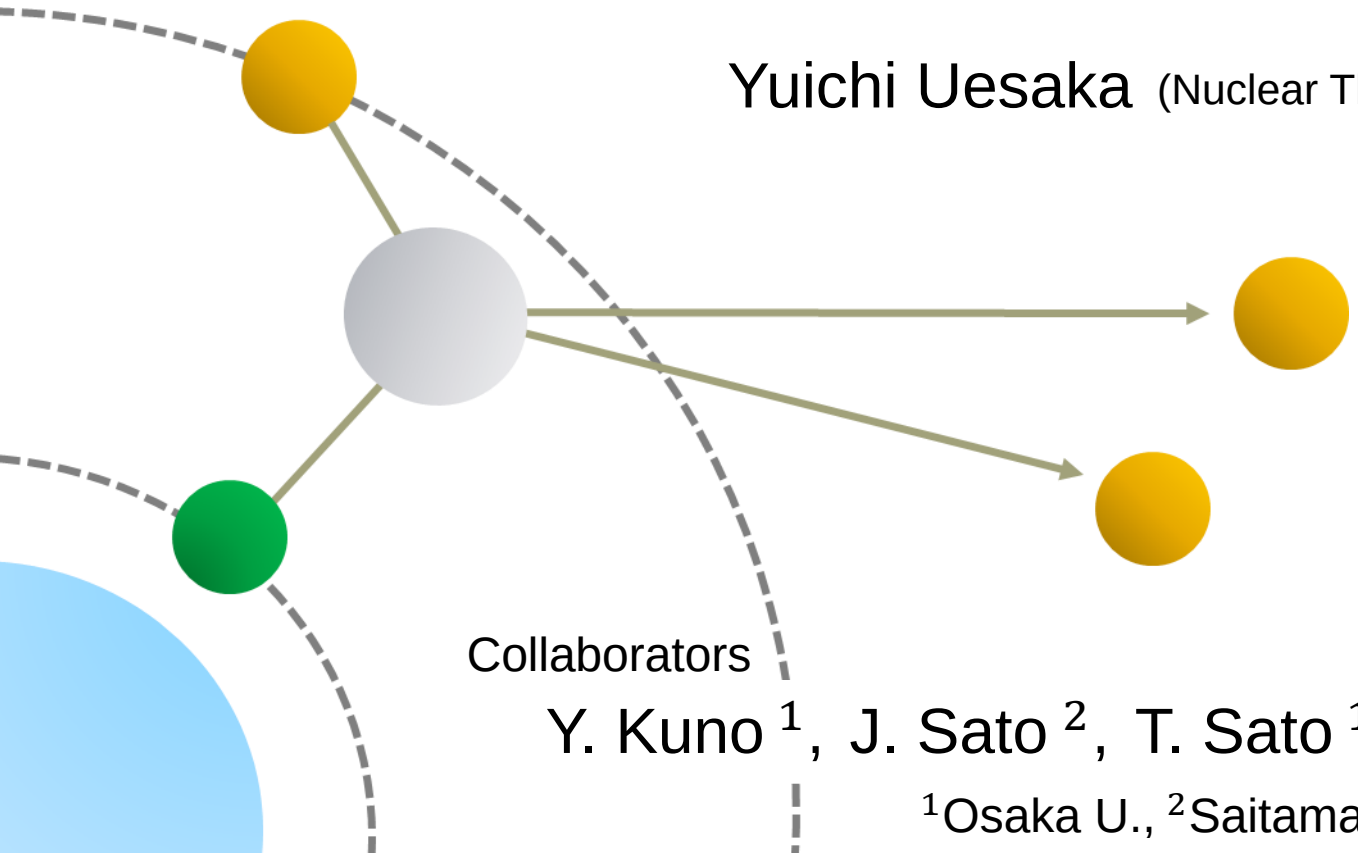
December 20, 2016 @ Particle Physics Theory Seminar, Osaka U.

Charged lepton flavor violating process, $\mu^- e^- \to e^- e^-$ in a muonic atom

YU, Y. Kuno, J. Sato, T. Sato & M. Yamanaka, Phys. Rev. D **93**, 076006 (2016).
YU, Y. Kuno, J. Sato, T. Sato & M. Yamanaka, in preparation.



Yuichi Uesaka (Nuclear Theory, Osaka U.)

Collaborators

Y. Kuno [1], J. Sato [2], T. Sato [1], M. Yamanaka [3]

[1]Osaka U., [2]Saitama U., [3]Kyoto Sangyo U.

Contents

1. Introduction

Charged Lepton Flavor Violation (CLFV)

- A probe for new physics -

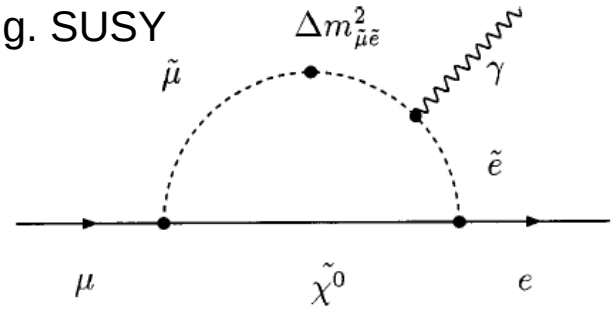
- lepton flavor #, L_e, L_μ, L_τ (Please distinguish them from lepton #, $L = L_e + L_\mu + L_\tau$.)

	e^-	μ^-	τ^-	ν_e	ν_μ	ν_τ	e^+	μ^+	τ^+	$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$	others
L_e	+1	0	0	+1	0	0	-1	0	0	-1	0	0	0
L_μ	0	+1	0	0	+1	0	0	-1	0	0	-1	0	0
L_τ	0	0	+1	0	0	+1	0	0	-1	0	0	-1	0

◆ lepton flavor violation in charged lepton sector = CLFV

- forbidden in SM
- predicted in many theories beyond SM
- contribution of neutrino oscilation → very small

e.g. SUSY



$\mathrm{Br}(\mu \to e\gamma) < 10^{-54}$

✓ cannot be observed by current technology

(<u>does not contaminate the search for new physics</u>)

➡ If seen, that is an evidence of new physics !! (not ν osci.)

CLFV search using muons

Advantages of using muon for rare process

1. long lifetime and simple kinematics
2. high intensity muon beam (~ 10^{8-9} muons per a second)

Examples of CLFV processes using muons

BR : Branching Ratio
(≡ signal rate / total decay rate)

a) $\mu^+ \to e^+\gamma$ BR $< 5.7 \times 10^{-13}$ by MEG
Phys. Rev. Lett. **110**, 201801 (2013).

b) $\mu^+ \to e^+e^-e^+$ BR $< 1.0 \times 10^{-12}$ by SINDRUM
Nucl. Phys. B **299**, 1 (1988).

c) $\mu^- N \to e^- N$ BR $< 7 \times 10^{-13}$ ($\mu^-\mathrm{Au} \to e^-\mathrm{Au}$) by SINDRUM II
Eur. Phys. J. C **47**, 337 (2006).

