Atomic Quantum Simulation of U(1) lattice gauge-Higgs model

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Topics:

1. What is Cold atom in an optical lattice?

2. Atomic Quantum Simulator I: Basic concepts and a feasible experimental method for U(1) lattice gauge-Higgs model

3. U(1) lattice gauge-Higgs model II: Numerical results by Monte-Carlo and Gross-Pitaevskii equation.
Useful References

Optical lattice:
• A. Leggett, “Quantum Liquids” (Oxford), Sec. 4.
• I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885.

Quantum Simulation of Lattice gauge theory (review article):

1. What is Cold atom in an optical lattice?

- What is Cold atom?
- Laser trapping technique
- Optical Lattice
- Bose-Hubbard model
By using two experimental methods: Laser Cooling and Evaporative Cooling

Alkali Bose gass nK order!

Alkali atom has a magnetic dipole.

The created ultra cold alkali gasses have densities between $10^{12} - 10^{15}$ particles per $cm^3$.

In such a trapping system, energy shift depending on a hyperfine-atomic levels becomes,

$$V_i(r) = E_i(||B(r)||)$$

This is simple Zeeman effect

A electric dipole of an alkali atom couples to a electric field induced by laser.

\[ F = \frac{1}{2} \alpha(\omega_L) \nabla(|E(r)|^2) \]

\[ \alpha(\omega_L) \approx \frac{|\langle e| \hat{d}_E |g\rangle|^2}{\hbar \Delta} \]

\[ V_{dip}(r) = \frac{3\pi c^2}{2\omega_2^3} \frac{\Gamma}{\Delta} I(r) \]

AC-Stark shift

\[ \Delta = \omega_L - \omega_0 \]

\( > 0 \) : blue detuning
\( < 0 \) : red detuning

Laser intensity

Detuning

Time averaged intensity

Second perturbation theory

Electric ground state \( \rightarrow \) a single doublet \( ^2S_{1/2} \)

First excited state

ns-np splitting

Laser intensity

Detuning
Optical lattice potential can be created by standing lasers. The created electric field becomes periodic potential for cold atoms.

A single linearly polarized laser give rise to an electric field,

\[ E(rt) = E_0 \cos(k \cdot r - \omega t) \]

Two counter-propagating lasers produce a standing wave,

\[ E(rt) = 2E_0 \cos\left(\frac{(k_1+k_2) \cdot r}{2} - \omega t\right) \]

By averaging over the laser frequency, we obtains an effective energy for a single atom. As one basic periodic potential, we can make the following 3D periodic potential.

\[ V_p(x, y, z) = V_0\left(\sin^2 kx + \sin^2 ky + \sin^2 kz\right) \]

Here, the potential amplitude can be controlled as follows:

\[ V_0 \propto \frac{2I \Gamma^2}{\omega_0 \Delta} \]

Trapping frequency up to 100kHz.

\[ \hbar \omega_0 = 2E_r(V_0/E_r)^{1/2} \]

This potential depends on detuning and laser intensity.

The cold atoms are tightly confined in each potential minima.
Optical lattices

By changing laser parameters, incident angles and the number of incident laser, one can make some of geometrical structures.

2D square lattice and 3D cubic lattice

Suitable for (2+1)D Lattice Gauge theories

Optical Super lattice

Suitable for Graphene physics and Haldane Hubbard model, etc..

Quantum Entanglement

Bell State…
Bose Hubbard model:


\[ H_{BH} = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \sum_i U n_i(n_i - 1) + \sum_i \mu n_i. \]

\[ t = const. \omega_0 \exp(-\alpha). \]

\[ U \sim (V_0/E_R)^{3/4}. \]

\[ \alpha \equiv (8V_0/E_R)^{1/2}. \]

\[ U/t \sim (a/d) \cdot \exp(2\sqrt{V_0/E_r}). \]

These parameters can be widely controlled!
Quantum phase transition occurs \( \rightarrow \) Mott insulator-Superfluid phase transition

In 2003, by using Rb alkali atoms, the model was realized by Greiner group.
These experimental results corresponded to many theoretical predictions.

