - \frac{\rho(t)\,\omega_n^2(t)}{2}\phi_n^2(t)\right]$$

with $\rho(t) = N^{-1}(t)A^{d-1}(t)$ **and** $\omega_n^2(t) = N^2(t)\left[\lambda_n A^{-2}(t) + m^2\right]$

"a system of decoupled harmonic oscillators"

Hamiltonian (in the Schrödinger picture)

$$H_s(t) = \sum_n H_{n,s}(t) = \sum_n \left[\frac{\pi_{n,s}^2}{2\rho(t)} + \frac{\rho(t)\,\omega_n^2(t)}{2}\phi_{n,s}^2 \right]$$

$$= \sum_n \omega_n(t)\, a_{n,s}{}^{\dagger}(t)\, a_{n,s}(t) \quad (+\text{const})$$

$$\left(a_{n,s}(t) \equiv \sqrt{\frac{\rho(t)\,\omega_n(t)}{2}}\phi_{n,s} + \frac{i}{\sqrt{2\rho(t)\,\omega_n(t)}}\pi_{n,s} \right)$$

time-dependent even in the Schr. picture

instantaneous ground state at t :

$$|0_t, t\rangle \text{ s.t. } a_{n,s}(t)\,|0_t, t\rangle = 0 \; (\forall n)$$

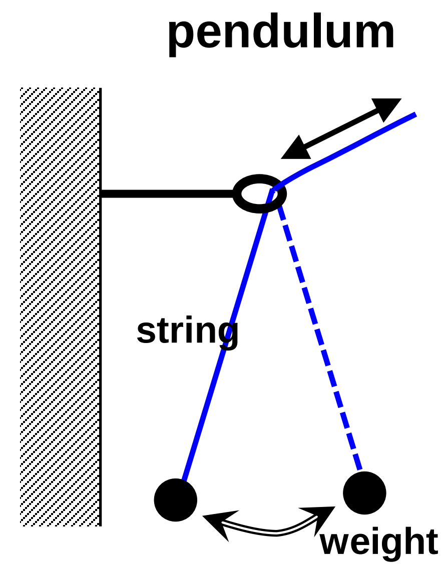
■ QM with time-dependent Hamiltonian

To simplify expressions, we set (for a while)

$$\phi_n \to q\,,\quad \pi_n \to p\,,\quad \omega_n(t) \to \omega(t)$$

$$H_{n,s}(t) \to H_s(t) = \frac{p_s^{\,2}}{2\rho(t)} + \frac{\rho(t)\,\omega^2(t)}{2} q_s^{\,2}$$

$$= \omega(t)\, a_s^{\dagger}(t)\, a_s(t)\ \ (+\text{const})$$



Time evolution operator

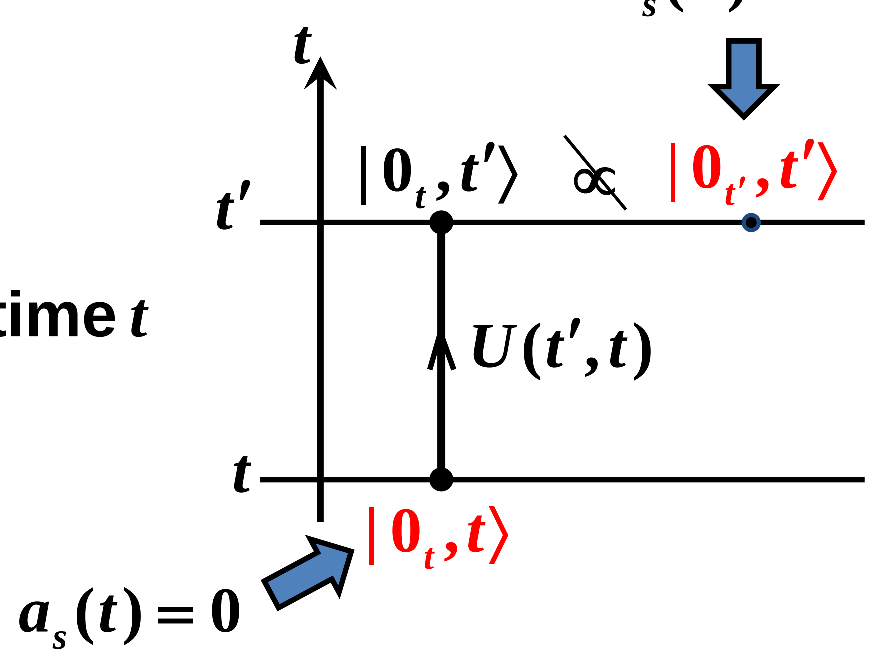
$$U(t',t) = \mathrm{T}\exp\left[-i\int_t^{t'} dt''\, H_s(t'')\right]$$

$$|\psi,t\rangle \ \to\ |\psi,t'\rangle = U(t',t)\,|\psi,t\rangle$$

Instantaneous ground state at time t

$$|0_t,t\rangle \ \text{ s.t. }\ a_s(t)\,|0_t,t\rangle = 0$$

NB: $|0_t,t'\rangle \not\propto |0_{t'},t'\rangle$



• **Heisenberg picture**

[Schrödinger]

$$|\psi,t\rangle = U\left(t,T_s\right)|\psi,T_s\rangle$$

$$O_s(t)$$

$\Leftrightarrow$

[Heisenberg]

$$|\psi\rangle = |\psi,T_s\rangle$$

$$O(t) = U^{-1}\left(t,T_s\right)O_s(t)\,U\left(t,T_s\right)$$

(e.g. $a_s(t) = \sqrt{\frac{\rho(t)\omega(t)}{2}}q_s + \frac{i}{\sqrt{2\rho(t)\omega(t)}}p_s$)

$$\left(\langle O\rangle_t = \langle\psi,t|O_s(t)|\psi,t\rangle = \langle\psi|O(t)|\psi\rangle\right)$$

$$\Rightarrow\quad H(t) = \frac{1}{2\rho(t)}p^2(t) + \frac{\rho(t)\omega^2(t)}{2}q^2(t)$$

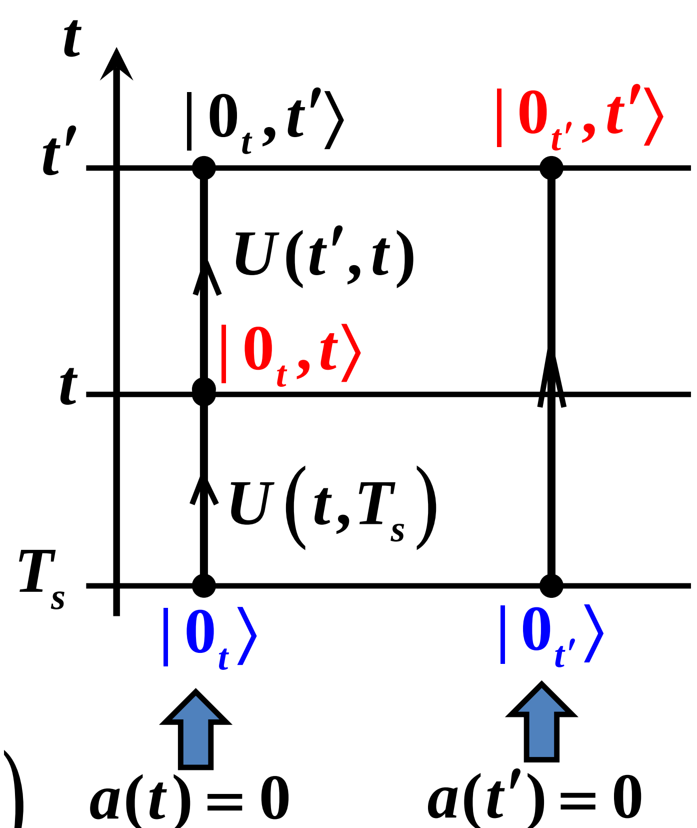
$$= \omega(t)\,a^\dagger(t)\,a(t)$$

Heisenberg eqn

$$\begin{cases} \dot{q}(t) = i\left[H(t), q(t)\right] = \dfrac{p(t)}{\rho(t)} \\ \dot{p}(t) = i\left[H(t), p(t)\right] = \rho(t)\,\omega^2(t)\,q(t) \end{cases}$$

$$\Leftrightarrow\quad \ddot{q}(t) + \frac{\dot{\rho}(t)}{\rho(t)}\dot{q}(t) + \omega^2(t)\,q(t) = 0$$

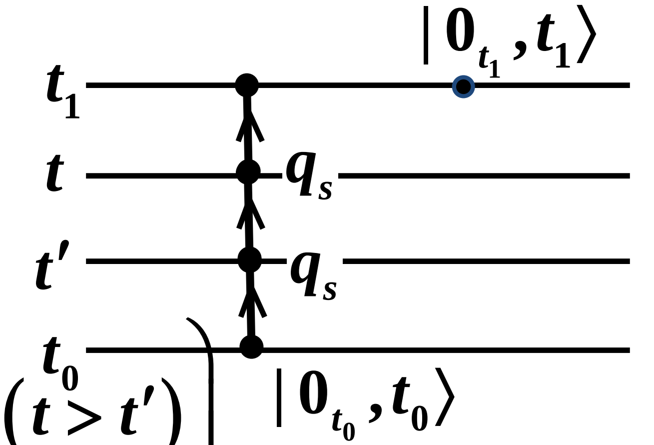
$\left(\Rightarrow\ W_\rho[f,g] \equiv \rho\left(f\dot{g} - \dot{f}g\right)\ :\ t\text{-indep}\right)$



- **2-pt function**

$$G_{10}(t,t';t_1,t_0) \equiv \frac{\langle 0_{t_1} | \mathrm{T}\, q(t)\, q(t') | 0_{t_0} \rangle}{\langle 0_{t_1} | 0_{t_0} \rangle}$$

$$\left(= \frac{\langle 0_{t_1}, t_1 | U(t_1,t)\, q_s\, U(t,t')\, q_s\, U(t',t_0) | 0_{t_0}, t_0 \rangle}{\langle 0_{t_1}, t_1 | U(t_1,t_0) | 0_{t_0}, t_0 \rangle} \quad (t > t') \right)$$



This can be easily calculated if one expresses $q(t)$ as

$\varphi(t;t_I)$: **"wave function"**

$$q(t) = \varphi(t;t_I)\, a(t_I) + \varphi^*(t;t_I)\, a^\dagger(t_I) \qquad (t_I = t_0, t_1)$$

$$G_{10}(t,t';t_1,t_0) \equiv \frac{\langle 0_{t_1} | a(t_1)\, a^\dagger(t_0) | 0_{t_0} \rangle}{\langle 0_{t_1} | 0_{t_0} \rangle} \varphi(t;t_1)\, \varphi^*(t';t_0)$$

$$= \frac{i}{W_\rho[\varphi(s;t_1), \varphi^*(s;t_0)]} \varphi(t;t_1)\, \varphi^*(t';t_0)$$

- How can one find the wave functions $\varphi(t;t_I)$?

