A Vectorlike Supersymmetric Grand Unified Model with Noncompact Horizontal Symmetry

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This talk is based on Ref. [1, N.Y.,PTEP,2013,123B01]

1. Introduction

How to understand Nature

Hints of Physics beyond the Standard Model

Matter sector: Quarks and Leptons

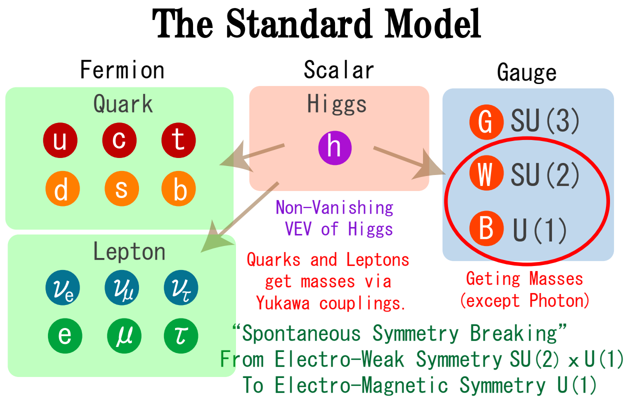
Three Chiral Generations
Hierarchical Mass Structures } ⇒ ①

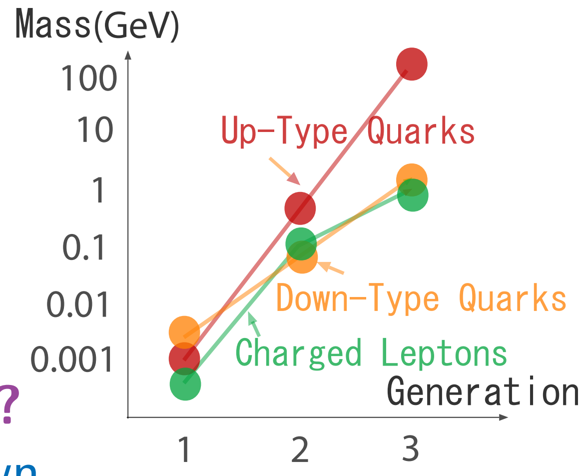
Charge Quantization
Anomaly Cancellations } ⇒ ②

Only Fundamental Representations ⇒ ③

⇒ **How can we understand these questions in a four dimensional effective field theory?**

① Horizontal Symmetry? ② GUT? ③ Unknown.





Approaches to understand generations using horizontal symmetry

Non-Abelian (e.g.,$SU(3)$, A_4)

Naturally explaining three generations.

[2, 3, S.F.King, G.G.Ross,PLB'01;H. Ishimori et al.; · · ·]

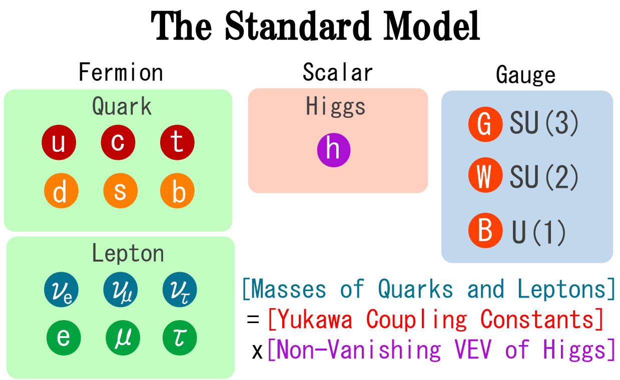
Abelian $U(1)$

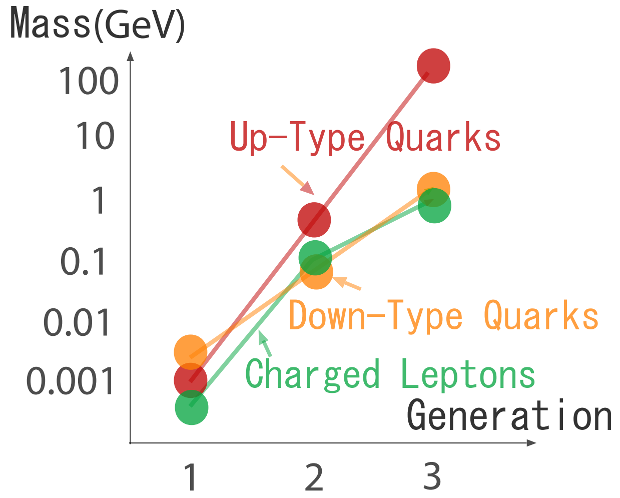
Naturally explaining the mass hierarchies.
Froggatt-Nielsen mechanism

[4, 5, C.D.Froggatt, H.B.Nielsen,NPB'79; N.Maekawa,PTP'01;· · ·]

Noncompact Non-Abelian $SU(1,1)$

[6–8, K.Inoue, PTP'95; K.Inoue, N.Y.,PTP'08; N.Yamatsu, PTEP'12;· · ·]





Noncompact Non-Abelian Group (minimal $SU(1,1)$)

