A Vectorlike Supersymmetric Grand Unified Model with Noncompact Horizontal Symmetry

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This talk is based on Ref. [1, N.Y., PTEP, 2013, 123B01]
1. Introduction

How to understand Nature
Hints of Physics beyond the Standard Model

Matter sector: Quarks and Leptons
Three Chiral Generations
Hierarchical Mass Structures \(\Rightarrow 1\)
Charge Quantization
Anomaly Cancellations \(\Rightarrow 2\)
Only Fundamental Representations \(\Rightarrow 3\)

⇒ How can we understand these questions in a four dimensional effective field theory?
1 Horizontal Symmetry? 2 GUT? 3 Unknown.
Approaches to understand generations using horizontal symmetry

**Non-Abelian** \((e.g., SU(3), A_4)\)
Naturally explaining three generations.
\[2, 3, S.F. King, G.G. Ross, PLB'01; H. Ishimori et al.; \cdots \]

**Abelian** \(U(1)\)
Naturally explaining the mass hierarchies.
Froggatt-Nielsen mechanism
\[4, 5, C.D. Froggatt, H.B. Nielsen, NPB'79; N. Maekawa, PTP'01; \cdots \]

**Noncompact Non-Abelian** \(SU(1, 1)\)
\[6–8, K. Inoue, PTP'95; K. Inoue, N.Y., PTP'08; N. Yamatsu, PTEP'12; \cdots \]
Noncompact Non-Abelian Group (minimal $SU(1, 1)$)

Noncompact Model ~ A Vectorlike MSSM with Horizontal Symmetry

Three Chiral Generations
Chiral matters produced from Vectorlike matters via “Spontaneous Generation of Generations”

Hierarchical Mass Structures
Hierarchical structure of Clebsch-Gordan coefficients

$\mathcal{N} = 1$ Supersymmetry [9, K.Inoue, H.Kubo & N.Y.,NPB’10]
Chirality of Scalar Fields and Stability of Vacuum Structures
Spontaneous Generation of Generations [6, K.Inoue, PTP’95]

We consider a vectorlike model such as QCD. ($q_x$ : e.g., quark doublets)

Left : $q_1$ $q_2$ $q_3$ $q_4$ $q_5$ $q_6$ * * * *
Right : $q_1$ $q_2$ $q_3$ $q_4$ $q_5$ $q_6$ * * * *

↓ Spontaneous Horizontal Symmetry Breaking

Left : $q_1$ $q_2$ $q_3$ $q_4$ $q_5$ $q_6$ * * * *
| | | | | | |
Right : $q_1$ $q_2$ $q_3$ $q_4$ $q_5$ $q_6$ *

Chiral matters cannot be realized from finite vectorlike matters.
To produce chiral generations spontaneously, theory must include infinite matters.

\[
\begin{align*}
\text{Left} : & \quad \hat{Q} = \{\hat{q}_1, \hat{q}_2, \hat{q}_3, \hat{q}_4, \hat{q}_5, \cdots \} \\
\text{Right}^c : & \quad \hat{Q}^c = \{\hat{q}_1^c, \hat{q}_2^c, \hat{q}_3^c, \hat{q}_4^c, \hat{q}_5^c, \cdots \}
\end{align*}
\]

I would like a horizontal symmetry $G_H$ to control the infinite matters. I would like quarks and leptons to belong to unitary reps. of $G_H$.

$\Rightarrow$ $G_H$ must be a noncompact group.

Noncompact non-Abelian groups include $SU(1,1)$ as a subgroup.

We (at least K. Inoue and I) hope that $SU(1,1)$ is useful to consider the mechanism of the spontaneous generation of generations.
Hierarchy of Yukawa coupling constants [6, 7, K.Inoue, PTP’95; K.Inoue & N.Y., PTP’08]

\[ W_{\text{Yukawa}}^{\text{chiral}} = \sum_{m,n=0}^{2} y_{u}^{mn} q_{m} u_{n}^{c} h_{u}, \quad y_{u}^{mn} \approx \left( \begin{array}{ccc} 1 & *\epsilon & *\epsilon^{2} \\ *\epsilon & *\epsilon^{2} & *\epsilon^{3} \\ *\epsilon^{2} & *\epsilon^{3} & *\epsilon^{4} \end{array} \right), \]

\*: Coefficient determined by \( SU(1, 1) \) CGCs.

