Gravity theory with the Gauss-Bonnet term, and Thermodynamic Black Hole stabilities

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1. Motivations

Low energy effective theory from string theory
→ Einstein Gravity + higher curvature terms
Gauss-Bonnet term is the simplest leading term.

Q : What is the physical effects of Gauss-Bonnet terms?

1) Effects to the Black Holes.

No-Hair Theorem of Black Holes

Stationary black holes (in 4-dim Einstein Gravity) are completely described by 3 parameters of the Kerr-Newman metric :
mass, charge, and angular momentum (M, Q, J)

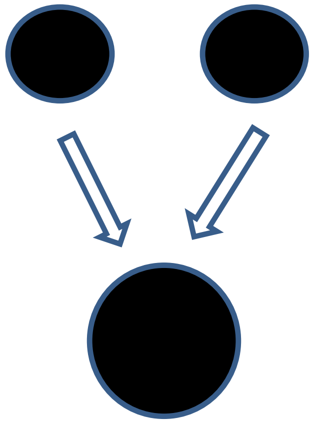
Werner Israel(1967), Brandon Carter(1971,1977), David Robinson (1975)

Hairy black hole solution ?
In the dilaton-Gauss-Bonnnet theory → Yes!

Exists the minimum mass of BH
Affects the stability, etc.

2) Effects in the Early Universe.

Motivations - continued



A Black Hole Merger

Colliding Black Holes :
A Black Hole Merger + Gravitational Wave

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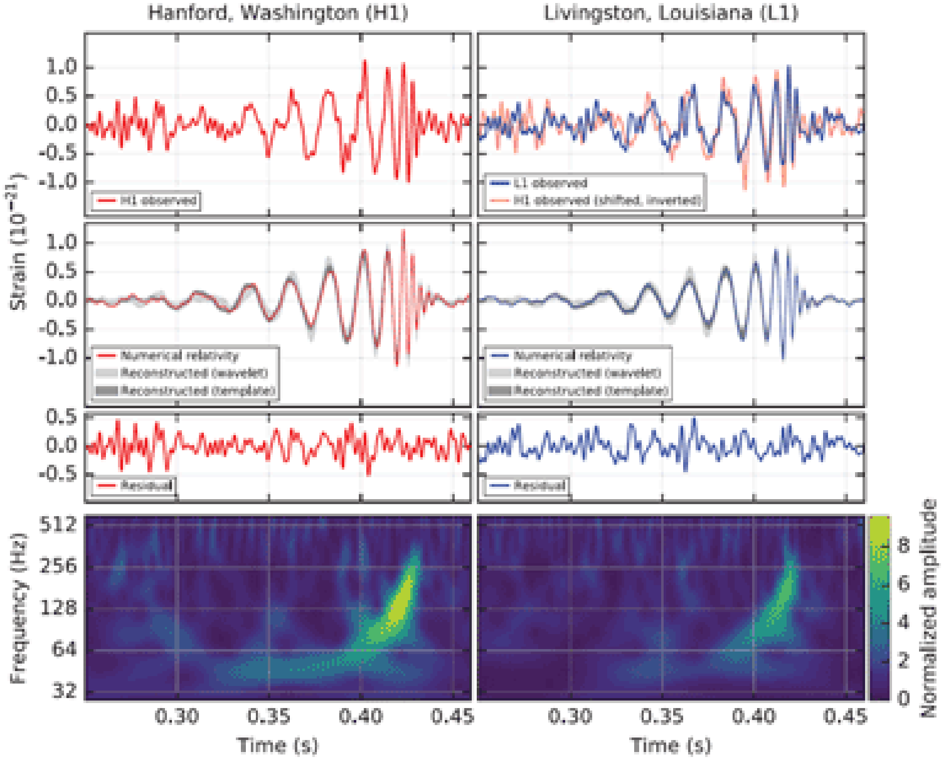
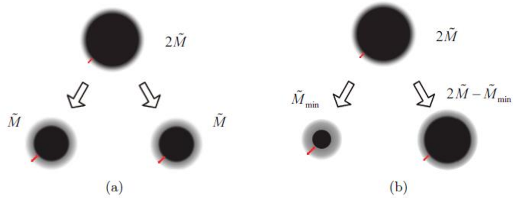


Q: A Black Hole unstable ?
splitting
into two Black Holes ?



Holography

(asymptotic) AdS Black Hole in d+1 dim

$\leftrightarrow$

Quantum System in d dim.

Instability of Black Holes

$\leftrightarrow$ instability of Quantum System

Hence,

instability of AdS BH

$\leftrightarrow$ phase transitions in Quantum System

* Black holes in higher dimensions are quite diverse !

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3. G-B gravity in Inflationary Universe

Model
Preheating, etc.

2. Black Holes in the Dilaton Gauss-Bonnet theory

W.Ahn, B. Gwak, BHL,
W.Lee,
Eur.Phys.J.C (2015)

Hairy black holes in Dilaton-Einstein-Gauss-Bonnet (DEGB) theory

Action

$$I = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla_\alpha \Phi \nabla^\alpha \Phi + \alpha e^{-\gamma\Phi} R^2_{\rm GB} \right] + \oint_{\partial\mathcal{M}} \sqrt{-h} d^3x \frac{K - K_o}{\kappa}, \quad (1)$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

Guo,N.Ohta & T.Torii, Prog.Theor.Phys. 120,581(2008);121 ,253 (2009);
N.Ohta &Torii,Prog.Theor.Phys.121,959; 122,1477(2009);124,207 (2010);
K.i.Maeda,N.Ohta Y.Sasagawa, PRD80, 104032(2009); 83,044051 (2011)
N. Ohta and T. Torii, Phys.Rev. D 88 ,064002 (2013).

The Gauss-Bonnet term :

$$R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Note :

1) The symmetry under $\gamma \to -\gamma, \Phi \to -\Phi.$

allows choosing γ positive values without loss of generality.

2) The coupling α dependency could be absorbed by the r → r/√α transformation.
with non-zero α coupling cases being generated by α scaling.
However, the behaviors for the α = 0 case cannot be generated in this way.
Hence, we keep the parameter α, to show a continuous change to α = 0.

The Einstein equations and the scalar field equation are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\Phi\partial^\rho\Phi + T^{GB}_{\mu\nu}\right), \qquad (2)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi] - \alpha\gamma e^{-\gamma\Phi}R^2_{\mathrm{GB}} = 0,$$

where

$$\begin{aligned} T^{GB}_{\mu\nu} &= -8\alpha(R_{\mu\rho\nu\sigma}\nabla^\rho\nabla^\sigma e^{-\gamma\Phi} - R_{\mu\nu}\Box e^{-\gamma\Phi} + 2\nabla_\rho\nabla_{(\mu}e^{-\gamma\Phi}R^\rho{}_{\nu)} - \frac{1}{2}R\nabla_\mu\nabla_\nu e^{-\gamma\Phi}) \\ &\quad +4\alpha(2R^{\rho\sigma}\nabla_\rho\nabla_\sigma e^{-\gamma\Phi} - R\Box e^{-\gamma\Phi})g_{\mu\nu}, \qquad (4) \end{aligned}$$

and $\Box \equiv \nabla_\mu\nabla^\mu$ is the d'Alembertian.

Note :

1. All the black holes in the DEGB theory with given non-zero couplings α and γ have hairs.
   I.e., there does not exist black hole solutions without a hair in DEGB theory.
   (If we have Φ = 0, dilaton equation of motion reduces to R^2_{GB} = 0, not satisfying the dilaton e.o. m..)

2. For the coupling α = 0, the solutions become a Schwarzschild black hole in Einstein gravity.

3. For $\gamma = 0$, DEGB theory becomes the Einstein-Gauss-Bonnet (EGB) theory.
   The EGB black hole solution is the same as that of the Schwarzschild one.
   However, the GB term contributes to the black hole entropy and influence stability.

We consider a spherically symmetric static spacetime with the metric

P. Kanti et al., PRD54, 5049 (1996).

$$ds^2 = -e^{X(r)}dt^2 + e^{Y(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{5}$$

Then the dilaton field equation turns out to be

$$\Phi'' + \Phi'\left(\frac{X'-Y'}{2} + \frac{2}{r}\right) = -\frac{4\alpha\gamma e^{-\gamma\Phi}}{r^2}\left[X'Y'e^{-Y} + (1-e^{-Y})\left(X'' + \frac{X'}{2}(X'-Y')\right)\right]\,, \tag{6}$$

Also, three Einstein equations for (tt), (rr), and (θθ) components as follows:

$$Y'\left(1 - \frac{4\alpha\gamma\kappa e^{-\gamma\Phi}\Phi'}{r}(1-3e^{-Y})\right) = \frac{\kappa r\Phi'^2}{2} + \frac{1-e^{Y}}{r} - \frac{8\alpha\gamma\kappa e^{-\gamma\Phi}}{r}(\Phi'' - \gamma\Phi'^2)(1-e^{-Y})\,, \tag{7}$$

$$X'\left(1 - \frac{4\alpha\gamma\kappa e^{-\gamma\Phi}\Phi'}{r}(1-3e^{-Y})\right) = \frac{\kappa r\Phi'^2}{2} + \frac{(e^{Y}-1)}{r}\,, \tag{8}$$

$$X'' + \left(\frac{X'}{2} + \frac{1}{r}\right)(X'-Y') = -\kappa\Phi'^2 - \frac{8\alpha\gamma\kappa e^{-\gamma\Phi-Y}}{r}\left(\Phi'X'' + (\Phi'' - \gamma\Phi'^2)X' + \frac{\Phi'X'}{2}(X'-3Y')\right)\,, \tag{9}$$

we choose three Eqs. (7) - (9) as dynamical equations and the remaining one Eq. (6) as the constraint equation.