New experiments for "$\mu^- - e^-$ conversion" are planned with higher sensitivity than previous ones.
(COMET, DeeMe @ J-PARC, Mu2e @ Fermilab)

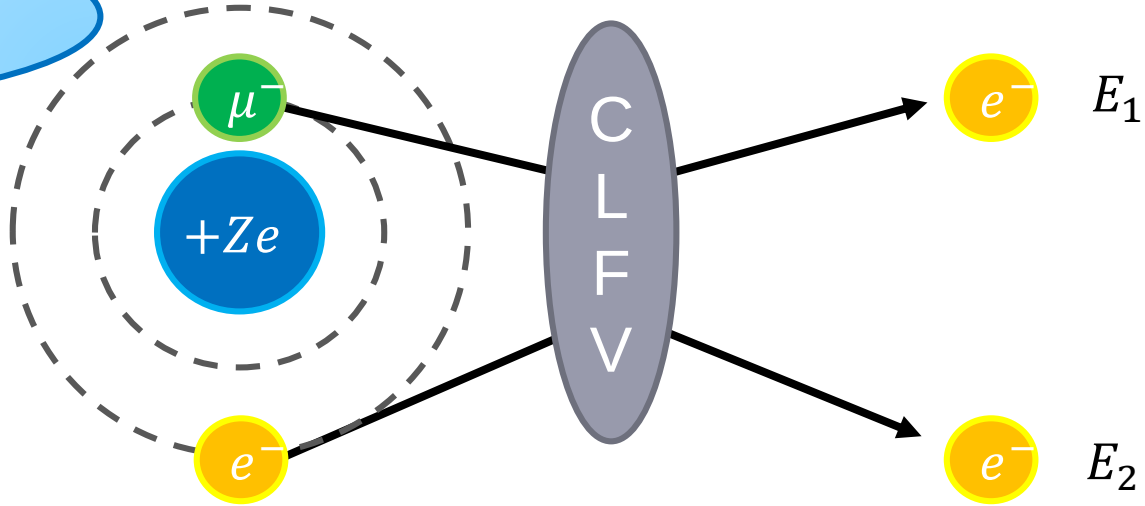
$\mu^- e^- \to e^- e^-$ in a muonic atom

M. Koike, Y. Kuno, J. Sato & M. Yamanaka,
Phys. Rev. Lett. **105**,121601(2010)

New CLFV search using muonic atoms

proposal in **COMET**

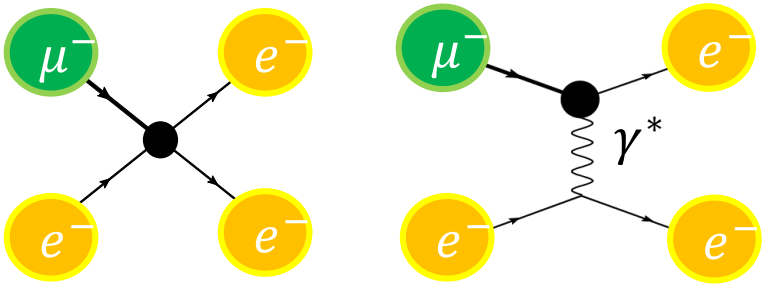
R. Abramishili et al.,
COMET Phase-I Technical Design Report,
KEK Report 2015-1 (2015)



Features

- clear signal : two e^-s ($E_1 + E_2 \simeq m_\mu + m_e - B_\mu - B_e$)
- 2 type CLFV interactions
  - ✓ μeee vertex
  - ✓ $\mu e\gamma$ vertex

  (similar to $\mu^+ \to e^+ e^+ e^-$)



- atomic # Z : large ⇒ decay rate Γ : large ($\Gamma \propto (Z-1)^3$)

(Rough) Estimation of decay rate

"flux"

$$\Gamma = \sigma v_{\mathrm{rel}} \int \mathrm{d}V \rho_\mu \rho_e$$

Suppose nuclear Coulomb potential is weak,

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\mathrm{rel}} |\psi_{1S}^e(0)|^2$$

(sum of two $1S$ e^-s)

Phys. Rev. Lett. **105**,121601 (2010).

σ : cross section of $\mu^- e^- \to e^- e^-$ (free particles')

v_{rel} : relative velocity of μ^- & e^-

$$\psi_{1S}^e(\vec{x}) = \sqrt{\frac{(m_e(Z-1)\alpha)^3}{\pi}} \exp(-m_e(Z-1)\alpha|\vec{x}|)$$

: wave function of $1S$ bound electron (non-rela)

$$\Gamma \propto (Z-1)^3$$

(the same Z dependence in the both contact & photonic cases)

Branching ratio

How many muons decay, compared to created # ?

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$

$\tilde{\tau}_\mu$: lifetime of a muonic atom

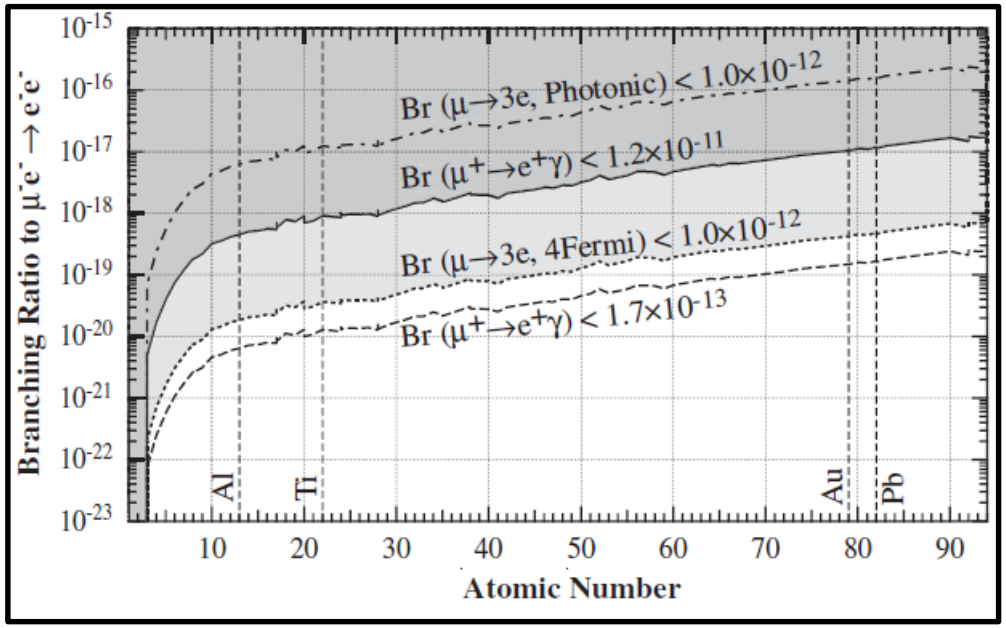
cf. $2.2\mu\mathrm{s}$ for a muonic H ($Z = 1$)
$80\mathrm{ns}$ for a muonic Pb ($Z = 82$)

- increasing as atomic # Z is larger

Using muonic atom with large Z is favored.

cf. Future experiment (COMET Phase-II) can product $\mathcal{O}(10^{18-19})$ muonic atoms.

Constraint from previous CLFV experiments



Phys. Rev. Lett. **105**,121601 (2010).

To improve the previous estimation

✓ Koike *et al.*

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\mathrm{rel}} |\psi_{1S}^{e}(0)|^2 \propto (Z-1)^3$$

(valid when nuclear Coulomb potential is sufficiently small)

Note

- not able to estimate differential decay rate
- "Z dependence" doesn't depend on CLFV int. $(\text{always } \Gamma \propto (Z-1)^3)$

approximations ($Z\alpha \ll 1$)

- spacial extension of bound lepton $\gg$ wave length of emitted e^-
- bound lepton : non-rela
- emitted e^- : plane wave

In atoms with large Z,

← *small orbital radius*

← *relativistic (especially, e^-)*

← *Coulomb distortion*

More quantitive estimation is needed ! (important for large Z)

Improvement and expectation

✓ this work

lepton wave function : relativistic Coulomb

the improvement contains… influence for $\Gamma_{\mu^- e^- \to e^- e^-}$

- finite orbit-size of bound leptons suppressed
- relativistic effects for bound leptons enhanced
- Coulomb distortion of emitted e^- ?

