Seminar @ Osaka University, 12 Jan. 2016

Fate of false vacuum at one-loop

~ To resolve the renormalization scale uncertainty ~

Yutaro Shoji
University of Tokyo

JHEP 1601 (2016) 031 (arXiv:1511.04860 [hep-ph])

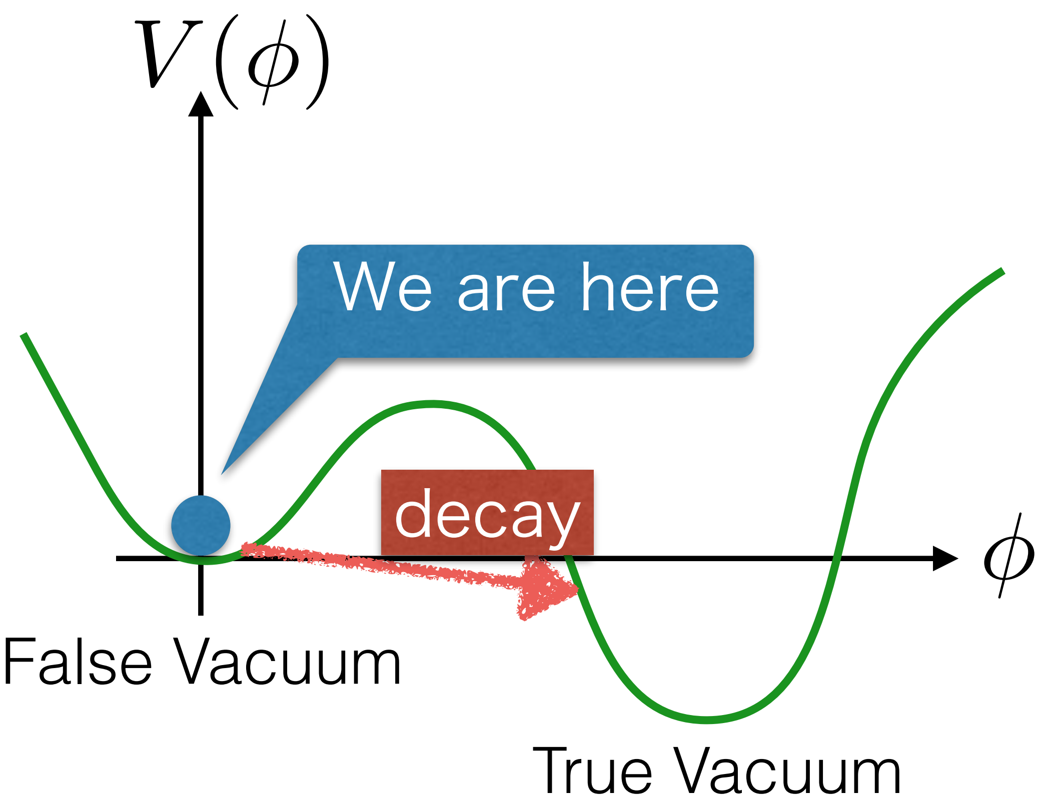
Collaborators: Motoi Endo, Takeo Moroi, Mihoko Nojiri (KEK)

Contents

- Introduction
- Calculation of the 1-loop factor
- SM + stau system
- Summary

Introduction

Vacuum decay



<u>Tree level potential</u>

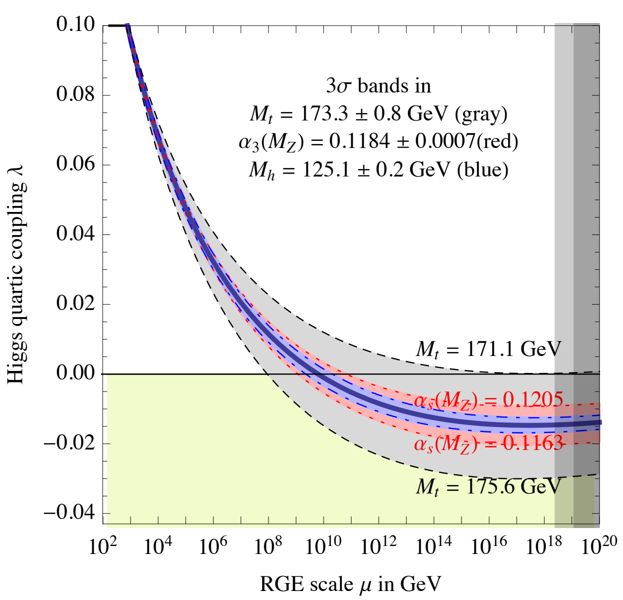
ex.) Supersymmetry

$h\tilde{t}_L\tilde{t}_R$ Higgs mass, hgg, hγγ, …

$h\tilde{\ell}_L\tilde{\ell}_R$ muon g-2, hγγ, …

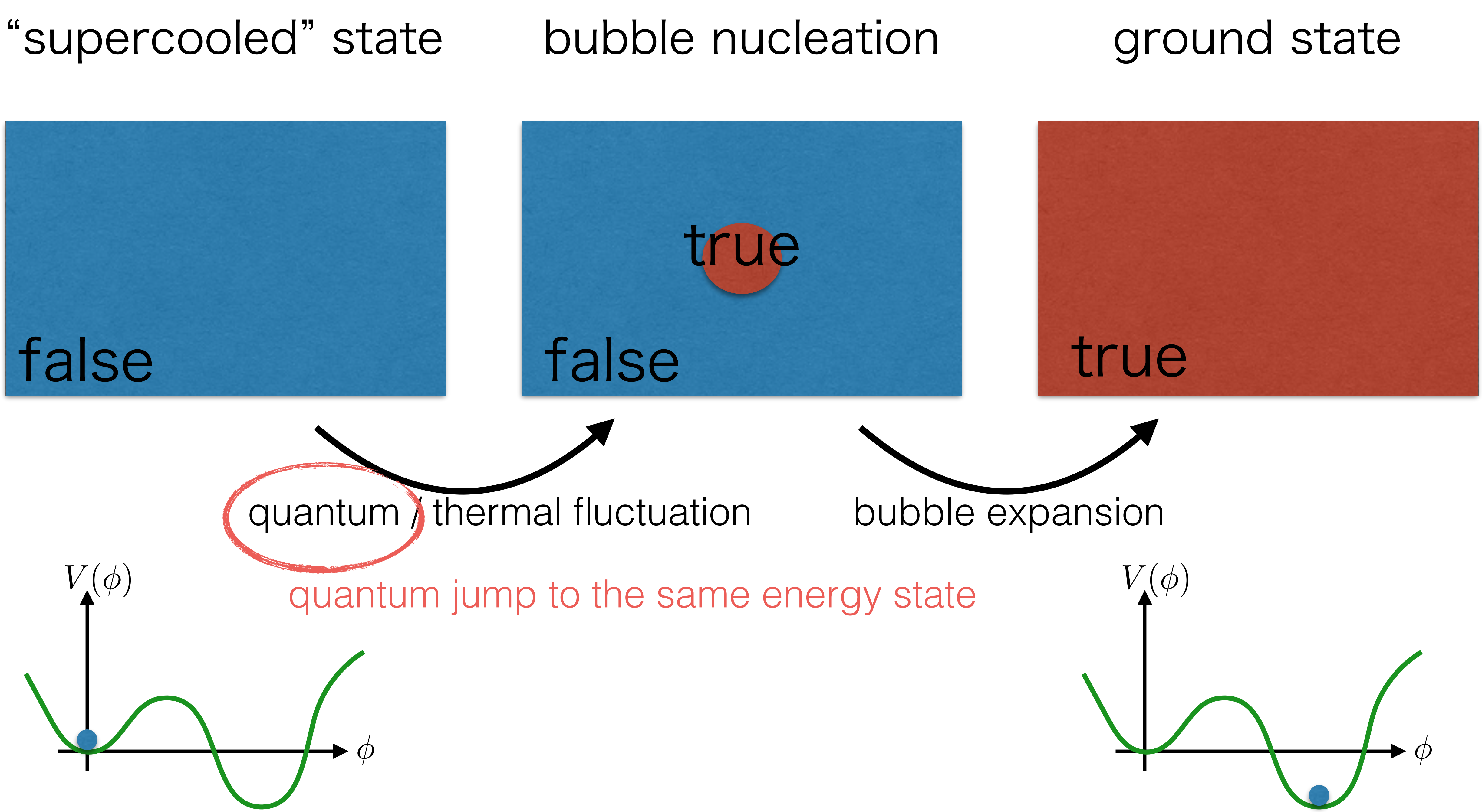
<u>Effective potential</u>

ex.) Standard Model



D. Buttazzo, et.al.,
1307.3536/hep-ph

Decay rate



"supercooled" state
bubble nucleation
ground state
false
true
false
true
quantum / thermal fluctuation
bubble expansion
quantum jump to the same energy state
$V(\phi)$
ϕ
$V(\phi)$
ϕ

