Atomic Quantum Simulation of U(1) lattice gauge-Higgs model

2015/6/2 Osaka University

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Topics:

1. What is Cold atom in an optical lattice?

2. Atomic Quantum Simulator Ⅰ:
   Basic concepts and a feasible experimental method for U(1) lattice gauge-Higgs model

3. U(1) lattice gauge-Higgs model Ⅱ:
   Numerical results by Monte-Carlo and Gross-Pitaevskii equation.

Useful References

Optical lattice:

- A.Leggett, “Quantum Liquids” (Oxford) , Sec.4.
- I.Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 .
- M. Greiner, et al, Nature **415**, 39–44 (2002).
- D. Jaksch, and P. Zoller, Ann. Phys. (N.Y.) **315**, 52 (2005).
- T. Lahaye et al, Rep. Prog. Phys. **72** 126401 (2009).

Quantum Simulation of Lattice gauge theory (review article):

- U.-J. Wiese, arXiv:1305.1602v1 (2013).
- E. Zohar, J. I. Cirac, B. Reznik, arXiv:1503.02312v1 (2015).
- J. I. Cirac and P. Zoller, Nature Physics **8**, 264–266 (2012)

- K. Kasamatsu, I. Ichinose, and T. Matsui, Phys. Rev. Lett. **111**, 115303 (2013).
- Y.K, K. Kasamatsu, Y. Takahashi, I. Ichinose, T. Matsui, arXiv:1412.7605.

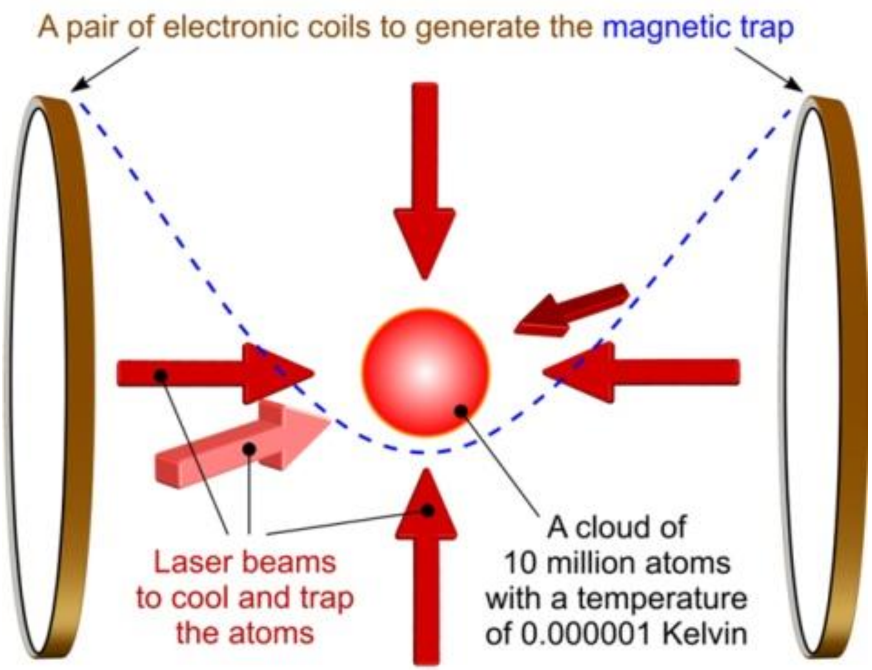
1. What is Cold atom in an optical lattice?

- What is Cold atom ?
- Laser trapping technique
- Optical Lattice
- Bose-Hubbard model

Creating ultra-cold atom in a magnetic trap

By using two experimental methods: Laser Cooling and Evaporative Cooling

Alkali Bose gass nK order !



http://publish.ucc.ie/boolean/2011/00/Russell/42/en

Alkali atom has a magnetic dipole.

The created ultra cold alkali gasses have densities between $10^{12} - 10^{15}$ particles per cm^3.

In such a trapping system, energy shift depending on a hyperfine- atomic levels becomes,

$$\mathrm{V}_i(\mathbf{r}) = E_i(\|\mathbf{B}(\mathbf{r})\|)$$

This is simple Zeeman effect

A electric dipole of an alkali atom couples to a electric field induced by laser.

Time averaged intensity

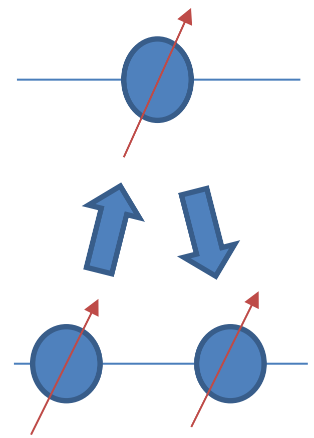
$$\mathbf{F} = \frac{1}{2}\alpha(\omega_L)\nabla(|\mathbf{E}(\mathbf{r})|^2)$$

laser frequency

$$\alpha(\omega_L) \approx \frac{|\langle e|\hat{d}_{\mathbf{E}}|g\rangle|^2}{\hbar\Delta}$$

Second perturbation theory

Electric ground state ➔ a single doublet($^2S_{1/2}$)



|e> First excited state

ω_0 ns-np splitting

|g>

$$V_{dip}(\mathbf{r}) = \frac{3\pi c^2}{2\omega_2^3}\frac{\Gamma}{\Delta}I(\mathbf{r})$$ AC-Stark shift

detuning Laser intensity

detuning

$$\Delta = \omega_L - \omega_0 \begin{cases} > 0 : \text{blue detuning} \\ < 0 : \text{red detuning} \end{cases}$$

Optical lattice

A Standing laser potential for creating Lattice field theory!

Optical lattice potential can be created by standing lasers.
The created electric field becomes periodic potential for cold atoms.

A single linearly polarized laser give rise to an electric field,

$$\mathbf{E}(rt) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

Two counter-propagating lasers produce a standing wave,

$$\mathbf{E}(rt) = 2\mathbf{E}_0 \cos\left(\frac{(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}{2} - \omega t\right)$$

By averaging over the laser frequency , we obtains an effective energy for a single atom.
As one basic periodic potential, we can make the following 3D periodic potential.

$$V_p(x,y,z) = V_0(\sin^2 kx + \sin^2 ky + \sin^2 kz)$$

Here, the potential amplitude can be controlled as follows:

$$V_0 \propto \frac{2I\Gamma^2}{w_0\Delta}$$

This potential depends on detuning and laser intensity.

The cold atoms are tightly confined in each potential minima.

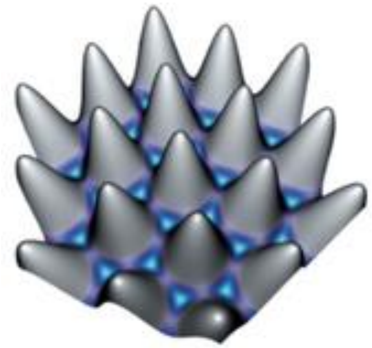
Trapping frequency up to 100kHz.

$$\hbar\omega_0 = 2E_r(V_0/E_r)^{1/2}$$

Optical lattices

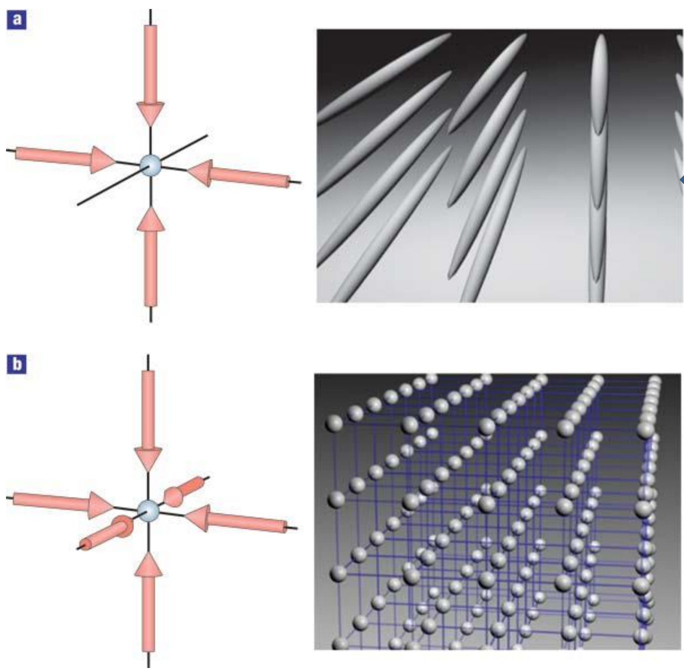
By changing laser parameters, incident angles
and the number of incident laser,
One can make some of geometrical structures.

Honeycomb lattice



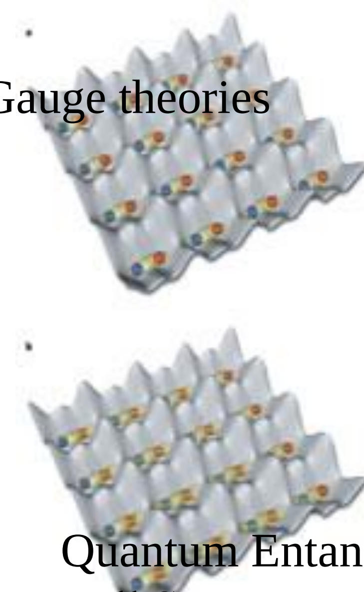
Suitable for Graphene physics and
Haldane Hubbard model, etc..

