Twisted fate of BTZ

Wen-Yu Wen (溫 文鈺)
CYCU, Taiwan (中原大學 台灣)
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Introduction

- Logarithmic correction to the black hole entropy could come from various sources; one of them is the loop correction to the surface gravity [D.V. Fursaev 95]

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}(1 + \frac{\sigma}{M^2})$$

$$S = \int dS = \int \frac{dM}{T_H} = \frac{A}{4} - 4\pi\sigma \,\mathrm{Log}(M^2 + \sigma) + \ldots$$

$$A = 4\pi(2M)^2$$

- If the logarithmic term remains at the Planck scale, for $\alpha<0$, one expects a remnant of mass $m_c = \sqrt{-\alpha}$ but with infinite entropy. This is the *trouble for remnants* [Susskind, 95].
- However, if the Hawking radiation can carry away information, a remnant only needs to store a finite amount of entropy (or zero entropy) by a choice of integral constant.

$$S' = 4\pi M^2 - \sigma \log(M^2 + \sigma) - C$$

$$C = 4\pi m_c^2 - \sigma \log(m_c^2 + \sigma)$$

Question to ask

- Does previous argument for remnant still work if a black hole possesses another phase at Planck scale?
- The small black hole in asymptotic Anti-de Sitter space becomes unstable and tunnels into thermal AdS vacuum. This is called the Hawking-Page phase transition.
- We will limit our discussion on the BTZ black hole in AdS_3.

outline

- Loop correction to surface gravity and existence of black hole remnant
- Hawking-Page phase transition and thermal AdS phase
- Competing thermal and remnant phases
- Winding strings and overcooling phase

BTZ black hole (non-rotating)

- The states which can tunnel into AdS vacuum are those BTZ without angular momentum

$$-(-M+\frac{r^2}{\ell^2})dt^2+(-M+\frac{r^2}{\ell^2})^{-1}dr^2+r^2d\phi^2$$

$$r_+ = \sqrt{M}\,\ell$$

$$T_H = \frac{\sqrt{M}}{2\pi\ell}$$

$$S = \frac{A}{4G} = \frac{\pi\ell\sqrt{M}}{2G}$$

$$E = \int T\,dS = \frac{M}{8G}$$

$$F_{BTZ} = E - TS = -\frac{M}{8G}$$

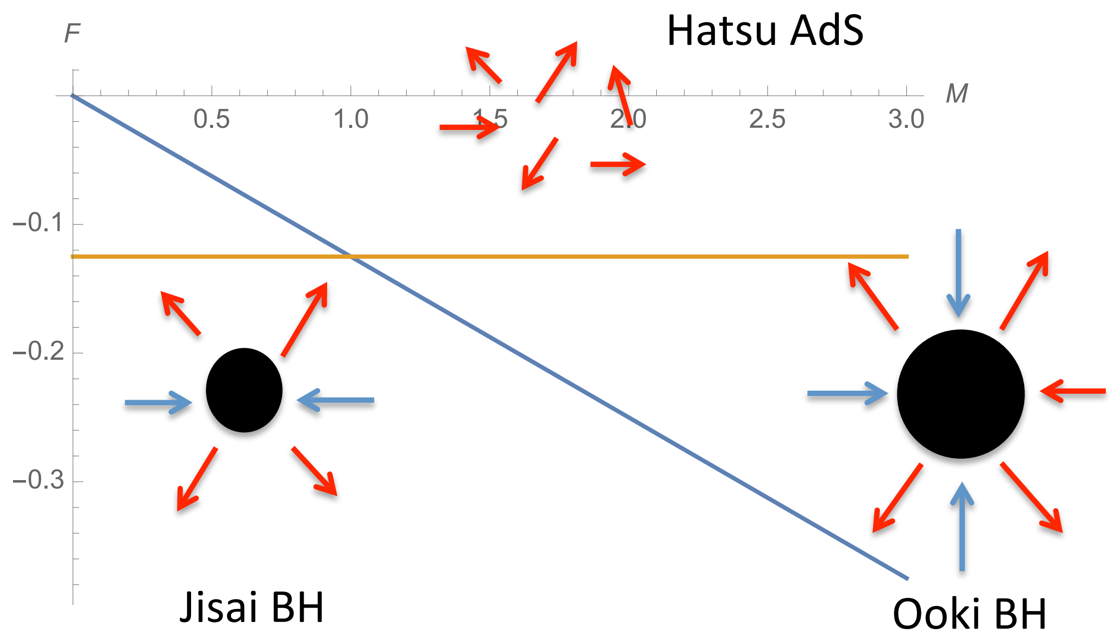
Thermal AdS and Hawking-Page phase transition