➡ Totally, how CLFV decay rates are changed ?

other expectation

- ➢ we can estimate energy & angular distribution of emitted e^- pair
- ➢ we may get different results, depending on CLFV interaction

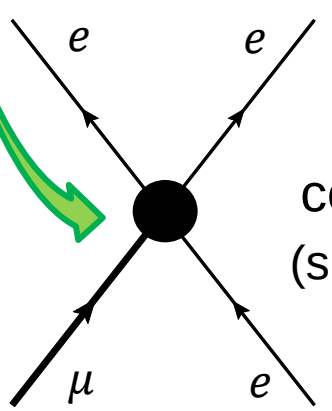
2. Formulation

Effective Lagrangian

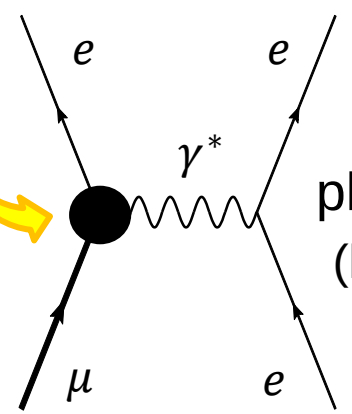
$$\mathcal{L}_I = \mathcal{L}_{contact} + \mathcal{L}_{photo}$$

$$\begin{aligned}\mathcal{L}_{contact} = {} & g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L) \\ & + g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L) \\ & + g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R) + [h.c.]\end{aligned}$$

$$\mathcal{L}_{photo} = g_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.]$$



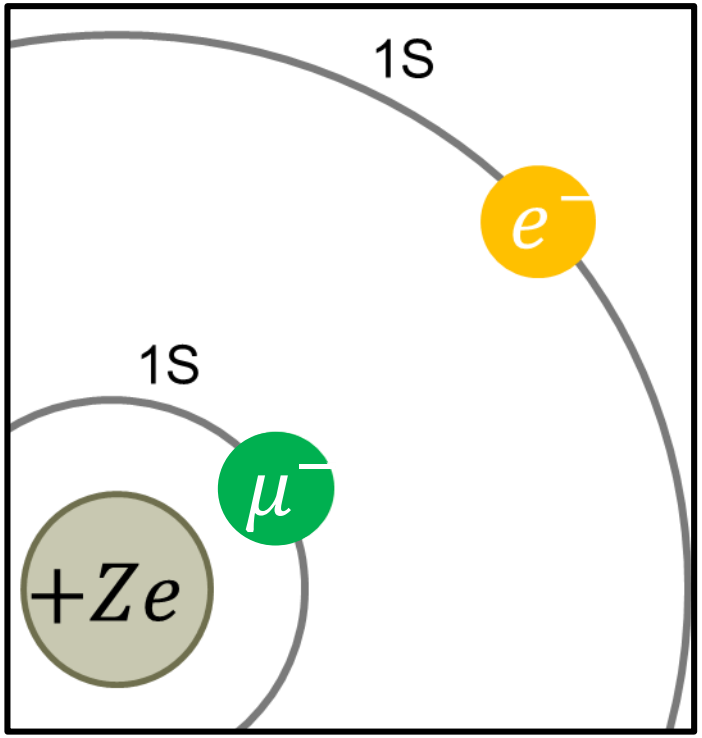
contact interaction
(short-range process)



photonic interaction
(long-range process)

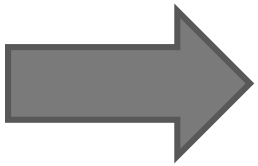
$\mu^- e^- \to e^- e^-$ process

initial state



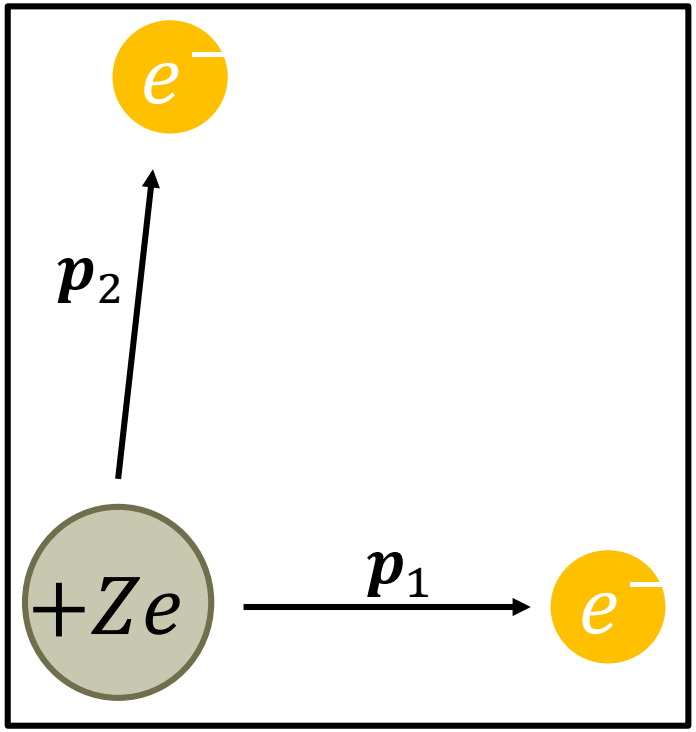
bound μ^- (1S) & bound e^-

CLFV



$\mu^- e^- \to e^- e^-$

final state



2 scattering e^-s (distorted wave)

$$E_1 + E_2 = m_\mu + m_e - B_\mu - B_e$$

- ignore nuclear recoil energy
- ignore interaction among e^-s

Decay rate

$$\Gamma = 2\pi \sum_f \sum_{\bar{\imath}} \delta(E_f - E_i) \left| \langle \psi_e^{s_1}(\boldsymbol{p}_1) \psi_e^{s_2}(\boldsymbol{p}_2) | H | \psi_\mu^{s_\mu}(1s) \psi_e^{s_e}(1s) \rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^s(\boldsymbol{p}) = \sum_{\kappa,\mu,m} 4\pi\, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y^*_{l_\kappa, m}(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

get radial functions by solving Dirac eq. numerically

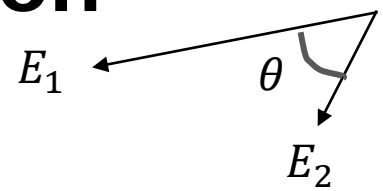
$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

ϕ : nuclear Coulomb potential

Energy & angular distribution



E_1 : energy of an emitted electron

θ : angle between two emitted electrons

$$\Gamma(\mu^-(1S)e^-(\alpha) \to e^-e^-) = \frac{1}{2}\int_{m_e}^{m_\mu - B_\mu^{1S} - B_e^\alpha} \mathrm{d}\,E_1 \int_{-1}^{1} \mathrm{d}\cos\theta \frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta}$$

differential decay rate :

P_l : Legendre polynomial

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta} = \sum_{\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l} M(E_1,\kappa_1,\kappa_2,J)M^*(E_1,\kappa_1',\kappa_2',J) \times w(\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l)P_l(\cos\theta)$$

$$M(E_1,\kappa_1,\kappa_2,J) = \underbrace{\sum_{i=1,\cdots,6} g_i\, M^i_{\mathrm{contact}}(E_1,\kappa_1,\kappa_2,J)}_{\text{contact}} + \underbrace{\sum_{j=L,R} g_j\, M^j_{\mathrm{photo}}(E_1,\kappa_1,\kappa_2,J)}_{\text{photonic}}$$

