Strong CP problem and axion on the lattice

Ryuichiro Kitano (KEK)

based on 1506.00370 with Nori Yamada (KEK)

Seminar@Osaka U., October 20, 2015

In the Standard Model,

all the possible renormalizable interactions are present in the Lagrangian,
except for one funny term called the θ term.

$$\mathcal{L}_\theta = \frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}\mathrm{tr}(F_{\mu\nu}F_{\rho\sigma}) = \frac{i\theta}{32\pi^2}F\tilde{F}$$

↑ gluon field strength

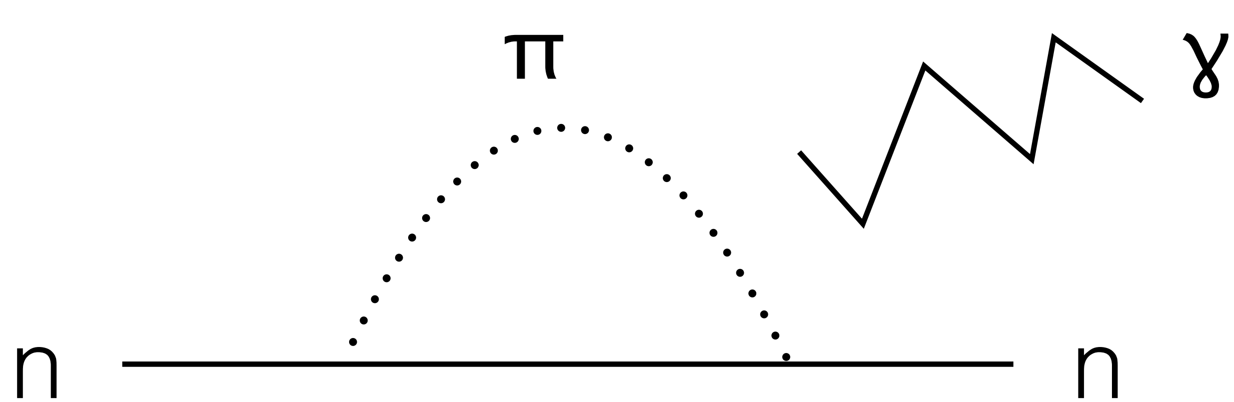
Maybe something deep is hiding behind this fact.

Strong CP problem

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar{\psi}]e^{-S_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2}F^2 + \frac{i\theta}{32\pi^2}F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

θ term breaks CP ['t Hooft '76]



$$d_n \sim 10^{-15}\theta e \cdot {\rm cm}$$

$$\theta \lesssim 10^{-10}$$????

[Crewther, Di Vecchia, Veneziano, Witten '79]

Is θ-term really physical?

—> Does the partition function Z depend on θ?

$$\frac{1}{iZ}\frac{dZ}{d\theta}\bigg|_{\theta=0} = \langle \int d^4x \frac{1}{32\pi^2} F\tilde{F} \rangle \bigg|_{\theta=0} = 0$$

(CP)

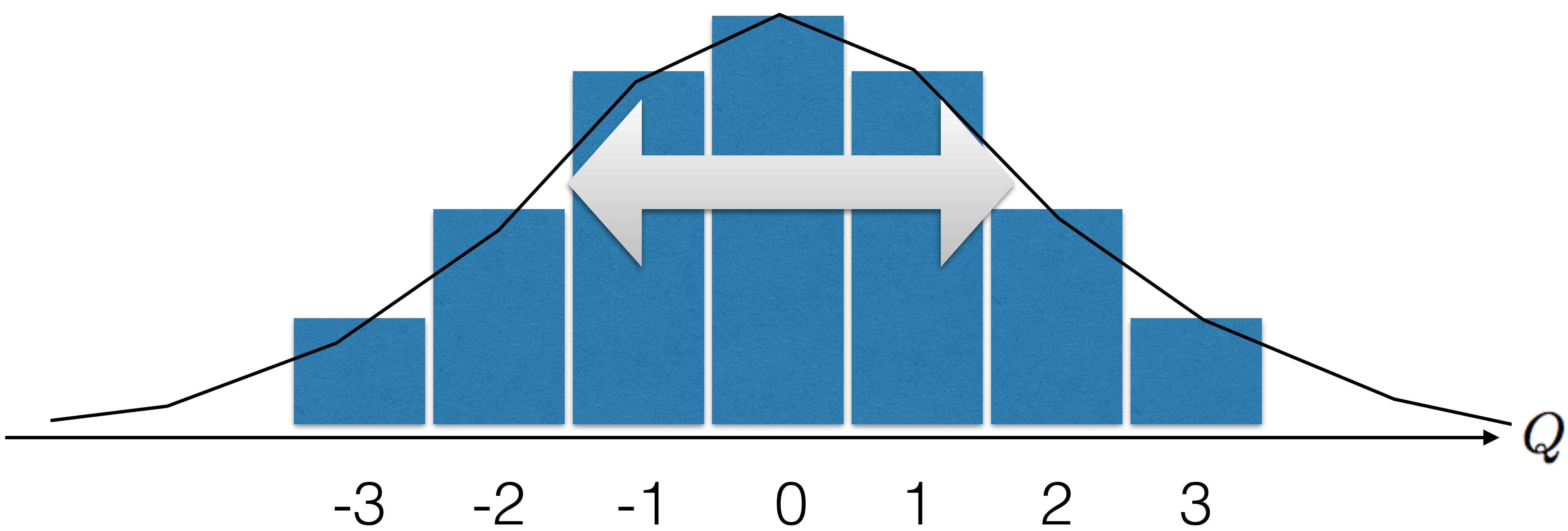
$$= Q$$

(topological charge = integers!)

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2Z}{d\theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2 \rangle}{V}$$

(topological susceptibility)

χ_t



$$\langle Q^2 \rangle = \chi_t V$$

χ_t measures how often instantons appear in the path integral.

If χ_t is nonzero, θ is physical.

χ_t and m_u

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar\psi] e^{-S_{\rm QCD}} = \int [dA][d\psi][d\bar\psi] e^{-S'_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar\psi (D+m)\psi \right)$$

$$S'_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \bar\psi (D + m e^{-i\gamma_5\theta})\psi \right)$$

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = -m_u \langle \bar u u \rangle + O(m_u^2/m_\pi^2)$$

If m_u is non zero, θ is physical.

If m_u=0, physics does **not** depend on θ.
—> no strong CP problem

m_u=0?

[PDG]

LIGHT QUARKS (u, d, s)

