Chirality, Topology, and Astrophysics

Naoki Yamamoto (Keio University)

Osaka University
June 7, 2016

Main question

Possible new roles of topology in quantum many-body and non-equilibrium problems in high-energy physics?

- Early Universe
- Heavy ion collisions
- Neutron stars and supernovae
- …

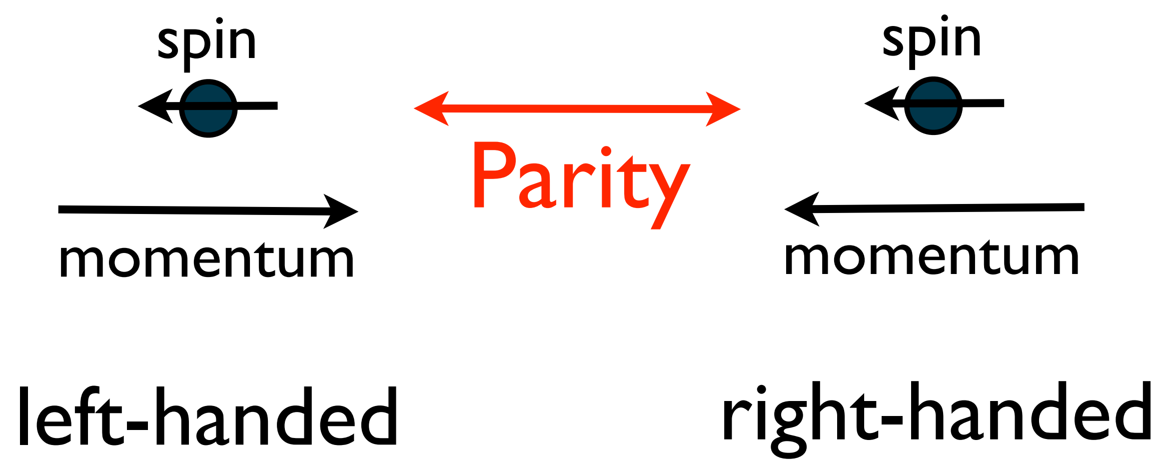
Beyond Nambu paradigm (spontaneous symmetry breaking)

Contents

- Chirality and Topology
- Chiral Kinetic Theory — Son-NY, PRL (2012), PRD (2013)
- Chiral Plasma Instability — Akamatsu-NY, PRL (2013), PRD (2014)
- Chiral transport in supernovae — NY, PRD (2016) & 1603.08864.