If you are interested in deriving the above model, see a famous text, A. Altland and, B. D. Simons, “Condensed Matter Field Theory” (Cambridge Univ. press), especially Sec.2.

In an optical lattice, both hopping \( t \) and on-site interaction \( U \) are easily tunable!
Summary I

1. Cold atom in an Optical lattice may be a versatile Quantum Simulator:
   - Control system condition
     → particle density, kind of particle, particle interaction, hopping amplitude, artificial magnetic fields, etc.
     → Geometry(square, triangular, and honeycomb, etc.), Dimension(1D,2D,3D, also 4D! by using internal degree of freedom ),
   - Optical lattice can trap both bosonic and fermionic atoms.

2. Real Experimental Simulator:
   Optical lattices can test Condensed Matter Physics and more Lattice Quantum Field Theory. ➔ Direct observation of many-body physics
   ➔ We can observe real time dynamics (I will show later)
2. Atomic Quantum Simulator I: Basic concepts and a feasible experimental method for U(1) lattice gauge-Higgs model
What is Quantum Simulator?

1. For some interesting Quantum systems,
   - To construct artificial, controllable, and versatile quantum system experimentally.
   - To detect these dynamics. (imaginary time $\Rightarrow$ real time)

2. For Lattice Gauge Theories and Strong-Correlation Systems...
   - Quantum Simulator compensates for classical simulation, and also endows new knowledge.

3. This real experimental simulator can realize many models which have been studied by academic interest.
Local Gauge invariant system must be created.

We need to set optimal Interactions, lattice geometry, and particle number, etc..

In this seminar, our proposal: U(1) gauge-Higgs model can be experimentally constructed from Bose Hubbard model in BEC state

Cold atom system

Local gauge symmetry

In one component cold atom system, difficult...

Higgs coupling model can be constructed.

This is a U(1) gauge-Higgs model

To construct U(1)GHM, we start to set a extended Bose Hubbard Model (BHM) with long range interaction.

\[
H = - \sum_{r,a \neq b} J_{a,b} \hat{\psi}_{r,a}^\dagger \hat{\psi}_{r,b} + \frac{V_0}{4} \sum_{r,a} \hat{\rho}_{r,a}^2 \\
+ \sum_{r,a \neq b} \frac{V_{ra,rb}}{2} \hat{\rho}_{r,a} \hat{\rho}_{r,b},
\]

This Bose-Hubbard model can be set on the below optical lattice system.

This interaction term generates a Gauss’s law. The tuning condition will be proposed.

The dashed red lines indicate the 2D optical lattice with the square geometry, and cold atoms reside on its sites denoted by black crosses. Its unit cell consists of a pair of white and blue squares (yellow region).

The filled black lines indicate the 2D gauge lattice on which the U(1) lattice GHM is defined.

Then the cold atoms are viewed to sit on each link of the gauge lattice to play the role of gauge field.
Assuming uniform atom densities in each link, the field operator can be decoupled into density fluctuation and phase.

\[
\hat{\Psi}_{r,i} = \sqrt{\hat{\rho}_{r,i}} \ e^{i\hat{\theta}_{r,i}} \ , \quad \hat{\rho}_{r,i} = \rho_0 + \hat{\eta}_{r,i}
\]

mean density fluctuation

We can notice that the canonical relation between density and phase is equivalent to that of U(1) gauge theory.

\[\{\hat{\eta}_{r,i}, \ \hat{\theta}_{r,i}\}\], conjugate variables

\[\hat{E}_{r,i} = -\hat{\eta}_{r,i}, \ \hat{A}_{r,i} = \hat{\theta}_{r,i}\]

Electric flux and U(1) gauge variables correspond to density fluctuation and U(1) phase variables of BEC at each link.

