Effective field theory approach for spacetime symmetry breaking

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mainly based on arXiv:1412.5601

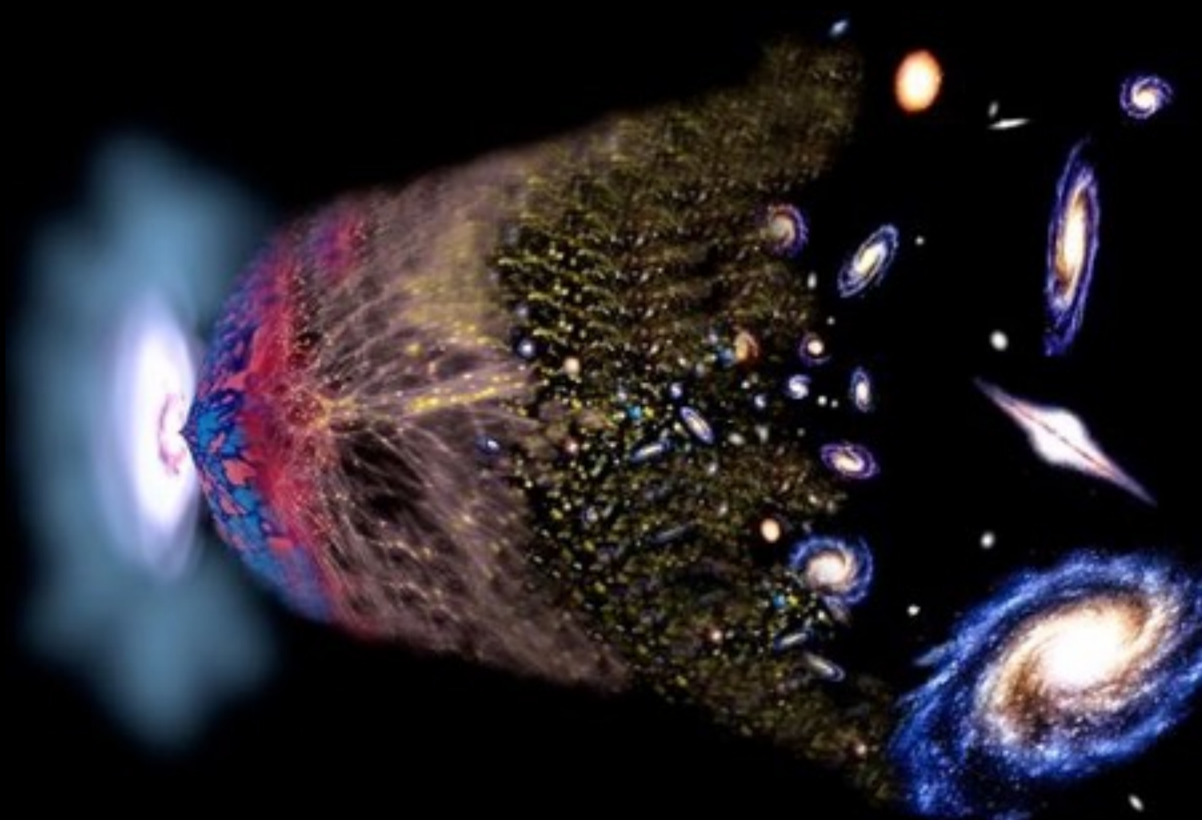
with Y. Hidaka (RIKEN) and G. Shiu (Wisconsin&HKUST)

@阪大 April 7th 2015

1. Introduction

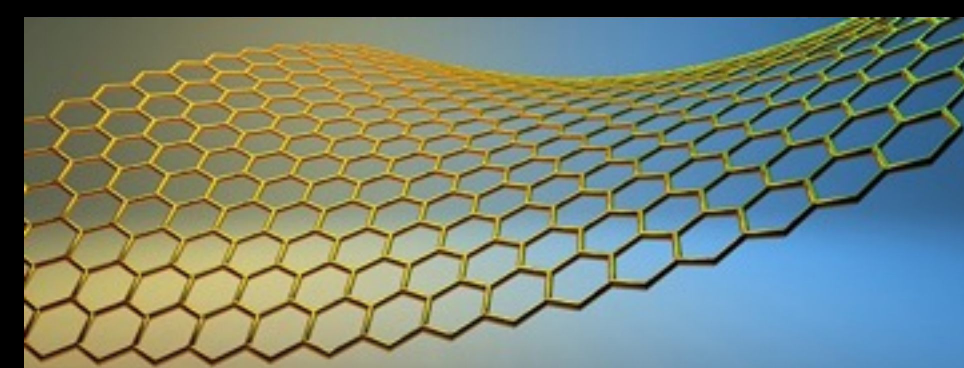
spacetime symmetry breaking

cosmology

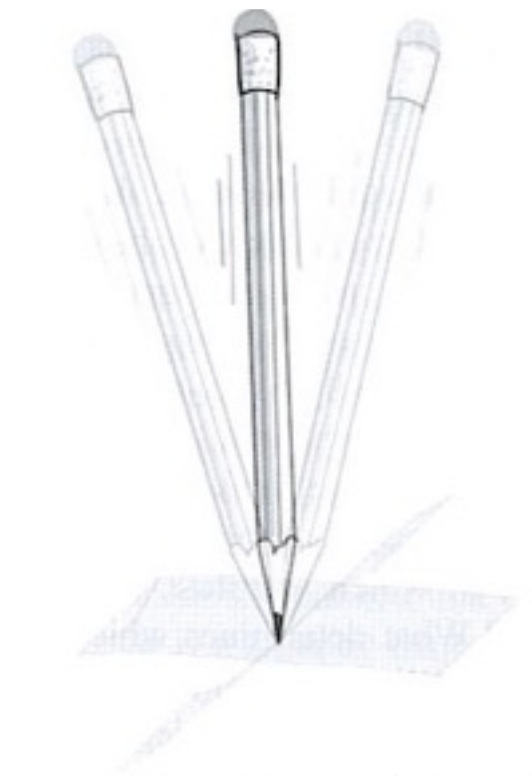


time-translation

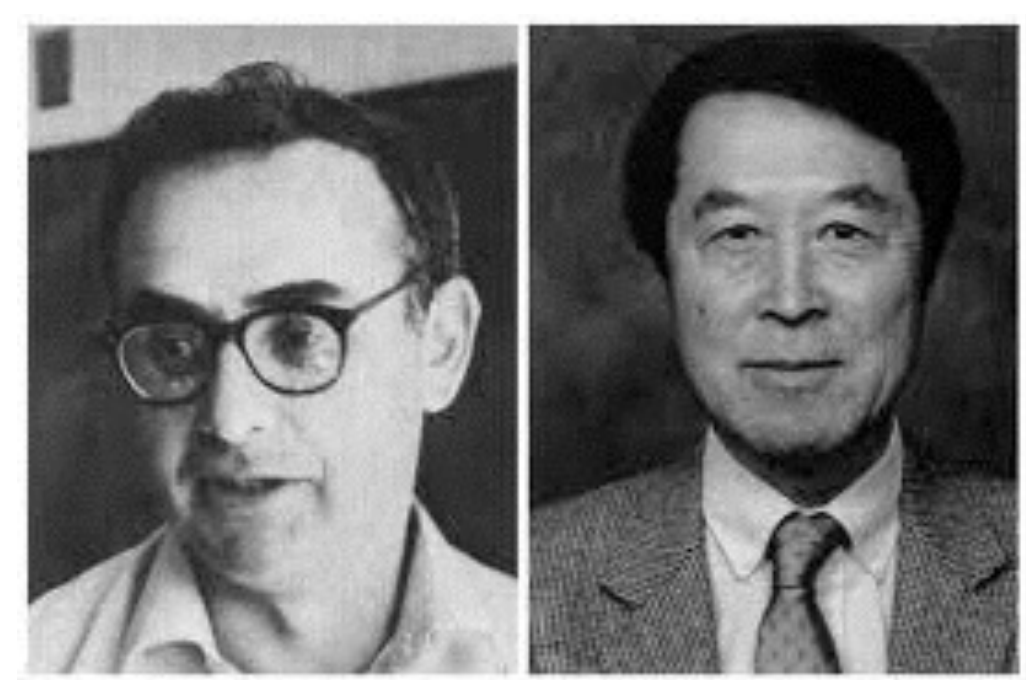
condensed matter



Poincare symmetry



Nambu-Goldstone (NG) modes

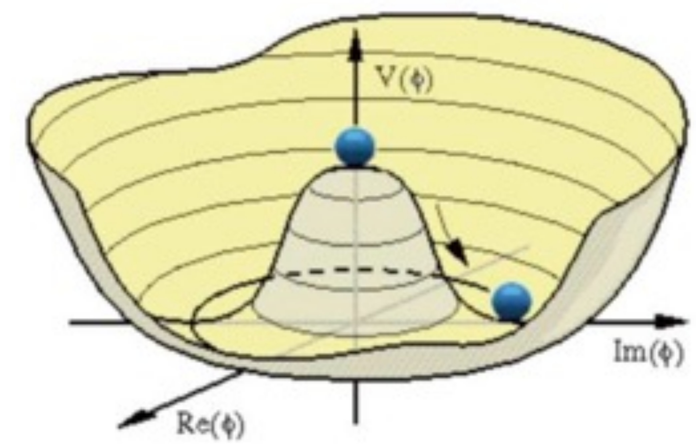


Nambu-Goldstone theorem

breaking of continuum symmetry

$\longrightarrow$ gapless mode (NG mode)

nonlinear rep of broken sym



internal symmetry in relativistic system

one broken symmetry $\longleftrightarrow$ one gapless mode with dispersion relation $\omega = k$

ex. pion as NG boson

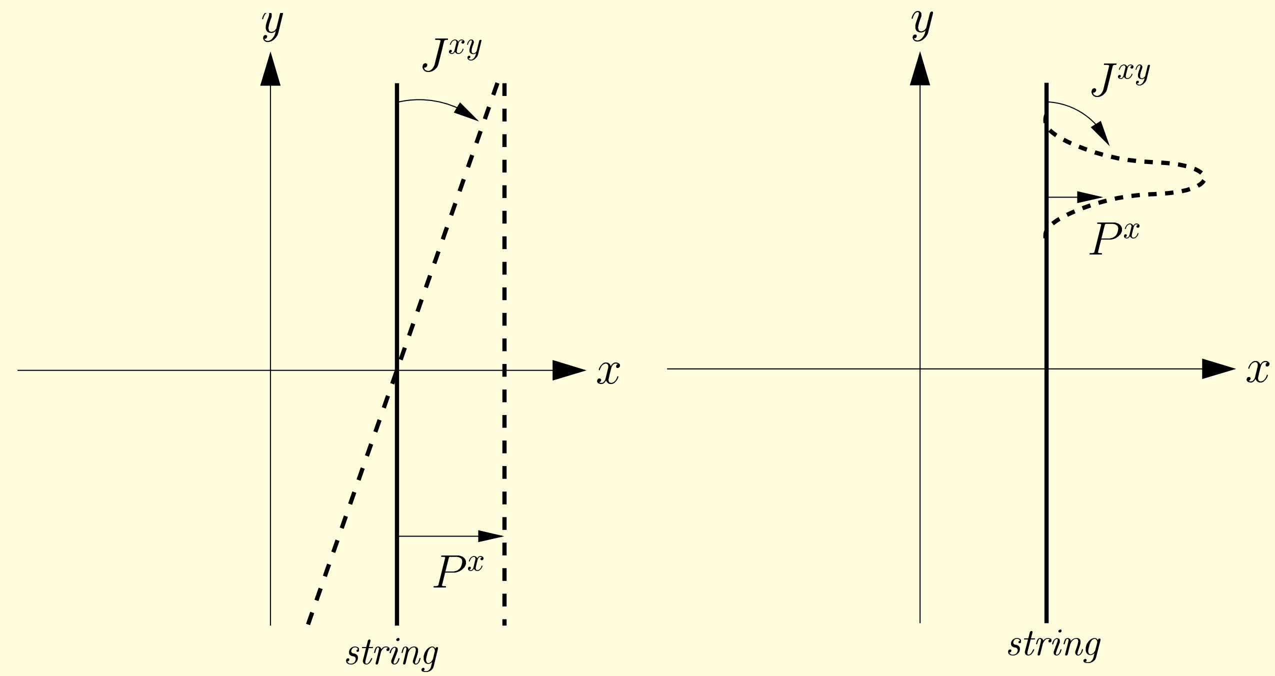
$$SU(2)_L \times SU(2)_R \to SU(2)_V \quad \longleftrightarrow \quad \pi^0,\ \pi^{\pm}$$

cf. recent progress in nonrela [Nambu '04, Hidaka '12, Watanabe-Murayama '12]