Decay rate

<u>Bubble nucleation rate</u>

$$\gamma = Ae^{-B}$$

"WKB" in QM

dim. = 4

1/(time*volume)

bounce action

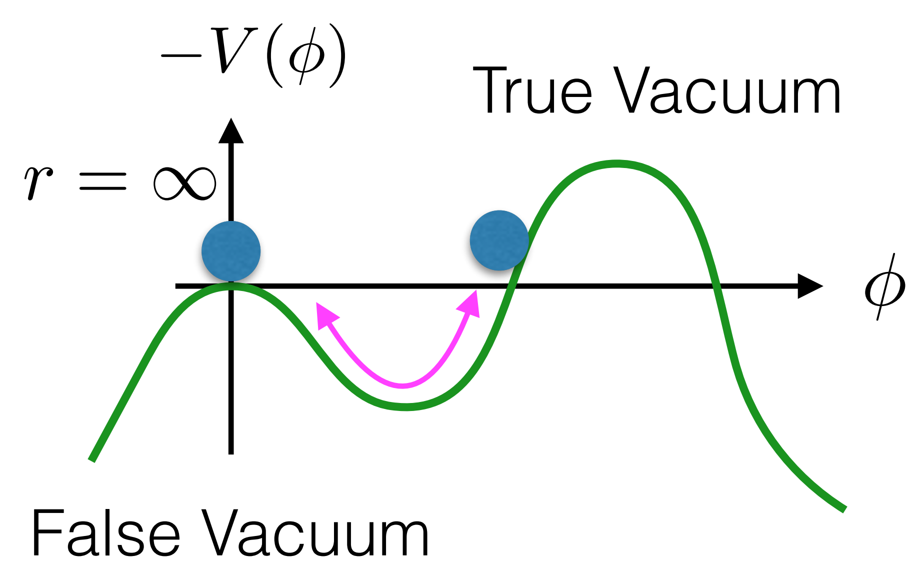
$$B = S(\phi_B)$$

Pre-exponential factor

$$A \sim \mu^4$$

typical scale

bounce solution



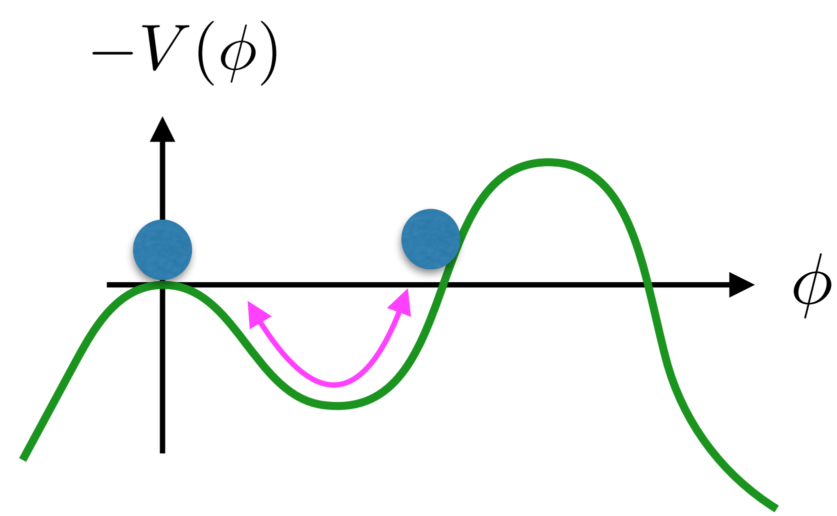
O(4) symmetric classical solution

$$\partial^2 \phi = V'(\phi)$$

Toy model

Potential

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$



Bounce

$$\partial^2\phi = V'(\phi)$$

Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B)\right]$$

Decay rate

$$\gamma \simeq m^4 e^{-B}$$

Toy model

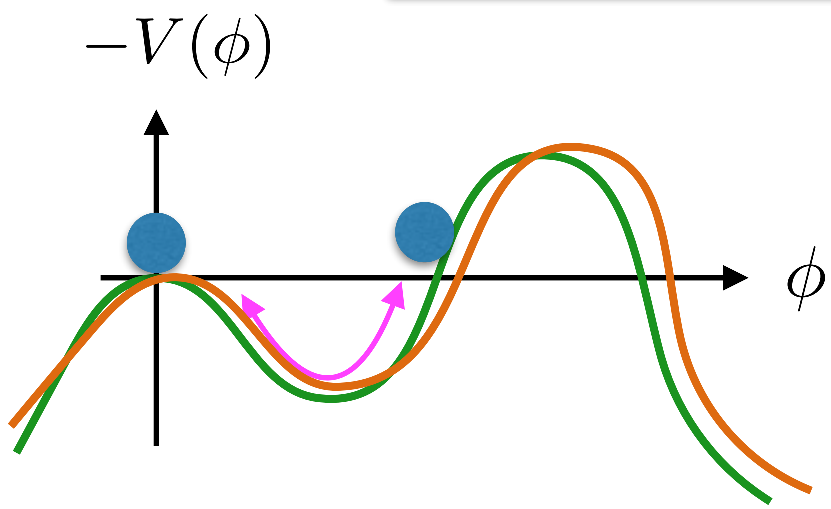
Potential

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Scale dependent

Bounce

$$\partial^2\phi = V'(\phi)$$



Action

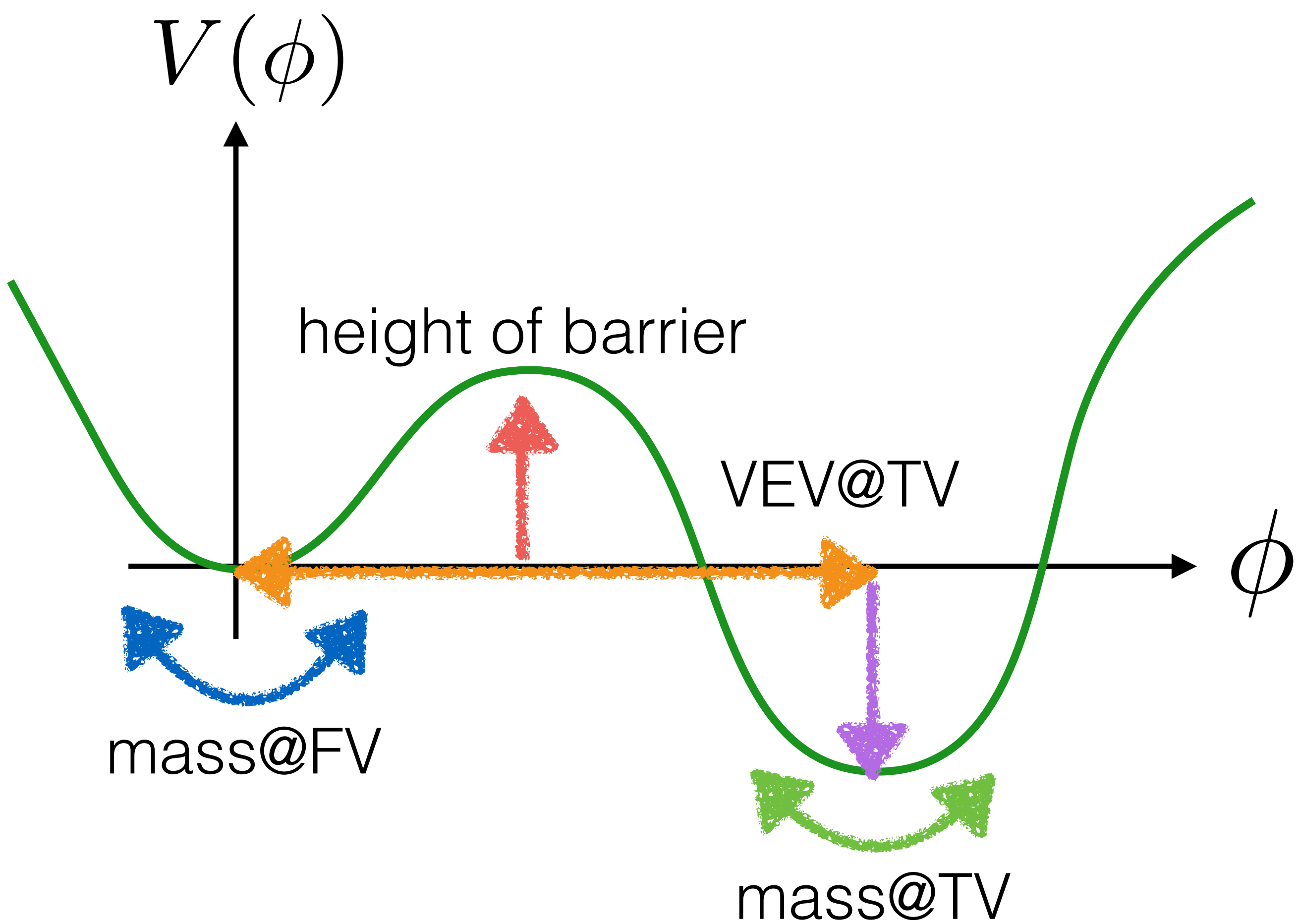
$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B)\right]$$

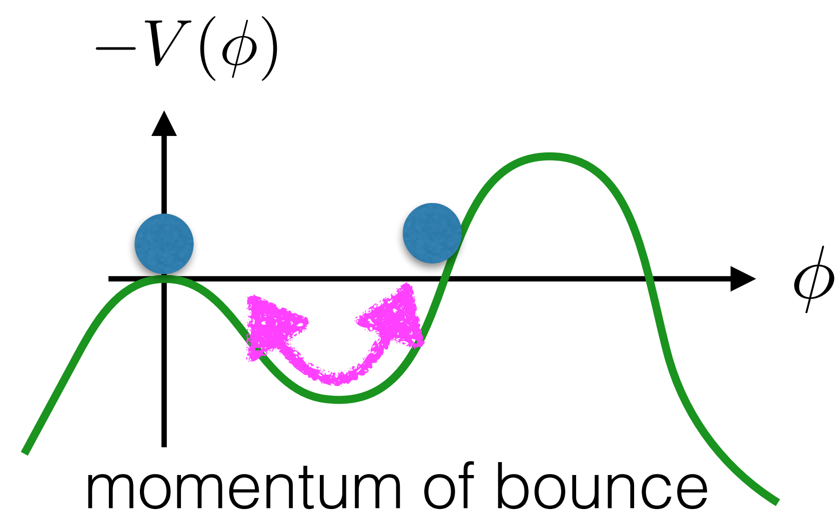
Decay rate

$$\gamma \simeq m^4 e^{-B}$$

OK, use the “typical” scale!

Renormalization scale





OK, use the “typical” scale!

But, I don’t know what is the best scale.

Does it change so much?

How large is the scale dependence?

