Gravitational waves from first order phase transition of the Higgs field at high energy scales

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Contents

• Introduction
• 1st order phase transition and GWs
• Phase transition of Higgs at high scale and GWs
Introduction
Recent discoveries

Higgs boson

Gravitational wave (GW)

Hanford, Washington (H1)

ATLAS (2012)

LIGO (2016)
• Unfortunately (?), both are **NOT** new physics beyond SM

• However, both (Higgs & GWs) **can be** a new probe of physics beyond SM

• In some extension of SM, electroweak phase transition can be 1st order

  • Electroweak baryogenesis
  • GWs from bubble collisions
    
    $f \sim 1$ mHz (LISA range)

• We show that Higgs phase transition can happen at much higher energy scale and resulting GWs may probe the nature of Higgs and physics beyond SM
GWs from 1st order phase transition
Phase transition (toy model)

\[ V(\phi) = V_0 - m_\phi^2 \phi^2 + \lambda \phi^4 \]

\[ \mathcal{L} = -g^2 \phi^2 \chi^2 \]

Suppose that \( \chi \) is in thermal equilibrium

\[ V_T(\phi) \approx \frac{g^2}{12} T^2 \phi^2 \quad (\phi \ll T) \]

\[ T \gtrsim m_\phi/g : \text{Symmetric phase} \]

\[ T \lesssim m_\phi/g : \text{Broken phase} \]
Phase transition (toy model)

Finite-temperature effective potential

$$V_T^B/F(\phi) = \frac{T^4}{2\pi^2} J_B/F\left(\frac{m^2}{T^2}\right)$$

$$m(\phi) = g\phi$$

$$J_B[m^2\beta^2] = \int_0^\infty dx \ x^2 \log \left[1 - e^{-\sqrt{x^2 + \beta^2 m^2}}\right]$$

High-temperature expansion \(m \ll T\)

$$J_B(m^2/T^2) = -\frac{\pi^4}{45} + \frac{\pi^2 m^2}{12 T^2} - \frac{\pi}{6} \left(\frac{m^2}{T^2}\right)^{3/2} - \frac{1}{32} \frac{m^4}{T^4} \log \frac{m^2}{a_b T^2}$$

Thermal mass term

$$V \sim g^2 T^2 \phi^2$$

Cubic term

$$V \sim -g^3 T \phi^3$$
• Zero + Finite-T potential

\[ V \sim (g^2T^2 - m_\phi^2)\phi^2 - AT\phi^3 + \lambda\phi^4 \]

\[ T \gg m_\phi/g \quad T < T_c \equiv \sqrt{\frac{m_\phi^2}{g^2 - A^2/\lambda}} \quad T \ll m_\phi/g \]

First order phase transition happens at \( T \sim m_\phi/g \)
**Bubble nucleation**  
Coleman (1977), Linde (1983)

- **Vacuum decay rate**
  \[ \Gamma \sim T^4 e^{-S_3/T} \]

- \( S_3 \): Action of O(3) symmetric bounce solution
  
  \[ S_3(T) = \int d^3x \left[ \frac{1}{2} (\nabla \Phi)^2 + V(\Phi, T) \right] \]

  \[ \frac{d^2\Phi}{dr^2} + \frac{2}{r} \frac{d\Phi}{dr} - \frac{\partial V}{\partial \Phi} = 0. \]

  Boundary condition:
  
  \[ \Phi(r = \infty) = \Phi_{\text{false}}, \]
  
  \[ \frac{d\Phi}{dr}(r = 0) = 0. \]

\~ Dynamics of scalar field with inverted potential \(-V\) with \(r\) being “time”
$V \sim (g^2 T^2 - m^2_\phi) \phi^2 - A T \phi^3 + \lambda \phi^4 \quad \left( \frac{m_\phi}{g} < T < T_c \right)$