Start from Heisenberg's eqn : $\ddot{q}(t)+\frac{\dot{\rho}(t)}{\rho(t)}\dot{q}(t)+\omega^2(t)\,q(t)=0$

⇩

Find a pair of independent c-# solns: $\{f(t),\ g(t)\}$

⇩

$q(t),\ p(t)$ then can be expressed as

$c_1,\ c_2$: const operators

$$q(t)=\big(f(t),g(t)\big)\begin{pmatrix}c_1\\c_2\end{pmatrix},\quad p(t)=\rho(t)\dot{q}(t)=\rho(t)\big(\dot{f}(t),\dot{g}(t)\big)\begin{pmatrix}c_1\\c_2\end{pmatrix}$$

$$\Leftrightarrow\ \begin{pmatrix}f(t) & g(t)\\ \dot{f}(t) & \dot{g}(t)\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}q(t)\\ \frac{p(t)}{\rho(t)}\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2\rho(t)\omega(t)}}\big(a(t)+a^\dagger(t)\big)\\ -i\sqrt{\frac{\omega(t)}{2\rho(t)}}\big(a(t)-a^\dagger(t)\big)\end{pmatrix}$$

$$=\frac{1}{\sqrt{2\rho(t)\omega(t)}}\begin{pmatrix}1 & 1\\ -i\omega(t) & i\omega(t)\end{pmatrix}\begin{pmatrix}a(t)\\a^\dagger(t)\end{pmatrix}$$

$$\therefore \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{\sqrt{2\rho(t)\omega(t)}} \begin{pmatrix} f(t) & g(t) \\ \dot{f}(t) & \dot{g}(t) \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ -i\omega(t) & i\omega(t) \end{pmatrix} \begin{pmatrix} a(t) \\ a^\dagger(t) \end{pmatrix}$$

$$= \underbrace{\frac{1}{W_\rho[f,g]} \begin{pmatrix} v(t) & \bar{v}(t) \\ -u(t) & -\bar{u}(t) \end{pmatrix}}_{\equiv C(t)} \begin{pmatrix} a(t) \\ a^\dagger(t) \end{pmatrix}$$

with $\begin{Bmatrix} u(t) \\ \bar{u}(t) \end{Bmatrix} \equiv \dot{f}(t) \pm i\omega(t) f(t)$, $\begin{Bmatrix} v(t) \\ \bar{v}(t) \end{Bmatrix} \equiv \dot{g}(t) \pm i\omega(t) g(t)$

c_1, c_2 are t-independent

⇓

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = C(t) \begin{pmatrix} a(t) \\ a^\dagger(t) \end{pmatrix} = C(t') \begin{pmatrix} a(t') \\ a^\dagger(t') \end{pmatrix}$$

⇓

Fundamental formula [MF-Sakatani-Sugishita]

$$\begin{pmatrix} a(t) \\ a^\dagger(t) \end{pmatrix} = B(t;t') \begin{pmatrix} a(t') \\ a^\dagger(t') \end{pmatrix}$$

with

$$B(t;t') \equiv C^{-1}(t) C(t') \equiv \begin{pmatrix} \alpha^*(t;t') & -\beta^*(t;t') \\ -\beta(t;t') & \alpha(t;t') \end{pmatrix}$$

"Bogoliubov coefficients"

$$= \frac{-i}{2W_\rho[f,g]\sqrt{\rho(t)\omega(t)\rho(t')\omega(t')}} \begin{pmatrix} \bar{v}(t)u(t') - \bar{u}(t)v(t') & \bar{v}(t)\bar{u}(t') - \bar{u}(t)\bar{v}(t') \\ u(t)v(t') - v(t)u(t') & u(t)\bar{v}(t') - v(t)\bar{u}(t') \end{pmatrix}$$

In particular,

$$q(t)=\frac{1}{\sqrt{2\rho(t)\omega(t)}}\left[a(t)+a^{\dagger}(t)\right]$$

$$=\frac{1}{\sqrt{2\rho(t)\omega(t)}}\left[\left(\alpha^{*}(t;t_I)-\beta(t;t_I)\right)a(t_I)+\left(\alpha(t;t_I)-\beta^{*}(t;t_I)\right)a^{\dagger}(t_I)\right]$$

$$\equiv \varphi(t;t_I)\,a(t_I)+\varphi^{*}(t;t_I)\,a^{\dagger}(t_I)$$

⇩

wave functions

$$\varphi(t;t_I)=\frac{1}{W_\rho[f,g]\sqrt{2\rho_I\omega_I}}\left[v_I\, f(t)-u_I\, g(t)\right]$$

$$\varphi^{*}(t;t_I)=\frac{1}{W_\rho[f,g]\sqrt{2\rho_I\omega_I}}\left[\bar{v}_I\, f(t)-\bar{u}_I\, g(t)\right]$$

$$\left(\begin{array}{l}\rho_I\equiv\rho(t_I),\quad \omega_I\equiv\omega(t_I),\\ \left\{\begin{array}{c}u_I\\ \bar{u}_I\end{array}\right\}\equiv \dot{f}(t_I)\pm i\omega(t_I)\,f(t_I),\quad \left\{\begin{array}{c}v_I\\ \bar{v}_I\end{array}\right\}\equiv \dot{g}(t_I)\pm i\omega(t_I)\,g(t_I)\end{array}\right)$$

- **Comments**

(1) Everything is determined automatically.
In particular, we need not to specify
the asymptotic forms of wave functions.

(2) Bogoliubov coefficients are independent of
the choice of indep solns $\{ f(t), g(t) \}$ (as they should).

(3) In order to ensure the convergence in the limit $\begin{cases} t_0 \to -\infty, \\ t_1 \to +\infty, \end{cases}$
we need to introduce a small imaginary part:

$$H(t) = e^{-i\varepsilon} H_{\text{original}}(t) \quad \Leftrightarrow \quad \begin{cases} \rho(t) = e^{i\varepsilon} \rho_{\text{original}}(t), \\ \omega(t) = e^{-i\varepsilon} \omega_{\text{original}}(t) \end{cases}$$

■ Summary

algorithm to obtain propagators [MF-Sakatani-Sugishita [1] 1301.7352]

- **Decompose** $\phi(x) = \phi(t,\mathbf{x}) = \sum_n \phi_n(t) Y_n(\mathbf{x})$
- **Solve EOM to obtain c-# soln:**

$$\ddot{\phi}_n(t) + \frac{\dot{\rho}(t)}{\rho(t)}\dot{\phi}_n(t) + \omega_n^2(t)\,\phi_n(t) = 0 \;\Rightarrow\; \{\, f_n(t),\;\; g_n(t)\}$$

- **Calculate wave functions** $\left(t_I = t_0\,,\, t_1\right)$

$$\begin{cases} \varphi_n(t;t_I) = \dfrac{1}{W_\rho[\,f_n, g_n]\sqrt{2\rho(t_I)\omega_n(t_I)}}\left[v_n(t_I)\,f_n(t) - u_n(t_I)\,g_n(t)\right] \\ \varphi_n^*(t;t_I) = \dfrac{1}{W_\rho[\,f_n, g_n]\sqrt{2\rho(t_I)\omega_n(t_I)}}\left[\bar{v}_n(t_I)\,f_n(t) - \bar{u}_n(t_I)\,g_n(t)\right] \end{cases}$$

$$\left(\begin{Bmatrix} u_n(t_I) \\ \bar{u}_n(t_I) \end{Bmatrix} \equiv \dot{f}_n(t_I) \pm i\omega_n(t_I)\,f_n(t_I),\quad \begin{Bmatrix} v_n(t_I) \\ \bar{v}_n(t_I) \end{Bmatrix} \equiv \dot{g}_n(t_I) \pm i\omega_n(t_I)\,g_n(t_I)\right)$$

- **2-pt functions for each mode**

$$G_{n,10}(t,t';t_1,t_0)=\frac{\langle 0_{t_1}\,|\,\mathrm{T}\,\phi_n(t)\,\phi_n(t')\,|\,0_{t_0}\rangle}{\langle 0_{t_1}\,|\,0_{t_0}\rangle}=\frac{i\,\varphi_n(t;t_1)\,\varphi_n^*(t';t_0)}{W_\rho[\varphi_n(s;t_1),\varphi_n^*(s;t_0)]}$$

$$G_{n,00}(t,t';t_0,t_0)=\frac{\langle 0_{t_0}\,|\,\mathrm{T}\,\phi_n(t)\,\phi_n(t')\,|\,0_{t_0}\rangle}{\langle 0_{t_0}\,|\,0_{t_0}\rangle}=\frac{i\,\varphi_n(t;t_0)\,\varphi_n^*(t';t_0)}{W_\rho[\varphi_n(s;t_0),\varphi_n^*(s;t_0)]}$$

- **Sum up over n:**

$$G_{10}(x,x';t_1,t_0)=\sum_n G_{n,10}(t,t';t_1,t_0)Y_n(\mathrm{x})Y_n(\mathrm{x}')$$

$$G_{00}(x,x';t_0,t_0)=\sum_n G_{n,00}(t,t';t_0,t_0)Y_n(\mathrm{x})Y_n(\mathrm{x}')$$

- **Take the limit $t_0\to-\infty,\ t_1\to+\infty$,**

  to obtain the in-out and in-in propagators:

$$G^{\text{out/in}}(x,x')\equiv\lim_{\substack{t_0\to-\infty\\ t_1\to+\infty}}G_{10}(x,x';t_1,t_0)$$

$$G^{\text{in/in}}(x,x')\equiv\lim_{t_0\to-\infty}G_{00}(x,x';t_0,t_0)$$

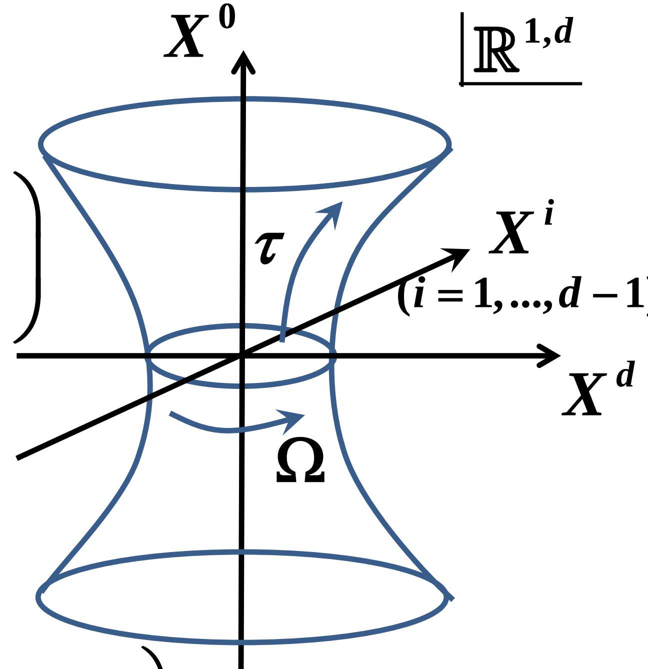
3. Propagators in de Sitter space

[1] MF-Sakatani-Sugishita [PRD88 (2013) 024041, 1301.7352]

■ Global patch

$$ds^2 = -d\tau^2 + \cosh^2\tau \, d\Omega_{d-1}^2 \qquad (\ell = 1)$$

$$= -\frac{dt^2}{(1-t^2)^2} + \frac{1}{1-t^2} d\Omega_{d-1}^2 \qquad \left(\begin{array}{l} t \equiv \tanh\tau; \\ -1 < t < +1 \end{array} \right)$$



