“Lagrangians” for N=2 SCFTs in 4d

Kazunobu Maruyoshi

(Seikei University)

w/ Jaewon Song, 1606.05632, 1607.04281
w/ Prarit Agarwal and Jaewon Song, 16XX.XXXXX

@Osaka University October 11, 2016

Introduction

Supersymmetry is powerful enough to determine some parts of quantum effects in field theory.

In particular, **the localization method** enables us to get exact answer of the partition function or correlation function of supersymmetric gauge theory. **[Pestun, Jafferis, Hama-Hosomichi-Lee]**

However, the localization can be only applied to the theory with a Lagrangian description, though it is known the existence of strongly-coupled theories which are difficult to write Lagrangians.

- 4d N=2 class S theories **[Gaiotto]**
- 4d N=2 Argyres-Douglas theory and its generalizations **[Argyres-Douglas, Argyres-Plesser-Seiberg-Witten...]**

UV Lagrangian

For example….

UV

N=1 SQCD
in conformal window

RG flow

**can compute
the partition function
from the UV Lagrangian!**

SCI [Spiridonov,Vartanov]

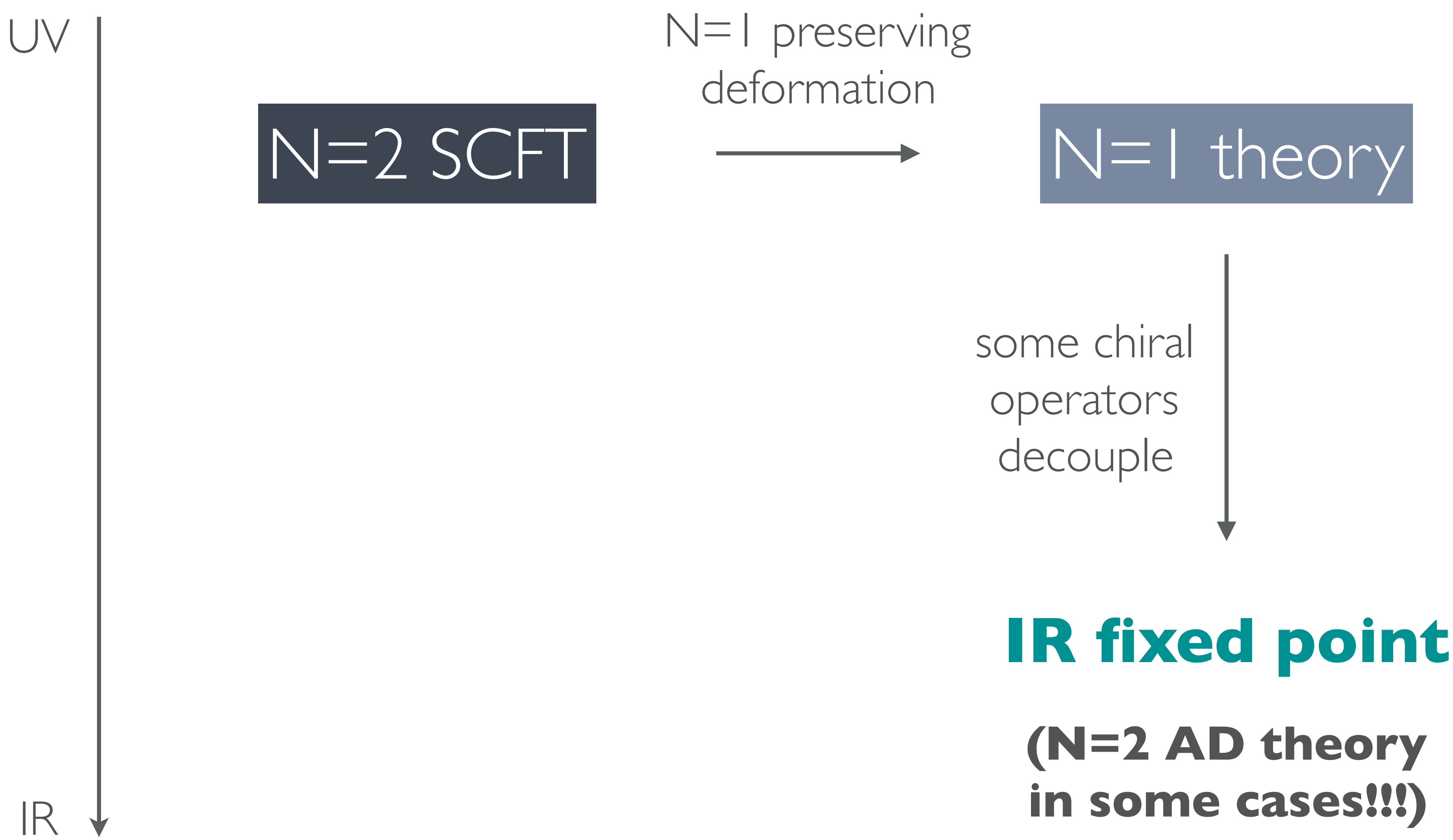
(N=1) strongly-coupled SCFT

IR

In this talk, I present Lagrangian descriptions which flow in the IR to N=2 Argyres-Douglas theories.

A remarkable phenomena is that the Lagrangian has only N=1 supersymmetry in 4d, which is enhanced to N=2 in the IR

How to get?


UV
IR
N=2 SCFT
N=1 preserving deformation
N=1 theory
some chiral operators decouple
IR fixed point
(N=2 AD theory in some cases!!!)

How useful?

To have Lagrangians gives a new handle to study the strongly-interacting theories. Non-BPS sector?

We can compute the partition functions of the SCFTs from the Lagrangians (superconformal index in the full generality).

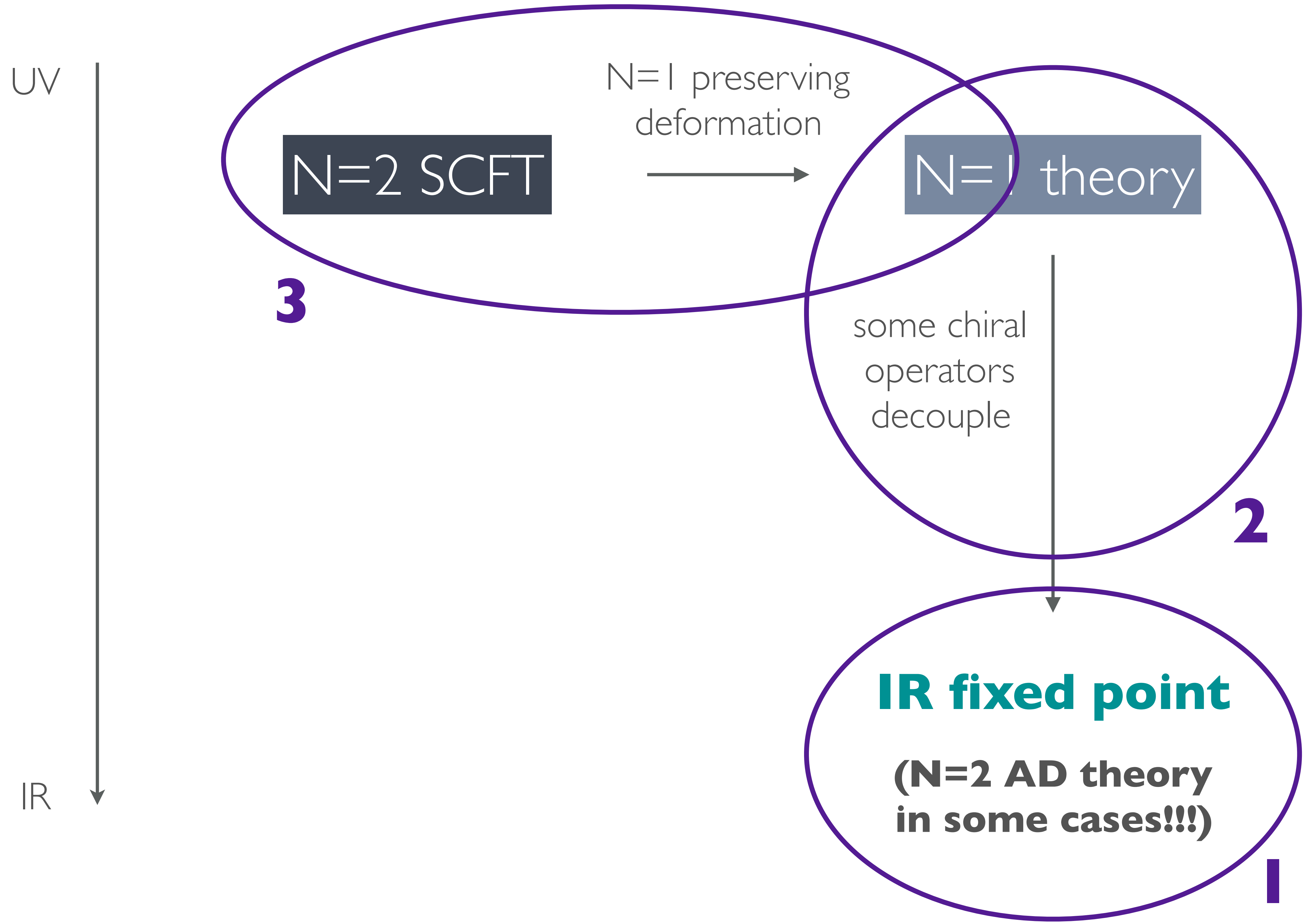
Indeed, the superconformal indices obtained in this way agree with the previous results in special limits by **[Buican-Nishinaka, Cordova-Shao, Song]**