3. Results

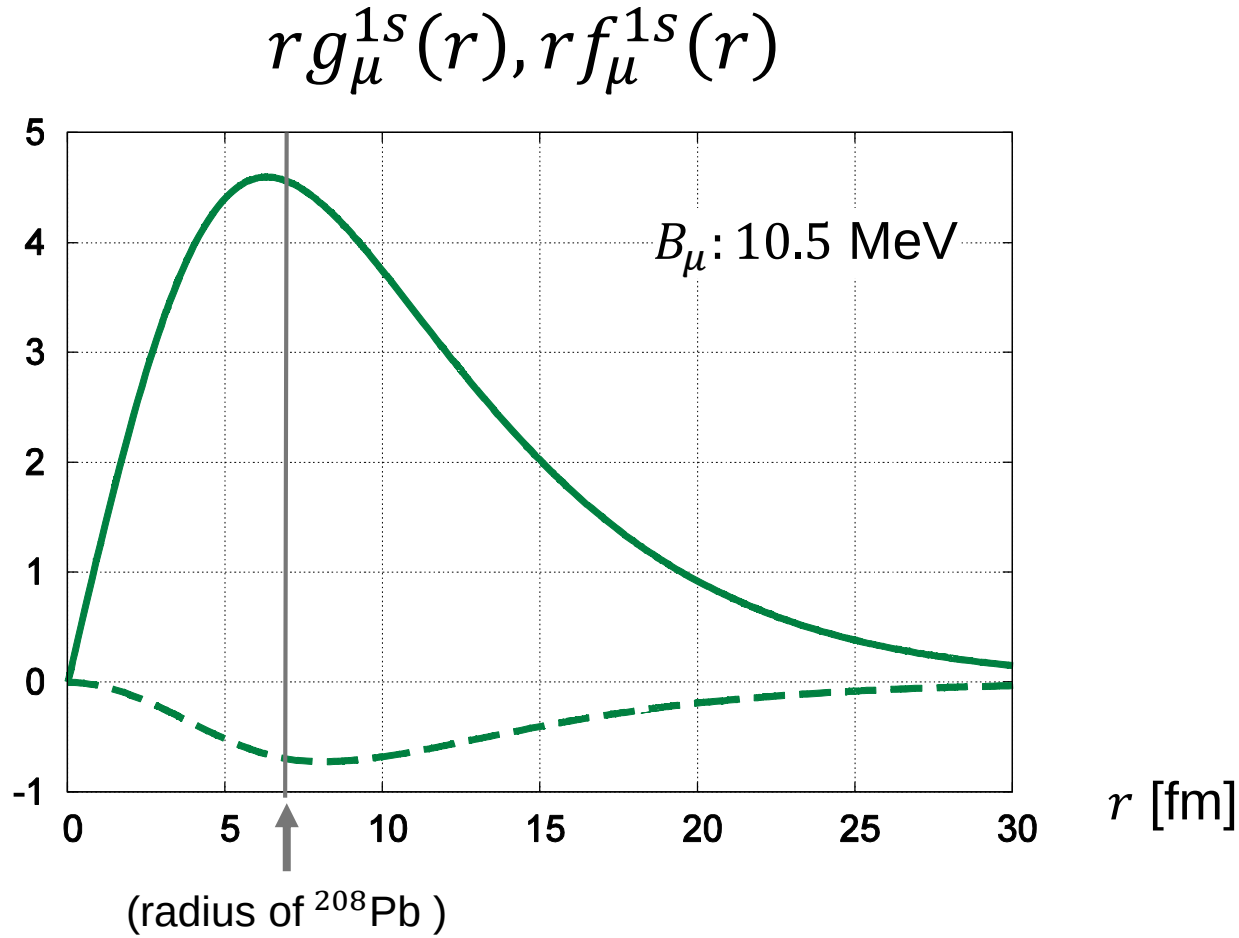
Radial wave function (bound μ^-)

^{208}Pb case $Z = 82$

$[\mathrm{MeV}^{-1/2}]$

$rg_\mu^{1s}(r)$: solid

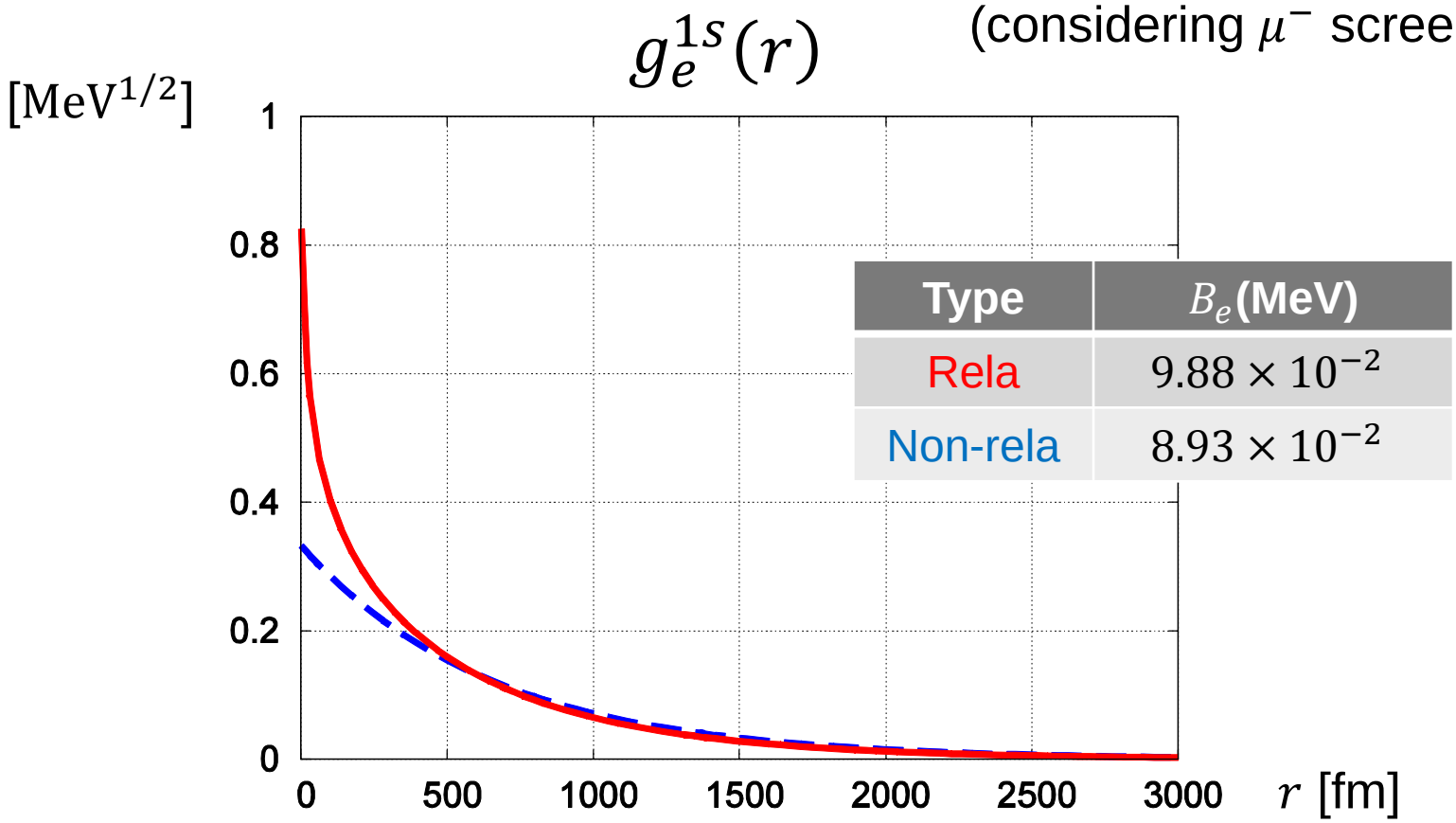
$rf_\mu^{1s}(r)$: dotted



Radial wave function (bound e^-)

^{208}Pb case $Z = 81$

(considering μ^- screening)



Type	B_e(MeV)
Rela	9.88×10^{-2}
Non-rela	8.93×10^{-2}

Relativity enhances the value near the origin.

