EFFECTIVE THEORY OF BLACK HOLES AT LARGE D

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based on some papers to be appeared

Collaborations with R.Emparan, D.Grumiller, K.Izumi,
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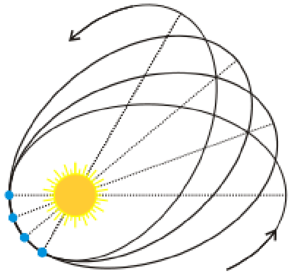
GENERAL RELATIVITY

- **Einstein equations describe gravitational physics**

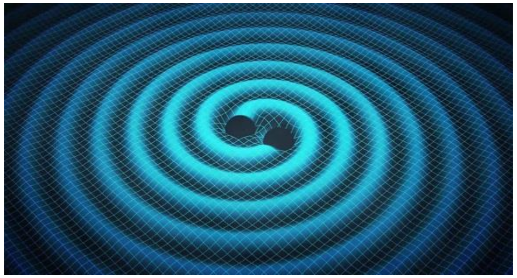
$$R_{\mu\nu} = 0$$

  - Equations have no scale **:** gravity from weak to strong region
  - Rich **observable** phenomena even in (4 dim) vacuum



ex) Precession of planets

Almost Newtonian (weak)

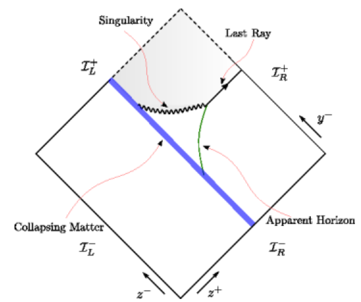


ex) Binary black hole mergers

Full GR effects (strong)

DIMENSIONS

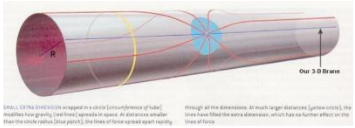
- **Gravity with matters in other dimensions**



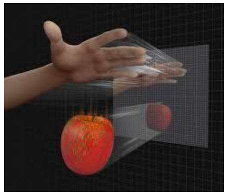
- In low dimensions (two or three)

No (local) gravitons: Toy models for quantum gravity
(*c.f.,* CGHS model, Dilaton gravity, BTZ black holes)

- In higher or general dimensions



Cosmology (Braneworld model as extra dimension scenario, String cosmology)



Holography (Matter physics via gauge/gravity correspondence, String theory)

Just for fun (No-uniqueness of black holes, Black hole instabilities,…)

MOTIVATION

- **Dimension *D* is a parameter of General Relativity**

  - Many problems can be formulated in general dimensions at the equation-level

    Black hole problem can be formulated in D>3

  - Various dimension-dependent (**parameter-dependent**) phenomena in gravitational physics

    No gravitational wave memory (supertranslation) in higher dimension (D>4)

    No uniqueness of black holes in D>4

    Rotating black holes are unstable in D>5

    Schwarzschild BH has a supersymmetric structure only in D=4

  - Understanding the parameter dependence is important (and just interesting)

DIFFICULTY

- **Solve the Einstein equations in various situations**

$$R_{\mu\nu} = 0$$

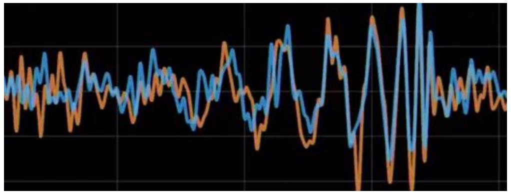
- Nonlinear partial differential equation system

Very hard system in general, even numerically

- No original scales in a system

Gravitons interact at all scales (weak and strong field are always coupled)

- QNMs contain information of horizon and infinity



e.g., GWs by aLIGO

➢ Reduce these difficulties by some assumptions (also in numerics)

Additional symmetries, Hierarchies in scales of a system (gradient expansions or WKB method), Taking limits of parameters in a system (PN, perturbations,..), …

PURPOSE

- **Give a new method to solve the Einstein equations**
  - Taking the limit of infinite spacetime dimension

$$D \to \infty$$

(Boundary of the parameter space)

  - This limit simplifies the equations in various ways

    - (Conformal) **Symmetry** enhancement at $D = \infty$

    - Natural **scale hierarchy** appears

      Weak and strong gravity fields are **decoupled (Effective theory description for black holes)**

  - We can solve the Einstein equations **analytically** as nonlinear PDE system in 1/D expansions

    - Dimensional-dependent phenomena can be seen in 1/D-corrections

CONTENTS

1. Large D limit of General Relativity

2. Effective theory of black holes

3. Summary

App. More details

CONTENTS

1. Large D limit of General Relativity
   - General properties of black holes at large D [Emparan-Suzuki-Tanabe (2013)]
2. Effective theory of black holes

3. Summary

App. More details

LARGE D LIMIT

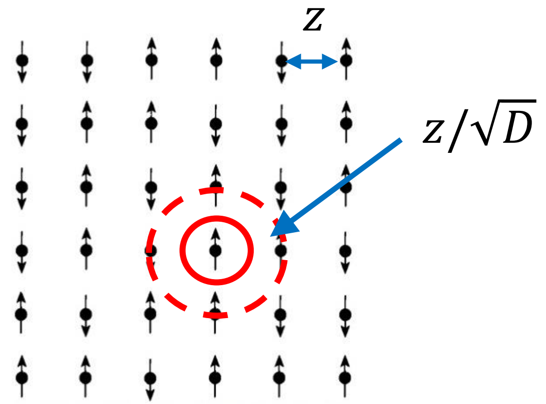
- ***Famous*** **Large D limit in statistical mechanics**

  - (Dynamical) Mean-field theory [Weiss (1907), Metzner and Vollhardt (1989), Georges, et.al. (1996)]

    ex) Ising model

$$H = -J\sum S_i S_j - \lambda \sum S_i$$

$$\rightarrow H_{\text{eff}} = -(\lambda + m)\sum S_i$$



- Interactions from next sites are absorbed into background fields

- This approximation becomes exact at $D = \infty$

  Interaction length scales as $z \rightarrow z/\sqrt{D}$ in D+1 spacetime dimension

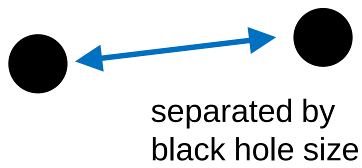
  No interaction between spins at $D = \infty$

- Dynamical mean field theory is a good approximation method for strongly correlated electron system in **two (spatial) dimensions**

LARGE D LIMIT OF GR

☐ *Our* Large D limit in General Relativity [Emparan-Suzuki-Tanabe (2013)]

- Effective description of black holes (if exists)



➢ Black holes do not interact each other

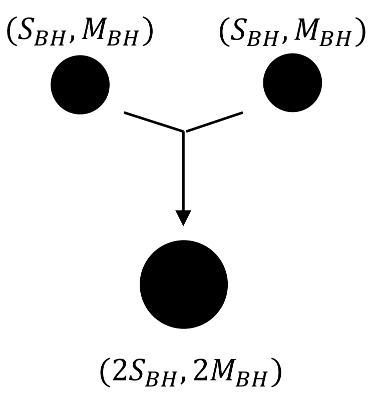
Black hole is just a “hole” in flat spacetime

- Gravitational potential is suppressed exponentially

$$\Phi = -\left(\frac{r_0}{r}\right)^{D-3} \to O(e^{-D})$$

- Hagedorn like behavior in thermodynamic quantities

$$S_{\rm BH} \propto M_{\rm BH}^{(D-2)/(D-3)} \to S_{\rm BH} \propto M_{\rm BH}$$



➢ Black hole merger does not need any entropy cost or energy loss by GWs (no interactions)

Almost same with mean field theory ? No gravitational interaction?