- **mode expansion**

$$\phi(x) = \sum_{L\geq 0} \sum_{M=1}^{N_L^{(d)}} \phi_{LM}(t) \, Y_{LM}(\Omega)$$

$$\left(\begin{array}{l} \Delta_{d-1} Y_{LM}(\Omega) = -L(L+d-2) Y_{LM}(\Omega) \\ Y_{LM}(\Omega) \in \mathbb{R}, \quad N_L^{(d)} = \dfrac{(L+d-2)!}{(d-2)!\,L!}(2L+d-2) \\ \int d^{d-1}\Omega \, Y_{LM}(\Omega) \, Y_{L'M'}(\Omega) = \delta_{LL'}\delta_{MM'} \end{array} \right)$$

- **EOM**

$$\ddot{\phi}_{LM}(t) + \frac{(d-3)t}{1-t^2}\dot{\phi}_{LM}(t) + \left[\frac{L(L+d-2)}{1-t^2} + \frac{m^2}{(1-t^2)^2} \right] \phi_{LM}(t) = 0$$

⇩

- **independent c-# solns**

$$\begin{cases} f_{LM}(t) = (1-t^2)^{(d-1)/4} \mathrm{P}^{\nu}_{L+(d-3)/2}(t) \\ g_{LM}(t) = (1-t^2)^{(d-1)/4} \mathrm{Q}^{\nu}_{L+(d-3)/2}(t) \end{cases}$$

$$\left(\nu \equiv \begin{cases} \sqrt{\left(\frac{d-1}{2}\right)^2 - m^2} & \left(m < \frac{d-1}{2}\right) \\ i\sqrt{m^2 - \left(\frac{d-1}{2}\right)^2} \equiv i\mu & \left(m > \frac{d-1}{2}\right) \end{cases} \right)$$

- **wave functions**

$$\begin{cases} \varphi_{LM}(t;t_1) \propto (1-t^2)^{(d-1)/4} \mathrm{P}^{-\nu}_{L+(d-3)/2}(t) & (t_1 \sim +1) \\ \varphi_{LM}(t;t_0) \propto (1-t^2)^{(d-1)/4} \mathrm{Q}^{\nu}_{L+(d-3)/2}(t) & (t_0 \sim -1) \end{cases}$$

- **2-pt functions**

$\left(\mathrm{P}^{\nu}_k(t), \mathrm{Q}^{\nu}_k(t) \right.$: associated Legendre functions defined on the interval $\left. -1 < t < +1 \right)$

$$G^{\text{out/in}}_{LM}(t,t')$$

$$= \begin{cases} \dfrac{i\pi}{2\sin\pi\nu} \left[(1-t_>^2)(1-t_<^2)\right]^{(d-1)/4} \mathrm{P}^{-\nu}_{L+(d-3)/2}(t_>)\, \mathrm{P}^{\nu}_{L+(d-3)/2}(t_>) & (d:\text{odd}) \\ \dfrac{1}{\cos\pi\nu} \left[(1-t_>^2)(1-t_<^2)\right]^{(d-1)/4} \mathrm{P}^{-\nu}_{L+(d-3)/2}(t_>)\, \mathrm{Q}^{\nu}_{L+(d-3)/2}(t_>) & (d:\text{even}) \end{cases}$$

• mode sum

$$G^{\text{out/in}}(x,x')$$

$$= \sum_L \sum_M G_{LM}^{\text{out/in}}(t,t')\, Y_{LM}(\Omega)\, Y_{LM}(\Omega') = \sum_L G_L^{\text{out/in}}(t,t') \underbrace{\sum_M Y_{LM}(\Omega)\, Y_{LM}(\Omega')}_{\frac{2L+d-2}{(d-2)|\Omega_{d-1}|} C_L^{(d-2)/2}\left(\Omega\cdot\Omega'\right)}$$

Gegenbauer polynomial

$$= \begin{cases} \dfrac{-e^{-i\pi(d-2)/2}}{2(2\pi)^{d/2}\sin\pi\nu}\left[\begin{array}{l}(u_+^2-1)^{-(d-2)/4} Q_{\nu-1/2}^{(d-2)/2}(u_+) \\ -(u_-^2-1)^{-(d-2)/4} Q_{\nu-1/2}^{(d-2)/2}(u_-)\end{array}\right] & (d:\text{odd}) \\[2ex] \dfrac{ie^{-i\pi(d-2)/2}}{2(2\pi)^{d/2}\cos\pi\nu}\left[\begin{array}{l}(u_+^2-1)^{-(d-2)/4} Q_{\nu-1/2}^{(d-2)/2}(u_+) \\ +(u_-^2-1)^{-(d-2)/4} Q_{\nu-1/2}^{(d-2)/2}(u_-)\end{array}\right] & (d:\text{even}) \end{cases}$$

($P_k^\nu(z), Q_k^\nu(z)$: associated Legendre functions
defined except for the cut $-\infty < z < +1$)

Here $u_\pm(x,x') = -Z(x,x') \pm i0$.

$G^{\text{out/in}}(x,x')$ **is de Sitter invariant!**

Similarly,

$$G_{\text{odd}}^{\text{in/in}}(x,x') = G_{\text{odd}}^{\text{out/in}}(x,x')$$

$$= \frac{-e^{-i\pi(d-2)/2}}{2(2\pi)^{d/2}\sin\pi\nu}\begin{bmatrix}(u_+^2-1)^{-(d-2)/4}Q_{\nu-1/2}^{(d-2)/2}(u_+) \\ -(u_-^2-1)^{-(d-2)/4}Q_{\nu-1/2}^{(d-2)/2}(u_-)\end{bmatrix}$$

$$G_{\text{even}}^{\text{in/in}}(x,x')$$

$$= \frac{e^{-i\pi(d-2)/2}}{2(2\pi)^{d/2}\sin\pi\nu}\left\{\begin{bmatrix}(u_+^2-1)^{-(d-2)/4}Q_{\nu-1/2}^{(d-2)/2}(u_+) \\ -(u_-^2-1)^{-(d-2)/4}Q_{\nu-1/2}^{(d-2)/2}(u_-)\end{bmatrix} + \frac{i\pi}{\cos\pi\nu}\begin{bmatrix}\mathrm{e}^{-i\pi\nu}(u_+^2-1)^{-(d-2)/4}P_{\nu-1/2}^{(d-2)/2}(u_+) \\ +e^{i\pi\nu}(u_-^2-1)^{-(d-2)/4}P_{\nu-1/2}^{(d-2)/2}(u_-)\end{bmatrix}\right\}$$

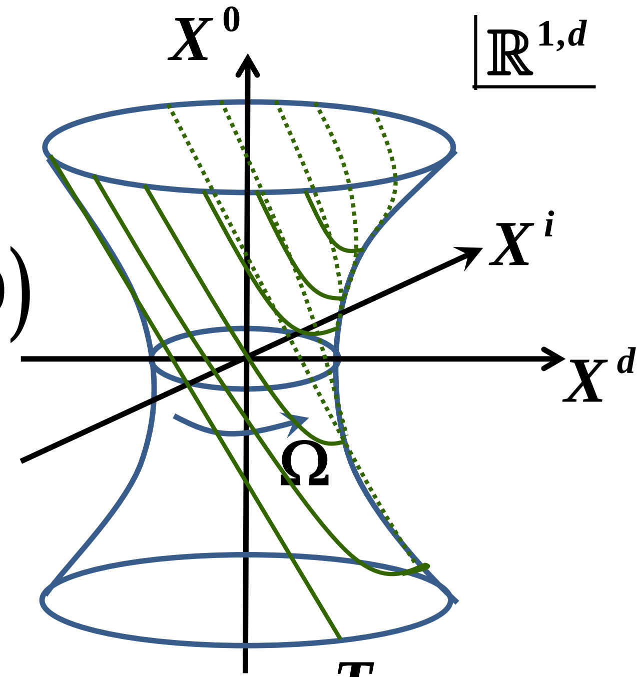
Again, $u_\pm(x,x') = -Z(x,x') \pm i0$

⇒ $G^{\text{in/in}}(x,x')$ is also de Sitter invariant!

■ Poincare patch

$$ds^2 = -d\tau^2 + e^{2\tau}\, d\mathbf{x}^2 \qquad (\ell = 1)$$

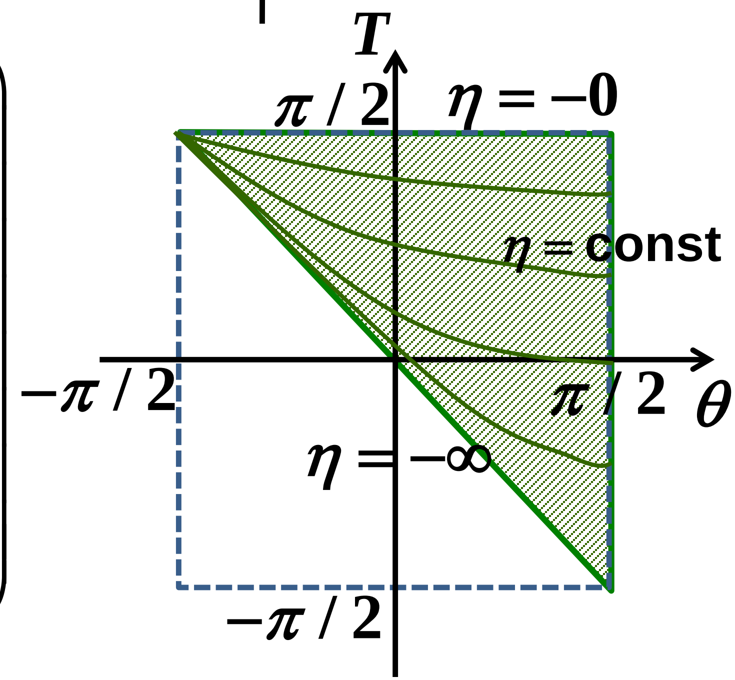
$$= \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2} \qquad \left(\eta \equiv -e^{-\tau};\ -\infty < \eta < 0\right)$$



• mode expansion

$$\phi(x) = \sum_{\mathrm{k}\geq 0} \sum_a \phi_{\mathrm{k},a}(t)\, Y_{\mathrm{k,a}}(\mathbf{x})$$

$$\left(\begin{array}{l} \Delta_{d-1} Y_{\mathrm{k},a}(\mathbf{x}) = -k^2 Y_{\mathrm{k},a}(\mathbf{x}) \quad \left(k = |\mathbf{k}|\right) \\ \left\{\begin{array}{l} Y_{\mathrm{k}=0,a=1} = \dfrac{1}{\sqrt{V}} \qquad \left(\begin{array}{l} \mathbf{k} = \left(k_1, \ldots, k_{d-1}\right) > 0 \\ \Leftrightarrow \text{1st nonzero element} > 0 \end{array}\right) \\ Y_{\mathrm{k}>0,a=1} = \sqrt{\dfrac{2}{V}} \cos \mathbf{k}\cdot\mathbf{x}, \quad Y_{\mathrm{k}>0,a=2} = \sqrt{\dfrac{2}{V}} \sin \mathbf{k}\cdot\mathbf{x} \end{array}\right. \\ \int_V d^{d-1}\mathbf{x}\, Y_{\mathrm{k},a}(\mathbf{x})\, Y_{\mathrm{k}',a'}(\mathbf{x}) = \delta_{\mathrm{k},\mathrm{k}'} \delta_{a,a'} \end{array}\right)$$



