Nucleon decay to test GUT models

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Phys.Rev.D88 (2013) 9, and Phys.Lett. B767 (2017) 398-402

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    - ii. Open question for GUT

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    etc.
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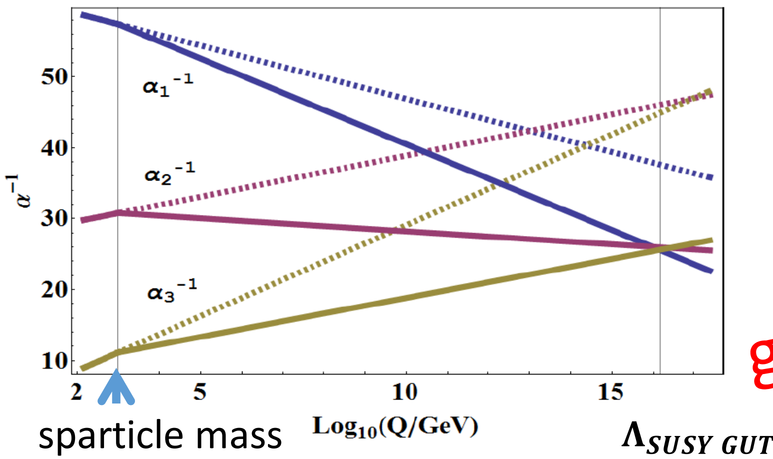
predicts testable nucleon decay

1. Introduction
   i. Advantage of GUT
   ii. Open question for GUT
   iii. Nucleon decay

GUT is marvelous !!!

• Unification of forces(interactions)



gauge coupling unification

⋯⋯ :SM

—— :MSSM

good!!

$\Lambda_{SUSY\ GUT} \sim 2 \times 10^{16}$ GeV

• Unification of particles

Realize simultaneously!!

Unification of SM fermions in $SU(5)$ GUT model

$$\mathbf{10} \to q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1 \qquad \bar{\mathbf{5}} \to d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$$

Unification of SM fermions in minimal $SO(10)$ GUT model

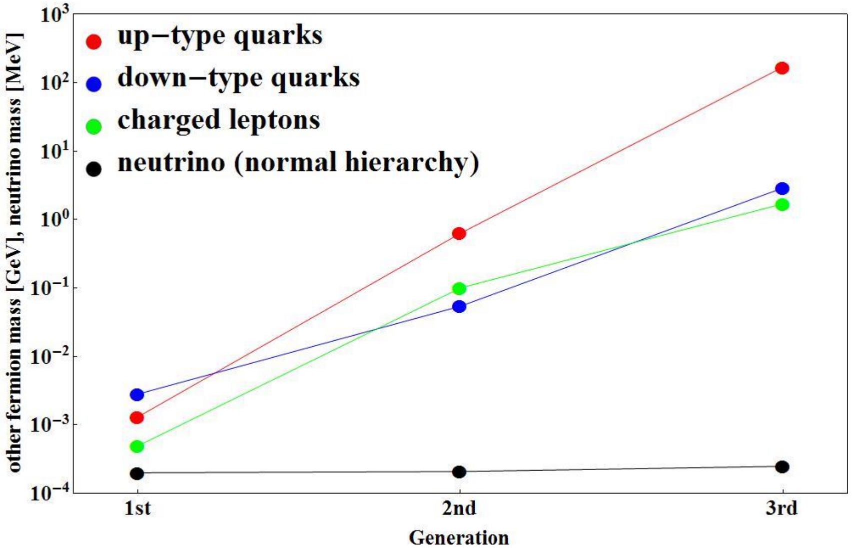
$$\mathbf{16} \to \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$

In gauge theory when gauge interactions are unified, particles (representation of gauge group) are unified simultaneously.

Advantage of particle unification

Unification of SM fermions in $SU(5)$ GUT model

$$\mathbf{10} \rightarrow q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1 \qquad \bar{\mathbf{5}} \rightarrow d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$$



$$|U_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.037 \\ 0.23 & 0.97 & 0.042 \\ 0.0087 & 0.041 & 1.0 \end{pmatrix}$$

$$|U_{MNS}| = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.47 & 0.52 & 0.71 \\ 0.31 & 0.65 & 0.69 \end{pmatrix}$$

mass hierarchy

M_u ➡ strong

M_d, M_e ➡ middle

M_ν ➡ weak

mixing

U_{CKM} ➡ small mixing

U_{MNS} ➡ large mixing

Particle unification explain measured quark and lepton masses and mixings !!! ⬅ explain this later

I explained advantages of GUTs and unification.

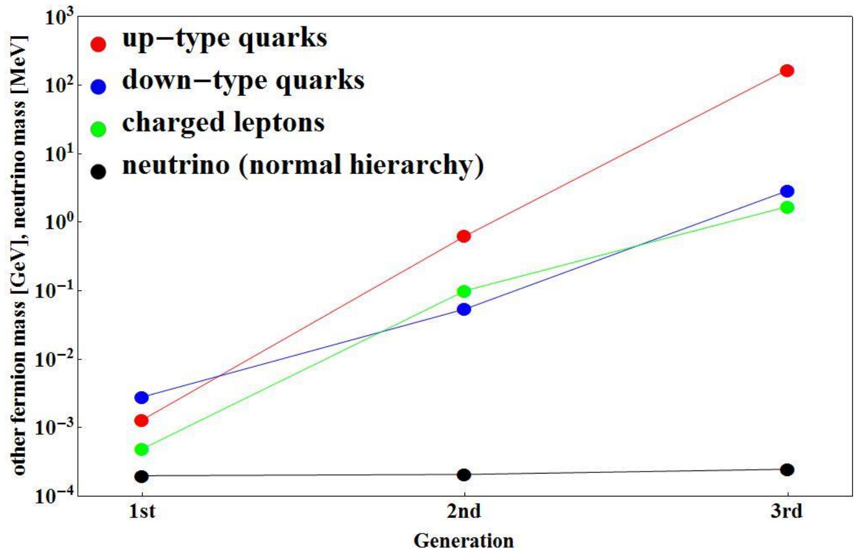
➡ but sometimes unification causes problem.

- Yukawa unification
  - SU(5) unification
  - SO(10) unification ➡ later
- doublet-triplet splitting

show mechanisms which solve these problems and are related to the nucleon decay

GUT short review | Yukawa unification in SU(5) | $+\alpha$

$$\mathbf{10} \to q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1 \qquad \bar{\mathbf{5}} \to d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$$



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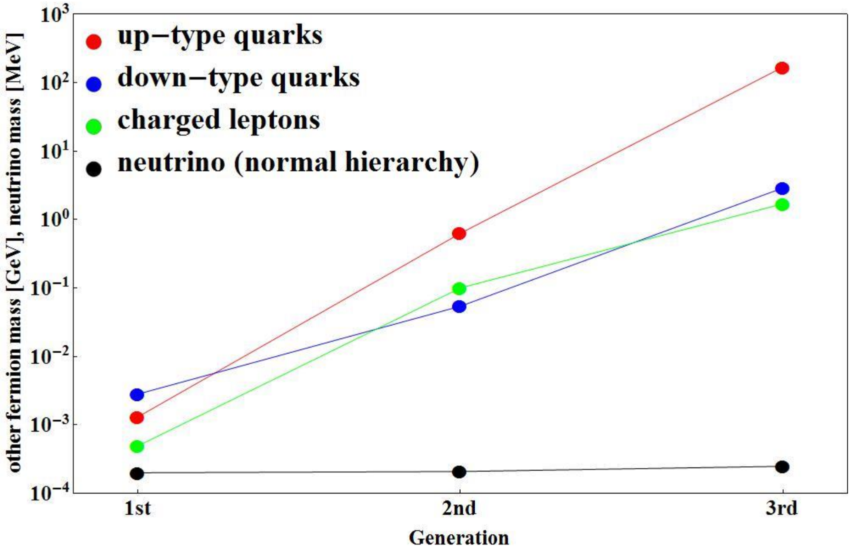
I said that particle unification explains measured quark and lepton masses and mixings, but …

$$Y_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$$

$$\underline{Y = Y_d = Y_e^T} \quad SU(5) \text{ Yukawa relation}$$

This relation is troublesome to explain measured quark and lepton masses and mixing.

Yukawa unification in SU(5)



$$\frac{m_s}{m_d} \sim 20 \neq 200 \sim \frac{m_\mu}{m_e}$$

@M_z scale

$$\frac{m_s}{m_d} = \frac{m_\mu}{m_e}$$

@GUT scale

Does RG eq. effect from GUT scale to M_z scale explain this difference?

$$\frac{m_s}{m_d} = \frac{y_s}{y_d} \qquad \frac{d}{dt} y_a = y_a^3 + \sum_i c_i g_i^2 y_a \sim \sum_i c_i g_i^2 y_a$$

$$\frac{d}{dt}\left(\frac{m_s}{m_d}\right) = \frac{y_d\left(\frac{d}{dt} y_s\right) - y_s\left(\frac{d}{dt} y_d\right)}{y_d^2} = 0$$

No, RG eq. effect does not affect this difference.

non-renormalizable $SU(5)$ GUT

SU(5) breaking VEV

$$Y_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H + \frac{Y'_{ij}}{\Lambda}\mathbf{10}_i\mathbf{24}_H\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H + \cdots = \underline{\left(Y + Y'\frac{\langle\mathbf{24}_H\rangle}{\Lambda}\right)_{ij}}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$$

$\Lambda = \Lambda_{\text{Planck}}$ or smaller

$$Y_d \neq Y_e^T$$

$$\frac{\langle\mathbf{24}_H\rangle}{\Lambda} \geq 10^{-3} \quad \Longleftrightarrow \quad \frac{m_u}{m_t} \sim 10^{-5}$$

non-renormalizable terms affect first- and second- generation masses.

→ solve problem at the GUT scale
Yukawa matrices get new degree of freedom

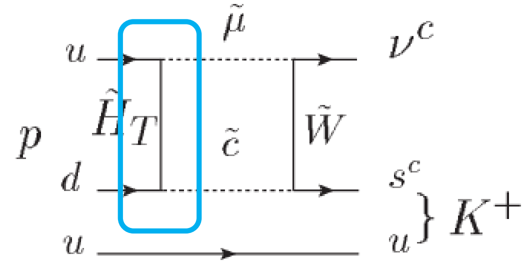
$$\mathbf{5}_H \to H_D(\mathbf{1},\mathbf{2})_{\frac{1}{2}} + H_T(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$$

SM Higgs doublet

$$M_{H_D} \sim 125\ \mathrm{GeV} \sim \Lambda_{EW}$$

Triplet (colored)

to suppress nucleon decay via dimension 5 and 6 operators



$$M_{H_T} \gg \Lambda_{EW}$$

How to realize this mass splitting?
Doublet-triplet splitting problem

DW mechanism realize it!

In $SO(10)$ GUT model

SM Higgs

$SO(10)$ adjoint Higgs

$$\mathbf{10} : \mathrm{H}, \mathrm{H}' \quad \mathbf{45} : \mathrm{A}$$

$$\mathrm{W} = cHAH' + c'\Lambda H'H'$$

$$\mathbf{10} \to \mathbf{5} + \overline{\mathbf{5}}\ (SO(10) \to SU(5))$$

DW form VEV

$$\langle \mathbf{45}_A \rangle = i\sigma_2 \times \begin{pmatrix} x & & & & \\ & x & & & \\ & & x & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

Dimopoulos, Wilczek (1982)

mass term for Higgs $\mathbf{10}$

$$(\mathbf{5}_H \quad \mathbf{5}_{H'})M_{10}\begin{pmatrix}\bar{\mathbf{5}}_H \\ \bar{\mathbf{5}}_{H'}\end{pmatrix}$$

M_{10} : mass matrix for Higgs $\mathbf{10}$

$$M_{10} = \begin{pmatrix} 0 & c\langle \mathbf{45}_A \rangle \\ c\langle \mathbf{45}_A \rangle & c'\Lambda \end{pmatrix}$$

doublet Higgs mass matrix M_D

$$M_D = \begin{pmatrix} 0 & 0 \\ 0 & c'\Lambda \end{pmatrix}$$

⇒
- one massless mode

  =SM doublet Higgs
- one massive mode

triplet Higgs mass matrix M_T

$$M_T = \begin{pmatrix} 0 & cx \\ cx & c'\Lambda \end{pmatrix}$$

⇒
- two massive modes

We can realize DT splitting through DW mechanism

…but, HH term and HH' term spoil this mechanism.

mass term for Higgs $\mathbf{10}$

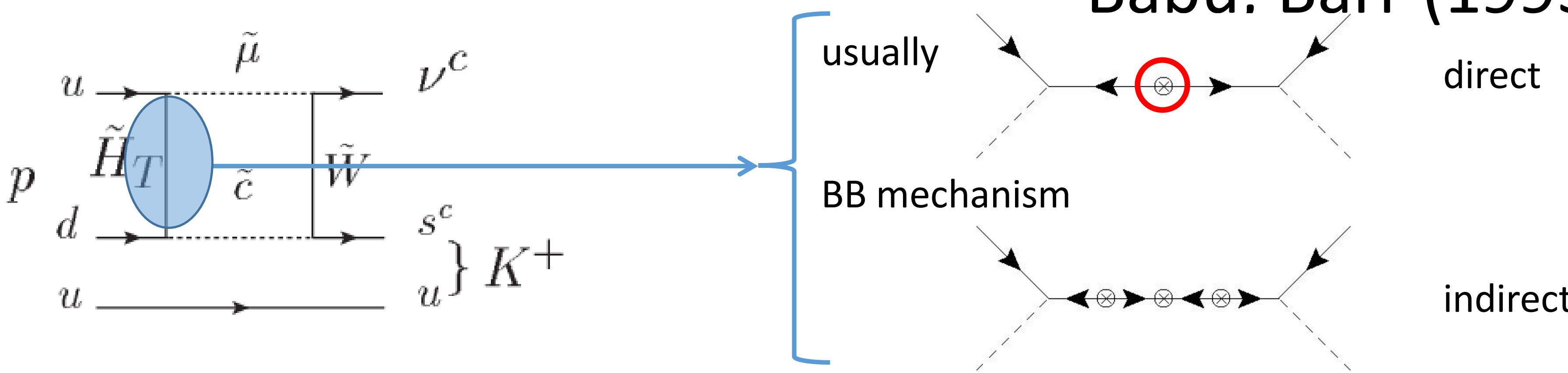
$$(\mathbf{5}_H \quad \mathbf{5}_{H'}) M_{10} \begin{pmatrix} \bar{\mathbf{5}}_H \\ \bar{\mathbf{5}}_{H'} \end{pmatrix}$$

M_{10} : mass matrix for Higgs $\mathbf{10}$

$$M_{10} = \begin{pmatrix} 0 & c\langle \mathbf{45}_A \rangle \\ c\langle \mathbf{45}_A \rangle & c'\Lambda \end{pmatrix}$$

suppress nucleon decay via dimension 5 operators

Babu. Barr (1993)



suppress dangerous nucleon decay process

Not Even Decoupling Can Save Minimal Supersymmetric SU(5)

Murayama, Pierce (2002)

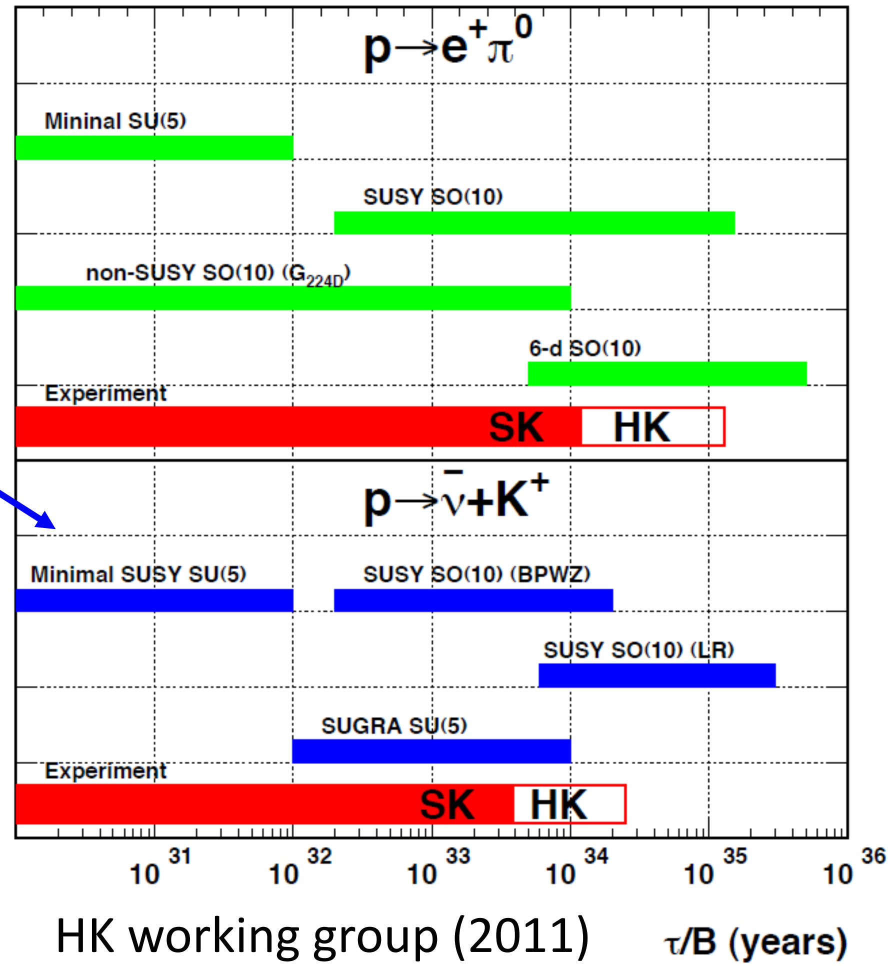
additional particles in minimal SU(5) model

X-type gauge boson : X

colored triplet Higgs : H_T

adjoint Higgs : Σ

gauge coupling unification



HK working group (2011)

Decoupling Can Revive Minimal Supersymmetric SU(5)
Hisano, Kobayashi, Kuwahara, Nagata (2013)

SUSY threshold correction in split SUSY model the "minimal" model

- minimal multiplets to satisfy the SM particle contents

↕

- minimal multiplets to satisfy the SM

GUT breaking effect

- to realize measured fermion masses and mixings
- solve a degeneracy of adjoint Higgs mass

⇒ new degree of freedom

Open question for GUT

Open questions for GUT models which realize SM

What kind of mechanism is used in the theory which describes our nature?