NG field for spacetime symmetry

\# mismatch in NG mode counting

fig. from Low-Manohar '02



ex.1 string [Low-Manohar '02]

broken symmetries:

rotation & translation

※ only one NG mode

ex.2 conformal symmetry

NG field for spacetime symmetry

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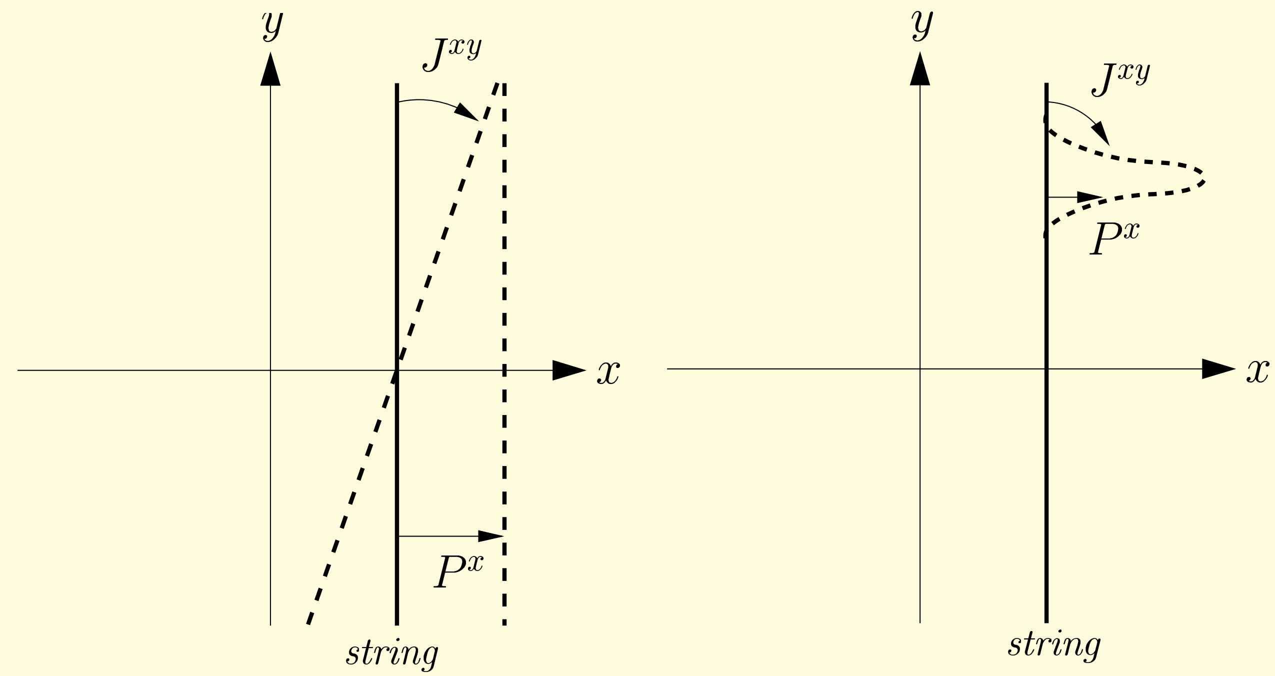

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broken symmetries:

rotation & translation

※ only one NG mode

ex.2 conformal symmetry

massive “NG” fields in a nonlinear rep

ex. smectic A phase of liquid crystals

- translation and rotational symmetries are broken

$$\pi \qquad\qquad \xi_{\hat{i}} \ \ (\hat{i}=1,2)$$

- $\xi_{\hat{i}}$: massive, in a nonlinear rep of rotation

coset construction for spacetime symmetry :

- based on global symmetry viewpoints
- remove redundant NG fields by inverse Higgs constraint
- recently revisited (ex. inverse Higgs [Low+ ’02, Endlich+ ’13, Brauner+ ’14], causality [Creminellli+ ’14], WZ terms [Delacretaz+ ’14])

in this talk, I discuss

effective theory from local symmetry point of view

- no redundant NG field from the beginning
- take into account massive “NG” fields also

plan of my talk:

1. Introduction ✔
2. Basic strategy
3. Single domain-wall
4. Physical applications
5. Summary and discussion

global symmetry vs gauge symmetry

~ internal symmetry ~

coset construction for internal symmetry breaking

consider an internal symmetry breaking $G \to H$

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{a} \begin{cases} \mathfrak{h}: \text{ residual symmetry} \\ \mathfrak{a}: \text{ broken symmetry} \end{cases}$$

- NG modes π^a = coordinates of G/H

$$\Omega = e^{\pi^a(x) T_a} \quad \text{with} \quad T_a \in \mathfrak{a} \quad \text{(broken symmetry)}$$

- ingredients of effective action:

Maurer-Cartan one form $J_\mu = \Omega^{-1}\partial_\mu \Omega$

- effective action is local right H invariant

※ coset construction provides general effective action

effective action for massive gauge boson A_μ :

$$\int d^4x \, \mathrm{tr} \left[-\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{v^2}{2} A_{\mathfrak{a}\,\mu} A_{\mathfrak{a}}^{\mu} + \ldots \right] \text{with} \quad A_{\mathfrak{a}\mu} \in \mathfrak{a}$$

- g : gauge coupling, v : order parameter

- NG modes are eaten by gauge boson (unitary gauge)

😊 dynamical dof = gauge field only

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introduce NG modes by Stuckelberg method:

$$A_\mu \to A'_\mu = \Omega^{-1}A_\mu\Omega + \Omega^{-1}\partial_\mu\Omega \text{ with} \qquad \Omega = e^{\pi^a(x)T_a}$$

※ global symmetry limit can be obtained by setting $A_\mu = 0$

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$A_\mu \to J_\mu = \Omega^{-1}\partial_\mu\Omega$ in the unitary gauge effective action

$$\int d^4x \, \mathrm{tr}\left[-\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} - \frac{v^2}{2}A_{\mathfrak{a}\,\mu}A^{\mu}_{\mathfrak{a}} + \ldots\right] \to \int d^4x \, \mathrm{tr}\left[-\frac{v^2}{2}J_{\mathfrak{a}\,\mu}J^{\mu}_{\mathfrak{a}} + \ldots\right]$$

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correspondence between global & local picture

- same # of broken symmetries
- same effective action

introd

$$A_\mu \to A'_\mu = \Omega^{-1}A_\mu\Omega + \Omega^{-1}\partial_\mu\Omega \text{ with} \qquad \Omega = e^{\pi^a(x)T_a}$$

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local properties of spacetime symmetry

let us start from rotation around origin

😣 difficult to write a figure well...

please let me use the blackboard

local properties of spacetime symmetry

consider a spacetime symmetry associated with $x'^{\mu} = x^{\mu} - \epsilon^{\mu}(x)$

its local properties around a point $x^{\mu} = x_{*}^{\mu}$ can be read off as

$$\epsilon^{\mu}(x) = \epsilon^{\mu}(x_{*}) + (x^{\nu} - x_{*}^{\nu})\nabla_{\nu}\epsilon^{\mu}(x) + \ldots$$

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- 1st term: shift of coord. system (translation)

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- 1st term: shift of coord. system (translation)
- 2nd term: deformations of coord. system

$$\nabla_{\mu}\epsilon^{\nu} = \delta_{\mu}^{\nu}\lambda + s_{\mu}{}^{\nu} + \omega_{\mu}{}^{\nu}$$

- trace part λ : isotropic rescaling
- symmetric traceless $s_{\mu\nu}$: anisotropic rescaling
- antisymmetric $\omega_{\mu\nu}$: Lorentz transformation

local properties of spacetime symmetry

consider a spacetime symmetry associated with $x'^{\mu} = x^{\mu} - \epsilon^{\mu}(x)$

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ex. special conformal on Minkowski space

$$\nabla_{\mu}\epsilon^{\nu} = 2\delta_{\mu}^{\nu}(b\cdot x) + 2(b_{\mu}x^{\nu} - b_{\nu}x^{\mu})$$

locally, a combination of Poincare & isotropic rescaling

relativistic symmetry	diffeomorphism	local Lorentz	isotropic Weyl
translation	✓		
isometry	✓	✓	
conformal	✓	✓	✓

Table 1: Embedding of spacetime symmetry in relativistic systems.

nonrelativistic symmetry	foliation preserving	local rotation	(an)isotropic Weyl	internal $U(1)$
translation	✓			
Galilean	✓	✓		✓
Schrödinger	✓	✓	✓	✓
Galilean conformal	✓	✓	✓	

Table 2: Embedding of spacetime symmetry in nonrelativistic systems.

as the local decomposition suggests,

any spacetime symmetry transformation can be embedded

into diffeomorphism, local Lorentz, (an)isotropic Weyl gauging spacetime symmetry

gauging spacetime symmetry

global spacetime symmetry $\in$ diffeo x local Lorentz x local Weyl

- diffeo & local Lorentz

  can be gauged by introducing curved spacetime action

$$\int d^4x\,\mathcal{L}[\Phi, \partial_m \Phi] \to \int d^4x \sqrt{-g}\,\mathcal{L}[\Phi, e_m^\mu \nabla_\mu \Phi]$$

gauging spacetime symmetry

global spacetime symmetry $\in$ diffeo x local Lorentz x local Weyl

- diffeo & local Lorentz

  can be gauged by introducing curved spacetime action

$$\int d^4x\,\mathcal{L}[\Phi, \partial_m \Phi] \to \int d^4x\sqrt{-g}\,\mathcal{L}[\Phi, e_m^\mu \nabla_\mu \Phi]$$

- Weyl symmetry

  1. Ricci gauging (not necessarily possible)

     introduce a local Weyl invariant curved spacetime action

  2. Weyl gauging (always possible)

     gauge global Weyl symmetry by introducing a gauge field W_μ

EFT recipe

diffeomorphism	local Lorentz	local Weyl	internal gauge
spacetime dependence	spin	scaling dimension	internal charge
metric $g_{\mu\nu}$	vierbein e_μ^m	Weyl gauge field W_μ	gauge field A_μ

symmetry breaking pattern based on local symmetries:

can be classified by condensation patterns $\langle \Phi^A(x) \rangle = \bar{\Phi}^A(x)$

once symmetry breaking patterns are given or identified,

we construct the effective action in the following way:

1. gauge the (broken) global symmetry
2. write down the unitary gauge effective action
3. introduce NG modes by Stuckelberg method
   and decouple the gauge sector

plan of my talk:

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2. Basic strategy ✔

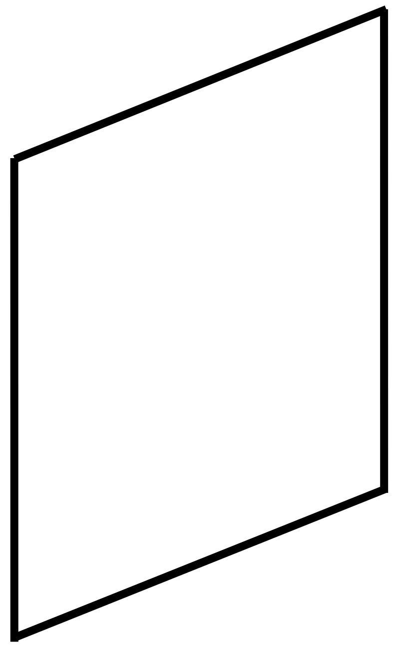
3. Single domain-wall

4. Physical applications

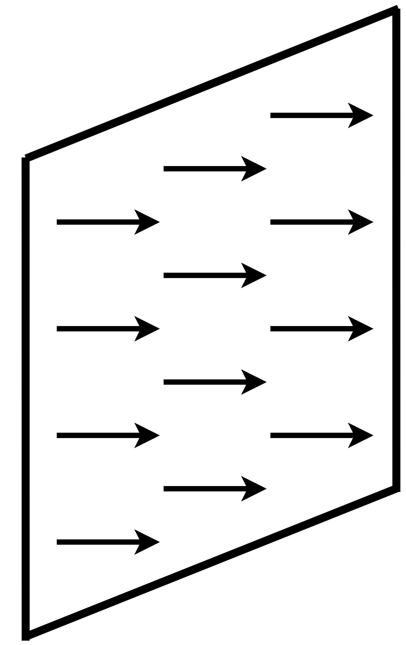
5. Summary and discussion

3. Single domain-wall

classification by the spin of domain-wall condensation:



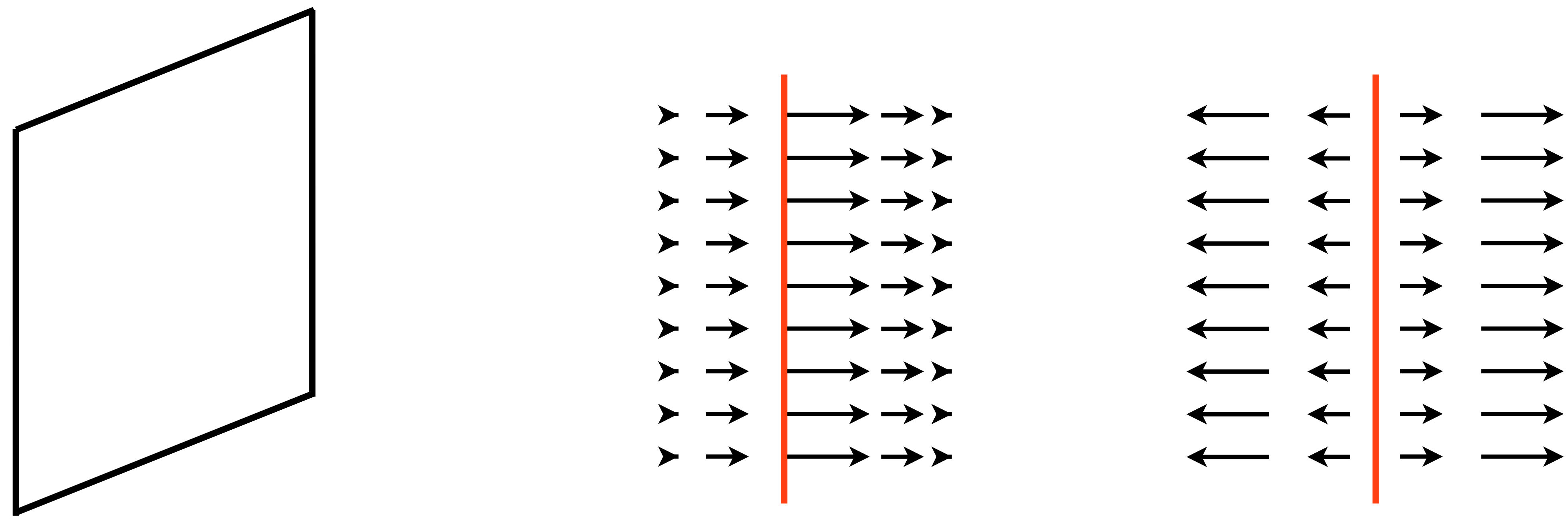
scalar domain-wall



nonzero spin domain-wall

	scalar	nonzero spin
global picture	both translation & rotation are broken	
local picture	diffs only	diffs & local rotation

nonzero spin domain-wall can be further classified:



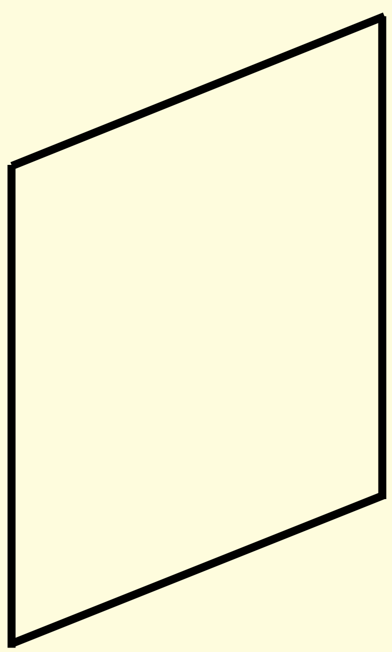
scalar domain-wall　　nonzero spin 1　　nonzero spin 2

		scalar	nonzero 1	nonzero 2
diffs	inside			
	outside	✔	✔	✔
local rotation	inside	✔		✔
	outside	✔	✔	

effective action for NG fields

diffs breaking: NG field π

$$S_P = \int d^4x \left[-\frac{1}{2}\alpha_1(z)\partial_\mu\pi\partial^\mu\pi - \frac{\alpha_2(z)}{2}(\partial_z\pi)^2 \right]$$

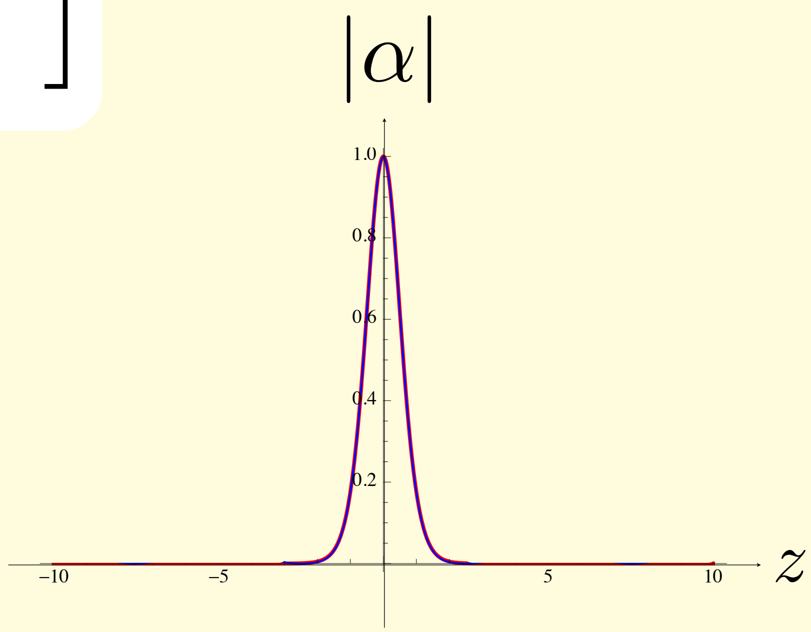


scalar domain-wall

※ scalar domain-wall case

- diffs NG field π only
- diffs are broken only on the brane
- no kinetic term outside the brane

$\rightarrow$ NG mode does not propagate in the bulk



free functions follow from breaking pattern

ex. α_i is nonzero only where diffs are broken

diffs breaking: NG field π

$$S_P = \int d^4x \left[-\frac{1}{2}\alpha_1(z)\partial_\mu \pi \partial^\mu \pi - \frac{\alpha_2(z)}{2}(\partial_z \pi)^2 \right]$$

local rotation breaking: NG field $\xi_{\widehat{\mu}}$ $(\widehat{\mu} = t, x, y)$

$$S_L = \int d^4x \left[-\frac{\beta_1(z)}{4}\left(\partial_{\widehat{\mu}}\xi_{\widehat{\nu}} - \partial_{\widehat{\nu}}\xi_{\widehat{\mu}}\right)^2 - \frac{\beta_2(z)}{2}\left(\partial^{\widehat{\mu}}\xi_{\widehat{\mu}}\right)^2 - \frac{\beta_3(z)}{2}\left(\partial_z \xi_{\widehat{\mu}}\right)^2 \right]$$