\( \epsilon \): a dimensionless parameter depending on \( SU(1, 1) \) breaking VEV.

The eigenvalues seem to be \( O(1) \), \( O(1)\epsilon^2 \), \( O(1)\epsilon^4 \), but \( \cdots \).

**Example**: to choose an \( SU(1, 1) \) weight set:

\[ y_{u}^{\text{diag}} = 1 + O(\epsilon^2), \quad \frac{1}{12}\epsilon^2 + O(\epsilon^4), \quad \frac{1}{720}\epsilon^4 + O(\epsilon^6). \]
Representations of $G_{SM}$ for Quarks and Leptons

- Chiral gauge anomalies are miraculously canceled out.
- $U(1)_Y$(and $U(1)_{em}$) charges are quantized.
- Only fundamental and trivial reps of $SU(3)_C$ and $SU(2)_L$ are included.
- When the SM gauge groups $G_{SM}(:= SU(3)_C \times SU(2)_L \times U(1)_Y)$ are embedded into a GUT group $SU(5)_{GUT}$, the SM chiral fermions can be embedded into $SU(5)$ fundamental reps. $10$ and $5^*$. When we consider a GUT group $SO(10)_{GUT}$, the SM chiral fermions can be embedded into a fundamental rep. $16$.

The above facts seem to suggest that the concept of grand unified theory is true.
Minimal $SU(5)$ SUSY GUT: Problem-I (Proton Decay)

- Colored Higgses with $M_C \simeq O(M_{\text{GUT}})$ produce too rapid proton decay (as long as we assume that sparticle masses are $O(1)$ TeV).

From Ref. [10, J.Hisano et al.,'13]

Superpotential quartic terms $\hat{Q}Q\hat{Q}L$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are generated by colored Higgses, and lead to dimension-6 operators $qqql$ and $u^cu^cd^ce^c$ causing proton decays.
Proton decays mediated by colored Higgses

From Ref. [10, J.Hisano et al.,'13]
(Recent Super-Kamiokande result $\tau(p \rightarrow K^+\bar{\nu}) > 3.3 \times 10^{33}$ yr [12, M.Miura,PoS'2010])
Minimal $SU(5)$ SUSY GUT: Problem-II (Higgses)

- How to realize doublet-triplet Higgs mass splitting.

Higgses (doublet) have masses $O(M_{EW}) \sim O(10^2)$ GeV, while the corresponding colored higgses (triplet) must have masses $O(M_{GUT}) \sim O(10^{16})$.

In the minimal $SU(5)$ SUSY GUT, the doublet part of the original $SU(5)$ $\mu$-term $\mu_5 \hat{H}_u \hat{H}_d^*$ is canceled by using the “$\mu$”-term $\langle \Phi_{24} \rangle \hat{H}_u \hat{H}_d^*$.

Even if we ignore the naturalness problem, the colored Higgs must have $O(10^{17})$ GeV regardless of the value of $\tan \beta$.

Note that since $SO(10)$ and $E_6$ GUT models lead to Yukawa coupling unification, $\tan \beta$ must be around $40 - 50$, and then the colored Higgs masses must exceed $O(M_{Planck})$. 
Minimal $SU(5)$ SUSY GUT: Problem-III (Yukawa couplings)

- In the minimal model, the Yukawa coupling matrix of down-type quark is the same as that of charged lepton.

Some known methods to avoid this problem

# To take into account of superpotential higher order terms, e.g., quartic terms and quintic terms.

$$W = \sum_{n=1}^{\infty} \kappa_n \frac{\langle \Phi_{24} \rangle^n}{\Lambda^n} \hat{F}_{10} \hat{G}_5^* \hat{H}_{d5^*}$$

$\kappa_n$: a dimensionless coupling constant, $\Lambda$: a dimensional parameter.

# To introduce higher dimensional representations, e.g., $45,45^*$ and $75$ representations of $SU(5)$ [13, H.Georgi, C.Jarlskog, PLB’79].
Some known methods suppressing proton decays via colored higgses

1. SUSY particle’s masses are much heavier than 1 TeV. \cite{10, J.Hisano et al., JHEP’13; \ldots}

2. In an orbifold extra dimension model $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$, when only doublet higgses have zero modes and colored higgses have no zero modes \cite{14, Y.Kawamura, PTP’01}, the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed \cite{15, L.J.Hall, Y.Nomura, PRD’01; \ldots}.