Why N=2 enhancement?

We lack the understanding of the mechanism of the supersymmetry enhancement. Even we don't prove the IR theory has N=2 supersymmetry.

Any comments and suggestions are helpful!!!

In the rest of talk



Review of Argyres-Douglas theories

Argyres-Douglas theory

- originally has been found at the special locus on the Coulomb branch of N=2 SU(3) pure SYM theory, where **mutually non-local massless particles appear** **[Argyres-Douglas]**.

- is **strongly coupled N=2 SCFT** with central charges **[Aharony-Tachikawa]**

$$a = \frac{43}{120}, \quad c = \frac{11}{30}$$

- has one-dimensional Coulomb branch parametrized by the chiral operator u of **scaling dimension 6/5**.

(The AD theory is the minimal nontrivial SCFT which saturates the central charge bound **[Liendo-Ramirez-Seo]**.)

Generalizations

We define Argyres-Douglas theories (generalization) by N=2 SCFTs with Coulomb branch operators with fractional dimensions.

[Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang]
[Cecotti-Neitzke-Vafa, Bonelli-KM-Tanzini, Xie]

- **(A_1, A_k) series**: conformal point of N=2 SU(k+1) pure SYM

$$\Delta(u_i) = \frac{2(k+3-i)}{k+3}$$

- **(A_1, D_k) series**: conformal point of N=2 SU(k-1) SQCD w/ 2 flavors

$$\Delta(u_i) = \frac{2i}{k}$$

An N=1 gauge theory

An N=1 gauge theory

Let us consider the following N=1 theory with **SU(2) vector multiplets** and **with the following chiral multiplets**:

	q	q'	ϕ	M
SU(2)	$\square$	$\square$	adj	1
$U(1)_{R0}$	1/2	-5/2	1	6
$U(1)_{\mathcal{F}}$	1/2	7/2	-1	-6

with the superpotential

$$W = \phi q q + M \phi q' q'$$

Gauge invariant chiral operators:

$$\mathrm{Tr}\phi^2, \; M, \; \ldots$$

Central charges

The central charges of the SCFT are determined from the anomaly coefficients of the IR R-symmetry: **[Anselmi-Freedman-Grisaru-Johansen]**

$$a = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}^3 - \mathrm{Tr}R_{\mathrm{IR}}), \quad c = \frac{1}{32}(9\mathrm{Tr}R_{\mathrm{IR}}^3 - 5\mathrm{Tr}R_{\mathrm{IR}})$$

In our case, the IR R-symmetry is a combination of two U(1)'s. Thus consider the following

$$R_{\mathrm{IR}}(\epsilon) = R_0 + \epsilon\mathcal{F}$$

The true R symmetry is determined by maximizing trial central charge **[Intriligator-Wecht]**

$$a(\epsilon) = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}(\epsilon)^3 - \mathrm{Tr}R_{\mathrm{IR}}(\epsilon))$$

decoupling of chiral multiplets

A caveat is that we have to check the chiral operators have dimension greater than one.