Noncompact Model ∼ A Vectorlike MSSM with Horizontal Symmetry

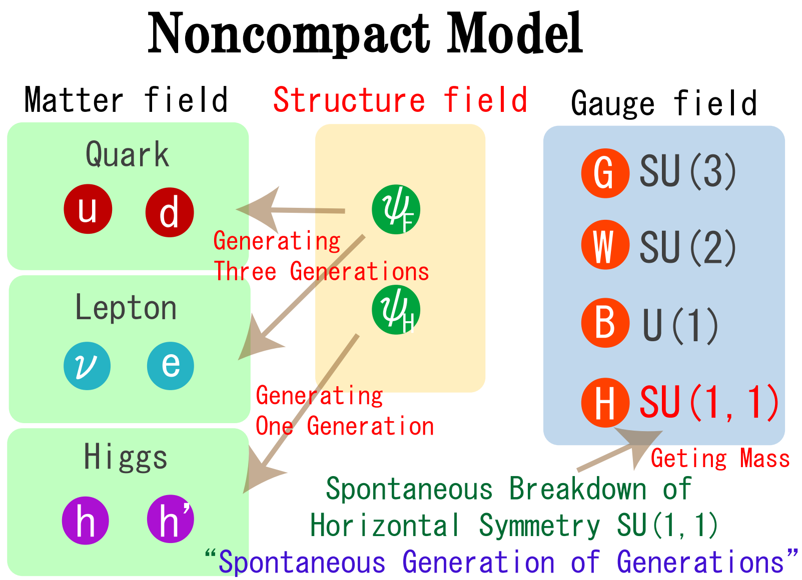
[6–8, K.Inoue,PTP'95; K.Inoue, N.Y.,PTP'08; N.Y., PTEP'13]

Three Chiral Generations

Chiral matters produced from Vectorlike matters via "**Spontaneous Generation of Generations**"

Hierarchical Mass Structures

Hierarchical structure of Clebsch-Gordan coefficients



$\mathcal{N} = 1$ Supersymmetry [9, K.Inoue, H.Kubo & N.Y.,NPB'10]

Chirality of Scalar Fields and Stability of Vacuum Structures

Spontaneous Generation of Generations [6, K.Inoue, PTP'95]

We consider a vectorlike model such as QCD. (q_x : e.g., quark doublets)

Left :	q_1	q_2	q_3	q_4	q_5	q_6	*	*	*	*
Right :	q_1	q_2	q_3	q_4	q_5	q_6	*	*	*	*

⇓ Spontaneous Horizontal Symmetry Breaking

Left :	q_1	q_2	q_3	q_4	q_5	q_6	*	*	*	*
				|	|	|	|	|	|	|
Right :				q_1	q_2	q_3	q_4	q_5	q_6	*

Chiral matters cannot be realized from finite vectorlike matters.

To produce chiral generations spontaneuously, theory must include infinite matters.

$$\text{Left} : \hat{Q} = \{\hat{q}_1,\ \hat{q}_2,\ \hat{q}_3,\ \hat{q}_4,\ \hat{q}_5,\ \cdots\cdots\}$$

$$\text{Right}^c : \hat{Q}^c = \{\hat{q}_1^c,\ \hat{q}_2^c,\ \hat{q}_3^c,\ \hat{q}_4^c,\ \hat{q}_5^c,\ \cdots\cdots\}$$

I would like a horizontal symmetry G_H to control the infinite matters.
I would like quarks and leptops to belong to unitary reps. of G_H.

$\Rightarrow G_H$ must be a noncompact group.

Noncompact non-Abelian groups include $SU(1,1)$ as a subgroup.

We (at least K. Inoue and I) hope that $SU(1,1)$ is usuful to consider the mechanism of the spontaneous generation of generaions.

Hierarchy of Yukawa coupling constants [6, 7, K.Inoue, PTP'95;K.Inoue & N.Y., PTP'08]

$$W_{\text{Yukawa}}^{\text{chiral}} = \sum_{m,n=0}^{2} y_u^{mn} q_m u_n^c h_u, \quad y_u^{mn} \underset{\sim}{\propto} \begin{pmatrix} 1 & *\epsilon & *\epsilon^2 \\ *\epsilon & *\epsilon^2 & *\epsilon^3 \\ *\epsilon^2 & *\epsilon^3 & *\epsilon^4 \end{pmatrix},$$

$*$: Coefficient determined by $SU(1,1)$ CGCs.

ϵ: a dimensionless parameter depending on $SU(1,1)$ breaking VEV.

The eigenvalues seem to be $O(1)$, $O(1)\epsilon^2$, $O(1)\epsilon^4$, but $\cdots$.

Example: to choose an $SU(1,1)$ weight set:

$$y_u^{\text{diag}} = 1 + O(\epsilon^2), \ \frac{1}{12}\epsilon^2 + O(\epsilon^4), \quad \frac{1}{720}\epsilon^4 + O(\epsilon^6).$$