The condition for the existence of a black hole solution.

$$e^{-\gamma\Phi_h} < \frac{r_h^2}{\alpha}\frac{1}{\gamma\sqrt{192\kappa}}\,.$$ from $$\Phi'_h = \frac{r_h e^{\gamma\Phi_h}}{8\alpha\gamma\kappa}\left(1 \pm \sqrt{1 - 192 e^{-2\gamma\Phi_h}\alpha^2\gamma^2\kappa/r_h^4}\right)$$

The asymptotic form of the solutions takes **(M = ADM mass, Q = scalar charge)**

$$e^X \simeq 1 - \frac{2M}{r} + \mathcal{O}(1/r^3)\,, \tag{18}$$

$$\Phi \simeq \Phi_\infty + \frac{Q}{r} + \mathcal{O}(1/r^2)\,, \tag{19}$$

The mass of a hairy black hole is represented as follows

$$M(r) = M(r_h) + M_{\rm hair}\,.$$

where $M(r_h) = \frac{1}{2}r_h$ **is the BH mass subtracting the scalar hair contribution.**
$\boldsymbol{M_{hair}}$ **represents the contribution from the scalar hair.**

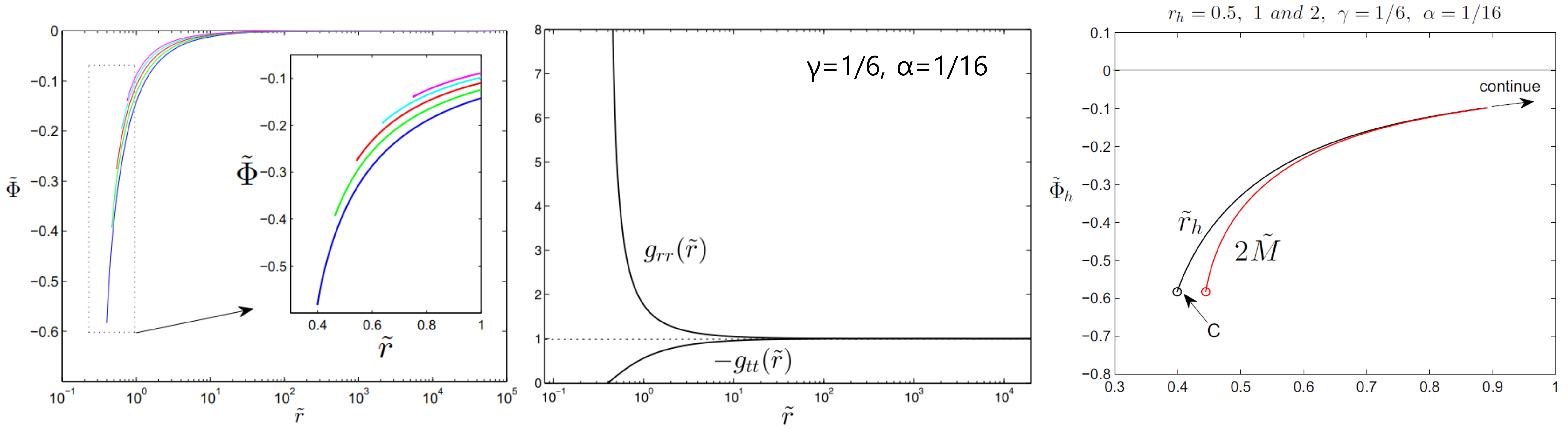
Note :

Hair Charge Q is not zero, and is not independent charge either.

DEGB Black Hole solutions

an event horizon at $g^{rr}(r_h) = 0$ or $g_{rr}(r_h) = \infty.$

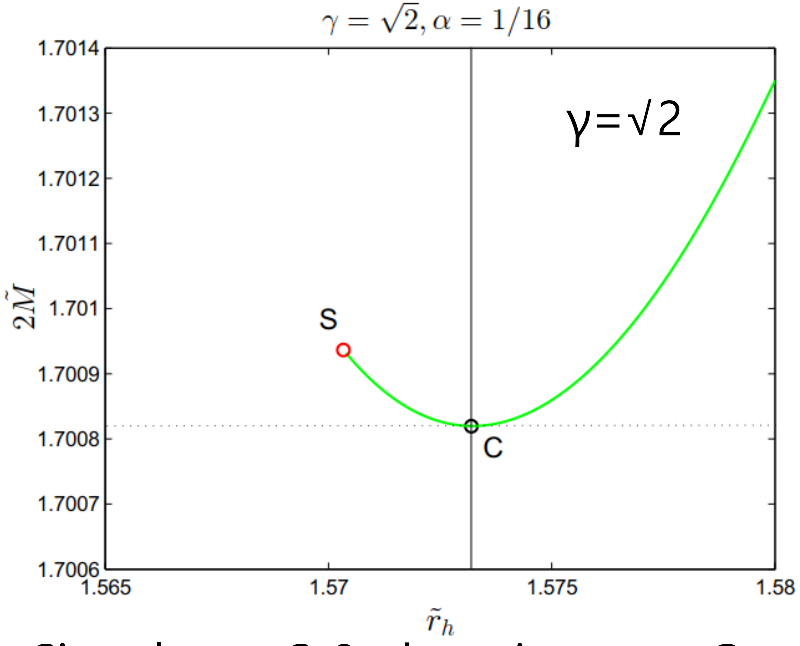
the rescaling $\tilde{\Phi} = \Phi - \Phi_\infty \quad r \to \tilde{r} = re^{\gamma\Phi_\infty/2} \quad M \to \tilde{M} = Me^{\gamma\Phi_\infty/2} \quad Q \to \tilde{Q} = Qe^{\gamma\Phi_\infty/2}.$



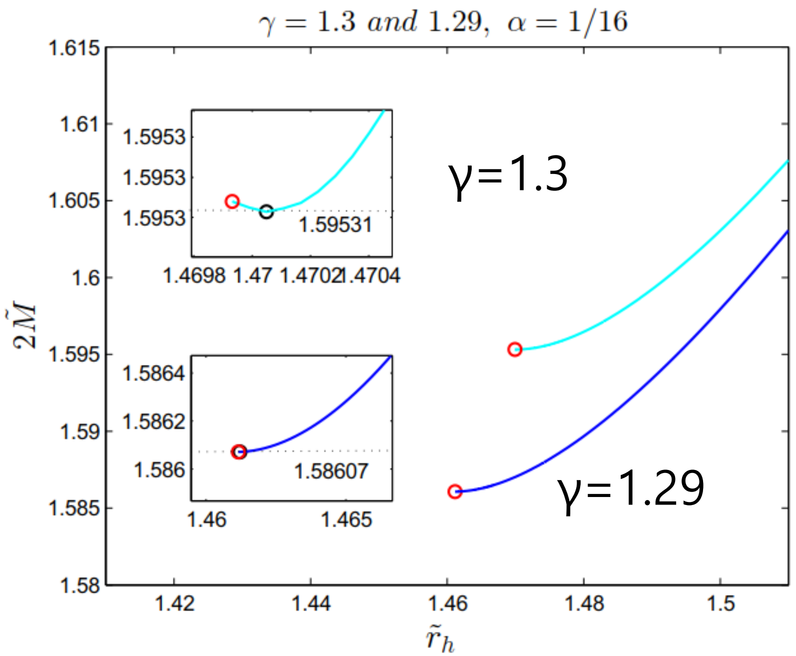
Note :

1. If DEGB black hole horizon becomes larger, the magnitude of the scalar field becomes smaller.
2. In the large horizon radius limit, the scalar field approaches zero, and then the black hole becomes a Schwarzschild black hole.

Coupling γ dependency of the minimum mass for fixed α 1/16.

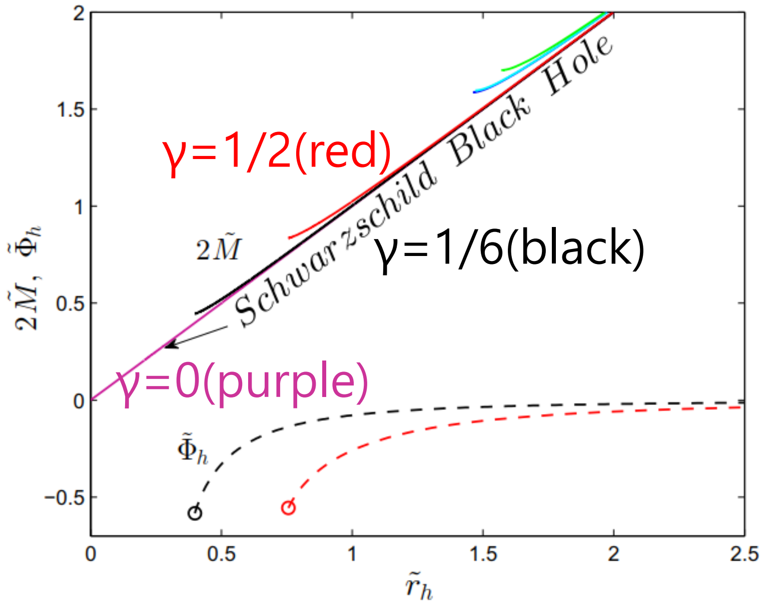


Singular pt S & the min. mass C



S overlaps w/ C btw γ=1.29 & 1.30.
No lower branch below γ=1.29

γ=1/2(red), 1/6(black), 0(purple)



As γ→0, the solution →Schw BH.

Note :

1. For large γ, sing. pt S & extremal pt C (with minimum mass $\tilde{M}$) exist.

2. The solutions between point S and C are unstable for perturbations and end at the singular point S , (which saturates the existence inequality Eq. In other words, there exist mass ranges of two black holes; the larger (smaller) one is perturbatively stable (unstable).

3. As γ smaller, the singular point S gets closer to the minimum mass point C.

4. Below γ=1.29, the unstable branch solutions disappears. The solution depends on the coupling γ and approaches to the Schwarzschild black hole as γ goes to zero.

Q: How about the properties, such as

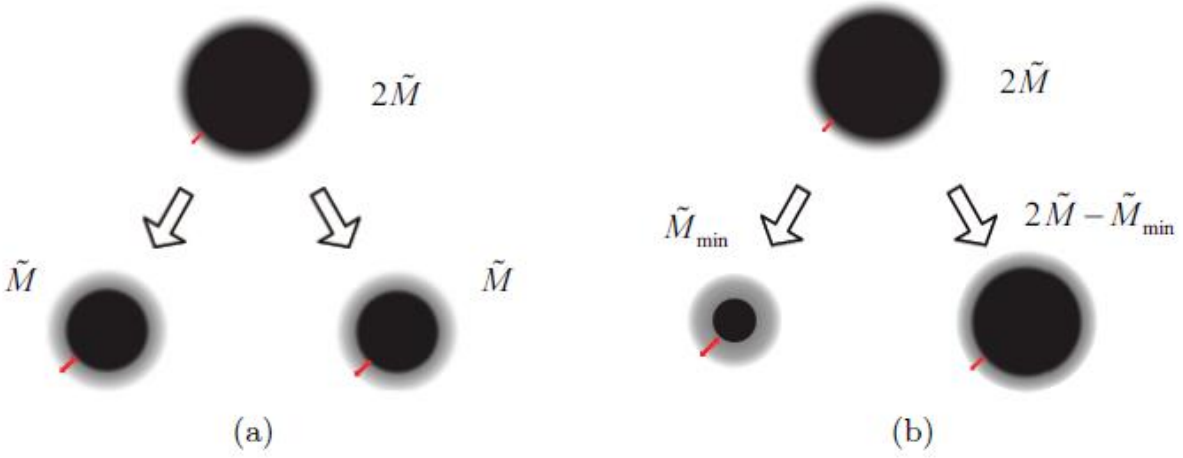
Stability
Implication to the cosmology
etc ?

Black Hole Stability

perturbative

non-perturbative

Fragmentation instability is based on the entropy preference between the solutions. Emparan and Myers, JHEP 0309, 025 (2003).



Perturbative Gravitational (in)stability

Perturbations of a black hole space-time

by adding fields or

by perturbing the metric.