- **Thick wall limit**

  Around temperature
  
  $T \sim T_c$

  $T_c \equiv \sqrt{\frac{m^2_\phi}{g^2 - A^2/\lambda}}$

  $R^{-2} \sim g^2 T^2 - m^2_\phi \sim \frac{(AT)^2}{\lambda}$

  $S_3 \sim \frac{4\pi}{3} R^3 V_m$

  $\frac{S_3}{T} \sim \frac{A}{\lambda^{3/2}}$

  $V_m \sim \frac{(AT)^4}{\lambda^3}$

  $\frac{AT}{\lambda}$

  $\frac{2AT}{\lambda}$
• Thin wall limit

\[ T = T_c - \Delta T \]

\[ \Delta V \sim \frac{g^2 A^2 T_c^3 \Delta T}{\lambda^2} \]

\[ \delta R^{-2} \sim g^2 T^2 - m_\phi^2 \sim \frac{(AT)^2}{\lambda} \]

\[ \frac{\delta R}{R} \sim \frac{\Delta V}{V_m} \sim \frac{g^2 \lambda \Delta T}{A^2 T_c} \]

\[ S_3 \sim 4\pi R^2 \delta RV_m \]

\[ \frac{S_3}{T} \sim \frac{A}{\lambda^{3/2}} \left( \frac{A^2 T_c}{\lambda g^2 \Delta T} \right)^2 \]

\[ V_m \sim \frac{(AT)^4}{\lambda^3} \]

\[ \frac{AT}{\lambda} \quad \frac{2AT}{\lambda} \]
Phase transition completes at

\[ \Gamma \sim H^4 \sim \frac{T^8}{M_P^4} \]

\[ \frac{S_3}{T} \sim 4 \ln \left( \frac{M_P}{T} \right) \sim 140 \]

if \( T \sim 100 \text{ GeV} \)
GWs from bubble collision  

Kamionkowski, Kosowski, Turner (1994)

- Collision of bubbles produce GWs
- Important parameter is bubble size at collision: $\beta^{-1}$
- Bubble size just after the production is $\sim T^{-1}$
- $\beta^{-1}$ is determined by duration of phase transition $\beta^{-1} \sim c\Delta t$

Note that $T^{-1} \ll \beta^{-1} \ll H^{-1}$
• **Duration of phase transition**

  • Tuning rate
    \[ \Gamma \sim T^4 e^{-S_3/T} \]

  • Phase transition happens at
    \[ \Gamma \sim H^4 \]
    \[ \frac{S_3}{T} \bigg|_{T=T_*} = 137 + 4 \log(100 \text{ GeV}/T_*) \]

  • Duration of phase transition \((S_* \equiv S_3/T)\)

    Typical duration: \(\frac{\Gamma}{H^4} = 1 \rightarrow 10\)

    \[ \frac{dS_*}{dt} \Delta t \sim O(1) \quad \rightarrow \quad \Delta t \sim \frac{1}{\beta} \sim \left( \frac{dS_*}{dt} \right)^{-1} \sim \frac{1}{H_*} \left( T \frac{dS_*}{dT} \right)^{-1} \]

    \[ \frac{\beta}{H_*} = T \frac{d(S_3/T)}{dT} \bigg|_{T=T_*} \]

    If \( S_3 \sim T^4, \quad \beta/H \sim 400 \).

    Actually the temperature dependence is complicated.

    Typically, the duration of PT is much shorter than the Hubble scale at PT.
**GWs from bubble collision**

Kosowsky et al. (92), Caprini et al. (2007), Jinno, Takimoto 1605.01403

- Frequency: \( f \sim \beta \frac{a_*}{a_0} \) 
  - **redshift**

- Strength: \( \dot{E}_{GW} \sim G(\dot{I})^2 \) (Quadra-pole formula)
  - \( I \sim MR^2 \sim \kappa \epsilon v^{-2} R^5 \)

- \( \rho_{GW} \sim N \frac{\dot{E}_{GW}\beta^{-1}}{(c\beta^{-1})^3} \sim G\kappa^2 \epsilon^2 v^2 \beta^{-2} \) (\( N \sim v^{-3} \))

- \( \Omega_{GW} \sim \Omega_{rad} \frac{\rho_{GW}}{\rho_{tot}} \sim \Omega_{rad} \kappa^2 \alpha^2 v^3 H_*^2 \beta^{-2} \)

\( \kappa \): fraction of bubble kinetic energy in latent heat
\( \alpha \): energy fraction of latent heat 

\( \kappa = \frac{(M/R^3)v^2}{\epsilon} \)

\( \alpha = \frac{\epsilon_*}{\frac{\pi^2}{30} g_* T_*^4} \)

- \( f_{\text{peak}} \approx 17 \left( \frac{f_*}{\beta} \right) \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{10^8 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}} \) [Hz]