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Beta functions

$$\beta_t = \frac{3Am^2}{16\pi^2} \qquad \beta_{m^2} = \frac{3}{16\pi^2}(\alpha m^2 + 3A^2)$$

$$\beta_A = \frac{9\alpha A}{16\pi^2} \qquad \beta_\alpha = \frac{9\alpha^2}{16\pi^2}$$

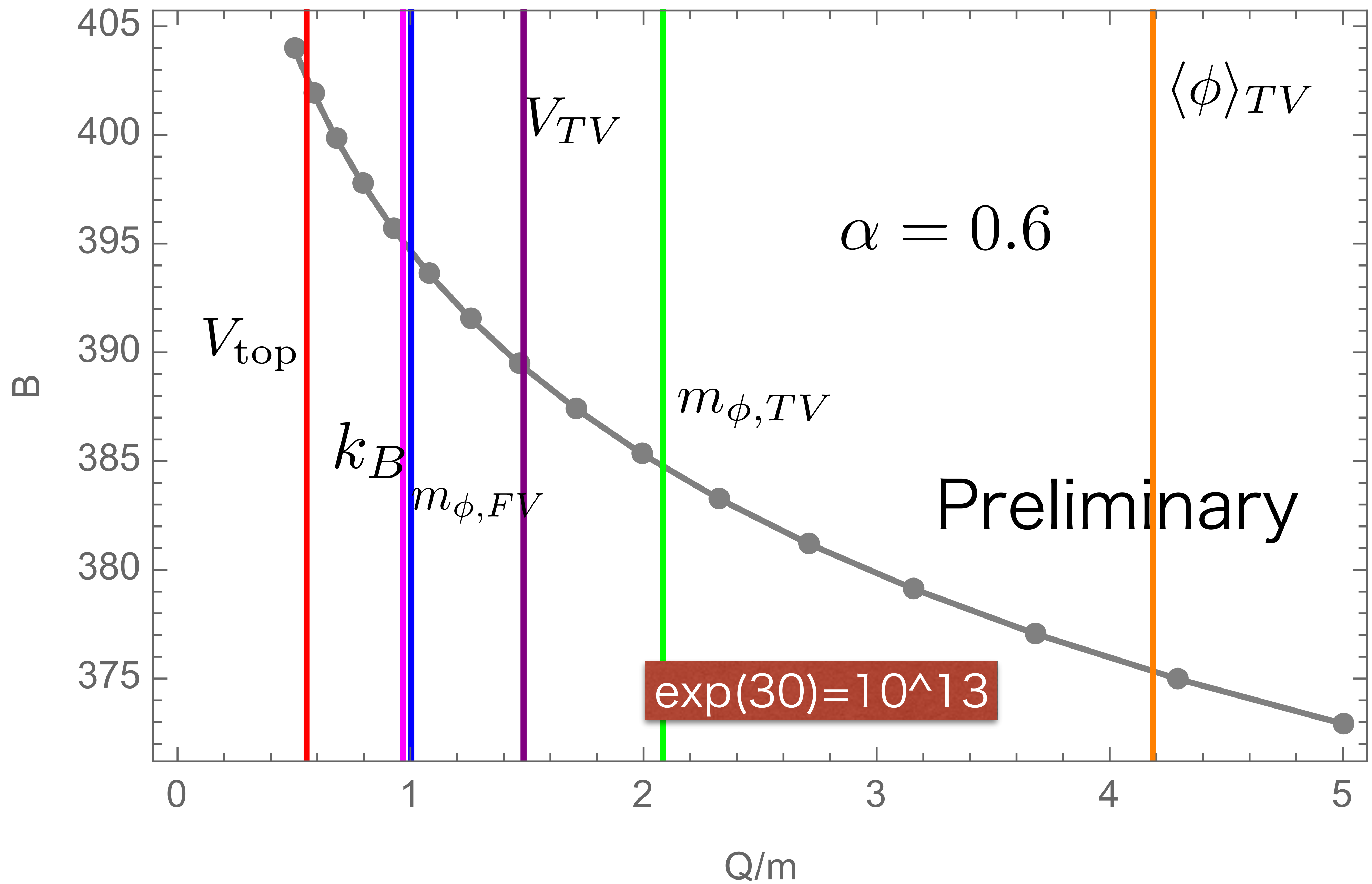
Renormalization conditions

@ $Q = m$

$$\bar{m}^2(m) = m^2,\ \bar{A}(m) = m,\ \bar{t}(m) = 0,\ \bar{\alpha}(m) = \alpha$$

How large is the scale dependence?

CosmoTransitions



can be much larger in a realistic model (top loop)

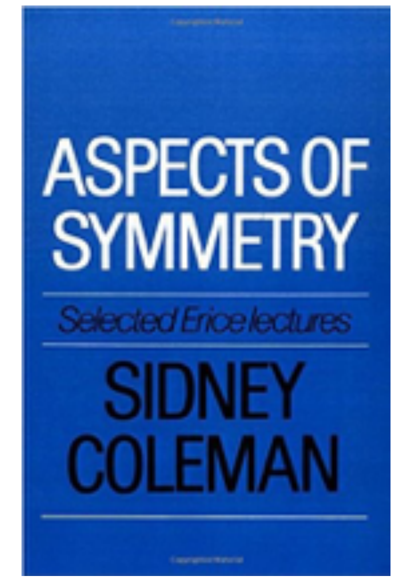
Calculation of the 1-loop factor

Pre-exponential factor

fluctuations around the bounce

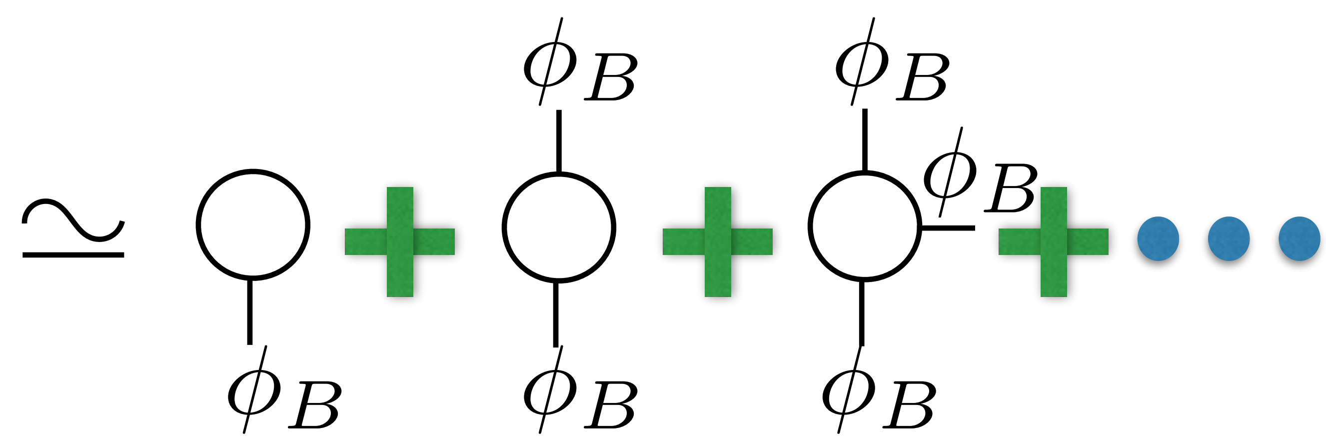
$$\gamma = Ae^{-B}$$

$$A = \frac{B^2}{4\pi^2}\left(\frac{\det' S''|_{\text{Bounce}}}{\det S''|_{\text{False}}}\right)^{-1/2}$$

