Charged lepton flavor violating process, \( \mu^- e^- \to e^- e^- \) in a muonic atom

Contents

1. Introduction
   - Charged Lepton Flavor Violation (CLFV)
   - CLFV search using muons
   - $\mu^{-}e^{-} \rightarrow e^{-}e^{-}$ in a muonic atom

2. Formulation
   - Partial wave expansion
   - Relativistic treatment for bound leptons

3. Results
   - Decay rates (Branching ratios)
   - Model-discriminating power

4. Summary
1. Introduction
1. Introduction

**Charged Lepton Flavor Violation (CLFV)**

- A probe for new physics -

- lepton flavor #, \( L_e, L_\mu, L_\tau \) (Please distinguish them from lepton #, \( L = L_e + L_\mu + L_\tau \).)

<table>
<thead>
<tr>
<th></th>
<th>( e^- )</th>
<th>( \mu^- )</th>
<th>( \tau^- )</th>
<th>( \nu_e )</th>
<th>( \nu_\mu )</th>
<th>( \nu_\tau )</th>
<th>( e^+ )</th>
<th>( \mu^+ )</th>
<th>( \tau^+ )</th>
<th>( \bar{\nu}_e )</th>
<th>( \bar{\nu}_\mu )</th>
<th>( \bar{\nu}_\tau )</th>
<th>others</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_e )</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( L_\mu )</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( L_\tau )</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

◆ lepton flavor violation in charged lepton sector = [CLFV]

- forbidden in SM
- predicted in many theories beyond SM
- contribution of neutrino oscillation → very small

\[ Br(\mu \to e\gamma) < 10^{-54} \]

✓ cannot be observed by current technology (does not contaminate the search for new physics)

If seen, that is an evidence of new physics !! (not \( \nu \) osci.)
1. Introduction

CLFV search using muons

Advantages of using muon for rare process

1. long lifetime and simple kinematics
2. high intensity muon beam ($\sim 10^{8-9}$ muons per a second)

Examples of CLFV processes using muons

<table>
<thead>
<tr>
<th>Process</th>
<th>BR $\leq$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\mu^+ \rightarrow e^+\gamma$</td>
<td>$5.7 \times 10^{-13}$</td>
<td>Phys. Rev. Lett. 110, 201801 (2013).</td>
</tr>
<tr>
<td>b) $\mu^+ \rightarrow e^+e^-e^+$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>Nucl. Phys. B 299, 1 (1988).</td>
</tr>
</tbody>
</table>

New experiments for “$\mu^- - e^-$ conversion” are planned with higher sensitivity than previous ones. (COMET, DeeMe @ J-PARC, Mu2e @ Fermilab)
1. Introduction

\[ \mu^- e^- \rightarrow e^- e^- \text{ in a muonic atom} \]


New CLFV search using muonic atoms

proposal in COMET


Features

• clear signal: two \( e^- \) s (\( E_1 + E_2 \approx m_\mu + m_e - B_\mu - B_e \))

• 2 type CLFV interactions
  √ \( \mu eee \) vertex
  √ \( \mu e\gamma \) vertex
  (similar to \( \mu^+ \rightarrow e^+ e^+ e^- \))

• atomic \# \( Z \): large \( \Rightarrow \) decay rate \( \Gamma \): large \( (\Gamma \propto (Z - 1)^3) \)
1. Introduction

(Rough) Estimation of decay rate

Suppose nuclear Coulomb potential is weak,

\[ \Gamma_{\mu^- e^- \rightarrow e^- e^-} = 2\sigma v_{\text{rel}}|\psi_{1S}^e(0)|^2 \]

(sum of two 1S e^-s)

\[ \sigma : \text{cross section of } \mu^- e^- \rightarrow e^- e^- \]

(free particles')

\[ \psi_{1S}^e(\vec{x}) = \sqrt{\frac{(m_e(Z - 1)\alpha)^3}{\pi}} \exp(-m_e(Z - 1)\alpha|\vec{x}|) \]

: wave function of 1S bound electron (non-rela)

\[ \Gamma \propto (Z - 1)^3 \]

(the same Z dependence in the both contact & photonic cases)

\[ \Gamma = \sigma v_{\text{rel}} \int dV \rho_{\mu} \rho_e \]

1. Introduction

Branching ratio

How many muons decay, compared to created #?

\[ \text{Br}(\mu^- e^- \rightarrow e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \rightarrow e^- e^-) \]

\( \tilde{\tau}_\mu \): lifetime of a muonic atom

cf. 2.2\(\mu\)s for a muonic H (\(Z = 1\))
80ns for a muonic Pb (\(Z = 82\))

- increasing as atomic # \(Z\) is larger

Using muonic atom with large \(Z\) is favored.