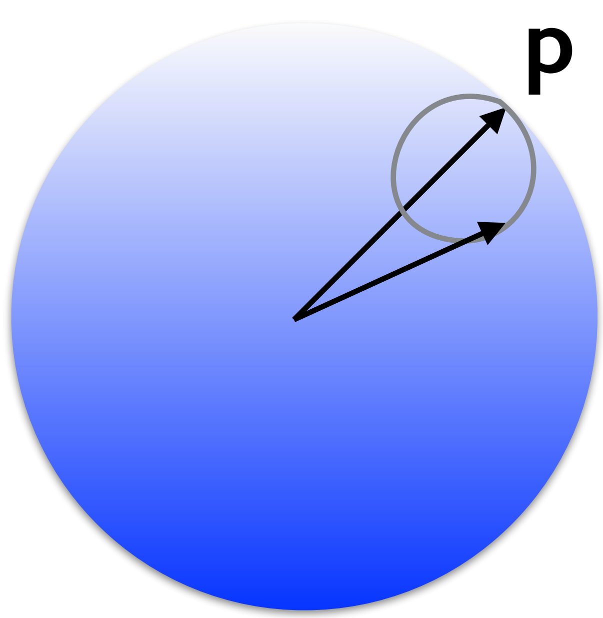
Chirality and Topology

Chirality of fermions

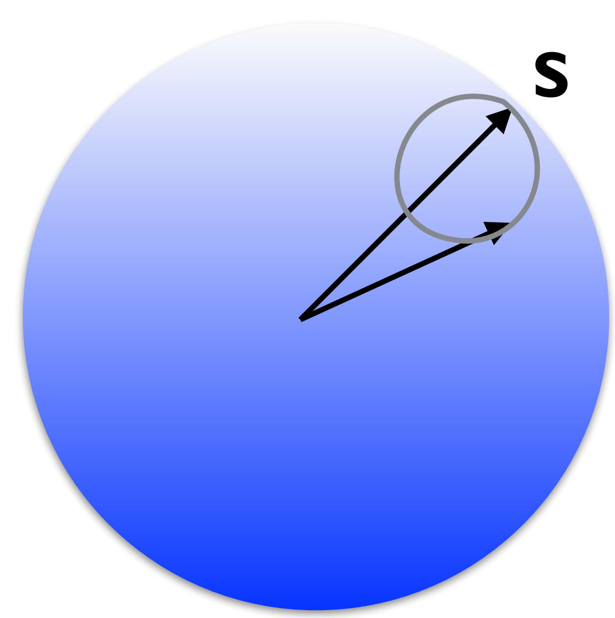


Chirality and topology

Right-handed fermions



momentum space



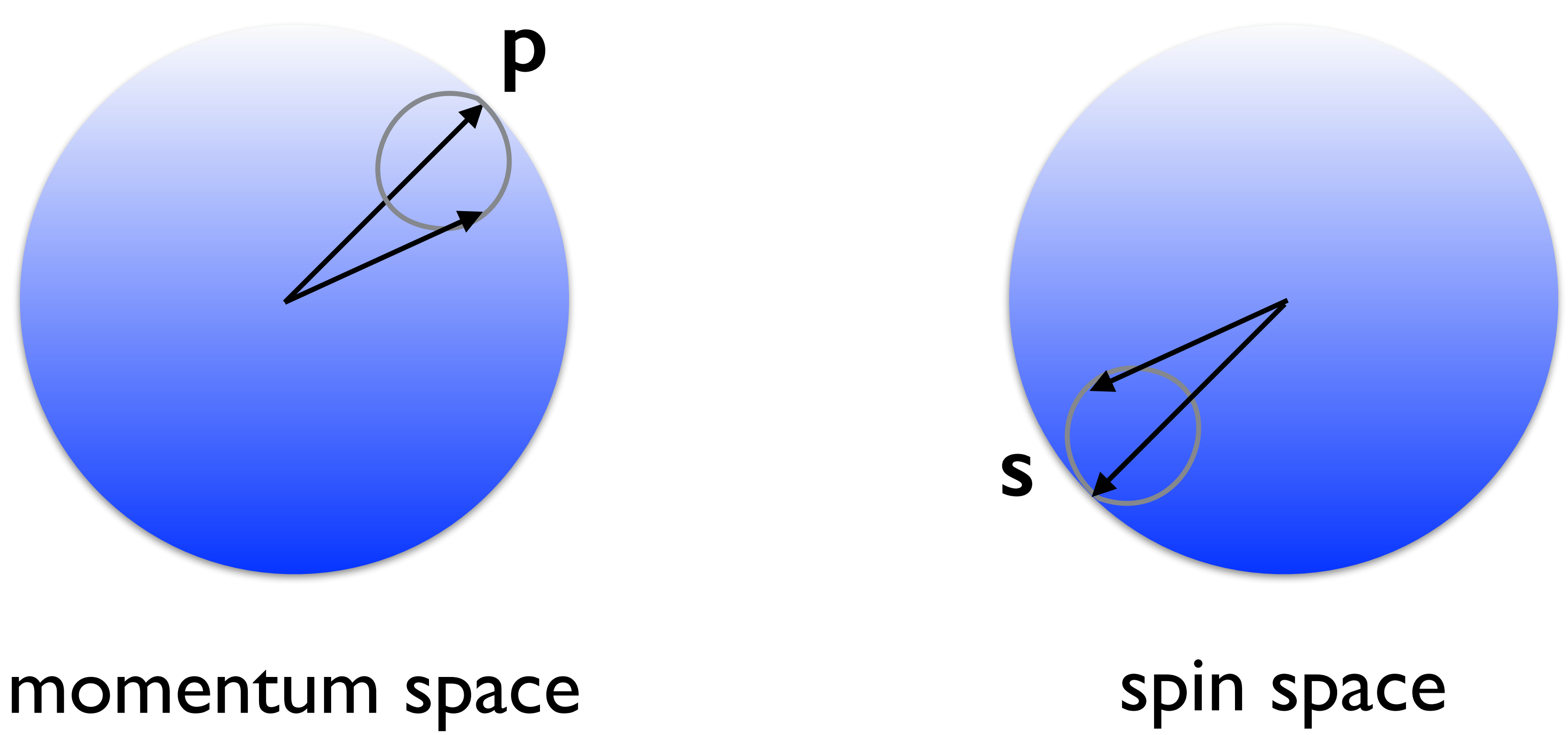
spin space

Mapping from S^2 (p-space) to S^2 (spin space):

winding number +1

Chirality and topology

Left-handed fermions

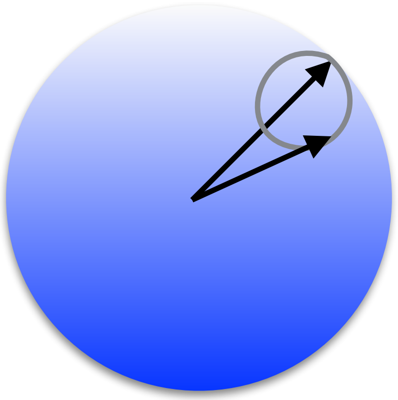


Mapping from S^2 (p-space) to S^2 (spin space):

winding number -1

Topology and Berry curvature

- Berry curvature Ω = Curvature of a Fermi surface
- Winding number = Area integral of Ω
- Equation of motion with Ω → Chiral kinetic theory
  Son-NY (2012, 2013); Stephanov-Yin (2012)



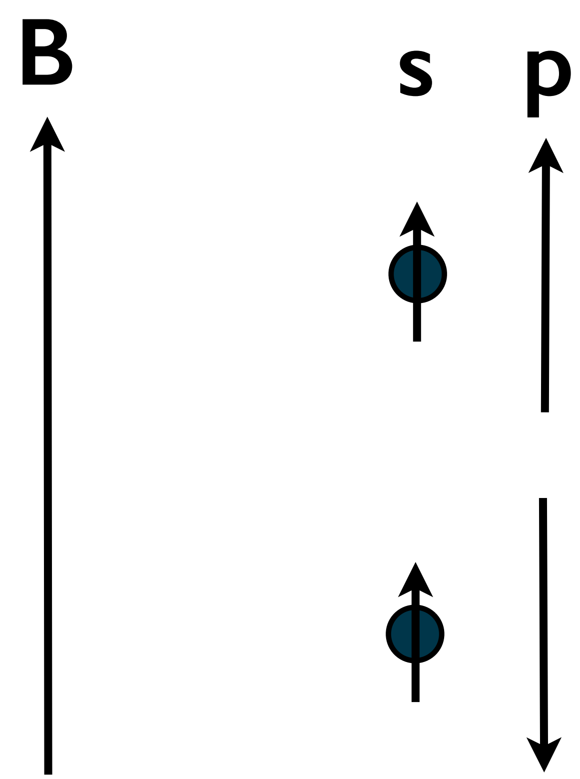
$$\boldsymbol{\Omega} = \pm \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$

p-space

Non-equilibrium dynamics is modified by topological effects

Chiral Magnetic Effect (CME)

- Current in the presence of μ_5: $\boldsymbol{j}_e \sim (\mu_R - \mu_L)\boldsymbol{B}$



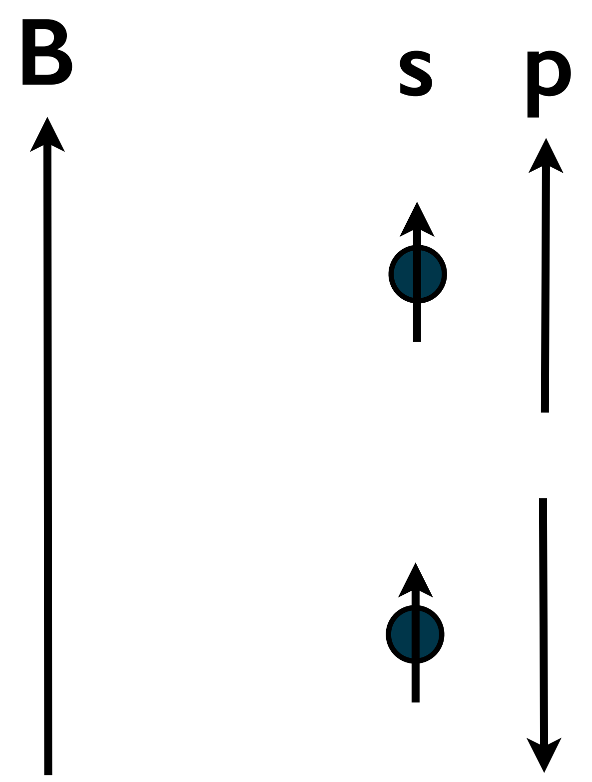
right-handed $\boldsymbol{j}_R = +\kappa\mu_R\boldsymbol{B}$

left-handed $\boldsymbol{j}_L = -\kappa\mu_L\boldsymbol{B}$

Vilenkin (1980); Nielsen, Ninomiya (1983); Son, Zhitnitsky (2004);
Kharzeev, Warringa, Fukushima (2008)

Chiral Magnetic Effect (CME)