- **EOM**

$$\ddot{\phi}_{\mathrm{k},a}(\eta)-\frac{d-2}{\eta}\dot{\phi}_{\mathrm{k},a}(\eta)+\left(k^2+\frac{m^2}{\eta^2}\right)\phi_{\mathrm{k},a}(\eta)=0$$

- **independent c-# solns**

$$\begin{cases} f_{LM}(\eta)=(-\eta)^{(d-1)/2}J_\nu(-k\eta) \\ g_{LM}(\eta)=(-\eta)^{(d-1)/2}N_\nu(-k\eta) \end{cases}$$

$$\left(\nu\equiv\begin{cases}\sqrt{\left(\frac{d-1}{2}\right)^2-m^2} & \left(m<\left(\frac{d-1}{2}\right)\right) \\ i\sqrt{m^2-\left(\frac{d-1}{2}\right)^2}=i\mu & \left(m>\left(\frac{d-1}{2}\right)\right)\end{cases}\right)$$

- **2-pt functions**

$$G^{\text{out/in}}_{\mathrm{k},a}(\eta,\eta')=\frac{\pi}{2}\left[(-\eta)(-\eta')\right]^{(d-1)/2}J_\nu(-k\eta_>)H^{(2)}_\nu(-k\eta_<)$$

$$G^{\text{in/in}}_{\mathrm{k},a}(\eta,\eta')=\frac{\pi}{4}\left[(-\eta)(-\eta')\right]^{(d-1)/2}H^{(1)}_\nu(-k\eta_<)H^{(2)}_\nu(-k\eta_<)$$

- **mode sum**

$$G^{\text{out/in}}(x,x') = \frac{e^{-i\pi(d-2)}}{(2\pi)^{d/2}}(u^2-1)^{-(d-2)/4} Q^{(d-2)/2}_{\nu-1/2}(u)$$

$$G^{\text{in/in}}(x,x')$$

$$= \frac{\Gamma\left(\frac{d-1}{2}+\nu\right)\Gamma\left(\frac{d-1}{2}-\nu\right)}{2(2\pi)^{d/2}}(u^2-1)^{-(d-2)/4} P^{-(d-2)/2}_{\nu-1/2}(u)$$

$$= \frac{\Gamma\left(\frac{d-1}{2}+\nu\right)\Gamma\left(\frac{d-1}{2}-\nu\right)}{(4\pi)^{d/2}\Gamma(d/2)} F\left(\frac{d-1}{2}+\nu, \frac{d-1}{2}-\nu, \frac{d}{2}, \frac{1-u}{2}\right)$$

Here, $u = -Z(x,x') + i0.$

⇨ $G^{\text{out/in}}(x,x'),\ G^{\text{in/in}}(x,x')$ **are de Sitter invariant!**

■ Properties

- **in-in propagators in the Poincare patch**
  - no massless limit (as is already known)
  - agrees with the propagators for the BD vacuum:

$$G^{\text{in/in}}(x,x') = G_{\text{BD}}(x,x') \quad \text{i.e.} \quad |0_{\eta_0}\rangle \xrightarrow{\eta_0 \to -\infty} |\text{BD}\rangle$$

⇒ $G^{\text{in/in}}(x,x')$: analytic continuation of Euclidean propagator on S^d

$$\left(ds^2 = -d\tau^2 + \cosh^2 \tau d\Omega_{d-1}^2 \;\Leftarrow\; ds_{\text{E}}^2 = d\tau_{\text{E}}^2 + \cos^2 \tau_{\text{E}}\, d\Omega_{d-1}^2 \right)$$

- **in-out propagators in the Poincare patch**
  - can take the massless limit
  - $G^{\text{out/in}}(x,x')$: analytic continuation of propagator on Euclidean AdS

$$\left(ds^2 = \ell^2 \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2} \;\Leftarrow\; ds_{\text{E}}^2 = \ell_{\text{E}}^2 \frac{d\eta_{\text{E}}^2 + d\mathbf{x}^2}{\eta_{\text{E}}^2} \quad \left(\ell_{\text{E}} = i\ell,\, \eta_{\text{E}} = i\eta \right) \right)$$

■ in-in and in-out

Which propagator should be used depends on the problem under consideration.

- **in-in**

$$G^{\text{in/in}}(x,x') = \frac{\langle \text{in} | \mathrm{T}\phi(x)\phi(x') | \text{in}\rangle}{\langle \text{in} | \text{in}\rangle}$$

In particular,

$$G^{\text{in/in}}(t,\mathbf{x};\, t,\mathbf{x}')$$
$$= \left(\phi_s(\mathbf{x})U(t,-\infty)\,|\,0_{-\infty},-\infty\rangle\right)^{\dagger}\phi_s(\mathbf{x}')U(t,-\infty)\,|\,0_{-\infty},-\infty\rangle$$

➡ **suitable when discussing expectation values and quantum fluctuations**

cf. quantum fluct. in inflation

- **in-out**

$$G^{\text{out/in}}(x,x') = \frac{\langle \text{out} | \mathrm{T}\phi(x)\,\phi(x') | \text{in}\rangle}{\langle \text{out} | \text{in}\rangle}$$

➡ **suitable when discussing the S-matrix**

■ More on the in-out propagators

- **relation to the functional integral with $i\varepsilon$**

$$G^{\text{out/in}}(x,x') = \frac{\int [d\phi(x)]\, e^{iS_\varepsilon[\phi]}\, \phi(x)\phi(x')}{\int [d\phi(x)]\, e^{iS_\varepsilon[\phi]}}$$

⇐ (proved and confirmed numerically)

with $S_\varepsilon[\phi] = \int d^d x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2 - i\varepsilon}{2}\phi^2 \right]$

- **relation to the random walks in de Sitter space**

∃ a heat kernel representation:

$$G^{\text{out/in}}(x,x') = \frac{1}{2}\int_0^\infty dT\, \langle x \,|\, e^{-i(T/2)(-\Box + m^2 - i\varepsilon)} \,|\, x' \rangle = \sum_{\substack{\text{path} \\ \text{from } x' \text{ to } x}} e^{\,i\,\text{length(path)}}$$

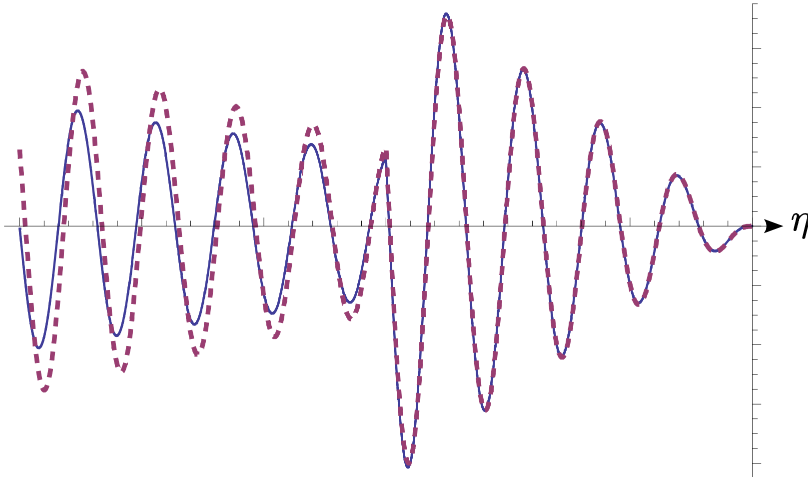
means

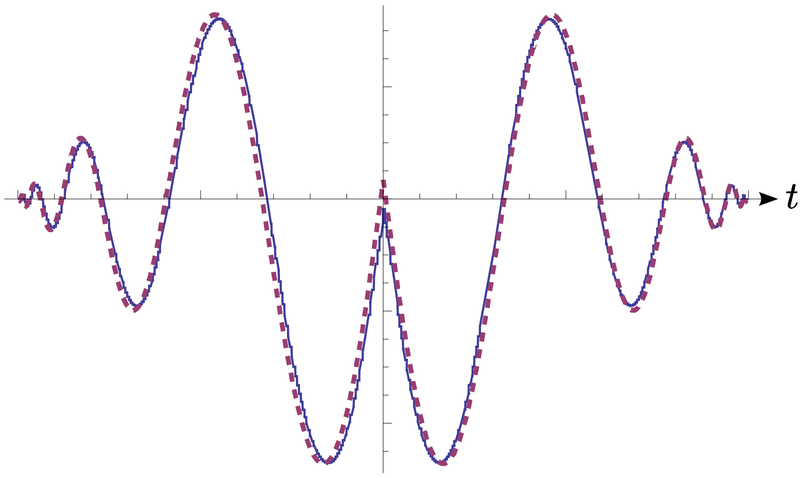
$$\left(= \langle x \,|\, \frac{-i}{-\Box + m^2} \,|\, x' \rangle \right)$$

⇒ $$\frac{\partial}{\partial m^2} G^{\text{out/in}}(x,x') = i \int d^d y \sqrt{-g(y)}\, G^{\text{out/in}}(x,y)\, G^{\text{out/in}}(y,x')$$

"composition law" [Polyakov]

■ Numerical results





$G_{\mathbf{k}}^{\text{out/in}}(\eta, 0)$ in Poincare

$(d = 4,\ m\ell = 0.5,\ k = 1)$

$G_{L}^{\text{out/in}}(t, 0)$ in global

$(d = 4,\ m\ell = 9,\ L = 1)$

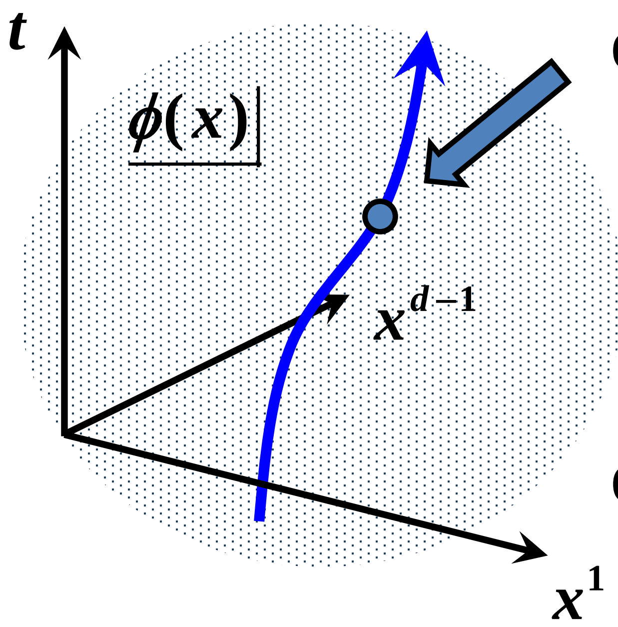
PART 2 :   Unruh-DeWitt detector in curved spacetime

[2] MF-Sakatani-Sugishita [PRD89 (2014) 064024, 1305.0256]

4.   Motivation and the result

[2] MF-Sakatani-Sugishita [PRD89 (2014) 064024, 1305.0256]

■ Unruh-DeWitt detector



detector

- interacting with $\phi(x)$
- moving along a classical path:

$$x(\tau) = \big(t(\tau), \mathbf{x}(\tau)\big) = \big(t, \mathbf{x}(t)\big)$$

detector Hamiltonian H^{d}

- const for proper time τ
- $H^{\mathrm{d}} \,|\, n\rangle = E_n \,|\, n\rangle \quad \big(n = 0, 1, 2, \ldots\big)$

For some spacetime,

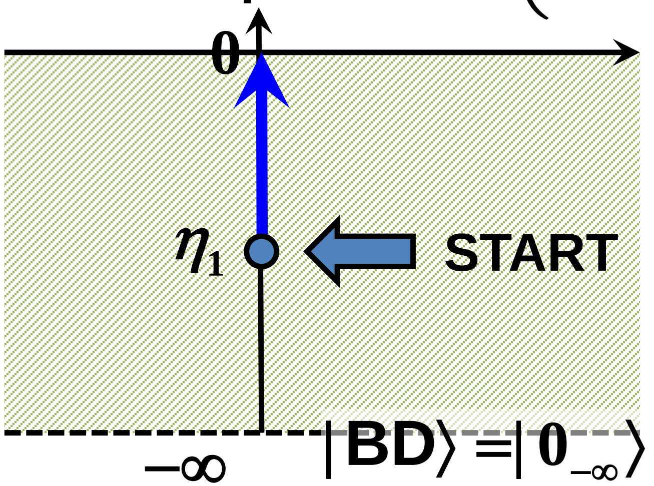
the detector behaves as if it is in a heat bath with temp $1/\beta$.