The $SU(5) \times SU(1, 1)$ Model

1. When only doublet higgses are chiral and colored higgses are vectorlike via spontaneous generation of generations \cite{7,16, K.Inoue, N.Yamashita, PTP’00; K.Inoue, N.Y., PTP’08}, the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed \cite{1, N.Y., PTEP’13}.
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2. A Vectorlike $SU(5) \times SU(1, 1)$ SUSY GUT model

Representations of $SU(1, 1)$ in the Noncompact Model

**Matter field** = infinite-dim. unitary reps. of the noncompact group

\[
\hat{F} = \{\hat{f}^\alpha, \hat{f}^{\alpha+1}, \hat{f}^{\alpha+2}, \cdots\},
\]

\[
\hat{F}^c = \{\hat{f}^c_{-\alpha}, \hat{f}^c_{-\alpha-1}, \hat{f}^c_{-\alpha-2}, \cdots\},
\]

$\alpha = \text{[real and positive]}$.

**Structure field** = finite-dim. reps.

\[
\hat{\Psi} = \{\hat{\psi}^{-S}, \hat{\psi}^{-S+1}, \cdots, \hat{\psi}^{-S-1}, \hat{\psi}^{-S}\},
\]

$S = \text{[Integer or half-integer]}$, called $SU(1, 1)$ spin.

Subscripts, such as $\alpha, S$, stand for the eigenvalue of $\tau_3$.

All Matter and Structure fields are chiral superfields in $\mathcal{N} = 1$ SUSY.

[8, N.Y., PTEP'13]
Matter Content in the $SU(5) \times SU(1,1)$

Matter fields

<table>
<thead>
<tr>
<th>Field</th>
<th>$\hat{F}_{10}$</th>
<th>$\hat{G}_{5^*}$</th>
<th>$\hat{H}_{u5}$</th>
<th>$\hat{H}_{d5}$^*</th>
<th>$\hat{F}_{10}^c$</th>
<th>$\hat{G}_{5}^c$</th>
<th>$\hat{H}_{u5}^c$</th>
<th>$\hat{H}_{d5}^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(5)$</td>
<td>10</td>
<td>5^*</td>
<td>5</td>
<td>5^*</td>
<td>10^*</td>
<td>5</td>
<td>5^*</td>
<td>5</td>
</tr>
<tr>
<td>$SU(1,1)$</td>
<td>$\alpha^{(i)}$</td>
<td>$\beta^{(i)}$</td>
<td>$-\gamma$</td>
<td>$-\delta$</td>
<td>$-\alpha^{(i)}$</td>
<td>$-\beta^{(i)}$</td>
<td>$+\gamma$</td>
<td>$+\delta$</td>
</tr>
</tbody>
</table>

Structure fields

<table>
<thead>
<tr>
<th>Field</th>
<th>$\hat{\Phi}_1$</th>
<th>$\hat{\Phi}_{24}'$</th>
<th>$\hat{\Psi}_{1/24}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(5)$</td>
<td>1</td>
<td>24</td>
<td>1 or 24</td>
</tr>
<tr>
<td>$SU(1,1)$</td>
<td>$S'$</td>
<td>$S'$</td>
<td>$S''$</td>
</tr>
</tbody>
</table>

Assuming that the nonvanishing VEVs of the structure fields break $SU(5) \times SU(1,1)$ into $G_{SM}(M_{GUT} \simeq O(10^{16})$ GeV): $\langle \hat{\Phi}_1 \rangle = \langle \phi_0 \rangle, \langle \hat{\Phi}_{24}' \rangle = \langle \phi_{+1} \rangle, \langle \hat{\Psi}_{1/24} \rangle = \langle \psi_{-3/2} \rangle$. 
3. Realization of Chiral Generations

The Spontaneous Generation of Generations

1. A structure field with an $SU(1,1)$ integer spin [6, K.Inoue, PTP’95]
2. Structure fields with $SU(1,1)$ integer spins [16, K.Inoue, N.Yamashita, PTP’00]
3. Structure fields with $SU(1,1)$ integer and half-integer spins [8, N.Y., PTEP’13]

In the current model, the discussion for quarks and leptons is corresponding to (3), and the discussion for higgses is corresponding to (2). We glance at them.
Three Chiral Generations of Quarks and Leptons [8, N.Y., PTEP’13]

The patterns of massless modes can be classified into three types by using the $SU(1, 1)$ weight of matter fields.