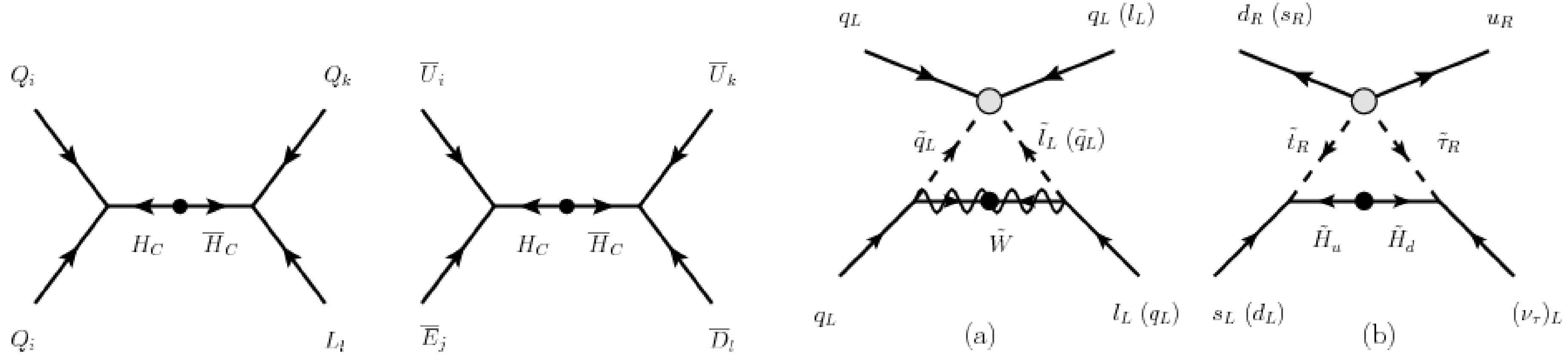
Representations of G_{SM} for Quarks and Leptons

- Chiral gauge anomalies are miraculously canceled out.
- $U(1)_Y$(and $U(1)_{em}$) charges are quantized.
- Only fundamental and trivial reps of $SU(3)_C$ and $SU(2)_L$ are included.
- When the SM gauge groups $G_{\mathrm{SM}}(:= SU(3)_C \times SU(2)_L \times U(1)_Y)$ are embedded into a GUT group $SU(5)_{\mathrm{GUT}}$, the SM chiral fermions can be embedded into $SU(5)$ fundamental reps. $\mathbf{10}$ and $\mathbf{5}^*$. When we consider a GUT group $SO(10)_{\mathrm{GUT}}$, the SM chiral fermions can be embedded into a fundamental rep. $\mathbf{16}$.

The above facts seem to suggest that the concept of grand unified theory is ture.

Minimal $SU(5)$ SUSY GUT: Problem-I (Proton Decay)

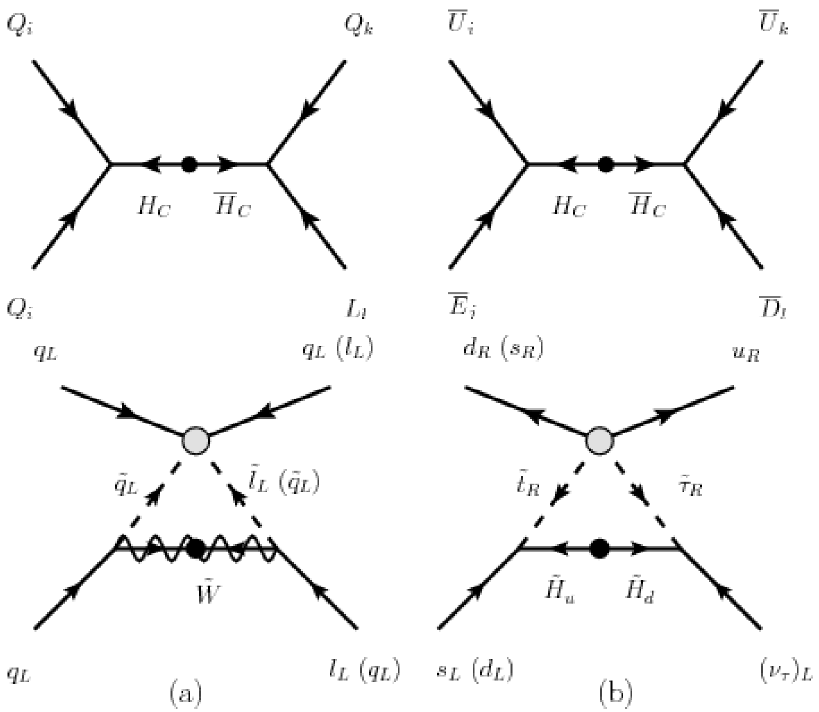
- Colored Higgses with $M_C \simeq O(M_{\rm GUT})$ produce too rapid proton decay (as long as we assume that sparticle masses are O(1) TeV).



From Ref. [10, J.Hisano et al.,'13]

Superpotential quartic terms $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are generated by colored Higgses, and lead to dimension-6 operators $qqq\ell$ and $u^cu^cd^ce^c$ causing proton decays.