- \( h_0^2 \Omega_{GW}(f_{\text{peak}}) \approx 1.7 \times 10^{-5} \kappa^2 \Delta \left( \frac{\beta}{H_*} \right)^{-2} \left( \frac{\alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} \)
We reexamined the spectrum of gravitational wave radiation in the parameters for the phase transition in the nMSSM [8]. Notice that the distance scale that is given by the average bubble size. In this case, the gravitational wave spectrum inherits the time scale of the source, very short compared to the relevant distance scale and the GW numerical simulations, our main finding is that the spectrum simplifies the distinction to the old and new formulas. The dependence of the peak frequency on the wall velocity much larger than the speed of light, is comparable to other sources of stochastic gravitational waves, such as turbulence [21, 22, 23, 24].

For small velocities, ref. [13], the dependence of the peak frequency on the wall velocity is slightly different. The peak frequency slightly depends on the expansion velocity of the bubble. In this approach, assumptions have to be made about the time-correlations of the velocity field. In their favored model, the authors obtain a scaling as $f \sim \beta / H$. This leads to a decrease in the peak frequency in the high frequency part of the spectrum. We suspect that this disparity is due to the thin wall and the envelope approximations. Even though the spectrum inherits the time scale of the source, very short compared to the relevant distance scale given by the average bubble size, the sensitivity of LISA and BBO is expected to be limited to low frequencies, below the best sensitivity range of planned or preexisting satellite experiments, such as turbulence [21, 22, 23, 24].

### Table I: Sets of parameters used in Fig. 3.

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<th>$\alpha$</th>
<th>$\beta / H$</th>
<th>$T_\star$ / GeV</th>
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Phase transition of Higgs at high energy and GWs
• Unfortunately, the electroweak phase transition in SM is NOT first order for $m_h = 125$ GeV

• We need extension of SM to realize 1st order PT
  Singlet extension
  2HDM
  MSSM
  Many studies on 1st order electroweak PT and GWs.

• Once extended, the scale of PT is not limited to electroweak scale. Much higher scale PT is possible.

  $\rightarrow$ Much wider range of GW frequency.
Basic idea

- Suppose that there is a scalar field whose VEV is much higher than EW scale
  \[ \phi_{NP} \text{ Peccei-Quinn field, B-L / GUT Higgs field etc.} \]

- EW Higgs can have huge mass term: \[ V \sim |\phi_{NP}|^2 |H|^2 \]

- EW scale is generated by tuning:
  \[ V \sim (|\phi_{NP}|^2 - v^2)|H|^2 = -m_H^2 |H|^2 \quad m_H \sim 100 \text{ GeV} \]

- Before \( \phi_{NP} \) gets VEV, SM Higgs has huge mass term.
  \[ V \sim -v^2 |H|^2 \]

- The scale of PT can be much different from EW scale!
Model

\[
V_0 = \lambda^2 (|\phi_{\text{NP}}|^2 - v_{\text{NP}}^2 - \delta_{\text{EW}}^2)|H|^2 + \frac{\lambda_H}{2} |H|^4 + \lambda_{\phi}^2 (|\phi_{\text{NP}}|^2 - v_{\text{NP}}^2)^2 + V_S
\]

\[
V_S = \sum_i \frac{\lambda_{S_H}^2}{2} S_i^2 |H|^2 + \sum_i \frac{\lambda_{S_{\phi}}^2}{2} S_i^2 |\phi_{\text{NP}}|^2
\]

- \(\phi_{\text{NP}}\): any scalar field having VEV of \(v_{\text{NP}}\) (Peccei-Quinn field, B-L Higgs, etc.)
- \(S_i\): singlet scalar having zero VEV

- At high temperature,
  \[H = \phi_{\text{NP}} = 0\]
- Phase transition happens at
  \[T \sim v_{\text{NP}} \gg v_{\text{EW}}\]
- GW frequency can be much higher: e.g. \(f \sim 1\,\text{Hz}\) (DECIGO)
Without singlets

FIG. 2: The temperature dependence of $\log_{10}(T_*[\text{GeV}])$

$\log_{10}(\alpha)$

$\log_{10}(\beta)$

$\log_{10}(h_0^{-2}\Omega_{GW}^{\text{peak}})$

$\log_{10}(f_{\text{peak}}[\text{Hz}])$

$(m_h[\text{GeV}], m_t[\text{GeV}])$

- (124.77, 174.32)
- (125.09, 173.34)
- (125.41, 172.36)
Why is GW small?