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

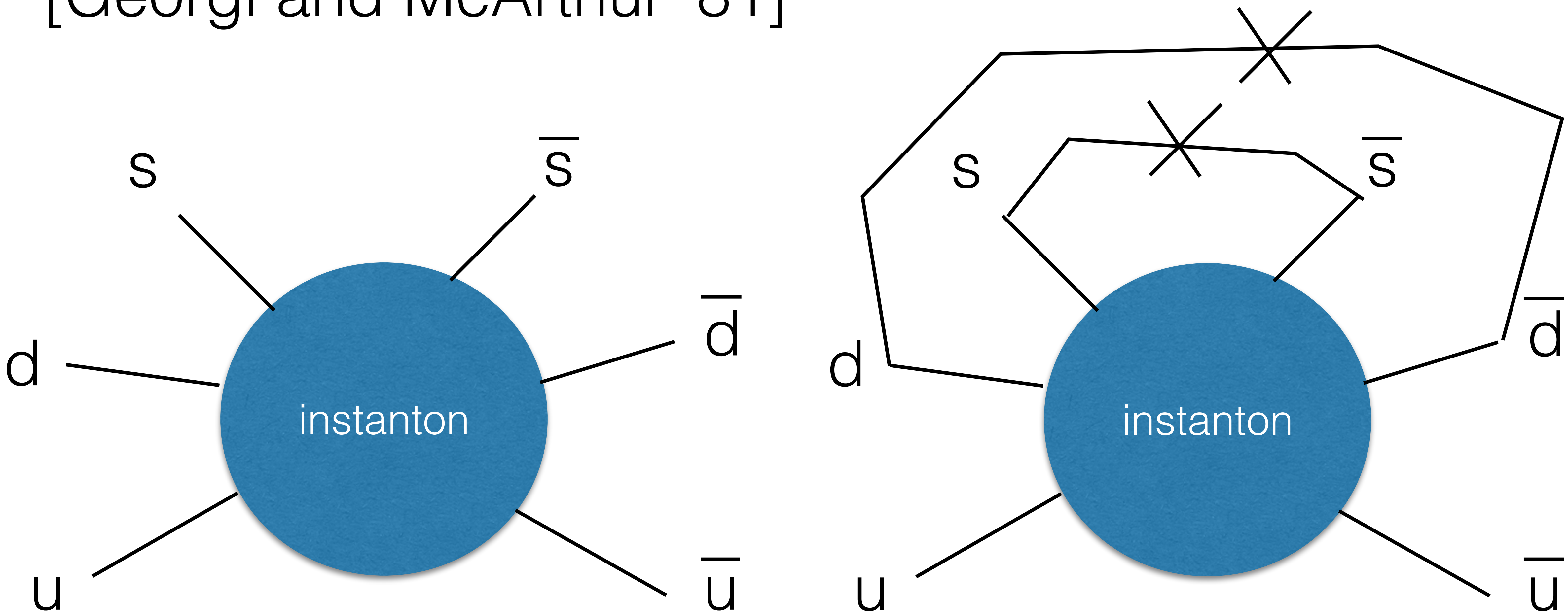
The u-, d-, and s-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\mathrm{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\mathrm{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
$2.3^{+0.7}_{-0.5}$ OUR EVALUATION	See the ideogram below.			
$2.15 \pm 0.03 \pm 0.10$	[1] DURR	11	LATT	$\overline{\mathrm{MS}}$ scheme
$2.24 \pm 0.10 \pm 0.34$	[2] BLUM	10	LATT	$\overline{\mathrm{MS}}$ scheme

Confusion 1

[Georgi and McArthur '81]



additive shift of $m_u \sim \frac{m_d m_s}{\Lambda_{\rm QCD}} \sim$ MeV

mimic the non-zero mass even if m_u=0?

Confusion 2

[Kaplan and Manohar '86]

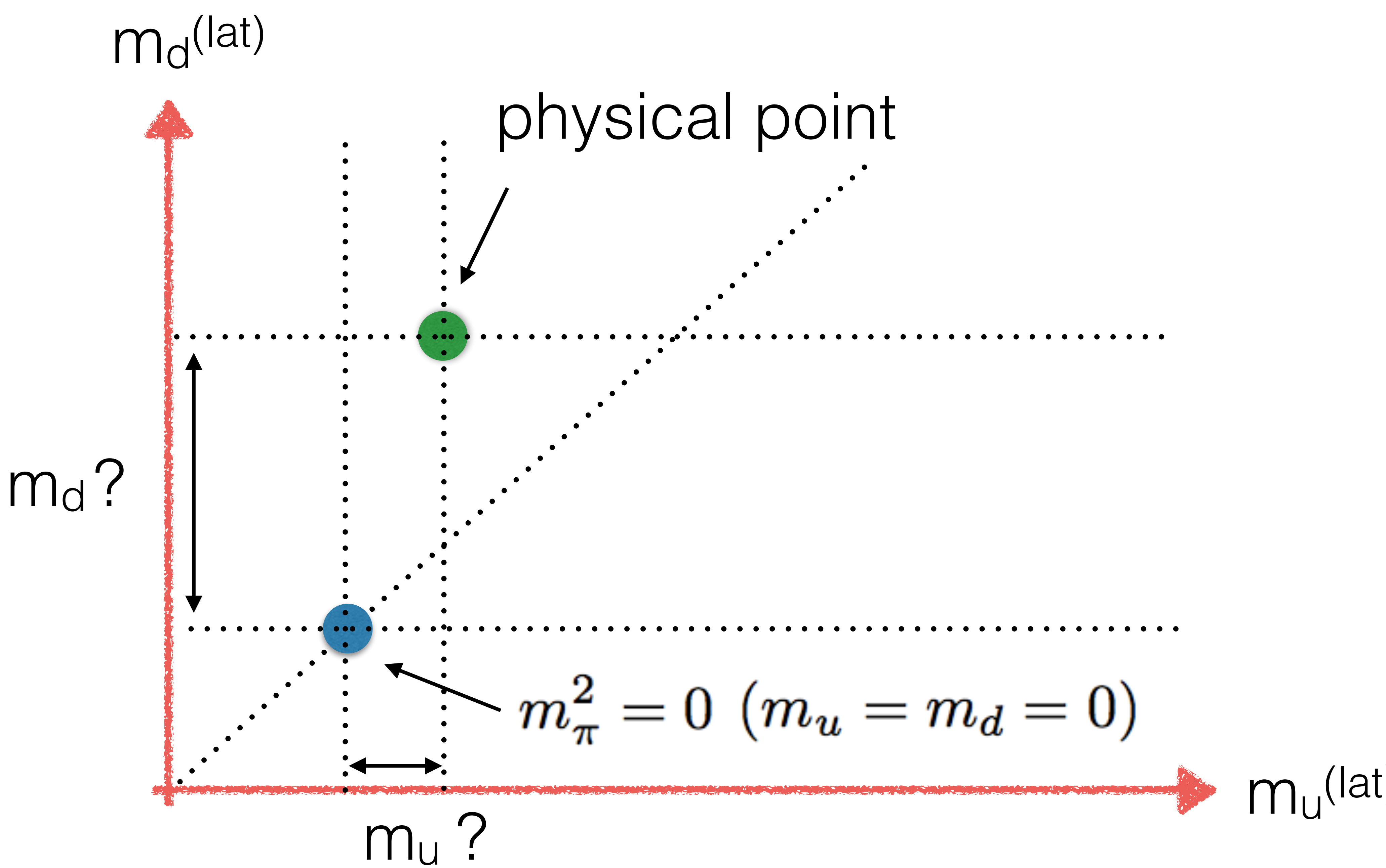
$$\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \qquad \begin{pmatrix} m_d m_s & & \\ & m_s m_u & \\ & & m_u m_d \end{pmatrix}$$