Proton decays mediated by colored Higgses



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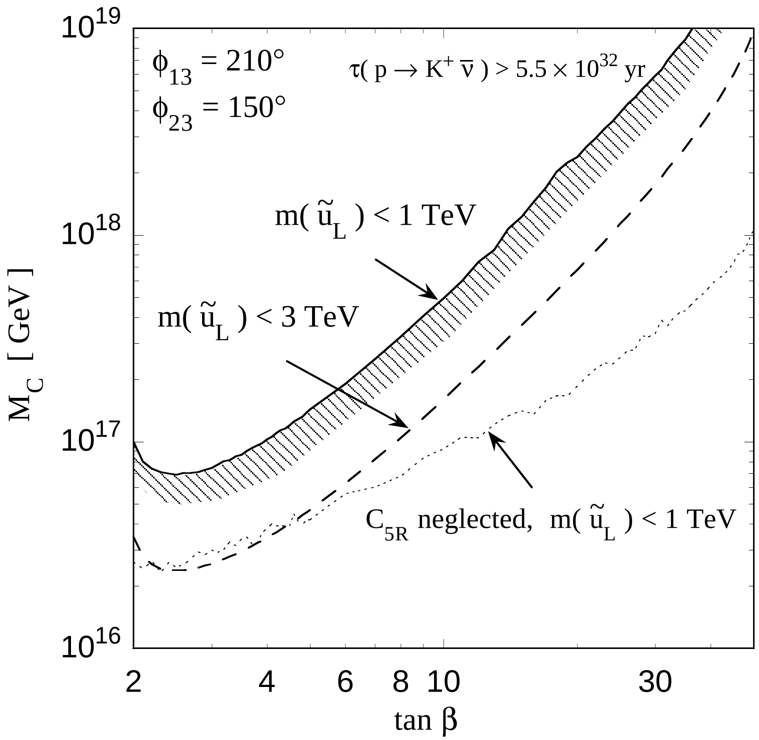

From Ref. [11, T.Goto, T.Nihei,PRD'99]

(Recent Super-Kamiokande result $\tau(p \to K^+\bar{\nu}) > 3.3 \times 10^{33}$ yr [12, M.Miura,PoS'2010])

Minimal $SU(5)$ SUSY GUT: Problem-II (Higgses)

- How to realize doublet-triplet Higgs mass splitting.

Higgses (doublet) have masses $O(M_{EW}) \sim O(10^2)$ GeV, while the corresponding colored higgses (triplet) must have masses $O(M_{\rm GUT}) \sim O(10^{16})$.

♯ In the minimal $SU(5)$ SUSY GUT, the doublet part of the original $SU(5)$ μ-term $\mu_{\mathbf{5}}\hat{H}_{u\mathbf{5}}\hat{H}_{d\mathbf{5}^*}$ is canceled by using the "μ"-term $\langle\hat{\Phi}_{\mathbf{24}}\rangle\hat{H}_{u\mathbf{5}}\hat{H}_{d\mathbf{5}^*}$.

♯ Even if we ignore the naturalness problem, the colored Higgs must have $O(10^{17})$ GeV regardless of the value of $\tan\beta$.
Note that since $SO(10)$ and E_6 GUT models lead to Yukawa coupling unification, $\tan\beta$ must be around $40-50$, and then the colored Higgs masses must exceed $O(M_{\rm Planck})$.

Minimal $SU(5)$ SUSY GUT: Problem-III (Yukawa couplings)

- In the minimal model, the Yukawa coupling matrix of down-type quark is the same as that of charged lepton.

Some known methods to avoid this problem

♯ To take into account of superpotential higher order terms, e.g., quartic terms and quintic terms.

$$W = \sum_{n=1} \kappa_n \frac{\langle \Phi_{\mathbf{24}} \rangle^n}{\Lambda^n} \hat{F}_{\mathbf{10}} \hat{G}_{\mathbf{5}^*} \hat{H}_{d\mathbf{5}^*}$$

κ_n: a dimensionless coupling constant, Λ: a dimensional parameter.

♯ To introduce higher dimensional representations, e.g., **45**,**45*** and **75** representations of $SU(5)$ [13, H.Georgi,C.Jarlskog,PLB'79].

Some known methods suppressing proton decays via colored higgses

♯ SUSY particle's masses are much heavier than 1 TeV. [10, J.Hisano et al.,JHEP'13;· · ·]

♯ In an orbifold extra dimension model $S^1/(Z_2 \times Z_2')$, when only doublet higgses have zero modes and colored higgses have no zero modes [14, Y.Kawamura,PTP'01], the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed [15, L.J.Hall,Y.Nomura,PRD'01;· · ·].

The $SU(5) \times SU(1,1)$ Model

♯ When only doublet higgses are chiral and colored higgses are vectorlike via spontaneous generation of generations [7, 16, K.Inoue,N.Yamashita,PTP'00;K.Inoue,N.Y.,PTP'08], the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed [1, N.Y.,PTEP'13].

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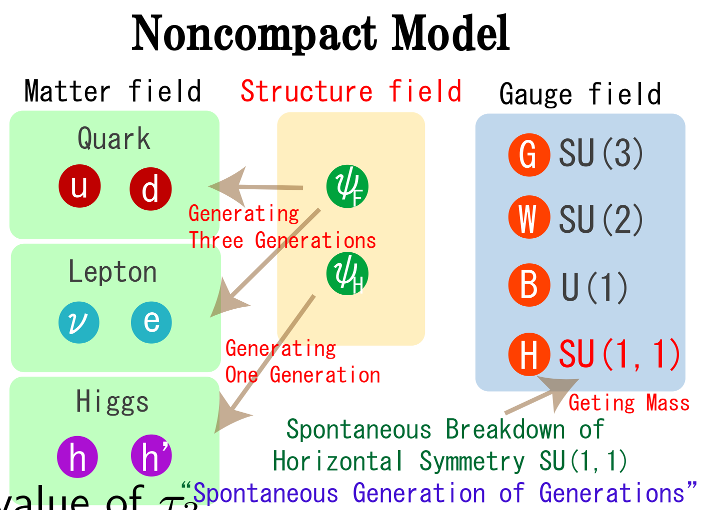
2. A Vectorlike $SU(5) \times SU(1,1)$ SUSY GUT model