- Which unification group is used?

  $SU(5), SO(10)$ or E_6?

- How to break unification group?

  Where are unification group breaking scales?

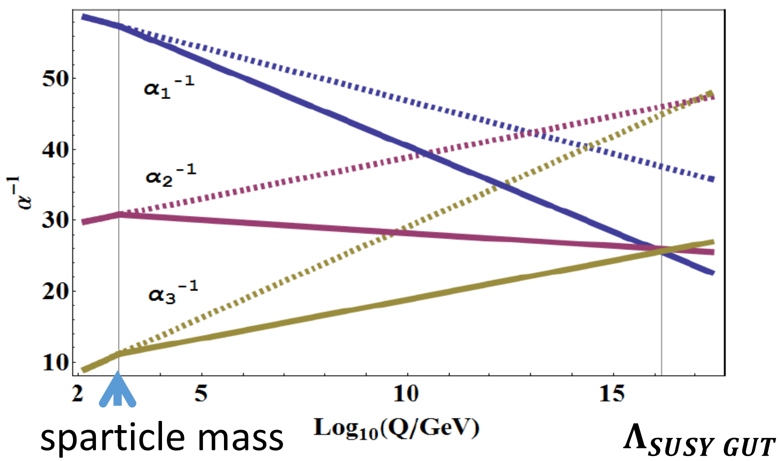
- Which diagonalizing matrix is large mixing matrix?

  How to realize measured CKM and MNS matrix?

etc.

In this talk I focus on relation between above three questions and nucleon decay.

To answer these questions we have to observe phenomena which come from GUT contribution.



$\underline{\Lambda_{SUSY\ GUT} \sim 2 \times 10^{16}\ \text{GeV}}$

$\gg$

$\underline{\Lambda_{LHC} \sim 10^{3-4}\ \text{GeV}}$

~~Direct detection~~

Another candidate?

Nucleon decay

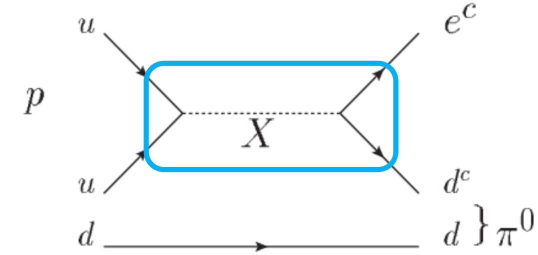
Limit from Super-Kamiokande

$\tau_{p\to\pi^0 e^+} \geq 1.4\times 10^{34}$ years $\Rightarrow$ $\Lambda_G \geq 10^{16}$ GeV

two operators which induce baryon number violation

- dimension 6 operators

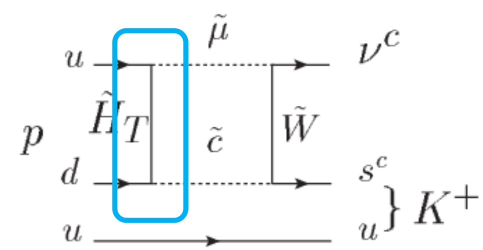
: X-type (colored doublet) gauge boson exchange



main decay mode $P \to \pi^0 e^+$

- dimension 5 operators : triplet Higgsino exchange

  SUSY contribution main decay mode $P \to K^+ \bar{\nu}$



$$\mathbf{5}_H \to \underbrace{H_D(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\text{SM Higgs doublet}} + H_T(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$$

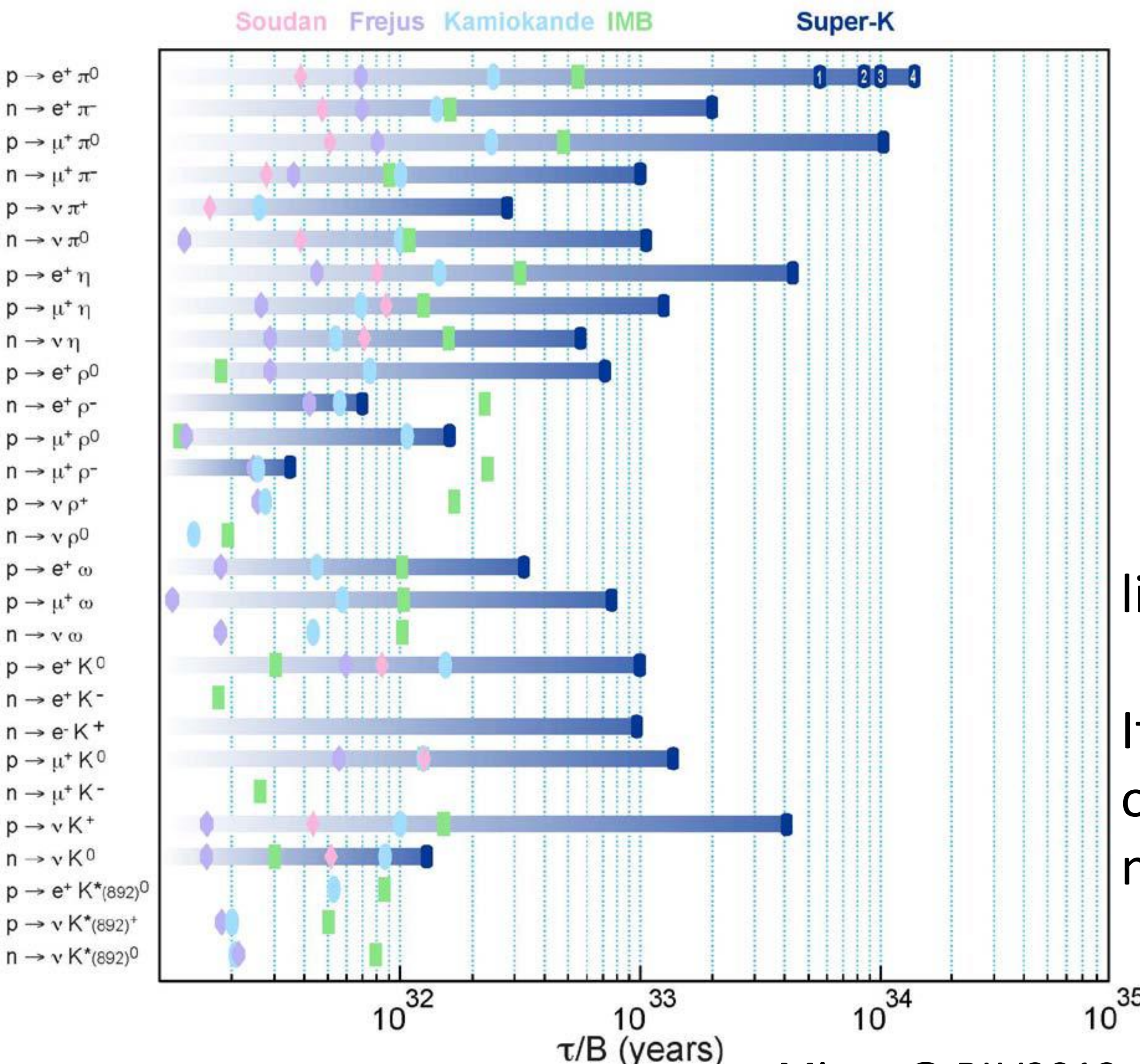
two uncertainties

- mechanism to realize doublet-triplet splitting
- SUSY particle contribution

$\Rightarrow$ not useful to test models and <u>able to suppress strongly</u>

$\Rightarrow$ I focus on nucleon decay via dimension 6 operator

Limit for nucleon decay

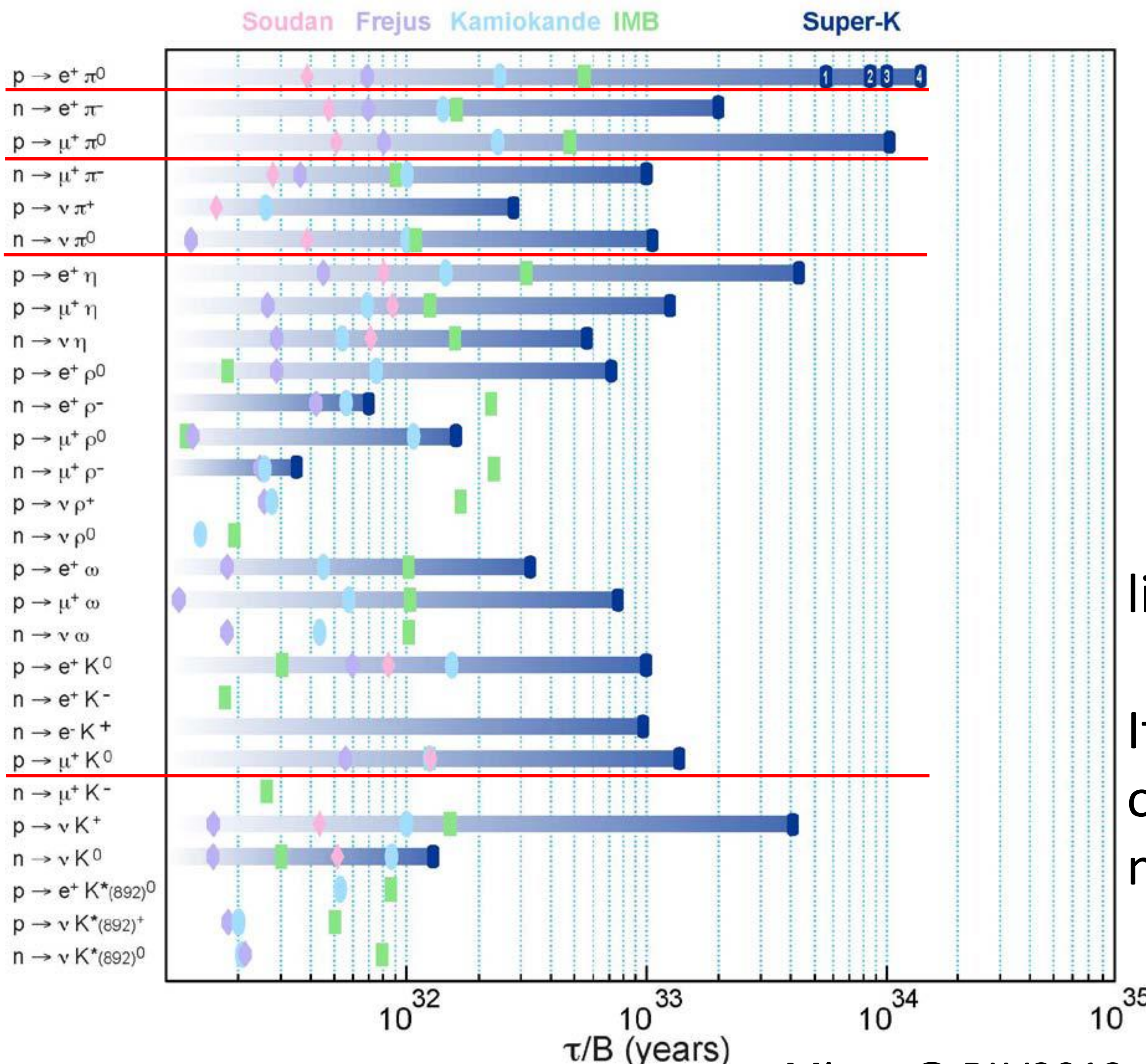


Miura @ BLV2013

limit for many decay modes

It is possible to observe not only proton decay but also neutron decay.

Limit for nucleon decay



Miura @ BLV2013

limit for many decay modes

It is possible to observe not only proton decay but also neutron decay.

Three important ratios

$$R_1 \equiv \frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\Gamma_{P\to\pi^0 e^+}} \qquad R_2 \equiv \frac{\Gamma_{P\to K^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}} \qquad R_3 \equiv \frac{\Gamma_{P\to\pi^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

test unification group

well known parameter to test unification group
Wilczek and Zee (1979)

diagonalizing matrix

R_1 becomes larger as rank of unification group becomes larger.

$$\begin{aligned}
\mathcal{L}_{eff} &= \frac{g_{GUT}^2}{M_{X_{SU(5)}}^2}\{(\overline{e^c_{R_i}}u_{Rj})(\overline{u^c_{L_j}}d_{Li}) + (\overline{e^c_{R_i}}u_{Rj})(\overline{u^c_{L_i}}d_{Lj}) \\
&+ (\overline{e^c_{L_i}}u_{Lj})(\overline{u^c_{R_j}}d_{Ri}) + (\overline{E^c_{L_i}}u_{Lj})(\overline{u^c_{R_j}}D_{Ri}) \\
&- (\overline{\nu^c_{L_i}}d_{Lj})(\overline{u^c_{R_j}}d_{Ri}) - (\overline{N^c_{L_i}}d_{Lj})(\overline{u^c_{R_j}}D_{Ri})\} \\
&+ \frac{g_{GUT}^2}{M_{X_{SO(10)}}^2}\{(\overline{e^c_{L_i}}u_{Lj})(\overline{u^c_{R_i}}d_{Rj}) - (\overline{\nu^c_{L_i}}d_{Lj})(\overline{u^c_{R_i}}d_{Rj})\} \\
&+ \frac{g_{GUT}^2}{M_{X_{E_6}}^2}\{(\overline{E^c_{L_i}}u_{Lj})(\overline{u^c_{R_i}}D_{Rj}) - (\overline{N^c_{L_i}}d_{Lj})(\overline{u^c_{R_i}}D_{Rj})\}
\end{aligned}$$