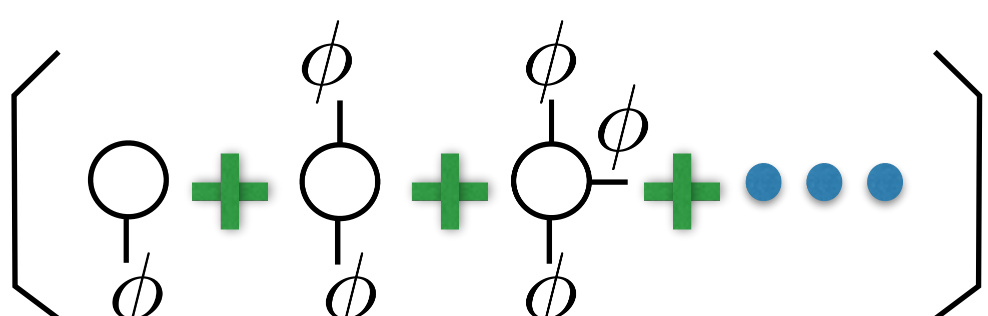


<u>Expectation</u>

cancellation of
the scale dependence
@1-loop



cf.) RGE is related to

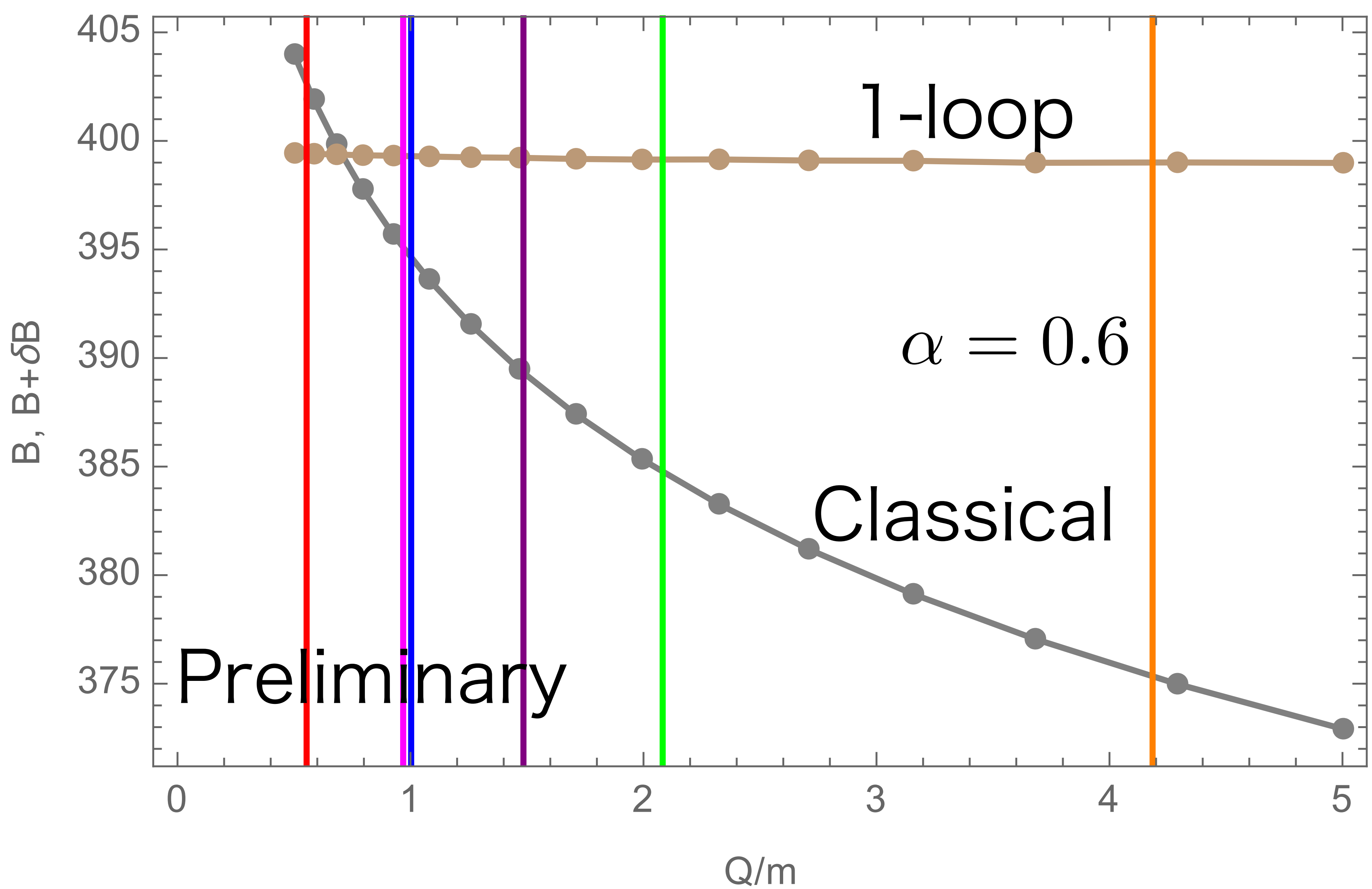


divergent part

After a complicated calculation…

Result

$$\gamma = Ae^{-B} \equiv m^4 e^{-B-\delta B}$$



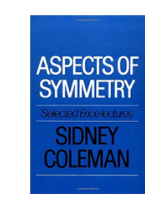
“ODE technique”

function of the bounce solution

$$\gamma = Ae^{-B}$$

$$\ln A^{-2} = \ln \frac{\det\left[-\partial^2 + m_0^2 + \delta W\right]}{\det\left[-\partial^2 + m_0^2\right]}$$

<u>Theorem</u> (Dirichlet BC) (J. H. van Vleck, ’28; R. H. Cameron and W. T. Martin, ’45; …)



The ratio of the determinant of differential operators,

$$L_j = -\frac{d^2}{dx^2} + R_j(x) \quad \text{on} \quad I = [0, 1]$$

w/ Diriclet BC $f(0) = 0,\ f(1) = 0$

is given by the ratio of the solutions of differential equations

$$\frac{\mathrm{Det} L_1}{\mathrm{Det} L_0} = \frac{y_1(1)}{y_0(1)} \qquad L_j y_j(x) = 0$$

$$y_j(0) = 0,\ y_j'(0) = 1$$

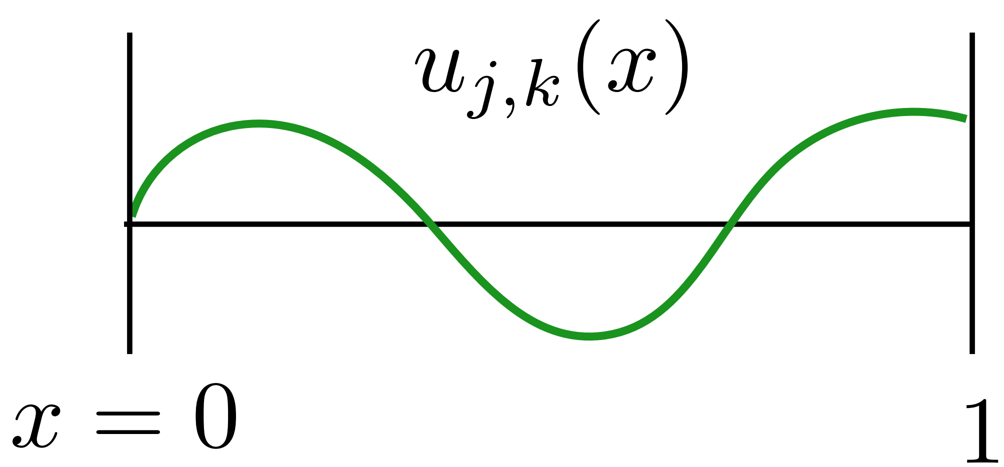
Proof

by K. Kirsten and A. J. McKane, '03

Differential eq.

$$\left(L_j - k^2\right) u_{j,k}(x) = 0$$

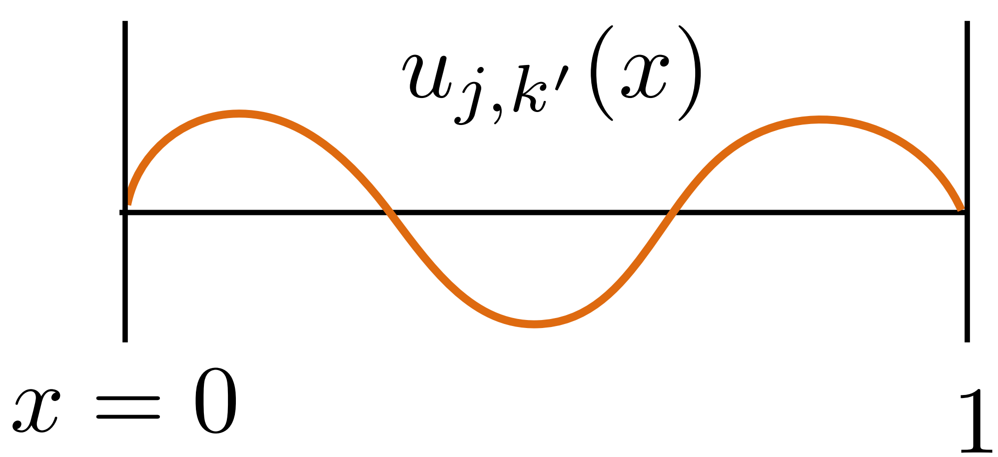
$$u_{j,k}(0) = 0, \;\; u'_{j,k}(0) = 1$$



$u_{j,k}(1) \neq 0$

does not satisfy the boundary conditions

➡ k^2 is not an eigenvalue of L_j

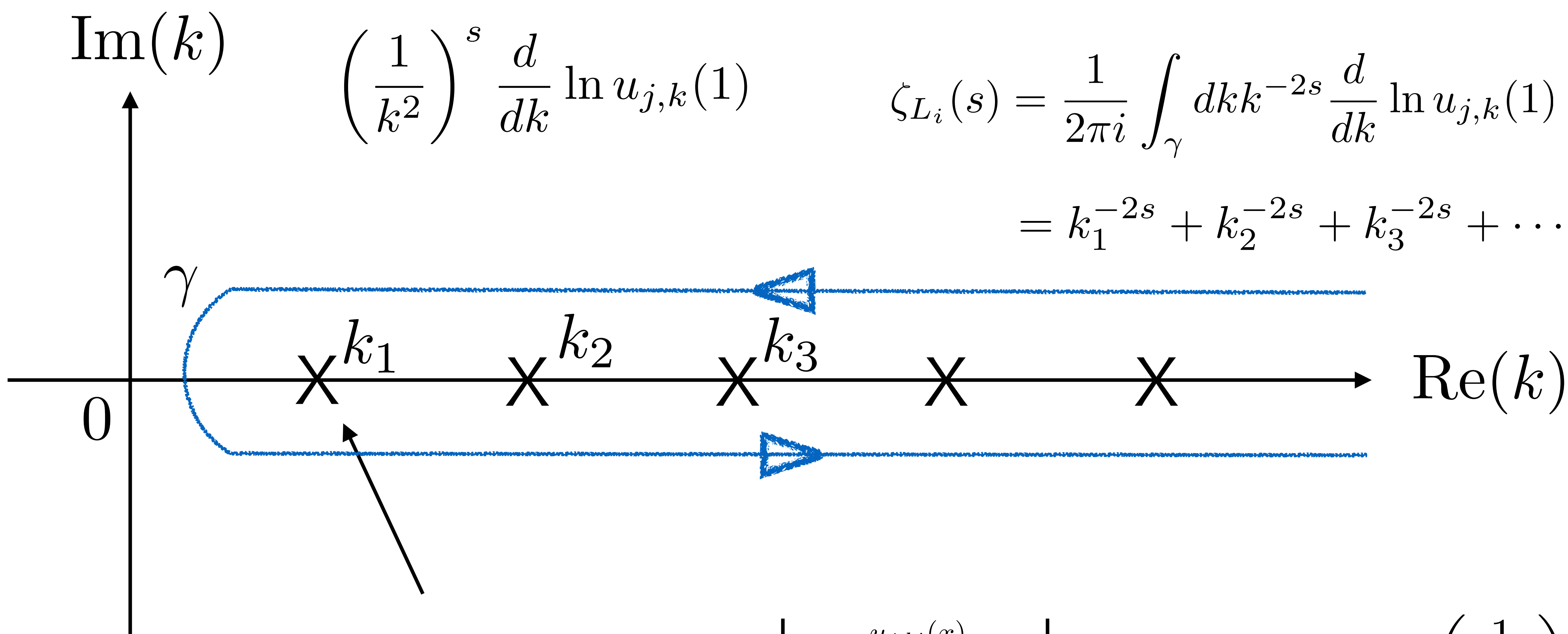


$u_{j,k'}(1) = 0$

satisfies the boundary conditions

➡ k'^2 is an eigenvalue of L_j

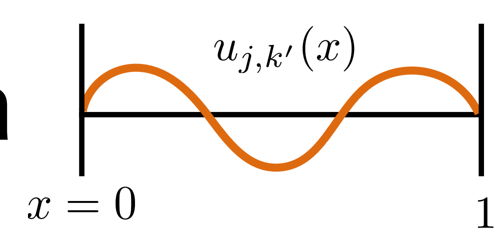
Proof



$$\zeta_{L_i}(s) = \frac{1}{2\pi i} \int_\gamma dk k^{-2s} \frac{d}{dk} \ln u_{j,k}(1)$$