2D square lattice and 3D cubic lattice



Suitable for
(2+1)D Lattice Gauge theories

Optical Super lattice



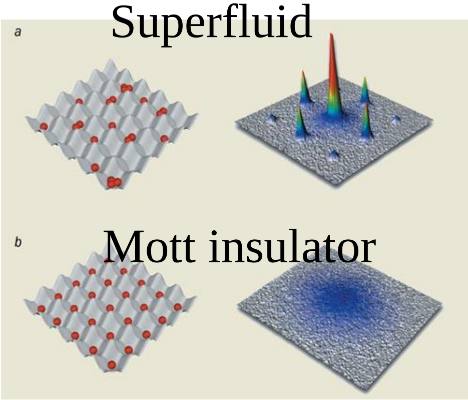
Quantum Entanglement
Bell State…

Bose Hubbard model:

They assumed that 2D superconducting islands linked each other: Each island includes the cooper pairing Boson induced by strong coupling regime.

- Theory: M.P.A.Fisher, et al., Phys. Rev. B **40**, 546 (1989).
- Experiment realization: M. Grainer, et al, Nature **415**, 39–44 (2002).

$$H_{BH} = -t \sum_{<i,j>} b_i^\dagger b_j + \sum_i U n_i(n_i - 1) + \sum_i \mu n_i .$$



momentum distribution

$$t = const.\omega_0 \exp(-\alpha).$$

$$\alpha \equiv (8V_0/E_R)^{1/2}.$$

$$U \sim (V_0/E_R)^{3/4}$$

$$U/t \sim (a/d)\cdot exp(2\sqrt{V_0/E_r})$$

These parameters can be widely controlled !
Quantum phase transition occurs → Mott insulator-Superfluid phase transition

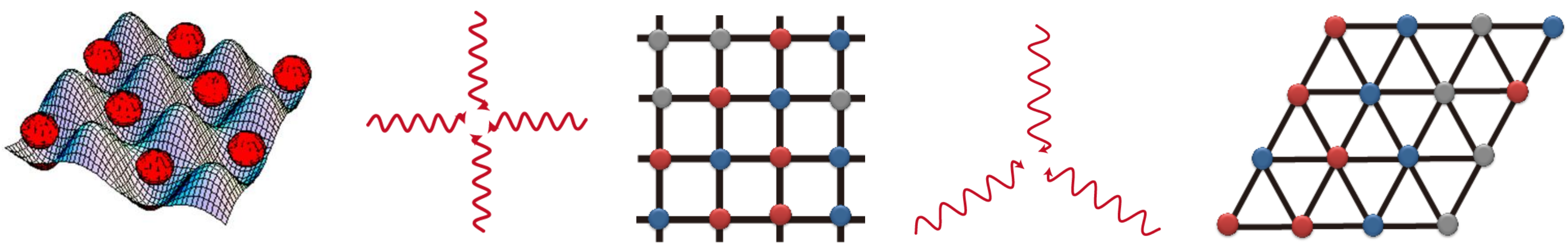
In 2003, by using Rb alkali atoms, the model was realized by Greiner group.
These experimental results corresponded to many theoretical predictions.

If you are interested in deriving the above model, see a famous text,
A. Altland and, B. D. Simons , "Condensed Matter Field Theory"
(Cambridge Univ. press), especially Sec.2.

In an optical lattice, both hopping t and on-site interaction U are easily tunable!

Summary I

1. Cold atom in an Optical lattice may be a versatile Quantum Simulator:
 ･Control system condition
 → particle density, kind of particle, particle interaction, hopping amplitude, artificial magnetic fields, etc.
 → Geometry(square, triangular, and honeycomb, etc.), Dimension(1D,2D,3D, also 4D! by using internal degree of freedom),
 ・ Optical lattice can trap both bosonic and fermionic atoms.

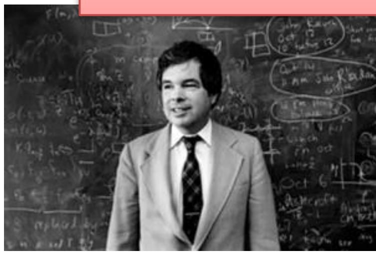


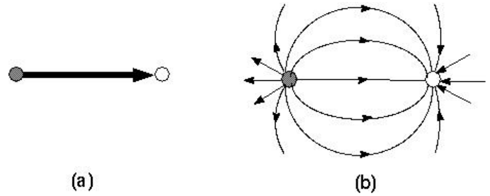
2. Real Experimental Simulator:
 Optical lattices can test Condensed Matter Physics and more Lattice Quantum Field Theory. ➔ Direct observation of many-body physics
 ⇒ We can observe real time dynamics (I will show later)

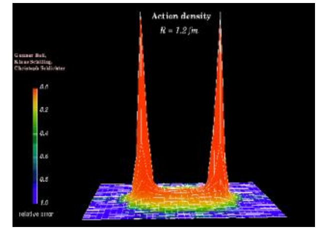
2. Atomic Quantum Simulator Ⅰ:
Basic concepts and a feasible experimental method for U(1) lattice gauge-Higgs model

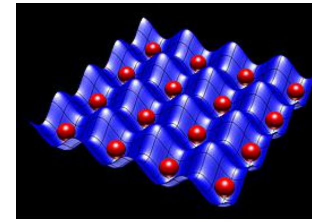
Introduction

J. Ignacio Cirac and Peter Zoller,
Nature Physics 8, 264–266 (2012)









- What is Quantum Simulator?

1. For some interesting Quantum systems,

･To construct artificial, controllable, and versatile quantum system experimentally.

･To detect these dynamics. (imaginary time ⇒ real time)

2. For Lattice Gauge Theories and Strong-Correlation Systems..,

･ Quantum Simulator compensates for classical simulation, and also endows new knowledge.

3. This real experimental simulator can realize many models which have been studied by academic interest.

To create Quantum Simulator for Gauge system.

Local Gauge invariant system must be created.

We need to set optimal Interactions, lattice geometry, and particle number, etc..

In this seminar, our proposal:
U(1) gauge-Higgs model can be experimentally constructed from Bose Hubbard model in BEC state

Local gauge symmetry

In one component cold atom system , difficult…

Cold atom system

Higgs coupling model can be constructed.

This is a U(1) gauge-Higgs model

K. Kasamatsu, I. Ichinose, and T. Matsui, Phys. Rev. Lett. **111**, 115303 (2013).

To construct U(1)GHM, we start to set a extended Bose Hubbard Model (BHM) with long range interaction.

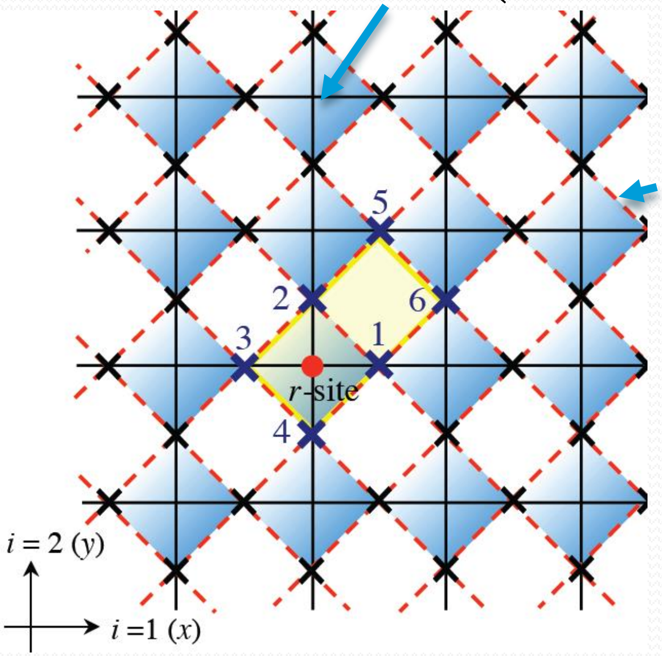
$$H = -\sum_{r,a\neq b} J_{a,b}\hat{\psi}^\dagger_{r,a}\hat{\psi}_{r,b} + \frac{V_0}{4}\sum_{r,a}\hat{\rho}^2_{r,a} + \sum_{r,a\neq b}\frac{V_{ra,rb}}{2}\hat{\rho}_{r,a}\hat{\rho}_{r,b},$$

Density-density interaction, dipole-dipole interaction, etc.

This interaction term generates a Gauss's law.
The tuning condition will be proposed.

This Bose-Hubbard model can be set on the below optical lattice system.

LGT (black filled line)



Optical Lattice (dashed red line)

The dashed red lines indicate the 2D optical lattice with the square geometry, and cold atoms reside on its sites denoted by black crosses. Its unit cell consists of a pair of white and blue squares (yellow region).

The filled black lines indicate the 2D gauge lattice on which the U(1) lattice GHM is defined.

Then the cold atoms are viewed to sit on each link of the gauge lattice to play the role of gauge field.

Assuming uniform atom densities in each link,
the field operator can be decoupled into density fluctuation and phase.

$$\hat{\Psi}_{ri} = \sqrt{\hat{\rho}_{ri}}\, e^{i\hat{\theta}_{ri}} \quad , \quad \hat{\rho}_{ri} = \rho_0 + \hat{\eta}_{ri}$$

mean density fluctuation

We can notice that the canonical relation between density and phase is equivalent to that of U(1) gauge theory.

$\{\hat{\eta}_{ri}, \quad \hat{\theta}_{ri}\}, \quad \text{conjugate variables}$

$$\boxed{\hat{E}_{r,i} = -\hat{\eta}_{r,i} \quad \hat{A}_{r,i} = \hat{\theta}_{r,i}}$$

Electric flux and U(1) gauge variables correspond to density fluctuation and U(1) phase variables of BEC at each link.