$SU(5)$ GUT

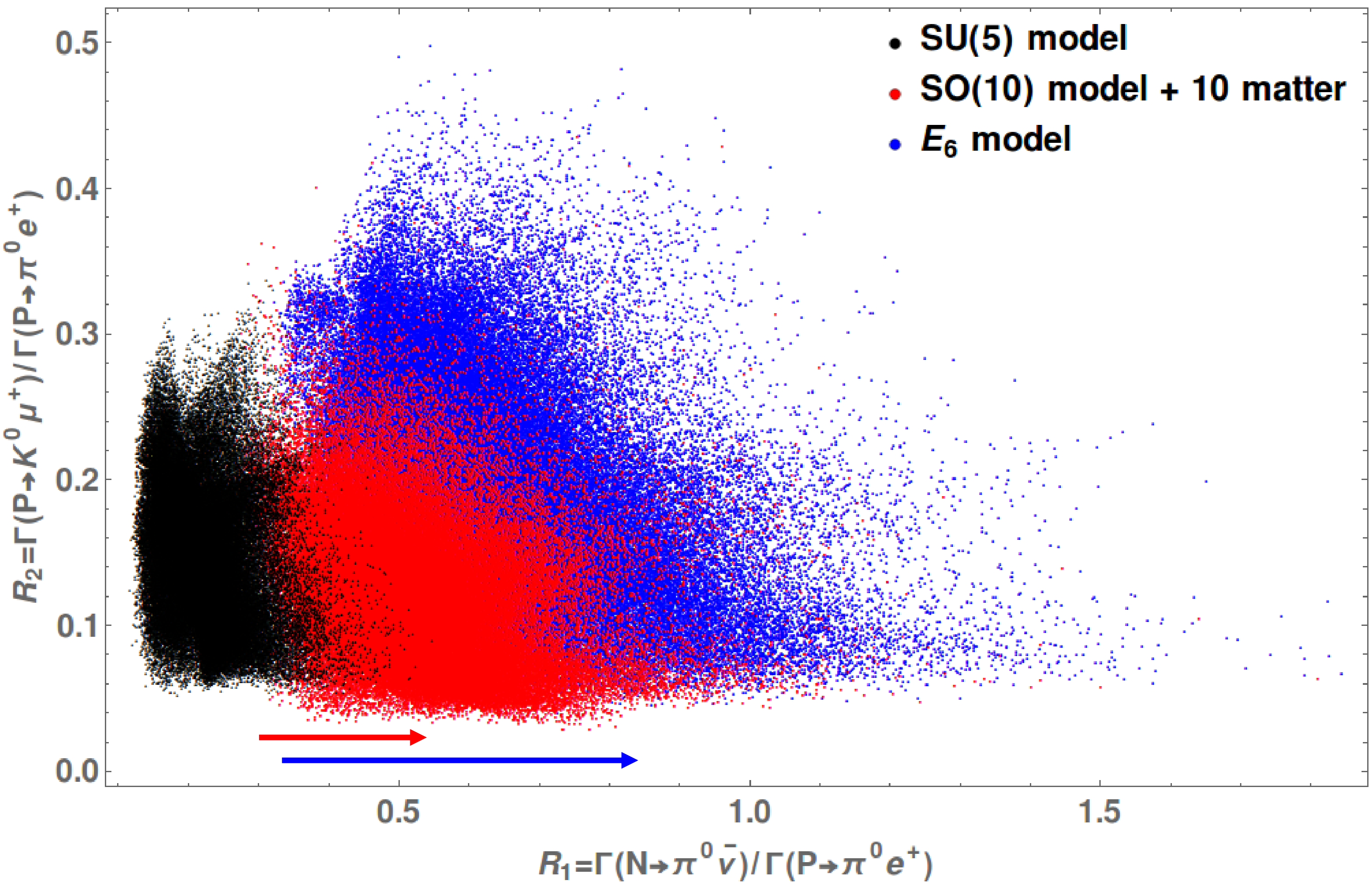
$SO(10)$ GUT

E_6 GUT

: include anti electron

: include anti neutrino

Result



R_1 becomes larger as rank of unification group becomes larger. ➡ useful to test unification group

Results of p→$\mu^+\pi^0$

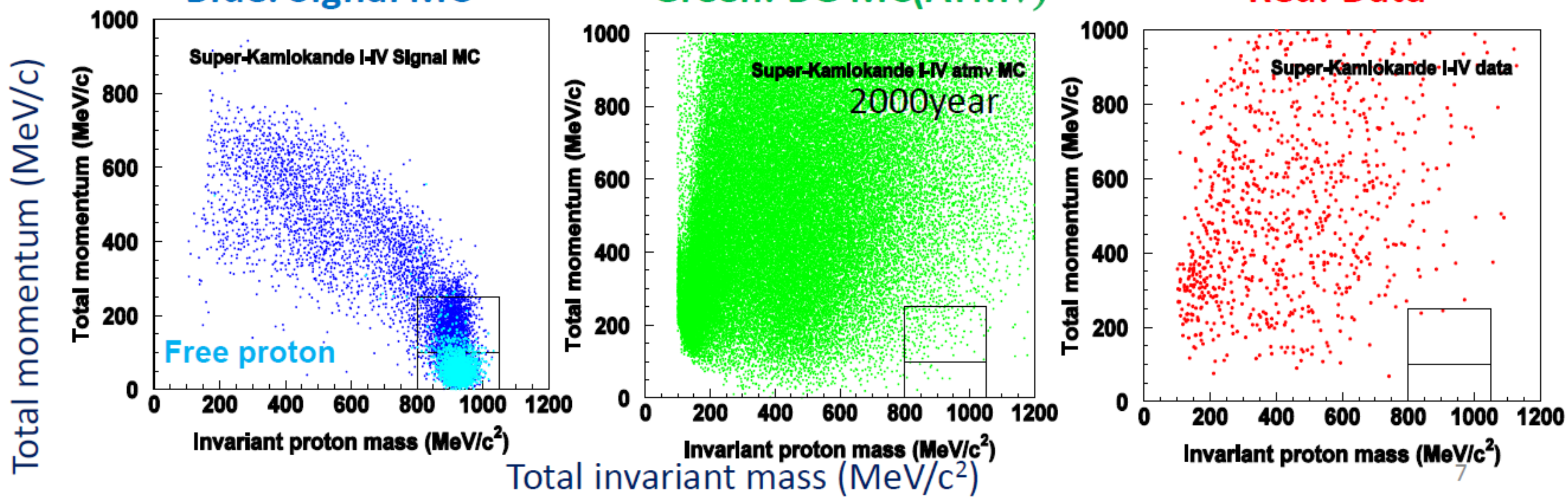
M. Ikeda @ NNN2015

(analysis proceeds as with $e^+\pi^0$ with additional requirement of 1 Michel-e)

- 306.3 kton·yrs (SKI-IV) (220kt·yrs in PRD)
- signal $\varepsilon(P_{tot}<250\text{MeV/c})$: 30-40%
- total expected #BKG:
  - $P_{tot}<100$: ~0.05, $100\leq P_{tot}<250$: ~0.82
- no significant data excess

$$\tau/B_{p\to\mu\pi 0} > 7.78\times 10^{33}\ \text{years (90\% CL)}$$

Blue: Signal MC   Green: BG MC(ATMν)   Red: Data



Results of $p \rightarrow \mu^+\pi^0$

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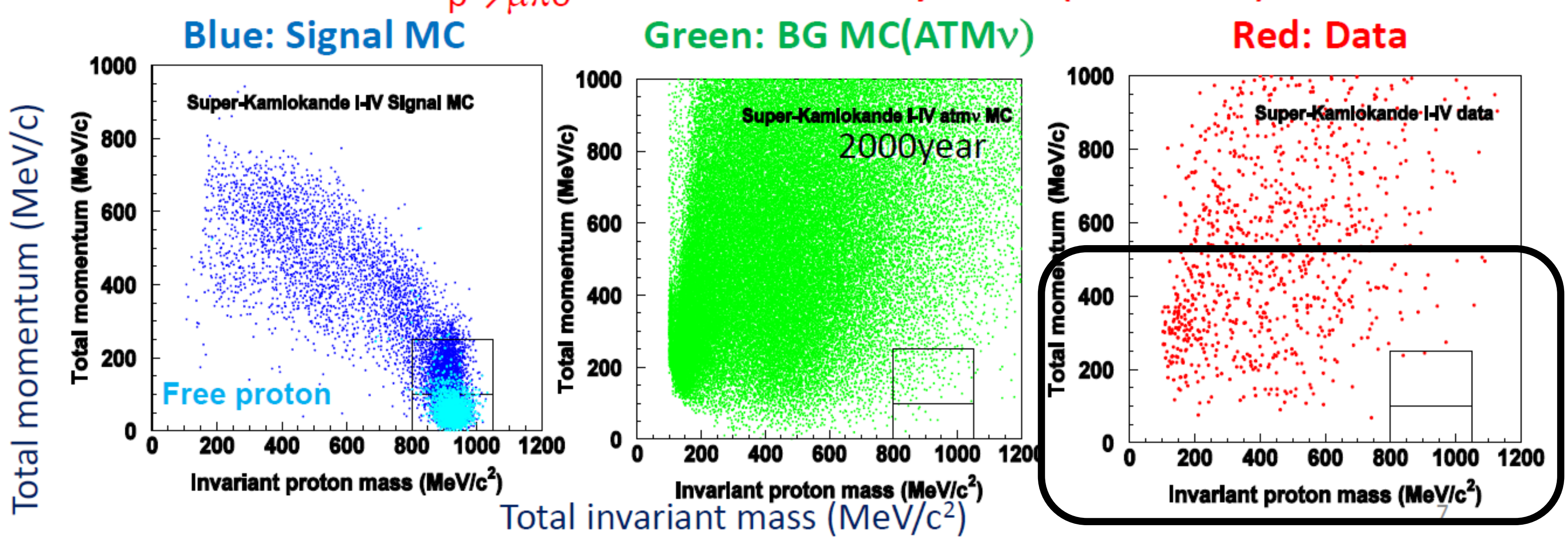
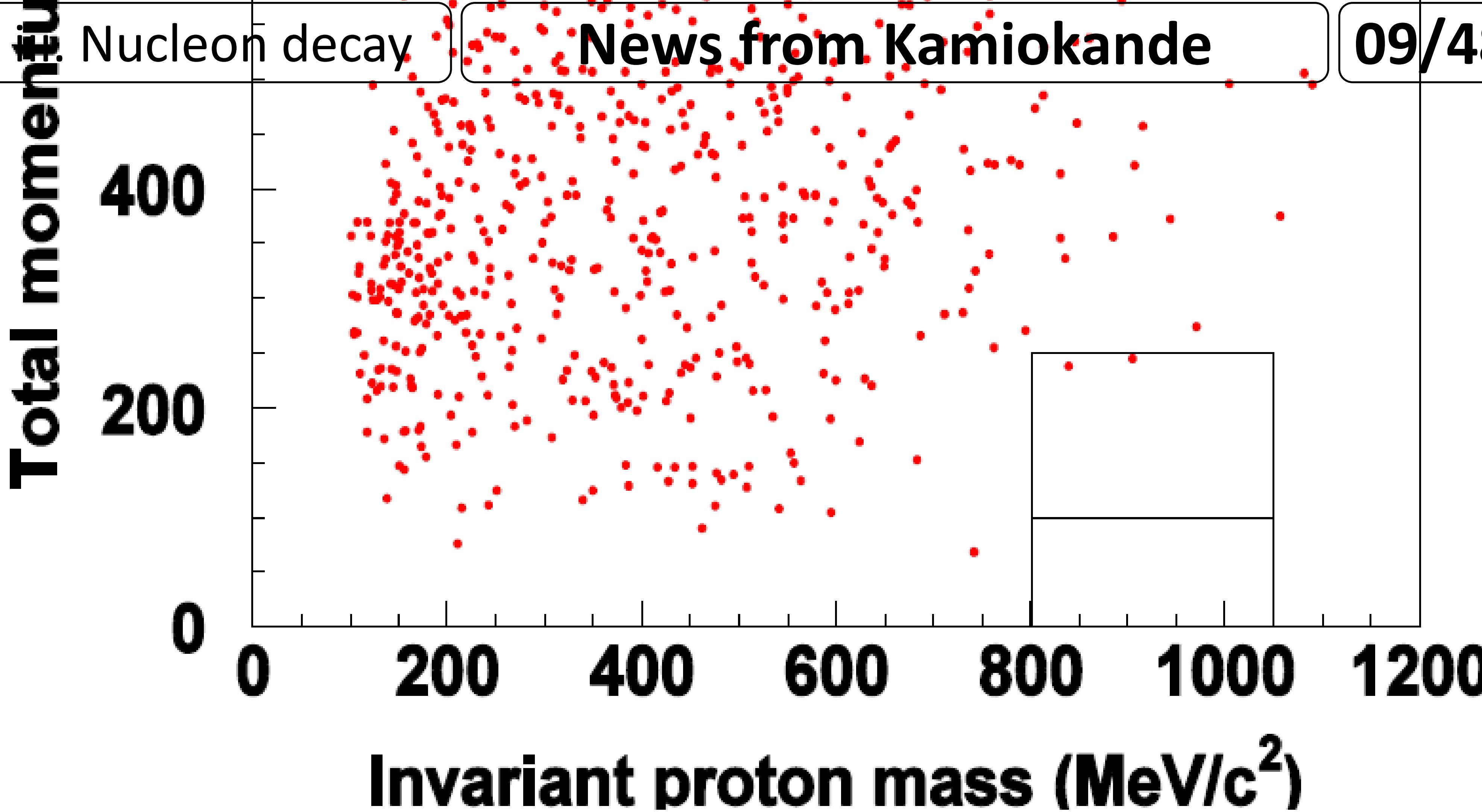




$P \to \pi^0 e^+$ decay mode

Total #BKG (SK I-IV)	0.61	0.87
Data (SK I-IV)	0	**2**

$P \to \pi^0 \mu^+$ decay mode

$\Gamma_{P\to\pi^0\mu^+} > \Gamma_{P\to\pi^0 e^+}$?

2 events !!!

Review of Nucleon Decay Searches at Super-Kamiokande

Volodymyr Takhistov arXiv:1605.03235

3.1 $p \to e^+\pi^0$ *and* $p \to \mu^+\pi^0$

The $p \to e^+\pi^0$ channel is often the most dominant nucleon decay mode in GUTs, with typical lifetime predictions of 10^{29-36} yrs. Previous searches for this channel have already excluded minimal $SU(5)$ [17, 18, 19, 20]. Within some models (e.g. flipped $SU(5)$ [21]), a similar channel, $p \to \mu^+\pi^0$, can also appear with a significant branching ratio.

Since e^+, $\mu^+(\to e^+\nu\nu)$ as well as $\pi^0(\to \gamma\gamma)$ produce visible Cherenkov rings, one can fully reconstruct the invariant mass and momentum of the parent proton. Figure 1 displays the signal MC, background MC and data (306 kiloton·yrs of exposure), after all the event selection criteria have been applied. The signal region consists of two portions, a "lower box" (free protons) and an "upper box" (bound protons), separated in the analysis for improved sensitivity. For $p \to e^+\pi^0$, the average signal efficiency as well as the total expected background within the selected region is 38.7% and 0.61 events, respectively. For $p \to \mu^+\pi^0$, it is 34.6% and 0.87 events, respectively. No data events pass the selection for $p \to e^+\pi^0$, while two events pass for $p \to \mu^+\pi^0$. The Poisson probability of observing two such events for a given exposure is 23%. Since both events also display background-like features, they are judged as coming from atmospheric-ν background. Hence, the 90% confidence level (C.L.) lower lifetime limits of 1.7×10^{34} yrs. and 7.8×10^{33} yrs. are placed on the $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ channels [22], respectively.

If we observe $P \to \pi^0\mu^+$ decay mode earlier than $P \to \pi^0 e^+$ decay mode

$$R_3 > 1$$

$$R_3 \equiv \frac{\Gamma_{P\to\pi^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

large mixing is favored

hint for diagonalizing matrices

hint for flavor physics in bSM

$P \to \pi^0\mu^+$ decay mode is very important.

gauge interaction induces nucleon decay via dimension 6 operators

weak interaction

- gauge coupling : g
- weak boson mass : M_W
- CKM and MNS matrix :

  U_{CKM} and U_{MNS}

⟺

new gauge interaction

- unified gauge coupling : g_{GUT}

  from gauge coupling unification
- X-type boson mass :

  $M_{SU(5)}$, $M_{SO(10)}$ and M_{E_6}
- diagonalizing matrix :

  L_ψ and R_ψ

2. Nucleon decay

i. Key for model test

ii. Diagonalizing matrix

iii. Symmetry breaking

Quark and lepton unification

$$E_6 \supset SO(10) \supset SU(5)$$

$$\mathbf{27} \to \mathbf{16} + \mathbf{10} + \mathbf{1} \qquad \times \text{three generations}$$

$$\mathbf{16} \to \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$

$$\mathbf{10} \to \underbrace{D_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}'} + \underbrace{\overline{D_R^c}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + \overline{L_L}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}$$

superheavy

six $\bar{\mathbf{5}}$s $\begin{cases} \text{three SM } \bar{\mathbf{5}}\text{s} \\ \text{three superheavy } \bar{\mathbf{5}}\text{s} \end{cases}$ explain this later

key: SM particles belong to $\mathbf{10}$ and $\bar{\mathbf{5}}$ of $SU(5)$ group
two $\bar{\mathbf{5}}$ matter ($\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}'$)

Gauge particle unification

$$E_6 \supset SO(10) \supset SU(5)$$

$$\mathbf{78} \to \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1}$$

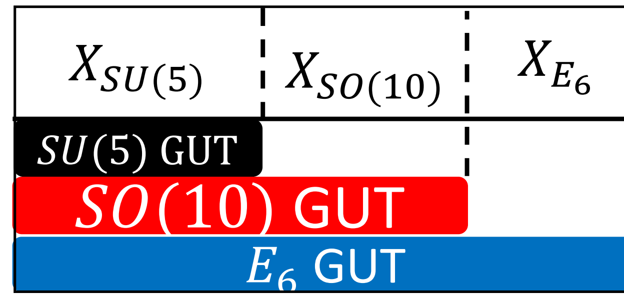
$$\mathbf{45} \to \underbrace{G(\mathbf{8},\mathbf{1})_0 + W(\mathbf{1},\mathbf{3})_0 + \overline{X_{SU(5)}}(\mathbf{3},\mathbf{2})_{-\frac{5}{6}} + X_{SU(5)}(\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}} + N^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{24}}$$

$$+ \underbrace{X_{SO(10)}(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + U'^c_R(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + E'^c_R(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}}$$

$$+ \underbrace{\overline{X_{SO(10)}}(\bar{\mathbf{3}},\mathbf{2})_{-\frac{1}{6}} + \overline{U'^c_R}(\mathbf{3},\mathbf{1})_{\frac{2}{3}} + \overline{E'^c_R}(\mathbf{1},\mathbf{1})_{-1}}_{\overline{\mathbf{10}}} + \underbrace{N'^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$