$$= k_1^{-2s} + k_2^{-2s} + k_3^{-2s} + \cdots$$

Has a pole when

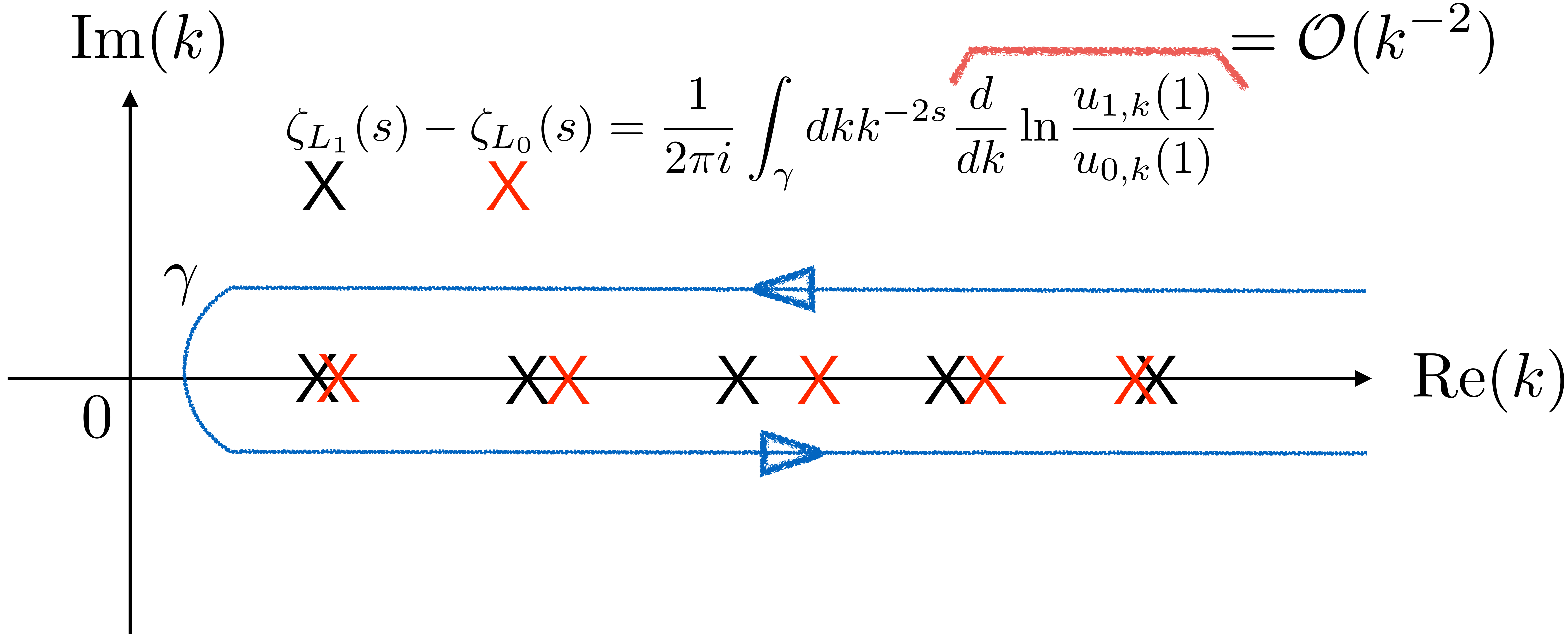


w/ residue $\left(\frac{1}{k'^2}\right)^s$

Proof

$$\zeta_{L_1}(s) - \zeta_{L_0}(s) = \frac{1}{2\pi i}\int_\gamma dk k^{-2s}\frac{d}{dk}\ln\frac{u_{1,k}(1)}{u_{0,k}(1)}$$

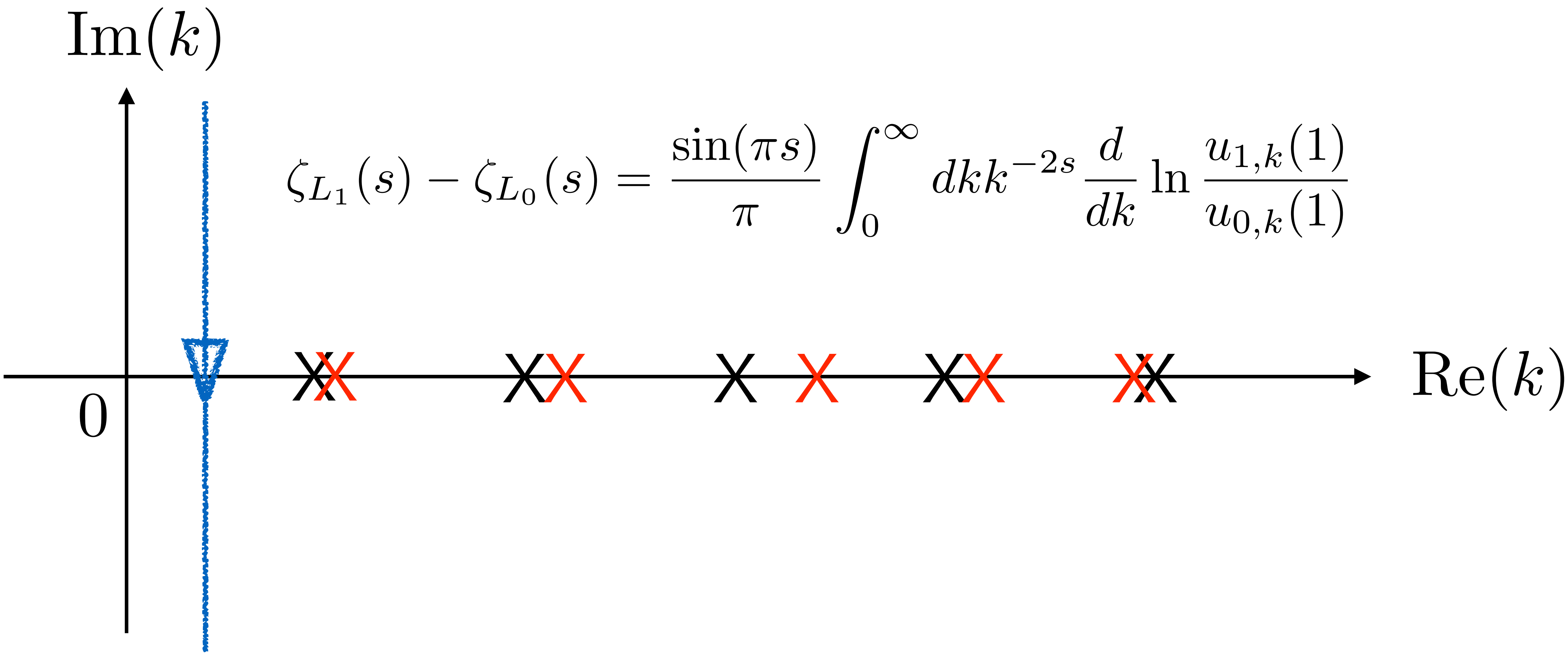
$\ln\frac{u_{1,k}(1)}{u_{0,k}(1)} \overset{|k|\to\infty}{=} \mathcal{O}(k^{-2})$



cf.)

$$\left[-\frac{d^2}{dx^2} - k^2\left(1 - \frac{R_j(x)}{k^2}\right)\right]u_{j,k}(x) = 0$$

Proof



$$\zeta_{L_1}(s) - \zeta_{L_0}(s) = \frac{\sin(\pi s)}{\pi} \int_0^\infty dk k^{-2s} \frac{d}{dk} \ln \frac{u_{1,k}(1)}{u_{0,k}(1)}$$

Proof

eigenvalues

$L_1 : k_i^2$

$L_0 : p_i^2$

$$\zeta_{L_1}(s) - \zeta_{L_0}(s) = \frac{\sin(\pi s)}{\pi} \int_0^\infty dk k^{-2s} \frac{d}{dk} \ln \frac{u_{1,k}(1)}{u_{0,k}(1)}$$

$$= k_1^{-2s} - p_1^{-2s} + k_2^{-2s} - p_2^{-2s} + \cdots$$

$$\frac{\mathrm{Det} L_1}{\mathrm{Det} L_0} = \frac{k_1^2 k_2^2 \cdots}{p_1^2 p_2^2 \cdots} = e^{-[\zeta'_{L_1}(0) - \zeta'_{L_0}(0)]} = \frac{u_{1,0}(1)}{u_{0,0}(1)}$$

Q.E.D.