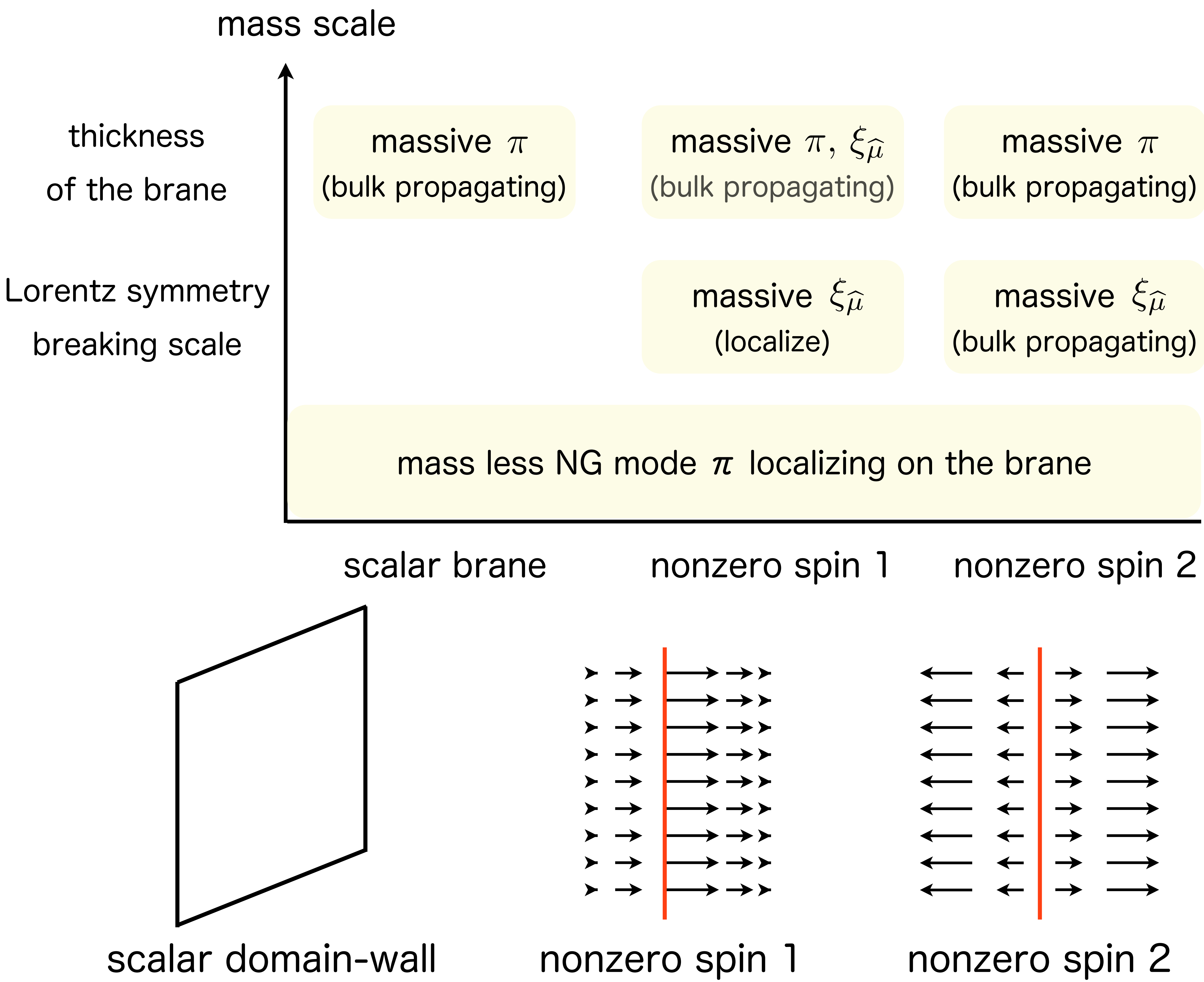
exists only if both are broken

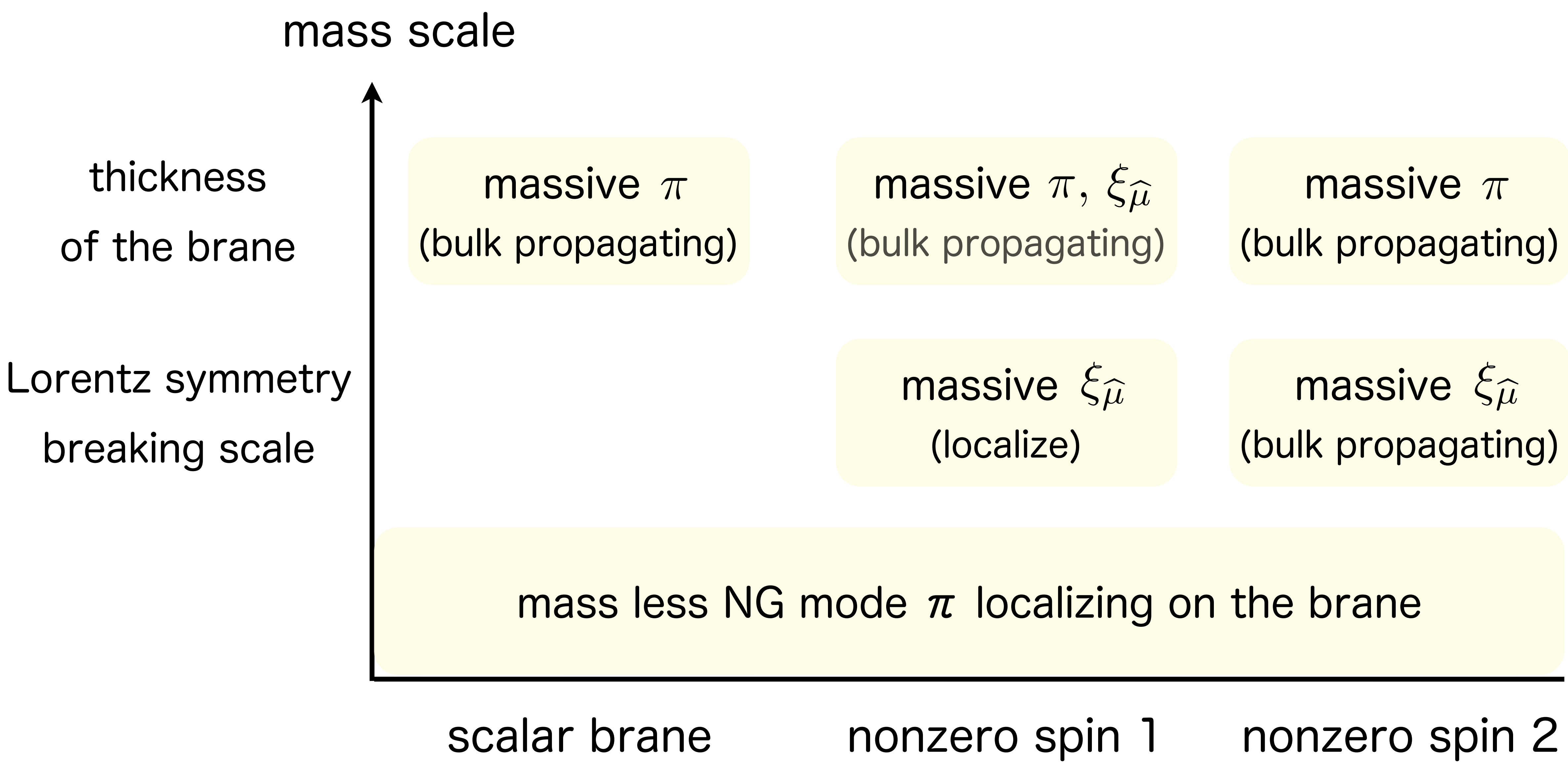
$$S_{PL} = \int d^4x \left[-\frac{m^2(z)}{2}\left(\xi_{\widehat{\mu}} - \partial_{\widehat{\mu}}\pi\right)^2 \right]$$

$\rightarrow$ mass term of $\xi_{\widehat{\mu}}$ and π-$\xi_{\widehat{\mu}}$ mixing

free functions follow from breaking pattern

ex. α_i is nonzero only where diffs are broken physical spectrum...


mass scale
thickness
of the brane
Lorentz symmetry
breaking scale
massive π
(bulk propagating)
massive π, $\xi_{\widehat{\mu}}$
(bulk propagating)
massive π
(bulk propagating)
massive $\xi_{\widehat{\mu}}$
(localize)
massive $\xi_{\widehat{\mu}}$
(bulk propagating)
mass less NG mode π localizing on the brane
scalar brane
nonzero spin 1
nonzero spin 2
scalar domain-wall
nonzero spin 1
nonzero spin 2



- NG fields can be massive for spacetime symmetry case
- same massless spectra, but massive spectra are different
- the mass scale is not necessarily high
- local picture is important at such an intermediate scale

plan of my talk:

1. Introduction ✔
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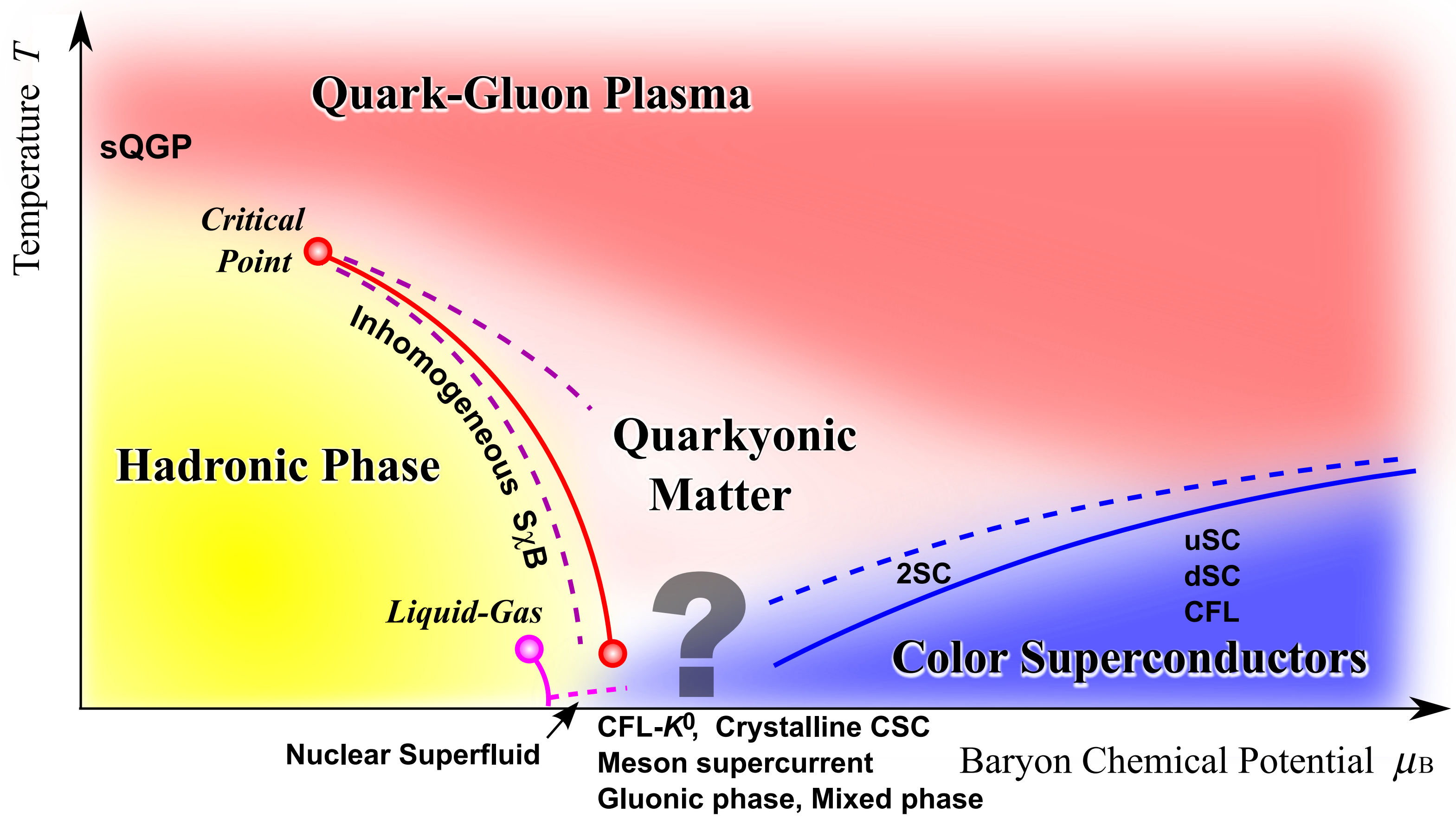
4. physical applications

4-1. inhomogeneous chiral condensate

4-2. cosmic inflation

4-1. inhomogeneous chiral condensate

inhomogeneous chiral condensate (in QCD phase diagram?)



$$\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\psi\rangle = \Delta(z)$$

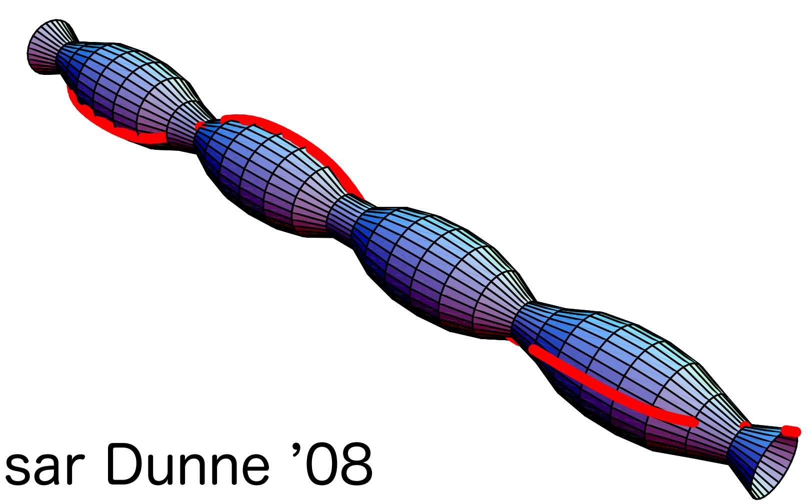
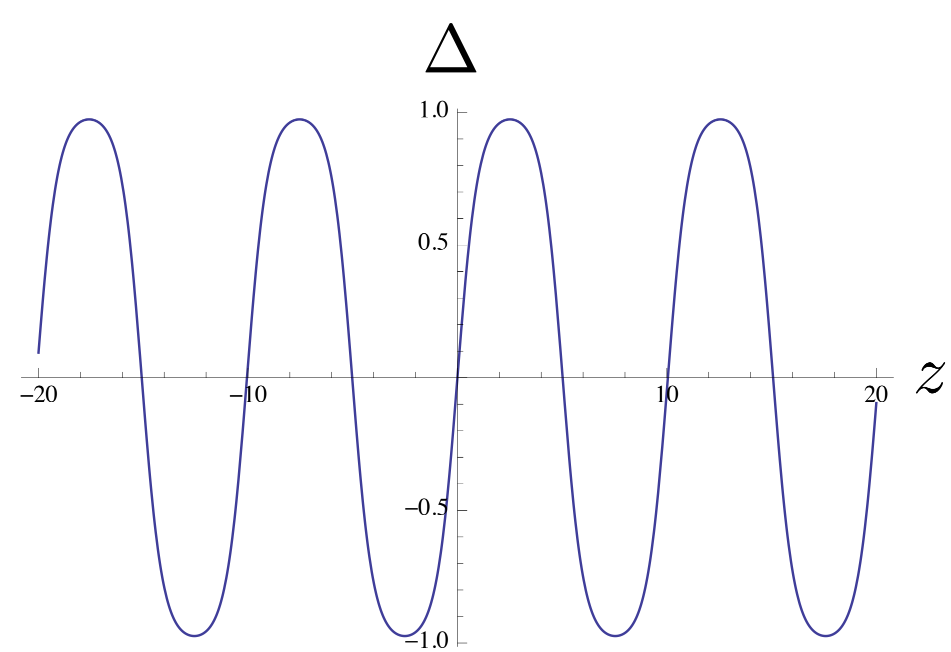