- Current in the presence of μ_5: $\boldsymbol{j}_e \sim (\mu_R - \mu_L)\boldsymbol{B}$



winding number +1

right-handed $\quad \boldsymbol{j}_R = +\kappa\mu_R\boldsymbol{B}$

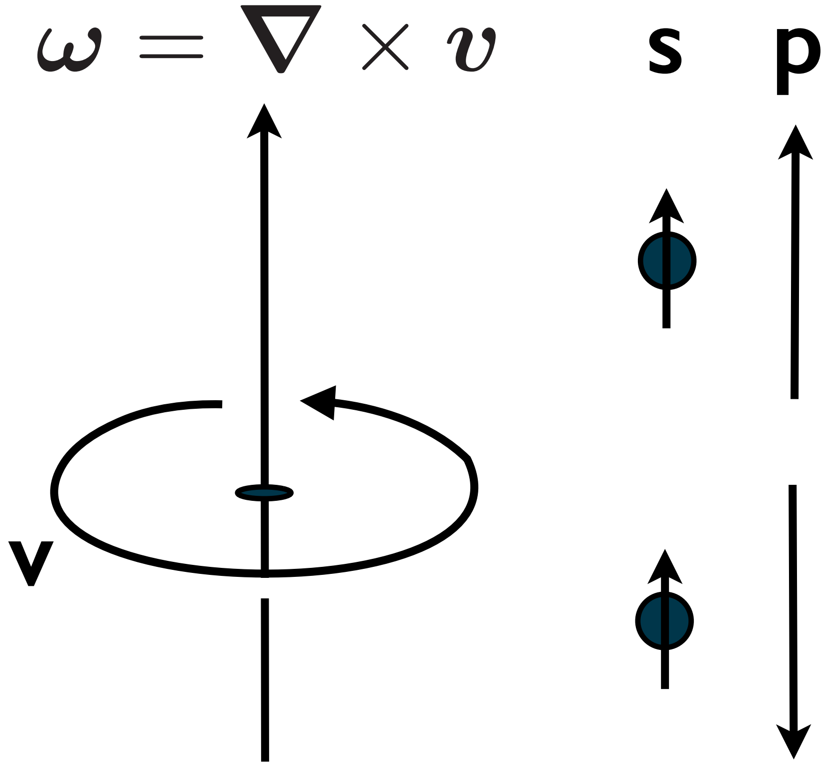
winding number -1

left-handed $\quad \boldsymbol{j}_L = -\kappa\mu_L\boldsymbol{B}$

Vilenkin (1980); Nielsen, Ninomiya (1983); Son, Zhitnitsky (2004);
Kharzeev, Warringa, Fukushima (2008)

Chiral vortical effect (CVE)

$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$



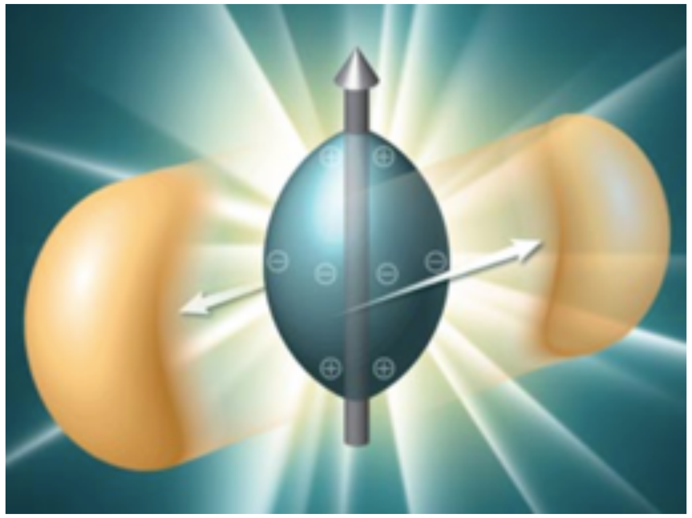
right-handed $\boldsymbol{j}_R = (C\mu_R^2 + DT^2)\boldsymbol{\omega}$

left-handed $\boldsymbol{j}_L = -(C\mu_L^2 + DT^2)\boldsymbol{\omega}$

Vilenkin (1979); Son-Surowka (2009); K. Landsteiner et al. (2011); K. Jensen et al. (2012)

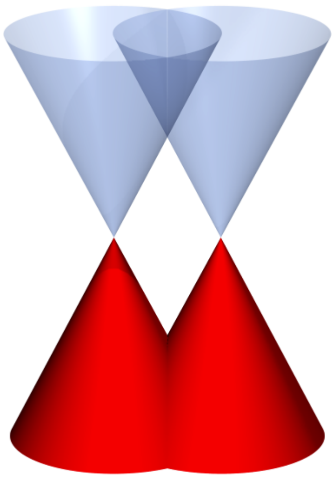
Chiral Matter

- Electroweak plasma in early Universe — Joyce-Shaposhnikov (1997), …
- Quark-gluon plasma in RHIC/LHC — Fukushima-Kharzeev-Warringa (2008), …
- Weyl semimetals ("3D graphene") — Nielsen-Ninomiya (1983), …
- Neutrino media in supernovae — NY (2015)

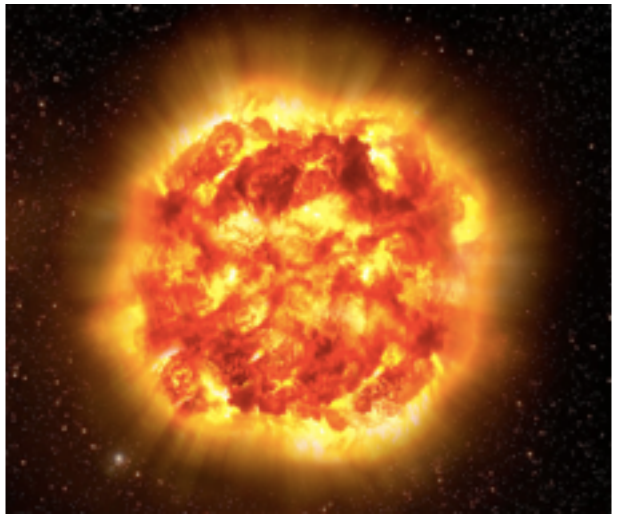


QGP?