The typical equations in the linear approximation :

$$-\frac{d^2R}{dr_*^2}+V(r,\omega)R=\omega^2R,$$

The quasinormal spectrum of a stable black hole is an infinite set of complex frequencies which describes damped oscillations.

If there is at least one growing mode, the space–time is unstable with the instability growth rate proportional to the imaginary part of the growing QNM.

4-dimensional BH : Perturbative Stability

Konoplya and Zhidenko, RMP (2011)
(arXiv:1102.4014)

Most of the 4-dim. black holes proved to be stable.
(Sch, SdS, SAdS, RNdS, Kerr, KdS, KAdS,

Extreme Kerr & RN BHs are unstable.

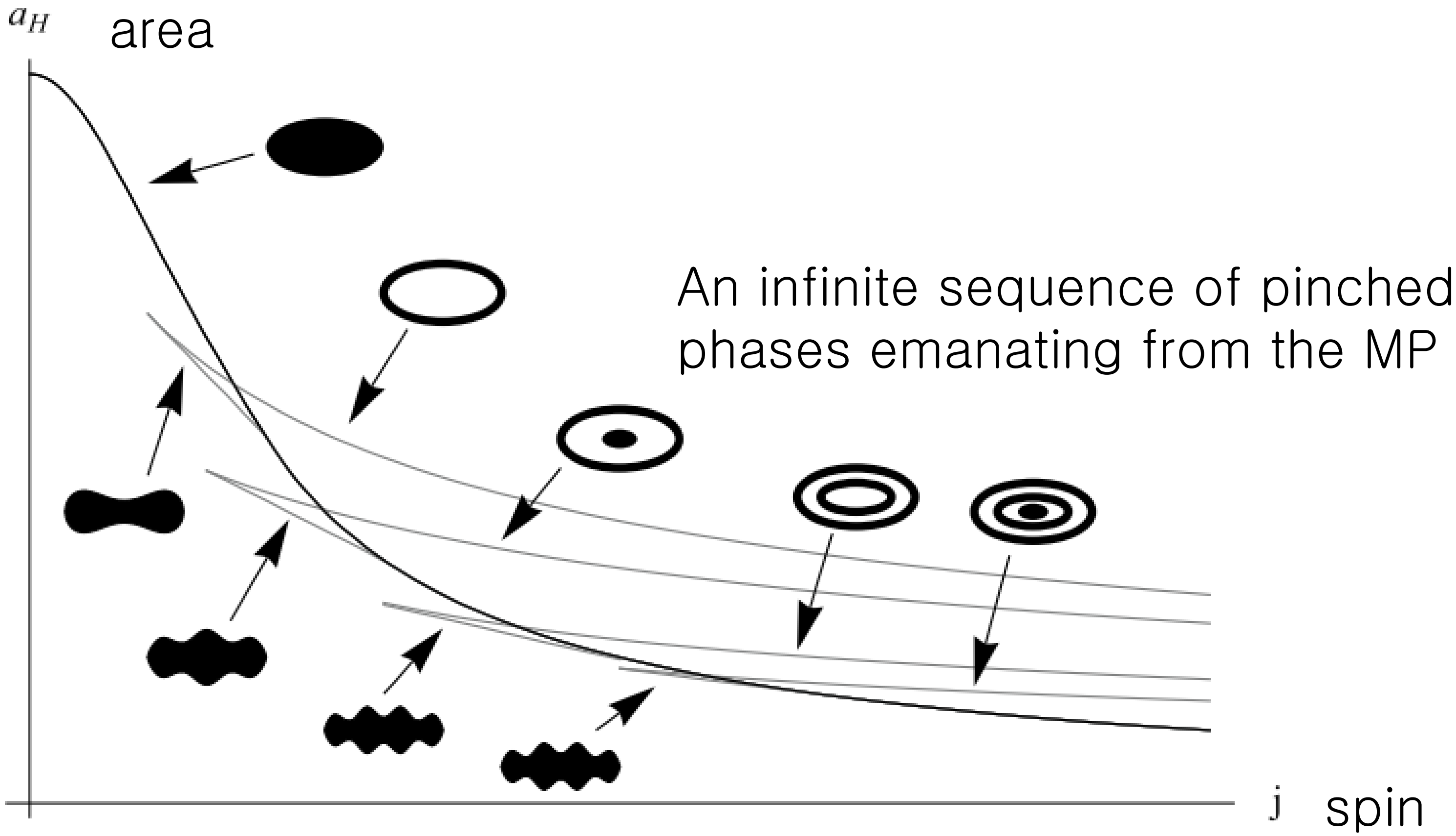
Higher (D ≥ 5) dim BH & stability

wide class of objects : black strings, black branes, black ring, saturn, etc.

There exists various instabilities : (non) Gregory-Laflamme instabilities, etc.

Q : How about nonperturbative stability?

The qualitative phase diagram for the black objects in D ≥ 6



An infinite sequence of pinched black hole phases emanating from the MP curve.

If thermal equilibrium is not imposed, multi-rings are possible in the upper region of the diagram.

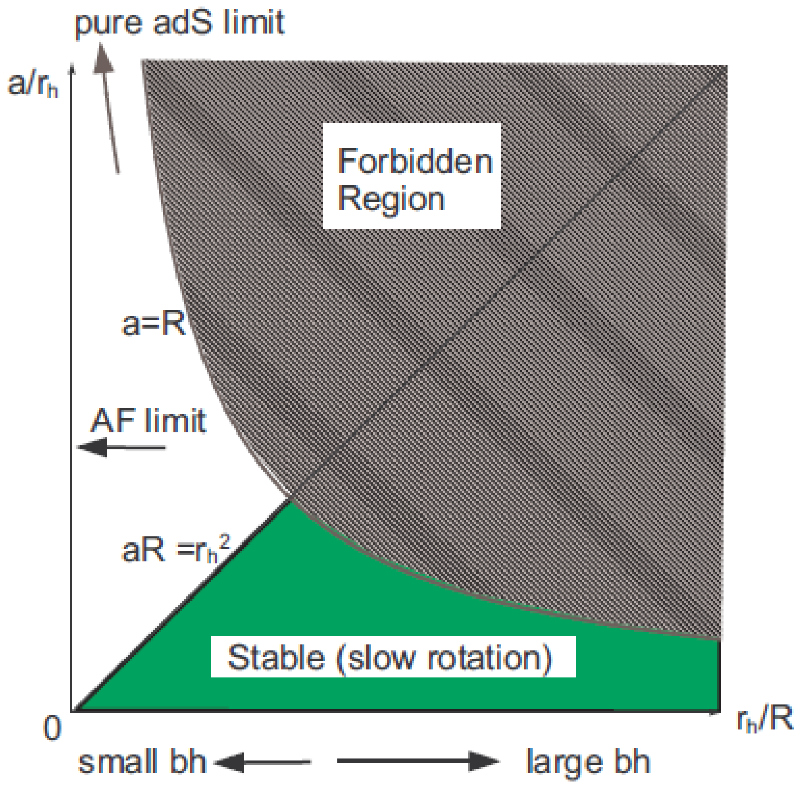
Black hole (perturbative) stability in Einstein gravity (Higher dim.)

Schwarzschild BH: stable

AdS Schwarzschild BH: stable

Kerr BH: stable for superradiance

AdS Kerr BH: stable in green region
→ superradiance instability

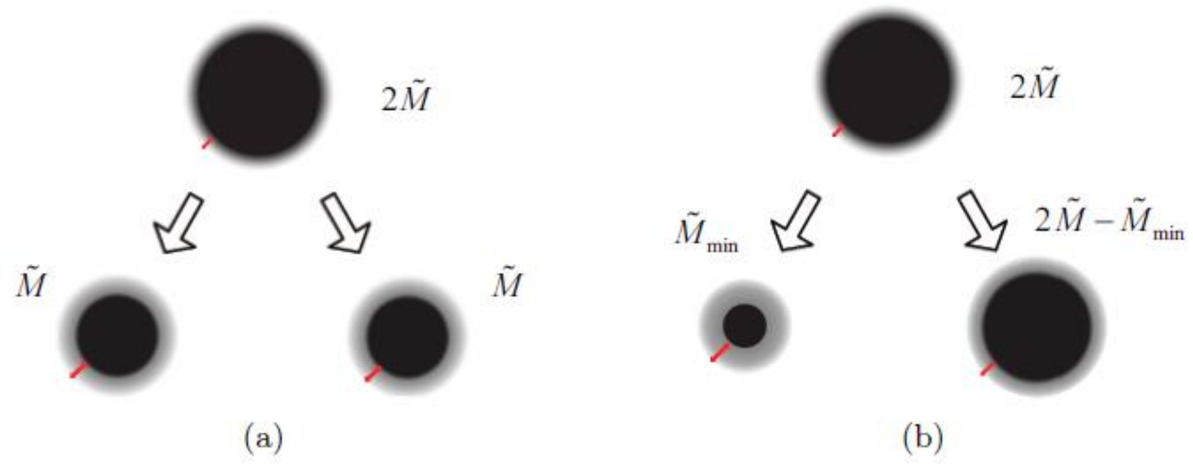


In Hideo Kodama, R.A. Konoplya, Alexander Zhidenko, Phys.Rev. D79 (2009) 044003

RN BH is stable under neutral and charged perturbations.
The stability of RN-AdS blackhole depends on the regions.

Fragmentation Instability

Can Black holes be broken apart into smaller black holes?
The initial phase is a single black hole.
The final phase is two black holes far from each other.



Apply thermodynamic 2nd law to initial (one black hole) and final(fragmented two black holes) phase.

entropy of 1 BH < entropy of 2 fragmented BHs

$\longrightarrow$ (transition to) instability

Investigation to unstable parameter region which is stable under linear perturbation.

Myers–Perry blackhole (Rotating Black hole in higher dimensions;
There doesn't exist any upper limit on the angular momentum)

becomes unstable for large angular momentum into fragmentation.

RN blackhole is also thermodynamically unstable in specific parameter region.

Instability of a charged AdS black hole
→ Instability of Schwarzschild-AdS black hole
Also the flat limit

The fragmentation instability of hairy black holes in the theory with a Gauss-Bonnet term in asymptotically flat space time

Fragmentation allows the upper or lower bound of black hole charges.
B. Gwak and B.-H. Lee, arXiv:1405.2803 PRD91 (2015) 6, 064020.

Let δ be the mass ratio $0 \le \delta \le \frac{1}{2}$.   : ($M \to M\delta + M(1-\delta)$)

There exists minimum mass ratio δ_{min} in DEGB Black Hole, because the minimum mass M_{min} of the black hole.

The black holes can be fragmented only when it exceeds twice of minimum mass. Black holes with mass below twice of minimum mass are absolutely stable.

These are due to effects of the Gauss-Bonnet terms.