From the above relation, we substitute the above decoupled operator into the BHM, and keep terms up to second order of density fluctuation. We obtain the following model,

\[
H_a = \frac{1}{2\gamma^2} \sum_r \left( \sum_i \nabla_i \hat{\eta}_{r,i} \right)^2 + \frac{V'_0}{2} \sum_{r,i} \hat{\eta}^2_{r,i} + H_L
\]

\[
H_L = -2J\rho_0 \sum_r \left[ \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4}) \\
+ \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4}) \right]
\]
Gauss’s Law

\[ H_a = \frac{1}{2\gamma^2} \sum_r \left( \sum_i \nabla_i \hat{n}_{r,i} \right)^2 + \frac{V'_0}{2} \sum_{r,i} \hat{n}_{r,i}^2 + H_L \]

Electric term

When we assume

\[ \gamma^2 \to 0 \Rightarrow \text{Gauss’s law appears!} \]

What is this hopping term?

\[ H_L = -2J \rho_0 \sum_r \left[ \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4}) \right. \\
+ \left. \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4}) \right] \]

Higgs coupling term…???
The Higgs field is a complex field defined on site $r$ with its radial excitation frozen (London limit):

$$\phi_r = e^{i\varphi_r}$$

If we translate the hopping terms into the following Higgs coupling term,

$$H'_L = -J\rho_0 \sum_{r, \mu < \nu} \left[ \cos(\varphi_{r+x} + \theta_{r,1} - \theta_{r,2} - \varphi_{r+y}) + \cos(\varphi_{r+x} + \theta_{r,1} - \theta_{r,4} - \varphi_{r-y}) \right]$$

$$+ \cos(\varphi_{r+y} + \theta_{r,2} - \theta_{r,3} - \varphi_{r-x}) + \cos(\varphi_{r-x} + \theta_{r,3} - \theta_{r,4} - \varphi_{r-y}) \right]$$

Furthermore, if we assume unitary gauge,

$$\varphi_r = 0$$

\[ • \text{Then, we can translate the hopping terms into a Higgs coupling term with unitary gauge.} \]

$$H_L = -2J\rho_0 \sum_r \left[ \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4}) \right]$$

$$+ \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4}) \right]$$

\[ • \text{In short, the Higgs field represents a fictitious charged matter field to describe the violation of charge-less Gauss's law in ultra-cold atoms.} \]
Proposal for a feasible experiment

Our motivation:
• To realize the above Gauss’s law, the density-density interaction must be tuned.
• We want to propose such a tuning condition as a feasible experimental method.
From now, Proposal for realizing the “Gauss’s Law”

\[ \sum_{r,a \neq b} \frac{V_{ra,rb}}{2} \hat{\rho}_{r,a} \hat{\rho}_{r,b} \rightarrow \frac{1}{2\gamma^2} \sum_r \left( \sum_i \nabla_i \hat{n}_{r,i} \right)^2 \]

From this density-density interaction, How to create the Gauss’s Law?

We prepares three different kind of dipolar atoms
\[ ^{52}Cr, ^{87}Rb, ^{168}Er \]
\[ 6\mu_{BM}, \mu_{BM}, 7\mu_{BM} \]

To make hoped interaction, we consider mutual interference of these dipole-dipole interactions (DDI).

To this end, we prepare a triple layer optical lattice system which have different lattice geometry.

Trapped atoms on the particular layer feel effective interactions from other atoms on different layers.

We carry out a perturbative calculation to make effective interactions.
1. We create triple layer system.

   Upper  Cr = B boson  
   Middle  Rb = A-boson  
   Lower   Er = C-boson

In the middle layer, the U(1)GHM system can be realized.

2. Each species of bosons is assumed to have a dipole, perpendicular to the plane of the layer. By treating the DDI between A-boson and B-boson as a perturbation, the second-order perturbation theory generates an effective inter-site interaction between the A-bosons.

3. Also, the DDI between A- and C-bosons generates another inter-site interactions between the A-bosons.
We show the two types inter-site interaction

In Cross section 2, the inter-site DDI becomes

\[ H_{AB} = U_{ab} \sum_{k,\delta} \rho_{A,k+\delta} n_{B,k} \]

In Cross section 3, the inter-site DDI becomes

\[ H_{AC} = U_{ac} \sum_{l,\bar{\delta}} \rho_{A,l+\bar{\delta}} n_{C,l} \]
By using the inter-site interactions, we want to change only A-boson long-range interaction (DDI). See the below table.