All that we need to solve

$$\frac{\mathrm{Det} L_1}{\mathrm{Det} L_0} = \frac{u_{1,0}(1)}{u_{0,0}(1)} \qquad L_j u_{j,0}(x) = 0$$

$$u_{j,0}(0) = 0, \; u'_{j,0}(0) = 1$$

Renormalization

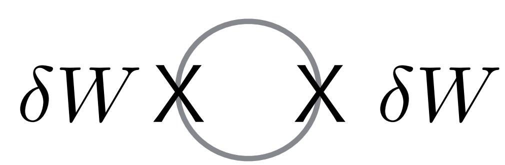
$$\gamma = Ae^{-B}$$

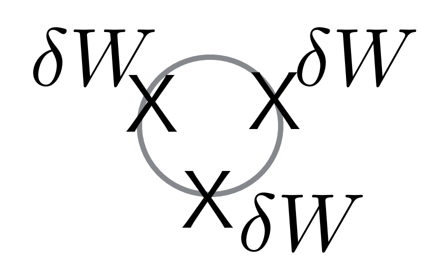
function of the bounce solution

$$\ln A^{-2} = \ln \frac{\det\left[-\partial^2 + m_0^2 + \delta W\right]}{\det\left[-\partial^2 + m_0^2\right]}$$

δ W expansion

$$= \mathrm{Tr}\left[(-\partial^2 + m_0^2)^{-1}\delta W - \frac{1}{2}(-\partial^2 + m_0^2)^{-1}\delta W(-\partial^2 + m_0^2)^{-1}\delta W + \cdots\right]$$





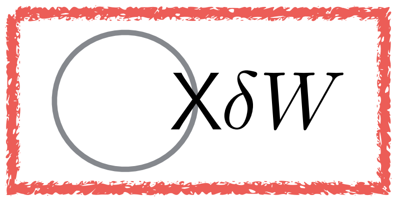
divergent

Analytical calculation (MS-bar)

convergent

ODE technique

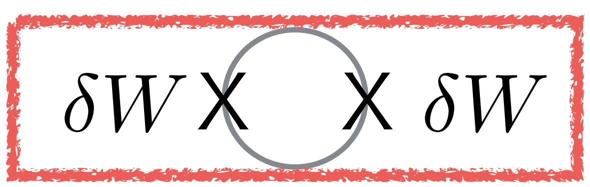
Analytical calculation (MS-bar)



$$\mathrm{Tr}\left[(-\partial^2+m_0^2)^{-1}\delta W\right]$$

$$=-\frac{m_0^2}{16\pi^2}\widetilde{\delta W}(0)\left[\frac{1}{\bar\epsilon}+1-\ln\frac{m_0^2}{\mu^2}\right]$$

↑ Fourier transform



$$\mathrm{Tr}\left[-\frac{1}{2}(-\partial^2+m_0^2)^{-1}\delta W(-\partial^2+m_0^2)^{-1}\delta W\right]$$

$$=-\frac{1}{256\pi^4}\int d|p||p|^3[\widetilde{\delta W}(p)]^2\left[\frac{1}{\bar\epsilon}+2-\ln\frac{m_0^2}{\mu^2}-\frac{\sqrt{|p|^2+4m_0^2}}{|p|}\ln\frac{\sqrt{|p|^2+4m_0^2}+|p|}{\sqrt{|p|^2+4m_0^2}-|p|}\right]$$

How can we match the analytical and the ODE results?

$$\ln A^{-2} = \ln \frac{\det\left[-\partial^2 + m_0^2 + \delta W\right]}{\det\left[-\partial^2 + m_0^2\right]} = \sum_{\ell=0}^{\infty} (\ell+1)^2 \lim_{r\to\infty} \ln \frac{\phi_\ell}{\phi_\ell^{(0)}}$$

$$\phi_\ell(x) = \Phi(r) Y_{\ell,m,m'}(\theta)$$

diverge after summing over ℓ

δW expansion of the ODE results

$$[-\partial^2 + m_0^2 + \delta W]\phi_\ell(x) = 0$$

$$\phi_\ell = \phi_\ell^{(0)} + \phi_\ell^{(1)} + \phi_\ell^{(2)} + \cdots$$

$$[-\partial^2 + m_0^2]\phi_\ell^{(0)}(x) = 0$$

$$[-\partial^2 + m_0^2]\phi_\ell^{(1)}(x) = -\delta W \phi_\ell^{(0)}(x)$$

$$[-\partial^2 + m_0^2]\phi_\ell^{(2)}(x) = -\delta W \phi_\ell^{(1)}(x)$$

$$\vdots$$

How can we match the analytical and the ODE results?

$$\ln A^{-2} = \ln \frac{\det\left[-\partial^2 + m_0^2 + \delta W\right]}{\det\left[-\partial^2 + m_0^2\right]} = \sum_{\ell=0}^{\infty} (\ell+1)^2 \lim_{r\to\infty} \ln \frac{\phi_\ell}{\phi_\ell^{(0)}}$$

$$= \sum_{\ell=0}^{\infty} (\ell+1)^2 \lim_{r\to\infty} \left[\frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}} + \frac{\phi_\ell^{(2)}}{\phi_\ell^{(0)}} - \frac{1}{2}\left(\frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}}\right)^2 + \cdots \right]$$

δ W expansion

$$= \mathrm{Tr}\left[(-\partial^2 + m_0^2)^{-1}\delta W - \frac{1}{2}(-\partial^2 + m_0^2)^{-1}\delta W (-\partial^2 + m_0^2)^{-1}\delta W + \cdots \right]$$

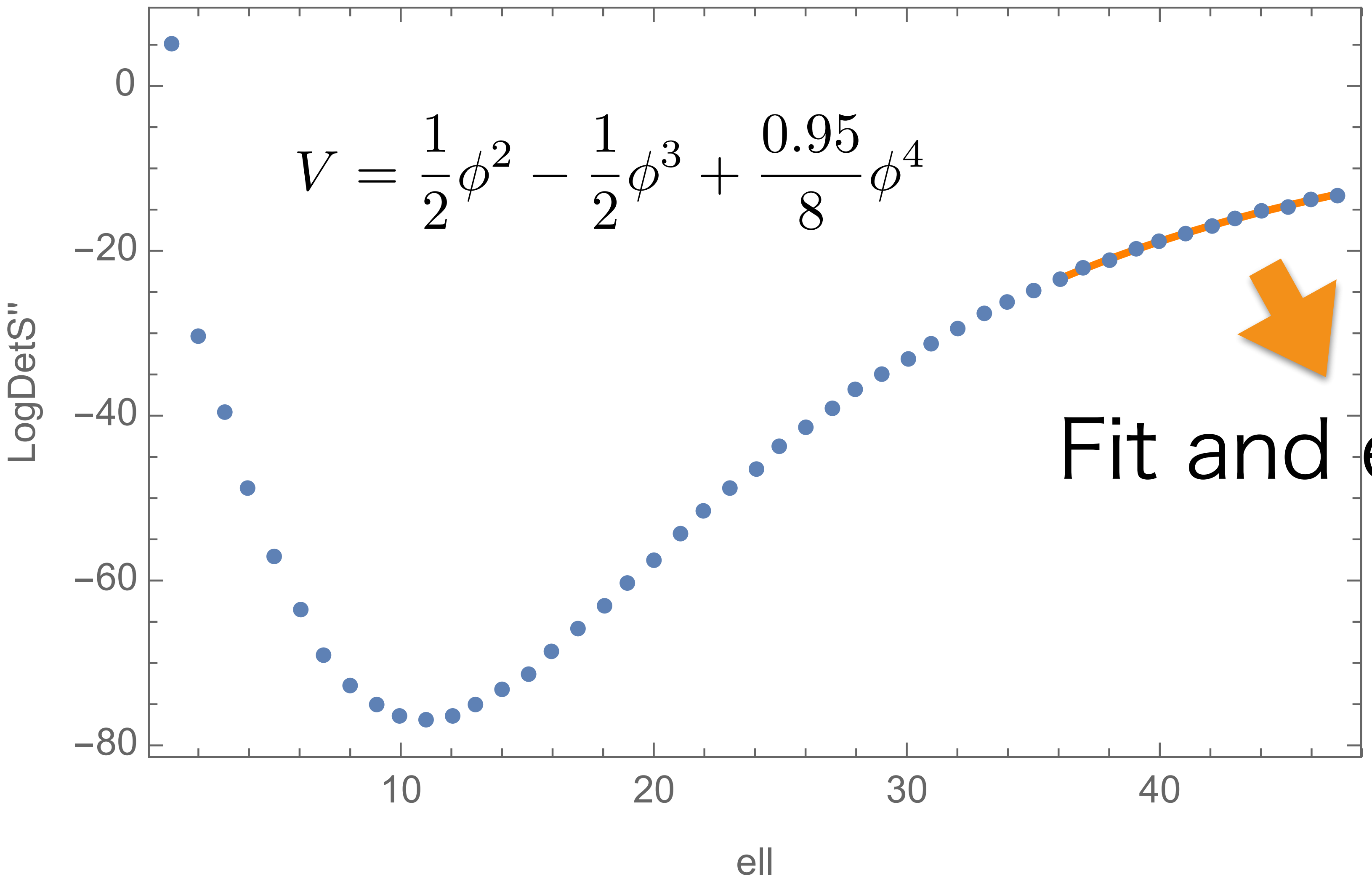
δ W expansion

Finite numerical result

$$\ln A^{-2}|_{\delta W^3, \delta W^4, \ldots} = \sum_{\ell=0}^{\infty} (\ell+1)^2 \lim_{r\to\infty} \left[\ln \frac{\phi_\ell}{\phi_\ell^{(0)}} - \frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}} - \frac{\phi_\ell^{(2)}}{\phi_\ell^{(0)}} + \frac{1}{2}\left(\frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}}\right)^2 \right]$$

Example

$$\ln A^{-2}|_{\delta W^3, \delta W^4, \ldots} = \sum_{\ell=0}^{\infty} (\ell+1)^2 \lim_{r\to\infty} \left[\ln \frac{\phi_\ell}{\phi_\ell^{(0)}} - \frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}} - \frac{\phi_\ell^{(2)}}{\phi_\ell^{(0)}} + \frac{1}{2}\left(\frac{\phi_\ell^{(1)}}{\phi_\ell^{(0)}}\right)^2 \right]$$