For Schwarzschild black hole

$$\frac{S_f}{S_i} = \frac{(\delta\, \tilde{r}_h)^2 + ((1-\delta)\, \tilde{r}_h)^2}{\tilde{r}_h^2} = \delta^2 + (1-\delta)^2\,, \tag{22}$$

The entropy ratio is always smaller than 1, and marginally approaches 1 as δ → 0.

Therefore, a Schwarzschild black hole is always stable under fragmentation, and marginally stable as δ → 0 (emission of the infinitesimal black hole).

These phenomena become different in the theory with the higher order of curvature term.

For a black hole in EGB theory

The EGB black hole solution is the same as the Schwarzschild one. However, the GB term contributes to the black hole entropy and influence stability.

The initial black hole entropy is

$$S_i = \frac{A_H}{4G}\left(1+\frac{8\alpha\kappa}{\tilde{r}_h^2}\right) = \frac{\pi}{G}\left(\tilde{r}_h^2 + 8\alpha\kappa\right) . \tag{24}$$

Unlike Schwarzschild black holes, the fragmentation instability occurs depending on the fragmentation ratio δ. **For the case of** $(\delta, 1-\delta)$ **fragmentation, the final phase entropy is given**

$$S_f = \frac{\pi}{G}\left((\delta\tilde{r}_h)^2 + 8\alpha\kappa\right) + \frac{\pi}{G}\left(((1-\delta)\tilde{r}_h)^2 + 8\alpha\kappa\right) . \tag{25}$$

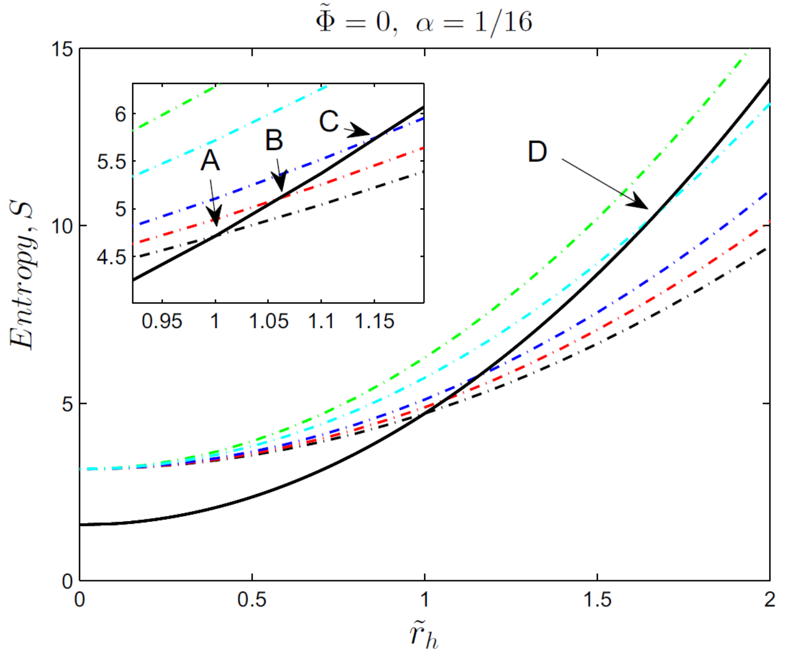
The EGB black hole is unstable if,

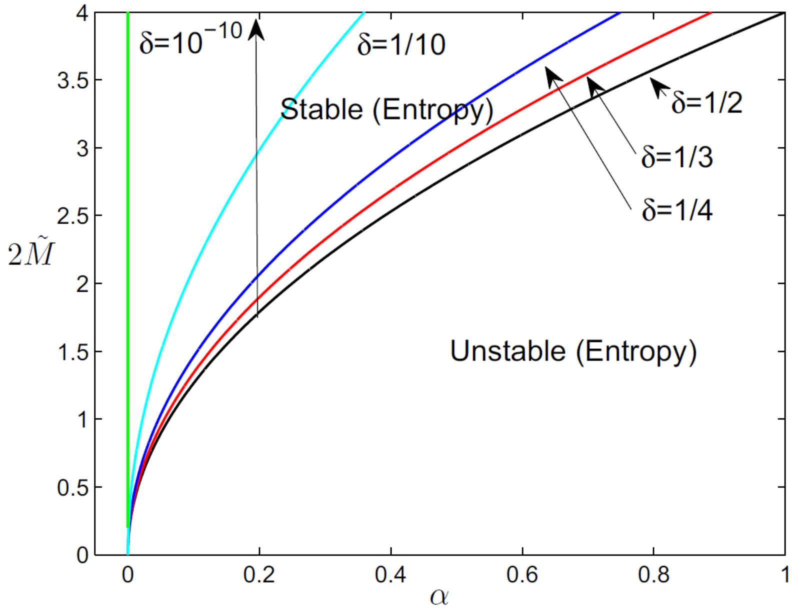
$$\frac{S_f}{S_i} = \frac{\left((\delta\tilde{r}_h)^2 + 8\alpha\kappa\right) + \left(((1-\delta)\tilde{r}_h)^2 + 8\alpha\kappa\right)}{\left(\tilde{r}_h^2 + 8\alpha\kappa\right)} > 1 . \tag{26}$$

EGB black hole entropy

$\kappa = 1$

Phase diagram of EGB black hole





Initial single BH entropy : black solid line

Final two-BH (δ, 1- δ) entropy

$(\delta, 1-\delta) = (\frac{1}{2}, \frac{1}{2})$: black dashed-dot lines ,

$(\delta, 1-\delta) = (\frac{1}{3}, \frac{2}{3})$: red

$(\delta, 1-\delta) = (\frac{1}{4}, \frac{3}{4})$: blue

$(\delta, 1-\delta) = (\frac{1}{10}, \frac{9}{10})$: cyan and

$(\delta, 1-\delta) = (10^{-10}, 1-10^{-10})$: green .

$\delta = \frac{1}{2}$ (black solid line),

$\delta = \frac{1}{3}$ (red solid line),

$\delta = \frac{1}{4}$ (blue solid line),

$\delta = \frac{1}{10}$ (cyan solid line),

$\delta = 10^{-10}$ (green solid line)

For the limit of $\delta \to 0$, all the EGB black holes may become unstable under fragmentation.

The crossing points go up from point A to D (decreasing the stable region) with smaller δ.

For a black hole in DEGB theory

The DEGB black hole has the additional entropy coming from the higher curvature term.

$$S = \frac{\pi \tilde{r}_h^2}{G}\left(1 + \frac{8\alpha\kappa}{\tilde{r}_h^2} e^{-\gamma\tilde{\Phi}_h}\right), \qquad (28)$$

Note : γ= 0 corresponds to EGB black hole case

The entropy ratio between the initial single BH and the final two BH states are

$$\frac{S_f}{S_i} = \frac{\delta^2 + (\delta-1)^2 + \frac{8\alpha\kappa e^{-\gamma\tilde{\Phi}_\delta} + 8\alpha\kappa e^{-\gamma\tilde{\Phi}_{1-\delta}}}{\tilde{r}_h^2}}{1 + \frac{8\alpha\kappa e^{-\gamma\tilde{\Phi}_h}}{\tilde{r}_h^2}}, \qquad (31)$$

Note : 1) As $\tilde{r}$ >> 1, the entropy ratio becomes that of Schwarschild BH,

$$\frac{S_f}{S_i} = \delta^2 + (1-\delta)^2 < 1. \qquad (30)$$

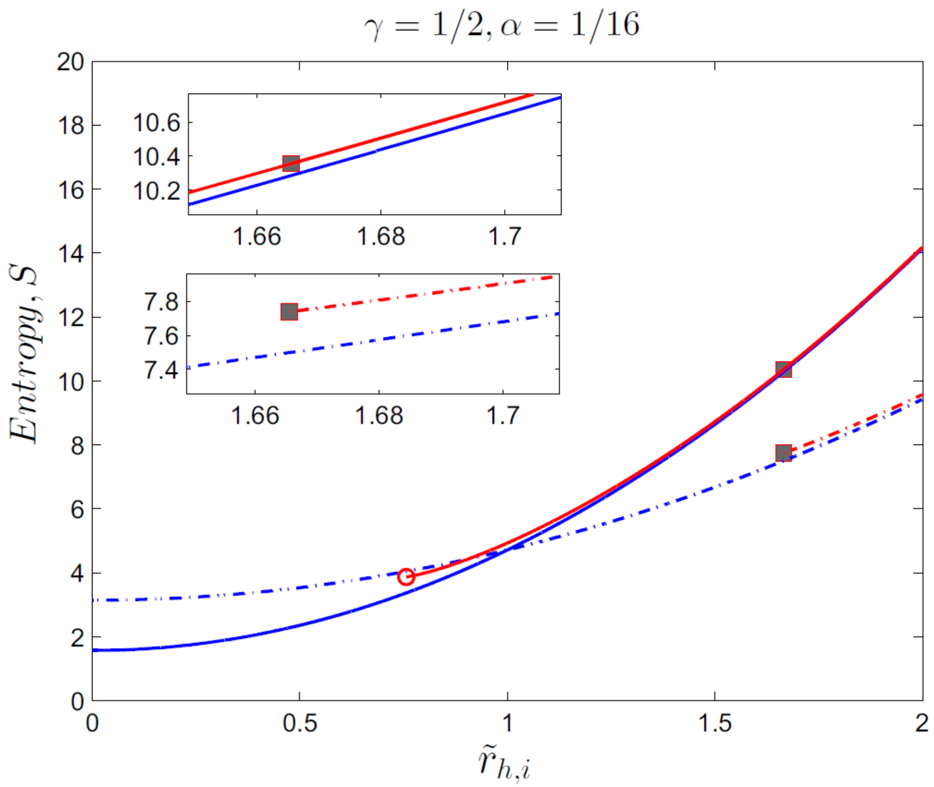
Thus, very massive DGB black holes are stable under fragmentation.