- Large $\lambda_H$
  - S3 is sensitive to T: short duration of PT (large $\beta$)
  - Small amount of latent heat $\alpha$

- Small $\lambda_H$
  - S3 is insensitive to T: long duration of PT (small $\beta$)
  - Large amount of latent heat $\alpha$

- Smaller $\lambda_H$ → Larger GW
• Singlet extension

\[ V_S = \sum_i \frac{\lambda_{SH}^2}{2} S_i^2 |H|^2 + \sum_i \frac{\lambda_{S\phi}^2}{2} S_i^2 |\phi_{NP}|^2 \]

• Change Higgs quartic coupling through RGE:

\[ \frac{d\lambda_H}{d \ln \mu} = \beta_H^{SM} + \frac{N_S}{16\pi^2} \lambda_{SH}^4 \]

• At T=0, \( m_S^0 = \lambda_S \phi v_{NP} \)

RGE is not affected below this scale.

• At high T, \( \phi_{NP} = 0 \)

hence \( m_S = 0 \)

RGE changes Higgs coupling at \( T=T^* \)
We have taken $N=10^{0.01}$ due to the coupling of the Higgs with additional singlets. In such a situation, the strength of $\omega_\phi$ becomes small. In such a situation, the strength of $\omega_\phi$ becomes large. In such a situation, the strength of $\omega_\phi$ becomes large.

The gravitational waves gets enhanced because the relative density of $N_H$ after the transition becomes relatively large. In such a situation, the strength of $\omega_\phi$ becomes large. In such a situation, the strength of $\omega_\phi$ becomes large.

In order to see the typical situation where the GW energy fraction $\Omega_{GW}^{peak}$ becomes $10^{-3}$, we consider two types of models in the Higgs sector. In the first model we have only the standard model Higgs field and such scalars exist in general. These couplings can cause the first order phase transition of the Higgs field at the physics scale, which is much higher than the weak scale.

Hence the peak position of the GWs as well as its strength can take broad range of values depending on the new physics scale. If the new physics contains scalar fields $\lambda_S$, the couplings between the standard model Higgs field and such scalars exist in general. These couplings can cause the first order phase transition of the Higgs field at the physics scale, which is much higher than the weak scale.

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FIG. 12: GW energy fraction $\Omega_{GW}$ as a function of $f$ [Hz]. Together with $\Omega_{GW} \propto f^{-2.5}$, $\Omega_{GW}$ may be detectable at DECIGO.

FIG. 13: The GW spectrum from bubble collisions for (solid) turbulence (dashed). See text for details.

In this paper, we have considered GWs generated by phase transition of the standard model Higgs field and such scalars exist in general. These couplings may cause the first order phase transition of the Higgs field as well as its strength $\mathcal{O}(10^{-9})$ due to the coupling of the Higgs with additional singlets.

As a final remark, in this paper we considered GWs as well as cosmic variability. Since the scale of phase transition of some other scalar fields that do or do not exist in the standard model may be possible if the number of the singlets is $\sim O(10^{3})$. The sensitivity of the DECIGO may open up a new possibility for probing new physics scale. If the new physics contains scalar fields with $m_{NP} \lesssim 10^{15} \text{GeV}$, the couplings between the standard model Higgs field and such scalar fields can cause the first order phase transition of the Higgs field and the generated GWs is too weak to detect by designed future experiments. Second model contains additional singlet fields $\phi^{i}$ with $m_{NP} \lesssim 10^{14} \text{GeV}$, which is much higher than the weak scale.

Hence the peak position of the GWs as well as its strength $\mathcal{O}(10^{-9})$ can take broad range of values depending on the new physics scale. If the new physics contains scalar fields associated with first order phase transition of the Higgs field and such scalars exist in general, these couplings can cause the first order phase transition of the standard model Higgs field at the temperature of the universe around the new physics scale. If the new physics contains scalar fields with $\mathcal{O}(10^{3})$, then the sensitivity of the DECIGO may open up a new possibility for probing new physics scale, which is much higher than the weak scale.
Summary

• 1st order phase transition can happen at much higher energy scale than electroweak scale (Peccei-Quinn, B-L, etc.)

• Frequency of GWs from bubble collisions can be significantly different from previously thought

• Typically GW amplitude is too small, but it is possible to enhance GW signal in singlet extended models.