From the above relation, we substitute the above decoupled operator into the BHM, and keep terms up to second order of density fluctuation. We obtain the following model,

$$H_a = \frac{1}{2\gamma^2}\sum_r \left(\sum_i \nabla_i \hat{\eta}_{r,i}\right)^2 + \frac{V_0'}{2}\sum_{r,i}\hat{\eta}^2_{r,i} + H_L$$

$$H_L = -2J\rho_0 \sum_r \Big[\cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4}) + \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4})\Big]$$

Gauss's Law Electric term

$$H_a = \frac{1}{2\gamma^2}\sum_r \Big(\underbrace{\sum_i \nabla_i \hat{\eta}_{r,i}}_{\text{Gauss's Law}}\Big)^2 + \underbrace{\frac{V_0'}{2}\sum_{r,i}\hat{\eta}_{r,i}^2}_{\text{Electric term}} + \underbrace{H_L}_{\text{Higgs coupling term....???}}$$

Higgs coupling term….???

When we assume

$$\gamma^2 \to 0 \quad \Rightarrow$$ Gauss's law appears !

What is this hopping term?

$$H_L = -2J\rho_0 \sum_r \Big[\cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4})$$
$$+ \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4})\Big]$$

Why can we regard hopping term as Higgs coupling term?

The Higgs field is a complex field defined on site r with its radial excitation frozen (London limit)

$$\phi_r = e^{i\varphi_r}$$

If we translate the hopping terms into the following Higgs coupling term,

$$H'_L = -J\rho_0 \sum_{r,\mu<\nu} \Big[\cos(\varphi_{r+x} + \theta_{r,1} - \theta_{r,2} - \varphi_{r+y}) + \cos(\varphi_{r+x} + \theta_{r,1} - \theta_{r,4} - \varphi_{r-y})$$
$$+ \cos(\varphi_{r+y} + \theta_{r,2} - \theta_{r,3} - \varphi_{r-x}) + \cos(\varphi_{r-x} + \theta_{r,3} - \theta_{r,4} - \varphi_{r-y}) \Big]$$

Furthermore, if we assume unitary gauge,

$$\varphi_r = 0$$

▪Then, we can translate the hopping terms into a Higgs coupling term with unitary gauge.

$$H_L = -2J\rho_0 \sum_{r} \Big[\cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1} - \hat{\theta}_{r,4})$$
$$+ \cos(\hat{\theta}_{r,2} - \hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3} - \hat{\theta}_{r,4}) \Big]$$

▪In short, the Higgs field represents a fictitious charged matter field to describe the violation of charge-less Gauss's law in ultra-cold atoms.

Proposal for a feasible experiment

Our motivation:

- To realize the above Gauss's law, the density-density interaction must be tuned.
- We want to propose such a tuning condition as a feasible experimental method.

From now, Proposal for realizing the “Gauss’s Law”

$$\sum_{r,a\neq b}\frac{V_{ra,rb}}{2}\hat{\rho}_{r,a}\hat{\rho}_{r,b}, \Rightarrow \frac{1}{2\gamma^2}\sum_r\Big(\sum_i \nabla_i\hat{\eta}_{r,i}\Big)^2.$$

From this density-density interaction,
How to create the Gauss’s Law ?

We prepares three different kind of dipolar atoms

$$^{52}Cr,\ ^{87}Rb,\ ^{168}Er$$

$$6\mu_{BM} \quad \mu_{BM} \quad 7\mu_{BM}$$

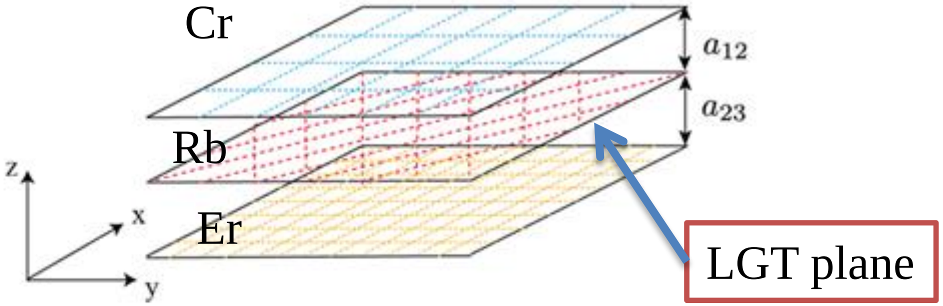
To make hoped interaction, we consider mutual interference of these dipole-dipole interactions (DDI).

To this end, we prepare a triple layer optical lattice system which have different lattice geometry.

Trapped atoms on the particular layer feel effective interactions from other atoms on different layers

$$\Uparrow$$

We carry out a perturbative calculation to make effective interactions.

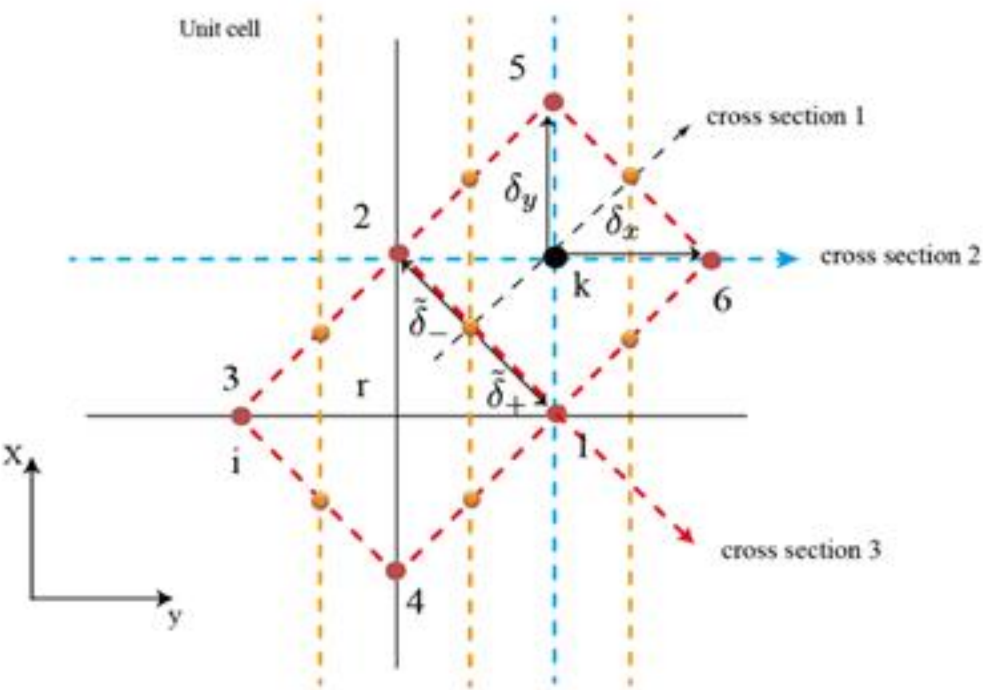


1. We create triple layer system.

Upper Cr = B boson
Middle Rb = A-boson
Lower Er = C-boson

In the middle layer, the U(1)GHM system can be realized.

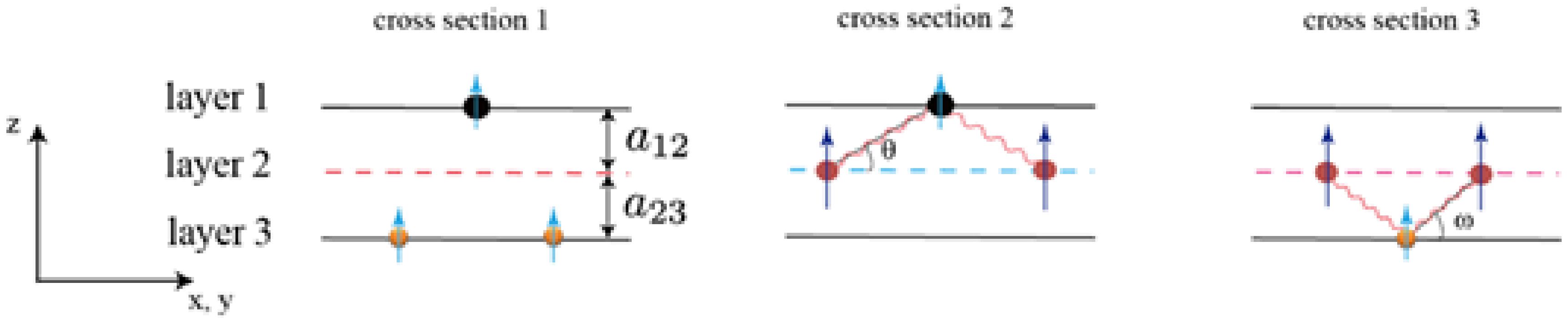
The unit cell of the projective mapping of the all layers.



2. Each species of bosons is assumed to have a dipole, perpendicular to the plane of the layer. By treating the DDI between A-boson and B-boson as a perturbation, the second-order perturbation theory generates an effective inter-site interaction between the A-bosons.