X_{E_6}

$$\mathbf{16} \to \underbrace{\boxed{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}}} + u^c_R(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e^c_R(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d^c_R(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu^c_R(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$



key: X-type gauge bosons belong to **24** (adjoint representation) and **10** of $SU(5)$ group

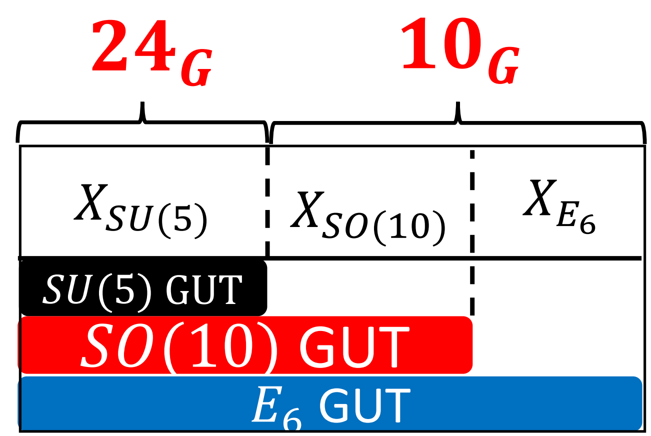
Gauge interaction

gauge interactions in E_6 models : $\mathbf{27}^\dagger \cdot \mathbf{78}_G \cdot \mathbf{27} \rightarrow$

X-type gauge interactions which induce nucleon decay

$SO(10)$ decomposition	$SU(5)$ decomposition
$\mathbf{16}^\dagger \cdot \mathbf{45}_G \cdot \mathbf{16}$	$\mathbf{10}^\dagger \cdot \mathbf{24}_G \cdot \mathbf{10}$
	$\overline{\mathbf{5}}^\dagger \cdot \mathbf{24}_G \cdot \overline{\mathbf{5}}$
	$\overline{\mathbf{5}}^\dagger \cdot \mathbf{10}_G \cdot \mathbf{10}$
$\mathbf{10}^\dagger \cdot \mathbf{45}_G \cdot \mathbf{10}$	$\overline{\mathbf{5}}'^\dagger \cdot \mathbf{24}_G \cdot \overline{\mathbf{5}}'$
$\mathbf{10}^\dagger \cdot \mathbf{16}_G \cdot \mathbf{16}$	$\overline{\mathbf{5}}'^\dagger \cdot \mathbf{10}_G \cdot \mathbf{10}$

key: $\mathbf{24}_G$ couples $\overline{\mathbf{5}}$ pair or $\mathbf{10}$ pair
$\mathbf{10}_G$ couples $\overline{\mathbf{5}}$ with $\mathbf{10}$



Dimension 6 operators

Dimension 6 operators which exchange $X_{SU(5)}$

$SO(10)$ decomposition	$SU(5)$ decomposition
$\mathbf{16}^\dagger \cdot \mathbf{45}_G \cdot \mathbf{16}$	$\mathbf{10}^\dagger \cdot \mathbf{24}_G \cdot \mathbf{10}$ $\overline{\mathbf{5}}^\dagger \cdot \mathbf{24}_G \cdot \overline{\mathbf{5}}$
$\mathbf{10}^\dagger \cdot \mathbf{45}_G \cdot \mathbf{10}$	$\overline{\mathbf{5}}'^\dagger \cdot \mathbf{24}_G \cdot \overline{\mathbf{5}}'$

$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \mathbf{10}^\dagger \cdot \mathbf{10}$$
$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \overline{\mathbf{5}}^\dagger \cdot \overline{\mathbf{5}}$$
$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \overline{\mathbf{5}}'^\dagger \cdot \overline{\mathbf{5}}'$$

Dimension 6 operators which exchange $X_{SO(10)}$

$SO(10)$ decomposition	$SU(5)$ decomposition
$\mathbf{16}^\dagger \cdot \mathbf{45}_G \cdot \mathbf{16}$	$\overline{\mathbf{5}}^\dagger \cdot \mathbf{10}_G \cdot \mathbf{10}$

$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \overline{\mathbf{5}}^\dagger \cdot \overline{\mathbf{5}}$$

key:

always include $\overline{\mathbf{5}}$

Dimension 6 operators which exchange X_{E_6}

$SO(10)$ decomposition	$SU(5)$ decomposition
$\mathbf{10}^\dagger \cdot \mathbf{16}_G \cdot \mathbf{16}$	$\overline{\mathbf{5}}'^\dagger \cdot \mathbf{10}_G \cdot \mathbf{10}$

$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \overline{\mathbf{5}}'^\dagger \cdot \overline{\mathbf{5}}'$$

Key for model test

$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \mathbf{10}^\dagger \cdot \mathbf{10}$$

only $SU(5)$ model

$$\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \bar{\mathbf{5}}^\dagger \cdot \bar{\mathbf{5}}$$

$SU(5)$ model
and
added operator
in $SO(10)$ and E_6 model

Key for test GUT model is

contribution from $\mathbf{10}^\dagger \cdot \mathbf{10} \cdot \bar{\mathbf{5}}^\dagger \cdot \bar{\mathbf{5}}$ operator

Diagonalizing matrix

Yukawa matrix diagonalization

$$\psi_{L\,i}\,(Y_\psi)_{ij}\,\psi^c_R = \left(L^\dagger_\psi \psi_L\right)_i \left(L^T_\psi Y_\psi R_\psi\right)_{ij} \left(R^\dagger_\psi \psi^c_R\right)_j$$

$$= \psi'_{L\,i}\,(Y^D_\psi)_{ij}\,\psi'^c_{R\,j}$$

$\Rightarrow$ diagonalizing matrix L_ψ, R_ψ

condition for 7 diagonalizing matrices

$\lambda = 0.22$

$$U_{CKM} = L^\dagger_u L_d \sim U_{CKM-type}$$

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

small mixing matrix

$$U_{MNS} = L^\dagger_\nu L_e \sim U_{MNS-type}$$

$$U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

large mixing matrix

It is impossible to fix all diagonalizing matrices.

We consider GUT model test with uncertainty of these diagonalizing matrices.

Diagonalizing matrix in GUT model

In $SU(5)$ GUT model

$$\mathbf{10} \to q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1$$

To realize CKM matrix diagonalizing matrix for $\mathbf{10}$ matter is small mixing

$$\bar{\mathbf{5}} \to d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$$

To realize MNS matrix diagonalizing matrix for $\bar{\mathbf{5}}$ matter is large mixing

In minimal $SO(10)$ GUT model

$$\mathbf{16} \to \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$

To realize CKM matrix, all diagonalizing matrices except neutrino diagonalizing matrix are small mixing

Through new degree of freedom from seesaw mechanism, only diagonalizing matrix for left-handed neutrino (~light neutrino) is large mixing

Diagonalizing matrix in GUT model

In $SO(10)$ GUT + 10 matter model

$$\mathbf{16} \to \underbrace{q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\nu_R^c(\mathbf{1},\mathbf{1})_0}_{\mathbf{1}}$$

+

$$\mathbf{10} \to \underbrace{D_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}'} + \underbrace{\overline{D_R^c}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + \overline{L_L}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}$$

$$\begin{pmatrix} \bar{\mathbf{5}} \\ \bar{\mathbf{5}}' \end{pmatrix} = U \begin{pmatrix} \bar{\mathbf{5}}^0 \\ \bar{\mathbf{5}}^h \end{pmatrix}$$

SM matters $\begin{pmatrix} \bar{\mathbf{5}}_1^0 & \bar{\mathbf{5}}_2^0 & \bar{\mathbf{5}}_3^0 \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{5}}_1 & \bar{\mathbf{5}}'_1 & \bar{\mathbf{5}}_2 \end{pmatrix}$

superheavy particles

To realize observed quark and lepton masses and mixings

U: 6×6 mixing matrix

Through this mixing diagonalizing matrix for $\bar{\mathbf{5}}^0$ matter can be large mixing

⇒ Diagonalizing matrix for $\mathbf{10}$ matter is small mixing
Diagonalizing matrix for $\bar{\mathbf{5}}^0$ matter is large mixing

Diagonalizing matrix in GUT model

E_6 GUT model

$$\mathbf{27} \to \mathbf{16} + \mathbf{10} + \mathbf{1}$$

E_6 fundamental representation includes added $\mathbf{10}$

$$\begin{pmatrix} \bar{\mathbf{5}} \\ \bar{\mathbf{5}}' \end{pmatrix} = U \begin{pmatrix} \bar{\mathbf{5}}^0 \\ \bar{\mathbf{5}}^h \end{pmatrix}$$

SM matters $\begin{pmatrix} \bar{\mathbf{5}}^0_1 & \bar{\mathbf{5}}^0_2 & \bar{\mathbf{5}}^0_3 \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{5}}_1 & \bar{\mathbf{5}}'_1 & \bar{\mathbf{5}}_2 \end{pmatrix}$

superheavy particles

6×6 mixing matrix

Dimension 6 operators which exchange X_{E_6}

$SO(10)$ decomposition	$SU(5)$ decomposition
$\mathbf{10}^\dagger \cdot \mathbf{16}_G \cdot \mathbf{16}$	$\bar{\mathbf{5}}'^\dagger \cdot \mathbf{10}_G \cdot \mathbf{10}$

$$\Rightarrow \quad \mathbf{10}^\dagger \cdot \mathbf{10} \cdot \bar{\mathbf{5}}'^\dagger \cdot \bar{\mathbf{5}}'$$

Added operator in E_6 GUT model includes $\bar{\mathbf{5}}'$

$SU(5)$ model
$SO(10)$ model+$\mathbf{10}$ matter
E_6 model

→ small mixing for $\mathbf{10}$ matter
large mixing for $\overline{\mathbf{5}}$ matter

---

minimal $SO(10)$ model → all diagonalizing matrix small mixing

---

small mixing matrix $\lambda = 0.22$

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.22 & 0.011 \\ 0.22 & 1 & 0.048 \\ 0.011 & 0.048 & 1 \end{pmatrix} \Leftrightarrow |U_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.037 \\ 0.23 & 0.97 & 0.042 \\ 0.0087 & 0.041 & 1.0 \end{pmatrix}$$

large mixing matrix

$$U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.47 & 0.22 \\ 0.47 & 1 & 0.47 \\ 0.22 & 0.47 & 1 \end{pmatrix} \Leftrightarrow |U_{MNS}| = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.47 & 0.52 & 0.71 \\ 0.31 & 0.65 & 0.69 \end{pmatrix}$$

we consider $O(1)$ uncertainty (0.5-2.0)

$$E_6 \underset{\langle\Phi\rangle}{\rightarrow} SO(10) \underset{\langle C\rangle}{\rightarrow} SU(5) \underset{\langle A\rangle}{\rightarrow} G_{SM}\ ?$$

unification scale

$$\mathcal{L}_{\rm eff} = -\frac{2\pi\alpha_{GUT}}{M^2_{X_{SU(5)}}}\left(\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\mathbf{10}_j^\dagger\cdot\mathbf{10}_j + \mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}_j^\dagger\cdot\bar{\mathbf{5}}_j + \mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}'^\dagger_j\cdot\bar{\mathbf{5}}'_j\right)$$
$$-\frac{2\pi\alpha_{GUT}}{M^2_{X_{SO(10)}}}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}_i^\dagger\cdot\bar{\mathbf{5}}_j - \frac{2\pi\alpha_{GUT}}{M^2_{X_{E_6}}}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}'^\dagger_i\cdot\bar{\mathbf{5}}'_j$$

- $\langle\Phi\rangle, \langle C\rangle \gg \langle A\rangle \quad \sim \quad M_{X_{E_6}}, M_{X_{SO(10)}} \gg M_{X_{SU(5)}}$

  all models become $SU(5)$ like model
- $\langle\Phi\rangle \sim \langle C\rangle \sim \langle A\rangle \quad \sim \quad M_{X_{E_6}}, M_{X_{SO(10)}} \geq M_{X_{SU(5)}}$

  small difference
- $\langle\Phi\rangle \sim \langle C\rangle < \langle A\rangle \quad \sim \quad M_{X_{E_6}}, M_{X_{SO(10)}} \leq M_{X_{SU(5)}}$

  large difference

easy to test

but because there are intermediation scales before unification, there is a possibility that gauge coupling unification is spoiled

3. GUT model test

i. Model point
ii. Advantage of ratio
iii. Result
iv. Summary 1

$SU(5)$ model
$SO(10)$ model+$\mathbf{10}$ matter
E_6 model

small mixing
for $\mathbf{10}$ matter
large mixing
for $\bar{\mathbf{5}}$ matter

---

minimal $SO(10)$ model

all diagonalizing matrix
small mixing

We calculate nucleon decay in above four models under $M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$.

Seven diagonalizing matrices

$$L_u, L_d, L_\nu, L_e, R_u, R_d, R_e$$

- Generate diagonalizing matrices within $O(1)$ uncertainty except L_u and L_ν.
- To realize measured CKM and MNS matrix,

$$L_u = L_d U_{CKM}^\dagger, L_\nu = L_e U_{MNS}^\dagger.$$

Test L_u and L_ν whether these matrices are matrices within $O(1)$ uncertainty or not.

Generate 10^5 model points with diagonalizing matrices which pass above tests.

Advantage of ratio

$$R_1 \equiv \frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\Gamma_{P\to\pi^0 e^+}} \qquad R_2 \equiv \frac{\Gamma_{P\to K^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}} \qquad R_3 \equiv \frac{\Gamma_{P\to\pi^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

- cancel part of X-type gauge boson mass dependence

proton lifetime$\propto M^4_{X_{SU(5)}}/g^4_{GUT}, M^4_{X_{SO(10)}}/g^4_{GUT}, M^4_{X_{E_6}}/g^4_{GUT}$

$$R_1, R_2, R_3 \propto M^4_{X_{SO(10)}}/M^4_{X_{SU(5)}}, M^4_{X_{E_6}}/M^4_{X_{SO(10)}}$$

- cancel form factor dependence in R_1 and R_3

Matrix element	W_0^{RL}, W_0^{LR}
$\langle\pi^0\|(ud)u\|p\rangle, \langle\pi^0\|(du)d\|n\rangle$	$-0.103(23)(34)$

$SU(2)$ isospin limit

calculated by lattice - Aoki, Shintani, Soni (2013)

One of the reasons that we use $N \to \pi^0\bar{\nu}$ in R_1

R_1

$$\mathcal{L}_{\rm eff} = -\frac{2\pi\alpha_{GUT}}{M_{X_{SU(5)}}^2}\left(\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\mathbf{10}_j^\dagger\cdot\mathbf{10}_j+\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}_j^\dagger\cdot\bar{\mathbf{5}}_j+\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}'^\dagger_j\cdot\bar{\mathbf{5}}'_j\right)$$

$$-\frac{2\pi\alpha_{GUT}}{M_{X_{SO(10)}}^2}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}_i^\dagger\cdot\bar{\mathbf{5}}_j-\frac{2\pi\alpha_{GUT}}{M_{X_{E_6}}^2}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}'^\dagger_i\cdot\bar{\mathbf{5}}'_j$$

$$\mathbf{10}^\dagger\cdot\mathbf{10}\cdot\mathbf{10}^\dagger\cdot\mathbf{10} \Rightarrow (\overline{e_R^c}u_R)(\overline{u_L^c}d_L) \quad \times 2$$

decay mode into charged lepton

$$\mathbf{10}^\dagger\cdot\mathbf{10}\cdot\bar{\mathbf{5}}^\dagger\cdot\bar{\mathbf{5}} \Rightarrow \begin{cases}(\overline{e_L^c}u_L)(\overline{u_R^c}d_R) & \text{decay mode into charged lepton}\\ (\overline{\nu_L^c}d_L)(\overline{u_R^c}d_R) & \text{decay mode into neutrino}\end{cases}$$

$$R_1 \equiv \frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\Gamma_{P\to\pi^0 e^+}}$$

Ratio of neutrino final state and charged lepton final state R_1 is useful to test GUT model, especially unification group.

R_1 becomes larger as rank of unification group becomes larger.