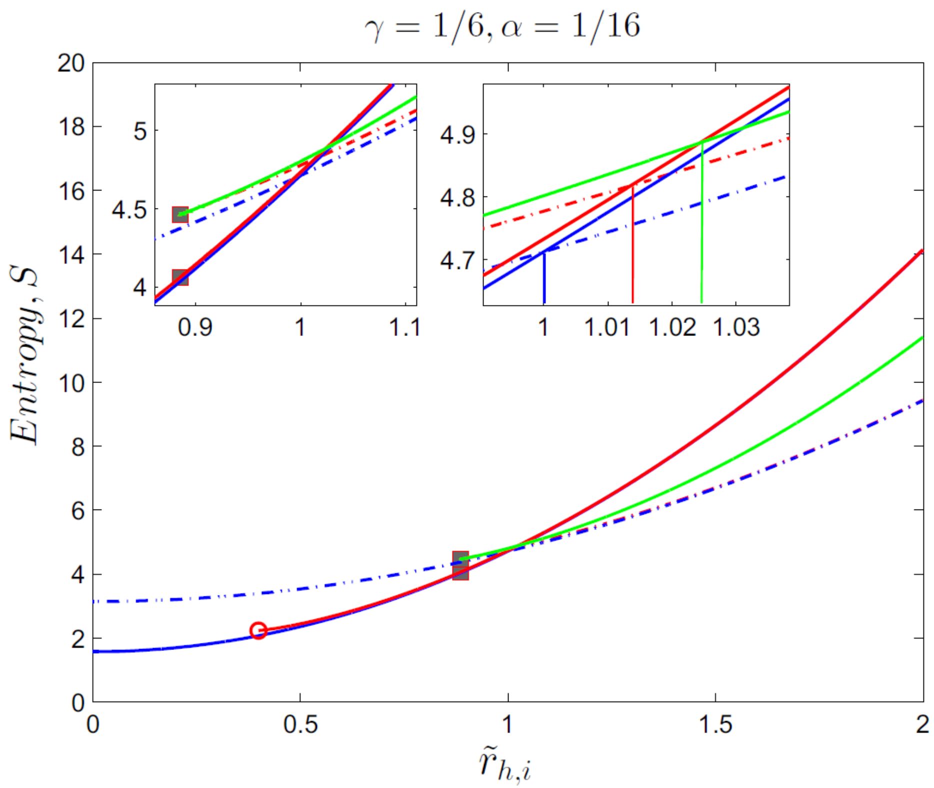
2) DGB black holes of masses smaller than $2M_{min}$ are absolutely stable, because there doesn't exist fragmented BH solutions.

Very large and very small black holes are absolutely stable !

Fragmentation Instability for DEGB Black Holes

The initial and final phase entropies with respect to $\tilde{r}_{h,i}$ for given couplings γ and α .



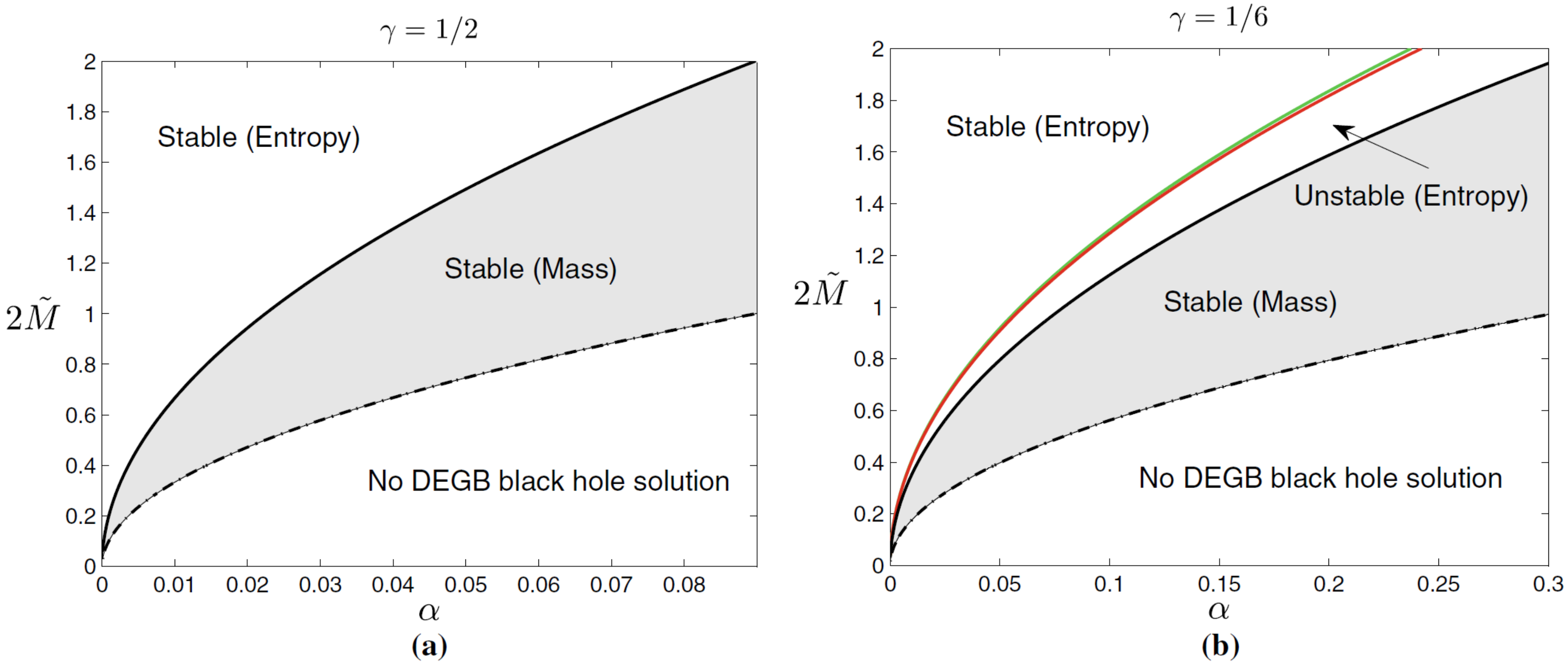


The — (–•–•–) lines are initial (final) phase entropies in EGB theory as a reference for $(\frac{1}{2}, \frac{1}{2})$.

The — (–•–•–) lines are initial (final) the phase entropies in DEGB theory for $(\frac{1}{2}, \frac{1}{2})$.

The initial phase exists above the red circle for the minimum mass. The final phase exists above the red box for $(\frac{1}{2}, \frac{1}{2})$. The — (green) represents fragmentation for marginal mass ratio $\bar{\delta}$.

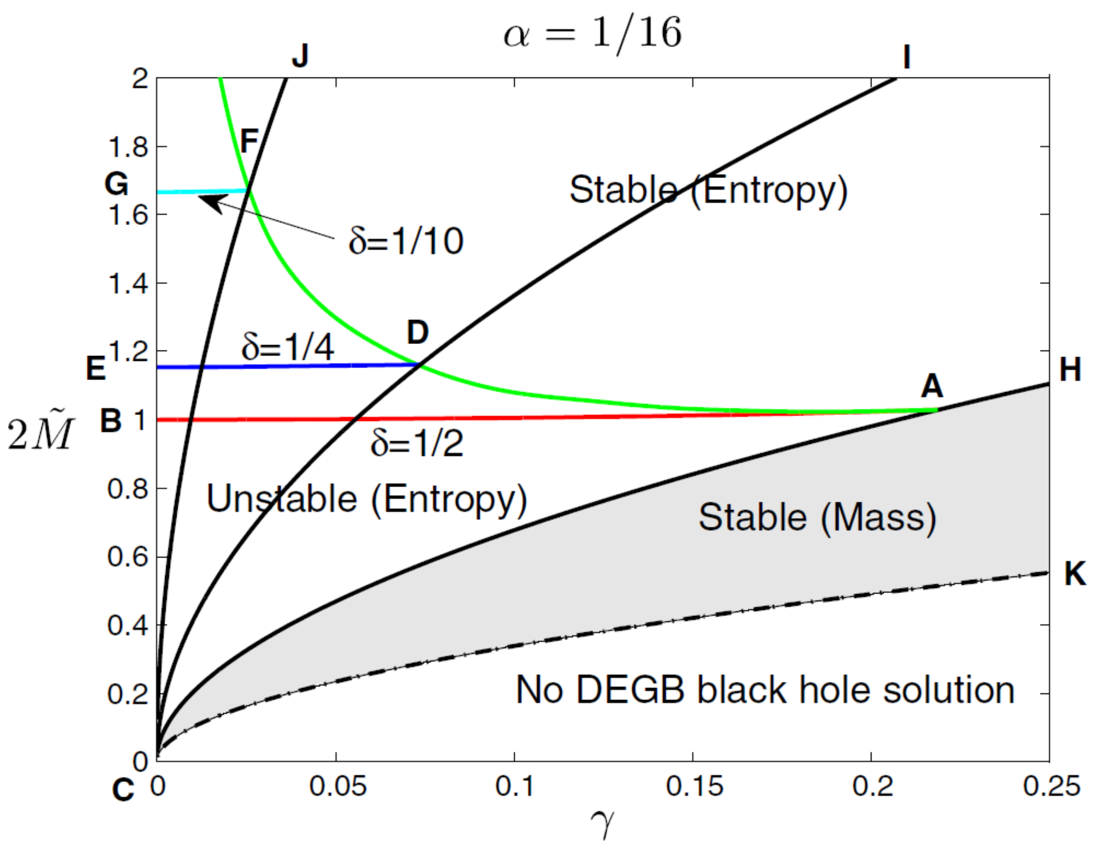
The phase diagrams with respect to α and $\tilde{M}$ in fixed γ.



The red solid line represents $(\frac{1}{2}, \frac{1}{2})$ fragmentation.

The green solid line represents $(\bar{\delta}, 1\text{-}\bar{\delta})$ fragmentation

The phase diagrams with respect to γ and $\tilde{M}$ in fixed α = 1/16



for

$(\frac{1}{2}, \frac{1}{2})$ (red solid line),
$(\frac{1}{4}, \frac{3}{4})$ (blue solid line),
$(\frac{1}{10}, \frac{9}{10})$ (cyan solid line), and
$(\bar{\delta}, 1-\bar{\delta})$ (green solid line)

fragmentation

Cosmological Effects

S. Koh, BHL, W. Lee, G. Tumurtushaa Phys.Rev. D90 (2014) no.6, 063527

S. Koh, BHL, W. Lee, G. Tumurtushaa arXiv:1610.04360

Inflation with a Gauss-Bonnet

- We consider an action with a Gauss-Bonnet term:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\frac{1}{2}\xi(\phi)\boxed{R^2_{\rm GB}}\right],$$

$$\boxed{R^2_{\rm GB}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2}$$

- Varying the action in flat FLRW Universe with metric:

$$ds^2=-dt^2+a(t)^2\delta_{ij}dx^idx^j,$$

- Einstein equation and Field equation yield:

$$H^2=\frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2+V+12\dot\xi H^3\right),$$

$$\dot H=-\frac{\kappa^2}{2}\left(\dot\phi^2-4\ddot\xi H^2-4\dot\xi H(2\dot H-H^2)\right),$$

$$\ddot\phi+3H\dot\phi+V_\phi+12\xi_\phi H^2\left(\dot H+H^2\right)=0,$$

Slow-roll inflation

- Inflaton potential and Gauss-Bonnet coupling satisfy:

$$\dot{\phi}^2/2 \ll V, \quad \ddot{\phi} \ll 3H\dot{\phi}, \quad 4\dot{\xi}H \ll 1, \quad \ddot{\xi} \ll \dot{\xi}H.$$

- The slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\ddot{H}}{H\dot{H}}, \quad \zeta = \frac{\dddot{H}}{H^2\dot{H}},$$
$$\delta_1 = 4\kappa^2\dot{\xi}H, \quad \delta_2 = \frac{\ddot{\xi}}{\dot{\xi}H}, \quad \delta_3 = \frac{\dddot{\xi}}{\dot{\xi}H^2}.$$