Representations of $SU(1,1)$ in the Noncompact Model

Matter field = infinite-dim. unitary reps. of the noncompact group

$$\begin{cases} \hat{F} = \{\hat{f}_{+\alpha}, \hat{f}_{+\alpha+1}, \hat{f}_{+\alpha+2}, \cdots\}, \\ \hat{F}^c = \{\hat{f}^c_{-\alpha}, \hat{f}^c_{-\alpha-1}, \hat{f}^c_{-\alpha-2}, \cdots\}, \end{cases}$$

α =[real and positive].



Structure field = finite-dim. reps.

$$\hat{\Psi} = \{\hat{\psi}_{-S}, \hat{\psi}_{-S+1}, \cdots, \hat{\psi}_{S-1}, \hat{\psi}_S\},$$

S =[Integer or half-integer], called $SU(1,1)$ spin.

Subscripts, such as α, S, stand for the eigenvalue of τ_3.

All Matter and Structure fields are chiral superfields in $\mathcal{N}=1$ SUSY.

[8, N.Y.,PTEP'13]

Matter Content in the $SU(5) \times SU(1,1)$

Matter fields

Field	$\hat{F}^{(\prime)}_{\mathbf{10}}$	$\hat{G}^{(\prime)}_{\mathbf{5}^*}$	$\hat{H}_{u\mathbf{5}}$	$\hat{H}_{d\mathbf{5}^*}$	$\hat{F}^{(\prime)c}_{\mathbf{10}^*}$	$\hat{G}^{(\prime)c}_{\mathbf{5}}$	$\hat{H}^c_{u\mathbf{5}^*}$	$\hat{H}^c_{d\mathbf{5}}$
$SU(5)$	$\mathbf{10}$	$\mathbf{5}^*$	$\mathbf{5}$	$\mathbf{5}^*$	$\mathbf{10}^*$	$\mathbf{5}$	$\mathbf{5}^*$	$\mathbf{5}$
$SU(1,1)$	$+\alpha^{(\prime)}$	$+\beta^{(\prime)}$	$-\gamma$	$-\delta$	$-\alpha^{(\prime)}$	$-\beta^{(\prime)}$	$+\gamma$	$+\delta$

Structure fields

Field	$\hat{\Phi}_{\mathbf{1}}$	$\hat{\Phi}'_{\mathbf{24}}$	$\hat{\Psi}_{\mathbf{1}/\mathbf{24}}$
$SU(5)$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$ or $\mathbf{24}$
$SU(1,1)$	S	S'	S''

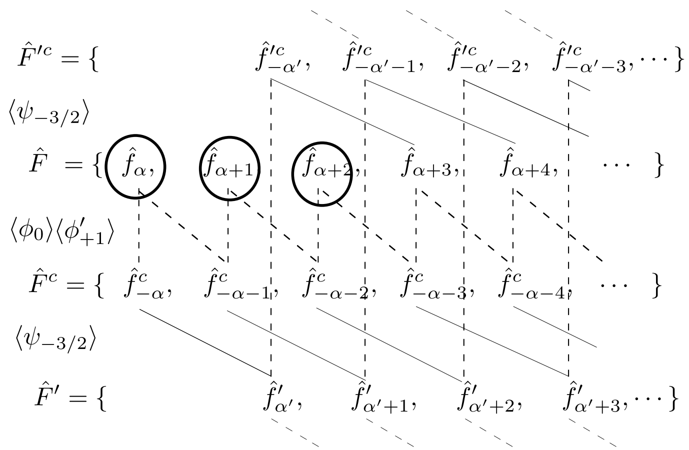
Assuming that the nonvanishing VEVs of the structure fields break $SU(5) \times SU(1,1)$ into $G_{SM}(M_{\rm GUT} \simeq O(10^{16})$ GeV): $\langle \hat{\Phi}_{\mathbf{1}} \rangle = \langle \phi_0 \rangle, \langle \hat{\Phi}'_{\mathbf{24}} \rangle = \langle \phi'_{+1} \rangle, \langle \hat{\Psi}_{\mathbf{1}/\mathbf{24}} \rangle = \langle \psi_{-3/2} \rangle$.

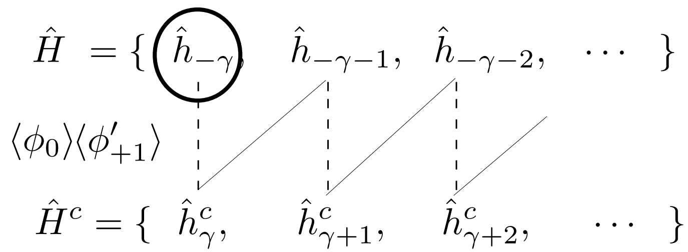
3. Realization of Chiral Generations

The Spontaneous Generation of Generations

① A structure field with an $SU(1,1)$ integer spin [6, K.Inoue,PTP'95]
② Structure fields with $SU(1,1)$ integer spins [16, K.Inoue,N.Yamashita,PTP'00]
③ Structure fields with $SU(1,1)$ integer and half-integer spins [8, N.Y.,PTEP'13]