This patterns determine structures of Yukawa couplings in part.
Whether massless modes appear or not depends on the VEVs of the structure fields and the coupling constants between structure fields and matter fields.

If the condition $\epsilon_2 < \epsilon_{cr} < \epsilon_3$ is satisfied, the doublet components appear at low energy without triplet components.

I.e., the lightest doublet higgs has no Dirac mass, while the lightest colored higgs has a Dirac mass $O(M_{GUT}) \sim O(10^{16})$ GeV.
4. Structures of Yukawa Couplings

Patterns of Yukawa Couplings:

1. Minimal Higgs sector [6, K.Inoue, PTP’95]
2. Extended Higgs sector [7, K.Inoue & N.Y., PTP’08]
3. Non-trivial quarks, leptons, and Higgs sector [8, N.Y., PTEP’13]

Short summary:

In the current model, chiral down-type quarks and charged leptons are affected by the $SU(5)$ breaking effect, i.e., the VEV of the structure fields with the adjoint representation of $SU(5)$. Thus, the Yukawa coupling matrix of the down-type quark is different from that of the charged lepton.
5. Proton Decay

Superpotential quartic terms $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are generated by colored higgses, and lead to dimension-6 operators causing proton decays.
Proton decays mediated by colored higgses

\[ W \sim \frac{1}{M_C} \left( \hat{Q} \hat{Q} \hat{Q} \hat{L} + \hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c \right) . \]

(Recent Super-Kamiokande result \( \tau(p \rightarrow K^+ \nu) > 3.3 \times 10^{33}\) yr [12, M.Miura, PoS'2010])


\[ \phi_{13} = 210^\circ \quad \tau(p \rightarrow K^+ \nu) > 5.5 \times 10^{32}\] yr
\[ \phi_{23} = 150^\circ \]

\[ m(\tilde{u}_L) < \text{1 TeV} \]
\[ m(\tilde{u}_L) < \text{3 TeV} \]

\[ C_{SR} \text{ neglected, } m(\tilde{u}_L) < \text{1 TeV} \]
The $SU(5) \times SU(1, 1)$ Model

- $\mu$-term $\hat{H}_u \hat{H}_d$, $\hat{Q} \hat{Q} \hat{Q} L$ and $\hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c$ are forbidden by the horizontal symmetry $SU(1, 1)$. (They are allowed by ordinary $R$-parity.)

- Spontaneous horizontal symmetry breaking can produce nonzero $\mu$-parameter $O(m_{\text{SUSY}})$, and then $\hat{Q} \hat{Q} \hat{Q} L$ and $\hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c$ are also generated.

- The masses of the colored higgses come from mass terms $O(M_{\text{GUT}})$ between the colored higgses and their conjugate fields as well as the effective $\mu$-term $O(m_{\text{SUSY}})$ between up- and down-type colored higgses.
The $SU(5) \times SU(1, 1)$ Model

\[
W \sim \frac{\mu}{M_{Tu}M_{Td}} \left( \hat{Q} \hat{Q} \hat{Q} \hat{L} + \hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c \right), \quad "M''_C := \frac{M_{Tu}M_{Td}}{\mu},
\]

where $\mu = O(m_{\text{SUSY}})$ and $M_{Tu} \simeq M_{Td} = O(M_{\text{GUT}})$.

$"M''_C = O(M_{\text{GUT}}^2/m_{\text{SUSY}}) \sim O(10^{29})$ GeV.

- The contribution to proton decay via colored higgses ($p \rightarrow K^+\bar{\nu}$) is highly suppressed. Thus the main contribution comes from the GUT gauge bosons ($p \rightarrow \pi^0e^+$).
6. Summary

A Vectorlike $SU(5) \times SU(1, 1)$ SUSY GUT model

1. Three chiral generations of quarks and leptons and one chiral generation of Higgses can be realized by using structure fields with integer and half-integer $SU(1, 1)$ spins discussed in Ref. [8, N.Y., PTEP’13].

2. In this case, the doublet-triplet mass splitting of Higgses can be naturally realized.

3. The Yukawa coupling matrix of down-type quark is different from that of charged lepton.

4. Since the colored Higgses have Dirac masses, the contribution to proton decay derived from colored Higgses is highly suppressed.
References


[7] K. Inoue and N. Yamatsu, “Charged Lepton and Down-Type Quark Masses in $SU(1, 1)$