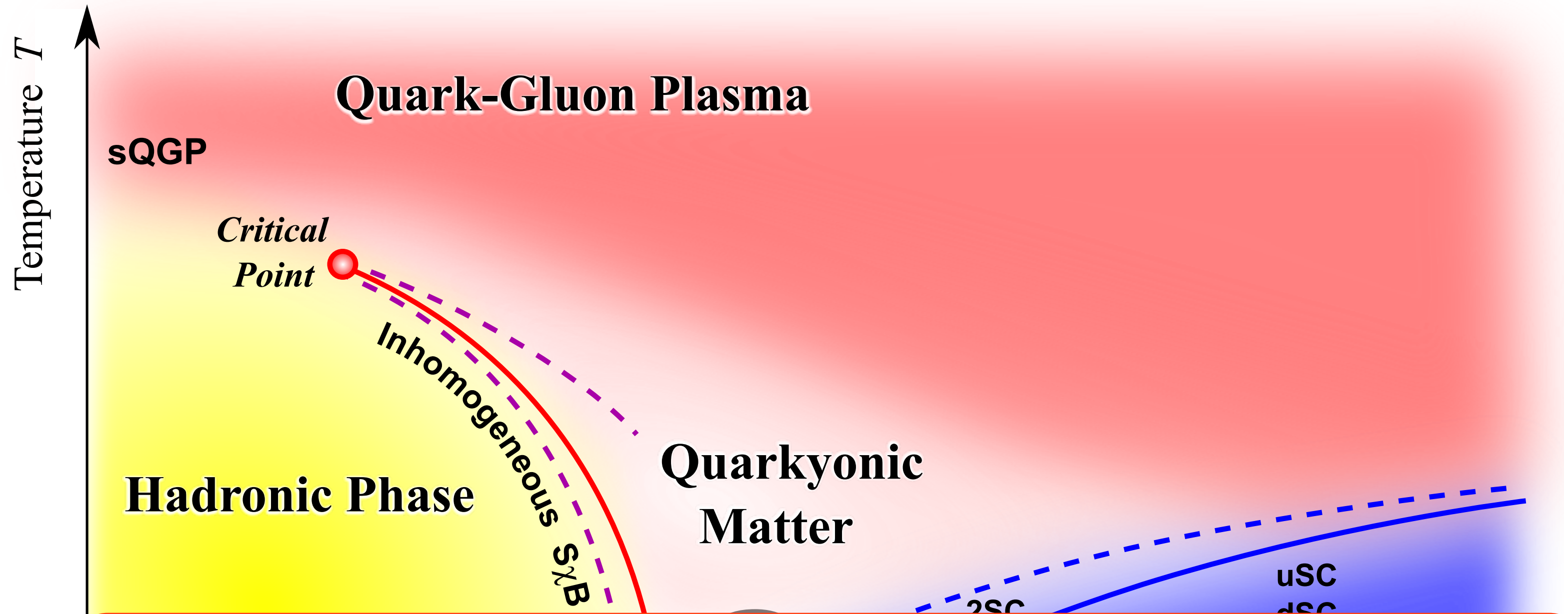
fig: Basar Dunne '08

chiral spiral (complex)



real kink

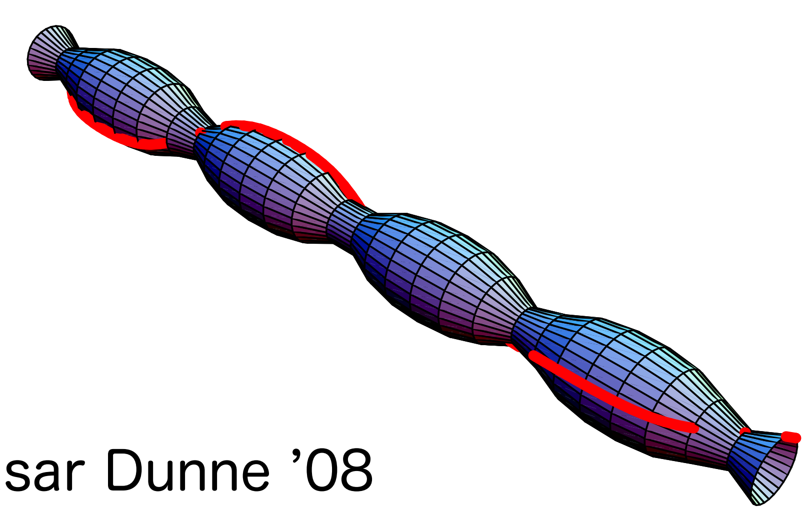
inhomogeneous chiral condensate (in QCD phase diagram?)



fluctuations around the condensation?

- e.g. dispersion relations of NG fields

$$\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\psi\rangle = \Delta(z)$$



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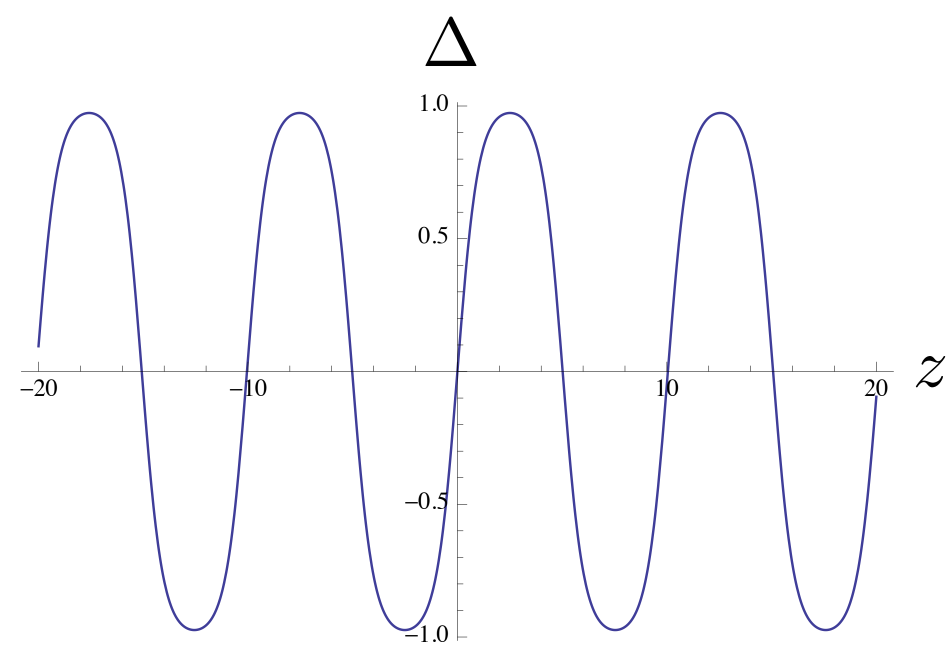

real kink

inhomogeneous chiral condensate (in QCD phase diagram?)

- effective action for NG field (nonrelativistic)

$$S = \int d^4x \left[\frac{\alpha_t(z)}{2}\dot{\pi}^2 - \frac{\alpha(z)}{2}(\partial_i \pi)^2 - \frac{\gamma(z)}{2}(\partial_z \pi)^2 + \mathcal{O}(\pi)^3 \right] + \int d^3x \left[\frac{\alpha(z)}{2}\pi + \mathcal{O}(\pi^2) \right]_{z=-\infty}^{z=\infty}$$

- free functions of z: nonvanishing where translation is broken
- boundary term linear in π (cf. vanishes for single domain-wall)

  ※ nonvanishing $\alpha(z)$ on the whole spacetime

inhomogeneous chiral condensate (in QCD phase diagram?)

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- free functions of z: nonvanishing where translation is broken
- boundary term linear in π (cf. vanishes for single domain-wall)

※ nonvanishing $\alpha(z)$ on the whole spacetime

→ existence of condensate with lower energy (cf. tadpoles)

→ $\alpha(z) = 0$ for minimum energy condensate

inhomogeneous chiral condensate (in QCD phase diagram?)

- effective action for NG field (nonrelativistic)

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- dispersion relation: $\omega^2 \sim c_1 k_z^2$

inhomogeneous chiral condensate (in QCD phase diagram?)

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※ quadratic dispersion in the transverse direction to modulation

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- dispersion relation: $\omega^2 \sim c_1 k_z^2 + c_2 k_z k_\perp^2 + c_3 k_\perp^4$

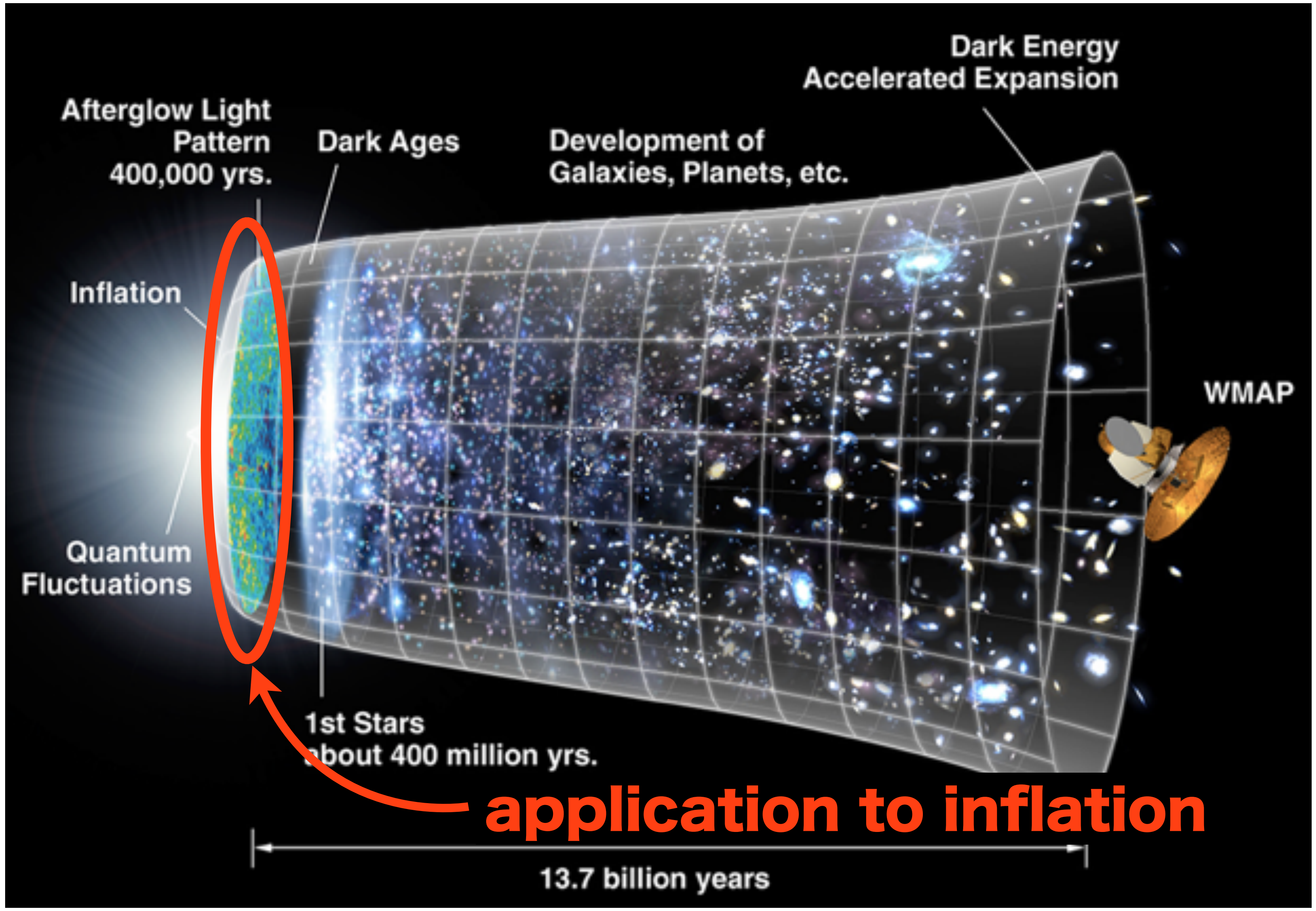
※ quadratic dispersion in the transverse direction to modulation

- thermal fluctuations generically kill the modulation

$\rightarrow$ constraint on volume and temp. for modulation to survive

(more detailed analysis in Hidaka-Kamikado-Kanazawa-Noumi to appear)