these two matrices have the **same** quantum numbers under the chiral symmetry

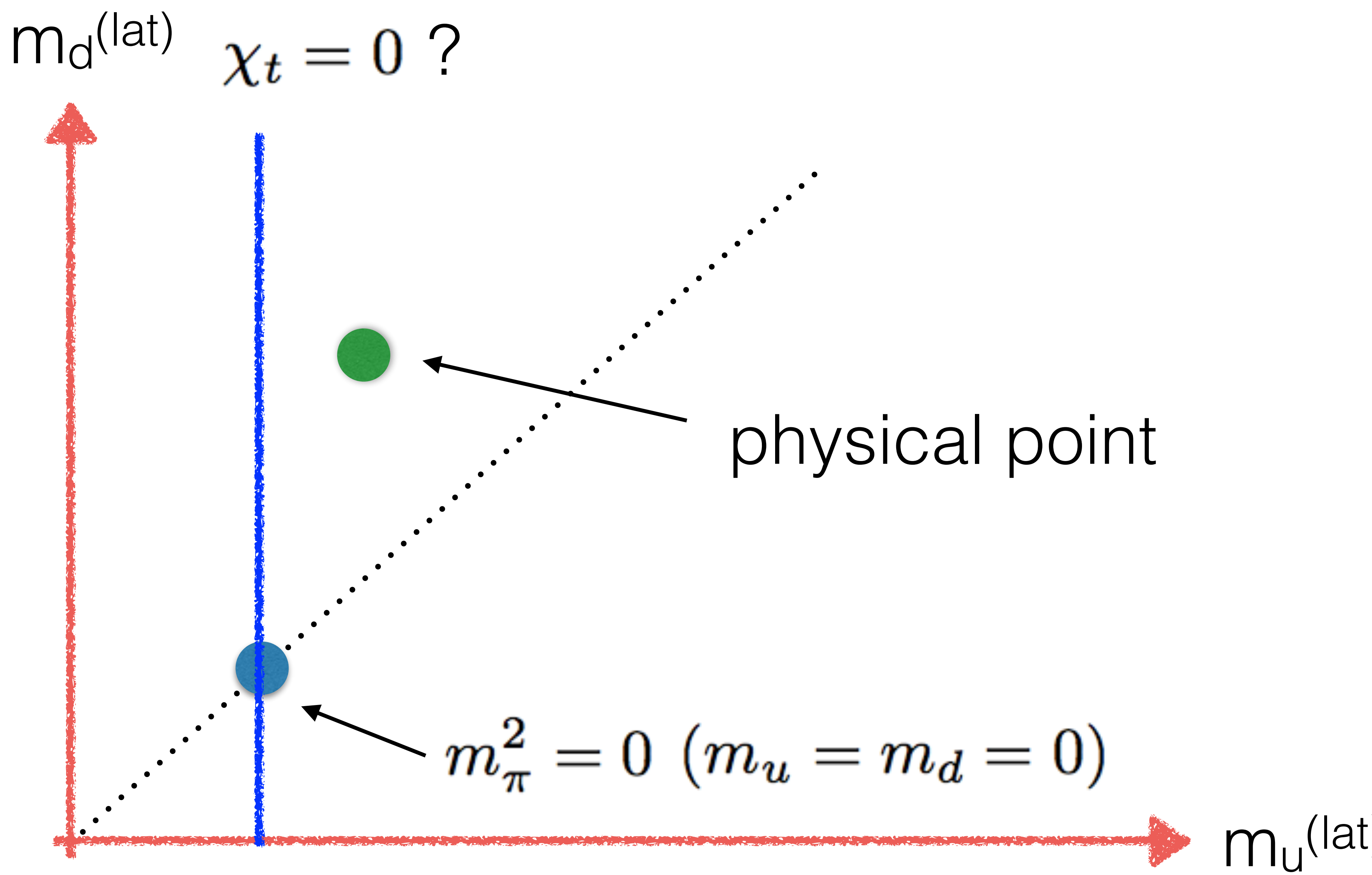
- the chiral Lagrangian cannot distinguish m_u from $m_u + c m_d m_s$
- again, mimic nonzero m_u?

Lattice?

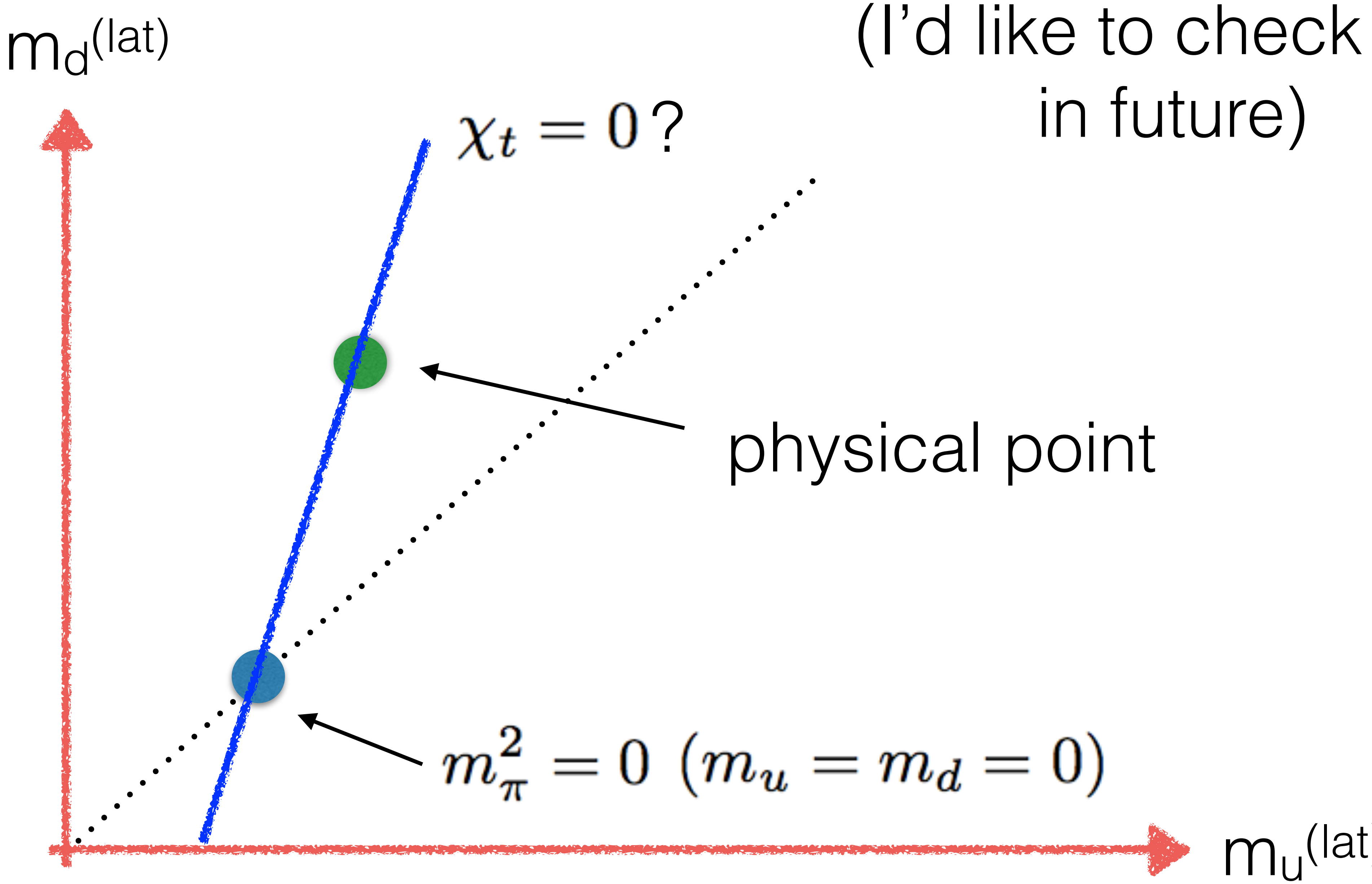
Lattice action has m_u and m_d as parameters.



I think the really important question is …



I think the really important question is …



(I'd like to check this in future)

[Peccei and Quinn '77]

Axion

OK, maybe m_u is non zero and θ is physical.

Then, why is θ so small?