BLACK HOLES

□ D dim Schwarzschild black hole metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3}$$

➢ Gravity by a black hole is localized in very near-horizon region

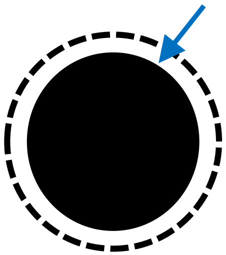
$$|r - r_0| \simeq O(r_0/D)$$

- In this region, the potential $(r_0/r)^{D-3}$ becomes $O(1)$

- Scale hierarchy appears naturally

$$r_0 \gg \frac{r_0}{D}$$

black hole size | interaction scale



POTENTIALS

☐ Probe scalar field analysis

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}$$

$$dr_* = f(r)^{-1}dr$$

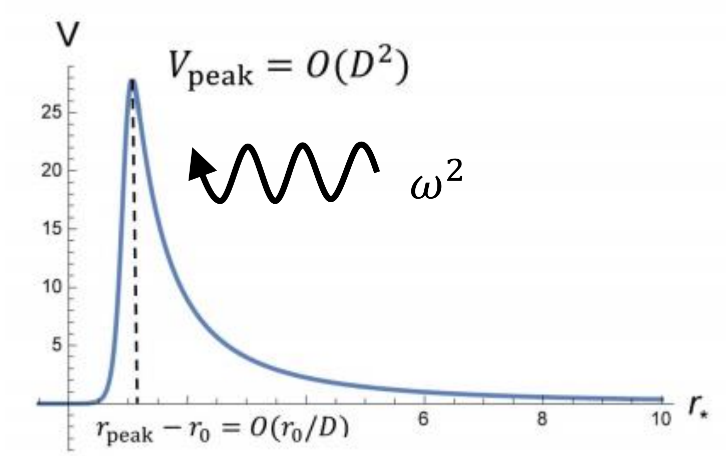
➢ Massless scalar field in Schwarzschild BH

$$\Box\Phi = 0$$

$$\Phi = r^{-(D-2)2}\phi(r)e^{-i\omega t}\mathrm{Y}_{\ell\, m}$$

⇩

$$\left[\frac{d^2}{dr_*^2} + (\omega^2 - V)\right]\phi(r) = 0$$



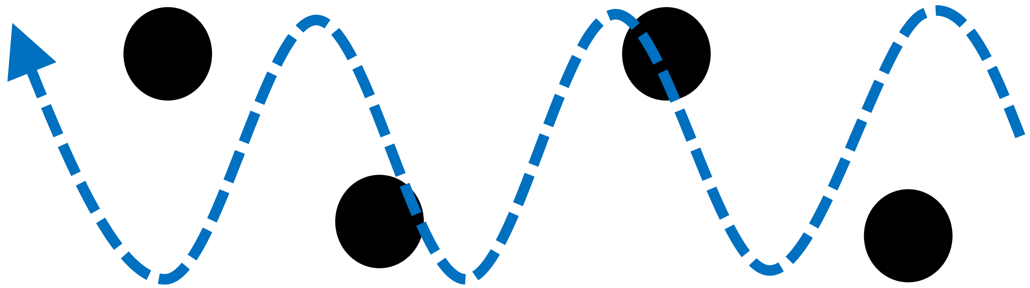
- To observe the black hole, scalar field needs very high energy

$$\omega r_0 = O(D)$$

- Low energy scalar fields ($\omega r_0 = O(1)$) cannot see the black hole

SCALAR FIELD PICTURE

- **Black hole is just a hole for (low energy) scalar fields**



- Black hole is stiff and not oscillated by (low energy) scalar fields

  Similar with mean field theory (we can treat many-BH system for scalar field dynamics)
  No interesting dynamics for black hole physics

- For high energy scalar fields we have no effective description

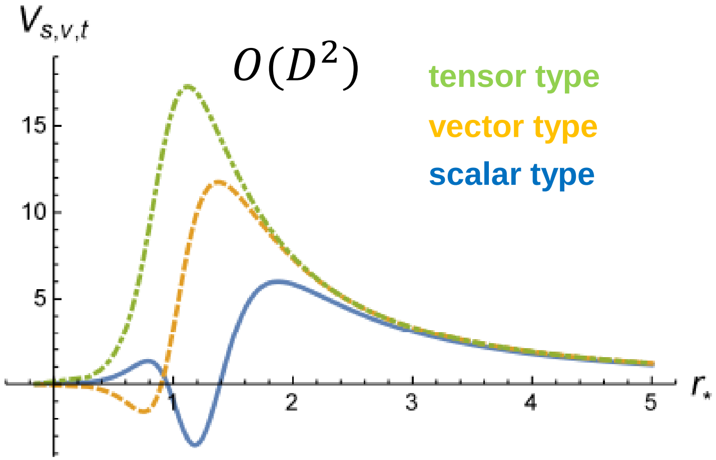
  Spacetime is very dynamical by scalar fields (*e.g.,* gravitational collapse)

ZERO MODE GRAVITONS

- **Graviton dynamics (gravity) is nontrivial at large D**

  - Gravitational perturbations of Schwarzschild BH

$$\left[\frac{d^2}{dr_*^2} + \left(\omega^2 - V_{s,v,t}\right)\right]\phi(r) = 0$$



- Scalar and vector perturbations have nontrivial structure

  *Tensor perturbation has same potential with scalar fields

- Low energy gravitons can probe this nontrivial structure

  Such modes are confined by the potential barrier in very near-horizon region

ZERO MODE GRAVITONS

☐ Graviton dynamics (gravity) is nontrivial at large D

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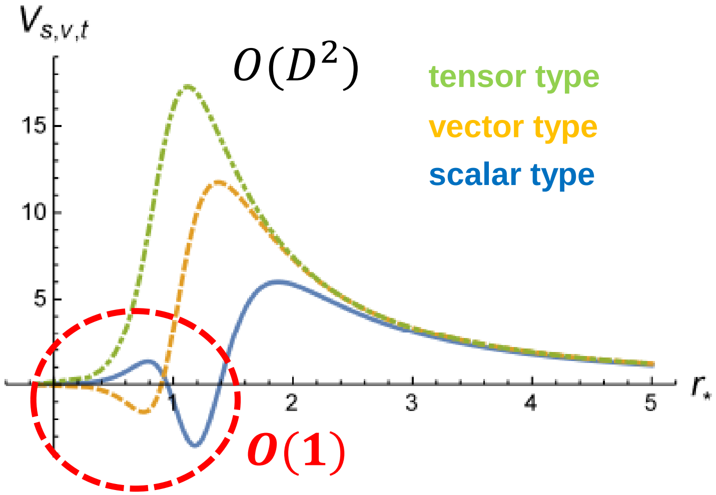