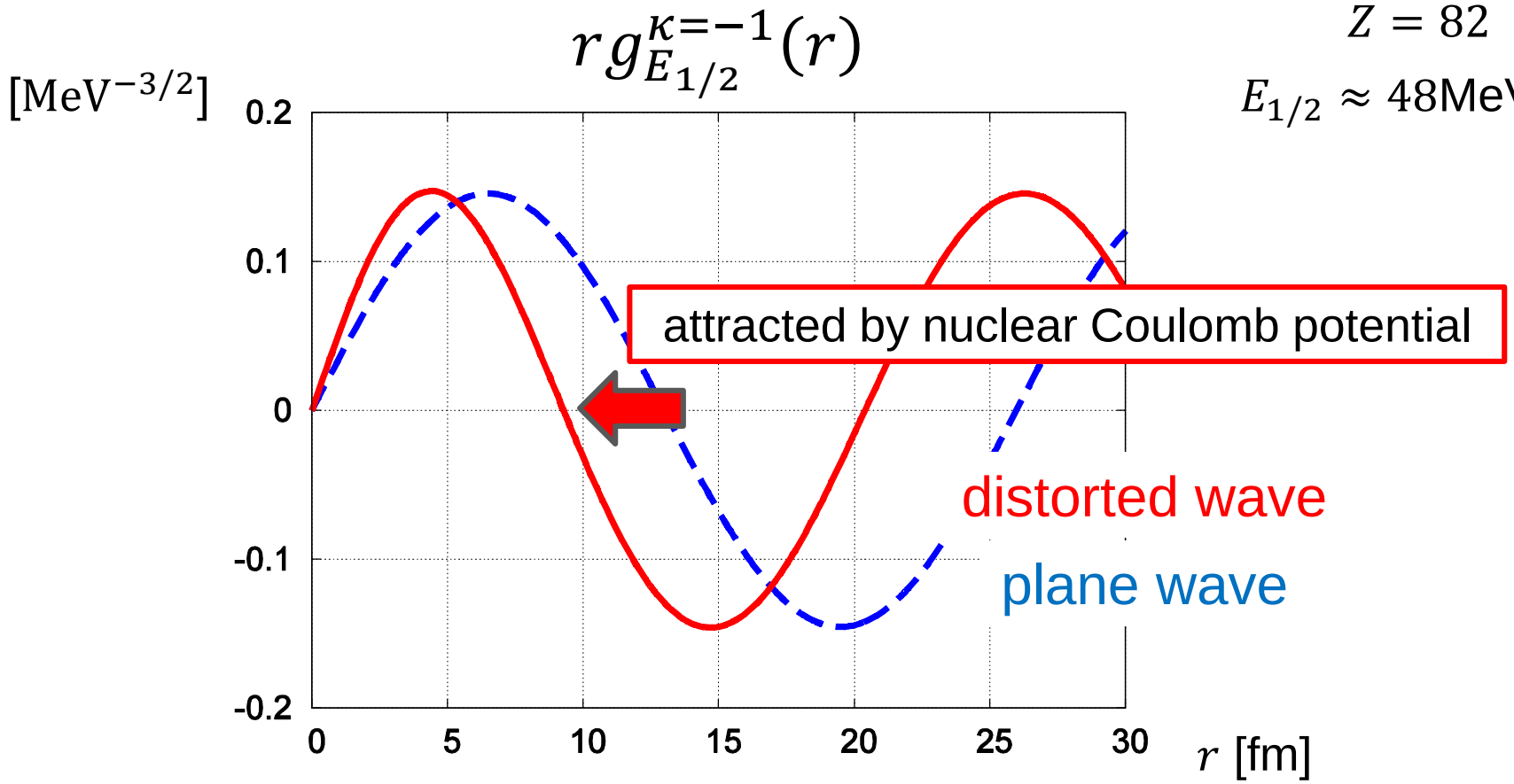
Radial wave function (scattering e^-)

e.g. $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

$E_{1/2} \approx 48$MeV



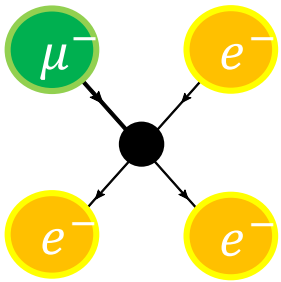
① enhanced value near the origin

② local momentum increased effectively

Contact process

✓ bound μ^-

✓ scattering e^-

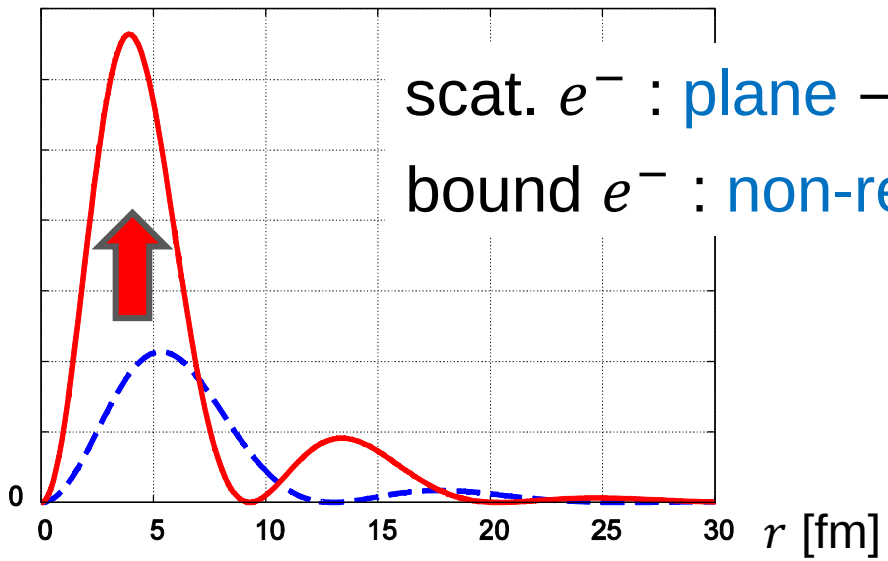


✓ bound e^-

✓ scattering e^-

◆ overlap of bound μ^-, bound e^-, and two scattering e^-s

$r^2 g_\mu^{1s}(r) g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) g_{E_{1/2}}^{\kappa=-1}(r)$