- Potential and GB coupling based SR parameters:

$$\epsilon = \frac{1}{2\kappa^2}\frac{V_\phi}{V}Q,$$
$$\eta = -\frac{V_{\phi\phi}Q}{\kappa^2 V_\phi} - \frac{1}{\kappa^2}Q_\phi,$$
$$\zeta = \frac{V_{\phi\phi\phi}Q^2}{\kappa^4 V_\phi} + \frac{V_{\phi\phi}Q^2}{2\kappa^4 V} + \frac{3V_{\phi\phi}Q_\phi Q}{\kappa^4 V_\phi} + \frac{V_\phi Q_\phi Q}{2\kappa^4 V} + \frac{1}{\kappa^4}Q_\phi^2 + \frac{1}{\kappa^4}Q_{\phi\phi}Q,$$

$$\delta_1 = -\frac{4\kappa^2}{3}\xi_\phi V Q,$$
$$\delta_2 = -\frac{\xi_{\phi\phi}Q}{\kappa^2\xi_\phi} - \frac{V_\phi Q}{2\kappa^2 V} - \frac{1}{\kappa^2}Q_\phi,$$
$$\delta_3 = \frac{\xi_{\phi\phi\phi}Q^2}{\kappa^4\xi_\phi} + \frac{3\xi_{\phi\phi}V_\phi Q2}{2\kappa^4\xi_\phi V} + \frac{3\xi_{\phi\phi}Q_\phi Q}{\kappa^4\xi_\phi} + \frac{V_{\phi\phi}Q^2}{2\kappa^4 V} + \frac{2V_\phi Q_\phi Q}{\kappa^4 V} + \frac{1}{\kappa^4}Q_\phi^2 + \frac{1}{\kappa^4}QQ_{\phi\phi}.$$

- Background EoM reduce to

$$H^2 \simeq \frac{\kappa^2}{3}V,$$
$$\dot{H} \simeq -\frac{\kappa^2}{2}(\dot{\phi}^2 + 4\dot{\xi}H^3),$$
$$3H\dot{\phi} + V_\phi + 12\xi_\phi H^4 \simeq 0,$$

- The number of e-folds:

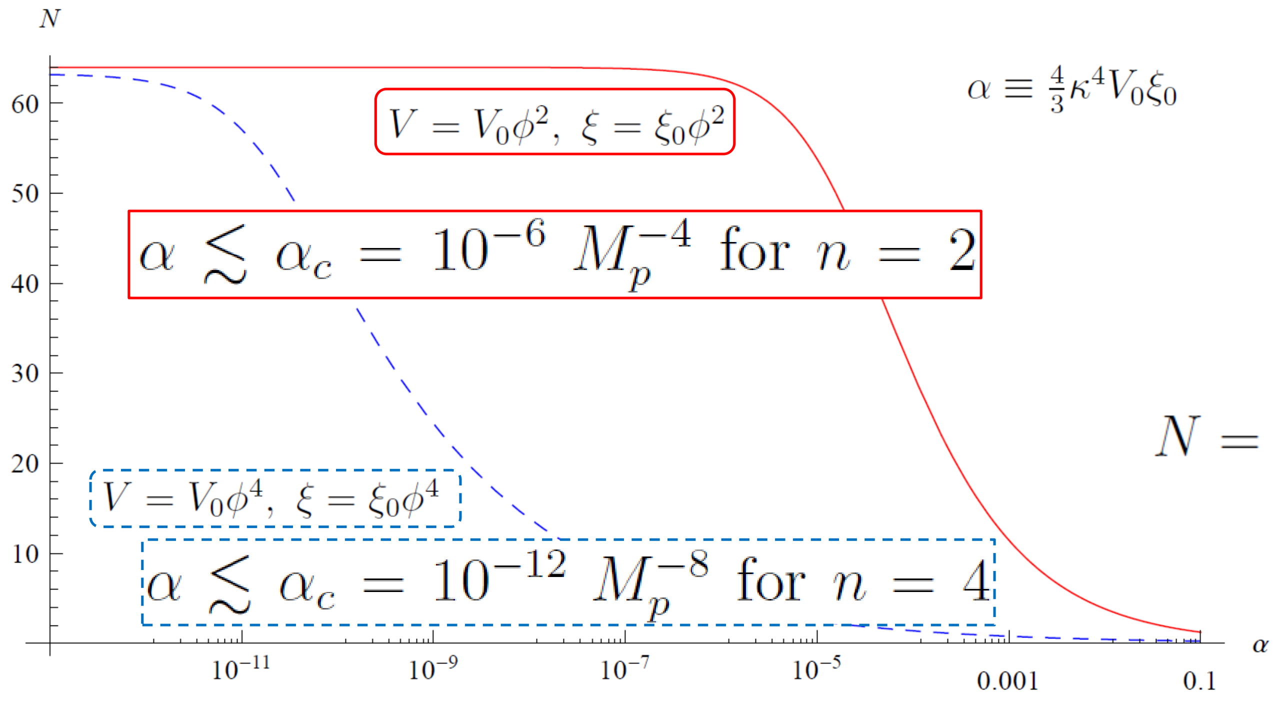
$$N(\phi) = \int_t^{t_e} H dt \simeq \int_{\phi_e}^{\phi} \frac{3\kappa^2 V}{3V_\phi + 4\kappa^4\xi_\phi V^2}d\phi \equiv \int_{\phi_e}^{\phi}\frac{\kappa^2}{Q}d\phi.$$

$$Q \equiv \frac{V_\phi}{V} + \frac{4}{3}\kappa^4\xi_\phi V.$$

Number of *e-folds*

$$N(\phi) = \int_t^{t_e} H dt \simeq \int_{\phi_e}^{\phi} \frac{3\kappa^2 V}{3V_{,\phi} + 4\kappa^4 \xi_{,\phi} V^2} d\phi \equiv \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi$$

$$Q \equiv \frac{V_{,\phi}}{V} + \frac{4}{3}\kappa^4 \xi_{,\phi} V.$$

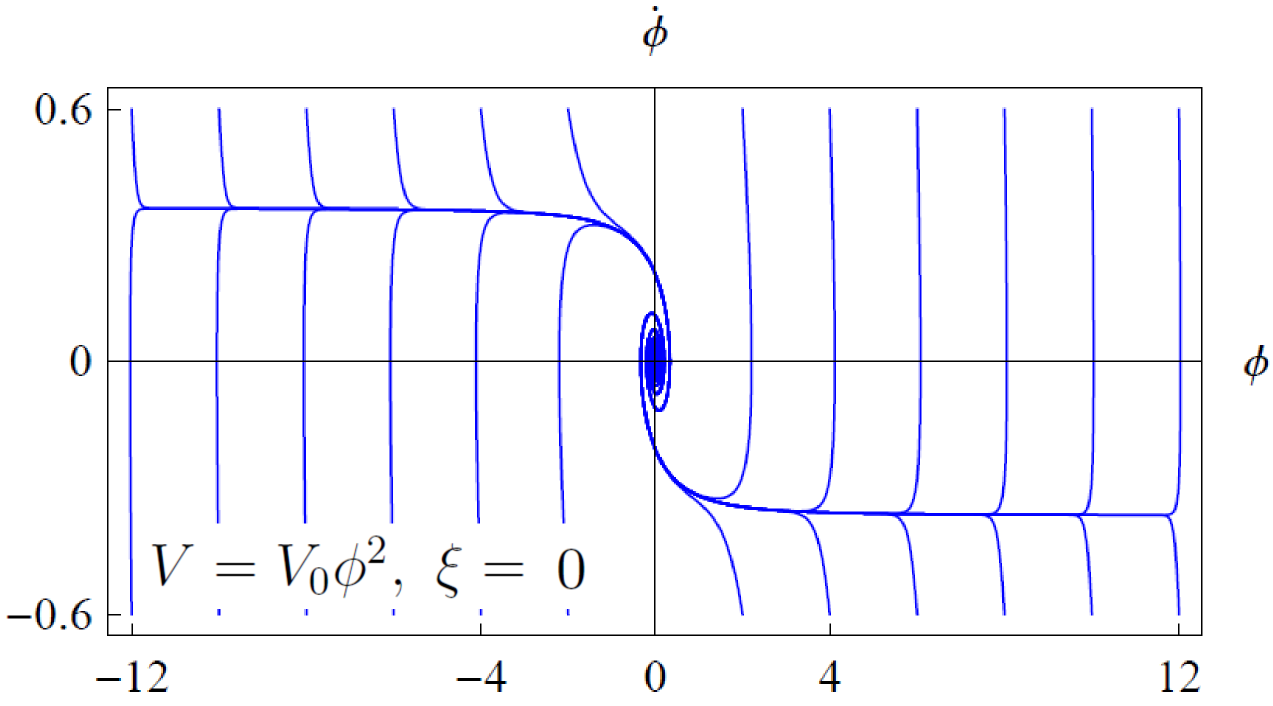


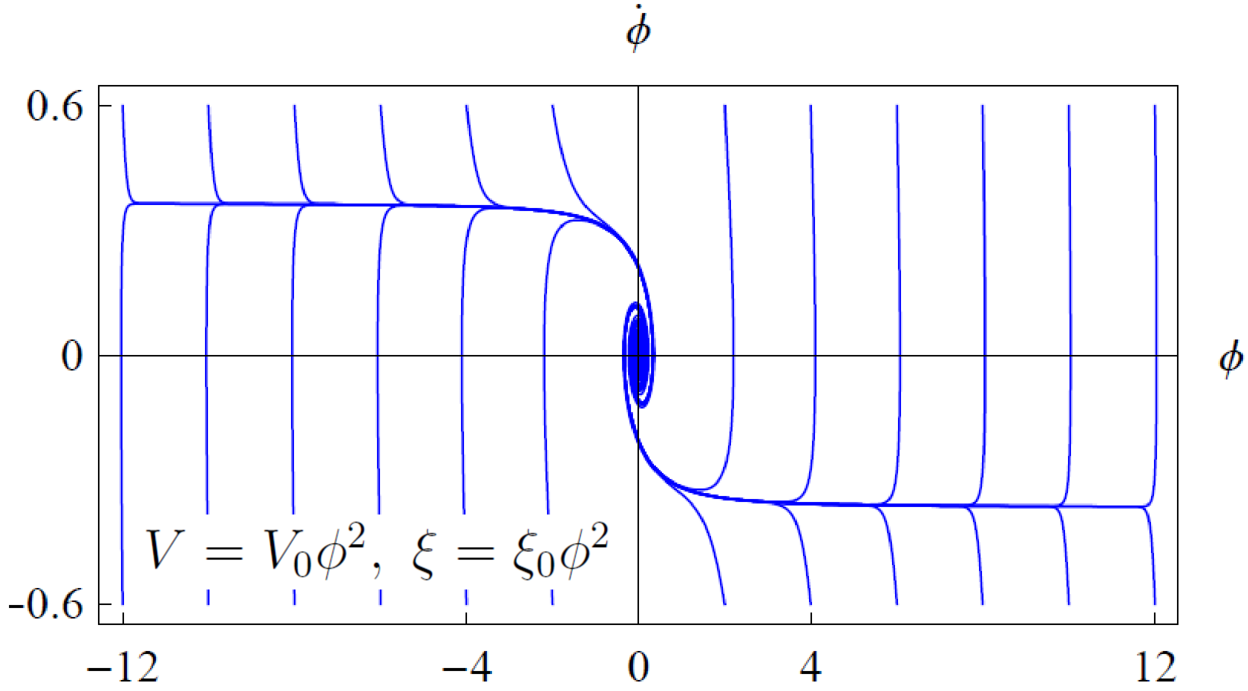
$$V = V_0\phi^n \text{ and } \xi = \xi_0\phi^n$$

$$N = \int_{\phi_e}^{\phi_i} \frac{\kappa^2}{Q} \simeq \frac{\kappa^2\phi_i^2}{2n} {}_2F_1\left(1, \frac{1}{n}, 1 + \frac{1}{n}; -\alpha\phi_i^{2n}\right)$$