- Scalar and vector perturbations have **nontrivial structure**

*Tensor perturbation has same potential with scalar fields

- **Low energy gravitons** can probe this nontrivial structure

Such modes are confined by the potential barrier in very near-horizon region

(NON)DECOUPLED MODES

◻ Decoupled modes: Quasi-bound state

- Decoupled modes: $\omega_{\text{dec}} r_0 = O(1)$

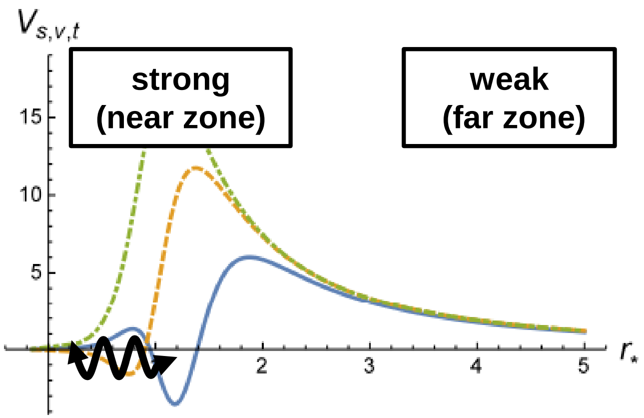
  - Localized and confined modes

  - Long-lived modes compared with black hole's characteristic time

$$\omega_{\text{dec}}^{-1} \gg \kappa^{-1} = O(r_0/D)$$

  - Decoupled from far zone dynamics

    Depending **only** on near horizon geometry



- Non-decoupled modes: $\omega_{\text{ndec}} r_0 = O(D)$

  - Fast modes, coupled with far zone dynamics

    Depending on surrounding matter and boundary conditions at infinity

TWO SECTORS

- **Black hole dynamics has two sectors at large D**
  - Perturbations excite two different modes

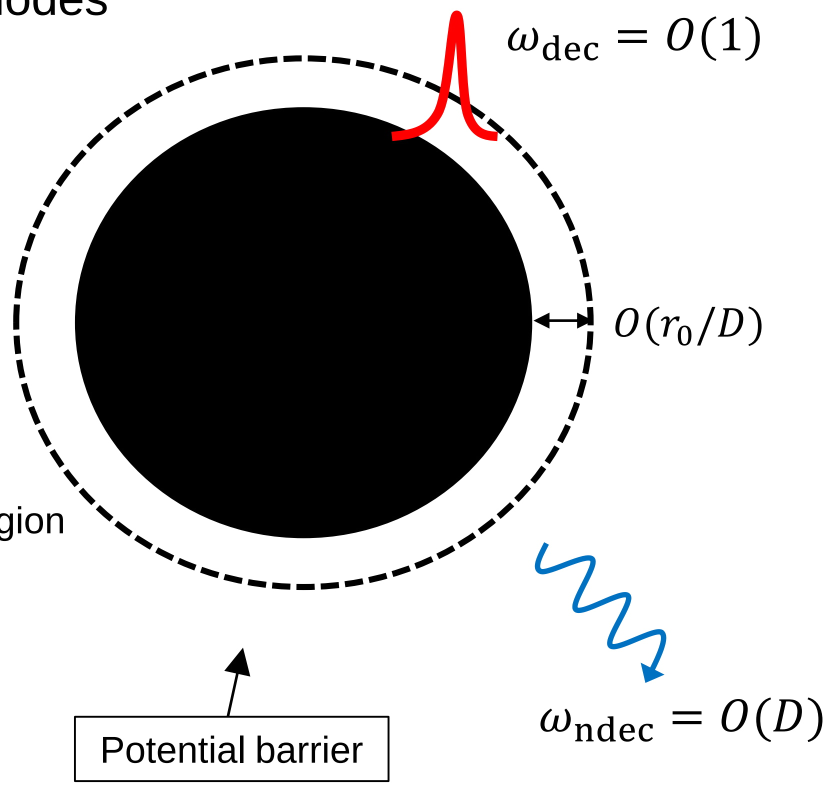
- Non-decoupled modes

- Fast oscillating modes, disperse into horizon and infinity instantly
- Universal mode for all non-extremal black holes (no interesting feature)

[Emparan-Tanabe (2013)]

- Decoupled modes

- Long-lived slow modes in near-horizon region
- Capture horizon dynamics of black holes
- Non-universal, important modes for black hole physics
- Effective description would exist



LINEAR ANALYSIS

- **Decoupled modes describe various black hole physics**

  - Various instabilities of black holes are in decoupled sector

    Instabilities of rotating black holes, black branes, black rings, charged black holes, and so on

  - Very easy to obtain higher order corrections in the 1/D expansions

    Only near horizon dynamics, almost stationary modes (zero modes)

e.g., Gregory-Laflamme modes

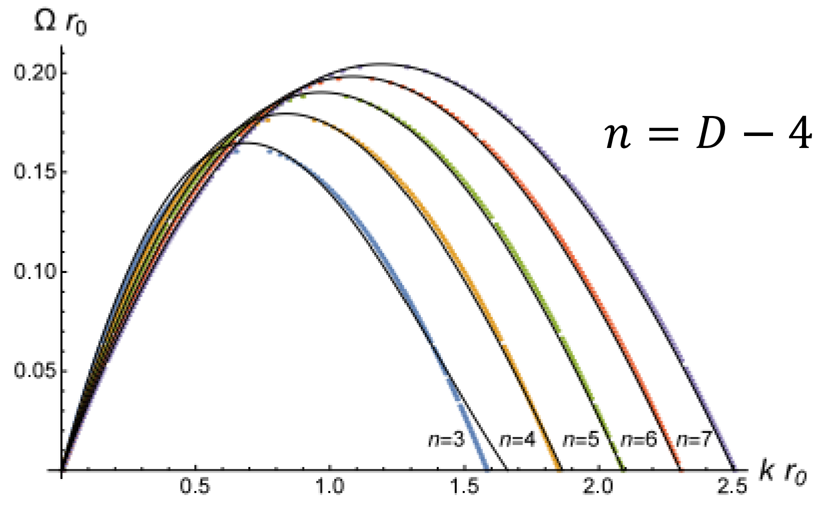
[Gregory-Laflamme (1993)]

Unstable mode of black branes $e^{\Omega t}$

Up to $O(1/D^3)$ corrections are included

$$\Omega r_0 = k(1-k) + O(1/D)$$

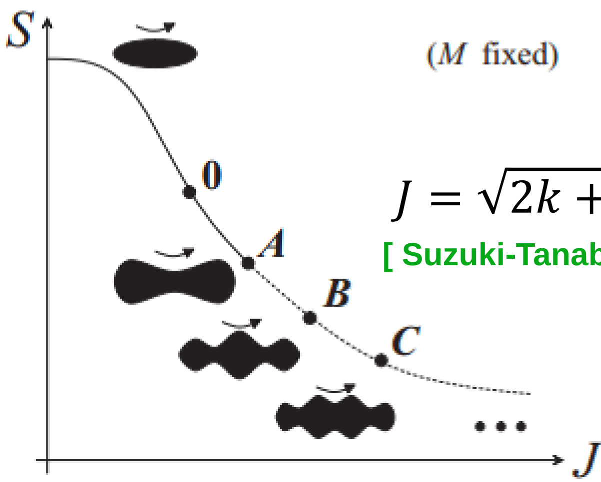
Good agreement with numerical results



$n = D - 4$

OTHER (LINEAR) RESULTS

☐ Reproduce various numerical results

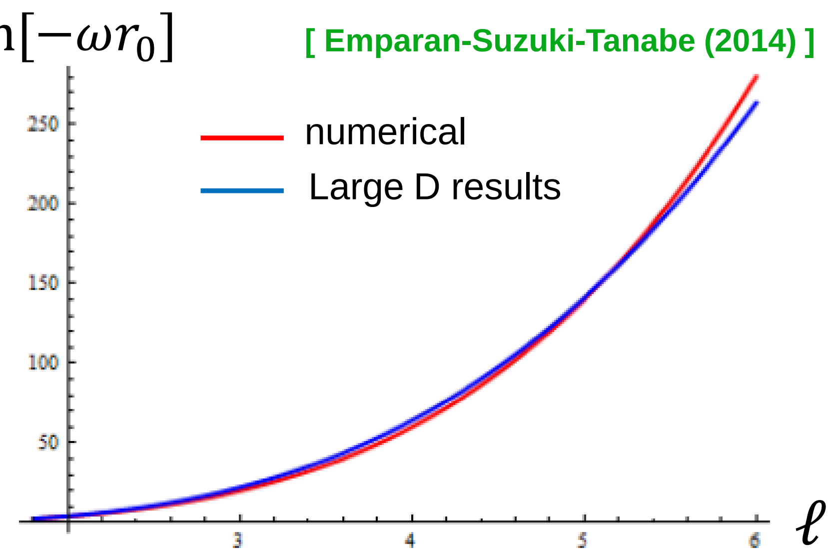


$J = \sqrt{2k+1}$

[Suzuki-Tanabe (2015)]