Constraint from previous CLFV experiments

\[ 10^{-15} \]
\[ 10^{-16} \]
\[ 10^{-17} \]
\[ 10^{-18} \]
\[ 10^{-19} \]

A
H
II
II
Au
Pb

Atomic Number


cf. Future experiment (COMET Phase-II)
can product \(\mathcal{O}(10^{18-19})\) muonic atoms.
To improve the previous estimation

\[ \Gamma_{\mu-e^{-}\rightarrow e^{-}e^{-}} = 2\sigma \nu_{\text{rel}} |\psi_{1S}^{e}(0)|^{2} \propto (Z - 1)^{3} \]

(valid when nuclear Coulomb potential is sufficiently small)

- not able to estimate differential decay rate
- "Z dependence" doesn’t depend on CLFV int. (always \( \Gamma \propto (Z - 1)^{3} \))

In atoms with large \( Z \),
- small orbital radius
- relativistic (especially, \( e^{-} \))
- Coulomb distortion

More quantitative estimation is needed! (important for large \( Z \))

1. Introduction

\( \bullet \) Koike et al.

- emitted \( e^{-} \): plane wave
- spacial extension of bound lepton \( \gg \) wave length of emitted \( e^{-} \)
- bound lepton: non-rela
- emitted \( e^{-} \): plane wave

\( \bullet \) emitted \( e^{-} \): plane wave

\( \bullet \) spacial extension of bound lepton \( \gg \) wave length of emitted \( e^{-} \)

\( \bullet \) bound lepton: non-rela

\( \bullet \) emitted \( e^{-} \): plane wave

\( \bullet \) approximations \( (Z\alpha \ll 1) \)
1. Introduction

Improvement and expectation

✓ this work

lepton wave function: relativistic Coulomb

the improvement contains...

• finite orbit-size of bound leptons
• relativistic effects for bound leptons
• Coulomb distortion of emitted $e^-$

influence for $\Gamma_{\mu^- e^- \rightarrow e^- e^-}$

- suppressed
- enhanced

Totally, how CLFV decay rates are changed?

other expectation

➢ we can estimate energy & angular distribution of emitted $e^-$ pair
➢ we may get different results, depending on CLFV interaction
2. Formulation
2. Formulation

**Effective Lagrangian**

\[ \mathcal{L}_I = \mathcal{L}_{\text{contact}} + \mathcal{L}_{\text{photo}} \]

**Contact interaction** (short-range process)

\[ \mathcal{L}_{\text{contact}} = g_1(\bar{e}_L\mu_R)(\bar{e}_Le_R) + g_2(\bar{e}_R\mu_L)(\bar{e}_Re_L) + g_3(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_R\gamma^\mu e_R) + g_4(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_L\gamma^\mu e_L) + g_5(\bar{e}_R\gamma_\mu\mu_R)(\bar{e}_L\gamma^\mu e_L) + g_6(\bar{e}_L\gamma_\mu\mu_L)(\bar{e}_R\gamma^\mu e_R) + [h.c.] \]

**Photonic interaction** (long-range process)

\[ \mathcal{L}_{\text{photo}} = g_R\bar{e}_L\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\bar{e}_R\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.] \]
2. Formulation

\[ \mu^- e^- \rightarrow e^- e^- \] process

**Initial State**
- Bound \( \mu^- \) (1S) & bound \( e^- \)

**Final State**
- 2 scattering \( e^- \)s (distorted wave)
  
  \[ E_1 + E_2 = m_\mu + m_e - B_\mu - B_e \]

- Ignore nuclear recoil energy
- Ignore interaction among \( e^- \)s
2. Formulation