The axion provides a nice solution.

$$\theta \to \theta + \frac{a(x)}{f_a} \qquad \left(\Delta\mathcal{L} = \frac{ia(x)}{32\pi^2 f_a} F\tilde{F}\right)$$

$$\frac{\chi_t}{f_a^2} = m_a^2$$

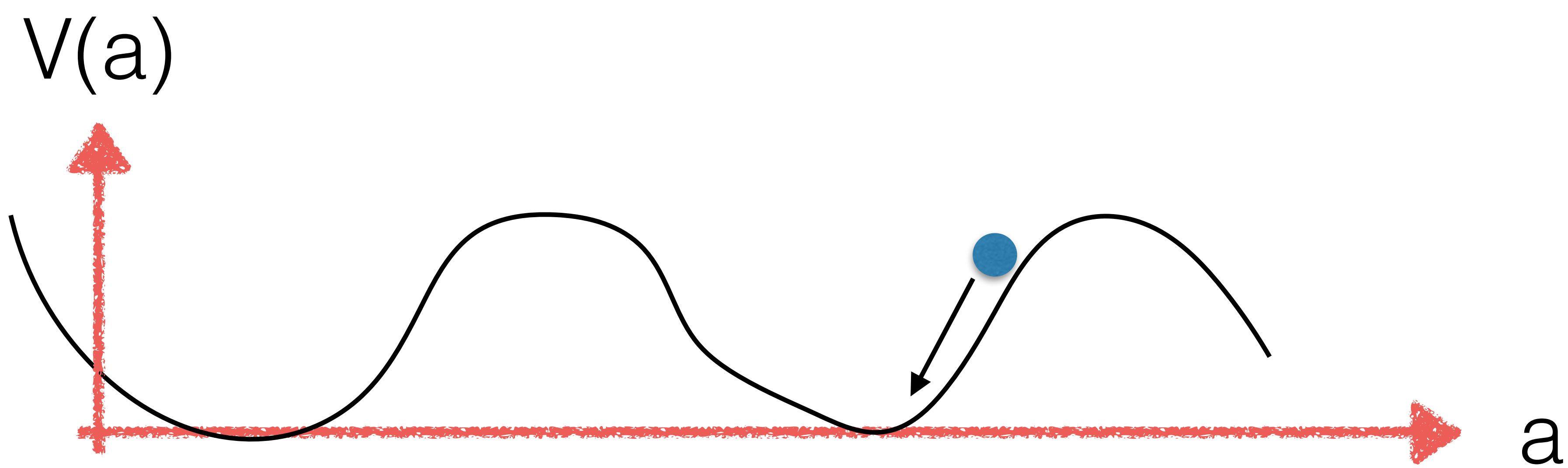
$$\mathcal{L}_{\rm eff} = \frac{\chi_t}{2}\theta^2 + \cdots \quad \Rightarrow \quad \mathcal{L}_{\rm eff} = (\partial_\mu a)^2 + \frac{\chi_t}{2}\left(\theta + \frac{a}{f_a}\right)^2 + \cdots$$

$$\chi_t > 0 \quad \Rightarrow \quad \theta + \frac{a}{f_a} = 0$$

(dynamically selected)

[Preskill, Wise, Wilczek ’83][Abbott, Sikivie ’83]
[Dine, Fischler ’83]

Axion Dark Matter



$$\ddot{a} + 3H\dot{a} = -V'(a) \sim -m_a^2 a$$

$$\left.\frac{n_a}{T^3}\right|_{\rm now} \sim \frac{m_a(T_*) f_a^2 \theta_{\rm ini}^2}{T_*^3}$$

where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

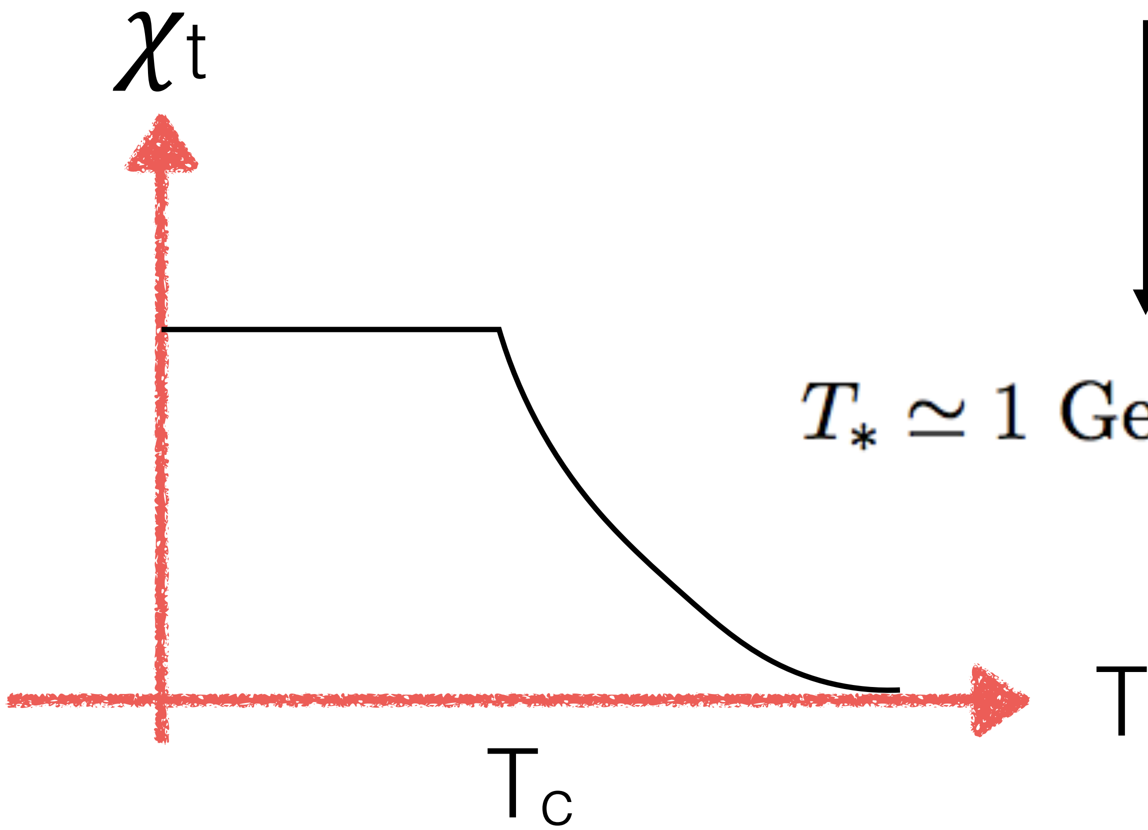
instanton paradigm

The standard way to calculate the temperature dependence of m_a is based on the dilute instanton gas approximation.

[Pisarsky, Yaffe '80]

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\rm QCD}^b T^{-8}$$

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f = 9$$



instanton action $e^{-8\pi^2/g^2}$

$$T_* \simeq 1~\mathrm{GeV} \cdot \left(\frac{m_a}{10^{-5}~\mathrm{eV}}\right)^{1/6}$$