- Instability of rotating BHs

[Emparan-Suzuki-Tanabe (2014)]



- QNMs of 4d Schwarzschild BH

➢ Also produce unknown results

Instability of charged (rotating) black holes, QNMs of charged (AdS) black brane,…

SHORT SUMMARY

- **Large D limit is useful to black hole physics**

  - Scale hierarchy naturally appears at large D

$$r_0 \quad \gg \quad r_0/D$$

Decoupled   Non-decoupled

  - There are localized low energy gravitons in near-horizon region

    Decoupled modes: Almost stationary modes and decoupled dynamics from far zone dynamics

    Decoupled modes capture horizon dynamics of black holes such as deformations and instabilities of horizon

    Easy to obtain higher order collections: good accuracies in not much higher dimensions

    e.g., a few % errors even in 4 dim for QNMs of Schwarzschild BHs

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   - Nonlinear dynamics of decoupled modes

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QUESTIONS

- **We focus on decoupled modes**

  - Can we construct a nonlinear model for decoupled modes ?

    Linear perturbations (linear analysis of decoupled modes) are well understood

    Simpleness of calculation implies an existence of simple nonlinear theory ?

    Effective Lagrangian or equations for zero modes ?

  - What mechanism classifies decoupled and non-decoupled modes ?

    gravitons = (decoupled gravitons) + (non-decoupled gravitons)

    How many decoupled modes exist ?

    e.g., Schwarzschild BHs has **three** decoupled modes (**two** in scalar type perturbation, and **one** in vector type perturbation)

    How do we know that black holes have decoupled modes or not ?

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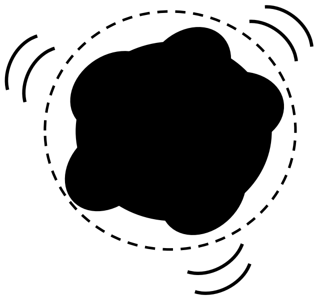
MEMBRANE PARADIGM

☐ Effective theory as membrane paradigm

- Black holes have slow and small deformations

$$\partial_t = O(1) \qquad \delta r_0 = O(1/D)$$



- Deformations are confined in near-horizon region

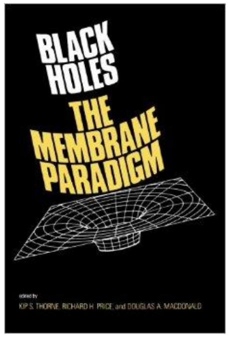
$$\partial_r \Phi = O(D)$$

- Deformed black holes are embedded dynamical objects in a flat spacetime (or fixed background geometry)

⇒ **membrane paradigm for black holes**

[Price-Thorne (1986)]

Horizon is a *physical object (viscous fluid)*



➢ Solving the Einstein equations, membrane paradigm is **derived at large D**

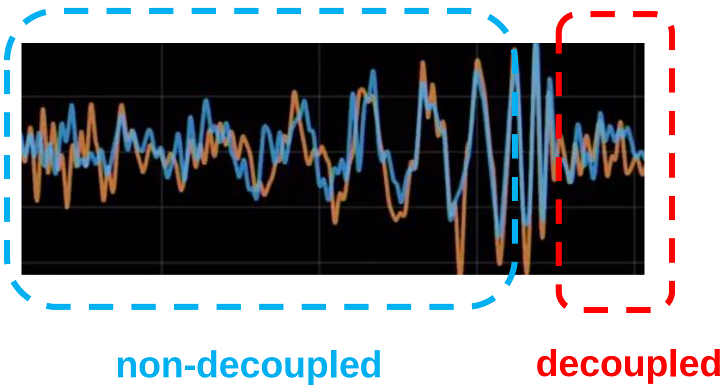
EFFECTIVE THEORY OF DECOUPLED MODES

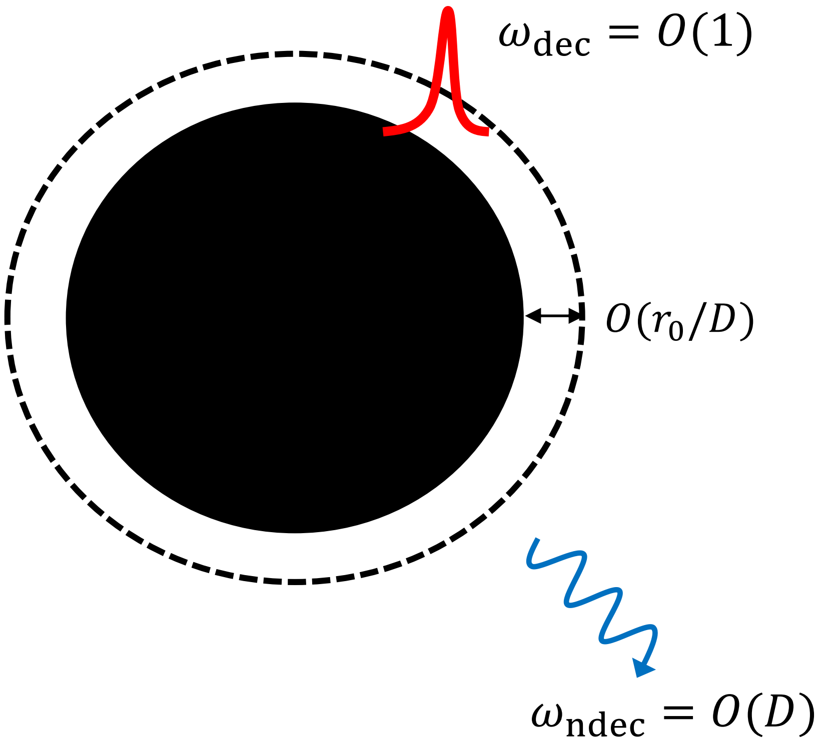
- **Integrating out (neglecting) of non-decoupled modes**

[Emparan-Suzuki-Tanabe (2015), Minwalla et.al. (2015)]