<table>
<thead>
<tr>
<th>group</th>
<th>range</th>
<th>((a, b))</th>
<th>(V_{ab})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>NN</td>
<td>((1,2), (2,3), (3,4), (1,4))</td>
<td>(\gamma^{-2})</td>
</tr>
<tr>
<td>(ii)</td>
<td>1st half of NNN</td>
<td>((1,3), (2,4))</td>
<td>(\gamma^{-2})</td>
</tr>
<tr>
<td>(iii)</td>
<td>2nd half of NNN</td>
<td>((1,5), (4,6))</td>
<td>0</td>
</tr>
</tbody>
</table>

1. NN and NNN interactions are same amplitude.
2. We want to vanish \((1,5), (4,6)\) link interactions

From now, we will derive effective interactions from path integral formulation.

- The B- and C-bosons are trapped in deep optical lattices with negligible hopping.
- second-order perturbation theory.
We start to define the Inter-layer dipole-dipole interaction,

\[ \int d\mathbf{r}d\mathbf{r}' \frac{1}{2} \psi^\dagger_c(\mathbf{r}) \psi^\dagger_a(\mathbf{r}') U_{DDI}(|\mathbf{r} - \mathbf{r}'|) \psi_c(\mathbf{r}) \psi_a(\mathbf{r}') \]

\[ \sim U_{ac} \sum_{l,\delta} a^\dagger_{l,+\delta} a_{l,+\delta} c^\dagger_l c_l = U_{ac} \sum_{l,\delta} n_{a,l,+\delta} n_{c,l}, \]

\[ U_{ac} = \frac{1}{2} \int d\mathbf{r}d\mathbf{r}' w^2_{c,l}(\mathbf{r}) U_{DDI}(\mathbf{r}, \mathbf{r}') w^2_{a,l,+\delta}(\mathbf{r}'), \]

\[ U_{DDI}(\mathbf{r}, \mathbf{r}') = \frac{d}{|\mathbf{r} - \mathbf{r}'|^3} \left[ 1 - \frac{3a^2_{23}}{|\mathbf{r} - \mathbf{r}'|^2} \right]. \]

Next, C-boson system can be defined as follows,

\[ Z_c \equiv \int [dc_l dc^\dagger_l] \exp\left( \int_0^\beta d\tau \sum_l (-c^\dagger_l(\tau) \partial_\tau c_l(\tau) + \mu c^\dagger_l(\tau) c_l(\tau) \right. \]

\[ -U_c n_{c,l}(\tau) (n_{c,l}(\tau) - 1) + U_{ac} \sum_{\delta} (n_{c,l}(\tau) - \rho_c) n_{a,l,+\delta}(\tau)) \right) \]
Integrate out C-boson system (Second perturbation theory)

\[ Z_c \sim 1 + U_{ac}^2 \int_0^\beta d\tau \sum_{l, \tilde{\delta}} \beta C(\mu_c, U_c, \beta) n_{a,l+\tilde{\delta}}(\tau) n_{a,l-\tilde{\delta}}(\tau) \equiv e^{-\int_0^\beta d\tau H_{qV_c}}. \]

This effective interaction depends on the following factor,

\[ C(\mu_c, U_c, \beta) \equiv \frac{\sum m_l e^{-\beta E_m l} m_l^2}{\sum m_l e^{-\beta E_m l}} - \rho_c^2 = \langle (\hat{n}_c - \langle n_c \rangle)^2 \rangle. \]

This factor is controllable in an optical lattice system!

Also, we can same calculation for B-boson sector.

In this situation, when we assume bare long-range interactions in A-boson system,

\[ \sum_{r, a \neq b} \frac{V_{ra,rb}}{2} \rho_{r,a} \rho_{r,b} = \begin{cases} 2V & (ra,rb) = NN \\ 2V_N & (ra,rb) = NNN \end{cases} \]

The above A-boson interactions can be modified by effective interactions from B- and C-bosons.

We obtain fine tuning relations for realizing Gauss's law.