⇔ The density distribution of the detector behaves as

$$\rho_n(t) \xrightarrow{t \to \infty} \frac{1}{Z} e^{-\beta E_n} \left(Z = \sum_{k \geq 0} e^{-\beta E_k} \right)$$

This transition is instantaneous for an "ideal" detector.

■ de Sitter case $\left(ds^2 = \ell^2 \dfrac{-d\eta^2 + dx^2}{\eta^2} \right)$



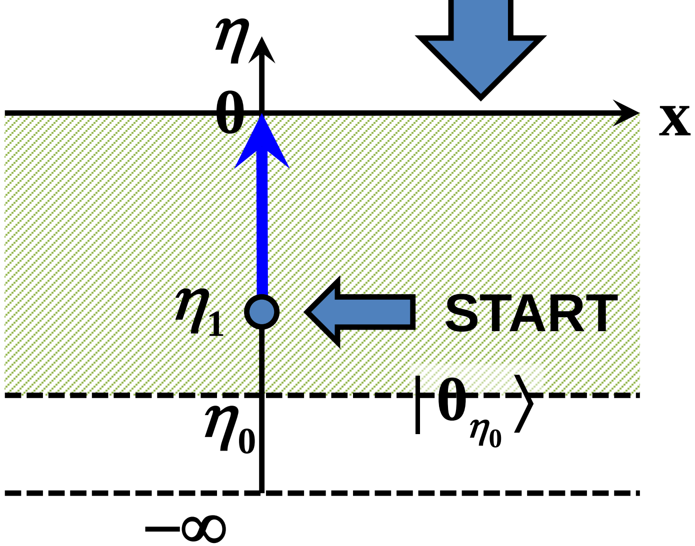
$$\rho_n(\eta) \xrightarrow[\text{relax.}]{\eta \gg \eta_1} \frac{1}{Z} e^{-2\pi \ell E_n} \quad \left(\begin{array}{l} \text{Gibbs} \\ \text{distribution} \end{array} \right)$$

Detector feels as if

it is in a thermal bath with $T = 1/2\pi\ell$

(Relaxation is instantaneous if the detector is "ideal".)

What if taking the instantaneous vacuum at a finite past?



Detector will feel as if

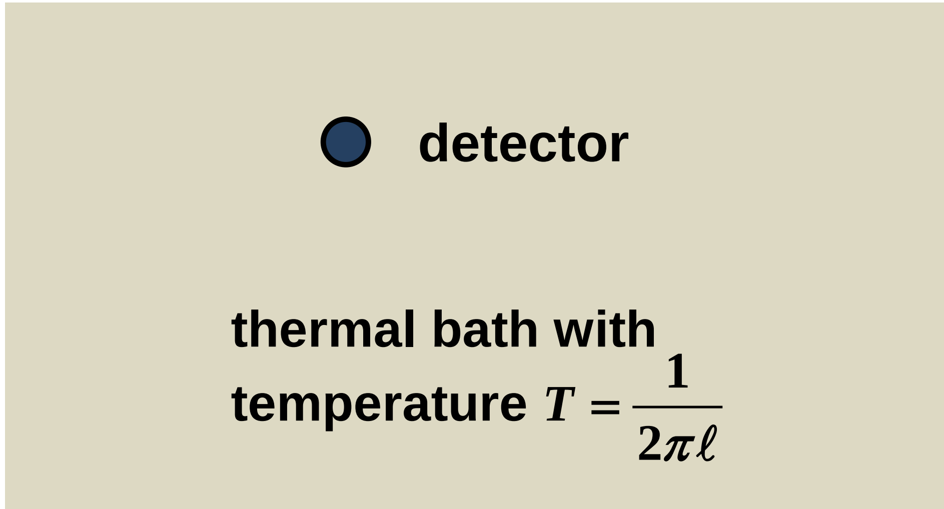
it is in a medium which

{ is not in thermal equilibrium

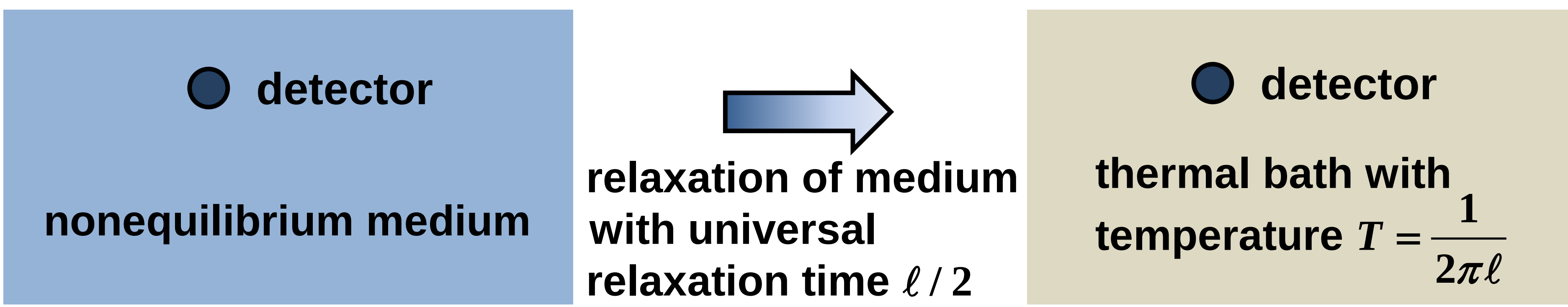
relaxes to the equilibrium corr. to |BD⟩ }

(The difference between $|0_{\eta_0}\rangle$ and |BD⟩ becomes irrelevant as time goes on.)

For $|0_{-\infty}\rangle = |\text{BD}\rangle$:



For $|0_{\eta_0}\rangle$:



■ Result

We find that

even for an ideal detector,

the density distribution $\{\rho_n(\eta)\}$ has

a finite relaxation time $\tau_{\text{relax}} = \ell / 2$.

$$\tau_{\text{relax}} = \frac{\ell}{2} : \begin{cases} \text{universal quantity} \\ \text{related to nonequilibrium dynamics} \\ \text{intrinsic to de Sitter space} \end{cases}$$

To show this, we first develop a framework...

5. Master equation for Unruh-DeWitt detector (general theory)

[2] MF-Sakatani-Sugishita [PRD89 (2014) 064024, 1305.0256]

■ Setup

detector $\phi(x)$

- total Hilbert space: $\mathscr{H}^{\text{tot}} = \mathscr{H}^{\text{d}} \otimes \mathscr{H}^{\phi}$
- Hamiltonian (Schrödinger picture)

$$H^{\text{tot}}(t) = H^{\text{d}}\frac{d\tau}{dt} \otimes 1 + 1 \otimes H^{\phi}(t) + V(t)$$

$$V(t) = \mu \frac{d\tau}{dt} \otimes \phi\big(\mathrm{x}(t)\big) \cdot \theta(t - t_1)$$

- time evolution operator

$$U^{\text{tot}}(t, t_1) = \mathrm{T}\exp\left[-i\int_{t_1}^{t} dt'\, H^{\text{tot}}(t')\right]$$

- total density matrix

$$\rho^{\text{tot}}(t) = U^{\text{tot}}(t,t_1)\, \rho^{\text{tot}}(t_1) \left[U^{\text{tot}}(t,t_1)\right]^{-1}$$

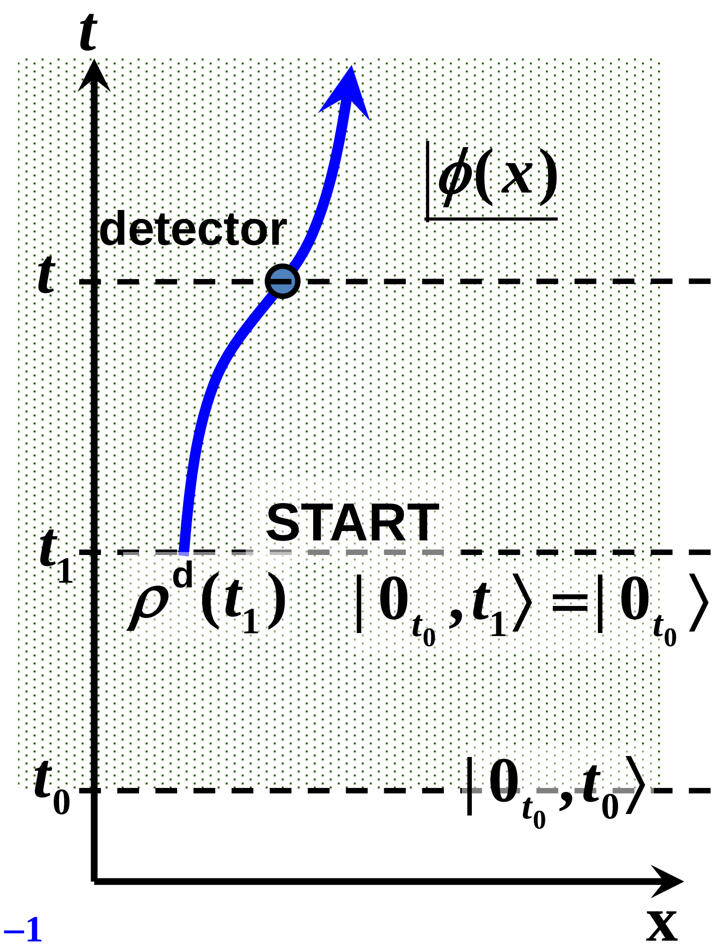
$$\Leftrightarrow \quad \dot{\rho}^{\text{tot}}(t) = -i\left[H^{\text{tot}}(t), \rho^{\text{tot}}(t)\right]$$

"von Neumann eqn"

- BC

$$\rho^{\text{tot}}(t_1) = \rho^{\text{d}}(t_1) \otimes \rho^{\phi}(t_1)$$

$$\left(\text{e.g. } \rho^{\phi}(t_1) = |0_{t_0}, t_1\rangle\langle 0_{t_0}, t_1| = |0_{t_0}\rangle\langle 0_{t_0}|\right) \Leftarrow T_s = t_1$$



Suppose that the behavior of the detector is only concerned.