http://www0.bnl.gov/rhic/news2/



Weyl semimetals



Supernovae

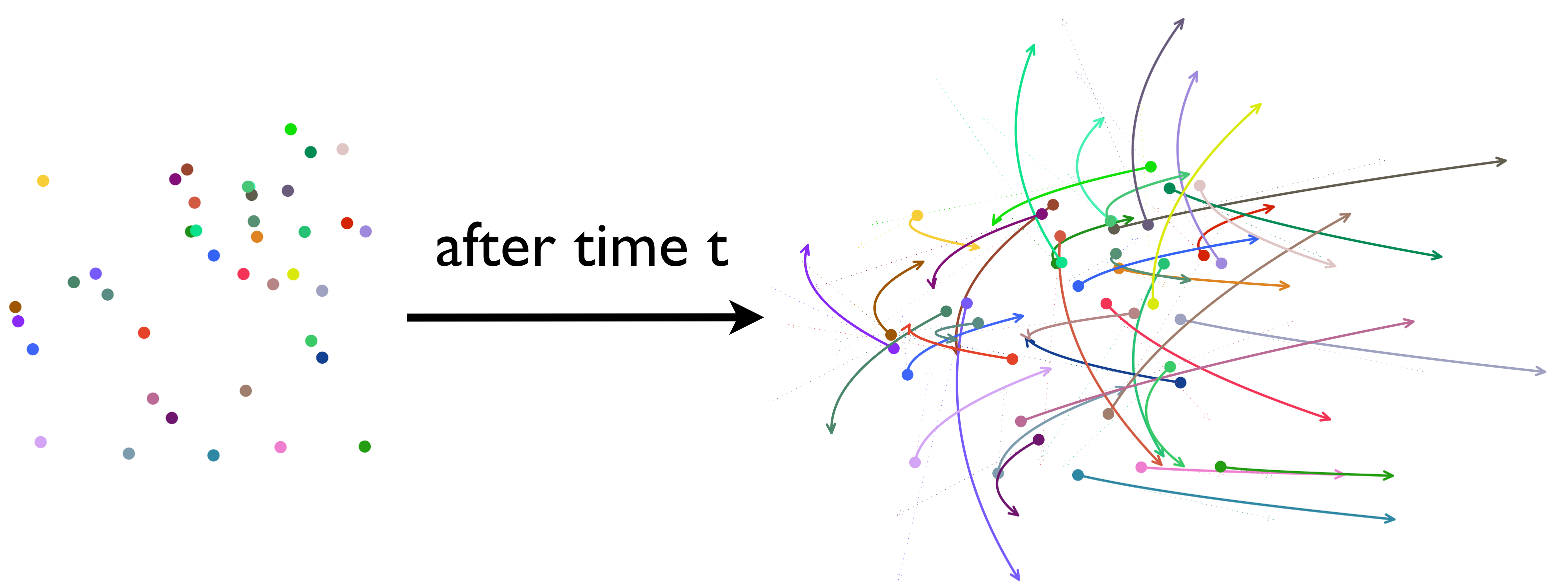
Chiral kinetic theory

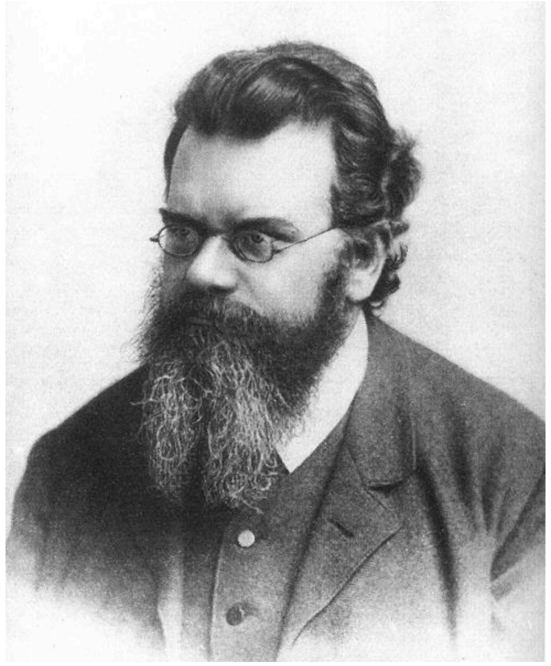
Son-NY (2012); Stephanov-Yin (2012)

Kinetic theory

Kinetic theory, typified by Boltzmann equation, describes the statistical behavior of a system in and out of equilibrium

$$\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = c[n_{\boldsymbol{p}}]$$





Ludwig Boltzmann

Boltzmann equation

- Formulated w.r.t. the distribution function: $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- First ignore collisions; Liouville's theorem implies

$$\frac{dn_{\boldsymbol{p}}}{dt} = \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \underline{\dot{\boldsymbol{x}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \underline{\dot{\boldsymbol{p}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = 0$$

Boltzmann equation

- Formulated w.r.t. the distribution function: $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- First ignore collisions; Liouville's theorem implies

$$\frac{dn_{\boldsymbol{p}}}{dt} = \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = 0$$

Lorentz force

Boltzmann equation

- Formulated w.r.t. the distribution function: $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- Add collisions:

$$\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = c[n_{\boldsymbol{p}}]$$

- This describes, e.g., Ohm's law:

$$\boldsymbol{j}_{\text{noneq}} = \int d\boldsymbol{p}\ \boldsymbol{v} \delta n_{\boldsymbol{p}} = \sigma \boldsymbol{E}$$

Boltzmann equation

- Formulated w.r.t. the distribution function: $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- Add collisions:

$$\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = c[n_{\boldsymbol{p}}]$$

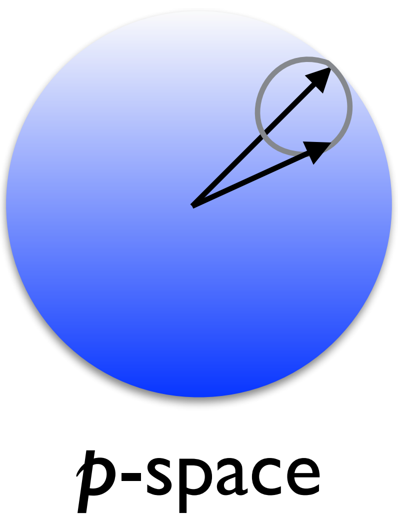
- This describes, e.g., Ohm's law:

$$\boldsymbol{j}_{\text{noneq}} = \int d\boldsymbol{p}\ \boldsymbol{v} \delta n_{\boldsymbol{p}} = \sigma \boldsymbol{E}$$

- CME? Quantum anomalies?

Topology and Berry curvature

- Berry curvature Ω = Curvature of a Fermi surface
- Winding number = Area integral of Ω
- Equation of motion with Ω → Chiral kinetic theory
  Son-NY (2012, 2013); Stephanov-Yin (2012)



$$\boldsymbol{\Omega} = \pm \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$

Non-equilibrium dynamics is modified by topological effects

Equations of motion

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

Lorentz force

Equations of motion

“Lorentz force” in ***p*-space**

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}$$

$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

Sundaram-Niu, PRB (1999)

Lorentz force in ***x*-space**

Equations of motion

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega_p} = \omega^{-1}[\hat{\boldsymbol{p}} + \boldsymbol{E} \times \boldsymbol{\Omega_p} + (\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega_p})\boldsymbol{B}]$$

$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B} = \omega^{-1}[\boldsymbol{E} + \hat{\boldsymbol{p}} \times \boldsymbol{B} + (\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega_p}]$$

$$\omega = 1 + \boldsymbol{B} \cdot \boldsymbol{\Omega_p}$$

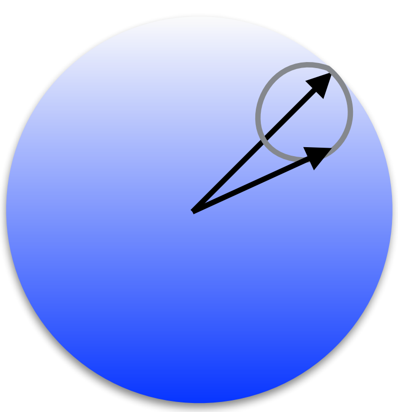
Boltzmann equation

- Formulated w.r.t. the distribution function: $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- First ignore collisions; Liouville's theorem implies

$$\frac{dn_{\boldsymbol{p}}}{dt} = \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \dot{\boldsymbol{x}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = 0$$