scat. e^- : plane → distorted

bound e^- : non-rela → rela

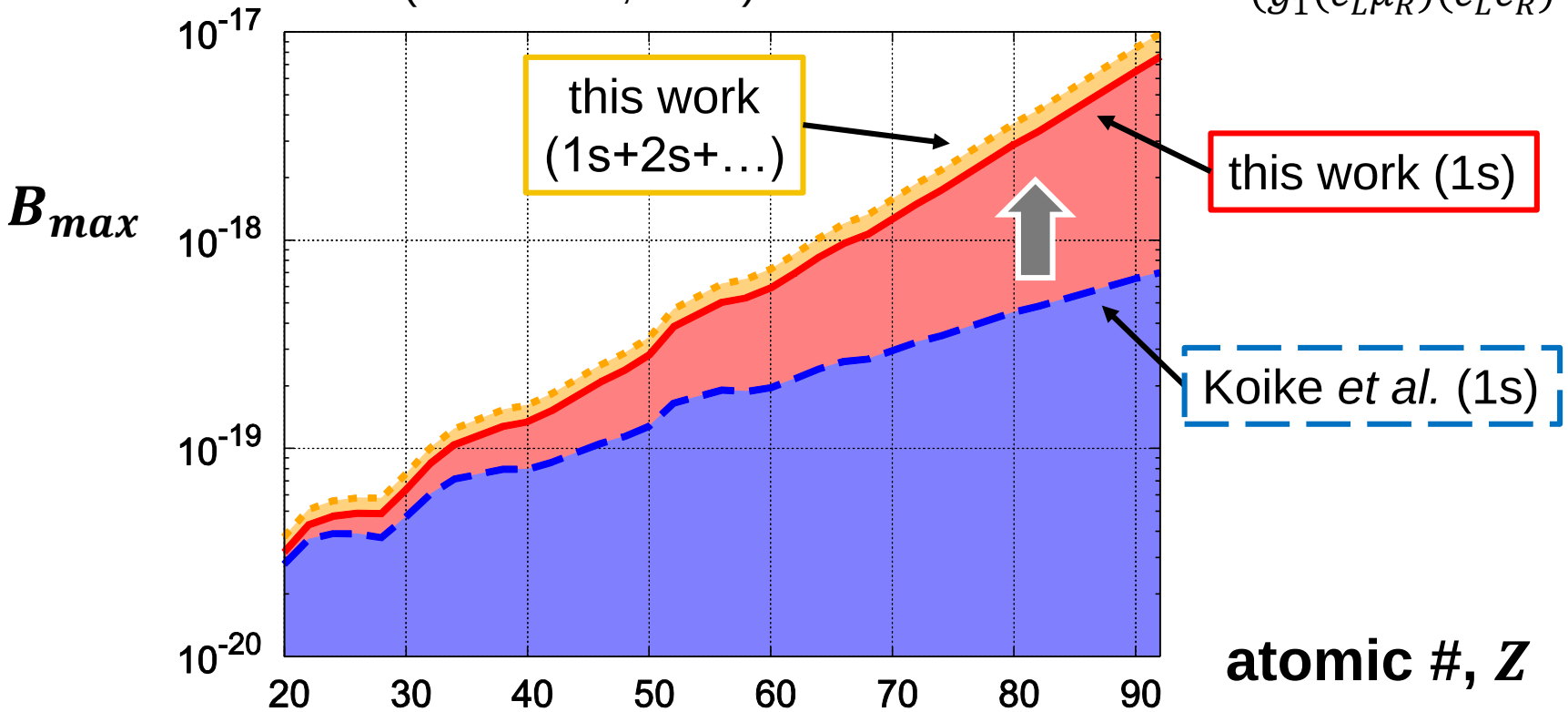
wave functions move to the center

transition rate UP!

Upper limits of BR (contact process)

$BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}$ (SINDRUM, 1988) → $BR(\mu^-e^- \to e^-e^-) < B_{max}$

$(g_1(\bar{e}_L\mu_R)(\bar{e}_L e_R))$



YU, Y. Kuno, J. Sato, T. Sato & M. Yamanaka, Phys. Rev. D **93**, 076006 (2016).

needed # of muonic atoms ($Z = 82$)