R_1

$$\mathcal{L}_{\rm eff} = -\frac{2\pi\alpha_{GUT}}{M^2_{X_{SU(5)}}}\left(\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\mathbf{10}_j^\dagger\cdot\mathbf{10}_j+\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}_j^\dagger\cdot\bar{\mathbf{5}}_j+\mathbf{10}_i^\dagger\cdot\mathbf{10}_i\cdot\bar{\mathbf{5}}'^\dagger_j\cdot\bar{\mathbf{5}}'_j\right)$$
$$-\frac{2\pi\alpha_{GUT}}{M^2_{X_{SO(10)}}}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}_i^\dagger\cdot\bar{\mathbf{5}}_j-\frac{2\pi\alpha_{GUT}}{M^2_{X_{E_6}}}\mathbf{10}_i^\dagger\cdot\mathbf{10}_j\cdot\bar{\mathbf{5}}'^\dagger_i\cdot\bar{\mathbf{5}}'_j$$

R_1 (strictly speaking $\Gamma_{N\to\pi^0\bar{\nu}}$) is very useful to test unification group.

When $V_{10}=U_{CKM-type}\sim\mathbf{1}_{3\times3}, V_{\bar{5}}=V_{MNS-type}$

always positive contribution

$$\frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\underline{(\Gamma_{N\to\pi^0\bar{\nu}})_{SU(5)}}}=1+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{SO(10)}}}(2+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{SO(10)}}})|(R_d)_{11}|^2+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{E_6}}}(2+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{E_6}}})|(R_d)_{21}|^2$$

➡ $\Gamma_{N\to\pi^0\bar{\nu}}$ in $SU(5)$ GUT model

This is because in neutrino final state neutrino flavors (from electron to tau) are summed up.

R_1 becomes larger as rank of unification group becomes larger.

R_2

$$\mathcal{L}_{\text{eff}} = -\frac{2\pi\alpha_{GUT}}{M^2_{X_{SU(5)}}}\left(\mathbf{10}_i^\dagger \cdot \mathbf{10}_i \cdot \mathbf{10}_j^\dagger \cdot \mathbf{10}_j + \mathbf{10}_i^\dagger \cdot \mathbf{10}_i \cdot \bar{\mathbf{5}}_j^\dagger \cdot \bar{\mathbf{5}}_j + \mathbf{10}_i^\dagger \cdot \mathbf{10}_i \cdot \bar{\mathbf{5}}'^\dagger_j \cdot \bar{\mathbf{5}}'_j\right)$$

$$-\frac{2\pi\alpha_{GUT}}{M^2_{X_{SO(10)}}}\mathbf{10}_i^\dagger \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}_i^\dagger \cdot \bar{\mathbf{5}}_j - \frac{2\pi\alpha_{GUT}}{M^2_{X_{E_6}}}\mathbf{10}_i^\dagger \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}'^\dagger_i \cdot \bar{\mathbf{5}}'_j$$

If no mixing

added operator in $SO(10)$ model

$$\mathbf{10}_i^\dagger \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}_i^\dagger \cdot \bar{\mathbf{5}}_j$$

$$(\overline{e^c_L} u_L)(\overline{u^c_R} d_R) \quad \cancel{(\overline{\mu^c_L} u_L)(\overline{u^c_R} s_R)}$$

$$R_2 \equiv \frac{\Gamma_{P\to K^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

smaller than R_2 in $SU(5)$ model

added operator in E_6 model

$$\mathbf{10}_i^\dagger \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}'^\dagger_i \cdot \bar{\mathbf{5}}'_j$$

2nd generation particles

$$\cancel{(\overline{e^c_L} u_L)(\overline{u^c_R} d_R)} \quad (\overline{\mu^c_L} u_L)(\overline{u^c_R} s_R)$$

$$R_2 \equiv \frac{\Gamma_{P\to K^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

lager than R_2 in $SO(10)$ model

R_3

$$\mathcal{L}_{\mathrm{eff}} = -\frac{2\pi\alpha_{GUT}}{M^2_{X_{SU(5)}}}\left(\mathbf{10}^\dagger_i \cdot \mathbf{10}_i \cdot \mathbf{10}^\dagger_j \cdot \mathbf{10}_j + \mathbf{10}^\dagger_i \cdot \mathbf{10}_i \cdot \bar{\mathbf{5}}^\dagger_j \cdot \bar{\mathbf{5}}_j + \mathbf{10}^\dagger_i \cdot \mathbf{10}_i \cdot \bar{\mathbf{5}}'^\dagger_j \cdot \bar{\mathbf{5}}'_j\right)$$

$$- \frac{2\pi\alpha_{GUT}}{M^2_{X_{SO(10)}}}\mathbf{10}^\dagger_i \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}^\dagger_i \cdot \bar{\mathbf{5}}_j - \frac{2\pi\alpha_{GUT}}{M^2_{X_{E_6}}}\mathbf{10}^\dagger_i \cdot \mathbf{10}_j \cdot \bar{\mathbf{5}}'^\dagger_i \cdot \bar{\mathbf{5}}'_j$$

$$P \to \pi^0 \mu^+$$

$$R_3 \equiv \frac{\Gamma_{P\to\pi^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$$

only 1st generation

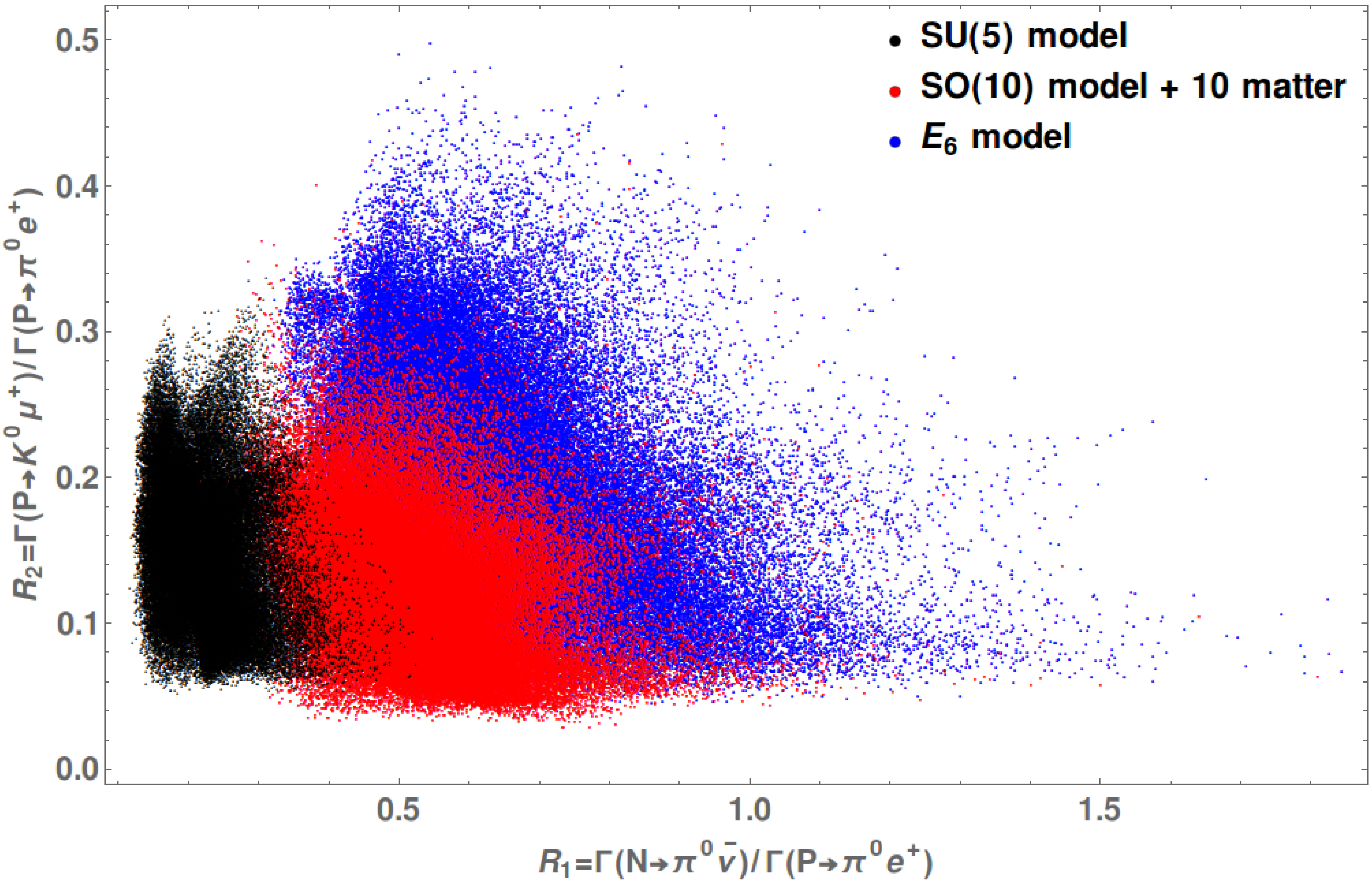
2nd generation

comes from mixing

large mixing comes from $\bar{\mathbf{5}}$ matter

R_3 becomes larger as rank of unification group becomes larger.

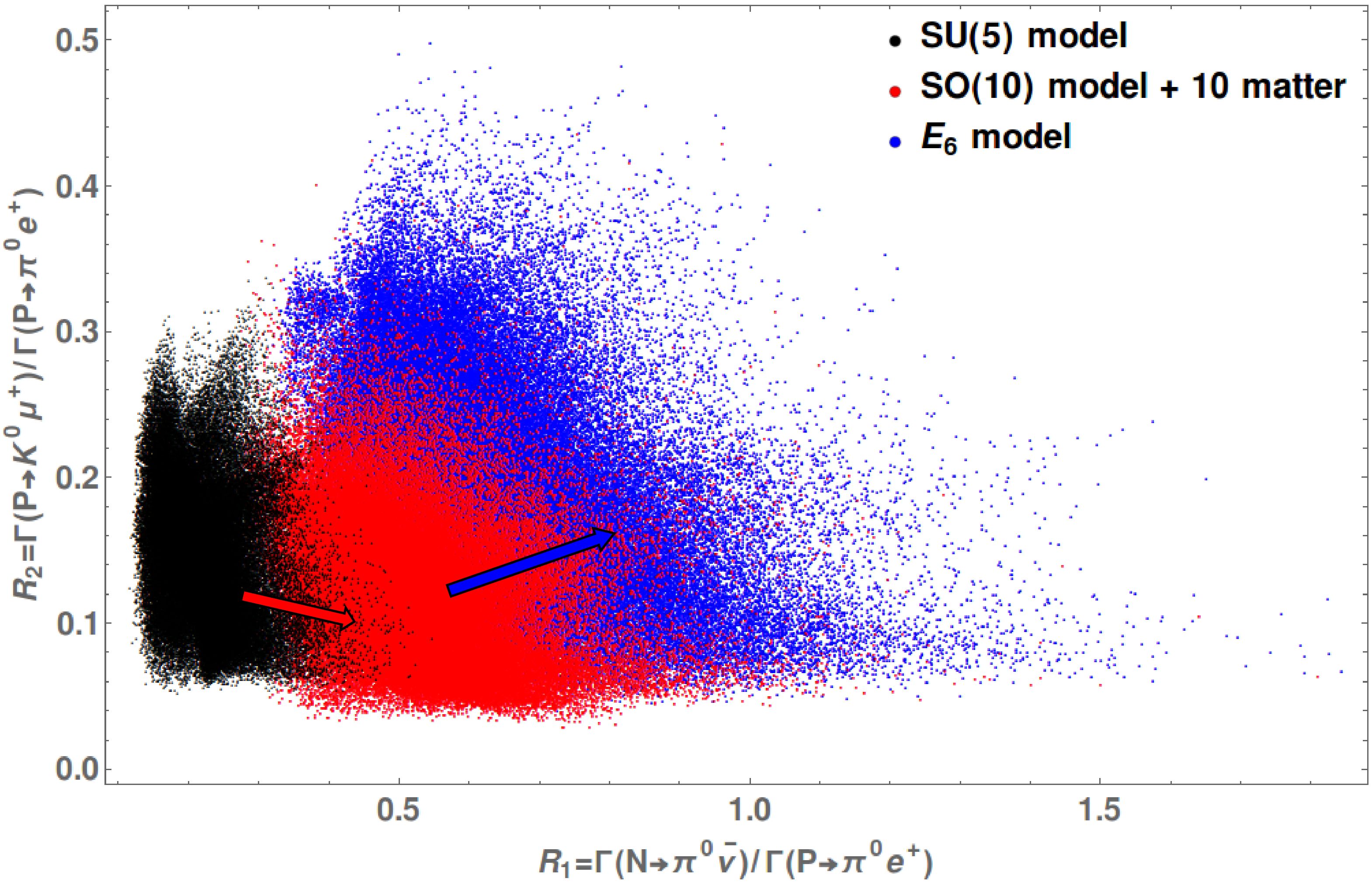
R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

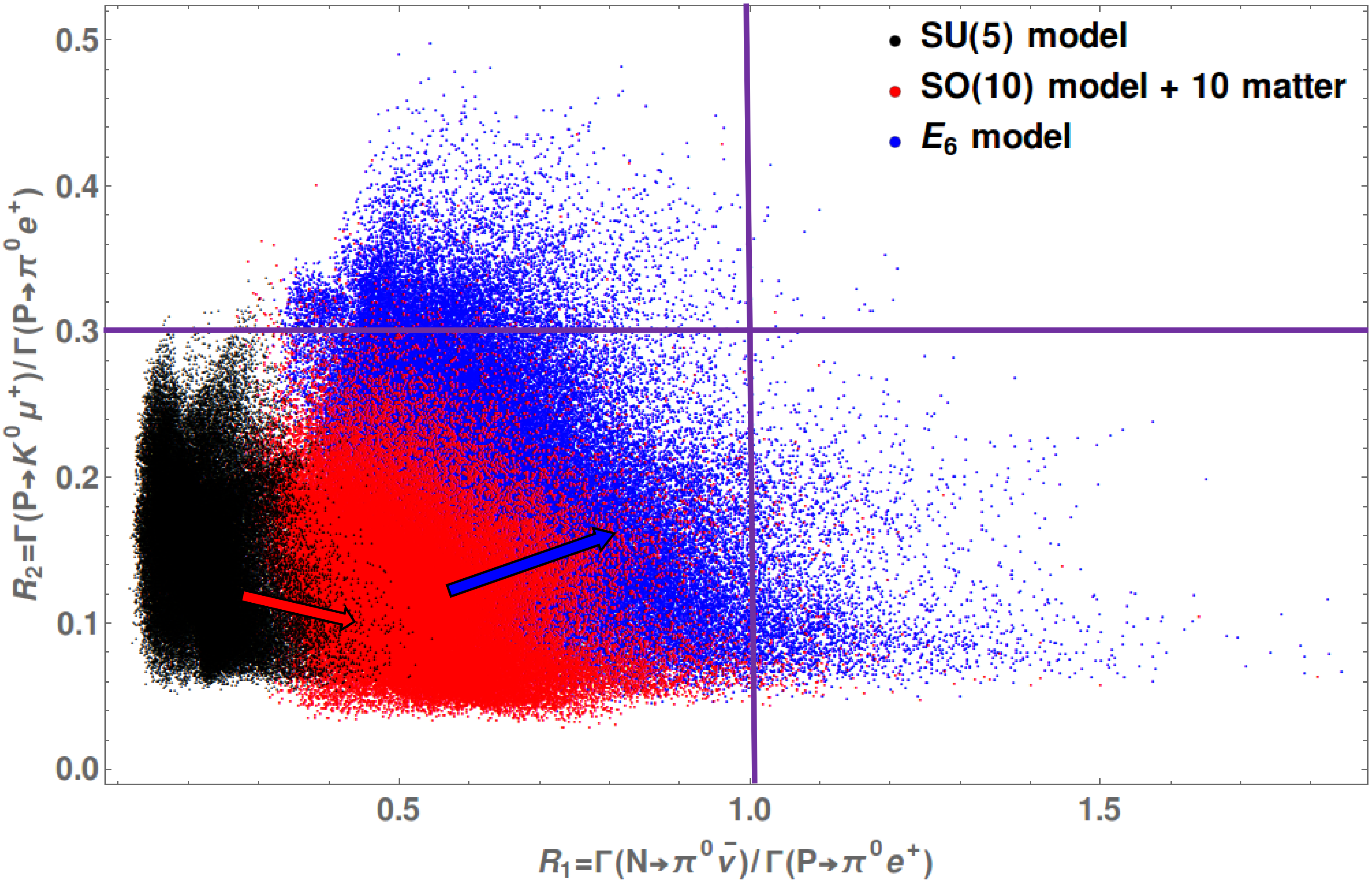
R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