- The pure AdS3 at finite temperature is given by compactified wick-rotated time of periodicity β=1/T
- Phase transition happens at M=1, where free energy of BTZ is higher than that of thermal AdS

$$I = -\frac{\beta}{8G} = \beta F$$

$$F_{AdS} = -\frac{1}{8G}$$

$$T_{HP} = \frac{1}{2\pi \ell}$$



Loop quantum corrections to BTZ

- Loop correction to Hawking temperature

$$T_H^q = T_H/(1+\sum_i \alpha_i \frac{\hbar^i}{r_+^i})$$

- Log-corrected entropy

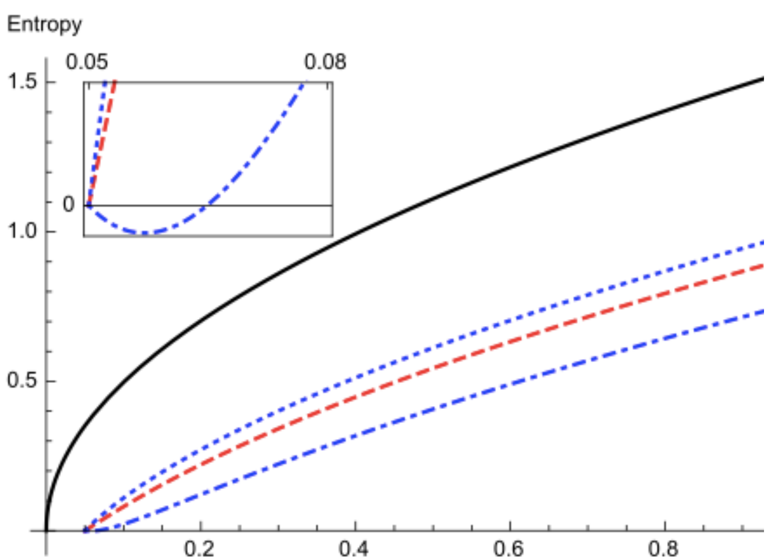
$$S_{BH}^{q'} = \int \frac{dM}{T_H^q} = \frac{\pi r_+}{2G} + \alpha_1 \hbar \frac{\pi}{2G} \ln r_+ - \alpha_2 \hbar^2 \frac{\pi}{2G r_+} + - \cdots$$