3. Also, the DDI between A- and C-bosons generates another inter-site interactions between the A-bosons.

We show the two types inter-site interaction



In Cross section 2, the inter-site DDI becomes

$$H_{AB} = U_{ab} \sum_{k,\delta} \rho_{A,k+\delta} n_{B,k}$$

In Cross section 3, the inter-site DDI becomes

$$H_{AC} = U_{ac} \sum_{l,\bar{\delta}} \rho_{A,l+\bar{\delta}} n_{C,l}$$

Gauss law tuning

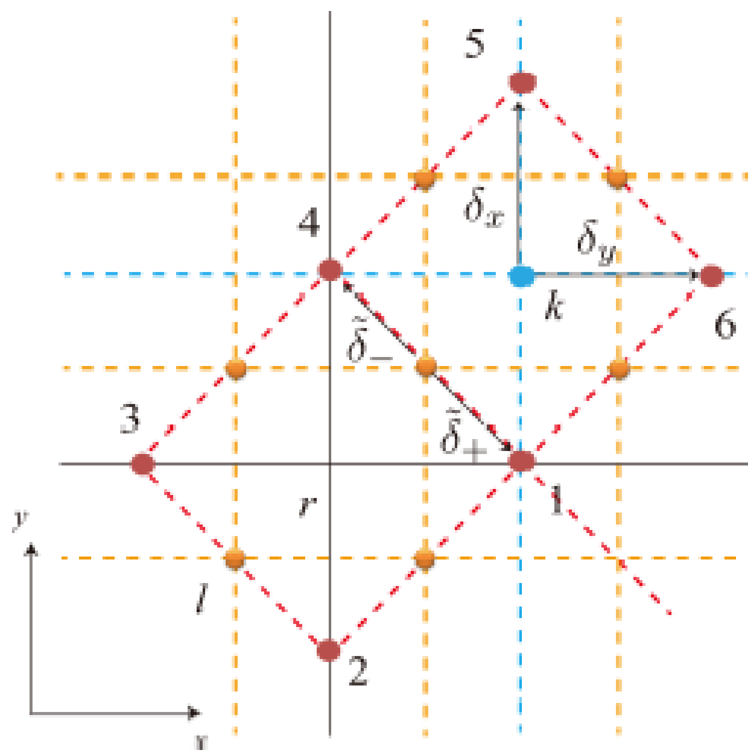
By using the inter-site interactions, we want to change only A-boson long-range interaction(DDI).
See the below table.

group	range	(a,b)	V_{ab}
(i)	NN	(1,2), (2,3), (3,4), (1,4)	γ^{-2}
(ii)	1st half of NNN	(1,3), (2,4)	γ^{-2}
(iii)	2nd half of NNN	(1,5), (4,6)	0

A-boson long-range interaction

1.NN and NNN interactions are same amplitude.

2.We want to vanish (1,5), (4,6) link interactions



From now,
we will derive effective interactions from path integral formulation.

・The B- and C-bosons are trapped in deep optical lattices with negligible hopping.

・second-order perturbation theory.

Calculation in Path-integral formulation(A-boson and C-boson)

We start to define the Inter-layer dipole-dipole interaction,

$$\int d\mathbf{r}d\mathbf{r}'\frac{1}{2}\psi_c^\dagger(\mathbf{r})\psi_a^\dagger(\mathbf{r}')U_{DDI}(|\mathbf{r}-\mathbf{r}'|)\psi_c(\mathbf{r})\psi_a(\mathbf{r}')$$

$$\sim U_{ac}\sum_{l,\tilde{\delta}} a^\dagger_{l+\tilde{\delta}}a_{l+\tilde{\delta}}c^\dagger_l c_l = U_{ac}\sum_{l,\tilde{\delta}} n_{a,l+\tilde{\delta}}n_{c,l},$$

$$U_{ac} = \frac{1}{2}\int d\mathbf{r}d\mathbf{r}' w^2_{c,l}(\mathbf{r})U_{DDI}(\mathbf{r},\mathbf{r}')w^2_{a,l+\tilde{\delta}}(\mathbf{r}'),$$

$$U_{DDI}(\mathbf{r},\mathbf{r}') = \frac{d}{|\mathbf{r}-\mathbf{r}'|^3}\left[1-\frac{3a_{23}^2}{|\mathbf{r}-\mathbf{r}'|^2}\right].$$

Berry phase

Next, C-boson system can be defined as follows,

$$Z_c \equiv \int [dc_l dc_l^\dagger]\exp\Big(\int_0^\beta d\tau\sum_l(-c_l^\dagger(\tau)\partial_\tau c_l(\tau)+\mu_c c_l^\dagger(\tau)c_l(\tau)$$

$$-U_c n_{c,l}(\tau)(n_{c,l}(\tau)-1)+U_{ac}\sum_{\tilde{\delta}}(n_{c,l}(\tau)-\rho_c)n_{a,l+\tilde{\delta}}(\tau))\Big)$$

On-site interaction

Inter-site DDI

Integrate out C-boson system (Second perturbation theory)

$$Z_c \sim 1 + U_{ac}^2 \int_0^\beta d\tau \sum_{l,\tilde{\delta}} \beta C(\mu_c, U_c, \beta) \underline{n_{a,l+\tilde{\delta}}(\tau) n_{a,l-\tilde{\delta}}(\tau)}$$
$$\equiv e^{-\int_0^\beta d\tau H_{qVc}}.$$

A-boson density-density interaction!

This effective interaction depends on the following factor,

$$C(\mu_c, U_c, \beta) \equiv \frac{\sum_{m_l} e^{-\beta E_{m_l}} m_l^2}{\sum_{m_l} e^{-\beta E_{m_l}}} - \rho_c^2 = \langle (\hat{n}_c - \langle n_c \rangle)^2 \rangle.$$

This factor is controllable in an optical lattice system!

Also, we can same calculation for B-boson sector.

In this situation, when we assume bare long-range interactions in A-boson system ,

$$\sum_{r,a\neq b} \frac{V_{ra,rb}}{2} \rho_{r,a}\rho_{r,b} \quad V_{ra,rb} = \begin{cases} 2V & (ra,rb) = \text{NN} \\ 2V_N & (ra,rb) = \text{NNN} \end{cases}$$

The above A-boson interactions can be modified by effective interactions from B- and C-bosons.

We obtain fine tuning relations for realizing Gauss's law.

Fine tuning relation

$$\begin{aligned} 2V - U_{ab}^2 \beta C(\mu_b, U_b, \beta) - U_{ac}^2 \beta C(\mu_c, U_c, \beta) &= 2V_N, \\ 2V_N - U_{ab}^2 \beta C(\mu_b, U_b, \beta) &= 0. \end{aligned}$$

3. U(1) lattice gauge-Higgs model Ⅱ:
Numerical results by Monte-Carlo and Gross-Pitaevskii equation.

Real time dynamics and proposal for feasible experiments of lattice gauge-Higgs model simulated by cold atoms

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[2] *Department of Physics, Kinki University, Higashi-Osaka, Osaka 577-8502, Japan*
[3] *Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*
(Dated: March 17, 2015)

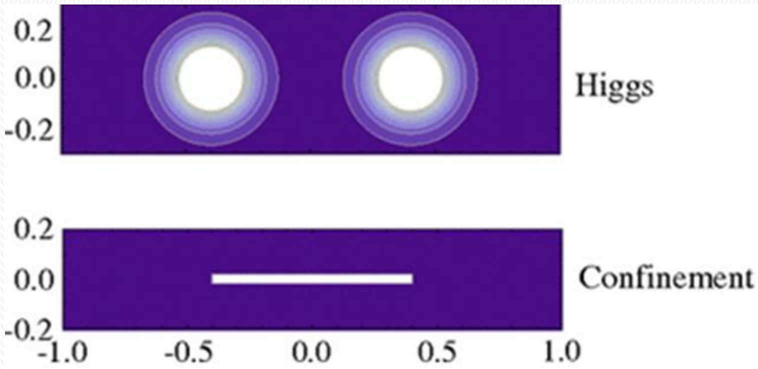
arXiv:1412.7605.

Expected phases of (2+1)D U(1) gauge-Higgs Model

$$H_a = \frac{1}{2\gamma^2}\sum_r\Big(\sum_i \nabla_i \hat{\eta}_{r,i}\Big)^2 + \frac{V_0'}{2}\sum_{r,i}\hat{\eta}_{r,i}^2 + H_L$$

$$H_L = -2J\rho_0\sum_r\Big[\cos(\hat{\theta}_{r,1}-\hat{\theta}_{r,2}) + \cos(\hat{\theta}_{r,1}-\hat{\theta}_{r,4}) + \cos(\hat{\theta}_{r,2}-\hat{\theta}_{r,3}) + \cos(\hat{\theta}_{r,3}-\hat{\theta}_{r,4})\Big]$$

Density modulation pattern



•(2+1)D GH model supports the confinement phase and the Higgs phase.
The confinement phase is characterized by the strong phase fluctuation.

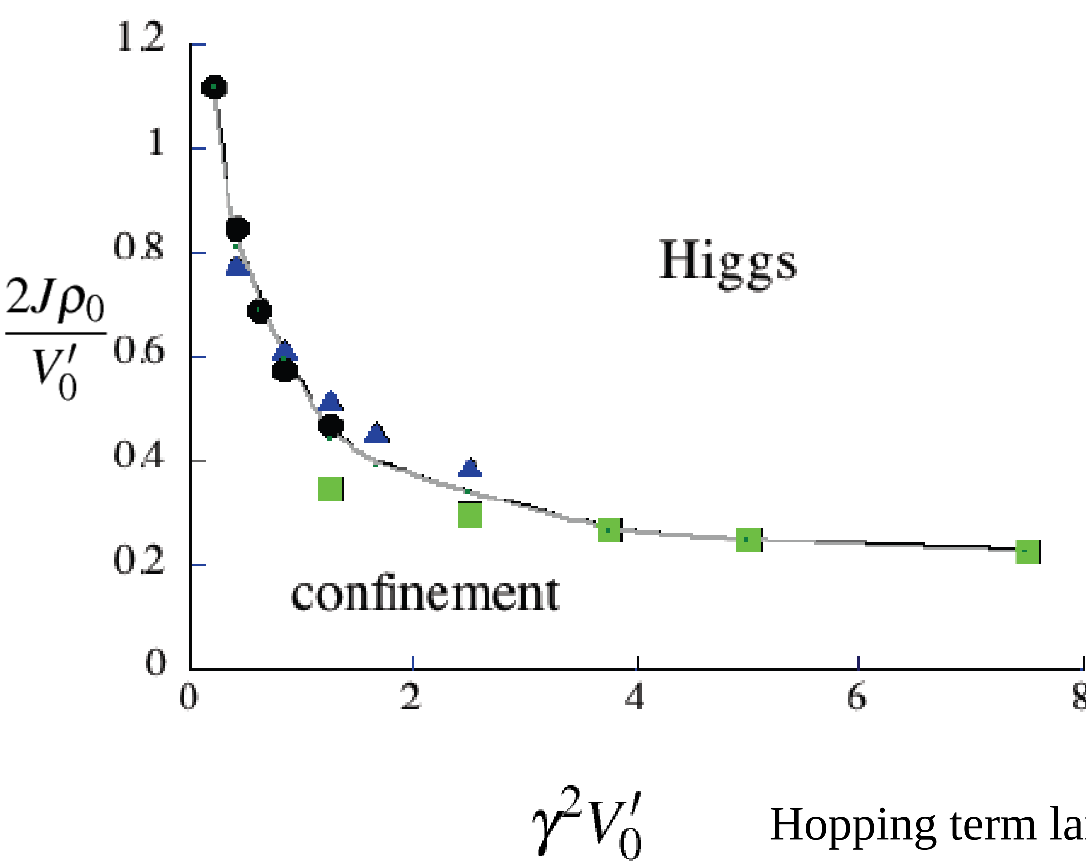
•In contrast, the Higgs phase possesses the phase coherence over the system and the system can be regarded as a superfluid phase; the density wave can propagate around the charges.