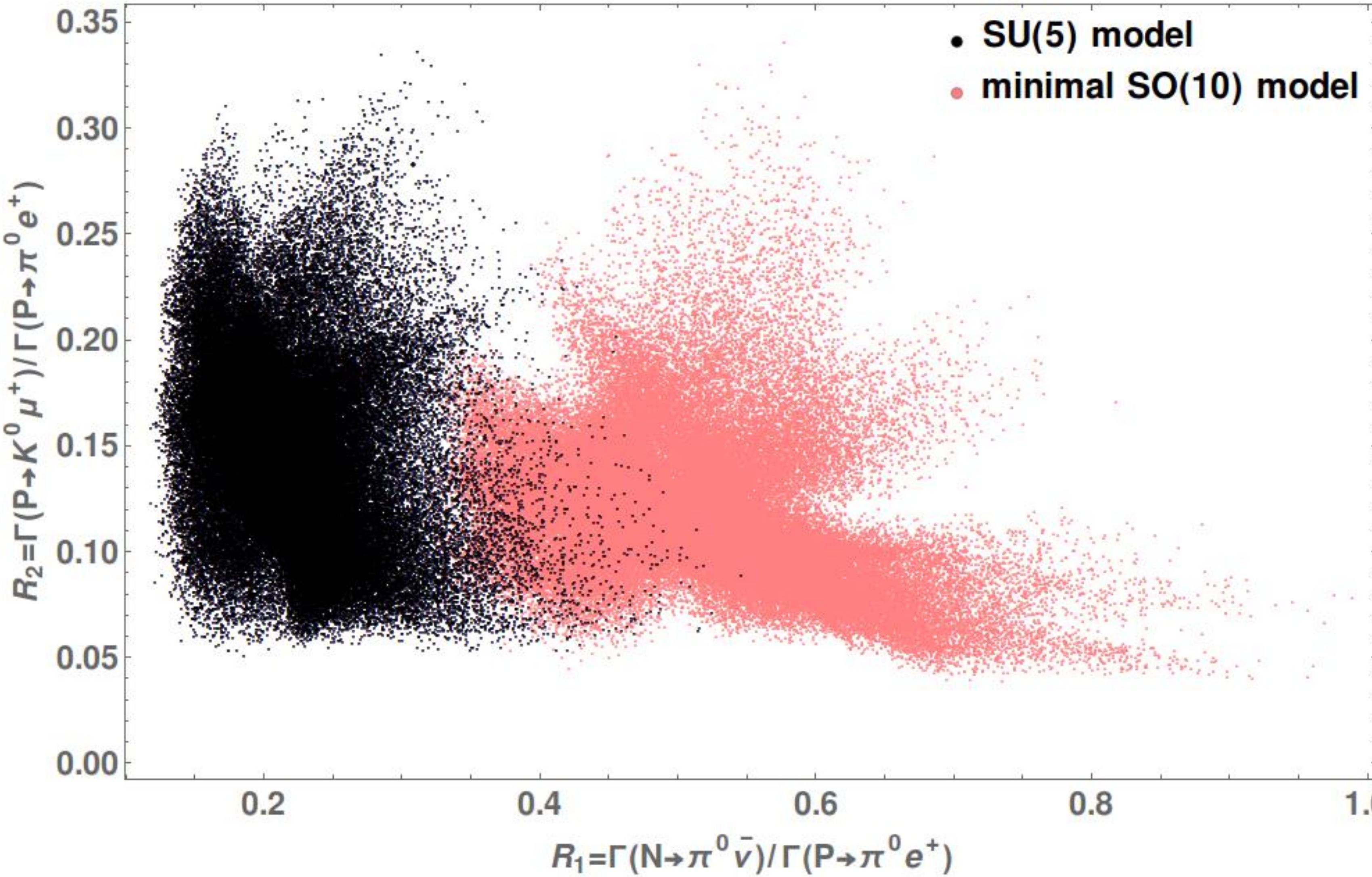
R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

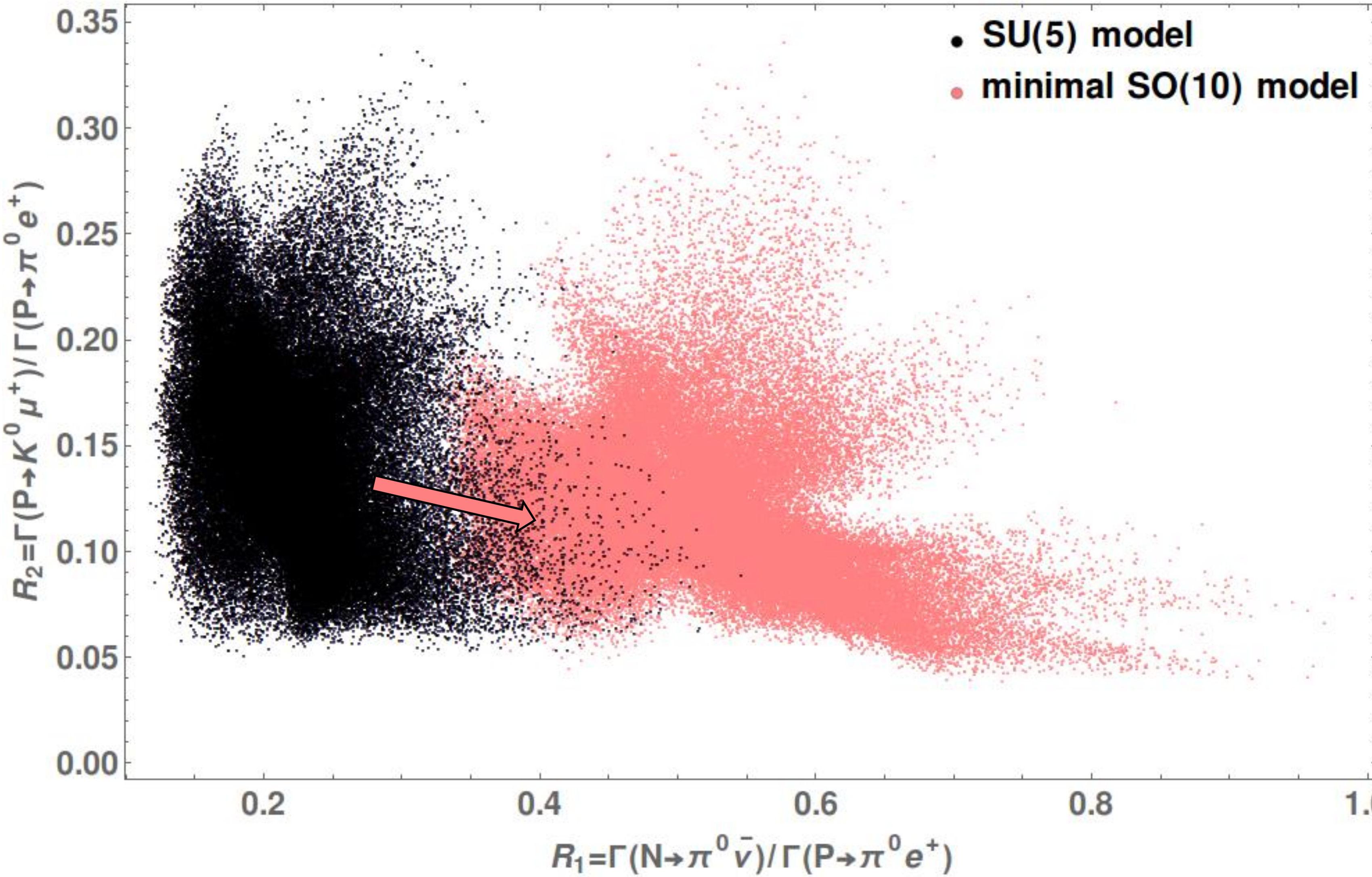
R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

all diagonalizing matrices are $U_{CKM-type}$ except neutrino one

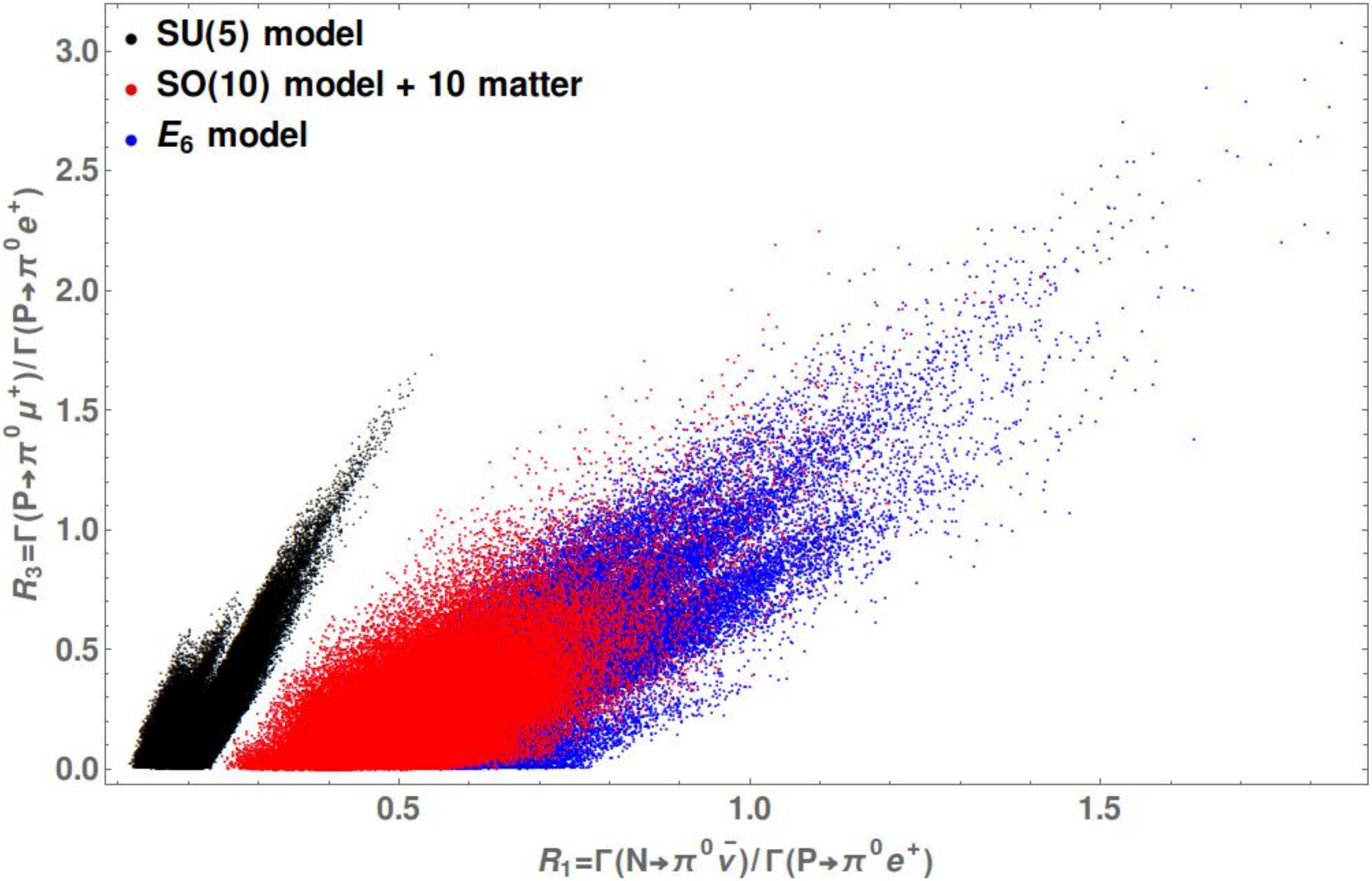
R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

all diagonalizing matrices are $U_{CKM-type}$ except neutrino one

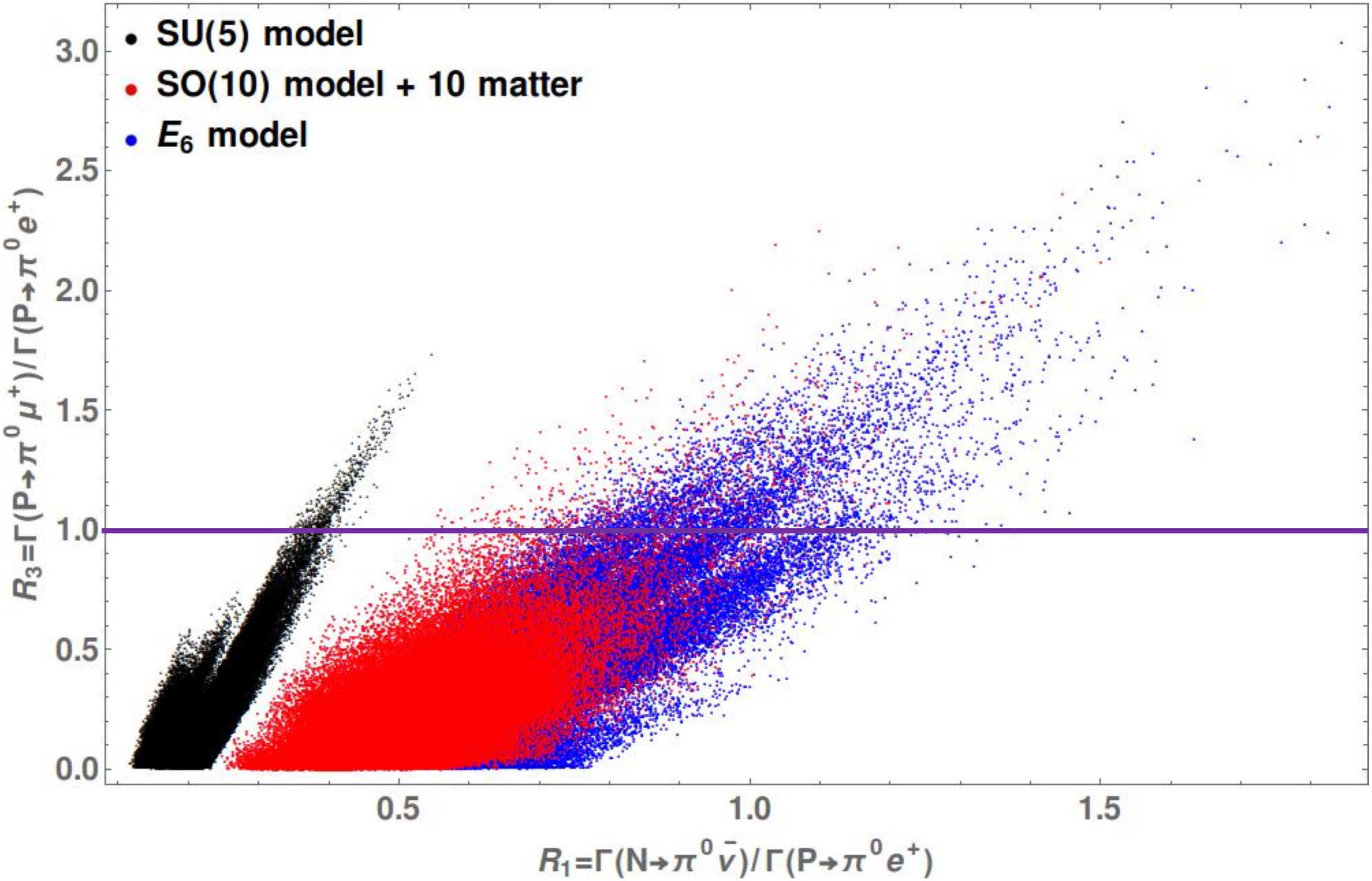
R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