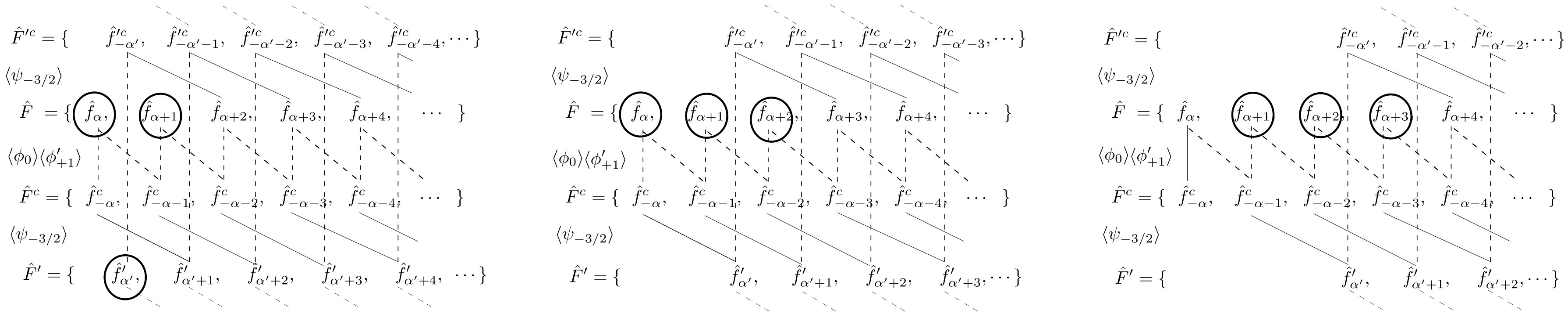
In the current model, the discussion for quarks and leptons is corresponding to ③, and the discussion for higgses is corresponding to ②. We glance at them.





Three Chiral Generations of Quarks and Leptons [8, N.Y.,PTEP'13]

The patterns of massless modes can be classified into three types by using the $SU(1,1)$ weight of matter fields.



This patterns determine structures of Yukawa couplings in part.

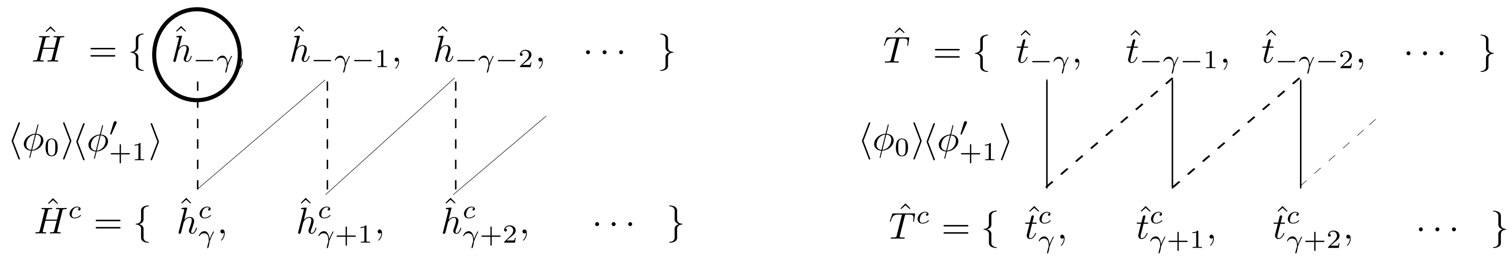
One Chiral Generation of Higgses [7, 16, K.Inoue,N.Yamashita,PTP'00;K.Inoue,N.Y.,PTP'08]

Whether massless modes appear or not depnds on the VEVs of the structure fields and the coupling constants between structure fields and matter fields.

If the condition $\epsilon_{\mathbf{2}} < \epsilon_{cr} < \epsilon_{\mathbf{3}}$ is satisfied, the doublet components appear at low energy without triplet components.

$\epsilon_{\mathbf{2}} < \epsilon_{\mathbf{3}}$: depending on VEVs and coupling constants; ϵ_{cr}: depending on $SU(1,1)$ spin.

I.e., the lightest doublet higgs has no Dirac mass, while the lightest colored higgs has a Dirac mass $O(M_{\mathrm{GUT}}) \sim O(10^{16})$ GeV.



4. Structures of Yukawa Couplings

Patterns of Yukawa Couplings:

① Minimal Higgs sector [6, K.Inoue,PTP'95]

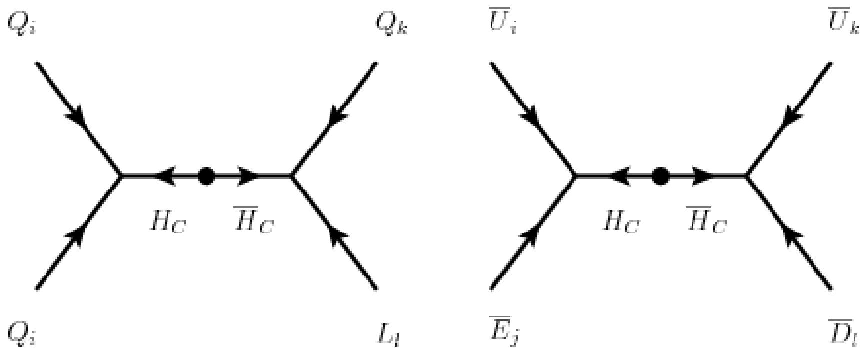
② Extended Higgs sector [7, K.Inoue & N.Y., PTP'08]