**Decay rate**

\[
\Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi^s_1(p_1)\psi^s_2(p_2) | H | \psi^{s\mu}_u(1s)\psi^{s\mu}_e(1s) \rangle \right|^2
\]

use partial wave expansion to express the distortion

\[
\psi^s_e(p) = \sum_{\kappa,\mu,m} 4\pi i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y^*_{l_\kappa,m}(\hat{r}) e^{-i\delta_\kappa} \psi^\kappa,\mu_p
\]

get radial functions by solving Dirac eq. numerically

\[
\begin{align*}
\frac{dg_\kappa(r)}{dr} + \frac{1 + \kappa}{r} g_\kappa(r) - (E + m + e\phi(r)) f_\kappa(r) &= 0 \\
\frac{df_\kappa(r)}{dr} + \frac{1 - \kappa}{r} f_\kappa(r) + (E - m + e\phi(r)) g_\kappa(r) &= 0
\end{align*}
\]

\[
\phi : \text{nuclear Coulomb potential}
\]
2. Formulation

Energy & angular distribution

\[ E_1: \text{energy of an emitted electron} \]
\[ \theta: \text{angle between two emitted electrons} \]

\[
\Gamma(\mu^{-}(1S)e^{-}(\alpha) \rightarrow e^{-}e^{-}) = \frac{1}{2} \int_{m_e}^{m_{\mu}} dE_1 \int_{-1}^{1} d\cos\theta \frac{d^2\Gamma}{dE_1 d\cos\theta}
\]

**differential decay rate:**

\[
\frac{d^2\Gamma}{dE_1 d\cos\theta} = \sum_{\kappa_1, \kappa_2, \kappa'_1, \kappa'_2, J, l} M(E_1, \kappa_1, \kappa_2, J)M^*(E_1, \kappa'_1, \kappa'_2, J) \times w(\kappa_1, \kappa_2, \kappa'_1, \kappa'_2, J, l) P_l(\cos\theta)
\]

\[
M(E_1, \kappa_1, \kappa_2, J) = \sum_{i=1, \ldots, 6} g_i M^i_{\text{contact}}(E_1, \kappa_1, \kappa_2, J) + \sum_{j=L,R} g_j M^j_{\text{photo}}(E_1, \kappa_1, \kappa_2, J)
\]

contact photonic
3. Results
3. Results

Radial wave function (bound $\mu^-$)

$Z = 82$

$^{208}$Pb case

$B_\mu: 10.5$ MeV

$r g_{\mu}^{1s}(r), r f_{\mu}^{1s}(r)$

$r$ [fm]

(radius of $^{208}$Pb)

$r g_{\mu}^{1s}(r)$: solid

$r f_{\mu}^{1s}(r)$: dotted

[MeV$^{-1/2}$]
3. Results

Radial wave function (bound $e^-$)

$g_e^{1s}(r)$

$^{208}\text{Pb case} \quad Z = 81$

(considering $\mu^-$ screening)

<table>
<thead>
<tr>
<th>Type</th>
<th>$B_e (\text{MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rela</td>
<td>$9.88 \times 10^{-2}$</td>
</tr>
<tr>
<td>Non-rela</td>
<td>$8.93 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Relativity enhances the value near the origin.
Radial wave function (scattering $e^-$)

e.g. $\kappa = -1$ partial wave

$$ r g_{E_{1/2}}^{\kappa=-1}(r) $$

$[\text{MeV}^{-3/2}]$

$0.2$

$0.1$

$0$

$-0.1$

$-0.2$

$0$

$5$

$10$

$15$

$20$

$25$

$30$

$r \text{ [fm]}$

$\text{attracted by nuclear Coulomb potential}$

$\text{distorted wave}$

$\text{plane wave}$

$\text{Pb case}$

$Z = 82$

$E_{1/2} \approx 48 \text{MeV}$

$\text{① enhanced value near the origin}$

$\text{② local momentum increased effectively}$

3. Results
Contact process

✓ bound $\mu^-$  
✓ scattering $e^-$  
✓ bound $e^-$  
✓ scattering $e^-$  

◆ overlap of bound $\mu^-$, bound $e^-$, and two scattering $e^-$s

$\frac{g_{1s}^{1s}(r) g_{1s}^{1s}(r) g_{E_{1/2}}^{k=-1}(r) g_{E_{1/2}}^{k=-1}(r)}{r^2}$

wave functions move to the center

scat. $e^-$: plane $\rightarrow$ distorted

bound $e^-$: non-rela $\rightarrow$ rela

transition rate UP!
3. Results

Upper limits of BR (contact process)

\[ BR(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12} \]

\[ BR(\mu^- e^- \rightarrow e^- e^-) < B_{\text{max}} \]