- Dynamics = (**non-decoupled**) + (**decoupled**)

  time scale : $O(r_0/D)$ time scale : $O(r_0)$

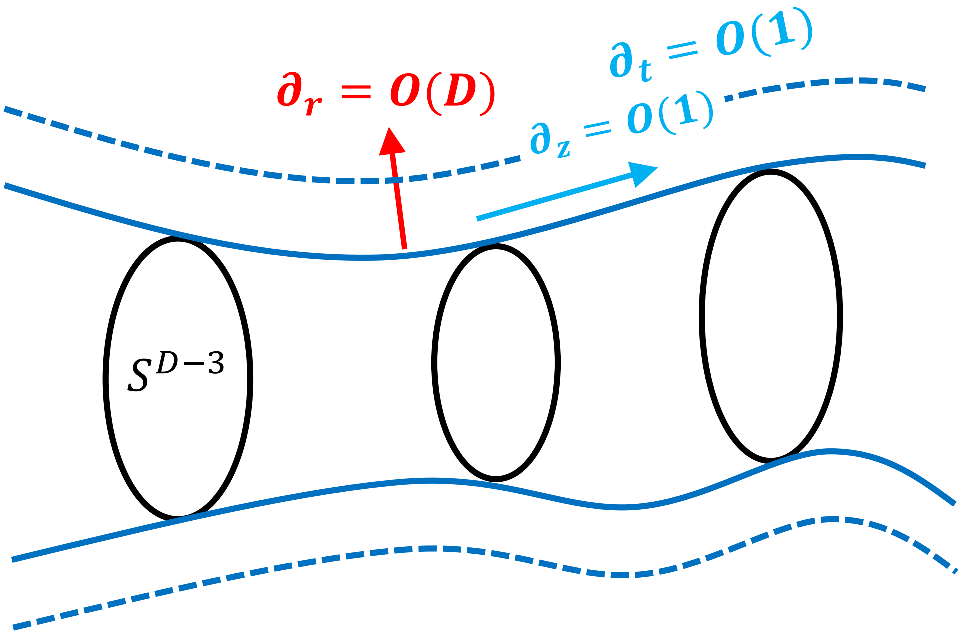




- Effective theory only for slow modes
- Black holes after drastic phenomena

SETUP FOR EFT

- **Solving the Einstein equations for decoupled modes**



- Equations are nonlinear **ODEs** w.r.t radial derivatives

- Physical quantities of BH such as Mass, (angular) momentum, charge, etc, are given by integration functions

- EoMs of membrane appear in the constraint equations

STATIONARY CASE

- **Stationary black holes without external fields**

[Emparan-Shiromizu-Suzuki-Tanabe-Tanaka (2015), Suzuki-Tanabe (2015)]

- membrane becomes a soap bubble

$$K = 2\gamma\kappa$$

K: mean curvature of the membrane

γ: redshift factor in the background

κ: surface gravity of the membrane

- Schwarzschild, Myers-Perry BHs (rotating BHs), (non-uniform) black brane, black droplets/funnels, …

- Black hole is just balanced only by its own surface tension (gravity)

External fields give polarizations and additional deformation to the membrane

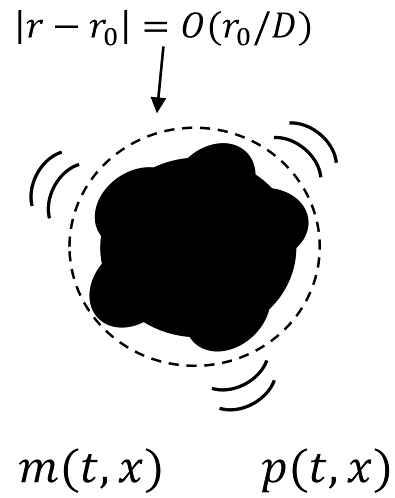
DYNAMICAL CASE

- **Time-dependent large D effective equations**

  - Solving the Einstein equations, we obtain

$$ds^2 = -\left(1 - \frac{\boldsymbol{m(t,x)}}{R}\right)dt^2 + 2dtdr - \frac{\boldsymbol{p(t,x)}}{R}dtdx + \cdots$$

$$R = (r/r_0)^{D-3}$$



  - Black hole is a membrane with time dependent mass and momentum

    Energy-momentum tensor of a membrane is read by the Brown-York method

  - Constraint equations in the Einstein equations give nonlinear dynamical effective equations for $m(t,x)$ and $p(t,x)$

    EoMs of the membrane is derived from the Einstein equation

    We can derive effective equations for general black holes

DYNAMICAL BLACK STRINGS

- **Explicit example: Dynamical black string**

[Emparan-Suzuki-Tanabe (2015)]

- D dim static black string solution (exact solution)

$$ds^2 = -\left(1 - \frac{m}{R}\right) dt^2 + 2dtdr + dZ^2 + r^2 d\Omega_{D-3} \qquad R = (r/r_0)^{D-3}$$

mass: $(D-3)mr_0^{D-3}$ tension: mr_0^{D-3}

- D dim dynamical black string solution at large D

$$ds^2 = -\left(1 - \frac{\boldsymbol{m(t,z)}}{R}\right) dt^2 + 2dtdr + \frac{dz^2}{D} - \frac{\boldsymbol{p(t,z)}}{R} \frac{dtdz}{\sqrt{D}} + r^2 d\Omega_{D-3}$$

$$Z = z/\sqrt{D}$$

$\sqrt{D}$ factor appears from the sound velocity order

$$v \sim (\text{tension}/\text{mass})^{1/2} \sim O(1/\sqrt{D})$$

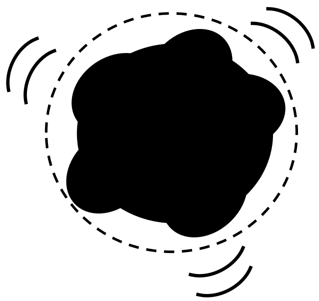
EFFECTIVE EQUATIONS

$$ds^2 = -\left(1 - \frac{\boldsymbol{m(t,z)}}{R}\right)dt^2 + 2dtdr + \frac{dz^2}{D} - \frac{\boldsymbol{p(t,z)}}{R}\frac{dtdz}{\sqrt{D}} + r^2 d\Omega_{D-3}$$

□ Effective equations of dynamical black strings

$$\partial_t m - \partial_z^2 m = -\partial_z p$$

$$\partial_t p - \partial_z^2 p = \partial_z m - \partial_z \left(\frac{p^2}{m}\right)$$



- Nonlinear coupled diffusion equations

Very easy to solve the equations numerically

- These equations can be rewritten in a hydrodynamic form

$$\partial_t m + \partial_z (m v_z) = 0$$

$$\partial_t (m v_z) + \partial_z \tau_{zz} = 0$$

$$p = m v_z - \partial_z m$$

- Mass and momentum are conserved (feature of decoupled modes)
- Truncation is occurred naturally by the large D limit

GREGORY-LAFLAMME

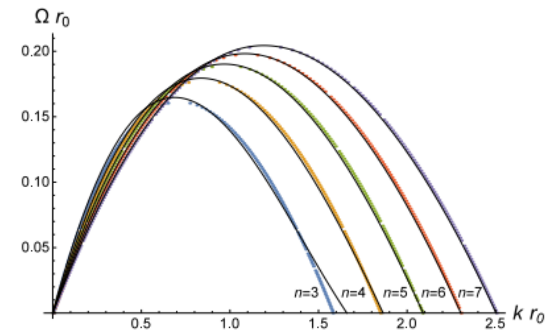
- **Black strings are dynamically unstable**

  - Linear analysis $\delta h_{\mu\nu} \propto e^{\Omega t}$

  - What is the final state ?

    Nontrivial static state ? Singularity ?