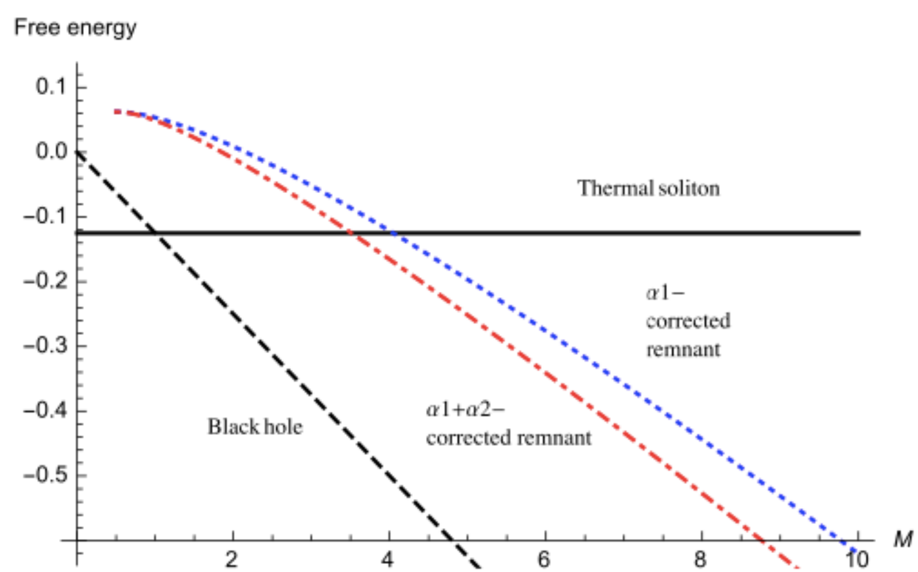
- Entropy and free energy for BTZ with remnant

$$S^q_{BH} = \frac{\pi l\sqrt{M}}{2G} - \frac{\pi l\sqrt{m_c}}{2G} + \alpha_1\hbar\frac{\pi}{4G}\ln(M/m_c) - \alpha_2\hbar^2\frac{\pi}{2Gl\sqrt{M}} + \alpha_2\hbar^2\frac{\pi}{2Gl\sqrt{m_c}} + - \cdots$$

$$T^q_H = T_H/(1 + \sum_i \alpha_i \frac{\hbar^i}{r^i_+})$$

$$F^q_{BH} = E - T^q_H S^q_{BH}$$





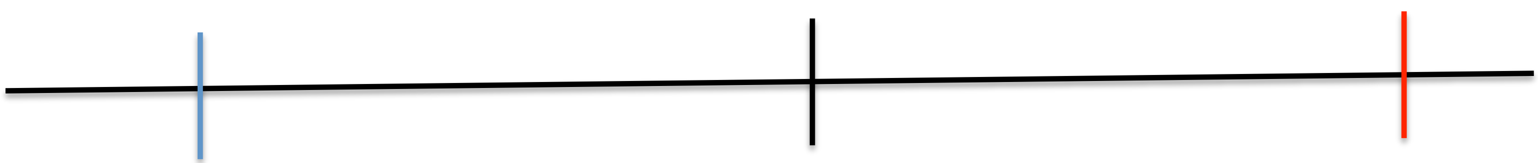
Conclusion for the moment

- The Hawking-Page phase transition happens at slightly higher mass for BTZ with a remnant
- Thermal AdS is still energetically favored than BTZ at low temperature

Implication of BTZ remnant scale

- Though the BTZ remnant phase is not energetically favored, this IR scale could have implied corresponding UV scale (S-duality)

$$Z_{BTZ}(T_{BTZ},\Omega) = Z_{AdS}(T_{AdS},\Omega), \qquad T_{AdS} = \frac{1-\Omega^2 l^2}{4\pi^2 T_{BTZ} l^2}$$

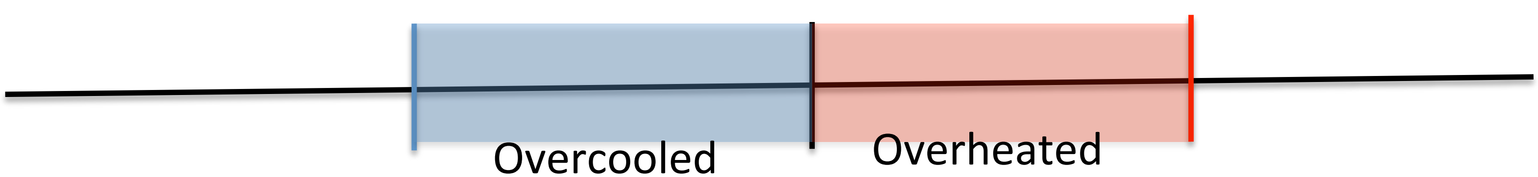


$$T_{IR} = \frac{\sqrt{m_c}}{2\pi\ell} \qquad T_{HP} = \frac{1}{2\pi\ell} \qquad T_{UV} = \frac{1}{2\pi\ell\sqrt{m_c}}$$