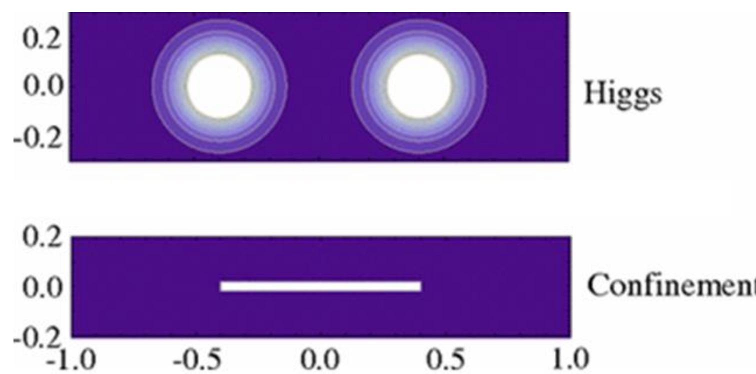
To check such a phase diagram, we carry out the path-integral Monte-Carlo Simulation by using effective theory (integrate out the density fluctuation field).

Sorry, let me skip the method due to time considerations, I shall propose numerical result.

Numerical result of (2+1)D U(1) gauge-Higgs model

By using Monte-Carlo Simulation





Hopping term large → Higgs phase (Superfluid)

Hopping term small
Gauss law coupling large } Confinement phase

Equation of motion: Gauge-Higgs equation (GHE)

The time-dependent equations can be derived from the real-time path integral formulation under saddle-point approximation.
The operators of the original Hamiltonian are replaced by the *c*-number fields.

$$\begin{aligned} H_a &= A \sum_{i \in odd} (\eta_i + \eta_{i+x} - \eta_{i+x+y} - \eta_{i+y})^2 \\ &\quad + \frac{V_0'}{2} \sum_i \eta_i^2 - \tilde{J}' \sum_{<i,j>} \cos(\theta_i - \theta_j) \end{aligned}$$

$A = \dfrac{1}{2\gamma^2}$ Gauss's law strength

$\dfrac{V_0'}{2}$ on-site repulsion

$\tilde{J}' = 2J\rho_0$ Hopping strength

$$L = -i \sum_i \eta_i \frac{d\theta_i}{dt} - H_a$$

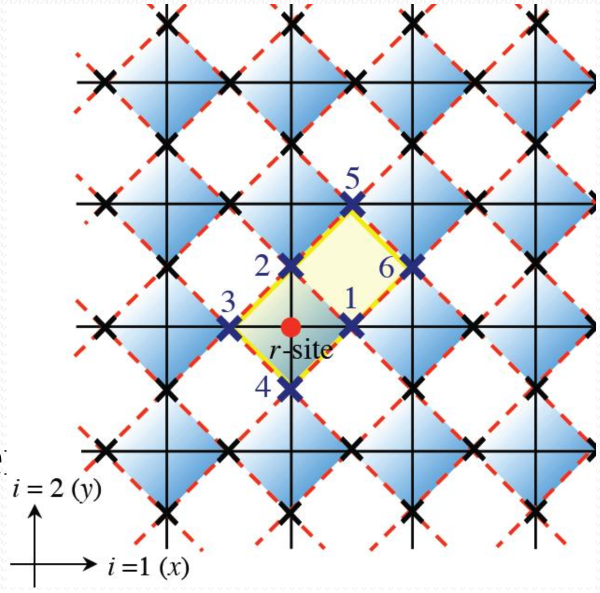
$$\frac{d}{dt}\eta_i = \tilde{J}' \sum_j \sin(\theta_i - \theta_j)$$

$$\begin{aligned} \frac{d}{dt}\theta_i &= -2A(\eta_i - \eta_{i+x} + \eta_{i+x+y} - \eta_{i+y}) \\ &- 2A(-\eta_{i-x-y} - \eta_{i-y} + \eta_i + \eta_{i-x}) - V_0'\eta_i \end{aligned}$$

i: odd

$$\begin{aligned} \frac{d}{dt}\theta_i &= -2A(-\eta_{i-x} - \eta_i + \eta_{i+y} + \eta_{i-x}) \\ &- 2A(-\eta_{i-y} - \eta_{i-y+x} + \eta_i + \eta_{i+x}) - V_0'\eta_i \end{aligned}$$

i: eve



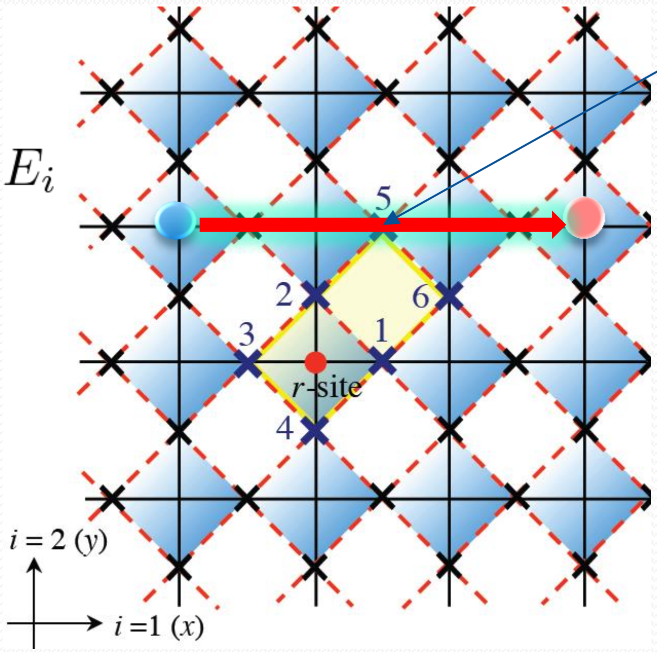
We can simulate the real-time dynamics!

How do we observe real time dynamics in each phase ?

First, we assume a single electric flux between two static charges!

To judge Confinement phase or Higgs phase,
we put on a single electric flux between static charges.

$$\eta_i = -E_i$$



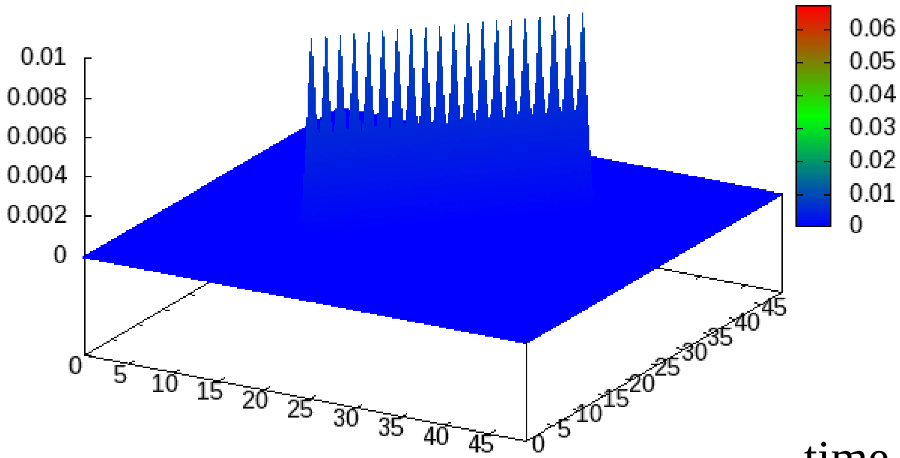
density fluctuation= Electric flux

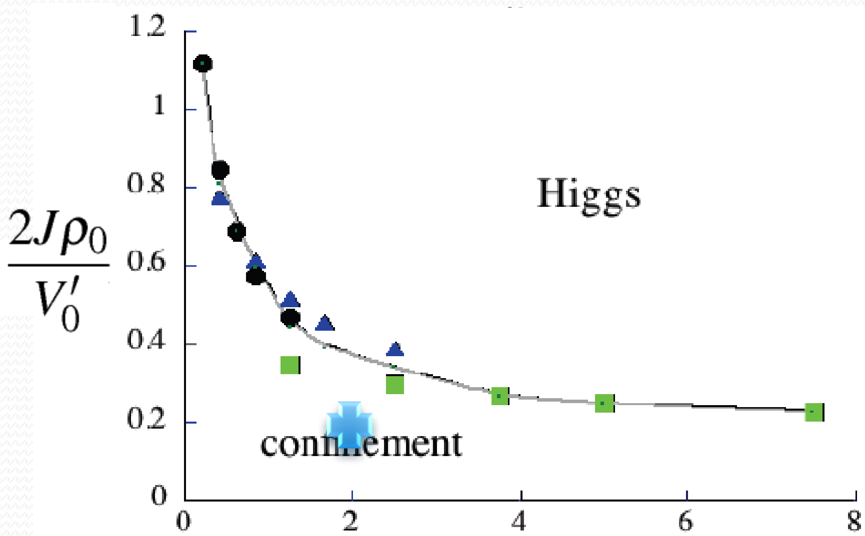
We consider the dynamical stability of the single flux prepared as an initial condition.