Chiral kinetic theory

$$(1 + \boldsymbol{B}\cdot\boldsymbol{\Omega})\frac{\partial n_{\boldsymbol{p}}}{\partial t} + [\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{\Omega} + (\boldsymbol{v}\cdot\boldsymbol{\Omega})\boldsymbol{B}]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + [\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} + (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = c[n_{\boldsymbol{p}}]$$



p-space

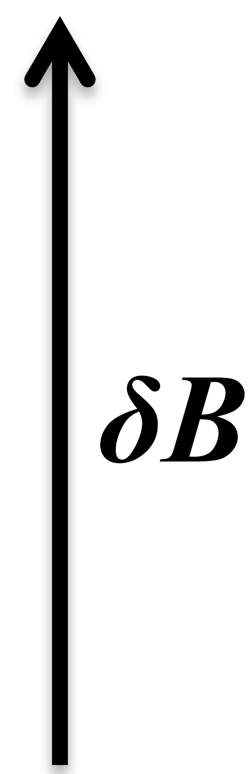
$$\boldsymbol{\Omega} = \pm\frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}$$

Son-NY (2012); Stephanov-Yin (2012)

Chiral Plasma Instability

Akamatsu-NY (2013, 2014)

Chiral Plasma Instability (CPI)



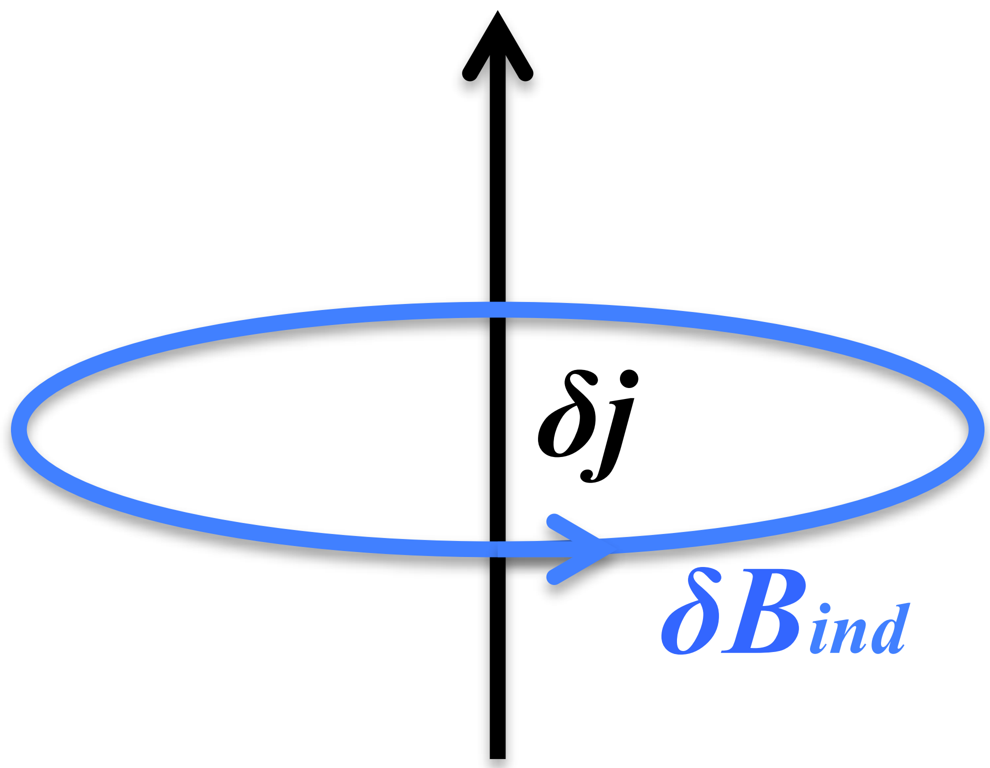
Assume homogeneous μ_5 initially

Chiral Plasma Instability (CPI)

Chiral magnetic effect

$$\boldsymbol{\delta j} \sim \mu_5 \boldsymbol{\delta B}$$

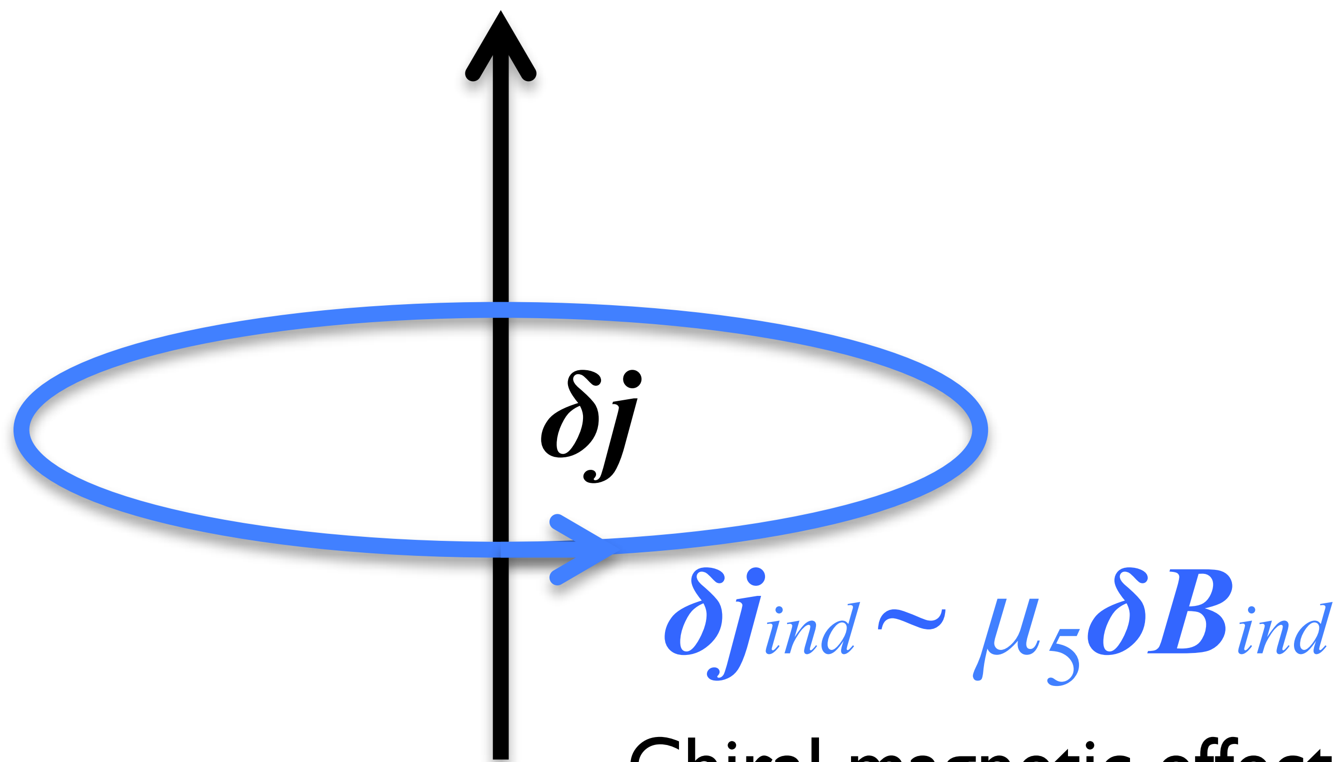
Chiral Plasma Instability (CPI)