⇓

- **reduced density matrix**

$$\rho(t) \equiv \mathrm{Tr}_\phi\, \rho^{\mathrm{tot}}(t)$$

$$\left(\mathbf{NB:}\ \rho(t_1) = \mathrm{Tr}_\phi\, \rho^{\mathrm{tot}}(t_1) = \mathrm{Tr}_\phi \left(\rho^{\mathrm{d}}(t_1) \otimes \rho^{\phi}(t_1) \right) = \rho^{\mathrm{d}}(t_1) \right)$$

- **interaction picture**

$$\rho_I^{\mathrm{tot}}(t) \equiv \left[U_0^{\mathrm{tot}}(t,t_1) \right]^{-1} \rho^{\mathrm{tot}}(t)\, U_0^{\mathrm{tot}}(t,t_1)$$

$$\left(U_0^{\mathrm{tot}}(t,t_1) = \mathrm{T}e^{-i\int_{t_1}^{t} dt'\, H_0^{\mathrm{tot}}(t')} = e^{-i\, H^{\mathrm{d}}\cdot(\tau-\tau_1)} \otimes \mathrm{T}e^{-i\int_{t_1}^{t} dt'\, H^{\phi}(t')} \right)$$

- **von Neumann equation**

$$\dot{\rho}_I^{\mathrm{tot}}(t) = -\,i \left[V_I(t), \rho_I^{\mathrm{tot}}(t) \right] = -i\, \mathrm{ad}_{V_I(T)} \rho_I^{\mathrm{tot}}(t)$$

$$\left(V_I(t) \equiv \left[U_0^{\mathrm{tot}}(t,t_1) \right]^{-1} V(t)\, U_0^{\mathrm{tot}}(t,t_1) = \mu_I(t) \frac{d\tau}{dt} \otimes \phi_I\left(t, \mathrm{x}(t)\right) \cdot \theta(t-t_1) \right)$$

- **BC**

$$\rho_I^{\mathrm{tot}}(t_1) = \rho^{\mathrm{tot}}(t_1) = \rho^{\mathrm{d}}(t_1) \otimes \rho^{\phi}(t_1)$$

Method of orthogonal projection [cf. Kubo-Toda-Hashitsume]

- For $O \in \mathrm{End}\,\mathcal{H}^{\mathrm{tot}}$, we introduce

$$\mathcal{P} : \mathrm{End}\,\mathcal{H}^{\mathrm{tot}} \to \mathrm{End}\,\mathcal{H}^{\mathrm{tot}}$$
$$O \mapsto \mathcal{P}O \equiv \left(\mathrm{Tr}_\phi\, O\right) \otimes X^\phi$$

X^ϕ : arbitrary as long as it is time indep, satisfying $\mathrm{Tr}_\phi\, X^\phi = 1$

$\Rightarrow \mathcal{P}^2 = \mathcal{P}$

$$\Rightarrow \quad \mathcal{P}\rho_I^{\mathrm{tot}}(t) \equiv \left(\mathrm{Tr}_\phi\, \rho_I^{\mathrm{tot}}(t)\right) \otimes X^\phi = \rho_I(t) \otimes X^\phi$$

- Choose X^ϕ s.t.

$$\begin{cases} X^\phi = \rho_I^\phi(t_1) = |\,0_{t_0}\rangle\langle 0_{t_0}\,| \\ \langle \phi_I(x)\rangle_X = \mathrm{Tr}_\phi\left(\phi_I(x) X^\phi\right) = \langle 0_{t_0}\,|\,\phi_I(x)\,|\,0_{t_0}\rangle \equiv 0 \end{cases}$$

$\Downarrow$

master equation [MF-Sakatani-Sugishita [2]] $\left(\mathcal{Q} \equiv 1 - \mathcal{P}\right)$

$$\frac{d\rho_I(t)}{dt} \otimes \rho_I^\phi(t_1) = -\mathcal{P}\,\mathrm{ad}_{V_I(t)} \int_{t_1}^{t} dt'\, \mathrm{T} e^{-i\int_{t'}^{t} dt'' \mathcal{Q}\, \mathrm{ad}_{V_I(t'')}}\, \mathrm{ad}_{V_I(t')}\, \rho_I(t') \otimes \rho_I^\phi(t_1)$$

proof Using $P+\mathcal{Q}\equiv 1$,

$$\frac{d}{dt}\rho_I^{\text{tot}}(t)=-i\,\text{ad}_{V_I(T)}\rho_I^{\text{tot}}(t)\Rightarrow\begin{cases}\dfrac{d}{dt}\mathcal{P}\rho_I^{\text{tot}}(t)=-i\,\mathcal{P}\,\text{ad}_{V_I(t)}\mathcal{P}\rho_I^{\text{tot}}(t)-i\,\mathcal{P}\,\text{ad}_{V_I(t)}\mathcal{Q}\rho_I^{\text{tot}}(t) & \text{--- (1)}\\[2ex] \dfrac{d}{dt}\mathcal{Q}\,\rho_I^{\text{tot}}(t)=-i\,\mathcal{Q}\,\text{ad}_{V_I(t)}\mathcal{P}\rho_I^{\text{tot}}(t)-i\,\mathcal{Q}\,\text{ad}_{V_I(t)}\mathcal{Q}_I^{\text{tot}}(t) & \text{--- (2)}\end{cases}$$

This is a coupled diff eqn for $\mathcal{P}\rho_I^{\text{tot}}(t)$ and $\mathcal{Q}\rho_I^{\text{tot}}(t)$.

First solve (2) w.r.t. $\mathcal{Q}\rho_I^{\text{tot}}(t)$:

$$\mathcal{Q}\rho_I^{\text{tot}}(t)=\text{T}e^{-i\int_{t_1}^{t}dt'\,\mathcal{Q}\,\text{ad}_{V_I(t')}}\mathcal{Q}\rho_I^{\text{tot}}(t_1)-i\int_{t_1}^{t}dt'\,\text{T}e^{-i\int_{t'}^{t}dt''\,\mathcal{Q}\,\text{ad}_{V_I(t'')}}\mathcal{Q}\,\text{ad}_{V_I(t')}\mathcal{P}\rho_I^{\text{tot}}(t')$$

Substitute this to (1) using $\mathcal{P}\rho_I^{\text{tot}}(t)=\rho_I(t)\otimes X^{\phi}=\rho_I(t)\otimes\rho_I^{\phi}(t_1)$:

$$\frac{d\rho_I(t)}{dt}\otimes\rho_I^{\phi}(t_1)=\underline{-i\,\mathcal{P}\,\text{ad}_{V_I(t)}\mathcal{P}\rho_I^{\text{tot}}(t)}-i\,\mathcal{P}\,\text{ad}_{V_I(t)}\text{T}e^{-i\int_{t_1}^{t}dt'\,\mathcal{Q}\,\text{ad}_{V_I(t')}}\underline{\mathcal{Q}\,\rho_I^{\text{tot}}(t_1)}$$
$$-\mathcal{P}\,\text{ad}_{V_I(t)}\int_{t_1}^{t}dt'\,\text{T}e^{-i\int_{t'}^{t}dt''\,\mathcal{Q}\,\text{ad}_{V_I(t'')}}\mathcal{Q}\,\text{ad}_{V_I(t')}\mathcal{P}\rho_I^{\text{tot}}(t')\quad\text{--- (*)}$$

We have chosen $X^{\phi}=\rho_I^{\phi}(t_1)$ and $\rho_I^{\text{tot}}(t_1)=\rho_I^{\text{d}}(t_1)\otimes\rho_I^{\phi}(t_1)$

$$\Rightarrow\ \mathcal{P}\rho_I^{\text{tot}}(t_1)=\rho_I(t_1)\otimes X^{\phi}=\rho_I^{\text{d}}(t_1)\otimes\rho_I^{\phi}(t_1)=\rho_I^{\text{tot}}(t_1)\ \Leftrightarrow\ \mathcal{Q}\,\rho_I^{\text{tot}}(t_1)=0$$

We have set $\text{Tr}_{\phi}\left(\phi_I(x)X^{\phi}\right)=\text{Tr}_{\phi}\left(\phi_I(x)\rho_I^{\phi}(t_1)\right)=0$

$$\Rightarrow\ \mathcal{P}\,\text{ad}_{V_I(t)}\mathcal{P}\rho_I^{\text{tot}}(t)=\text{Tr}_{\phi}\left(\phi_I(x)\rho_I^{\phi}(t_1)\right)\otimes\rho_I^{\phi}(t_1)=0$$

Only the third term in (*) survives. ■

Approximation of master equation

master equation [MF-Sakatani-Sugishita [2]] $\left(\mathcal{Q}\equiv 1-\mathcal{P}\right)$

$$\frac{d\rho_I(t)}{dt}\otimes\rho_I^\phi(t_1)=-\mathcal{P}\,\mathrm{ad}_{V_I(t)}\int_{t_1}^{t}dt'\,\mathrm{T}e^{-i\int_{t'}^{t}dt''\mathcal{Q}\,\mathrm{ad}_{V_I(t'')}}\mathrm{ad}_{V_I(t')}\rho_I(t')\otimes\rho_I^\phi(t_1)$$

- **perturbation to** $O(\mu^2)$

$$\Rightarrow\quad\frac{d\rho_I(t)}{dt}=-\mathrm{Tr}_\phi\left[\mathrm{ad}_{V_I(t)}\int_{t_1}^{t}dt'\,\mathrm{ad}_{V_I(t')}\rho_I(t')\otimes\rho_I^\phi(t_1)\right]+O(\mu^3)$$

With

$$\begin{cases}V_I(t)=\mu_I(t)\dfrac{d\tau}{dt}\otimes\phi_I\left(x(\tau)\right)\cdot\theta(t-t_1)\\ X^\phi=\rho_I^\phi(t_1)=|0_{t_0}\rangle\langle 0_{t_0}|\end{cases}$$

$$G_X^+(x,x')\equiv\mathrm{Tr}_\phi\left(\phi_I(x)\,\phi_I(x')X^\phi\right)=\langle 0_{t_0}|\phi_I(x)\,\phi_I(x')|0_{t_0}\rangle$$

we have

$$\dot\rho_I(\tau)=\int_{\tau_1}^{\tau}d\tau'\begin{pmatrix}\left[\mu_I(\tau),\rho_I(\tau')\mu_I(\tau')\right]G_X^+\left(x(\tau'),x(\tau)\right)\\-\left[\mu_I(\tau),\mu_I(\tau')\rho_I(\tau')\right]G_X^+\left(x(\tau),x(\tau')\right)\end{pmatrix}+O(\mu^3)$$