**Fine tuning relation**

\[ 2V - U_{ab}^2 \beta C(\mu_b, U_b, \beta) - U_{ac}^2 \beta C(\mu_c, U_c, \beta) = 2V_N, \]

\[ 2V_N - U_{ab}^2 \beta C(\mu_b, U_b, \beta) = 0. \]
3. U(1) lattice gauge-Higgs model II:
Numerical results by Monte-Carlo and Gross-Pitaevskii equation.
Expected phases of (2+1)D U(1) gauge-Higgs Model

\[ H_a = \frac{1}{2\gamma^2} \sum_r \left( \sum_i \nabla_i \hat{n}_{r,i} \right)^2 + \frac{V_0'}{2} \sum_{r,i} \hat{n}_{r,i}^2 + H_L \]

\[ H_L = -2J \rho_0 \sum_r \left[ \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4}) \right. \\
+ \left. \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4}) \right] \]

- (2+1)D GH model supports the confinement phase and the Higgs phase. The confinement phase is characterized by the strong phase fluctuation.

- In contrast, the Higgs phase possesses the phase coherence over the system and the system can be regarded as a superfluid phase; the density wave can propagate around the charges.

To check such a phase diagram, we carry out the path-integral Monte-Carlo Simulation by using effective theory (integrate out the density fluctuation field).
Sorry, let me skip the method due to time considerations, I shall propose numerical result.

Numerical result of (2+1)D U(1) gauge-Higgs model

By using Monte-Carlo Simulation

\[ \frac{2J \rho_0}{V_0'} \]

Hopping term large → Higgs phase (Superfluid)

\[ \gamma^2 V_0' \]

Hopping term small

Gauss law coupling large

Confinement phase
Equation of motion: Gauge-Higgs equation (GHE)

The time-dependent equations can be derived from the real-time path integral formulation under saddle-point approximation.

The operators of the original Hamiltonian are replaced by the \(c\)-number fields.

\[
H_a = A \sum_{i \in \text{odd}} \left( \eta_i + \eta_{i+x} - \eta_{i+x+y} - \eta_{i+y} \right)^2 \\
+ \frac{V_0'}{2} \sum_i \eta_i^2 - \tilde{J}' \sum_{<i,j>} \cos(\theta_i - \theta_j)
\]

\[
L = -i \sum_i \eta_i \frac{d\theta_i}{dt} - H_a
\]

\[
\frac{d}{dt} \eta_i = \tilde{J}' \sum_j \sin(\theta_i - \theta_j)
\]

\[
\frac{d}{dt} \theta_i = -2A(\eta_i - \eta_{i+x} + \eta_{i+x+y} - \eta_{i+y}) \\
- 2A(-\eta_{i-x-y} - \eta_{i-y} + \eta_i + \eta_{i-x}) - V_0'\eta_i
\]

\[
\frac{d}{dt} \theta_i = -2A(-\eta_{i-x} - \eta_i + \eta_{i+y} + \eta_{i-x}) \\
- 2A(-\eta_{i-y} - \eta_{i-y+x} + \eta_i + \eta_{i+x}) - V_0'\eta_i
\]

\[
A = \frac{1}{2\gamma^2} \quad \text{Gauss’s law strength}
\]

\[
\frac{V_0'}{2} \quad \text{on-site repulsion}
\]

\[
\tilde{J}' = 2J\rho_0 \quad \text{Hopping strength}
\]

We can simulate the real-time dynamics!
How do we observe real time dynamics in each phase?

First, we assume a single electric flux between two static charges!

To judge Confinement phase or Higgs phase, we put on a single electric flux between static charges.

We consider the dynamical stability of the single flux prepared as an initial condition.

If the flux decays $\rightarrow$ we can specify Higgs phase.
If the flux is stable $\rightarrow$ we can specify Confinement phase.
First, we will show a numerical result in confinement regime.

\[ \gamma^2 = 2, \quad V_0' = 1, \quad J = 0.1 \quad \rho_0 = 1 \]

Flux string is conserved with oscillating, and the frequency and amplitude depend on the strength of Gauss’s law, on-site interaction and hopping strength.
Next, we will show a numerical result in Higgs regime.