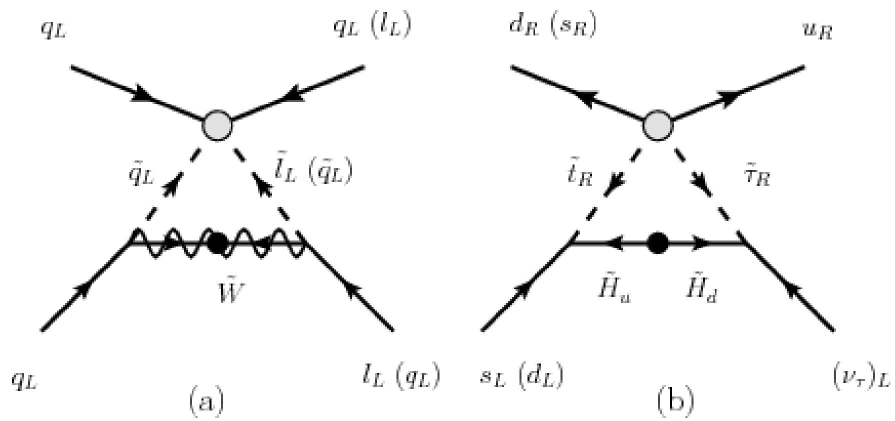
③ Non-trivial quarks, leptons, and Higgs sector [8, N.Y., PTEP'13]

Short summary:

♯ In the current model, chiral down-type quarks and charged leptons are affected by the $SU(5)$ breaking effect, i.e., the VEV of the structure fields with the adjoint representation of $SU(5)$. Thus, the Yukawa coupling matrix of the down-type quark is different from that of the charged lepton.

5. Proton Decay

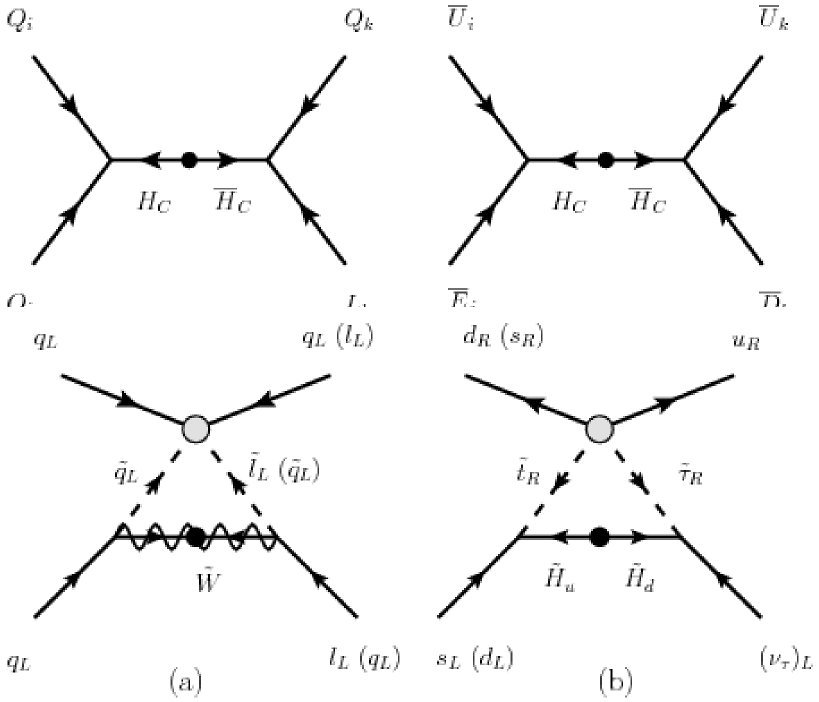




From Ref. [10, J.Hisano et al.,JHEP'13]

Superpotential quartic terms $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are generated by colored higgses, and lead to dimension-6 operators causing proton decays.