If it is less than one, it is interpreted as being decoupled. Thus we subtract its contribution from central charge, and re-a-maximize

$$\cancel{\mathrm{Tr}\phi^2}, \;\; M, \;\; \ldots$$

dimension 6/5

$$\epsilon = \frac{13}{15}, \quad a = \frac{43}{120}, \quad c = \frac{11}{30}$$

$\mathcal{N}=1$ deformations of $\mathcal{N}=2$ SCFTs

N=1 deformation

Suppose we have an N=2 SCFT ***T*** with **non-Abelian flavor symmetry F.** (We use the convention for R symmetry $(2I_3, r) = (J_+, J_-)$.)

Then let us

- **couple N=1 chiral multiplet M in the adjoint rep of F by the superpotential**

$$W = \mathrm{tr}\mu M$$

M: $(J_+, J_-)=(0,2)$

- **give a nilpotent vev to M (which is specified by the embedding ρ: SU(2)→F), which breaks F.**

(For F=SU(N), this is classified by a partition of N or Young diagram.)

This gives IR theory $T_{IR}[T, \rho]$, which is generically N=1 supersymmetric.

More on N=1 deformation

M and **μ** in the adjoint representation of F are decomposed into su(2) spin j-reps $M_{j,j_3}, \ \mu_{j,j_3}$.

By the vev, the superpotential is deformed as

$$W = \mu_{1,-1} + \sum_{j,j_3} \mu_{j,j_3} M_{j,-j_3}$$

Due to the first term the U(1) symmetry is broken, but the following combinations are preserved:

$$J'_+ := J_+, \quad J'_- := J_- - 2\rho(\sigma_3)$$

An argument shows that for each su(2) rep, only the lowest component survives: **[Gadde-KM-Tachikawa-Yan]**

$$M \to \{M_{j,-j_3(=-j)}\}$$

$$W = \sum_j \mu_{j,j} M_{j,-j}$$

a-maximization

Input data:

1. **the central charges of *T***
2. **flavor central charge of F**
3. **operator spectrum of *T***

With these one can determine the central charges by a-maximization.

Note that this is applied to non-Lagrangian theories!!!

T = SU(2) w/ 4 flavors

<u>**In this case, F = SO(8)**</u>

We consider the principal embedding of SO(8), the vev which breaks SO(8) completely.

The adjoint rep decomposes as

28 → 3, 7, 7, 11

$$M_{1,-1}, M_{3,-3}, M'_{3,-3}, M_{5,-5}$$

→ after integrating out the massive fields,
we get the Lagrangian in the previous slide

$$W = \phi qq + M_1\phi^2 qq' + M_3 qq' + M_5\phi q'q' + M'_3\phi^3 q'q',$$

T = SU(2) w/ 4 flavors

Other choices of embeddings:

- **[5,1³], [4,4]** (SU(2)) → **(A1, A3) theory** (SU(2) flavor sym.)

$$a = \frac{11}{24}, \quad c = \frac{1}{2}$$

- **[3²,1²]** (U(1)xU(1)) → **(A1, D4) theory** (SU(3) flavor sym.)

$$a = \frac{7}{12}, \quad c = \frac{2}{3}$$

- **[5,3]** (no symmetry) → **N=1? or N=2? SCFT**

$$a = \frac{6349}{13872}, \quad c = \frac{3523}{13872}$$

- other embeddings → N=1 SCFTs

T = SU(N) w/ 2N flavors

In this case, **F = SU(2N) x U(1)**

- **[N]** (no sym.) → **(A_1, A_{2N-1}) theory** (U(1) sym.)

$$a = \frac{12N^2 - 5N - 5}{24(N+1)}, \quad c = \frac{3N^2 - N - 1}{6(N+1)}$$

- **[N-1,1]** (U(1)) → **(A_1, D_{2N}) theory** (SU(2)xU(1) sym.)

$$a = \frac{6N - 5}{12}, \quad c = \frac{3N - 2}{6}$$

- others….