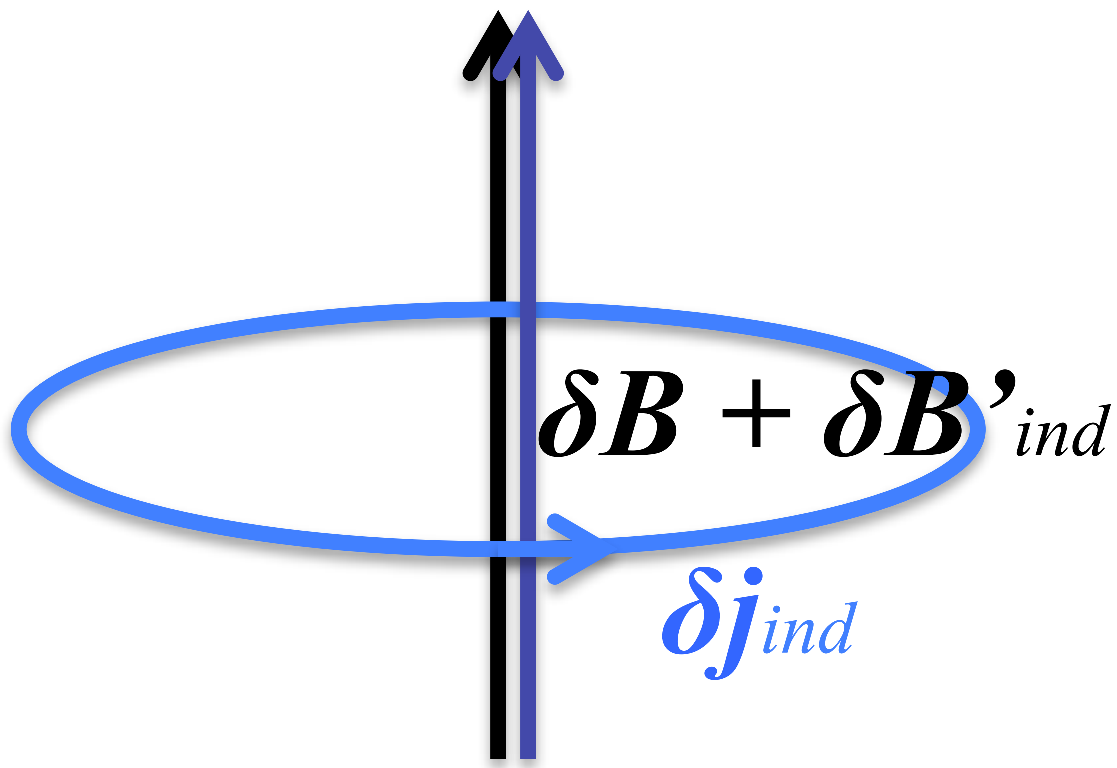
Ampere's law

$$\nabla \times B = j$$

Chiral Plasma Instability (CPI)



Chiral Plasma Instability (CPI)

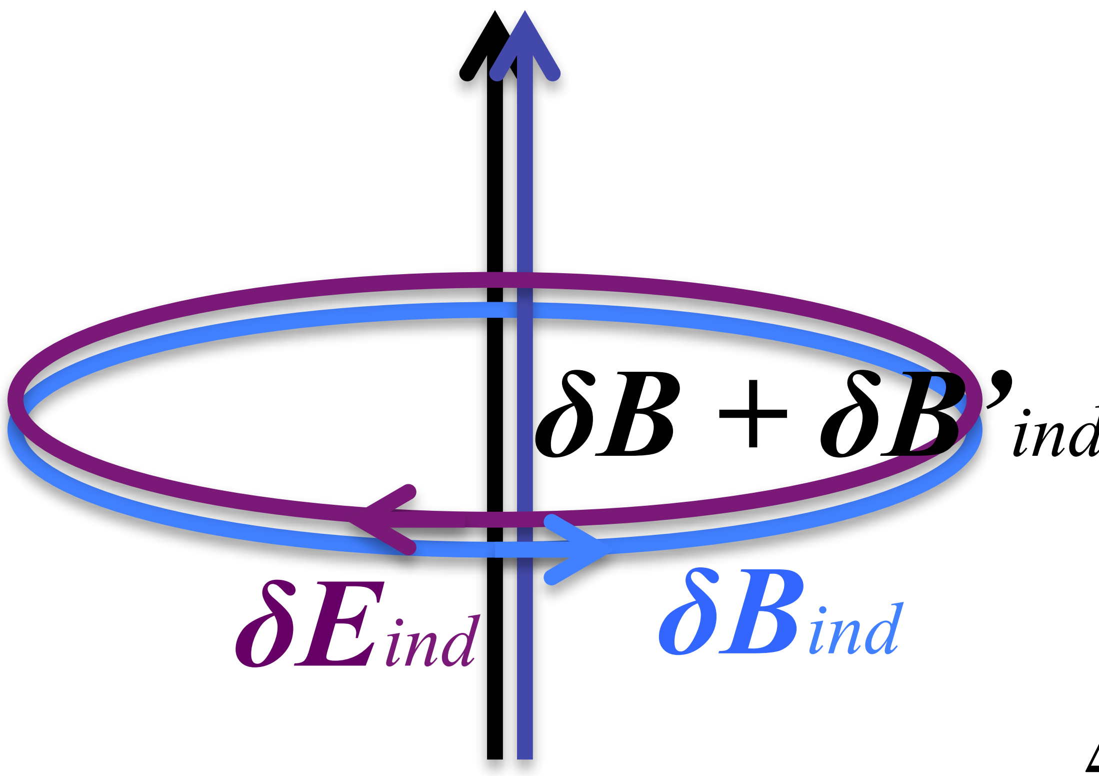


Ampere's law

$$\nabla \times B = j$$

Positive feedback: instability

Chiral Plasma Instability (CPI)