    One of unresolved problem in GR



➢ Very hard problem of GR

- Only one example

  1 month for one solution

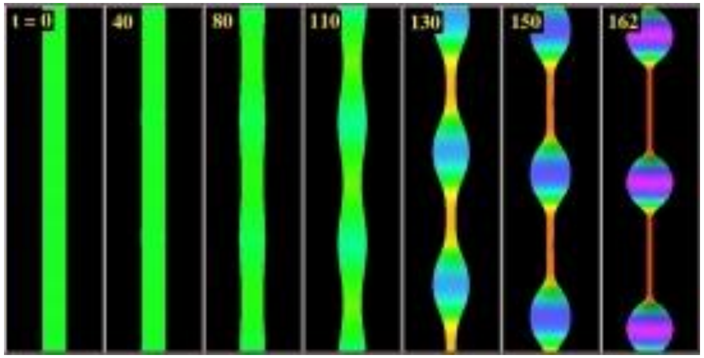
- How universal ?

  Initial condition dependence

  Dimension dependence

  Effects of other matter such as gauge fields

Evolution of 5D unstable black string



[Lehner-Pretorius (2010)]

LARGE D RESULTS

- Large D effective theory gives simple analysis

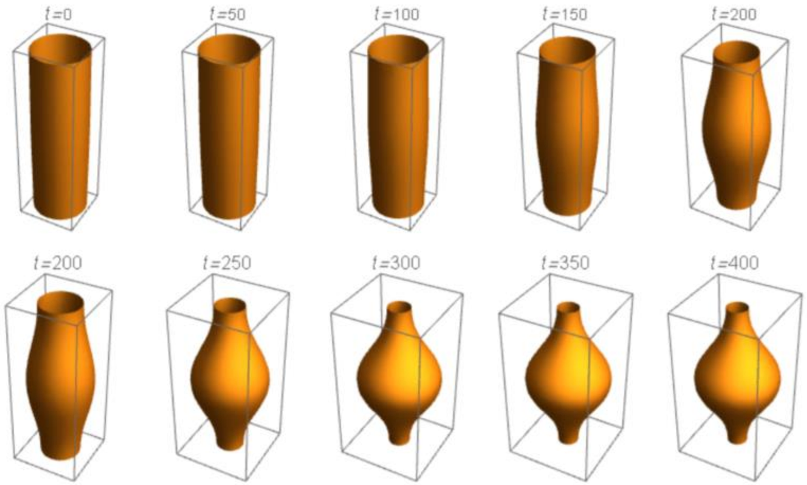
$$\partial_t m - \partial_z^2 m = -\partial_z p$$

$$\partial_t p - \partial_z^2 p = \partial_z m - \partial_z \left(\frac{p^2}{m}\right)$$

```
eq1 = ∂t m[t, z] - ∂z,z m[t, z] + ∂z p[t, z];
eq2 = ∂t p[t, z] - ∂z,z p[t, z] - ∂z m[t, z] + ∂z (p[t, z]^2 / m[t, z]);

tmax = 1455;
k = 0.995;
Ls = 2 π / k;
pertm = 0.05 Cos[k z];
pertp = 0;

pde = {eq1 == 0, eq2 == 0};
icbc = {m[0, z] == 1 + pertm, p[0, z] == pertp, m[t, -Ls / 2] == m[t, Ls / 2], p[t, -Ls / 2] == p[t, Ls / 2]};

sol = NDSolve[{pde, icbc}, {m, p}, {t, 0, tmax}, {z, -Ls / 2, Ls / 2}, MaxStepSize → 0.1];
```

LARGE D RESULTS

- **We can solve equations easily**



- Final state is **non-uniform black string** in large dimensions (as expected)

- A few seconds for one calculation (systematic analysis is possible)

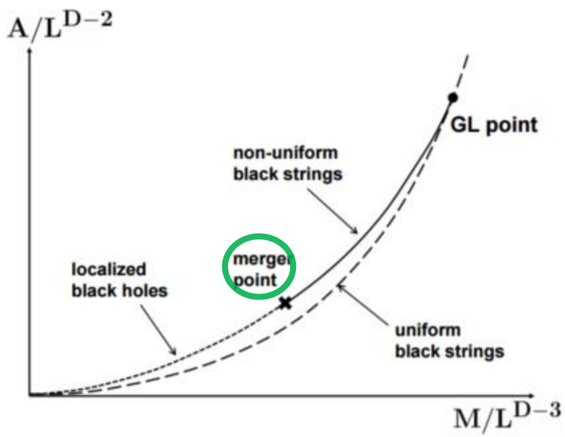
- Inclusion of higher order corrections in 1/D expansions gives dimensional dependence of results

Observation of critical dimensions, cusp in the phase diagram,…

NUBS

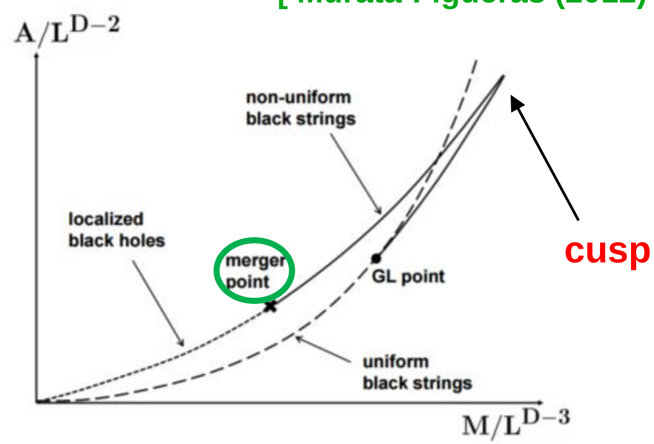
- **Phase diagram of Non-uniform black strings (NUBS)**

[Murata-Figueras (2012)]



Higher dim (D>13)

Critical dimensions (D~13)



Lower dim (D<13)

- Dimensional dependence of finale state of GL instability
- Large D calculation can capture these phenomena

[Emparan-Luna-Martinez-Suzuki-Tanabe (to be appeared)]

SHORT SUMMARY

- **Large D method is very powerful**
  - For decoupled mode analysis (even in nonlinear region)
  - Dynamical analysis is possible systematically
  - Derivation of membrane paradigm from the Einstein equations

- What can we do ? (advertisement of our (future) work)
  - Final state of instabilities of black holes

    Rotating black holes, black rings, charged black holes

    Also for final state of the superradiant instability
  - (Dynamical) deformation of black objects

    Wave collisions on planar AdS black brane (AdS version of GL analysis)

    Polarized black holes/branes by external electric fields

    Dynamics of braneworld black holes or black droplets/funnels

    Construction of unknown black holes with new horizon topology

CONTENTS

1. Large D limit of General Relativity

2. Effective theory of black holes

3. Summary

App. More details

SUMMARY

- **Gravity always has a dimensionless parameter in its theory, spacetime dimension D**

  - Large D limit can be always taken if we have gravity

    Simplification of the theory on the boundary of the parameter space

  - We could show its powerfulness in black hole physics

    Black holes have a few low energy excitation (decoupled modes)

    Effective theory of decoupled modes is the revival of the membrane paradigm

    Nonlinear and dynamical analysis become possible systematically

  - Gravity and black holes are now tools, and so does the large D limit

    (Possible) Application to other fields (Holographic superconductor, Drude model, Riemann problem, Cosmology,…)

    Current procedure of the large D method is a bit technical, but very easy (if one can calculate Riemann tensors)

OUTLOOK

□ **Large D limit exists always if gravity exists**

➢ More systematic and general

- Understanding the application range in black hole physics

[Emparan-Izumi-Suzuki-Tanabe, Emparan-Suzuki-Tanabe (to be appeared)]

➢ Application to other gravitational physics

- Other classical gravitational phenomena: Cosmology, gravitational wave physics, gravitational collapse, many-body system of BHs,…

- Quantum gravity : Analogy with the large N limit of gauge theories

➢ Deeper understanding in black hole physics

- Simpleness of decoupled modes and (conformal) symmetry

[Tanabe (to be appeared)]

- What can we do in non-decoupled sector ?