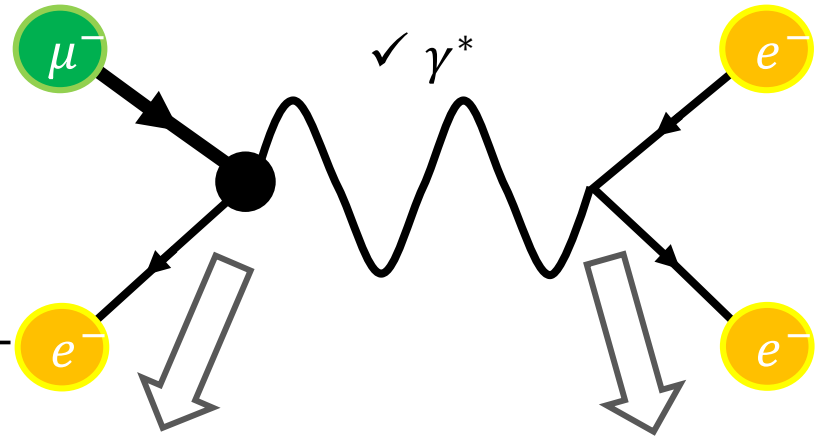
2.1×10^{18} → 3.0×10^{17}

Photonic process

✓ bound μ^-

✓ γ^*

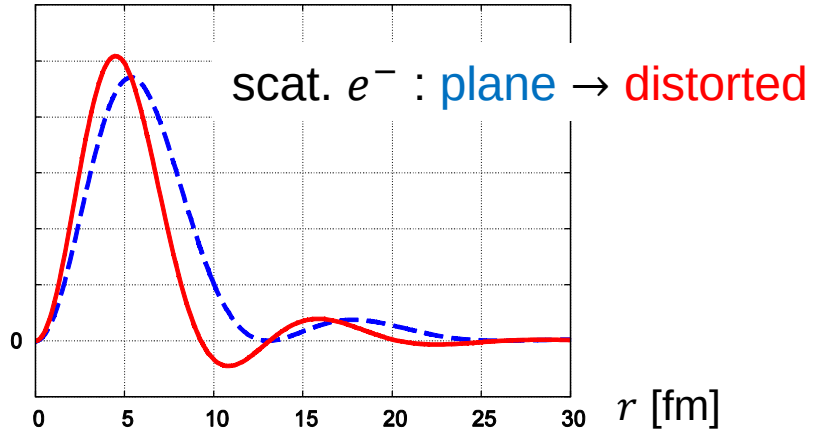
✓ bound e^-



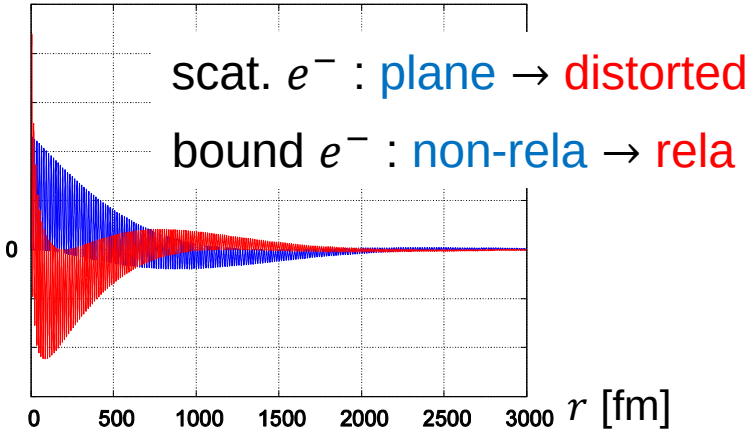
✓ scattering e^-

✓ scattering e^-

$r^2 g_\mu^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$



$r^2 g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$



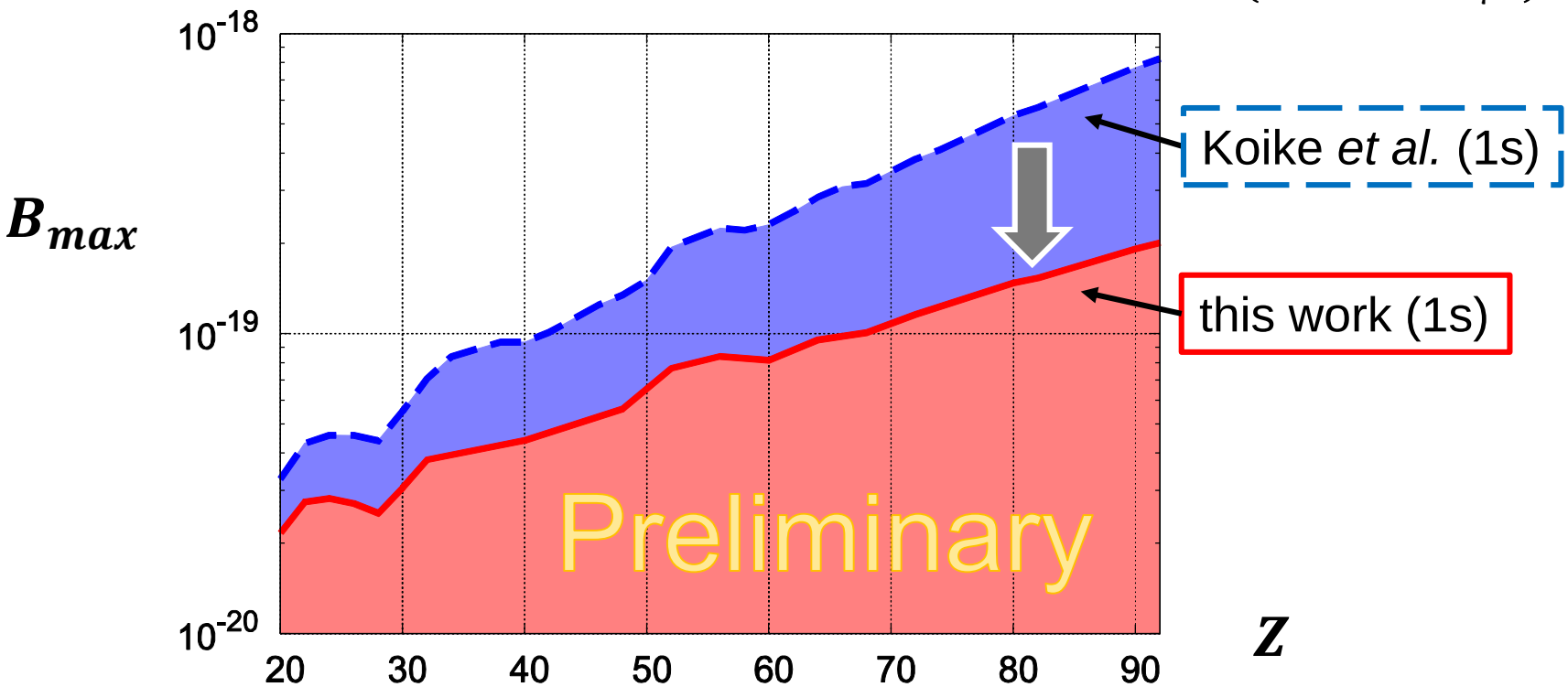
distortion of scattering e^- ⇒ overlap integral down

Upper limits of BR (photonic process)

$BR(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(MEG, 2013)

$(g_L \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu})$



needed # of muonic atoms ($Z = 82$)

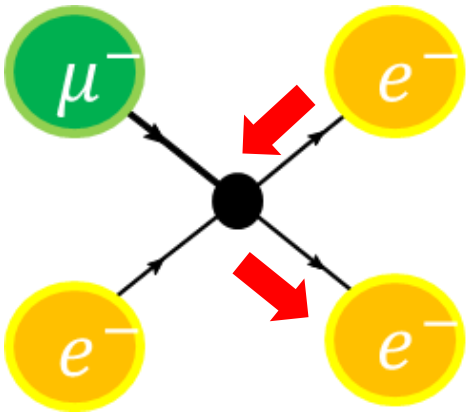
1.8×10^{18} → 6.6×10^{18}

Phase shift effect of distortion

(makes a momentum of scattering e^- larger effectively)

contact process

bound μ^- emitted e^-

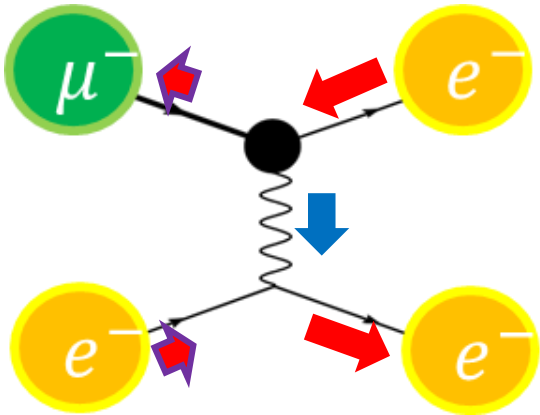


bound e^- emitted e^-

- no momentum mismatches

photonic process

bound μ^- emitted e^-



bound e^- emitted e^-

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!

suppressed…

Model-discriminating power

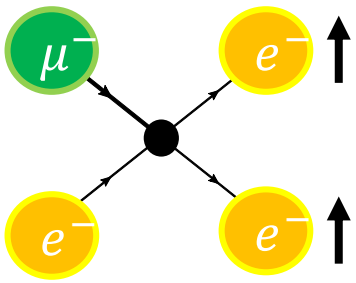
After finding CLFV, can the CLFV source be decided ?

- Here, only 3 very simple model will be considered.

Model 1 : contact (same chirality)

$$\mathcal{L}_I = g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + [h.c.]$$

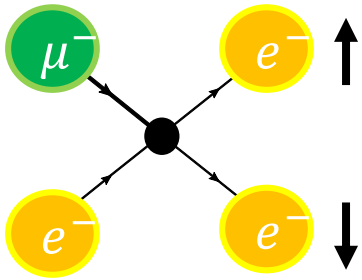
$$g_1 \neq 0, g_{else} = 0$$



Model 2 : contact (opposite chirality)

$$\mathcal{L}_I = g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + [h.c.]$$

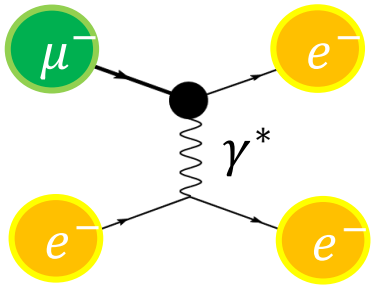
$$g_5 \neq 0, g_{else} = 0$$



Model 3 : photonic

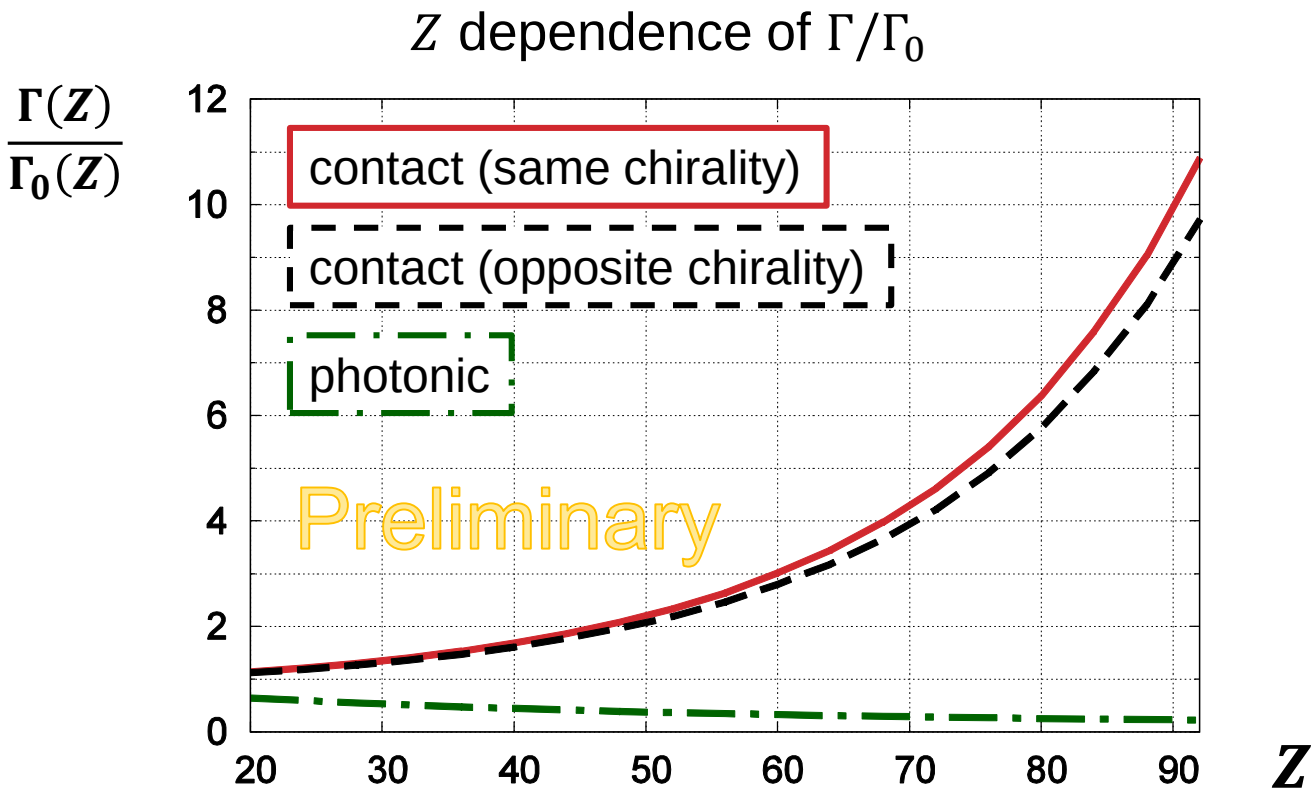
$$\mathcal{L}_I = g_R\overline{e_R}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + [h.c.]$$