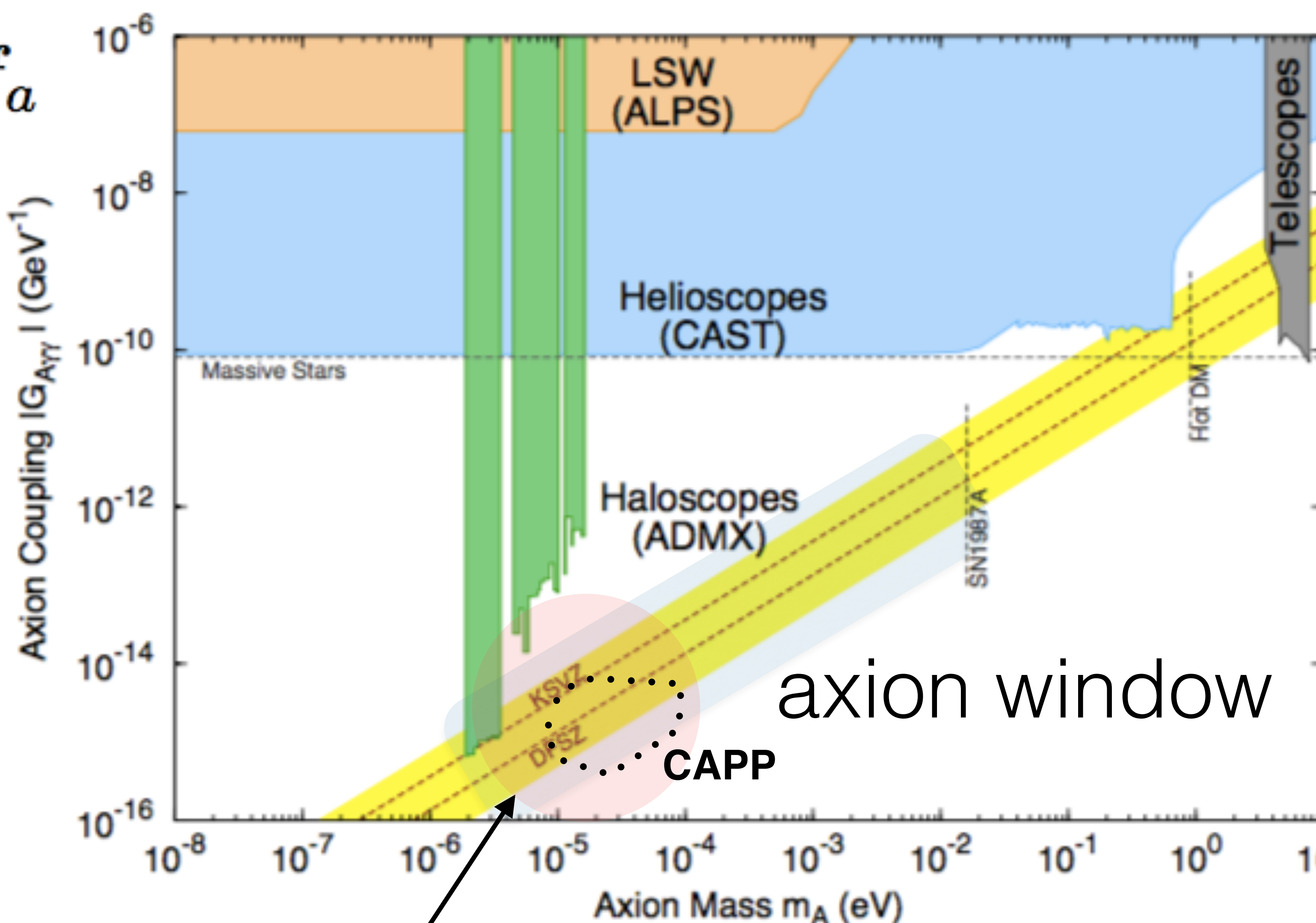
$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}~\mathrm{eV}}\right)^{-7/6}$$

Axion Dark Matter

$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$

[PDG]

$\sim 1/f_a$



good DM abundance

[Cohen '96][Aoki, Fukaya, Taniguchi '12]

Is instanton correct?

Based on $\langle\bar{q}q\rangle = O(m_q)$ at high temperatures and the Ward identities, Cohen has argued

$$\chi_t(T) = O(m_q^4) \qquad \text{for } N_f=2$$

whereas the instanton says

$$\chi_t(T) = O(m_q^2) \qquad \text{for } N_f=2$$

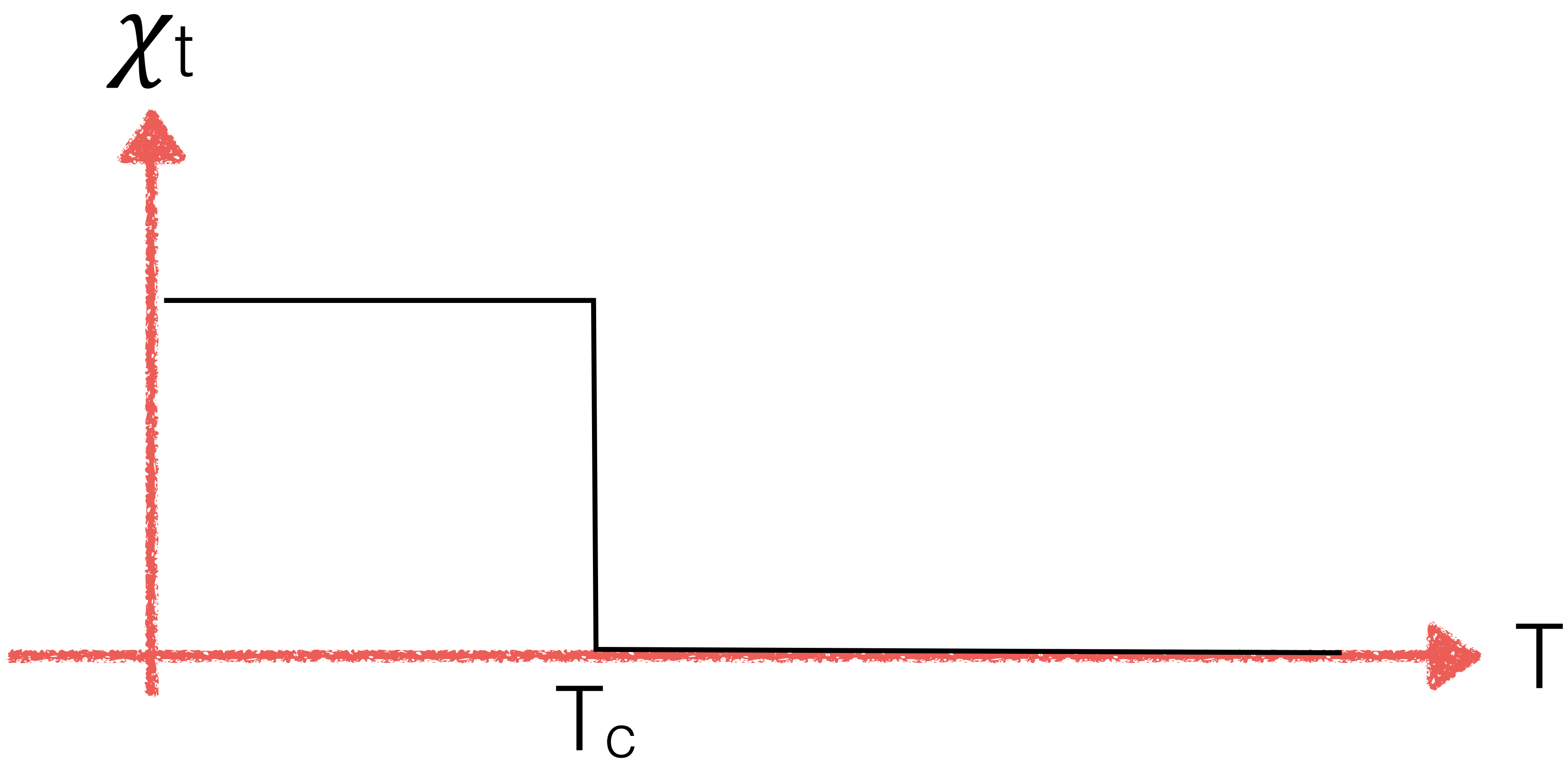
Aoki et al refined the Cohen's analysis and argued

$$\chi_t(T) = 0 \qquad \text{for small but finite } m_q$$

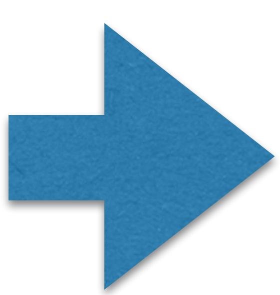
in any case, it is clearly inconsistent with instantons.