We found an intermittent density-wave emission.

Since the amplitude fluctuation of the Higgs field is absent, the phase boundary becomes less clear because the two phases connect with each other through crossover.
The Bose-Hubbard Hamiltonian satisfied with Gauss’s law tuning.

\[
H = - \sum_{r,a \neq b} J_{a,b} \hat{\psi}_{r,a}^\dagger \hat{\psi}_{r,b} + \frac{V_0}{4} \sum_{r,a} \hat{\rho}_{r,a}^2 \\
+ \sum_{r,a \neq b} \frac{V_{ra,rb}}{2} \hat{\rho}_{r,a} \hat{\rho}_{r,b}.
\]

By deriving the saddle-point equation, we obtain a dynamical equation

\[
i \frac{\partial \psi_{r,i}}{\partial t} = -J(\psi_{r,i} + \psi_{r-i,i} + \psi_{r+i,i} + \psi_{r+i-i,i}) \\
+ \left[ \left( V'_0 + \frac{2}{\gamma^2} \right) |\psi_{r,i}|^2 + \frac{1}{\gamma^2} (|\psi_{r-i,i}|^2 + |\psi_{r-i,i}|^2) \\
+ |\psi_{r+i,i}|^2 + |\psi_{r+i-i,i}|^2 + |\psi_{r-i,i}|^2 + |\psi_{r+i,i}|^2 \right] \psi_{r,i}
\]

The amplitude fluctuation in the BH model can give rise to a similar effect of the fluctuation of the Higgs coupling.

The Higgs-confinement transition may become first order and its boundary can be sharp.


We do not intact

London limit

\[|\psi_{r,i}|^2 \approx \rho \sim \rho_0 + \eta\]
In confinement phase, the flux dynamics generated by GPE is much similar to that by GHE.

\[ \gamma^2 = 0.5, V'_0 = 1, J = 1 \]

\[ \gamma^2 V'_0 = \frac{V'_0}{2A} \]
In Higgs regime, the structure of the density flux is gradually lost by emitting the density waves from the charge.

The density waves are generated in a different way:
  - successively in the GPE
  - intermittently in the GHE.
The real observation of single electric flux in a cold atomic system can be done: Higgs phase → flux string vanishes.

Confinement phase → flux string is stable with oscillating.

It is important to note that the Gross-Pitaevskii dynamical approach can give a new method to explore the phase structure and real time dynamics of the LGT.

By using GPE, we must observe such as dynamical features of various configurations of an electric flux or many fluxes (= flux tube).

Recent experiment progress may realize various kind of lattice field models. In particular, so far, I think one of the final goals is to realize a complete QCD simulating system.
Future outlook

• We may directly measure topological objects in Gauge theory ⇒ monopole, instantons.

By using GP equation, the dynamics of such a object may be simulated.

• To construct the \((3+1)\)D U(1) gauge-Higgs model, we need to construct a BCC crystalline type optical lattice.

\[
V = uE^2[\cos^2(k_xx) + \cos^2(k_yy) + \cos^2(k_zz) + 2c(\cos(k_xx)\cos(k_yy) + \cos(k_xx)\cos(k_zz) + \cos(k_yy)\cos(k_zz))] \]

However, to realize faithful Gauss law, more complicated tuning of interactions is needed!

• We will try to measure two types of vortex; One is the BEC phase vortices,

\[
B_r = \Omega_r = rot\theta_{i,j} = \sum_{i,j \in \mathcal{P}} \theta_{i,j} \]

• The other is the magnetic vortices,

\[
\Omega_{M,r} = rot F_{\mu\nu} \quad F_{\mu\nu} = \theta_{r+\mu,\nu} + \theta_{r,\nu} - \theta_{r+\nu,\mu} - \theta_{r,\mu} \]

These topological objects will be able to measured by GPE, and predict real dynamical phenomena on Cold atomic Quantum Simulator.
Appendix
原子と“素粒子”

☆原子は内部構造を持つ

1) \( a + b \rightarrow a^* + b^* \) : “別な粒子”に変化
   内部状態が異なると、別の光学ポテンシャルを感じる
   ➔ 質量が変化した