(SINDRUM, 1988)

\[ (g_1(e_L \mu_R)(e_L e_R)) \]

\[ B_{\text{max}} \]

atomic #, Z

needed # of muonic atoms (Z = 82)

\[ 2.1 \times 10^{18} \]

\[ 3.0 \times 10^{17} \]

3. Results

**Photonic process**

- ✓ **bound \( \mu^- \)**
- ✓ **\( \gamma^* \)**
- ✓ **bound \( e^- \)**

- ✓ **scattering \( e^- \)**

\[
\begin{align*}
\frac{r^2 g_{\mu}^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)}{\gamma} &= - j_0(q_0 r) \\
\frac{r^2 g_{e}^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)}{\gamma} &= - j_0(q_0 r)
\end{align*}
\]

- **distortion of scattering \( e^- \)**
- **overlap integral down**

**scat. \( e^- \): plane \rightarrow distorted**

**bound \( e^- \): non-rela \rightarrow rela**
3. Results

Upper limits of BR (photonic process)

\[
BR(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}
\]
(MEG, 2013)

\[
BR(\mu^- e^- \rightarrow e^- e^-) < B_{max}
\]

\[
(g_L \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu})
\]

Koike et al. (1s)

this work (1s)

Preliminary

needed # of muonic atoms \((Z = 82)\)

\[
1.8 \times 10^{18} \rightarrow 6.6 \times 10^{18}
\]
3. Results

Phase shift effect of distortion

(makes a momentum of scattering $e^-$ larger effectively)

contact process

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

$\triangleright$ no momentum mismatches

photonic process

bound $\mu^-$  emitted $e^-$

bound $e^-$  emitted $e^-$

$\triangleright$ momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!  suppressed…
3. Results

Model-discriminating power

After finding CLFV, can the CLFV source be decided?

- Here, only 3 very simple model will be considered.

---

Model 1: contact (same chirality)

\[ \mathcal{L}_I = g_1 (\bar{e}_L \mu_R) (\bar{e}_L e_R) + [\text{h. c.}] \]

\[ g_1 \neq 0, g_{\text{else}} = 0 \]

Model 2: contact (opposite chirality)

\[ \mathcal{L}_I = g_5 (\bar{e}_R \gamma_\mu \mu_R) (\bar{e}_L \gamma^\mu e_L) + [\text{h. c.}] \]

\[ g_5 \neq 0, g_{\text{else}} = 0 \]

Model 3: photonic

\[ \mathcal{L}_I = g_R \bar{e}_R \sigma^{\mu\nu} \mu_R F_{\mu\nu} + [\text{h. c.}] \]

\[ g_R \neq 0, g_{\text{else}} = 0 \]
3. Results

Discriminating method 1

~ atomic # dependence of decay rates ~

\[ \frac{\Gamma(Z)}{\Gamma_0(Z)} \]

- The \( Z \) dependences are different among interactions.
- Compared to \((Z - 1)^3\), that of contact process is larger, while that of photonic process is smaller.
3. Results

Discriminating method 2

~ energy and angular distributions ~

\[ E_1 : \text{energy of an emitted electron} \]
\[ \theta : \text{angle between two emitted electrons} \]

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 d\cos\theta} \quad \text{[MeV}^{-1}] \\
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 d\cos\theta} \quad \text{[MeV}^{-1}]
\]

\begin{center}
\begin{tabular}{c}
\text{contact process} \\
\text{photonic process}
\end{tabular}
\end{center}
4. Summary
4. Summary

Conclusion

- $\mu^- e^- \rightarrow e^- e^-$ process in a muonic atom
  - interesting candidate for CLFV search
  - Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron are important in calculating decay rates.

Distortion makes difference between 2 processes.

- contact process: decay rate Enhanced (7 times in $Z = 82$)
- photonic process: decay rate suppressed (1/4 times in $Z = 82$)

- How to identify interaction types, found by this analyses
  - atomic # dependence of the decay rate
  - energy and angular distributions of emitted electrons
4. Summary

Future work

- further analysis for experimental setup
  - estimation of background
    - main noise: Decay In Orbit (DIO)
  - moderate setting of experiment
    - What atom is favored?
    - Is a better muon beam pulse or DC?

- calculation using various models beyond SM
  - interference among interactions