if χ_t=0 above T_c~150MeV,

the axion suddenly starts to oscillate at $T=T_c$

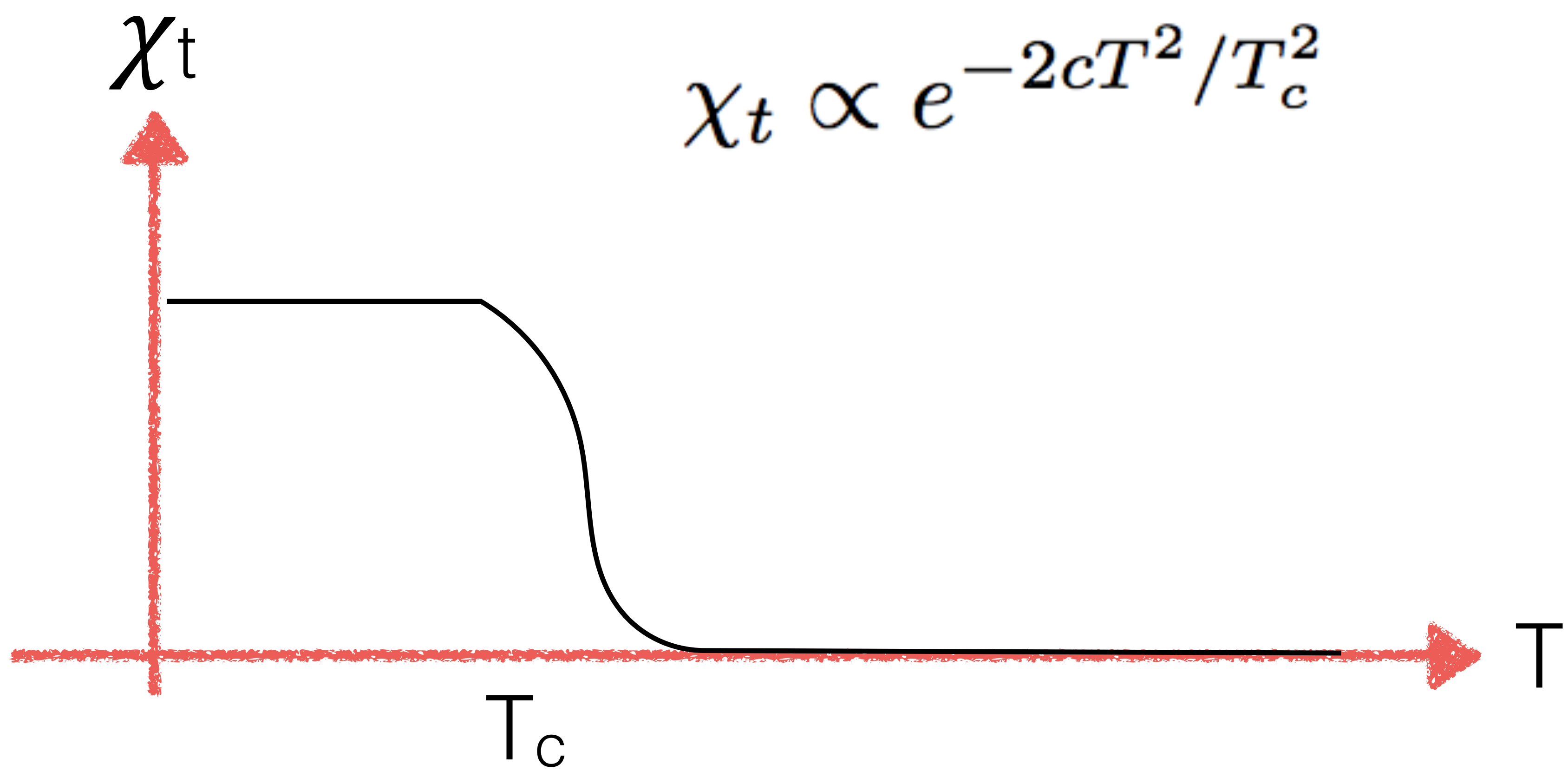


$$\Omega_a \sim 2 \times 10^5 \theta_{\rm ini}^2$$ independent of m_a



axion window is gone.

a bit milder case



$$\Omega_a \sim 0.2\theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-1} \times 2.5c \qquad (c \gg 1)$$

enhancement due to the non-adiabatic evolution of the potential.

It seems that
the lattice determination of
χ_t is important

χ_{t} on the lattice

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

we just need to measure Q in each configuration.

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$
$$= \int d^4x \ \mathrm{tr}\gamma_5$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

There are two ways to measure Q.

Bosonic definition

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$

on the lattice, one would not get integers
due to the ambiguities in the definition of F.

—> The techniques called Cooling or Wilson flow
can make it possible to identify Q.

Fermionic definition

$$Q = \int d^4x \ \mathrm{tr}\gamma_5$$

With a properly defined γ_5, one can get integers.

This method gives unambiguous Q, but the cost of the calculation is high.

Somehow,

in 2015, three independent calculations appeared.

(in the SU(3) Yang-Milles theory, **no quarks yet**)

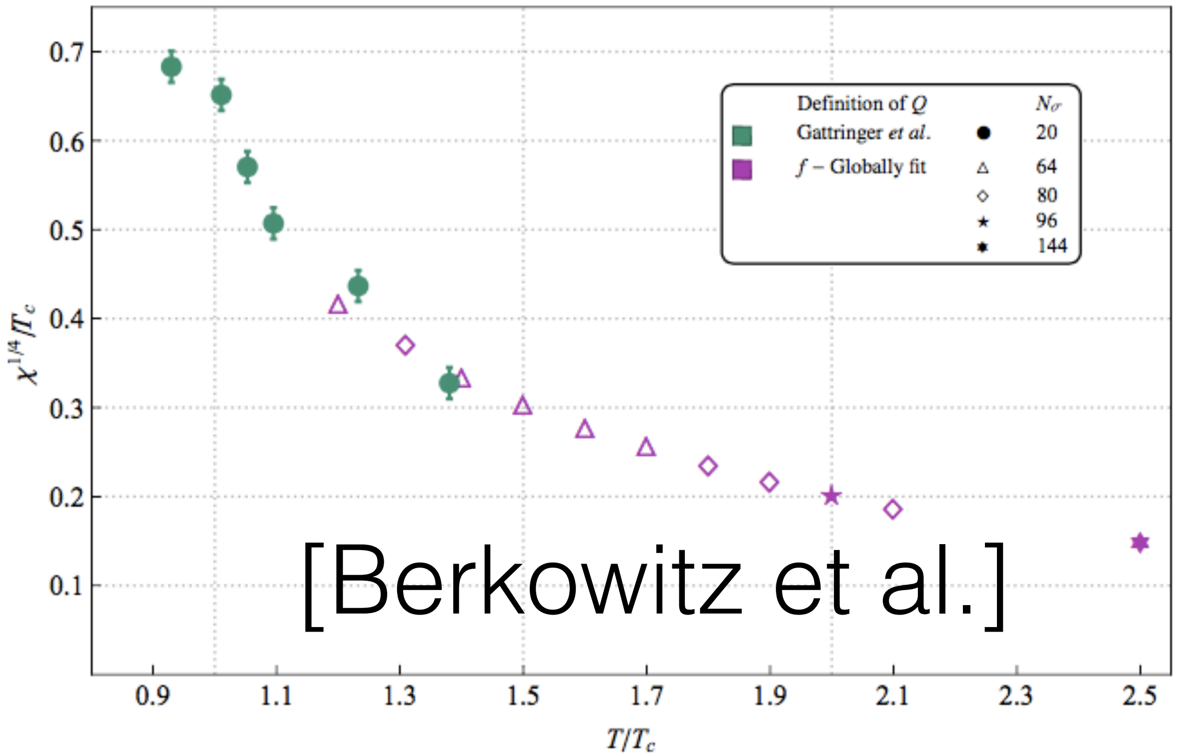
E. Berkowiz, M. Buchoff, E. Rinaldi (LLNL)