$$g_R \neq 0, g_{else} = 0$$



Discriminating method 1

~ atomic # dependence of decay rates ~



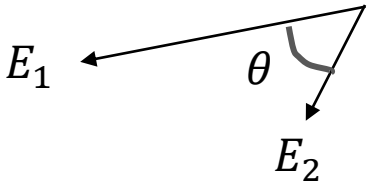
- The Z dependences are different among interactions.
- Compared to $(Z-1)^3$, that of contact process is larger, while that of photonic process is smaller.

Discriminating method 2

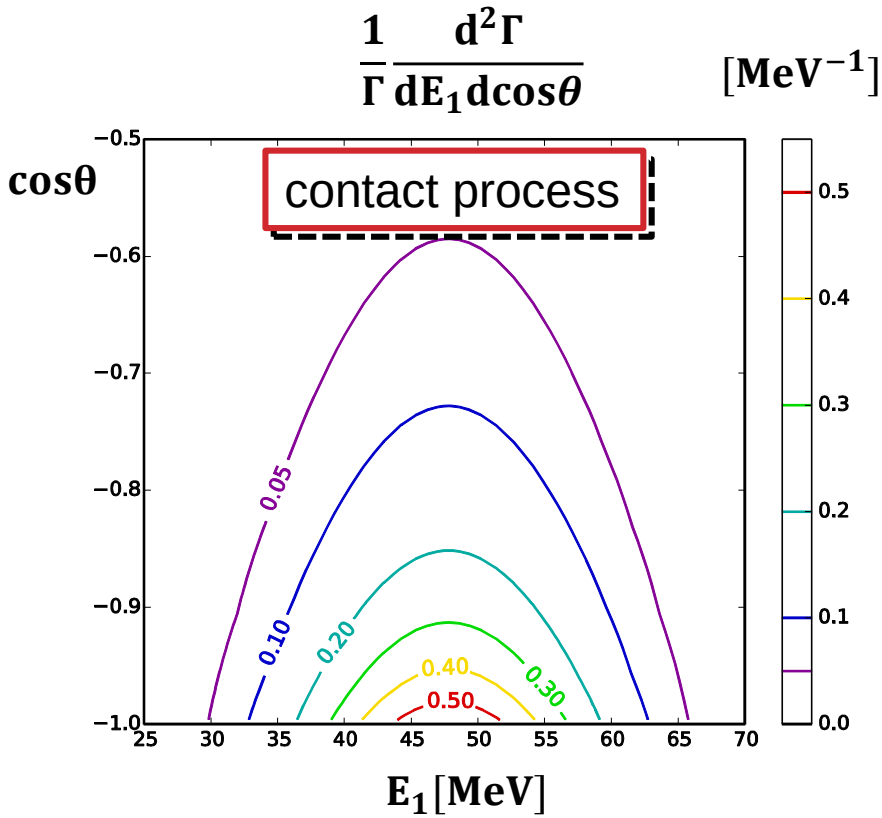
~ energy and angular distributions ~

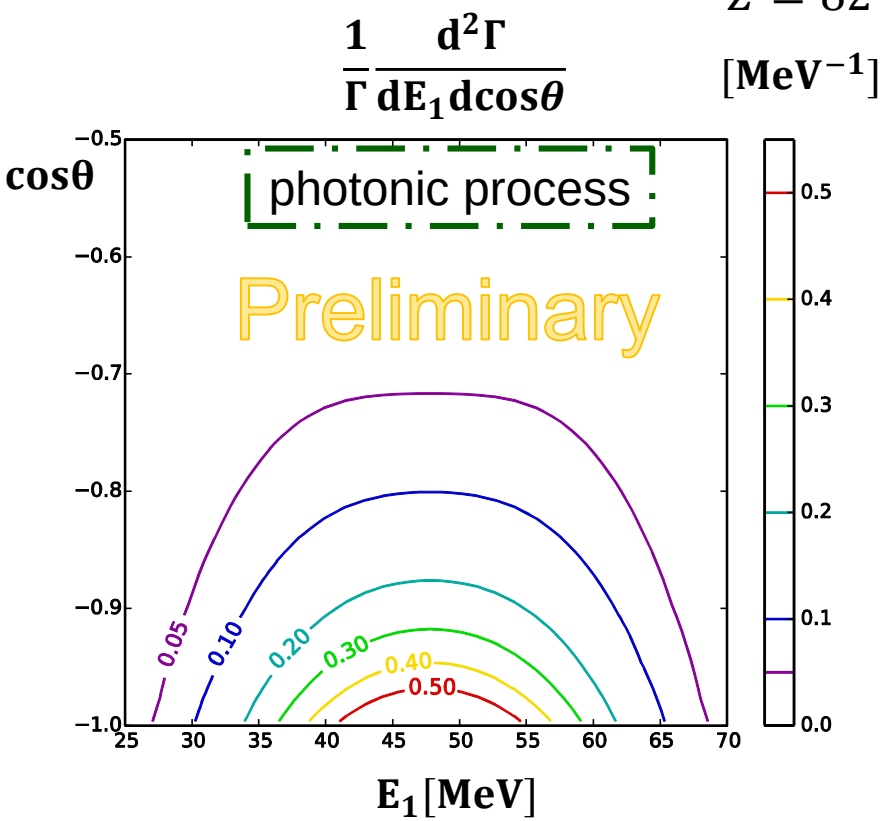
E_1 : energy of an emitted electron

θ : angle between two emitted electrons



$Z = 82$





4. Summary

Conclusion

- $\mu^- e^- \to e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron

are important in calculating decay rates.

⇩ Distortion makes difference between 2 processes.

- ■ contact process : decay rate Enhanced (7 times in $Z = 82$)
- ■ photonic process : decay rate suppressed (1/4 times in $Z = 82$)

◆ How to identify interaction types, found by this analyses

- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons

Future work

- further analysis for experimental setup
  - ✓ estimation of background
    - main noise : Decay In Orbit (DIO)
  - ✓ moderate setting of experiment
    - What atom is favored ?
    - Is a better muon beam pulse or DC ?
- calculation using various models beyond SM
  - ✓ interference among interactions