If the flux decays $\longrightarrow$ we can specify Higgs phase.
If the flux is stable $\longrightarrow$ we can specify Confinement phase.

First, we will show a numerical result in confinement regime.

Confinement phase





time $\sim$**100** msec

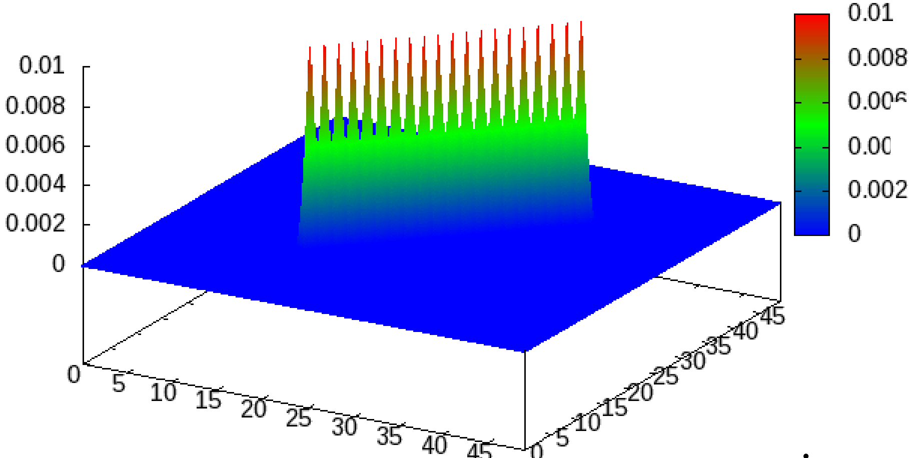
$$\gamma^2 V_0' = \frac{V_0'}{2A}$$

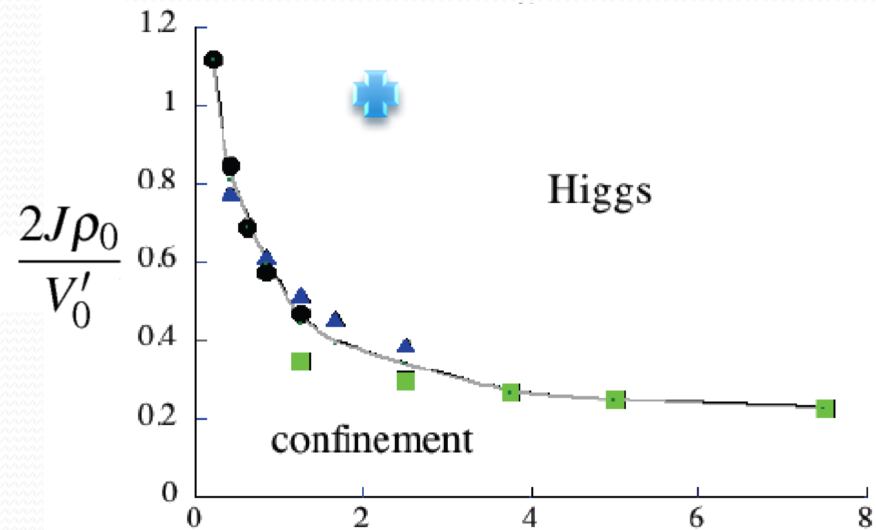
$$\gamma^2 = 2, V_0' = 1, J = 0.1 \quad \rho_0 = 1$$

Flux string is conserved with oscillating, and the frequency and amplitude depend on the strength of Gauss’s law, on-site interaction and hopping strength.

Next, we will show a numerical result in Higgs regime

Higgs phase phase





$$\gamma^2 = 2, V_0' = 1, J = 1$$

time $\sim$**100** msec

$$\rho_0 = 1$$

$$\gamma^2 V_0' = \frac{V_0'}{2A}$$

We found an intermittent density-wave emission.

Since the amplitude fluctuation of the Higgs field is absent, the phase boundary becomes less clear because the two phases connect with each other through crossover.

Full Gross-Pitaevskii equations (GPE)

The Bose-Hubbard Hamiltonian satisfied with Gauss’s law tuning.

$$H = -\sum_{r,a\neq b} J_{a,b}\hat{\psi}^{\dagger}_{r,a}\hat{\psi}_{r,b} + \frac{V_0}{4}\sum_{r,a}\hat{\rho}^2_{r,a} + \sum_{r,a\neq b}\frac{V_{ra,rb}}{2}\hat{\rho}_{r,a}\hat{\rho}_{r,b},$$

By deriving the saddle-point equation,
we obtain a dynamical equation

$$i\frac{\partial\psi_{r,i}}{\partial t} = -J(\psi_{r,\bar{i}} + \psi_{r-\bar{i},\bar{i}} + \psi_{r+i,\bar{i}} + \psi_{r+i-\bar{i},\bar{i}}) + \left[\left(V_0' + \frac{2}{\gamma^2}\right)|\psi_{r,i}|^2 + \frac{1}{\gamma^2}(|\psi_{r,\bar{i}}|^2 + |\psi_{r-\bar{i},\bar{i}}|^2 + |\psi_{r+i,\bar{i}}|^2 + |\psi_{r+i-\bar{i},\bar{i}}|^2 + |\psi_{r-i,i}|^2 + |\psi_{r+i,i}|^2)\right]\psi_{r,i}$$

We do not intact

London limit

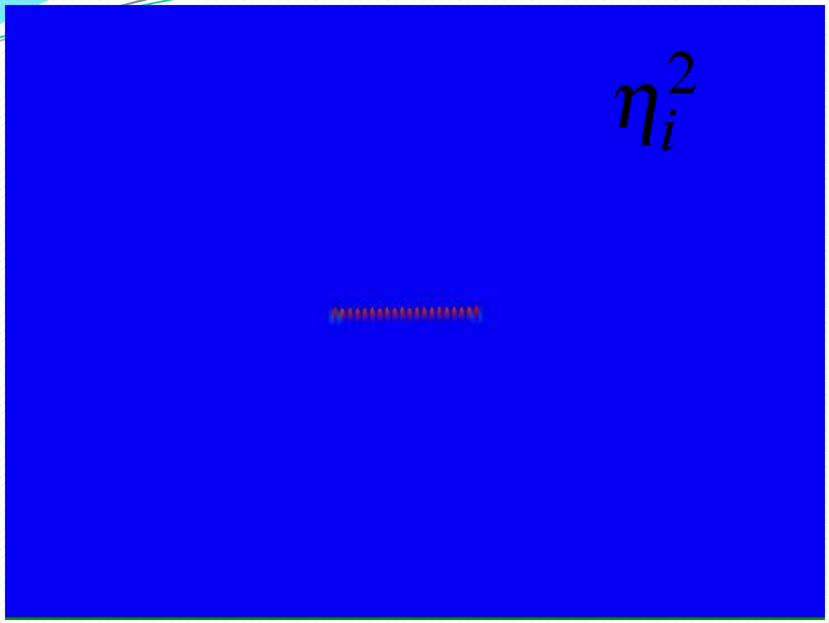
$$\xcancel{|\psi_{r,i}|^2 = \rho \sim \rho_0 + \eta}$$

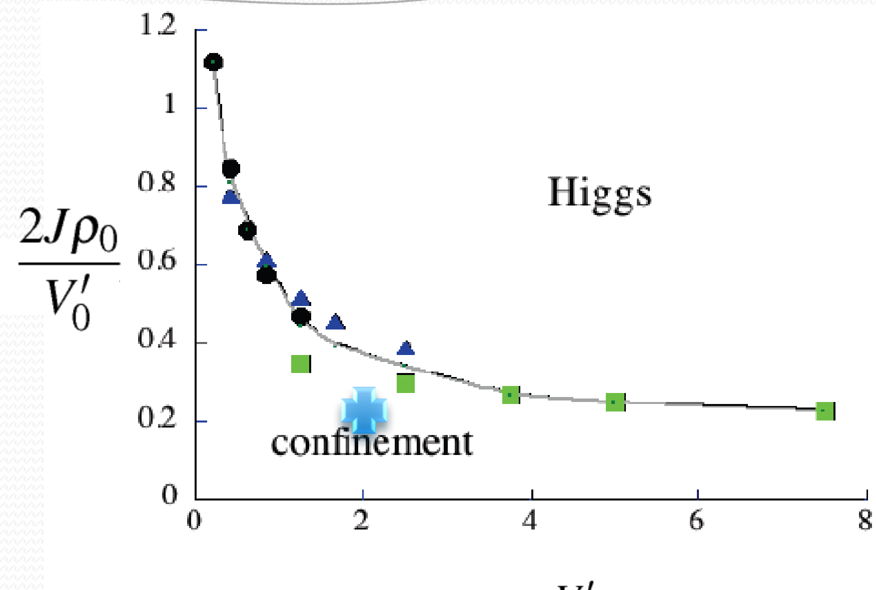
The amplitude fluctuation in the BH model
can give rise to a similar effect of the fluctuation of the Higgs coupling.
The Higgs-confinement transition may become first order and its boundary can be sharp .