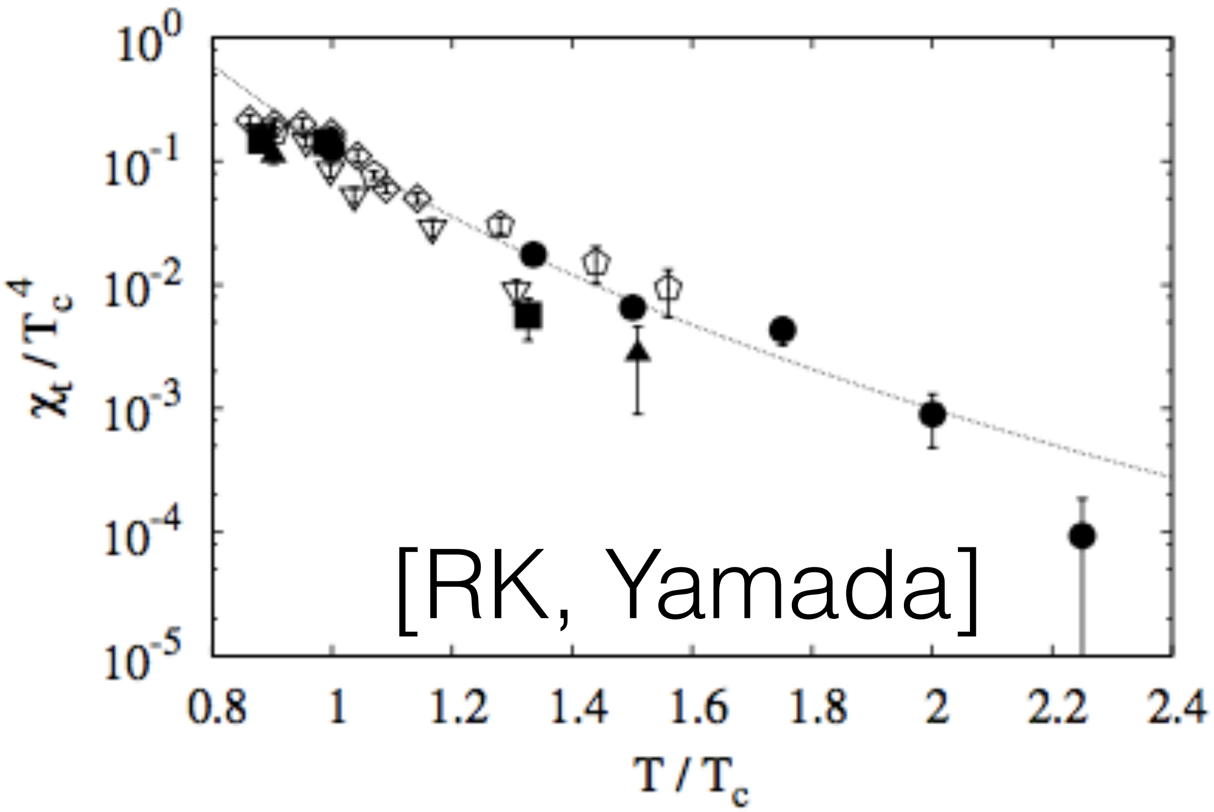
Bosonic (cooling)

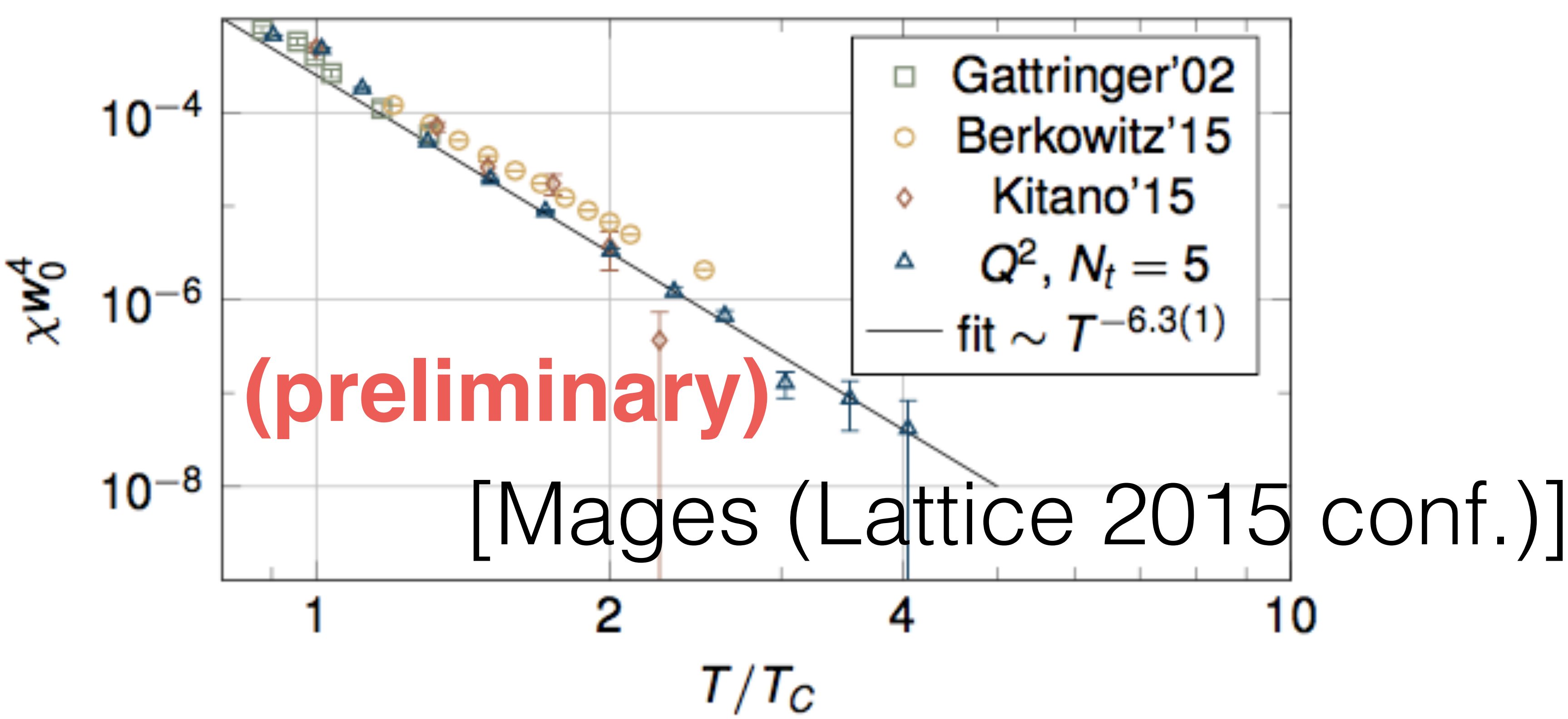
RK and N. Yamada (KEK) Fermionic (overlap)