Slow-Roll Inflation with the GB

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\boxed{-\frac{1}{2}\xi(\phi)R_{GB}^2}\right]$$





Linear Perturbation

- Perturbed metric in comoving gauge, with $\delta\phi = 0$, is given:

$$ds^2 = a(\tau)^2\{-d\tau^2 + [(1 - \boxed{2\mathcal{R}})\delta_{ij} + \boxed{h_{ij}}]dx^i dx^j\}, \qquad \text{where} \qquad h^i_i = 0 = h^i{}_{j,i}$$

- Fourier transform of $\mathcal{R}$ and h_{ij} are:

$$\mathcal{R}(\tau, \mathbf{x}) = \frac{1}{z_s}\int \frac{d^3k}{(2\pi)^{3/2}} v_s(\tau, k) e^{i\mathbf{k}\cdot\mathbf{x}},$$

$$h_{ij}(\tau, \mathbf{x}) = \frac{2}{z_t}\sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}} v_t^\lambda(\tau, k)\epsilon_{\lambda,ij} e^{i\mathbf{k}\cdot\mathbf{x}},$$

- Sasaki-Mukhanov equation:

$$v_s'' + \left(c_s^2 k^2 - \frac{z_s''}{z_s}\right) v_s = 0,$$

$$v_t'' + \left(c_t^2 k^2 - \frac{z_t''}{z_t}\right) v_t = 0,$$

$$z_s \equiv \sqrt{\frac{a^2(\dot\phi^2 + 6\dot\xi H^3\Delta)}{H^2(1 - \frac{1}{2}\Delta)^2}}, \quad z_t \equiv \sqrt{\frac{a^2}{\kappa^2}(1 - 4\kappa^2\dot\xi H)}, \quad \Delta = \frac{4\kappa^2\dot\xi H}{1 - 4\kappa^2\dot\xi H}, \quad c_s^2 \equiv 1 + \frac{2(\dot H - \kappa^2\dot\xi H(H^2 + 4\dot H) + \kappa^2\ddot\xi H^2)\Delta^2}{\kappa^2\dot\phi^2 + 6\kappa^2\dot\xi H^3\Delta}, \quad c_t^2 \equiv 1 - \frac{4\kappa^2(\ddot\xi - \dot\xi H)}{1 - 4\kappa^2\dot\xi H}.$$

$$z_s = \sqrt{\frac{a^2}{\kappa^2}\frac{2\epsilon - \delta_1(1 + 2\epsilon - \delta_2) + \frac{3}{2}\delta_1\Delta}{(1 - \frac{1}{2}\Delta)^2}}, \quad z_t = \sqrt{\frac{a^2}{\kappa^2}(1 - \delta_1)}, \quad \Delta = \frac{\delta_1}{1 - \delta_1}, \quad c_s^2 = 1 - \frac{(4\epsilon + \delta_1(1 - 4\epsilon - \delta_2))\Delta^2}{4\epsilon - 2\delta_1 - 2\delta_1(2\epsilon - \delta_2) + 3\delta_1\Delta}, \quad c_t^2 = 1 + \frac{\delta_1(1 - \delta_2)}{1 - \delta_1},$$

Power spectrum

- Leading order term in slow-roll parameters: $\tau = -\frac{1}{aH}\frac{1}{1-\epsilon}.$

$$v_A'' + \left(c_A^2 k^2 - \frac{\nu_A^2 - 1/4}{\tau^2}\right) v_A = 0,$$ where $$\nu_s \simeq \frac{3}{2} + \epsilon + \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{4\epsilon - 2\delta_1}, \qquad \nu_t \simeq \frac{3}{2} + \epsilon.$$

- General solutions: $$v_A = \frac{\sqrt{\pi|\tau|}}{2}\left[c_1^A(k) H_{\nu_A}^{(1)}(c_A k|\tau|) + c_2^A(k) H_{\nu_A}^{(2)}(c_A k|\tau|)\right],$$

- The Bunch-Davies vacuum for the initial fluctuation modes at $c_A k|\tau| \gg 1$

$$v_A = \frac{\sqrt{\pi|\tau|}}{2} e^{i(\nu_A + \frac{1}{2})\frac{\pi}{2}} H_{\nu_A}^{(1)}(c_A k|\tau|).$$ where $$H_{\nu_A}^{(1)} \sim \frac{2}{1 - e^{2i\nu_A\pi}}\left\{\frac{1}{\Gamma(1+\nu_A)}\left(\frac{x}{2}\right)^{\nu_A} - \frac{e^{i\nu_A\pi}}{\Gamma(1-\nu_A)}\left(\frac{x}{2}\right)^{-\nu_A}\right\},$$

- Power spectra for the scalar and tensor modes:

$$\mathcal{P}_s = \frac{k^3}{2\pi^2}\left|\frac{v_s}{z_s}\right|^2 \simeq \frac{\csc^2 \nu_s\pi}{\pi\mathcal{D}_s^2\Gamma^2(1-\nu_s)}\frac{1}{c_s^3|\tau|^2 a^2}\left(\frac{c_s k|\tau|}{2}\right)^{3-2\nu_s},$$

$$\mathcal{P}_t = 2\frac{k^3}{2\pi^2}\left|\frac{2v_t}{z_t}\right|^2 \simeq 8\frac{\csc^2 \nu_t\pi}{\pi\mathcal{D}_t^2\Gamma^2(1-\nu_t)}\frac{1}{c_t^3|\tau|^2 a^2}\left(\frac{c_t k|\tau|}{2}\right)^{3-2\nu_t},$$

$$z_A \equiv \mathcal{D}_A a^2, \qquad \mathcal{D}_s^2 = \frac{2\epsilon - \delta_1(1 + 2\epsilon - \delta_2) + \frac{3}{2}\delta_1\Delta}{\kappa^2(1 - \frac{1}{2}\Delta)^2}, \qquad \mathcal{D}_t^2 = \frac{1-\delta_1}{\kappa^2}.$$

Linear perturbations and Power spectrum

The spectral indices of the scalar and tensor modes and the tensor-to-scalar ratio are given by

$$
\begin{aligned}
n_s - 1 &\equiv \frac{d\ln\mathcal{P}_s}{d\ln k} = 3 - 2\nu_s \approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2-\epsilon)}{2\epsilon-\delta_1} \\
n_t &\equiv \frac{d\ln\mathcal{P}_t}{d\ln k} = 3 - 2\nu_t \approx -2\epsilon \\
r &= \frac{\mathcal{P}_t}{\mathcal{P}_s} \approx 8(2\epsilon - \delta_1)
\end{aligned}
$$

$$
\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\ddot{H}}{H\dot{H}},
$$

$$
\delta_1 = 4\kappa^2\dot{\xi}H, \quad \delta_2 = \frac{\ddot{\xi}}{\dot{\xi}H},
$$

$$
\zeta = \frac{\dddot{H}}{H^2\dot{H}}, \qquad \delta_3 = \frac{\dddot{\xi}}{\dot{\xi}H^2}.
$$

We can also calculate the running of spectral indices of scalar and tensor modes

$$
\begin{aligned}
\frac{dn_s}{d\ln k} &\approx -2\epsilon(2\epsilon+\eta) + \frac{(2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2-\epsilon))^2}{(2\epsilon-\delta_1)^2} \\
&\quad - \frac{2\epsilon(8\epsilon^2 + 7\epsilon\eta + \zeta) + \delta_1(\epsilon^2 + \epsilon\eta + \epsilon\delta_2 - \delta_3)}{2\epsilon - \delta_1} \\
\frac{dn_t}{d\ln k} &\approx -2(2\epsilon^2 + \epsilon\eta)
\end{aligned}
$$

Model-1

$$V(\phi) = V_0 e^{-\lambda\phi}, \quad \xi(\phi) = \xi_0 e^{-\lambda\phi}$$

$$N \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi = -\frac{1}{2\lambda^2} \ln\left(\alpha + e^{2\lambda\phi}\right) \qquad n_s - 1 = \lambda^2 \left(\frac{3\alpha}{e^{-2\lambda^2 N} - \alpha} - 1\right), \quad r = \frac{8\lambda^2 e^{-4\lambda^2 N}}{\left(e^{-2\lambda^2 N} - \alpha\right)^2}$$

$$\phi = \frac{1}{2\lambda} \ln\left(e^{-2\lambda^2 N} - \alpha\right)$$

Since λ^2 is always positive ($\lambda^2 > 0$) and α can be negative or positive, we can reach to the following results: if $\alpha > 0$, $0 < \alpha < e^{-2\lambda^2 N}$ with $-\sqrt{2} < \lambda < \sqrt{2}$. Or if $\alpha < 0$, $-e^{-2\lambda^2 N} < \alpha < 0$ with $\lambda < -\sqrt{2}$ and $\lambda > \sqrt{2}$. With these parameter ranges, we can freely choose the model parameters α and λ which are valid for inflation to occur. Unfortunately, these parameter range of α and λ do not favored by Planck data.