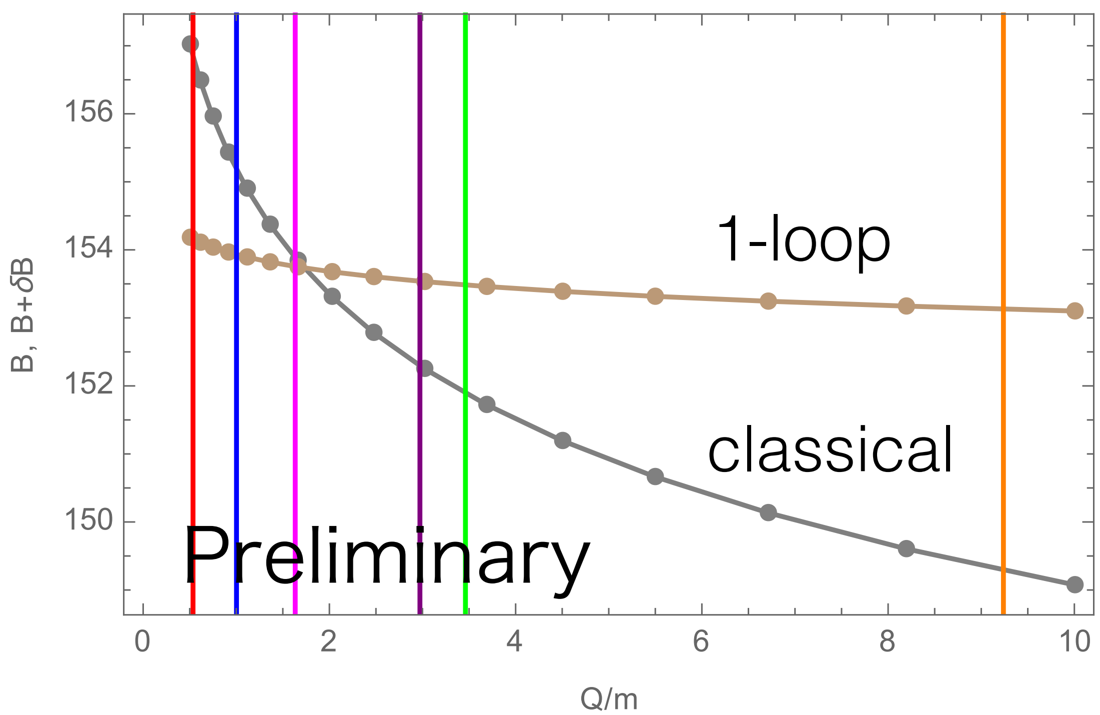


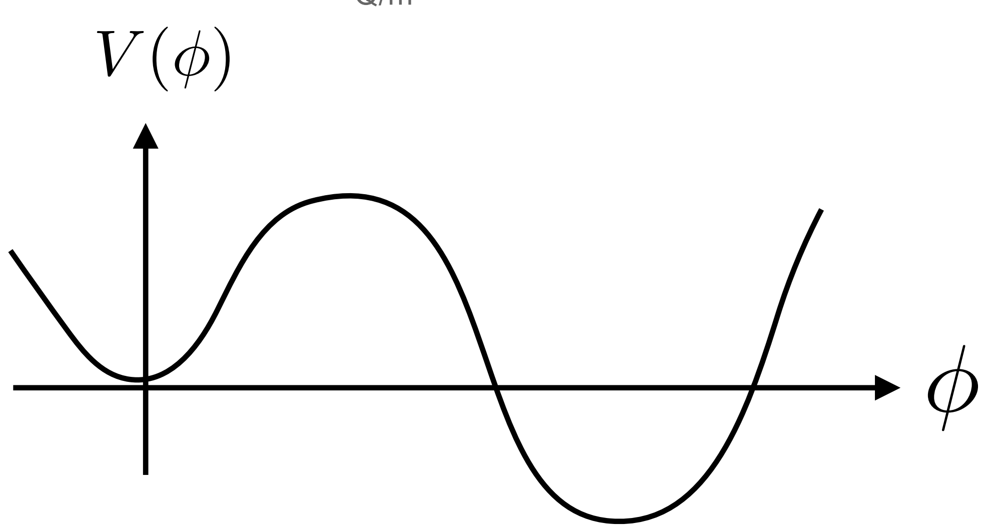
Fit and extrapolate

Results

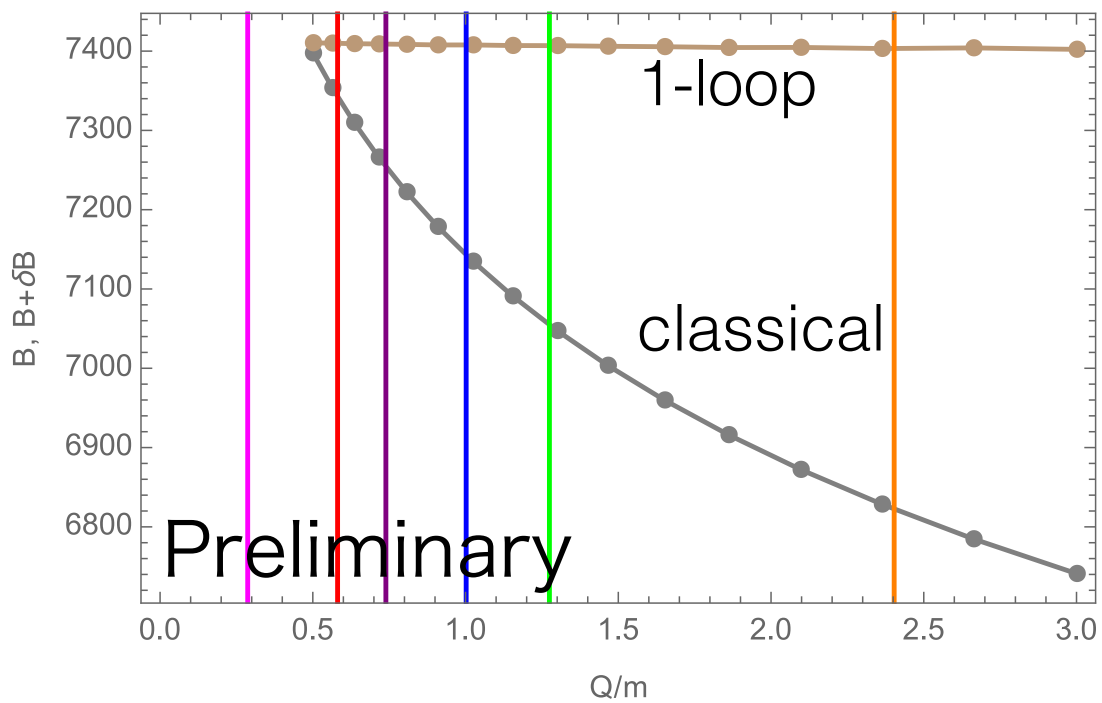
$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

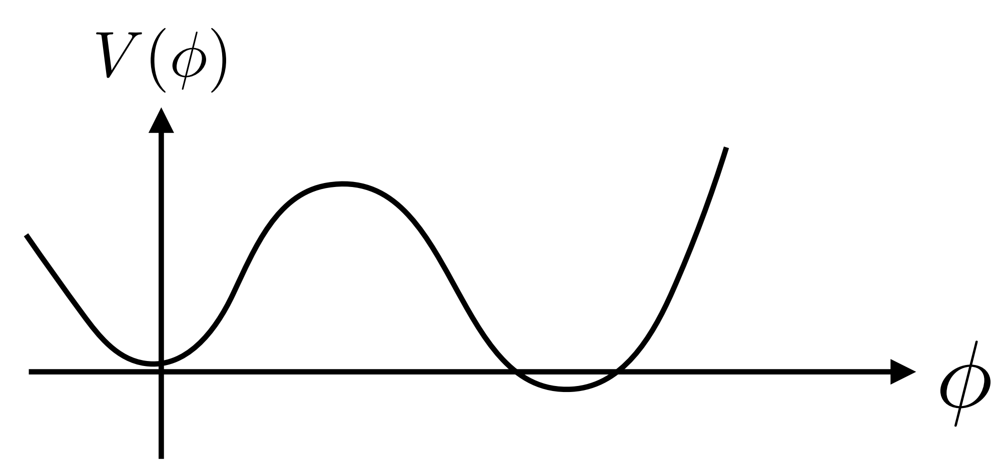
$\alpha = 0.3$





$\alpha = 0.9$





Comments

The bounce is calculated with `CosmoTransitions`, (C. L. Wainwright) which can deal with multiple fields.

We correctly subtracted the zero modes corresponding to the translational invariance.

Fermion determinant becomes more complicated because of the mixing between different ℓ states.

SM + stau system

Light stau

Stau can be light

$$m_{\tilde{\tau}} > 103.5\mathrm{GeV}$$ (LEP)

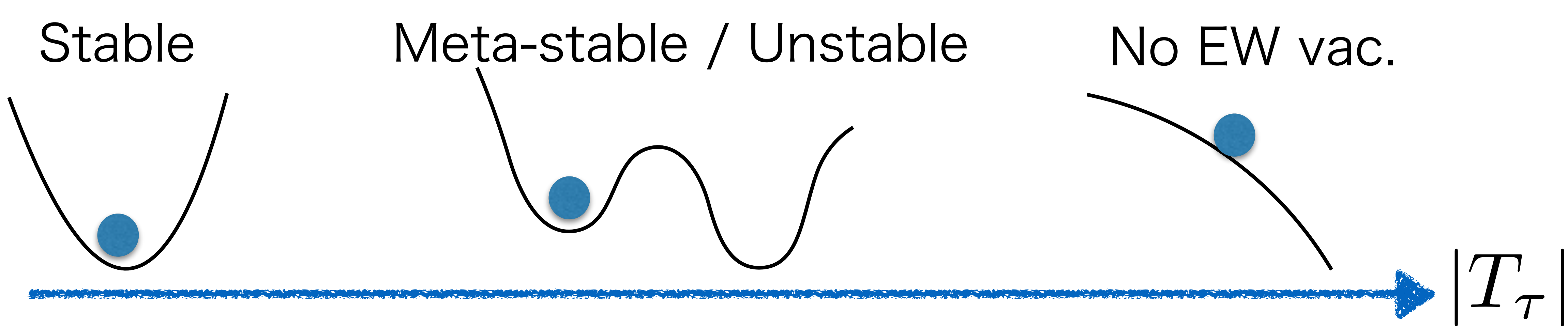
h γ γ coupling, co-annihilation with bino, ⋯