T = Sp(N) w/ 4N+4 flavors

In this case, **F = SO(4N+4)**

- **[4N+3,1]** (no sym.) → **(A_1, A_{2N}) theory** (U(1) sym.)

$$a = \frac{N(24N+19)}{24(2N+3)}, \quad c = \frac{N(6N+5)}{6(2N+3)}$$

- **[4N+1,111]** (SO(3)) → **(A_1, D_{2N+1}) theory** (SU(2) sym.)

$$a = \frac{N(8N+3)}{16N+8}, \quad c = \frac{N}{2}$$

- others….

For principal embedding

Theories with the IR N=2 enhancement when T =

- rank-one theories H_1, H_2, D_4, E_6, E_7, E_8 → H_0
- SU(N) SQCD with 2N flavors → (A_1, A_{2N})
- Sp(N) SQCD with 2N+2 flavors → (A_1, A_{2N+1})
- (A_1, D_k) theory [Cecotti-Neitzke-Vafa] → (A_1, A_{k-1})

Theories with no IR N=2 enhancement when T =

- other rank-one theories [Argyres et al.]
- T_N, and $R_{0,N}$ theories of class S [Gaiotto, Chacaltana-Distler]
- N=4 SU(2) SYM theory

What's the pattern of these?

principal embedding: we conjecture that for T to have 2d chiral algebra with the Sugawara construction **[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]** is the condition of the enhancement.

The Sugawara condition is related to the following:

$$\frac{\mathrm{dim}F}{c} = \frac{24h^\vee}{k_F} - 12$$

next-to-principal embedding: we conjecture that for T to have 2d chiral algebra with the Sugawara construction and saturating the flavor central charge bound is the condition of the enhancement.

$$\begin{aligned} k_F &\geq N, \quad SU(N) \\ k_F &\geq N+2, \quad Sp(N) \\ k_F &\geq N-4, \quad SO(N) \end{aligned}$$

Full superconformal index

Superconformal index

Now we had Lagrangian theories which flow to SCFTs in the IR. **Thus the superconformal indices of the latter can be simply computed from the matter content.**

The index of our N=1 theory is defined by

$$I = \mathrm{Tr}_{\mathcal{H}_{S^3}} (-1)^F p^{j_1+j_2-R/2} q^{j_1-j_2-R/2} \xi^{\mathcal{F}}$$

where j_1 and j_2 are rotation generators of the maximal torus $U(1)_1$ and $U(1)_2$ of $SO(4)=SU(2)_1 \times SU(2)_2$ and R and F is the generators of the $U(1)_R$ and $U(1)_{\mathcal{F}}$.

(If S^3 is described by equation $|x_1|^2+|x_2|^2=1$, j_1+j_2 and j_1-j_2 rotate x_1 and x_2 by phase.)

Index of H_0 theory

For instance one could calculate the index of the Argyres-Douglas (H_0) theory from the Lagrangian:

$$I=\kappa\frac{\Gamma((pq)^3\xi^{-6})}{\Gamma((pq)^1\xi^{-2})}\oint\frac{dz}{2\pi iz}\frac{\Gamma(z^{\pm}(pq)^{\frac{1}{4}}\xi^{\frac{1}{2}})\Gamma(z^{\pm}(pq)^{-\frac{5}{4}}\xi^{\frac{7}{2}})\Gamma(z^{\pm2,0}(pq)^{\frac{1}{2}}\xi^{-1})}{\Gamma(z^{\pm2})}$$

$\xi:$ fugacity for $\mathrm{U}(1)_{\mathcal{F}}$

(We subtract the contributions of the decoupled operators!)

We substitute $\xi\to t^{\frac{1}{5}}(pq)^{\frac{3}{10}}$ for the correct IR R symmetry. After that

- basically one can compute the integral
- Coulomb index limit (pq/t=u, p,q,t→0): $I_C=\frac{1}{1-u^{\frac{6}{5}}}$
- Macdonald limit (p→0) agrees with the index by **[Song]**

Questions

- ✪ other type of Argyres-Douglas theories (quiver gauge theories?)
- ✪ What is the condition of UV theory T for the enhancement?
- ✪ Why the enhancement?
- ✪ The IR Coulomb branch comes from M, gauge-singlet in the UV…
- ✪ string/M-theory realization?