Faraday's law

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$

anomaly relation

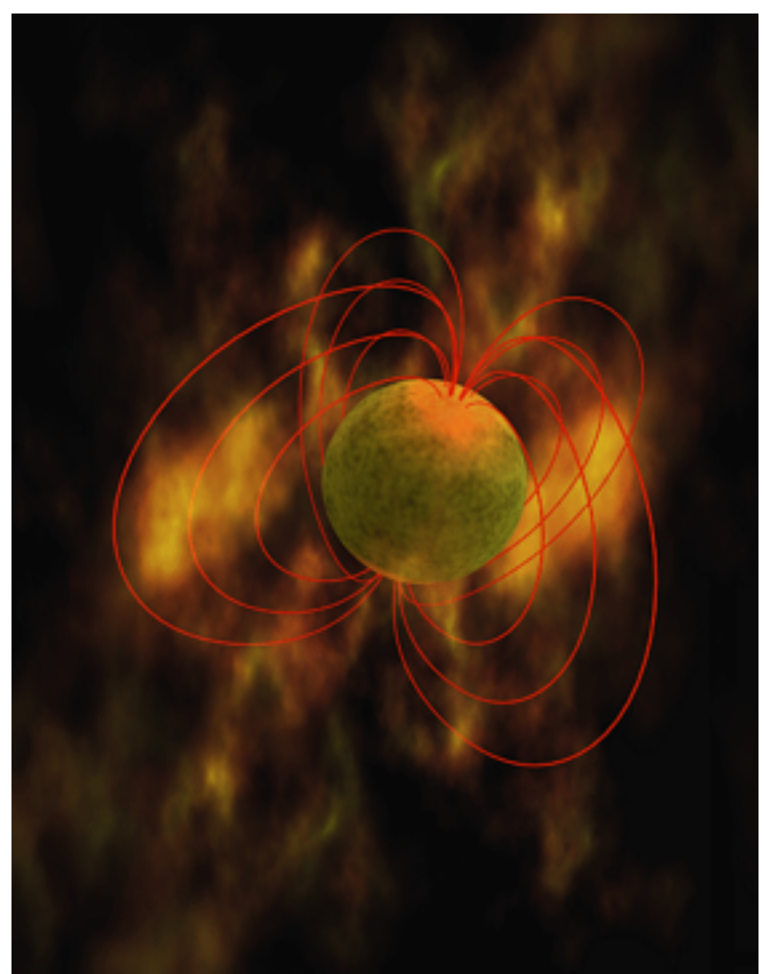
$$\Delta Q_5 = \# \boldsymbol{E} \cdot \boldsymbol{B} < 0$$

Quantum anomaly (non-linear effects) tends to make L and R equal

Some applications

Magnetar

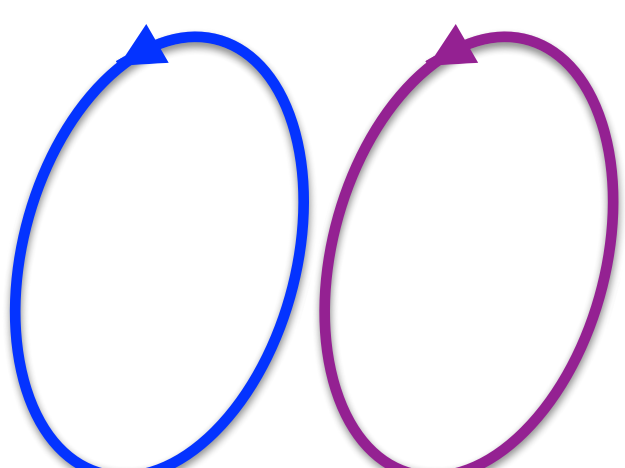
- Magnetar: the strongest "magnet" in the Universe
- ~10^{15} G at the surface
- How is the stable and strong magnetic field generated?



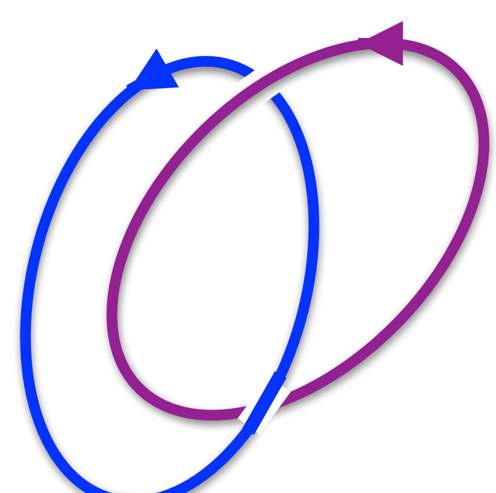
Magnetic helicity

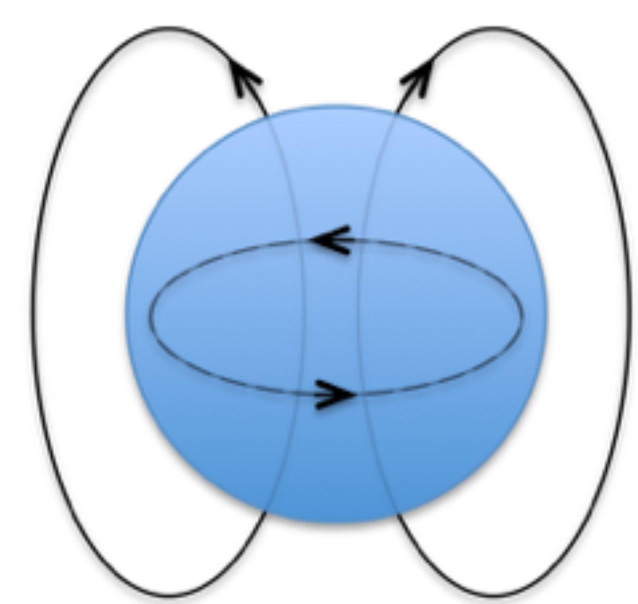
- Magnetic helicity (Chern-Simons number): $\mathcal{H}_{\mathrm{mag}} = \int_V \boldsymbol{A} \cdot \boldsymbol{B}$
- Proportional to linking number: (approximate) topological stability
- Assumed as an initial condition in magneto-hydrodynamics (MHD)
- However, its origin is not trivial (P-odd quantity).