Overcooled/heated phase and Atick-Witten winding mode

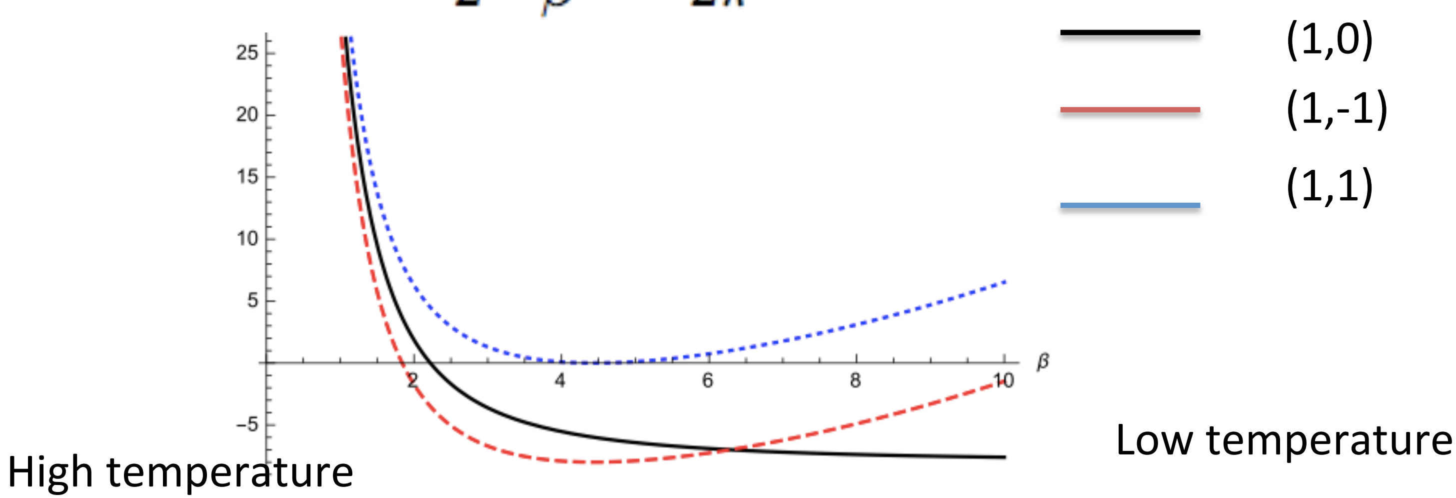
- BTZ can be regarded as effective theory of string at low energy limit. Now we include the string excitation, given by SL(2,R) WZW at level k.
- It was known that there appear two Hawking-Page temperatures T_H and T_L due to Atick-Witten tachyonic winding (momentum) mode [Atick-Witten, 88]

$$T_L = T_{HP}\frac{1}{\sqrt{k}}(4-\frac{1}{k-2})^{1/2} = \frac{4\pi^2 l^2}{T_H}$$



- If remnant scale happens to be located between T_L and T_{HP}, then it might still survive in the overcooled phase for some time. The tachyonic momentum mode is stabilized by the winding mode of mass scale m_c.

$$\frac{M^2}{4} = N + \frac{1}{2}\left(\frac{m\pi}{\beta} + \frac{n\beta}{2\pi}\right)^2 - 1 + \tilde{N} + \frac{1}{2}\left(\frac{m\pi}{\beta} - \frac{n\beta}{2\pi}\right)^2 - 1$$



Conclusion

- The remnant state is less energetically favored than thermal AdS below Hawking-Page temperature. However, it might be metastable in the overcooled phase thanks to stringy excitation.

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感謝