[Emparan-Grumiller-Tanabe (on going)]

CONTENTS

1. Large D limit of General Relativity

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App. More details

- Origin of decoupled modes
- Other examples

QUESTIONS

- **We focus on decoupled modes**

  - Can we construct a nonlinear model for decoupled modes ?

    Linear perturbations (linear analysis of decoupled modes) are well understood

    Simpleness of calculation implies an existence of simple nonlinear theory ?

    Effective Lagrangian or equations for zero modes ?

  - What mechanism classifies decoupled and non-decoupled modes ?

    gravitons = (decoupled gravitons) + (non-decoupled gravitons)

    How many decoupled modes exist ?

    e.g., Schwarzschild BHs has **three** decoupled modes (**two** in scalar type perturbation, and **one** in vector type perturbation)

QUESTIONS

- **We focus on decoupled modes**

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  - What mechanism classifies decoupled and non-decoupled modes ?

    gravitons = (decoupled gravitons) + (non-decoupled gravitons)

    How many decoupled modes exist in general ?

    ***e.g.,* Schwarzschild BHs has three decoupled modes (two in scalar type perturbation, and one in vector type perturbation)**

QNMS

- **Quasinormal modes are characterized by quantum numbers of perturbations**

ex) QNMs of Schwarzschild black holes

$$\Psi = \psi(r) e^{-i\omega t} Y_{\ell m}$$

- Quantum numbers: $\ell, \boldsymbol{m}$ and overtone number $\boldsymbol{n}$

$$\omega = \omega(\ell, m, n)$$

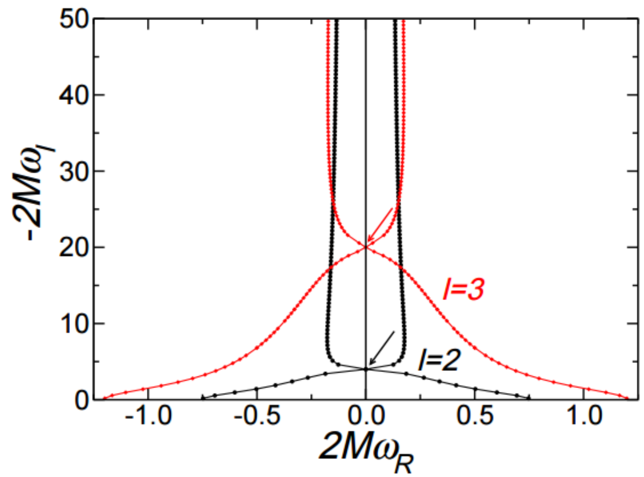
Overtone number : number of nodes of radial function $\psi(r)$

- Distribution of QNMs **:** Overtone number dependence of QNMs

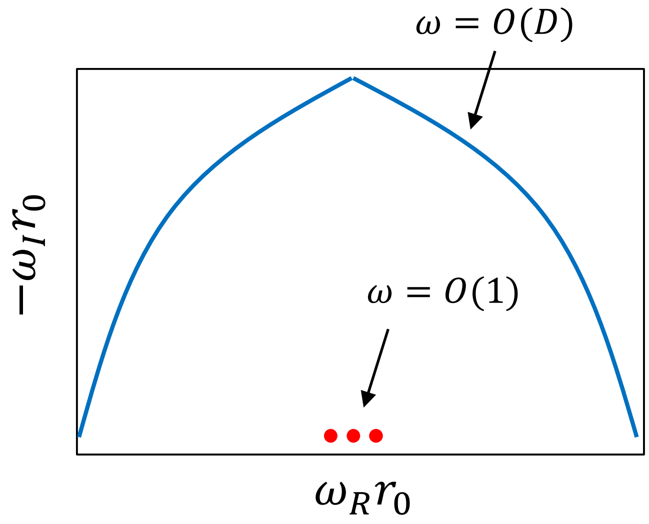
SYMMETRY PROTECTION ?

☐ Distribution of QNMs of Schwarzschild black holes

[Berti-Cardoso-Starinets (2009)]



4 dim Schwarzschild



Large D Schwarzschild

- Distribution of non-decoupled modes become dispersing with O(D)
- A few decoupled modes keep their positions at O(1) in $D \to \infty$

Zero modes, Symmetry protection ? Decoupled modes = Nambu-Goldstone modes ?

ORIGIN OF DECOUPLED MODES

☐ Expected picture

- Decoupled modes (zero modes) are Nambu-Goldstone modes

  - Black holes have an enhanced symmetry at the **large D limit**
  - The symmetry is spontaneously broken at finite D (by 1/D corrections)
  - # decoupled modes = # generators of broken symmetry

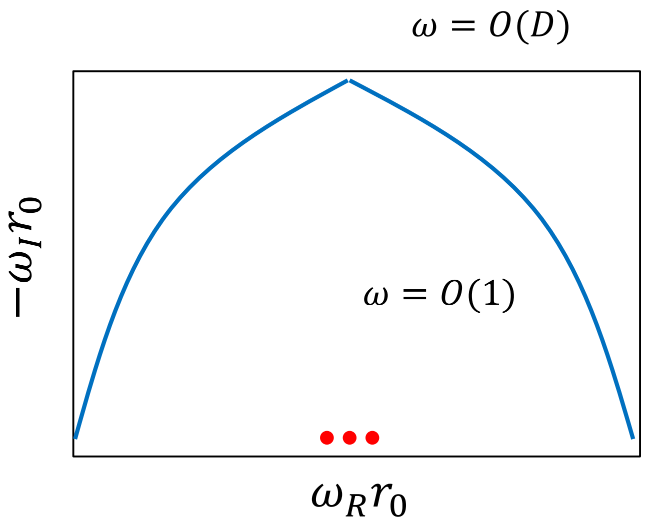
    ex) Schwarzschild BH has **three** broken generators

Non-decoupled modes

Dynamical gravitons even at the large D limit

Decoupled modes

Gauge modes (non-dynamical modes) at the large D limit



2D STRING

- **2D string action appears at $D = \infty$**

[Soda (1992), Grumiller-Kummer-Vassilevich (2002), Emparan-Grumiller-Tanabe (2013)]

- Dimensional reduction on a sphere

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + r_0^2 e^{-4\Phi/(D-2)} d\Omega_{D-2}$$

- Einstein-Hilbert action becomes 2D string action

$$S = \frac{1}{16\pi G} \int \sqrt{-g} d^D x \, R \qquad \lambda = D/r_0$$

$$= \frac{\Omega_{D-2} r_0^{D-2}}{16\pi G} \int \sqrt{-g} d^2 x \, e^{-2\Phi} (R + 4(\nabla\Phi)^2 + 4\lambda^2)$$

- Action for 2D string BH describing the SL(2,R)/U(1) coset model [Witten (1992)]
- D dim spherical symmetric modes = 2D graviton +Dilaton
- This fact suggests the appearance of a conformal symmetry at $D = \infty$?