S. Wenzel, et.al,
Phys. Rev. Lett. **95**, 051601 (2005).

Let’s see our result

Confinement phase





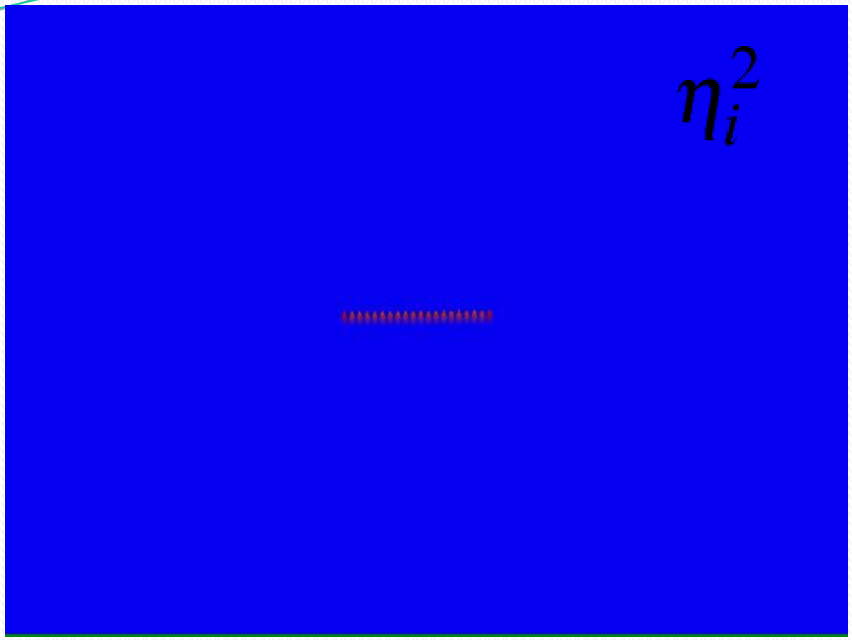
$$\gamma^2 V_0' = \frac{V_0'}{2A}$$

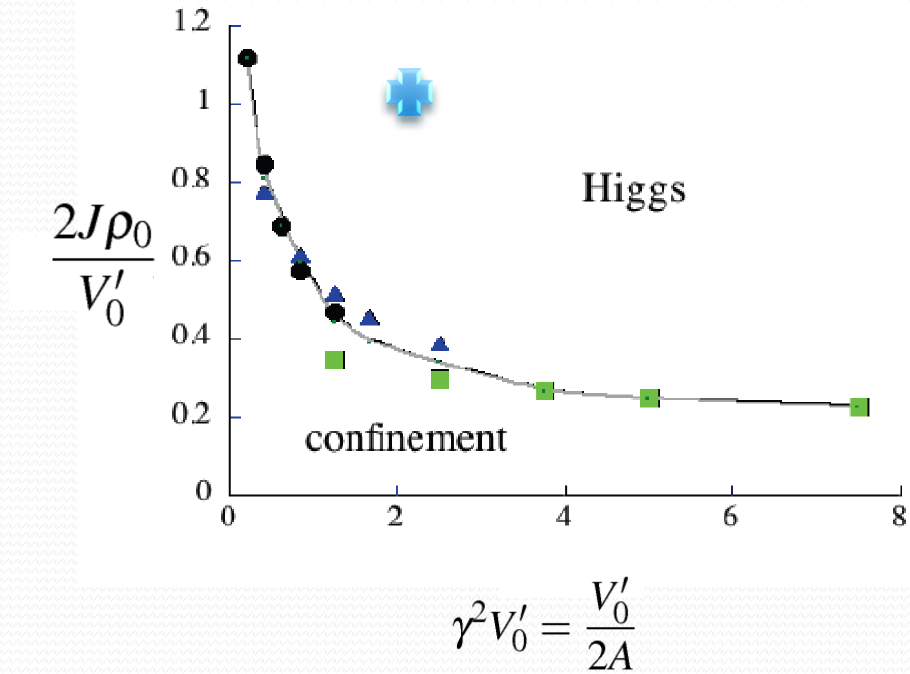
$$\gamma^2 = 0.5, V_0' = 1, J = 1$$

- In confinement phase, the flux dynamics generated by GPE is much similar to that by GHE.

Higgs phase

time ~100 msec





$$\gamma^2 = 1.2, V_0' = 1, J = 1$$

•In Higgs regime, the structure of the density flux is gradually lost by emitting the density waves from the charge.

•The density waves are generated in a different way:

successively in the GPE

intermittently in the GHE.

Summary

- The real observation of single electric flux in a cold atomic system can be done:

Higgs phase $\rightarrow$ flux string vanishes.

Confinement phase $\rightarrow$ flux string is stable with oscillating.

- It is important to note that the Gross-Pitaevskii dynamical approach can give a new method to explore the phase structure and real time dynamics of the LGT.

- By using GPE, we must observe such as dynamical features of various configurations of an electric flux or many fluxes (= flux tube).

- Recent experiment progress may realize various kind of lattice field models.

In particular, so far, I think one of the final goals is to realize a complete QCD simulating system.

Future outlook

•We may directly measure topological objects in Gauge theory $\Rightarrow$ monopole, instantons.

By using GP equation, the dynamics of such a object may be simulated.

•To construct the $(3+1)$D U(1) gauge-Higgs model, we need to construct a BCC crystalline type optical lattice.

$$V = uE^2[\cos^2(k_x x) + \cos^2(k_y y) + \cos^2(k_z z)$$
$$+ 2c(\cos(k_x x)\cos(k_y y) + \cos(k_x x)\cos(k_z z) + \cos(k_y y)\cos(k_z z))]$$

However, to realize faithful Gauss law, more complicated tuning of interactions is needed!

•We will try to measure two types of vortex;
One is the BEC phase vortices,

$$B_r = \Omega_r = rot\theta_{i,j} = \sum_{i,j\in P} \theta_{i,j}$$

•The other is the magnetic vortices,

$$\Omega_{M,r} = rotF_{\mu\nu} \qquad F_{\mu\nu} = \theta_{r+\mu,\nu} + \theta_{r,\nu} - \theta_{r+\nu,\mu} - \theta_{r,\mu}$$

These topological objects will be able to measured by GPE, and predict real dynamical phenomena on Cold atomic Quantum Simulator.

Appendix

原子と``素粒子”

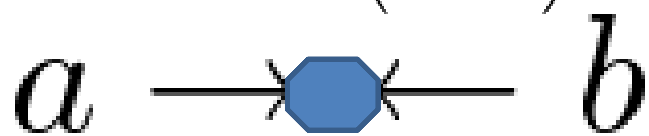
☆原子は内部構造を持つ

1) $a + b \longrightarrow a^* + b^*$: ``別な粒子”に変化

内部状態が異なると、別の光学ポテンシャルを感じる

➔ 質量が変化した

2) $a + b \rightarrow a^* + b\,(res.) \rightarrow a + b$

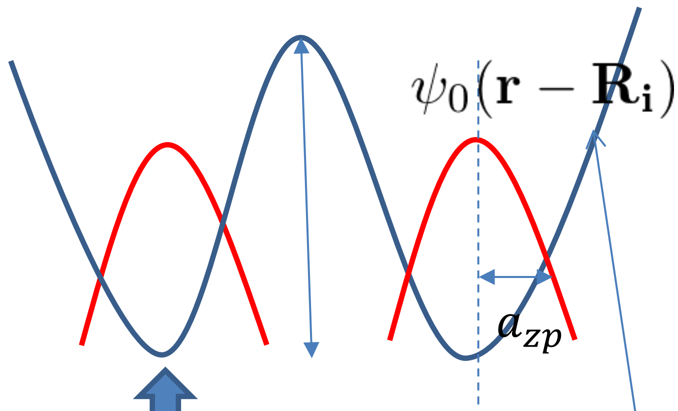


相互作用を自在にコントロール (Feshbach 共鳴)

3) $a_\alpha \longrightarrow a^* \longrightarrow a_\beta$ laser-assisted tunneling

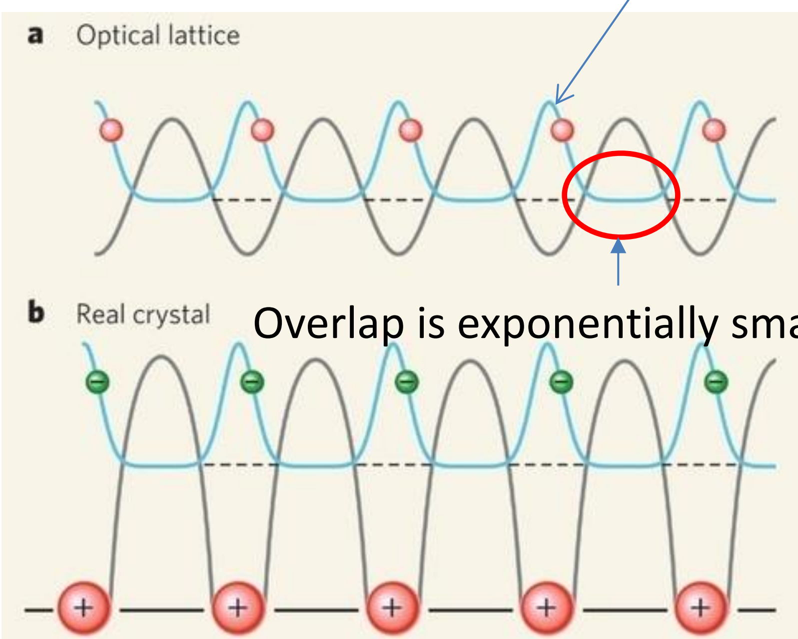
サイト リンク サイト $\alpha,\ \beta = \mathrm{spin}\ \ z\text{-comp.}$

Tight-binding picture



$$V = V_0 \sin^2 kx \approx a(kx)^2$$

Lowest Bloch state



Overlap is exponentially small.