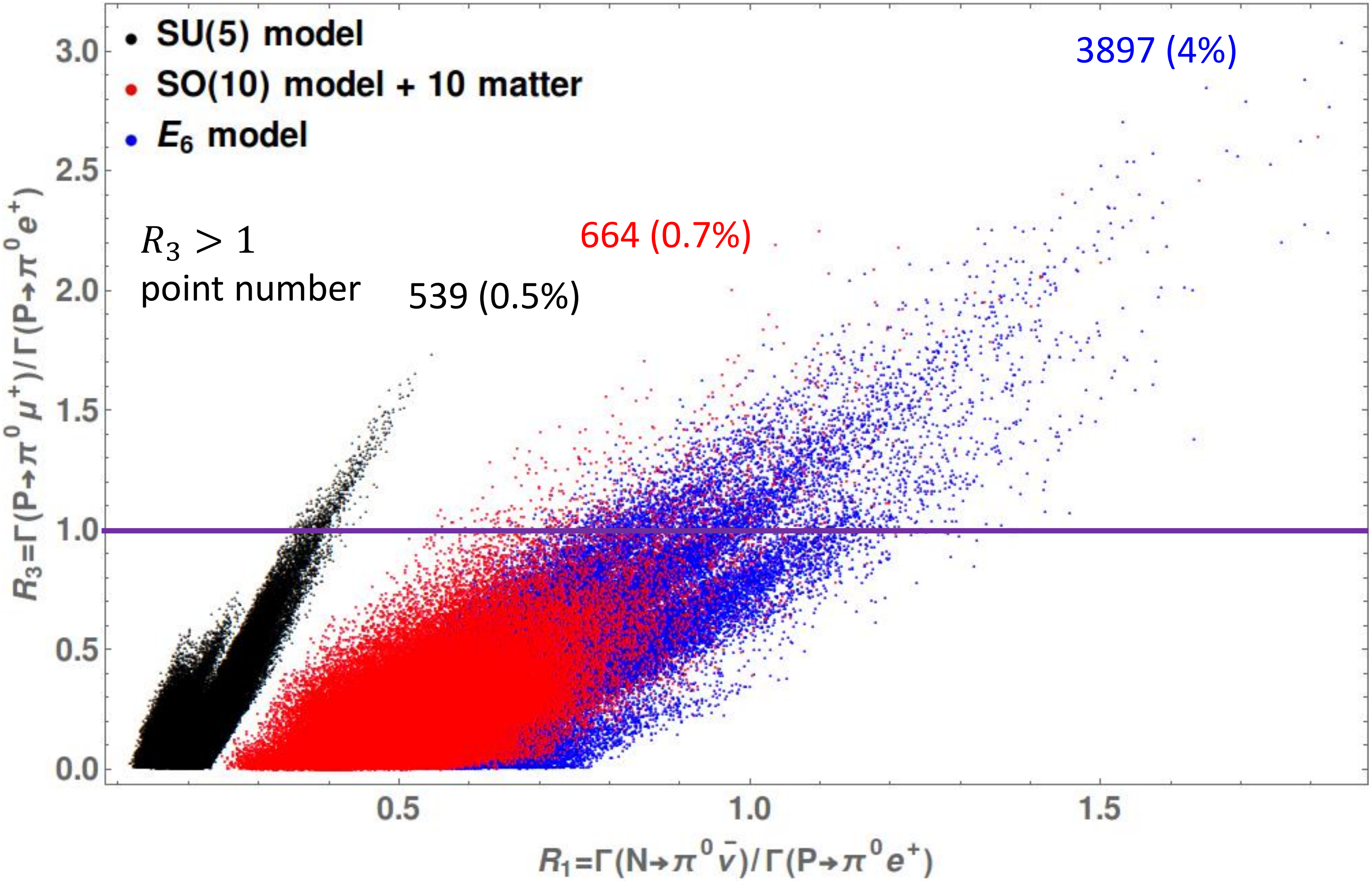
$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

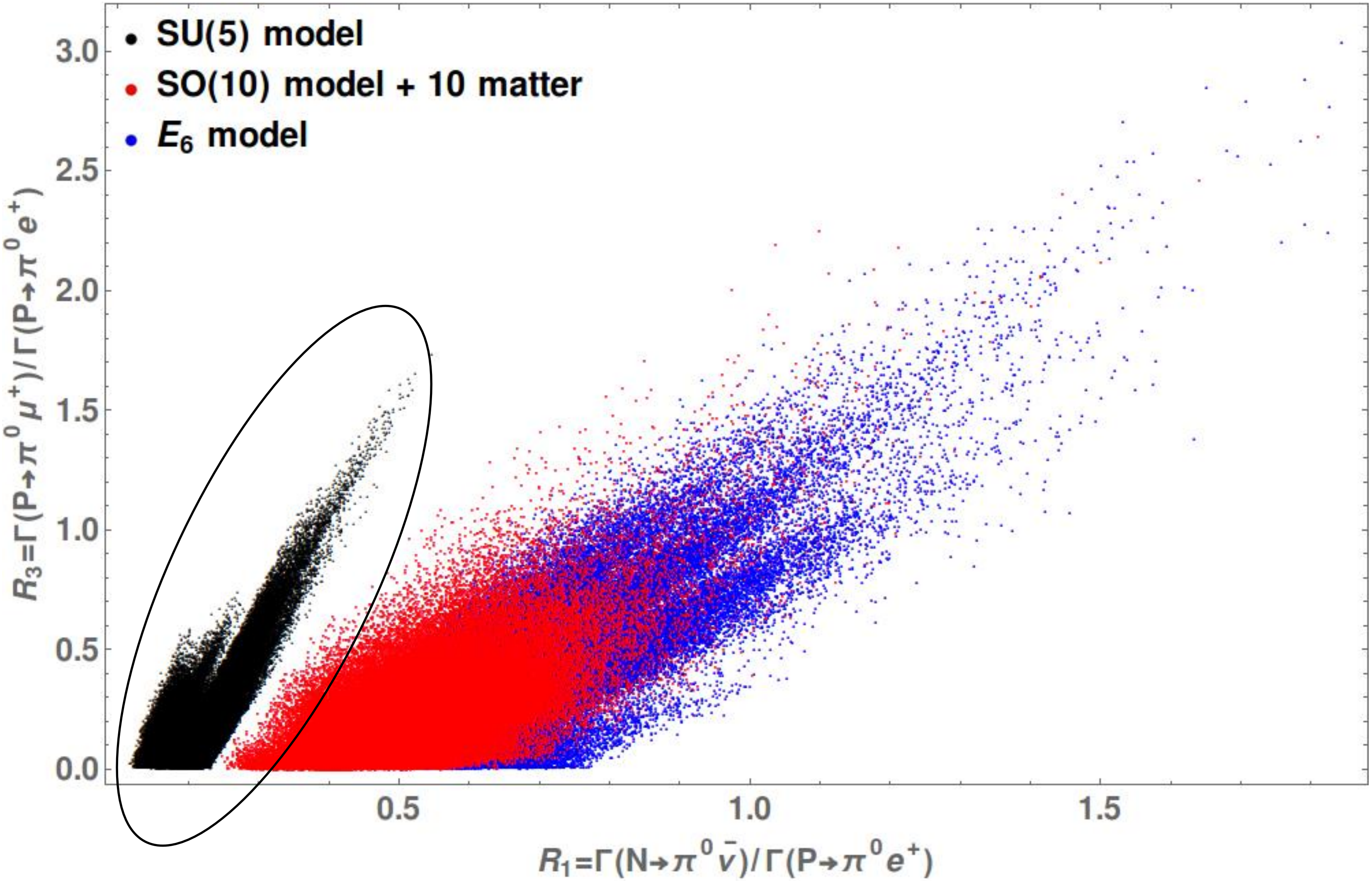
$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

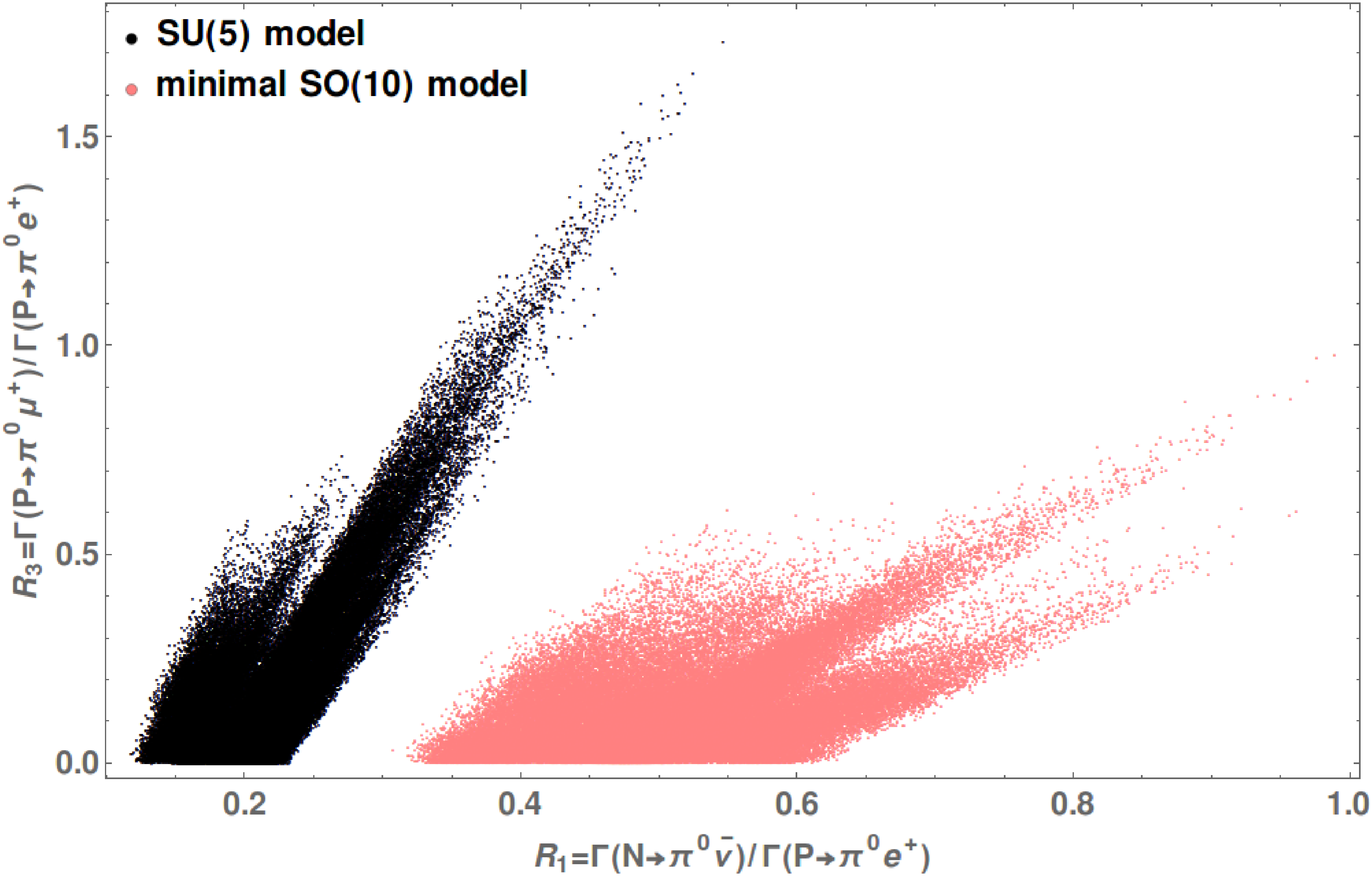
R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

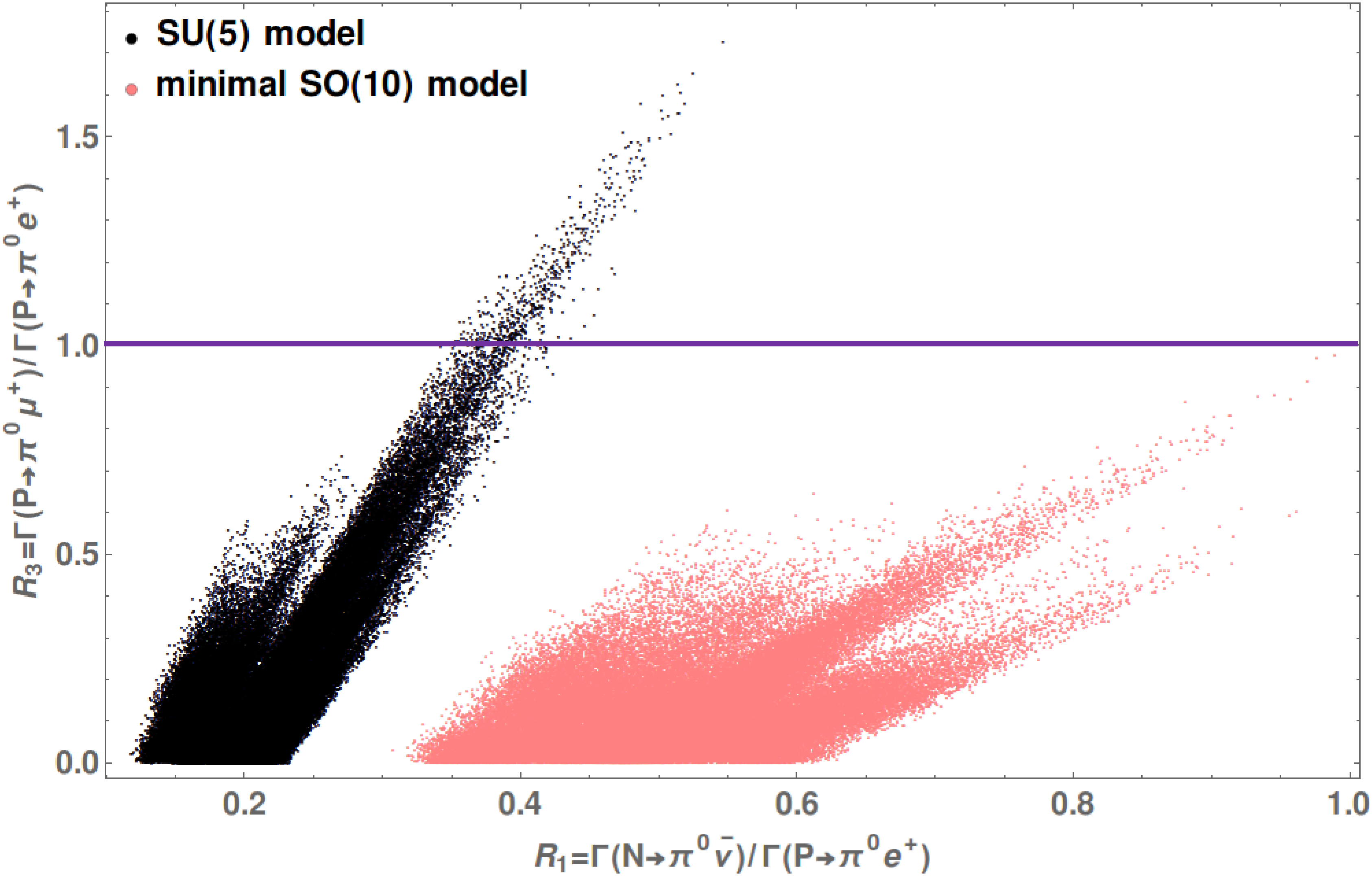
R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

all diagonalizing matrices are $U_{CKM-type}$ except neutrino one

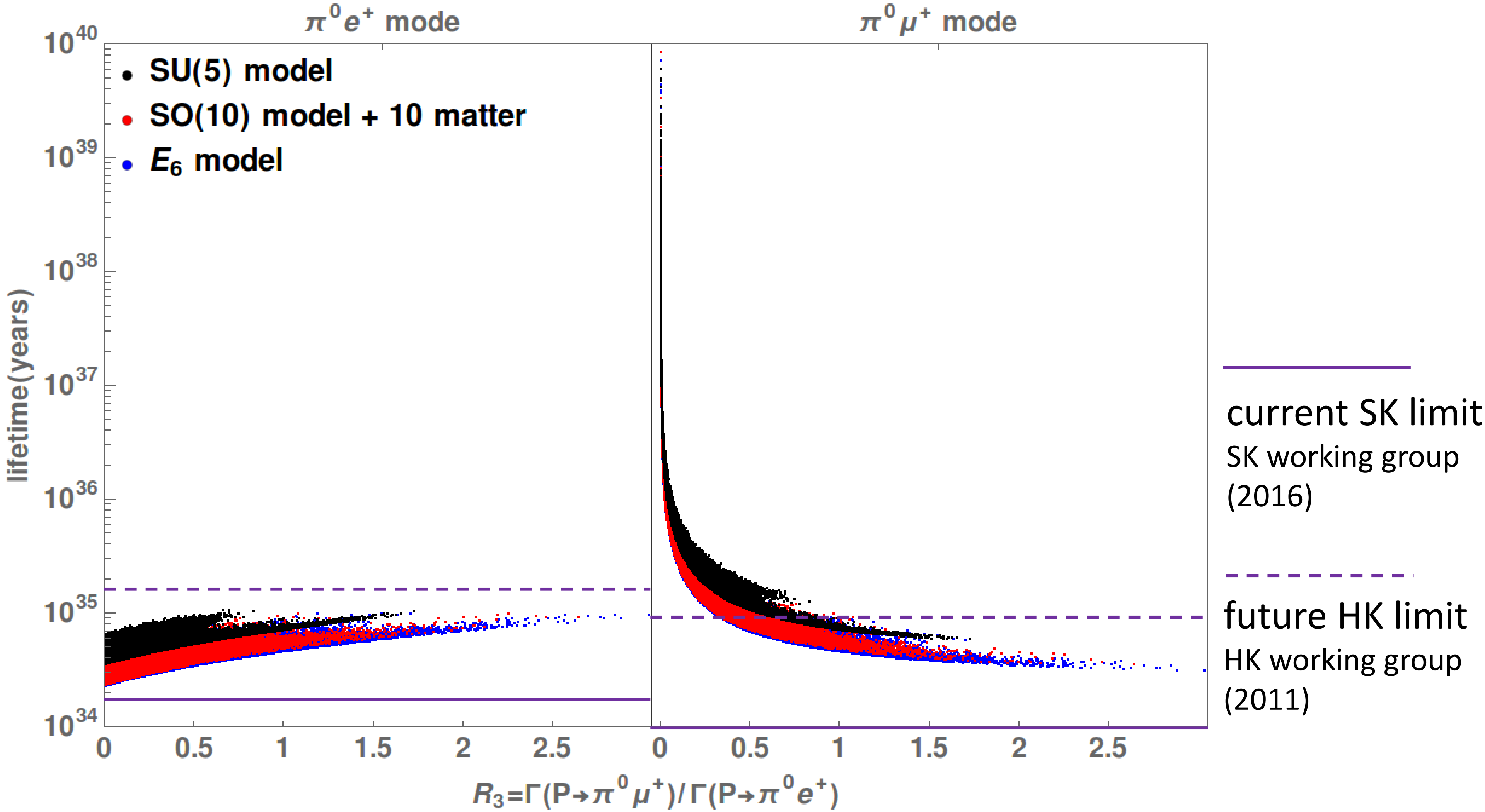
R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

all diagonalizing matrices are $U_{CKM-type}$ except neutrino one

proton lifetime



$$\frac{M_{X_{SU(5)}}}{g_{GUT}} = \frac{M_{X_{SO(10)}}}{g_{GUT}} = \frac{M_{X_{E_6}}}{g_{GUT}} = 10^{16}\ \mathrm{GeV}$$

minimal SUSY SU(5) GUT model

$$\frac{M_{X_{SU(5)}}}{g_{GUT}} \sim 3\times 10^{16}\ \mathrm{GeV}$$

We can observe nucleon decay in HK !!!