$\mathcal{H} = 0$



$\mathcal{H} = -2\Phi_1\Phi_2$



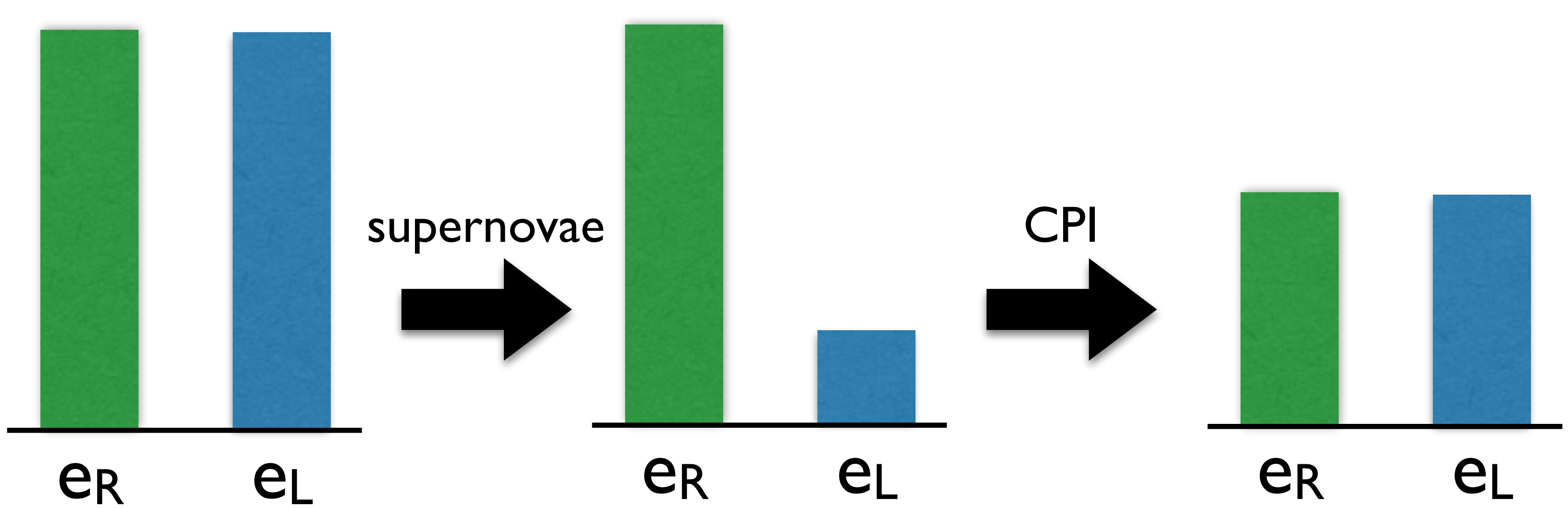


poloidal/toroidal B

Magnetic field from CPI?

- Neutrino emission at supernovae: $p + e_L^- \to n + \nu_e^L$
- More right-handed electrons remain, which is unstable (CPI)
- Magnetic field ~10^{18} G at the core (at most)
- Helicity conservation: fermion's helicity → magnetic helicity

Ohnishi-NY (2014)



+ helical B

Evidence of toroidal magnetic field?

Synopsis: Internal Magnetic Field Causes Neutron Star to Go Wobbly

L. Calçada/ESO

Possible Evidence for Free Precession of a Strongly Magnetized Neutron Star in the Magnetar 4U 0142+61

K. Makishima, T. Enoto, J. S. Hiraga, T. Nakano, K. Nakazawa, S. Sakurai, M. Sasano, and H. Murakami

Phys. Rev. Lett. **112**, 171102 (2014)

Published April 30, 2014

Possible Evidence for Free Precession of a Strongly Magnetized Neutron Star in the Magnetar 4U 0142+61

K. Makishima,[1,2,3] T. Enoto,[4,5] J. S. Hiraga,[2] T. Nakano,[1] K. Nakazawa,[1] S. Sakurai,[1] M. Sasano,[1] and H. Murakami[1]

[1]*Department of Physics, Graduate School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[2]*Research Center for the Early Universe, Graduate School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[3]*MAXI team, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*

[4]*High Energy Astrophysics Laboratory, RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*

[5]*NASA Goddard Space Flight Center, Astrophysics Science Division, Code 662, Greenbelt, MD 20771, USA*

(Dated: April 22, 2014)

Magnetars are a special type of neutron stars, considered to have extreme *dipole* magnetic fields reaching $\sim 10^{11}$ T. The magnetar 4U 0142+61, one of prototypes of this class, was studied in broadband X-rays (0.5–70 keV) with the *Suzaku* observatory. In hard X-rays (15–40 keV), its 8.69 sec pulsations suffered slow phase modulations by ± 0.7 sec, with a period of ~ 15 hours. When this effect is interpreted as free precession of the neutron star, the object is inferred to deviate from spherical symmetry by $\sim 1.6 \times 10^{-4}$ in its moments of inertia. This deformation, when ascribed to magnetic pressure, suggests a strong *toroidal* magnetic field, $\sim 10^{12}$ T, residing inside the object. This provides one of the first observational approaches towards toroidal magnetic fields of magnetars.

Chiral transport of neutrinos in supernovae

Neutrinos in supernovae

NY, arXiv:1511.00933 (astro-ph.HE)

- Neutrino production at supernovae: $p + e_L^- \to n + \nu_e^L$
- Neutrino mean free path ~ 1cm when $\rho_N \sim 10^{15}$ g/cm^3.

$$l^{\nu}_{\mathrm{mfp}} = \frac{1}{\sigma_A n_A}$$

cross section (computable in SM) → σ_A

number density of nuclei → n_A

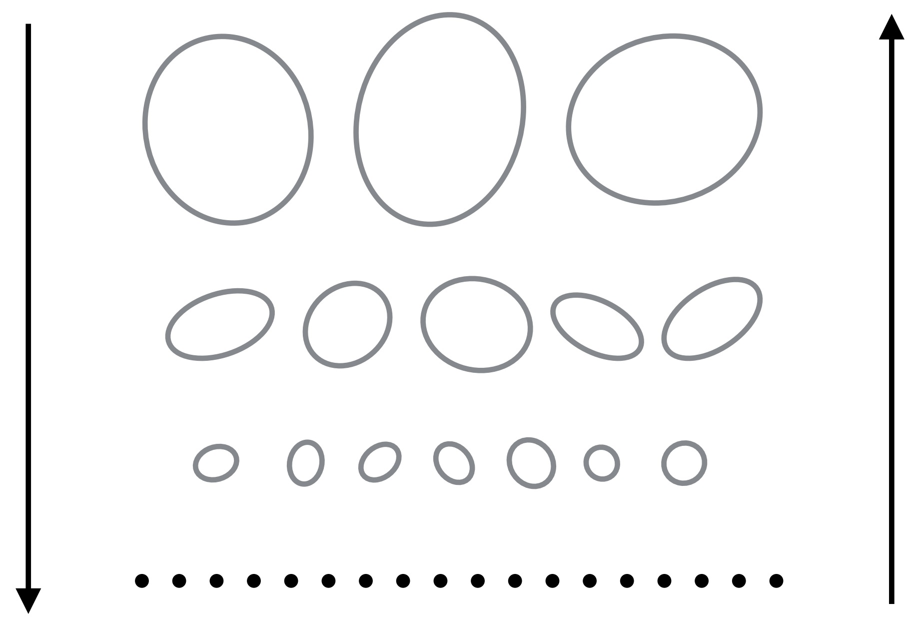
- Even neutrinos can make up matter in supernovae
→ *chiral* quantum liquids ($\mu_\nu \sim 200$ MeV)

Chiral turbulence

- Possibility of supernova exp. ⇄ cascade direction of turbulence

Direct cascade
(3D usual matter)

explosion difficult



Inverse cascade
(2D usual matter)

explosion easier

- 3D *chiral* matter: inverse cascade → explosion becomes easier

Left-handedness of neutrinos flips the cascade direction of turbulence!

NY, arXiv:1603.08864 (hep-th) & work in progress

Summary

- Chirality = Topology in relativistic many-body systems.
- Relevance of chiral transport in astrophysics:
  magnetars and supernova (SN) explosions.
- Future simulations of SN must include Berry curvature of ν
- More applications of chiral transport theories to cond-mat,
  nuclear, and astro physics.