4-2. cosmic inflation



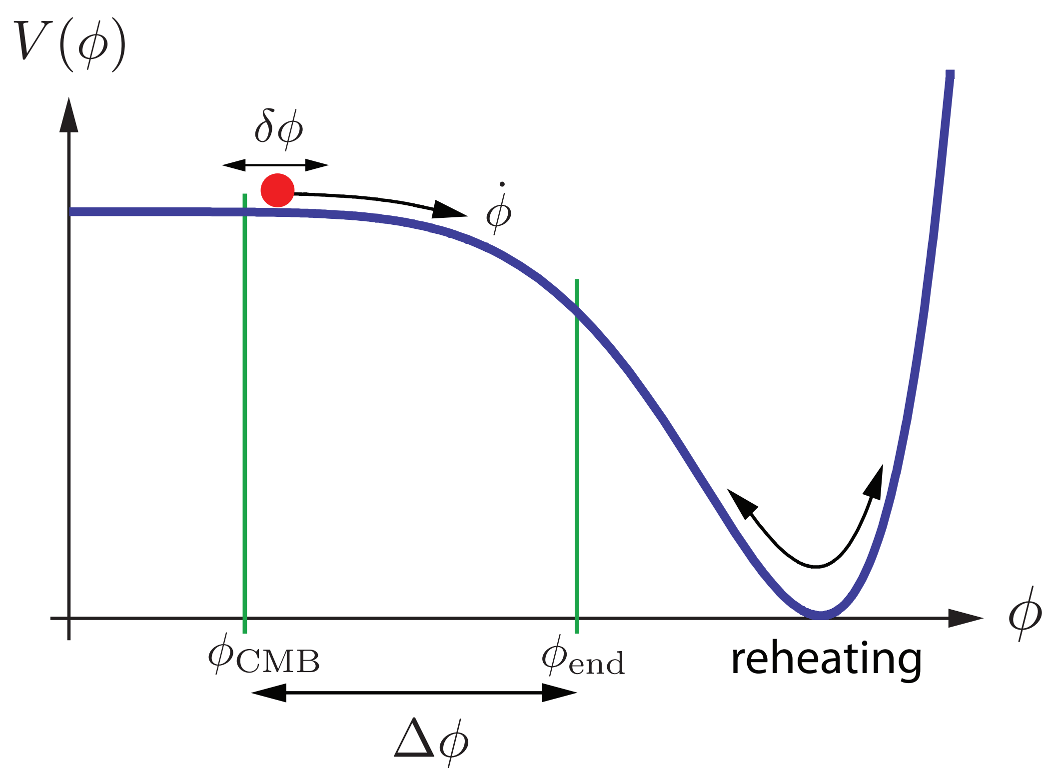
FRW spacetime $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

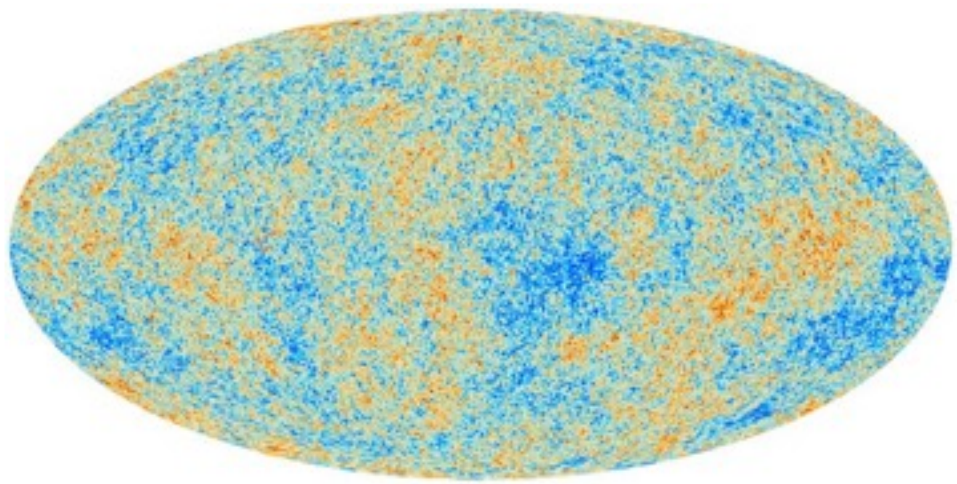
※ local symmetry picture is convenient in cosmology

- cosmology = gravitational theory
- isometry is not fixed in gravitational theory

cf. dS: 3dim conformal SO(4,1), Minkowski: Poincare symmetry

1. EFT approach for single field inflation [Cheung et al ’07]

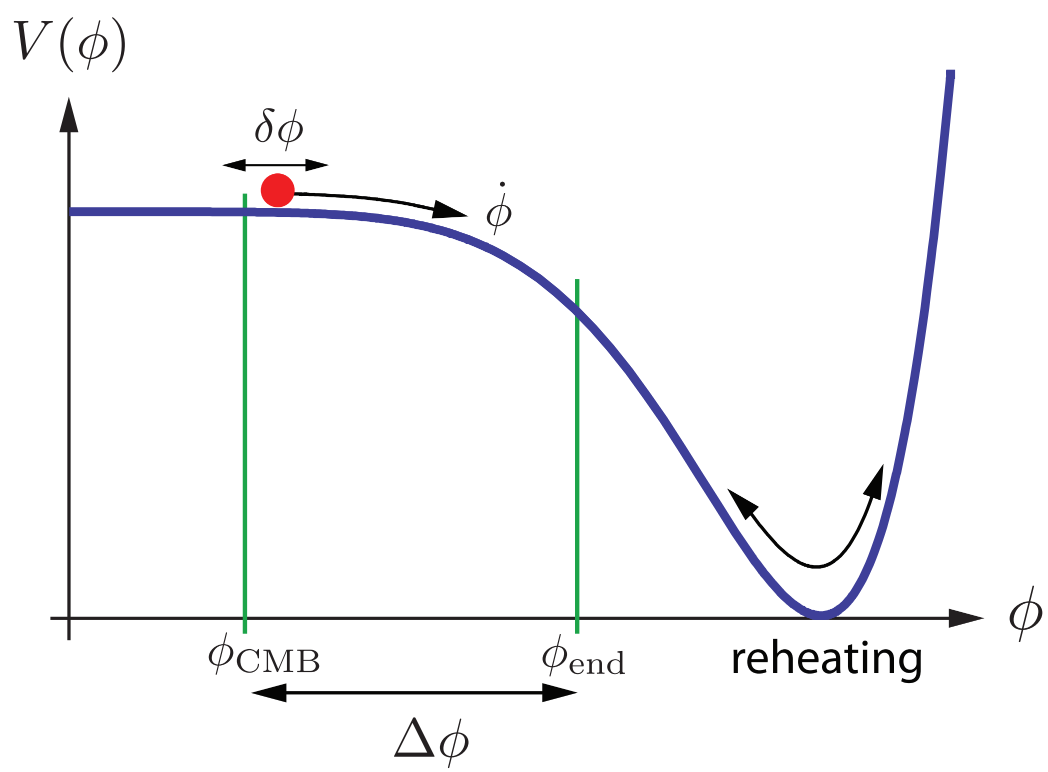


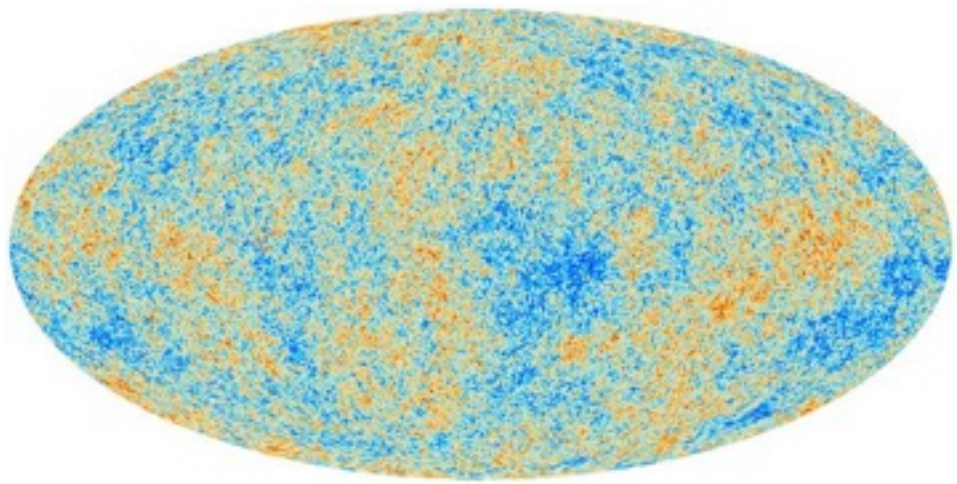


CMB temp fluctuation by Planck

- time evolution of inflaton breaks time diffs: $\langle \phi(x) \rangle = \bar{\phi}(t)$

1. EFT approach for single field inflation [Cheung et al '07]





CMB temp fluctuation by Planck

- time evolution of inflaton breaks time diffs: $\langle \phi(x) \rangle = \bar{\phi}(t)$

  NG field $\rightleftarrows$ fluctuation of time (cf. inflaton is clock)

  $\rightleftarrows$ temp fluctuation (cosmic expansion cools down universe)

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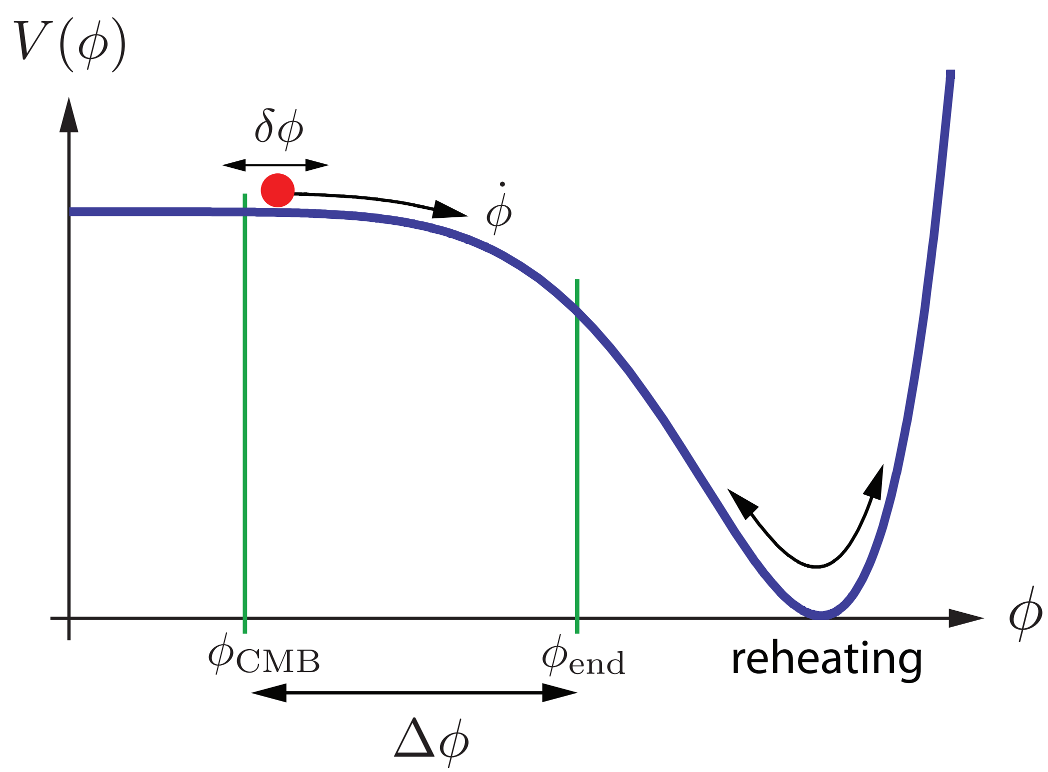