Harmonic frequency

$$\left(\frac{V''}{M}\right)^{1/2} = \left(\frac{2k^2V_0}{M}\right)^{1/2}$$

(Potential depth)/(Zero-point energy)= $\left(\frac{V_0}{E_R}\right)^{1/2}$

$$V_0 << E_R$$

The periodic potential is a small perturbation.

$$V_0 >> E_R$$

Wannier states localized in individual wells

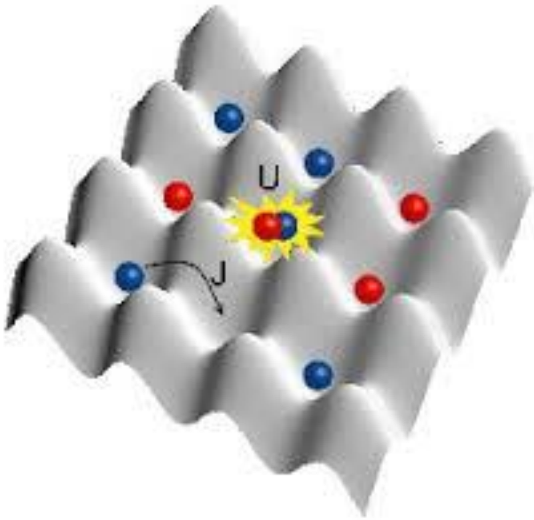
$$a_{zp}/a \sim (\hbar/M\omega_0)^{1/2}/(\lambda_{laser}/2) = \pi^{-1}(E_R/V_0)$$

Bloch waves の広がりと格子間隔の比

“Tight-binding state” Bloch states

$$\psi_{\mathbf{q}}(\mathbf{r}) = \sum_i exp(i\mathbf{q}\cdot\mathbf{r})\psi_0(\mathbf{r}-\mathbf{R_i})$$

On-site interaction



The pseudo potential approximation in boson system

$$H_{pp} = \frac{g}{2}\int d^3r \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}).$$

Tight-binding description

$$H_{pp} = U\sum_i \hat{n}_i\hat{n}_i$$

If $a_s \ll a_{zp}$,
this prohibits to mix higher Wannier states

$$U = g\int d^3r|w(\mathbf{r})|^4$$

The Gaussian grand state in the local oscillator potential. This is not the exact Wannier wave function of the lowest band.

If there are two particle in a well,

$$U = \sqrt{8/\pi}kaE_R\left(\frac{V_0}{E_R}\right)^{3/4}$$

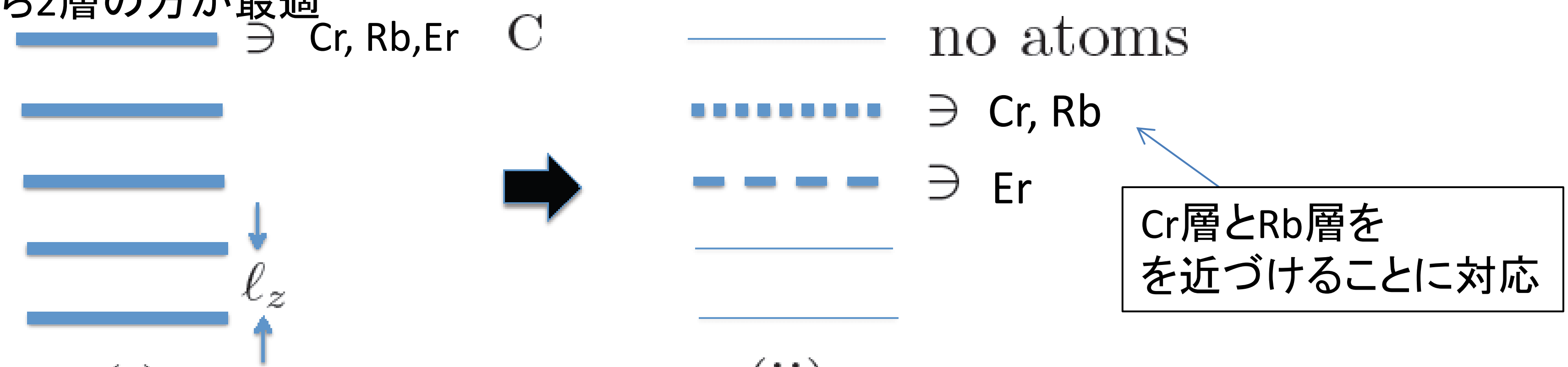
This parameter U can be easily controlled.

---> Feshbach resonance

Gauss's law tuning の具体例

具体的なdipoleの大きさから
3層から2層の方が最適

特定の層だけ原子を残す
Position-dependent microwave transfer



(i) (ii)

相互作用計算からtuningの一つの例

Rbの光格子の格子間隔 $\lambda_2 \sim 580[nm]$

$$V \sim \frac{36\mu_0\mu_{\mathrm{BM}}^2}{4\pi\lambda_2^3}, \qquad \ell_{\mathrm{AC}} = \ell_z \sim 580\ [\mathrm{nm}],$$

$$\beta \sim \frac{1}{2V}, \qquad U_{\ B} \sim 0.3V, \qquad \mu_{\ B} \sim 2.5V,$$

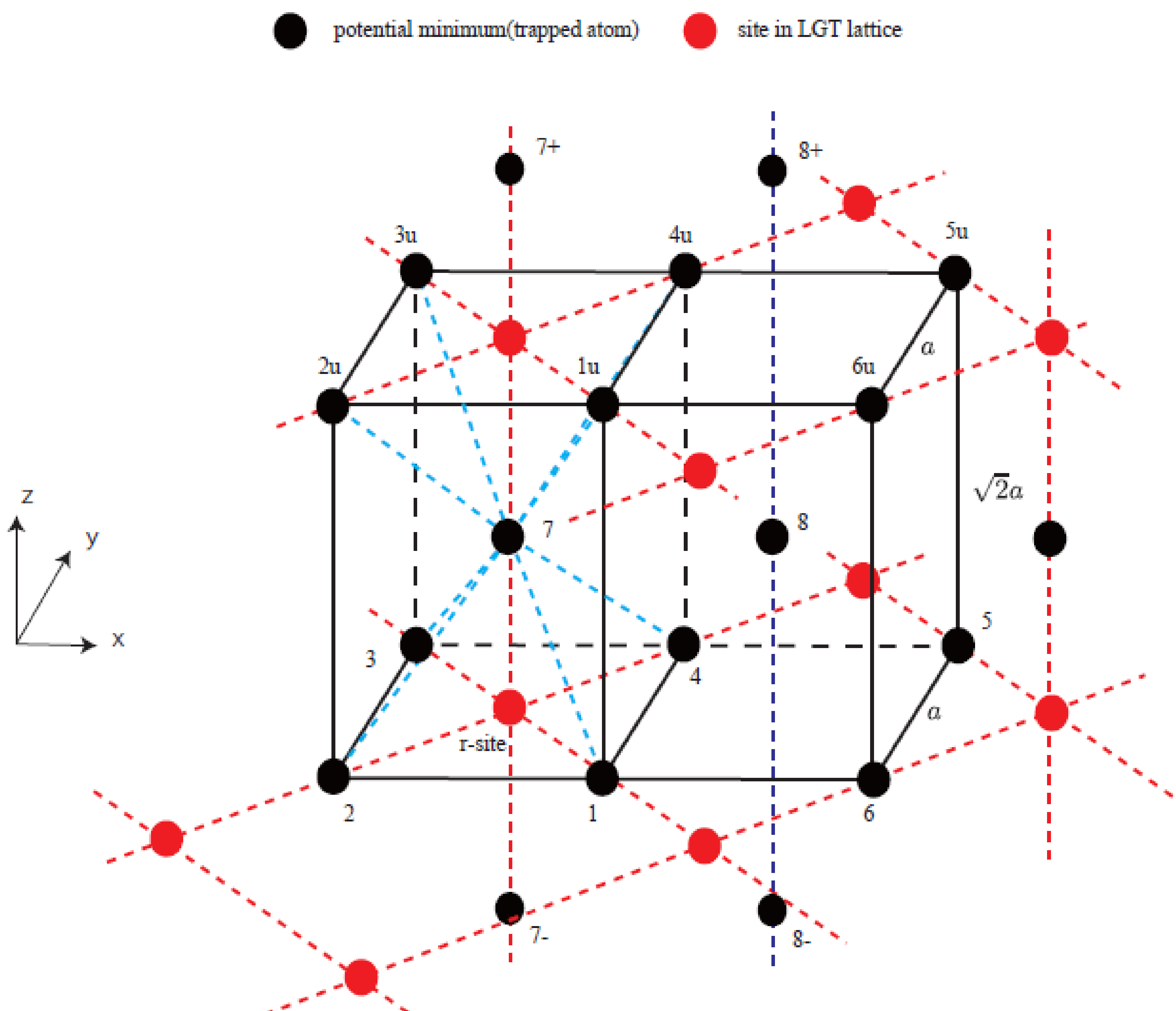
$$U_{\ C} \sim 1.3V, \qquad \mu_{\ C} \sim 2V.$$

V Rbが存在する層の NNのDDI相互作用

$U_{B(C)}$ Cr（Er）のon-site斥力

$\mu_{B(C)}$ Cr（Er）の 化学ポテンシャル

(3＋1)D LGT構成のための BCC potical lattice



Summary future outlook:

・現実の原子系でU(1) gauge-Higgs modelの構築方法を提案
特に原子間相互作用のチューニング（**ガウス則の構成に成功**）

⇒ インフレーション宇宙を記述する理論モデルへの寄与に期待

The real observation of electric flux in a cold atomic system.

Higgs phase → The flux string vanishes.

Confinement phase → The flux string is stable with oscillating.

⇒ 閉じ込めfluxの形状や長さ、張力をさらに詳しく解析する必要

- 最終目標は、やはり、**SU(N) lattice gauge theory** を目指すべき （もちろんN=3）
光格子の実験技術をもとに提案できるか？（様々な分野から**アイデア募集中**！）

- 格子ゲージ理論（特に格子QED, QCD）の数値計算における**難問を回避する**
(fermionの数値計算（負符号問題）, 等)