2) \( a + b \rightarrow a^* + b \) (res.) \( \rightarrow a + b \)
   \[ a \rightarrow \] \[ b \]

   相互作用を自在にコントロール (Feshbach 共鳴)

3) \( a_\alpha \rightarrow a^* \rightarrow a_\beta \)
   laser-assisted tunneling
   サイト リンク サイト

\( \alpha, \beta = spin \ z\text{-comp.} \)
Tight-binding picture

$V = V_0 \sin^2 kx \approx a(kx)^2$

$\psi_0(\mathbf{r} - \mathbf{R}_i)$

$\alpha_{zp}$

Lowest Bloch state

The periodic potential is a small perturbation.

$V_0 \ll E_R$

Harmonic frequency

$\left( \frac{V''}{M} \right)^{1/2} = \left( \frac{2k^2 V_0}{M} \right)^{1/2}$

(Potential depth)/(Zero-point energy) = $\left( \frac{V_0}{E_R} \right)^{1/2}$

$V_0 \gg E_R$

Wannier states localized in individual wells

$\alpha_{zp}/a \sim (\hbar/M\omega_0)^{1/2}/(\lambda_{\text{激光}}/2) = \pi^{-1}(E_R/V_0)$

Bloch waves の広がりと格子間隔の比

“Tight-binding state” Bloch states

$\psi_\mathbf{q}(\mathbf{r}) = \sum_i \exp(i\mathbf{q} \cdot \mathbf{r})\psi_0(\mathbf{r} - \mathbf{R}_i)$
On-site interaction

The pseudo potential approximation in boson system

\[ H_{pp} = \frac{g}{2} \int d^3r \hat{\psi}^\dagger(r) \hat{\psi}^\dagger(r) \hat{\psi}(r) \hat{\psi}(r). \]

Tight-binding description

\[ H_{pp} = U \sum_i \hat{n}_i \hat{n}_i \]

\[ U = g \int d^3r |w(r)|^4 \]

If there are two particle in a well,

\[ U = \sqrt{\frac{8}{\pi kaER}} \left( \frac{V_0}{ER} \right)^{3/4} \]

The Gaussian grand state in the local oscillator potential.
This is not the exact Wannier wave function of the lowest band.

If \( a_s \ll a_{zp} \),
this prohibits to mix higher Wannier states

This parameter U can be easily controlled.

--- Feshbach resonance
Gauss’s law tuning の具体例

具体的なdipoleの大きさから
3層から2層の方が最適

(i)  ⇔  Cr, Rb, Er

Position-dependent microwave transfer

(ii)

特定の層だけ原子を残す

Cr層とRb層を

を近づけることに対応

相互作用計算からtuningの一つの例

Rbの光格子の格子間隔 \( \lambda_2 \sim 580 [nm] \)

\[
V \sim \frac{36\mu_0\mu_B^2}{4\pi\lambda_2^3}, \quad \ell_{AC} = \ell_z \sim 580 [nm], \\
\beta \sim \frac{1}{2V}, \quad U_B \sim 0.3V, \quad \mu_B \sim 2.5V, \\
U_C \sim 1.3V, \quad \mu_C \sim 2V.
\]
(3+1)D LGT 構成のための BCC potential lattice
Summary future outlook:

・現実の原子系でU(1) gauge-Higgs modelの構築方法を提案
　特に原子間相互作用のチューニング(ガウス則の構成に成功)

⇒ インフレーション宇宙を記述する理論モデルへの寄与に期待

The real observation of electric flux in a cold atomic system.

Higgs phase → The flux string vanishes.

Confinement phase → The flux string is stable with oscillating.

⇒ 閉じ込めfluxの形状や長さ、張力をさらに詳しく解析する必要

・ 最終目標は、やはり、SU(N) lattice gauge theoryを目指すべき（もちろんN=3）
　光格子の実験技術をもとに提案できるか？（様々な分野からアイデア募集中！）

・ 格子ゲージ理論（特に格子QED, QCD）の数値計算における難問を回避する
　（fermionの数値計算（負符号問題）, 等）