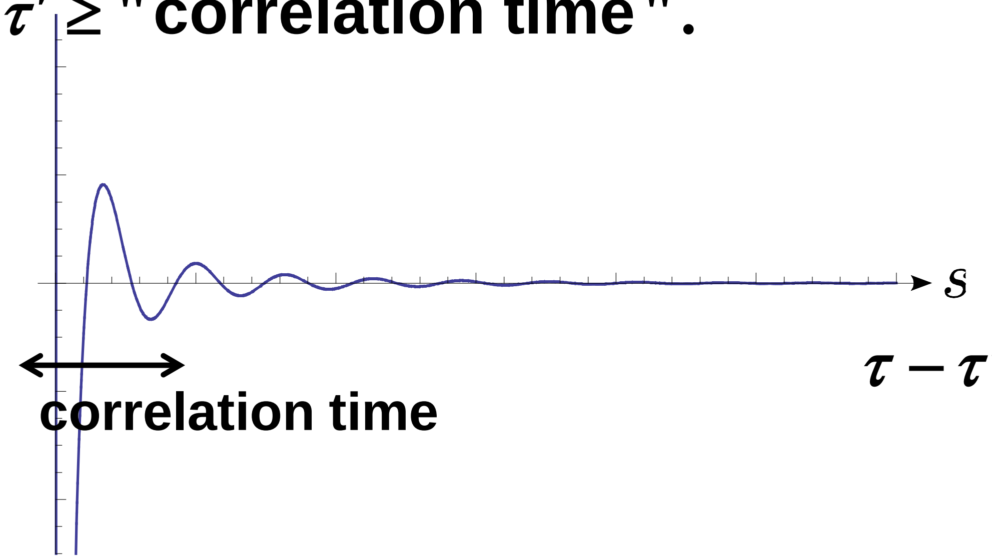
- **Return to the Schrödinger picture:**

$$\dot{\rho}_{mn}(\tau)+i(E_m-E_n)\,\rho_{mn}(\tau)$$

$$=\sum_{k,\ell}\int_{\tau_1}^{\tau}d\tau'\begin{pmatrix} e^{-(E_m-E_k)(\tau'-\tau)}\mu_{mk}\,\mu_{\ell n}\,\rho_{k\ell}(\tau')G_X^{+}\left(x(\tau'),\,x(\tau)\right)\\ +e^{-(E_m-E_\ell)(\tau-\tau')}\mu_{mk}\,\mu_{\ell n}\,\rho_{k\ell}(\tau')G_X^{+}\left(x(\tau),\,x(\tau')\right)\\ -e^{-(E_\ell-E_m)(\tau'-\tau)}\mu_{k\ell}\,\mu_{\ell n}\,\rho_{mk}(\tau')G_X^{+}\left(x(\tau'),\,x(\tau)\right)\\ -e^{-(E_k-E_n)(\tau-\tau')}\mu_{mk}\,\mu_{k\ell}\,\rho_{\ell n}(\tau')G_X^{+}\left(x(\tau),\,x(\tau')\right)\end{pmatrix}+O(\mu^3)$$

The Wightman function $G_X^{+}\left(x(\tau),\,x(\tau')\right)$ has a correlation in the time direction and almost vanishes if $\tau-\tau'\geq$ "correlation time".

$\rho_{k\ell}(t')$ in the integrand can be replaced by $\rho_{k\ell}(t)$



Further assuming that off-diagonal elements can be neglected, we have a closed equation for the density distribution $\rho_{mm}(t)$:

(to be the case in examples below)

$$\dot{\rho}_{mm}(\tau) = \sum_{k \neq m} \left[w_{mk}(\tau;\tau_1)\,\rho_{kk}(\tau) - w_{km}(\tau;\tau_1)\,\rho_{mm}(\tau) \right]$$

with $w_{mk}(\tau;\tau_1) = |\mu_{mk}|^2 \, \dot{F}\left(E_m - E_k;\, \tau; \tau_1\right)$

$$\dot{F}\left(\Delta E\,; \tau; \tau_1\right) \equiv \int_{\tau_1}^{\tau} d\tau' \begin{bmatrix} e^{-i\Delta E(\tau'-\tau)}\, G_X^+\left(x(\tau'), x(\tau)\right) \\ +e^{-i\Delta E(\tau-\tau')}\, G_X^+\left(x(\tau), x(\tau')\right) \end{bmatrix}$$

NB: Suppose $\dot{F}^{\text{eq}}\left(\Delta E\right) \equiv \lim_{\tau-\tau_1 \to \infty} \dot{F}\left(\Delta E; \tau; \tau_1\right)$ satisfies the relation

$$\dot{F}^{\text{eq}}\left(\Delta E\right) = e^{-\beta \Delta E}\, \dot{F}^{\text{eq}}\left(-\Delta E\right) \quad \left(\Leftrightarrow \quad \frac{w_{mk}(\tau;\tau_1)}{w_{km}(\tau;\tau_1)} = e^{-\beta(E_m - E_k)} \right)$$

$\Rightarrow$ Detailed balance: $\lim_{\tau-\tau_1 \to \infty} w_{mk}(\tau;\tau_1)\rho_{kk}^{\text{eq}} = \lim_{\tau-\tau_1 \to \infty} w_{km}(\tau;\tau_1)\rho_{mm}^{\text{eq}}$

$$\Rightarrow \quad \frac{\rho_{mm}^{\text{eq}}}{\rho_{kk}^{\text{eq}}} = e^{-\beta(E_m - E_k)} \quad \Rightarrow \quad \rho_{mm}^{\text{eq}} = \frac{1}{Z} e^{-\beta E_m}$$

6. Unruh-DeWitt detector in de Sitter space

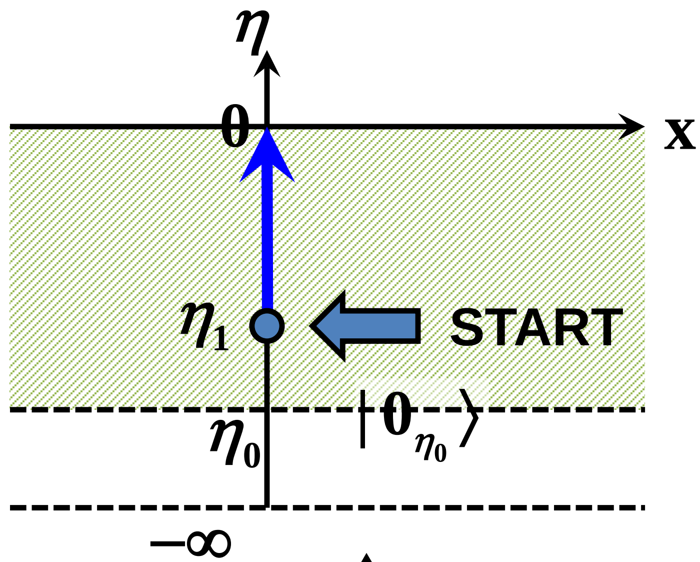
[2] MF-Sakatani-Sugishita [PRD89 (2014) 064024, 1305.0256]

■ Setup

- **classical path**

$$\left(ds^2 = \ell^2 \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2} \right)$$

$$x^\mu(\tau) = \big(\eta(\tau), \mathbf{x}(\tau)\big) = \left(-\ell\, e^{-\tau/\ell}, \mathbf{0}\right)$$

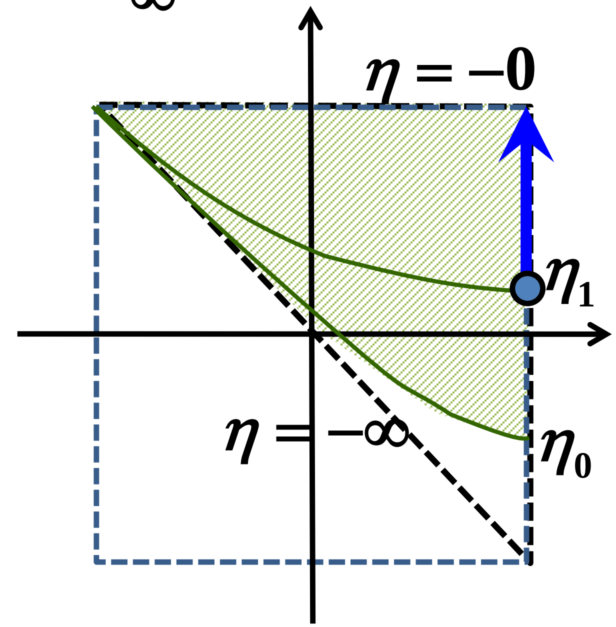


- **Wightman function**

$$G^+(x, x'; \eta_0) = \langle 0_{\eta_0} \,|\, \phi_I(x)\, \phi_I(x') \,|\, 0_{\eta_0} \rangle$$

$$= \int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} G_k^+(\eta, \eta'; \eta_0) \cos \mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')$$

- **What we need to do:**
  - **(1) Calculate** $G_k^+(\eta, \eta'; \eta_0)$
  - **(2) Expand it around** $\eta_0 = -\infty \;\left(\Leftrightarrow \text{BD}\right)$
  - **(3) Integrate over** k **to obtain**



$$\dot{F}\left(\Delta E; \tau; \tau_1; \eta_0 = -e^{-\tau_0}\right)$$

$$\equiv \int_{\tau_1}^{\tau} d\tau' \left[\begin{array}{l} e^{-i\Delta E(\tau' - \tau)}\, G_X^+\left(x(\tau'), x(\tau); \eta_0\right) \\ + e^{-i\Delta E(\tau - \tau')}\, G_X^+\left(x(\tau), x(\tau'); \eta_0\right) \end{array} \right] = \sum_{n \geq 0} e^{-n(\tau - \tau_0)} \dot{F}^{(n)}\left(\Delta E; \tau; \tau_1\right)$$

$$\left(\tau_0 \equiv -\ln(-\eta_0)\right)$$

■ Result

$$\dot{F}\left(\Delta E\,;\tau;\tau_1;\eta_0=-e^{-\tau_0}\right)$$

$$=\dot{F}^{(0)}\left(\Delta E\,;\tau;\tau_1\right)+e^{-2(\tau-\tau_0)}\dot{F}^{(2)}\left(\Delta E\,;\tau;\tau_1\right)+\cdots$$

with

$$\dot{\mathcal{F}}^{(0)} = \frac{e^{-\pi\Delta E}\,\Gamma\left(\frac{\frac{d-1}{2}+\nu+\mathrm{i}\Delta E}{2}\right)\Gamma\left(\frac{\frac{d-1}{2}-\nu+\mathrm{i}\Delta E}{2}\right)\Gamma\left(\frac{\frac{d-1}{2}+\nu-\mathrm{i}\Delta E}{2}\right)\Gamma\left(\frac{\frac{d-1}{2}-\nu-\mathrm{i}\Delta E}{2}\right)}{8\,\pi^{\frac{d+1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)}$$

$$+\frac{e^{\mathrm{i}\pi\frac{d-1}{2}}\,e^{\mathrm{i}\pi\nu}\,e^{-(\frac{d-1}{2}+\nu-\mathrm{i}\Delta E)(\tau-\tau_1)}}{8\,\pi^{\frac{d-1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)\,\sin(\pi\nu)}\;{}_3\widehat{F}_2\left(\begin{matrix}\frac{d-1}{2},\ \frac{\frac{d-1}{2}+\nu-\mathrm{i}\Delta E}{2},\ \frac{d-1}{2}+\nu\\ 1+\nu,\ \frac{\frac{d+3}{2}+\nu-\mathrm{i}\Delta E}{2}\end{matrix};\,e^{-2(\tau-\tau_1)}\right)$$

$$+\frac{e^{\mathrm{i}\pi\frac{d-1}{2}}\,e^{-\mathrm{i}\pi\nu}\,e^{-(\frac{d-1}{2}-\nu-\mathrm{i}\Delta E)(\tau-\tau_1)}}{8\,\pi^{\frac{d-1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)\,\sin(-\pi\nu)}\;{}_3\widehat{F}_2\left(\begin{matrix}\frac{d-1}{2},\ \frac{\frac{d-1}{2}-\nu-\mathrm{i}\Delta E}{2},\ \frac{d-1}{2}-\nu\\ 1-\nu,\ \frac{\frac{d+3}{2}-\nu-\mathrm{i}\Delta E}{2}\end{matrix};\,e^{-2(\tau-\tau_1)}\right)$$