But, the potential may become unstable towards the stau direction

$$V = T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + h.c.) + m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 + \cdots$$

$$T_\tau = y_\tau (A_\tau - \mu \tan\beta)$$

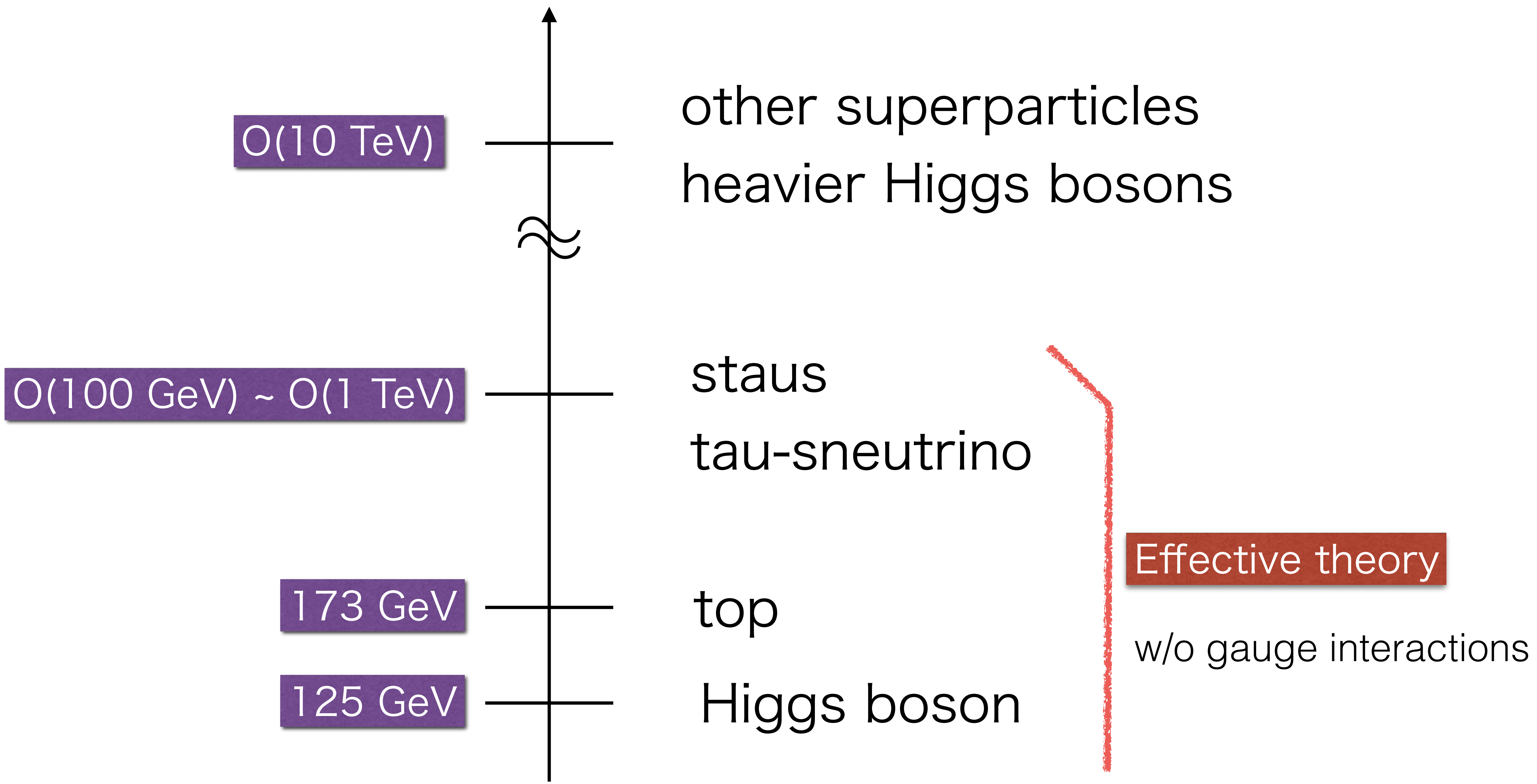
$$\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$$



Spectrum

For simplicity,

we assume only the staus are light



Effective theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm kin} - y_t(Hq_Lt_R^c + {\rm h.c.}) - m_H^2|H|^2 - \frac{1}{4}\lambda_H|H|^4$$
$$- m_{\tilde{\ell}_L}^2|\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2|\tilde{\tau}_R|^2 - T_\tau(H^\dagger\tilde{\ell}_L\tilde{\tau}_R^* + {\rm h.c.}) - \frac{1}{4}\kappa^{(1)}|\tilde{\ell}_L|^4 - \frac{1}{4}\kappa^{(2)}|\tilde{\tau}_R|^4$$
$$- \frac{1}{4}\lambda^{(1)}|H|^2|\tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(2)}|H^\dagger\tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(3)}|H|^2|\tilde{\tau}_R|^2 - \frac{1}{4}\kappa^{(3)}|\tilde{\ell}_L|^2|\tilde{\tau}_R|^2,$$

Boundary conditions

EW scale

$$y_t = \frac{M_t}{v},$$
$$m_H^2(M_t) = -\frac{1}{2}M_h^2,$$
$$\lambda_H(M_h) = \frac{M_h^2}{2v^2},$$

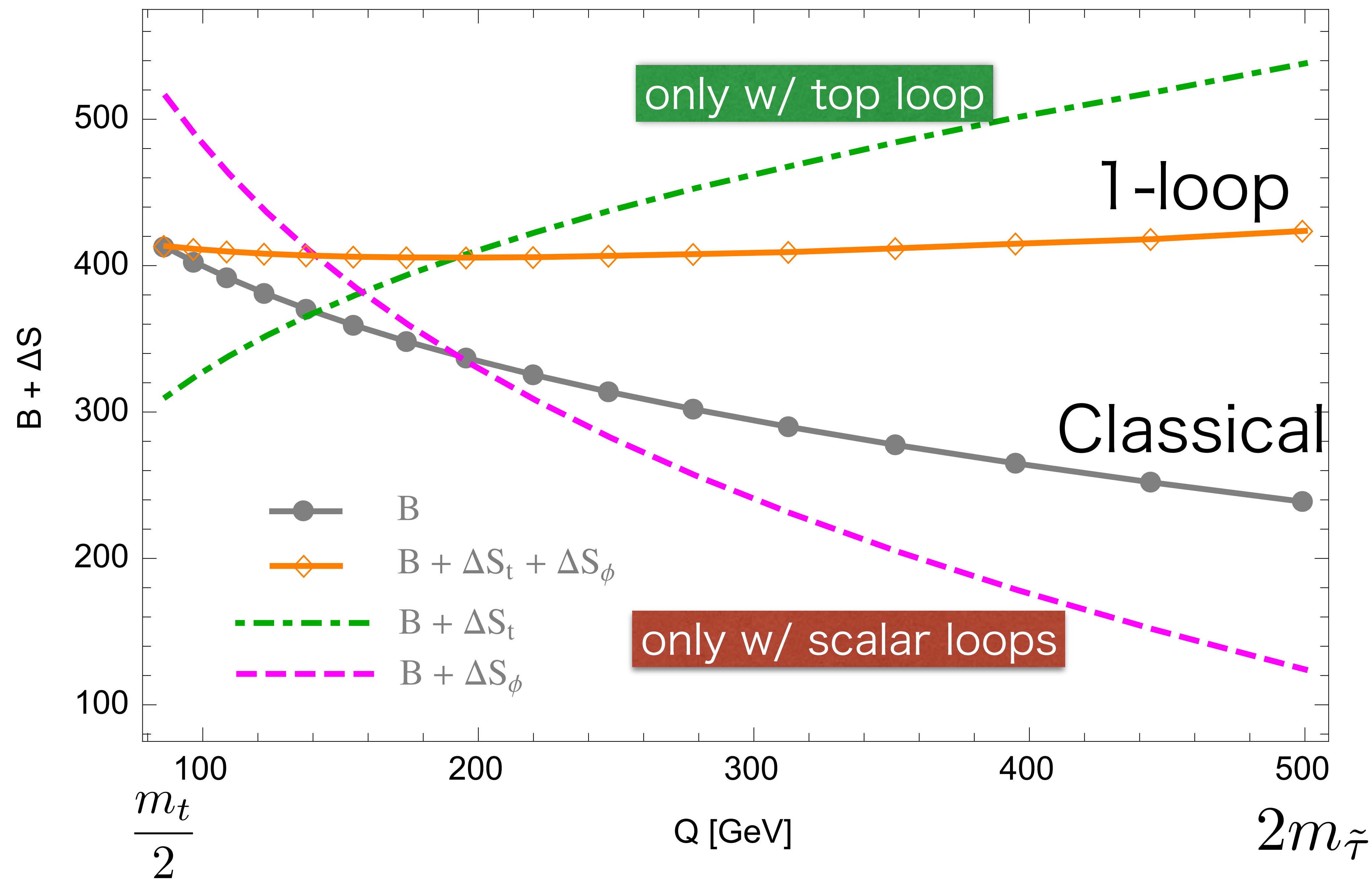
stau mass

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250\,{\rm GeV},$$
$$T_\tau = 300\,{\rm GeV}.$$

SUSY scale (10TeV)

$$\lambda^{(1)}(M_{\rm SUSY}) = (g^2 + g'^2)\cos 2\beta,$$
$$\lambda^{(2)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g^2\cos 2\beta,$$
$$\lambda^{(3)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g'^2\cos 2\beta,$$
$$\kappa^{(1)}(M_{\rm SUSY}) = \frac{1}{2}(g^2 + g'^2),$$
$$\kappa^{(2)}(M_{\rm SUSY}) = -\kappa^{(3)}(M_{\rm SUSY}) = 2g'^2,$$
$$\tan\beta = 20$$

Result


only w/ top loop
1-loop
Classical
only w/ scalar loops
B
$B + \Delta S_t + \Delta S_\phi$
$B + \Delta S_t$
$B + \Delta S_\phi$
$B + \Delta S$
500
400
300
200
100
100
200
300
400
500
Q [GeV]
$\frac{m_t}{2}$
$2m_{\tilde{\tau}}$

Summary

- The bubble nucleation rate has often been estimated without calculating the pre-exponential factor.
- This estimate involves uncertainty in the renormalization scale, which, we showed, results in O(10%) uncertainty in the exponent of the bubble nucleation rate.
- To reduce the uncertainty, we explicitly calculated the pre-exponential factor and showed that it is greatly reduced.
- Scalars and fermions have already been implemented, but the gauge bosons are now ongoing.