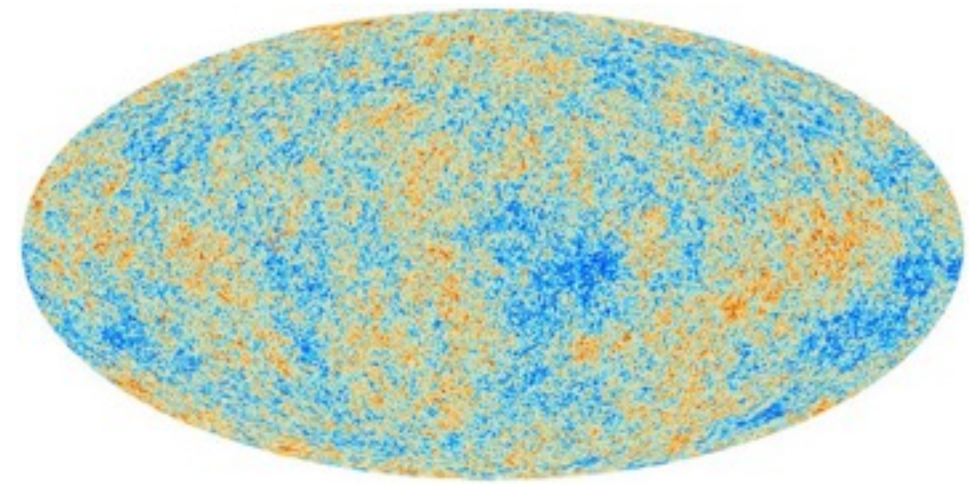

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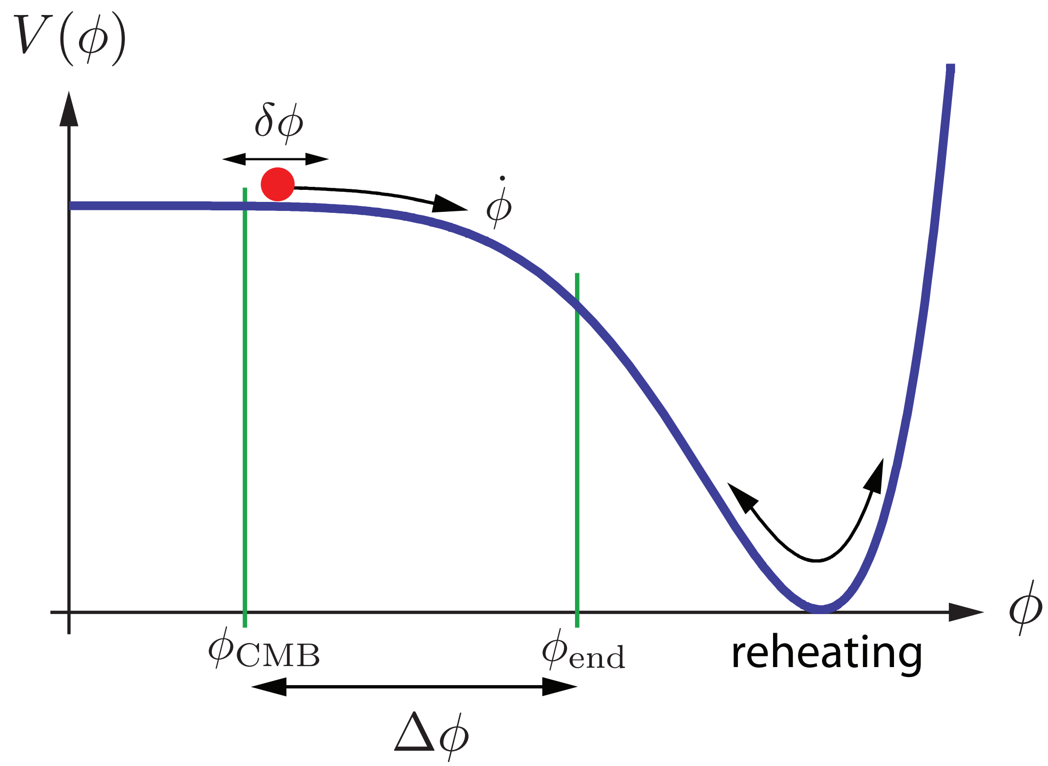

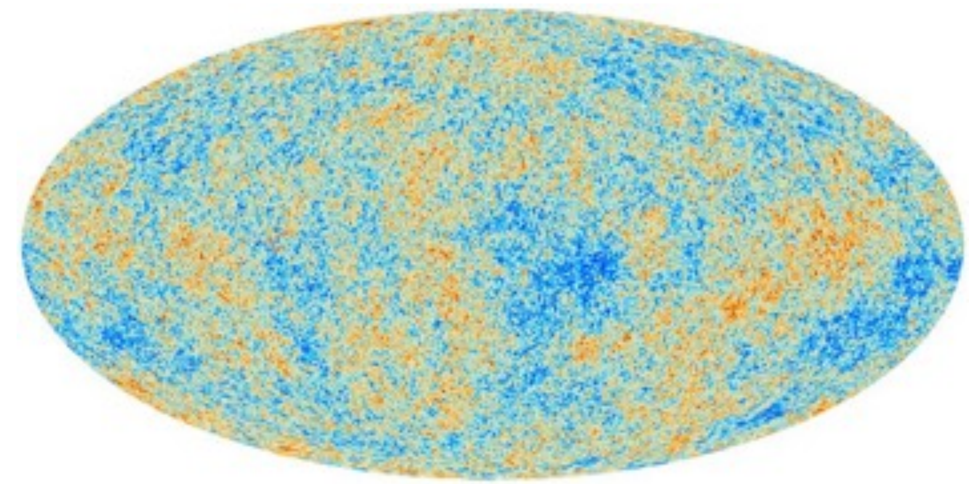

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  ※ NG field correlators are directly related to experiments

- provides a template for e.g. primordial non-Gaussianities

  cf. effects of heavy fields on non-Gaussianity [Noumi-Yamaguchi-Yokoyama '12]

2. other local symmetry breaking patterns for FRW?

let us recall "scalar domain-wall vs nonzero spin domain-wall"

- same global symmetry structure, but different local one
- same massless spectra, but different massive spectra

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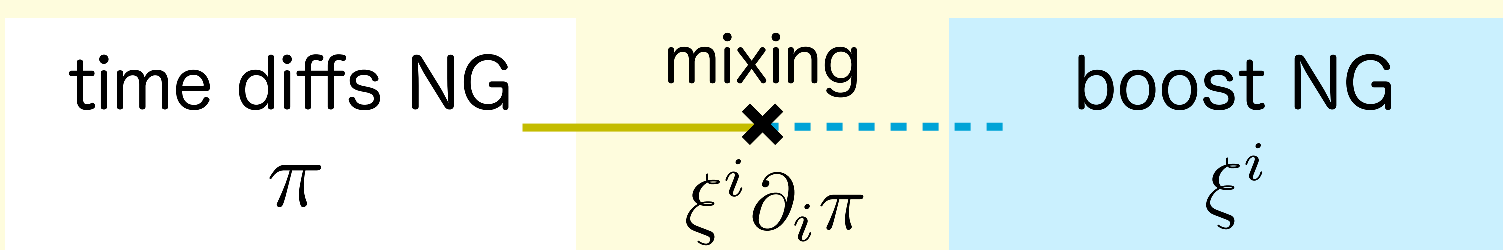
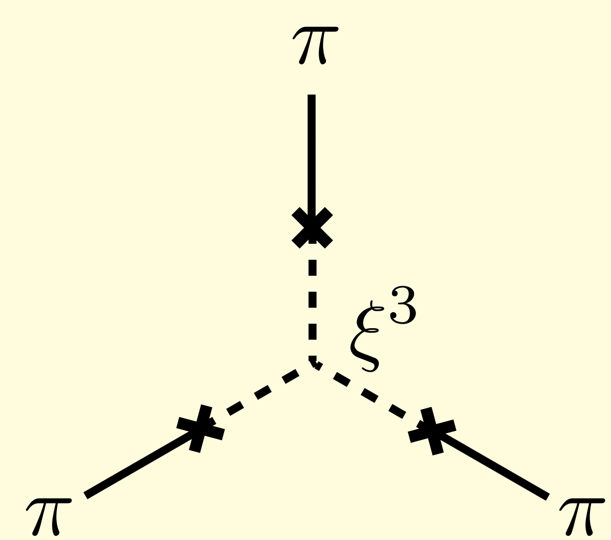
- effects on ex. primordial non-Gaussianity





5. Summary and prospects

summary

・EFT approach for spacetime symmetry breaking

- from local symmetry picture
- spacetime symmetry $\in$ diffeo x local Lorentz x (an)isotropic Weyl
- effective action from gauge symmetry breaking

・global vs local picture of spacetime symmetry breaking

- vector branes $\rightarrow$ massive Lorentz NG modes
- local picture is necessary to take into account massive NG fields

・coset construction revisited

- MC one form vs connections
- classification of physical meaning of inverse Higgs constraints

future directions

- application of EFT based on local picture
  - cosmological applications ex. primordial gravitational waves
  - inhomogeneous chiral condensate, liquid crystal, fluid dynamics ...
- more on nonrelativistic case
  - finite temperatures, finite densities, ...

Thank you!

4. Coset construction revisited

4-1 MC form vs connections

4-2 Role of inverse Higgs constraints

4-1. MC form vs connections

origin of Maurer-Cartan one form for internal symmetry breaking

origin of MC forms [e.g. Weinberg's textbook]

suppose that a condensation $\langle \Phi^A \rangle = \bar{\Phi}^A$ breaks $G \to H$

$$\mathfrak{g} = \mathfrak{a} \oplus \mathfrak{h} \;\; (\mathfrak{a} : \text{broken}, \; \mathfrak{h} : \text{unbroken})$$

1. decompose the NG field dof as

$$\Phi^A = \Omega^A{}_B \widetilde{\Phi} \text{ with } \; \Omega = e^{\pi(x)} \, (\pi \in \mathfrak{a} : \text{NG field})$$

2. its derivative is given by

$$\partial_\mu \Phi^A = \left[\Omega \boxed{\left(\partial_\mu + \Omega^{-1} \partial_\mu \Omega \right)} \right]^A{}_B \, \widetilde{\Phi}^B$$

※ $\partial_\mu + J_\mu \;\; \leftrightarrow \;\; \partial_\mu + A_\mu$: covariant derivative

- if we drop mater fields and concentrate on NG fields,

$$\partial_\mu \Phi^A = \Omega^A{}_B \boxed{\left(\Omega^{-1} \partial_\mu \Omega \right)^B{}_C \, \bar{\Phi}^C}$$

※ projection onto the broken sector: $J_\mu^{\mathfrak{a}} \;\; \leftrightarrow \;\; A_\mu^{\mathfrak{a}}$

nonlinear realization and the MC form

for spacetime symmetry

nonlinear realization and the MC form

- suppose that a condensation $\langle \Phi^A \rangle = \bar{\Phi}^A(x)$ breaks $G \to H$

$$\mathfrak{g} = \mathfrak{a} \oplus \mathfrak{h} \ \ (\mathfrak{a} : \text{broken}, \ \mathfrak{h} : \text{unbroken})$$

convenient to classify symmetry generators by scaling dim

$$\mathfrak{g} = \mathfrak{g}_P \oplus \widehat{\mathfrak{g}} = \mathfrak{g}_P \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1 + \dots$$

$$(\widehat{\mathfrak{g}}\text{: non-translation, } \ \mathfrak{g}_n\text{: dim } n)$$

e.g. $L, D \in \mathfrak{g}_0, \ \ K \in \mathfrak{g}_1$

- introduce a representative of the coset $G/\widehat{H}$

$$\Omega = \Omega_P \Omega_0 \Omega_1 \dots \ \text{with} \ \ \Omega_P = e^{Y^m(x) P_m}, \ \Omega_n = e^{\pi_n(x)} \ \ (\pi_n \in \mathfrak{a}_n)$$

※ treat translations generators as if they are broken symmetries

※ $Y^m(\bar{x})$ is the Minkowski coordinate defined by

$$Y^{\widehat{m}} = \bar{x}^{\widehat{m}} \ \text{(unbroken direction)}, \ Y^a = \bar{x}^a + \pi^a(\bar{x}) \ \text{(broken direction)}$$

note: $\bar{x}^m$ is the unitary gauge coordinate (π's are eaten)

nonlinear realization and the MC form

- introduce a representative of the coset $G/\widehat{H}$

$$\Omega = \Omega_P \Omega_0 \Omega_1 \ldots \text{ with } \Omega_P = e^{Y^m(x) P_m},\ \Omega_n = e^{\pi_n(x)}\ (\pi_n \in \mathfrak{a}_n)$$

※ Ω_n generates transformations around $Y^m(\bar{x})$

e.g. NG field for Lorentz ⇔ local Lorentz transf. parameter

→ $\pi_n\,(n \geq 1)$ does not generate physical dof (redundant NG field)

cf. $K_m \Phi(0) = 0$ for primary fields

- MC form

$$J_\mu = \Omega^{-1}\partial_\mu \Omega = \widehat{\Omega}^{-1}(\partial_\mu Y^m P_m)\widehat{\Omega} + \widehat{\Omega}^{-1}\partial_\mu \widehat{\Omega} \text{ with } \widehat{\Omega} = \Omega_0 \Omega_1 \ldots$$

in particular, its P_m can be interpreted as the vierbein

$$e_\mu^m = [J_\mu]_{P_m} = \left[\widehat{\Omega}^{-1}\left(\partial_\mu Y^n P_n\right)\widehat{\Omega}\right]_{P_m} = \left[\Omega_0^{-1}\left(\partial_\mu Y^n P_n\right)\Omega_0\right]_{P_m}$$