S. Mages et al (BMW) Bosonic (Wilson Flow)

lattice results

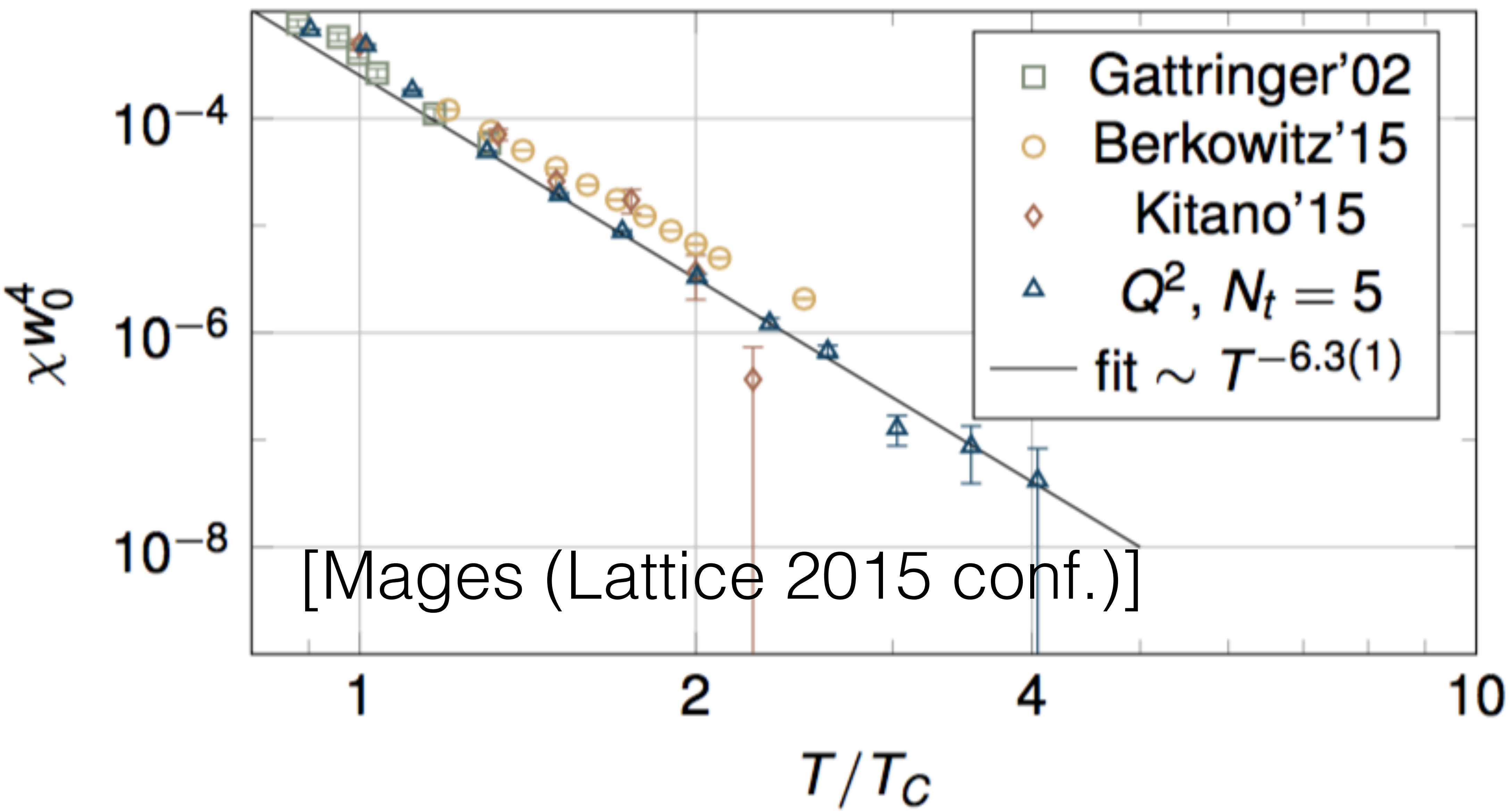






All look consistent

(at least qualitatively)



[Mages (Lattice 2015 conf.)]

We see a clear power law even at a very low temperature.

instanton?

The instanton predicts $\chi_t \propto T^{-7}$ for $T \gg T_c$

in SU(3) YM theory

at one-loop level

The lattice says

$\chi_t \propto T^{-6\pm0.?}$ T ~ 2-4Tc

It seems that the semiclassical instanton picture
is qualitatively good in YM theories.

But for the axion study, we need to include quarks.

problem at high temperature and/or with small quark masses

at high temperatures
and/or small quark masses

$$\langle Q^2 \rangle = \chi_t V \ll 1$$

→ We only see Q=0 configurations

→ We cannot calculate <Q^2>

Probably we need some method to improve the calculation further.

a trick

$$Z_{\mathrm{QCD}} = \int [dA][d\psi][d\bar{\psi}]e^{-S_{\mathrm{QCD}}}$$

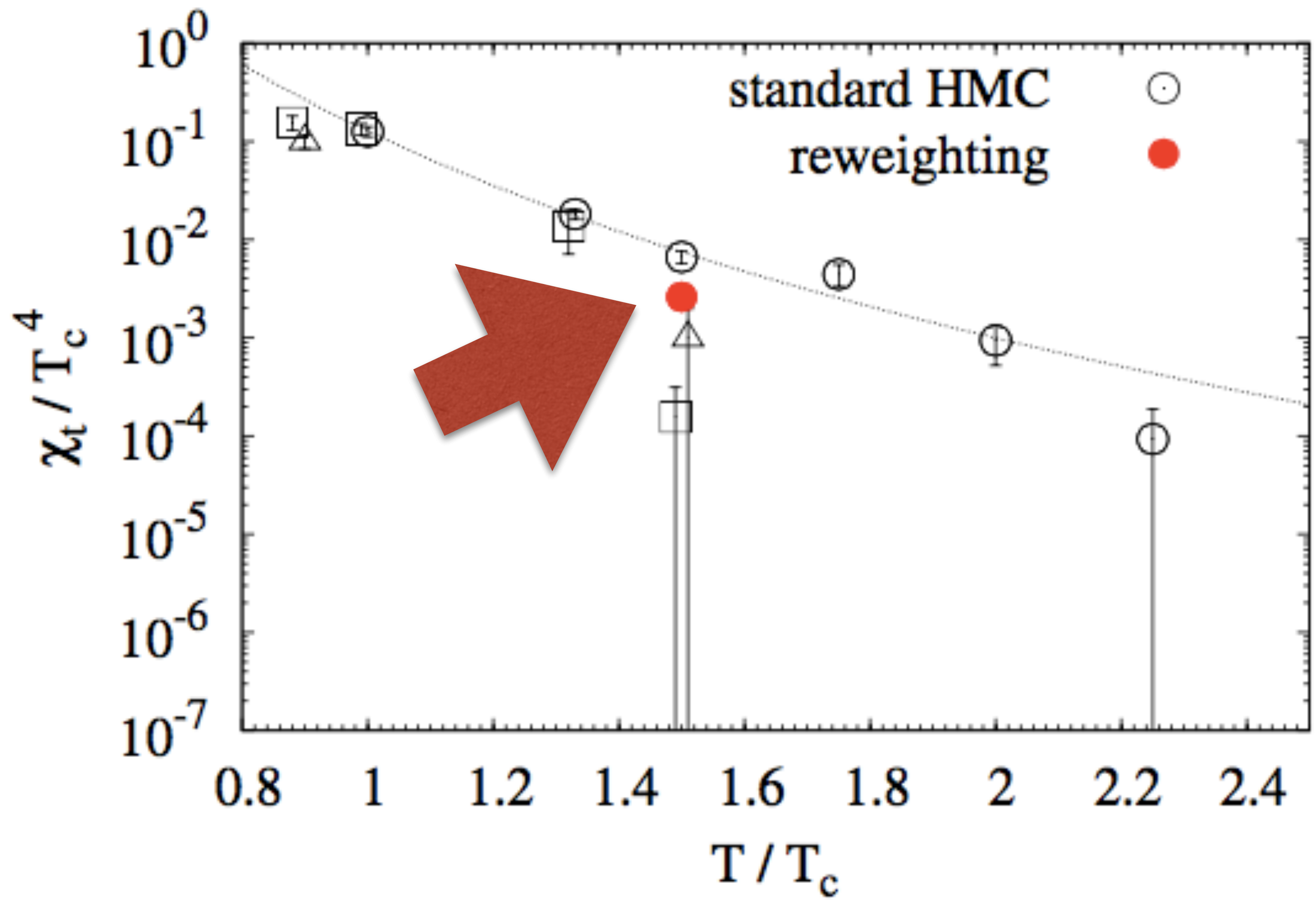
$$\Rightarrow \quad Z_X = \int [dA][d\psi][d\bar{\psi}]Xe^{-S_{\mathrm{QCD}}}$$

$$X = \det\left(\frac{H_W^2+\mu^2}{H_W^2+\epsilon^2}\right)^{N_\phi}$$

X enhances to generate nonzero Q confs.

$$\Rightarrow \quad \chi_t = \frac{\langle (Q^2/V)XX^{-1}\rangle}{\langle XX^{-1}\rangle} = \frac{\langle (Q^2/V)X^{-1}\rangle_X}{\langle X^{-1}\rangle_X}$$

trial



Wmm. Not too bad, but we need more studies.

Summary

χ_t is a fundamental quantity in QCD which measures the effects of topology.

→ very much related to Strong CP problem

The calculation in YM seems to support the instanton picture, **but anything can happen when we include dynamical quarks**.

more lattice simulations at both zero and high temperatures are necessary to make things clear.