Proton decays mediated by colored higgses



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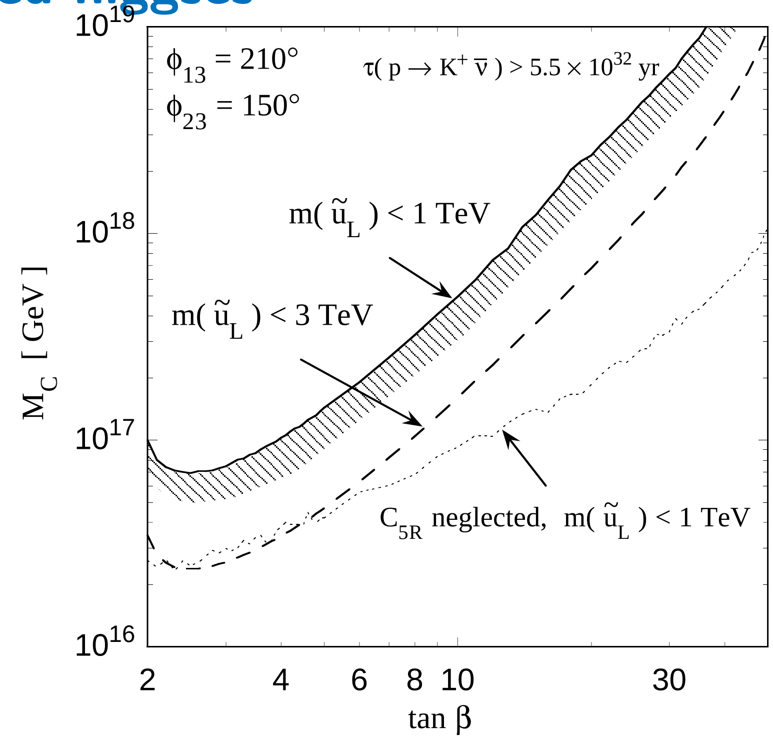
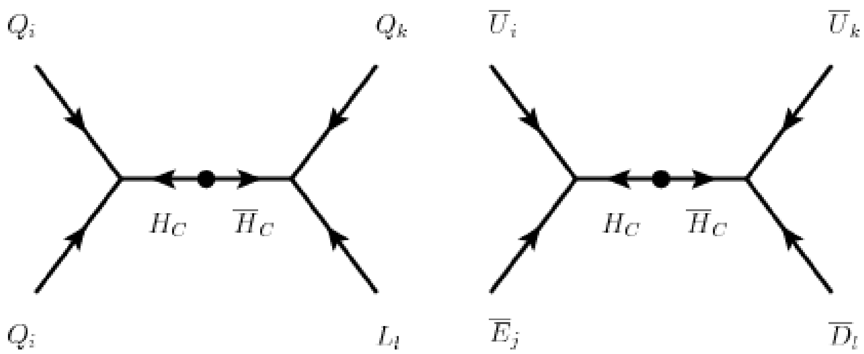


From Ref. [11, T.Goto, T.Nihei,PRD'99]

$$W \sim \frac{1}{M_C}\left(\hat{Q}\hat{Q}\hat{Q}\hat{L} + \hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c\right).$$

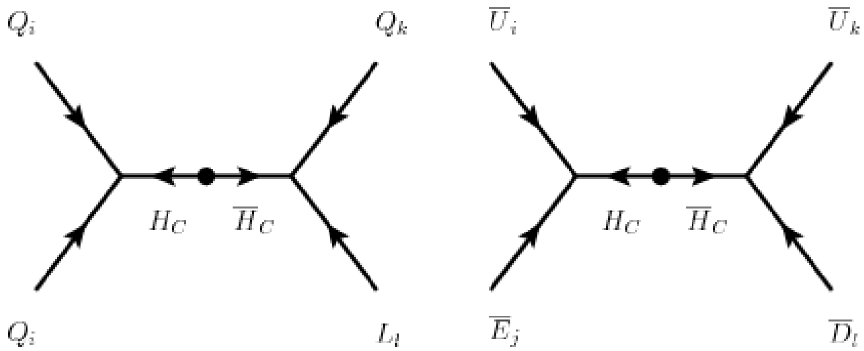
(Recent Super-Kamiokande result $\tau(p \to K^+\bar{\nu}) > 3.3 \times 10^{33}$ yr [12, M.Miura,PoS'2010])

The $SU(5) \times SU(1,1)$ Model



- μ-term $\hat{H}_u\hat{H}_d$, $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are forbidden by the horizontal symmetry $SU(1,1)$. (They are allowed by ordinary R-parity.)
- Spontaneous horizontal symmetry breaking can produce nonzero μ-parameter $O(m_{\mathrm{SUSY}})$, and then $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are also generated.
- The masses of the colored higgses come from mass terms $O(M_{\mathrm{GUT}})$ between the colored higgses and their conjugate fields as well as the effective μ-term $O(m_{\mathrm{SUSY}})$ between up- and down-type colored higgses.

The $SU(5) \times SU(1,1)$ Model



$$W \sim \frac{\mu}{M_{T_u} M_{T_d}} \left(\hat{Q}\hat{Q}\hat{Q}\hat{L} + \hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c \right), \quad ''M_C'' := \frac{M_{T_u} M_{T_d}}{\mu},$$

where $\mu = O(m_{\rm SUSY})$ and $M_{T_u} \simeq M_{T_d} = O(M_{\rm GUT})$.

$$\Rightarrow \quad ''M_C'' = O(M_{\rm GUT}^2 / m_{\rm SUSY}) \sim O(10^{29}) \text{ GeV}.$$

- The contribution to proton decay via colored higgses ($p \to K^+ \bar{\nu}$) is highly suppressed. Thus the main contribution comes from the GUT gauge bosons ($p \to \pi^0 e^+$).

6. Summary

A Vectorlike $SU(5) \times SU(1,1)$ SUSY GUT model

① Three chiral generations of quarks and leptons and one chiral generation of Higgses can be realized by using structure fields with integer and half-integer $SU(1,1)$ spins discussed in Ref. [8, N.Y., PTEP'13].

② In this case, the doublet-triplet mass splitting of Higgses can be naturally realized.

③ The Yukawa coupling matrix of down-type quark is different from that of charged lepton.

④ Since the colored Higgses have Dirac masses, the contribution to proton decay derived from colored Higgses is highly suppressed.

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