LARGE D BH = 2D BH

- **2D Witten black hole appears**
  - D dim Schwarzschild BH becomes the Witten BH

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2} \qquad f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3}$$

- 2D part (t,r) becomes dominant in the metric

$$ds^2 = \frac{1}{D^2}\left[-\left(1-\frac{1}{R}\right)d\hat{t}^2 + \frac{dR^2}{R(R-1)}\right] + r_0^2 d\Omega_{D-2} \qquad \hat{t} = D\,t, \quad R = \cosh^2\rho$$

$$= \frac{\mathbf{1}}{\boldsymbol{D}^2}\left[-\tanh^2\boldsymbol{\rho}\, \boldsymbol{d}\hat{\boldsymbol{t}}^2 + \boldsymbol{d\rho}^2\right] + \boldsymbol{r}_0^2 \boldsymbol{d\Omega}_{\boldsymbol{D}-2}$$

2D Witten black hole

- 2D Witten black hole (gauged WZW model : SL(2,R)/U(1) coset model)

* Strings can propagate on this geometry at the 1-loop level

[Witten (1992)]

GENERALITY

- **Black holes become Witten BH generically**

  - Rotating black holes also have 2D structure (boosted Witten BH)

    The large D limit of Myers-Perry black holes:

$$ds^2 = \frac{4r_0^2}{D^2}(1-v^2)\left(-d\hat{t}^2 + d\rho^2 + dy^2 + \frac{(d\hat{t} - v dy)^2}{(1-v^2)\cosh^2\rho} + \cdots\right)$$

  - More generically, (non-extremal) black holes have 2D structure universally [Emparan-Grumiller-Tanabe (2013), Emparan-Suzuki-Tanabe (to be appeared)]

    - **General properties of black holes at the large D limit**

      Graviton on black holes = 2D graviton + dilatons

      2D gravitons is not dynamical (zero modes)

CONFORMAL SYMMETRY

- 2D geometry has conformal symmetry

$$ds^2 = \frac{1}{D^2}[-\tanh^2\rho\, d\hat{t}^2 + d\rho^2] + r_0^2 d\Omega_{D-2}$$

$$= \frac{1}{D^2}\frac{dudv}{1-uv} + r_0^2 d\Omega_{D-2}$$

$$u = e^{\hat{t}}\sinh\rho$$
$$v = -e^{-\hat{t}}\sinh\rho$$

- Conformal transformation exists in coordinate transformation

$$u \to f(u) \qquad v \to g(v)$$

This is a reminiscent of SL(2,R) × SL(2,R) symmetry of gauged WZW model

Gauged (Local) symmetry : Time translation $\hat{t} \to \hat{t} + \epsilon$

$$u \to u(1+\epsilon) \qquad v \to v(1-\epsilon)$$

- Associated Virasoro operators

$$L_n = u^{n+1}\frac{\partial}{\partial u} \qquad \bar{L}_n = v^{n+1}\frac{\partial}{\partial v}$$

ENHANCED CONFORMAL SYMMETRY

□ Conformal structure is uplifted to the symmetry

$$ds^2 = \frac{1}{D^2}\frac{dudv}{1-uv} + r_0^2 d\Omega_{D-2}(x^A)$$

- Symmetry transformation on **2D black hole metric**

$$u \to u + \epsilon u^{n+1} \qquad L_n = u^{n+1}\frac{\partial}{\partial u}$$

$$u \to u + \boldsymbol{\epsilon(x^A)\ u^{n+1}} \qquad L_n = u^{n+1}\frac{\partial}{\partial u}$$

- This is still the (conformal) symmetry transformation of **D dim black holes metric**

- Deformations on 2D part does not affect S^{D-2} part
- This is **not** a coordinate transformation in D dim spacetime
- L_0 and $L_{\pm 1}$ reproduce the hidden conformal structure of D dim Schwarzschild black hole

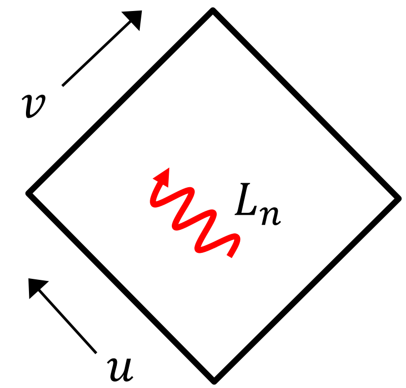
INTERPRETATIONS

- Each generators excite propagating gravitons in D dim black hole spacetime

$$ds^2 = \frac{1}{D^2}\frac{dudv}{1-uv} + r_0^2 d\Omega_{D-2}(x^A)$$

- "Ingoing gravitons" are generated by

$$u \to u + \boldsymbol{\epsilon}\left(\boldsymbol{x^A}\right)\ \boldsymbol{u^{n+1}} \qquad L_n = u^{n+1}\frac{\partial}{\partial u}$$



The energy of gravitons can be estimated by the Casimir operator of SL(2,R)

$$E_n \sim D^2 n^2$$ absorbed or dispersing

Zero modes exist, and they are not propagating (long lived bound state)

$$L_0 = u\frac{\partial}{\partial u} \qquad \bar{L}_0 = v\frac{\partial}{\partial v}$$

NG MODES

- **The symmetry is broken by 1/D corrections**

$$ds^2 = \frac{1}{D^2}\frac{du dv}{1-uv} + r_0^2 d\Omega_{D-2}(x^A)$$

$$u \to u + \boldsymbol{\epsilon(x^A)\ u^{n+1}} \qquad L_n = u^{n+1}\frac{\partial}{\partial u}$$

- Not the coordinate transformation (difference appears in 1/D corrections)

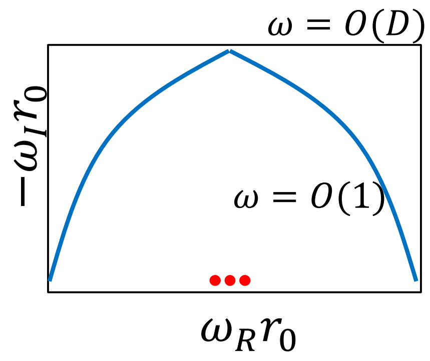
- Zero modes become Nambu-Goldstone modes of spontaneously broken conformal symmetry

$$L_0 = u\frac{\partial}{\partial u} \qquad \bar{L}_0 = v\frac{\partial}{\partial v}$$

- Comparing perturbation results, excited zero modes are identified with the decoupled modes **[Tanabe (to be appeared)]**

Excitations are scalar type perturbations (two decoupled modes in scalar type perturbations)

DECOUPLED MODES =NG MODES



☐ Decoupled modes = NG modes with zero energy

- There are two SL(2,R) in black holes generically
  - Existence of two decoupled modes in black holes
- Schwarzschild BH has three decoupled modes
  - Two in scalar type perturbations : Broken SL(2,R) × SL(2,R)
  - One in vector type perturbations : Broken U(1) ?

$$ds^2 = \frac{4r_0^2}{D^2}(1-v^2)\left(-d\hat{t}^2 + d\rho^2 + dy^2 + \frac{(d\hat{t} - v dy)^2}{(1-v^2)\cosh^2\rho} + \cdots\right)$$

Rotation is generated by the boost transformation

$$\hat{t} \to \hat{t}\cosh\alpha \ - y\sinh\alpha \quad \Rightarrow \quad \hat{t} \to \hat{t}\cosh\alpha(x^A) \ - y\sinh\alpha\,(x^A)$$

This boost symmetry is also broken in 1/D corrections (give NG modes)

SUMMARY

- **Decoupled modes are NG modes of large D BHs**

  - Black holes generically have 2D structure in the metric

  - 2D structure has two conformal symmetries in coordinate transformations

  - Rotations are introduced as boost transformation

  - Broken generators with zero energy give decoupled modes

    Scalar type : Conformal symmetry, Vector type: Boost symmetry

  - This understanding gives complete counting of number of decoupled modes

  - Useful or some help in application of (non)decoupled modes ?