$$+\frac{e^{-\mathrm{i}\pi\frac{d-1}{2}}\,e^{-\mathrm{i}\pi\nu}\,e^{-(\frac{d-1}{2}+\nu+\mathrm{i}\Delta E)(\tau-\tau_1)}}{8\,\pi^{\frac{d-1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)\,\sin(\pi\nu)}\;{}_3\widehat{F}_2\left(\begin{matrix}\frac{d-1}{2},\ \frac{\frac{d-1}{2}+\nu+\mathrm{i}\Delta E}{2},\ \frac{d-1}{2}+\nu\\ 1+\nu,\ \frac{\frac{d+3}{2}+\nu+\mathrm{i}\Delta E}{2}\end{matrix};\,e^{-2(\tau-\tau_1)}\right)$$

$$+\frac{e^{-\mathrm{i}\pi\frac{d-1}{2}}\,e^{\mathrm{i}\pi\nu}\,e^{-(\frac{d-1}{2}-\nu+\mathrm{i}\Delta E)(\tau-\tau_1)}}{8\,\pi^{\frac{d-1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)\,\sin(-\pi\nu)}\;{}_3\widehat{F}_2\left(\begin{matrix}\frac{d-1}{2},\ \frac{\frac{d-1}{2}-\nu+\mathrm{i}\Delta E}{2},\ \frac{d-1}{2}-\nu\\ 1-\nu,\ \frac{\frac{d+3}{2}-\nu+\mathrm{i}\Delta E}{2}\end{matrix};\,e^{-2(\tau-\tau_1)}\right)\Bigg]$$

$$\dot{F}^{(2)}\left(\Delta E\,;\tau;\tau_1\right)=\cdots$$

We are interested in time scales larger than $1/\Delta E$.

Taking an average over τ_1
with the duration $\Delta\tau \simeq 1/\Delta E$

$$\dot{F}\left(\Delta E;\tau;\eta_0=-e^{-\tau_0}\right) \equiv \frac{1}{\Delta\tau}\int_{\tau_1-\Delta\tau/2}^{\tau_1+\Delta\tau/2} d\tau_1' \,\dot{F}\left(\Delta E;\tau;\tau_1';\eta_0=-e^{-\tau_0}\right)$$

$$= \dot{F}^{\text{eq}}\left(\Delta E\right) + e^{-2(\tau-\tau_0)}\dot{F}^{(2)}\left(\Delta E\right) + O\left(e^{-4(\tau-\tau_0)}\right)$$

with

$$\dot{F}^{\text{eq}}\left(\Delta E\right)$$

$$= \frac{e^{-\pi\ell\Delta E}}{8\pi^{(d+1)/2}\,\Gamma\left(\frac{d-1}{2}\right)}\Gamma\left(\frac{d-1+2(\nu+i\ell\Delta E)}{4}\right)\Gamma\left(\frac{d-1+2(-\nu+i\ell\Delta E)}{4}\right)$$

$$\times\Gamma\left(\frac{d-1+2(\nu-i\ell\Delta E)}{4}\right)\Gamma\left(\frac{d-1-2(\nu+i\ell\Delta E)}{4}\right)$$

$$\dot{\mathcal{F}}^{(2)}(\Delta E) = \left(\frac{d-2}{4}\right)^2 \frac{1}{2^{3+n}\,\pi^{\frac{d-1}{2}}\,\Gamma\left(\frac{d-1}{2}\right)}$$

$$\times\Bigg[\mathrm{i}G^{2,2}_{3,3}\left(e^{\mathrm{i}\pi}\middle|\begin{matrix}-\frac{d-3-n}{2},\ -\frac{\frac{d-5}{2}-\nu-n-\mathrm{i}\Delta E}{2},\ -\frac{d-3-n}{2}+\nu\\ 0,\ \nu,\ -\frac{\frac{d-1}{2}-\nu-n-\mathrm{i}\Delta E}{2}\end{matrix}\right)$$

$$-\,\mathrm{i}G^{2,2}_{3,3}\left(e^{-\mathrm{i}\pi}\middle|\begin{matrix}-\frac{d-3-n}{2},\ -\frac{\frac{d-5}{2}-\nu-n-\mathrm{i}\Delta E}{2},\ -\frac{d-3-n}{2}+\nu\\ 0,\ \nu,\ -\frac{\frac{d-1}{2}-\nu-n-\mathrm{i}\Delta E}{2}\end{matrix}\right)$$

$$-\,G^{2,3}_{4,4}\left(e^{\mathrm{i}\pi}\middle|\begin{matrix}-\frac{d-3-n}{2},\ -\frac{d-3-n}{2}+\nu,\ -\frac{\frac{d-5}{2}-\nu-n-\mathrm{i}\Delta E}{2},\ -\frac{d-4-n}{2}+\nu\\ 0,\ \nu,\ -\frac{d-4-n}{2}+\nu,\ -\frac{\frac{d-1}{2}-\nu-n-\mathrm{i}\Delta E}{2}\end{matrix}\right)$$

$$-\,G^{2,3}_{4,4}\left(e^{-\mathrm{i}\pi}\middle|\begin{matrix}-\frac{d-3-n}{2},\ -\frac{d-3-n}{2}+\nu,\ -\frac{\frac{d-5}{2}-\nu-n-\mathrm{i}\Delta E}{2},\ -\frac{d-4-n}{2}+\nu\\ 0,\ \nu,\ -\frac{d-4-n}{2}+\nu,\ -\frac{\frac{d-1}{2}-\nu-n-\mathrm{i}\Delta E}{2}\end{matrix}\right)\Bigg]$$

$$+\,(\Delta E \to -\Delta E)\,.$$

$$\left(G^{m,n}_{p,q}\left(z\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right):\ \ \textbf{Meijer's } G\textbf{-function}\right)$$

■ Properties

(1) $\dot{F}^{\text{eq}}(\Delta E) = e^{-2\pi\ell\,\Delta E}\,\dot{F}^{\text{eq}}(-\Delta E)$

$$\Rightarrow \quad \rho_{mm}^{\text{eq}} = \frac{1}{Z} e^{-2\pi\ell\, E_m} \qquad \left(\text{i.e. } \; T = \frac{1}{\beta} = \frac{1}{2\pi\ell} \right)$$

(2) $A(\Delta E) \equiv \dot{F}^{\text{eq}}(\Delta E) + \dot{F}^{\text{eq}}(-\Delta E)$

$$\simeq \frac{|\,\Delta E\,|^{d-3}}{2^{d-2}\pi^{(d-3)/2}\Gamma\big((d-1)/2\big)} \qquad \big(|\,\Delta E\,|\,\ell \to +\infty\big)$$

(3) $\dot{F}^{(2)}(\Delta E) = \dot{F}^{(2)}(-\Delta E)$

■ Example: two-level detector

Let us consider a two-level system with

(1) $\Delta E \equiv E_2 - E_1 > 0$

(2) $\Delta E \gg 1/\ell$

(3) $\mu = (\mu_{mn}) = \begin{pmatrix} 0 & \mu_{12} \\ \mu_{12}^* & 0 \end{pmatrix}$

→ **master equation is decomposed to the diagonal and off-diagonal parts**

(4) detector is "ideal" : $\lambda^2 |\mu_{12}|^2 (\Delta E)^{d-3} \ell \gg 1$

↓

$$\begin{pmatrix} \dot{\rho}_{11}(\tau) \\ \dot{\rho}_{22}(\tau) \end{pmatrix} = \begin{pmatrix} -w_+(\tau;\tau_0) & w_-(\tau;\tau_0) \\ w_+(\tau;\tau_0) & -w_-(\tau;\tau_0) \end{pmatrix} \begin{pmatrix} \rho_{11}(\tau) \\ \rho_{22}(\tau) \end{pmatrix}$$

with $w_\pm(\tau;\tau_0) \equiv \lambda^2 |\mu_{12}|^2 \left[\dot{F}^{\text{eq}}(\pm\Delta E) + e^{-2(\tau-\tau_0)/\ell} \dot{F}^{(2)}(\pm\Delta E) \right]$

Expand $\rho_{mm}(\tau)$ around the equilibrium distribution:

$$\rho_{mm}(\tau) = \rho_{mm}^{\text{eq}} + \Delta\rho_{mm}(\tau) \quad \left(\rho_{mm}^{\text{eq}} = \frac{e^{-e\pi\ell E_{mm}}}{e^{-e\pi\ell E_{11}} + e^{-e\pi\ell E_{22}}};\ m = 1,2 \right)$$

$$\Delta\rho_{11}(\tau) + \Delta\rho_{22}(\tau) = 0$$

$$\Delta\rho_{11}(\tau)$$

$$= e^{-\lambda^2 |\mu_{12}|^2 A(\Delta E)(\tau - \bar{\tau}_1)} \Delta\rho_{11}(\bar{\tau}_1) \qquad (\bar{\tau}_1 \simeq \tau_1)$$

$$- \frac{\lambda^2 |\mu_{12}|^2 \dot{F}^{(2)}(\Delta E)}{\lambda^2 |\mu_{12}|^2 A(\Delta E) - 2} \tanh(\pi\ell\Delta E) \left(e^{-2(\tau - \bar{\tau}_1)/\ell} - e^{-\lambda^2 |\mu_{12}|^2 A(\Delta E)(\tau - \bar{\tau}_1) - 2(\bar{\tau}_1 - \tau_0)/\ell} \right)$$

$$\left(\textbf{NB:}\ A(\Delta E) = \dot{F}^{\text{eq}}(\Delta E) + \dot{F}^{\text{eq}}(-\Delta E) \sim |\Delta E|^{d-3} \right)$$

∴ **For an <u>ideal</u> detector,**

$$\Delta\rho_{11}(\tau) = -\frac{\dot{F}^{(2)}(\Delta E)}{A(\Delta E)} \tanh(\pi\ell\Delta E)\, e^{-2(\tau - \bar{\tau}_1)/\ell}$$

Properties

$$\Delta\rho_{11}(\tau) = -\frac{\dot{F}^{(2)}(\Delta E)}{A(\Delta E)} \tanh(\pi\ell\Delta E)\, e^{-2(\tau-\bar{\tau}_1)/\ell}$$

- Everything is in a universal form.

  (i.e., the expression does not depend on details of the detector.)

- The relaxation time is given by the universal quantity,

$$\tau_{\text{relax}} = \frac{\ell}{2}$$

This gives an example of the nonequilibrium-thermodynamic quantities intrinsic to de Sitter space.

7. Conclusion and outlook

■ What we have done:

[PART 1]

- **Proposed to take the vacuum in curved spacetime to be the set of instantaneous ground states.**
- **Constructed and developed a framework to calculate various propagators for the vacuum.**

[PART 2]

- **Wrote down the master equation which completely determines a finite time evolution of the density matrix of an Unruh-DeWitt detector in $\forall$spacetime.**
- **Applied the framework to uncover a universal nonequilibrium property intrinsic to de Sitter space.**

■ Future direction

(1) It should be important to establish the framework for interacting quantum fields, espcially for those in de Sitter space.

(2) ----- to apply the present framework to investigate the cosmological constant problem.

(3) ----- to extend the framework to cases where a horizon exists.

⇨ This may be helpful in understanding the thermodynamic aspects of dynamical BH.

(4) . . .

Thank you.