Model-2

$$\boxed{V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^n}$$

The number of e-folds before the end of inflation

$$N(\phi) \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi \simeq \frac{\kappa^2\phi^2}{2n}\, {}_2F_1\left(1; \frac{1}{n}; 1+\frac{1}{n}; -\alpha\phi^{2n}\right)$$

$$\phi \simeq \sqrt{\frac{2nN}{\kappa^2}}\left[1 + \frac{\alpha(2nN)^n}{2(n+1)\kappa^{2n}}\right]$$

$$n_s - 1 \simeq -\frac{n+2}{2N} + \frac{n(3n+2)(2nN)^n\alpha}{2(1+n)N\kappa^{2n}},$$

$$n_t \simeq -\frac{n}{2N} - \frac{n^2(2nN)^n\alpha}{2(1+n)N\kappa^{2n}},$$

$$r \simeq \frac{4n}{N} + \frac{4n(2n+1)(2nN)^n\alpha}{(1+n)N\kappa^{2n}},$$

$$\frac{dn_s}{d\ln k} \simeq -\frac{n+2}{2N^2} - \frac{n(n-1)(3n+2)(2nN)^n\alpha}{2(1+n)N^2\kappa^{2n}},$$

$$\frac{dn_t}{d\ln k} \simeq -\frac{n}{2N^2} + \frac{n^2(n-1)(2nN)^n\alpha}{2(1+n)N^2\kappa^{2n}}$$

Model-2

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^n$$

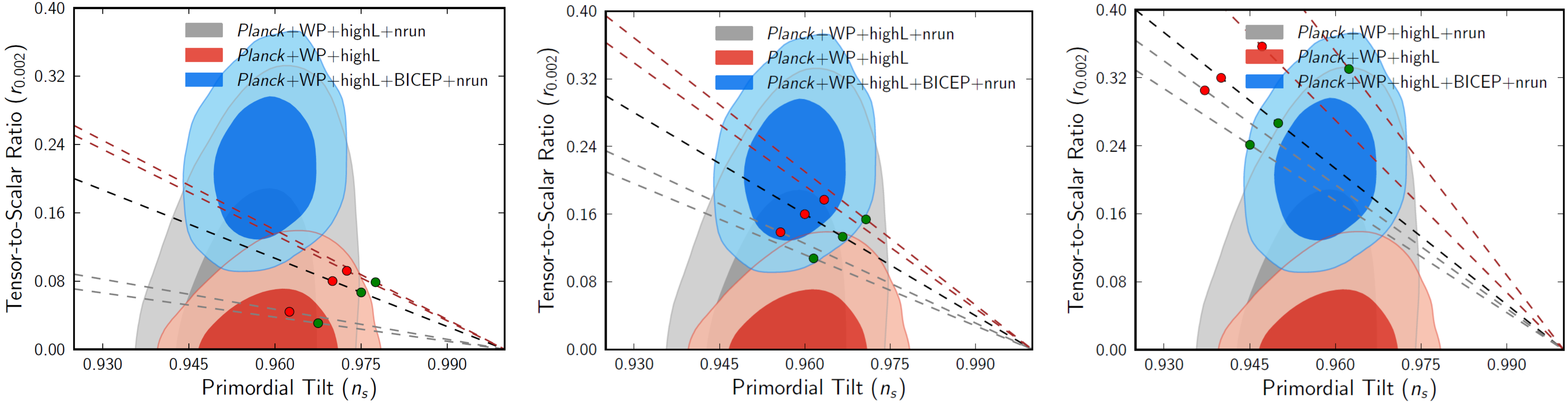
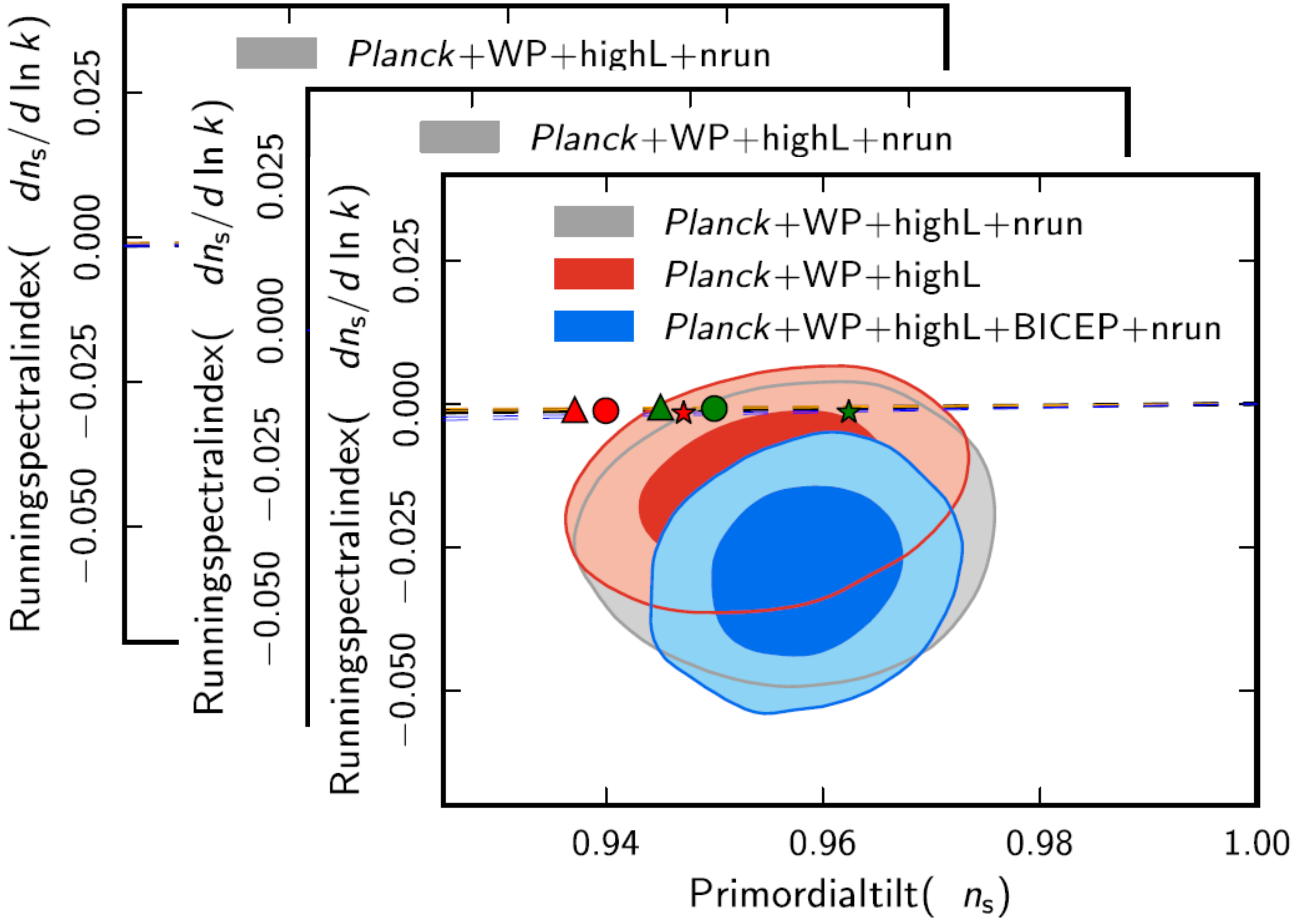


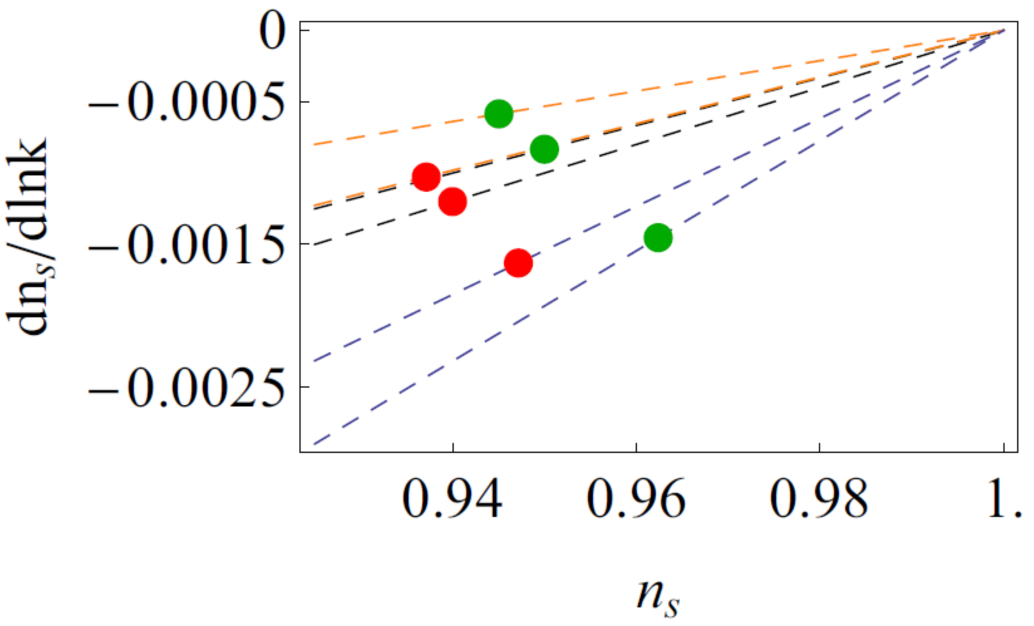
TABLE I: Observationally favored range of model parameter α for different values of n and N from observational data set.

Model n	Parameter range for N=50	Parameter range for N=60
n=1	$-6.6\times10^{-3} \le \alpha \le 2\times10^{-3}$	$-5.5\times10^{-3} \le \alpha \le 4\times10^{-4}$
n=2	$-5.2\times10^{-6} \le \alpha \le 6\times10^{-6}$	$-3.2\times10^{-6} \le \alpha \le 1.5\times10^{-6}$
n=4	lies outside of 2σ boundary	$0 \le \alpha \le 2.5\times10^{-12}$

➢ Model-2

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^n$$





Summary

We have studied the Black Hole with Gauss-Bonnet term

- Numerically constructed the static DGB **hairy** black hole in asymptotically flat spacetime

  There exists **minimum mass**, etc.
- **Fragmentation instability of black holes:**

  When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

  The amount of black hole hair decreases as the DGB black hole mass increases. DGB black hole configurations go to EGB black hole cases for small α **and** γ.

  The DGB black hole phase is unstable under fragmentation, even if these phases are stable under perturbation.

  We have found the phase diagram of the fragmentation instability for a black hole mass and two couplings.

GB term in inflation

- We have investigated the slow-roll inflation with the GB term which coupled to the inflaton field nonminimally. We have considered the potential and coupling functions as

$$V(\phi) = V_0 e^{-\lambda\phi}, \quad \xi(\phi) = \xi_0 e^{-\lambda\phi} \qquad V(\phi) = V_0 \phi^n, \quad \xi(\phi) = \xi_0 \phi^n$$

- First, we have applied our general formalism to the large-field inflationary model with exponential potential and exponential coupling. In this case, we could find the valid model parameter range for inflation to happen, unfortunately, these parameter ranges are not favored by the data sets.

- Second, we have studied models with monomial potential and monomial coupling to GB term. In this case, r is enhanced for $\alpha > 0$ while it is suppressed for $\alpha < 0$.

- N≈60 condition requires that $\alpha \approx 10^{-6}$ for V$\sim\varphi^2$ $\alpha \approx 10^{-12}$ for V$\sim\varphi^4$.

- In this work, running spectral index turns out to be inconsistent with BICEP2+Planck data. It would be interesting to search for the alternatives to reconcile Planck data with BICEP2 besides consideration of the running spectral index.

Other Effects such as the reheating under investigation

Thank You!