※ $\pi_n\,(n \geq 1)$ does not affect the vierbein

※ other components contain redundant NG fields $\pi_n\,(n \geq 1)$

ingredients of the effective action

ingredients of effective action

suppose a background condensation $\langle \Phi^A \rangle = \bar{\Phi}^A(x)$

※ not necessarily homogeneous

1. decompose the NG field dof as

$$\Phi^A(x) = \Omega_P(\pi) \left[\Omega_0(x, \pi_0)\right]^A{}_B \tilde{\Phi}^B(x)$$

※ $\Omega_n \; (n \geq 1)$ does not generate physical dof

- take the unitary gauge coordinate, where diff NG fields are eaten

$$\Phi^A(\bar{x}) = \left[\Omega_0(\bar{x}, \pi_0)\right]^A{}_B \tilde{\Phi}^B(\bar{x})$$ ($\bar{x}^m$: unitary gauge coordinate)

2. its derivative reproduces MC-type one form

$$\partial_\mu \Phi^A(\bar{x}) = \left[\Omega_0 \left(\partial_\mu + \Omega_0^{-1} \partial_\mu \Omega_0\right)\right]^A{}_B \tilde{\Phi}^B(\bar{x})$$

※ the one form is made only from Ω_0

※ $\bar{\Phi}^A(\bar{x})$ is not a constant in general $\rightarrow$ $\partial_\mu \bar{\Phi} \neq 0$

\# ingredients of effective action

- the one form can be identified with connections

in the relativistic case $\Omega_0 = \Omega_{\text{int}}\Omega_L\Omega_D$,

gauge and Weyl connections:

$$A_\mu = \Omega_{\text{int}}^{-1}\partial_\mu\Omega_{\text{int}},\ W_\mu = \Omega_D^{-1}\partial_\mu\Omega_D$$

spin connections:

$$S_\mu = \Omega_L^{-1}\partial_\mu\Omega_L + \frac{1}{2}\left(e_\mu^m e_\nu^n - e_\mu^n e_\nu^m\right) W^\nu L_{mn}$$

※ spin connection is not Weyl invariant

vierbein: $e_\mu^m = \left[\Omega_D^{-1}\Omega_L^{-1}(\partial_\mu Y^m P_m)\Omega_L\Omega_D\right]_{P_m}$

summary so far:

- parameterization of NG fields in coset construction
  - $\mathfrak{g}_0$ = local Lorentz, (an)isotropic Weyl, internal symmetries
  - $\pi_n \; (n \geq 1)$ does not generate physical dof (redundant NG field)

- ingredients of effective action
  - Ω_0 MC form $\sim$ connections: $\Omega_0^{-1}\partial_\mu \Omega_0 \sim A_\mu, \; W_\mu, \; S_\mu$
  - P_m component of MC form $\sim$ vierbein e_μ^m
  - functions of coordinates without translation invariance

  ※ same ingredients as gauge symmetry breaking picture

- main differences from the internal symmetry case
  - $\Omega_n \; (n \geq 1)$ does not appear explicitly $\rightarrow$ inverse Higgs
  - EFT parameters are promoted to functions of coordinates

4-2. Role of inverse Higgs constraints

inverse Higgs constraints

inverse Higgs constraints

a standard recipe:

1. take a commutator of broken symmetries

   $[P_m, A] \sim B + C$ where $B \in \mathfrak{a},\ C \in \mathfrak{h}$

2. remove the NG field for A if $B \neq 0$,

   by imposing the condition $[\Omega^{-1}\partial_\mu\Omega]_B = 0$

we can classify them by scaling dimension n :

(1) $n \geq 1$: redundant NG fields to be removed

ex. NG field for special conformal symmetry

(2) $n = 0$: further classified by local symmetry breaking patterns

ex. codimension one brane

conformal symmetry breaking

conformal symmetry breaking

- global symmetry breaking pattern:

$$P_m,\, L_{mn},\, D,\, K_m \;\to\; P_m,\, L_{mn}$$

- nonlinear realization: $\Omega = \Omega_P \Omega_0 \Omega_1$ with $\Omega_1 = e^{\chi^m K_m}$
- relevant commutator: $[P, K] \sim D + L$
  - essentially the same as local decomposition of symmetries
  - remove χ^m by imposing the condition $[J_\mu]_D = 0$
- MC form vs connections

$$[J_\mu]_D = W_\mu - 2\chi_\mu$$

$$[J_\mu]_{L_{mn}} = S_\mu^{mn} - (e_\mu^m W^n - e_\mu^n W^m) + 2(e_\mu^m \chi^n - e_\mu^n \chi^m)$$

$$[J_\mu]_{K_m} = \nabla_\mu \chi^m + W^m \chi_\mu + (\chi^2 - W^\nu \chi_\nu) e_\mu^m + (W_\mu - 2\chi_\mu)\, \chi^m$$

conformal symmetry breaking

- global symmetry breaking pattern:

$$P_m,\, L_{mn},\, D,\, K_m \;\to\; P_m,\, L_{mn}$$

- nonlinear realization: $\Omega = \Omega_P \Omega_0 \Omega_1$ with $\Omega_1 = e^{\chi^m K_m}$
- relevant commutator: $[P, K] \sim D + L$
  - essentially the same as local decomposition of symmetries
  - remove χ^m by imposing the condition $[J_\mu]_D = 0$
- MC form vs connections

$$[J_\mu]_D = W_\mu - 2\chi_\mu = 0$$

$$[J_\mu]_{L_{mn}} = S_\mu^{mn}$$

$$[J_\mu]_{K_m} = e^{m\nu}\Big(\nabla_\mu W_\nu + W_\mu W_\nu - \frac{1}{2} g_{\mu\nu} W^2\Big)$$

※ Weyl gauge field appears in a special conformal invariant way

Weyl gauging vs Ricci gauging

- K_m component of MC form:

$$[J_\mu]_{K_m} = e^{m\nu}\left(\nabla_\mu W_\nu + W_\mu W_\nu - \frac{1}{2} g_{\mu\nu} W^2\right)$$

- its Weyl transformation property can be rephrased as [Iorio et al '96]

$$\Delta\left[\nabla_\mu W_\nu + W_\mu W_\nu - \frac{1}{2} g_{\mu\nu} W_\rho W^\rho\right] = \Delta\left[\frac{1}{2-d}\left(R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R\right)\right]$$

- if we are originally working on conformally flat backgrounds,

$$[J_\mu]_{K_m} = \frac{e^{m\nu}}{2(2-d)}\left(R_{\mu\nu} - \frac{1}{2(d-1)} g_{\mu\nu} R\right)$$

※ inverse Higgs constraints promote Weyl gauge field to Ricci tensor

※ the same ingredients as Ricci gauging case

- we can perform Ricci gauging for conformal theories
- not necessary to introduce Weyl gauge fields explicitly

codimension one brane

codimension one brane

- global symmetry breaking pattern: $P_m,\, L_{mn} \to \;\, P_{\widehat{m}},\, L_{3\widehat{n}}$
- nonlinear realization: $\Omega = \Omega_P \Omega_0$ with $\;\Omega_0 = \Omega_L$

  ※ no higher order generators $\Omega_n \; (n \geq 1)$

- relevant commutator: $[P_{\widehat{m}}, L_{3\widehat{n}}] \sim \delta_{\widehat{m}\widehat{n}} P_3$

  - remove NG field for $L_{3\widehat{m}}$ by imposing $[J_\mu]_{P_3} = e^3_\mu = 0$

    ※ it is convenient to decompose it as

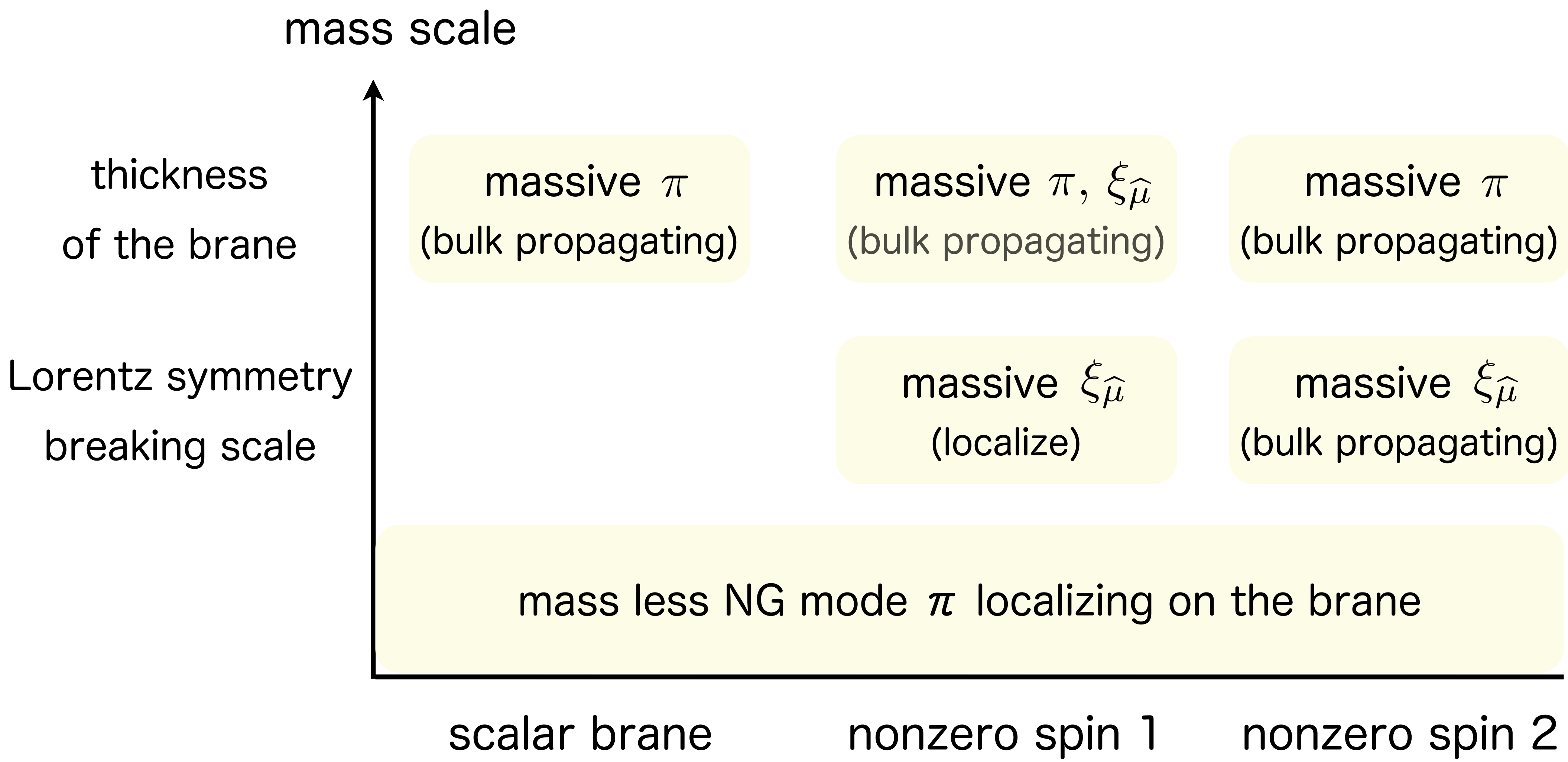
    $h^\nu_\mu e^3_\nu = 0 \;\; \Leftrightarrow \;\; e^3_\mu = n_\mu \; \to$ freeze out local Lorentz NG fields

    ※ same mass & mixing terms $\sim (\xi_{\widehat{\mu}} - \partial_{\widehat{\mu}} \pi)^2$

    $n^\mu e^3_\mu = 0 \;\to\; \partial_z \pi = 0 \to$ prohibit propagation in z-direction

    ※ constraint not on $\xi_{\widehat{m}}$, but rather on π

- the resulting dof is π localizing on the brane



- nonzero spin brane 1 & 2

  inverse Higgs constraints integrate out all the massive modes

- for scalar brane

  one may interpret $\xi_{\widehat{m}}$ as redundant NG fields

  inverse Higgs constraints remove redundant & massive NG fields

  ※ we can formulate without introducing $\xi_{\widehat{m}}$ consistently