Summary

- Nucleon decay is useful to test GUT.
  To reduce uncertainty ratio of partial decay width is useful.
- Especially $R_1 \equiv \frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\Gamma_{P\to\pi^0 e^+}}$ is very useful.
  R_1 becomes larger as rank of unification group becomes larger.
- The decay modes which comes form mixing are also useful.
  $SU(5)$ model points can be separated from other model points in R_1 vs $R_3 \equiv \frac{\Gamma_{P\to\pi^0\mu^+}}{\Gamma_{P\to\pi^0 e^+}}$ plane.
  When $R_3 > 1$, large mixing is favored.
- When $M_X/g_{GUT} = 10^{16}$ GeV, we can expect observation of nucleon decay in Hyper-Kamiokande.

Conditions to realize testable nucleon decay

- E_6 unification group
- Nucleon decay via dimension 6 operators is dominant.
- realize large mixing through $\bar{\mathbf{5}}$ mixing

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

- X-type gauge boson mass condition $M_{X_{SU(5)}} \geq M_{X_{SO(10)}}, M_{X_{E_6}}$
- $M_X/g_{GUT} = 10^{16}$ GeV

Are there any GUT models which realize above conditions?

Anomalous $U(1)_A$ E_6 SUSY GUT model

Anomalous $U(1)_A$ SUSY GUT

realize doublet-triplet splitting under “natural assumptions”

- consider all operators which are allowed by symmetry
- magnitude of operator’s coefficients are $O(1)$
- consider all higher dimensional operators

right then

hierarchical coefficient(e.g. difference between first generation particle mass and third generation particle mass)

mechanism to forbid unfavorable operators

come from?

anomalous $U(1)_A$ symmetry

anomalous $U(1)_A$ symmetry

anomalous $U(1)_A$ symmetry

determine VEV of GUT singlet Higgs field $H^{\pm}$($\mathrm{U}(1)_{\mathrm{A}}$ charge $h^{\pm}$)

$$\begin{cases} \langle H^+ \rangle = 0 \ (h^+ > 0) \\ \langle H^- \rangle = \lambda^{-h^-} \Lambda_{\text{SUSY GUT}} \ (h^- < 0) \end{cases} \qquad \lambda = 0.22 < 1$$

hierarchical coefficient(e.g. difference between first generation particle mass and second generation particle mass)

Froggatt-Nielsen field Θ (GUT singlet, $U(1)_A$ charge -1)

mechanism to forbid unfavorable operators

SUSY zero mechanism

forbid $U(1)_A$ charge negative and GUT singlet operators

$U(1)_A$ charge positive and GUT singlet operators can satisfy $U(1)_A$ symmetry by compensating with FN field

connect various phenomena

Nucleon decay

Nucleon decay via dimension 6 operators $\begin{cases} \langle H^+ \rangle = 0 \; (h^+ > 0) & \lambda = 0.22 < 1 \\ \langle H^- \rangle = \lambda^{-h^-} \Lambda_{\text{SUSY GUT}} \; (h^- < 0) & \end{cases}$

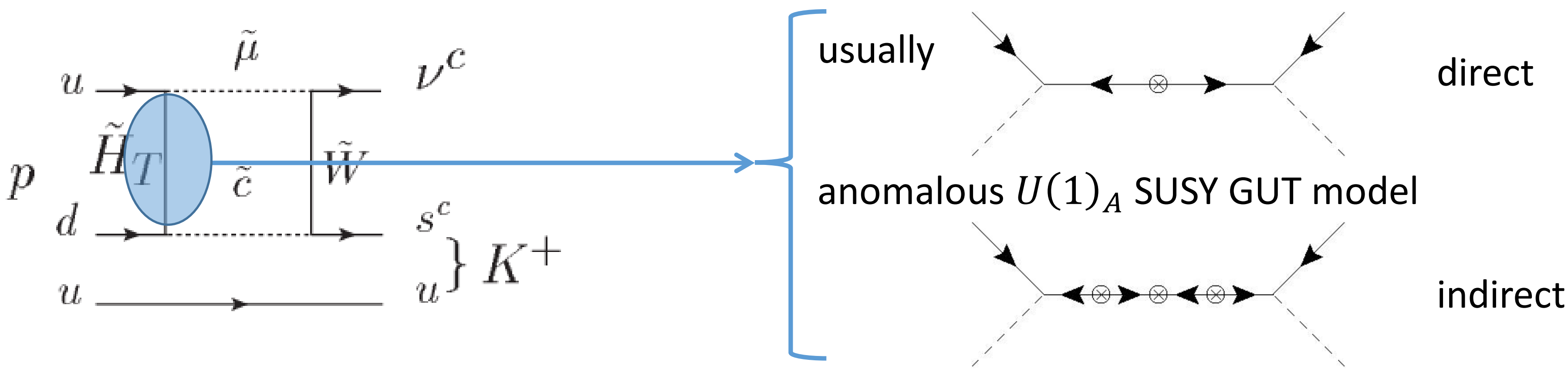
GUT scale of anomalous $U(1)_A$ SUSY GUT model: Λ_a

$$\Lambda_a \equiv x < \Lambda_{SUSY\ GUT}$$

nucleon decay via dimension 6 operators is enhanced

When $x = 10^{16}$ GeV, $M_X/g_{GUT} = 10^{16}$ GeV

Nucleon decay via dimension 5 operators



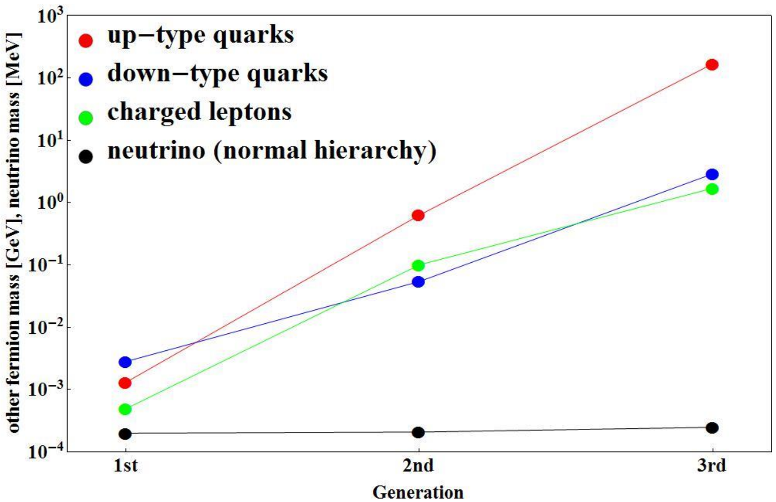
nucleon decay via dimension 5 operators is suppressed

nucleon decay via dimension 6 operators is dominant

Yukawa matrix in $SU(5)$

How to realize observed quark and lepton masses and mixings in $SU(5)$ GUT model

$$\mathbf{10} \to q_L(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1},\mathbf{1})_1 \qquad \bar{\mathbf{5}} \to d_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$$



$$|U_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.037 \\ 0.23 & 0.97 & 0.042 \\ 0.0087 & 0.041 & 1.0 \end{pmatrix}$$

$$|U_{MNS}| = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.47 & 0.52 & 0.71 \\ 0.31 & 0.65 & 0.69 \end{pmatrix}$$

mass hierarchy

M_u → strong

M_d, M_e → middle

M_ν → weak

mixing

U_{CKM} → small mixing

U_{MNS} → large mixing

assumption The **10** quark and lepton induces stronger hierarchies for Yukawa coupling than the $\bar{\mathbf{5}}$ quark and lepton.

$$\begin{pmatrix} \mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^{2.5} \\ \lambda^2 \end{pmatrix}$$

Yukawa matrix in $SU(5)$

$$Y_u \mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5_H} + Y_d \mathbf{10}\cdot\bar{\mathbf{5}}\cdot\bar{\mathbf{5}}_{\mathbf{H}} + \boxed{Y_\nu' \bar{\mathbf{5}}\cdot\mathbf{1}\cdot\mathbf{5_H} + M_{\nu_R}\mathbf{1}\cdot\mathbf{1}}$$

M_u $\quad$ M_d, M_e $\quad$ $\Rightarrow Y_\nu \bar{\mathbf{5}}\cdot\bar{\mathbf{5}}\cdot\mathbf{5_H}\cdot\mathbf{5_H}$ $\Rightarrow M_\nu$

$$M_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \left[\begin{array}{l} M_{u\,diag} \sim \begin{pmatrix} \lambda^6 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix} \langle H_u \rangle \\ L_u \sim R_u \sim U_{CKM-type} \end{array} \right.$$

$\Rightarrow$ strong hierarchy

$\Rightarrow$ middle hierarchy

$$M_d = M_e^t = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^{3.5} & \lambda^3 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_d \rangle \left[\begin{array}{l} M_{d\,diag} = M_{e\,diag} \sim \lambda^2 \begin{pmatrix} \lambda^4 & & \\ & \lambda^{2.5} & \\ & & 1 \end{pmatrix} \langle H_d \rangle \\ L_d \sim R_e \sim U_{CKM-type}, R_d \sim L_e \sim U_{MNS-type} \end{array} \right.$$

$$M_\nu = \frac{\lambda^{-5}}{\Lambda} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 \left[\begin{array}{l} M_{\nu\,diag} \sim \frac{\lambda^{-5}}{\Lambda} \begin{pmatrix} \lambda^2 & & \\ & \lambda & \\ & & 1 \end{pmatrix} \langle H_u \rangle^2 \\ L_\nu \sim U_{MNS-type} \end{array} \right.$$

$\Rightarrow$ weak hierarchy

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

Yukawa matrix in E_6

$$\mathbf{27} \to \mathbf{16} + \mathbf{10} + \mathbf{1}$$

$\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}'$

$$W = \lambda^{\psi_i+\psi_j+c}\Psi_i\Psi_j C + \lambda^{\psi_i+\psi_j+\phi}\Psi_i\Psi_j\Phi$$

$$E_6 \xrightarrow[\langle\Phi\rangle]{} SO(10) \xrightarrow[\langle C\rangle]{} SU(5) \xrightarrow[\langle \mathrm{A}\rangle]{} G_{SM}$$

the capital letter denotes the superfield and small letter denotes the corresponding $U(1)_A$ charge

superpotential for Yukawa interaction and $\bar{\mathbf{5}}$ mixing

$$\lambda^r \equiv \frac{\lambda^c\langle C\rangle}{\lambda^\phi\langle\Phi\rangle}$$

control $\bar{\mathbf{5}}$ mixing

When $r = 0.5$, following assumption are realized

The $\mathbf{10}$ quark and lepton induces stronger hierarchies for Yukawa coupling than the $\bar{\mathbf{5}}$ quark and lepton.

We get relation between $\langle C\rangle$ and $\langle\Phi\rangle$.

X-type gauge boson mass

$$E_6 \underset{\langle\Phi\rangle}{\rightarrow} SO(10) \underset{\langle C\rangle}{\rightarrow} SU(5) \underset{\langle A\rangle}{\rightarrow} G_{SM}$$

This adjoint VEV $\langle A\rangle$ is $SO(10)$ group notation and useful to realize DT splitting.

$$\langle \mathbf{45}_A\rangle = i\sigma_2 \times \begin{pmatrix} x & & & & \\ & x & & & \\ & & x & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

σ_i : Pauli matrix

$\langle C\rangle \equiv v_c, \langle\Phi\rangle \equiv v_\phi$

$$M^2_{X_{SU(5)}} = g^2_{GUT} x^2,$$

$$M^2_{X_{SO(10)}} = g^2_{GUT}(x^2 + v_c^2), M^2_{X_{E_6}} = g^2_{GUT}(\tfrac{1}{4}x^2 + v_\phi^2)$$

Here,

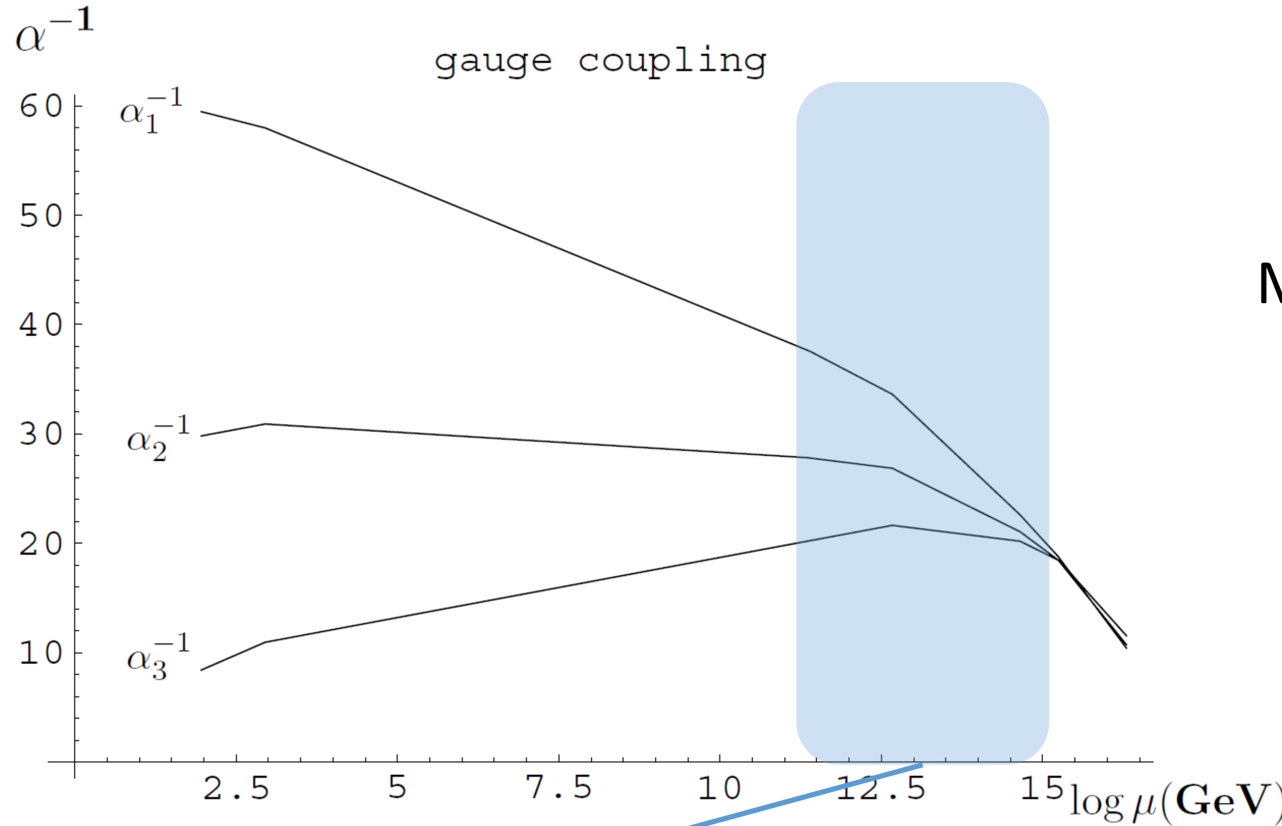
$x = 1 \times 10^{16}$GeV, $v_c = 5 \times 10^{14}$GeV, $v_\phi = 5 \times 10^{15}$GeV.

*$x > v_c$ is to realize VEV form $\langle A\rangle$.

$$M^2_{X_{SO(10)}} \sim M^2_{X_{SU(5)}} \sim 2M^2_{E_6}$$

$X_{SO(10)}$ and X_{E_6} contribution are large

gauge coupling unification



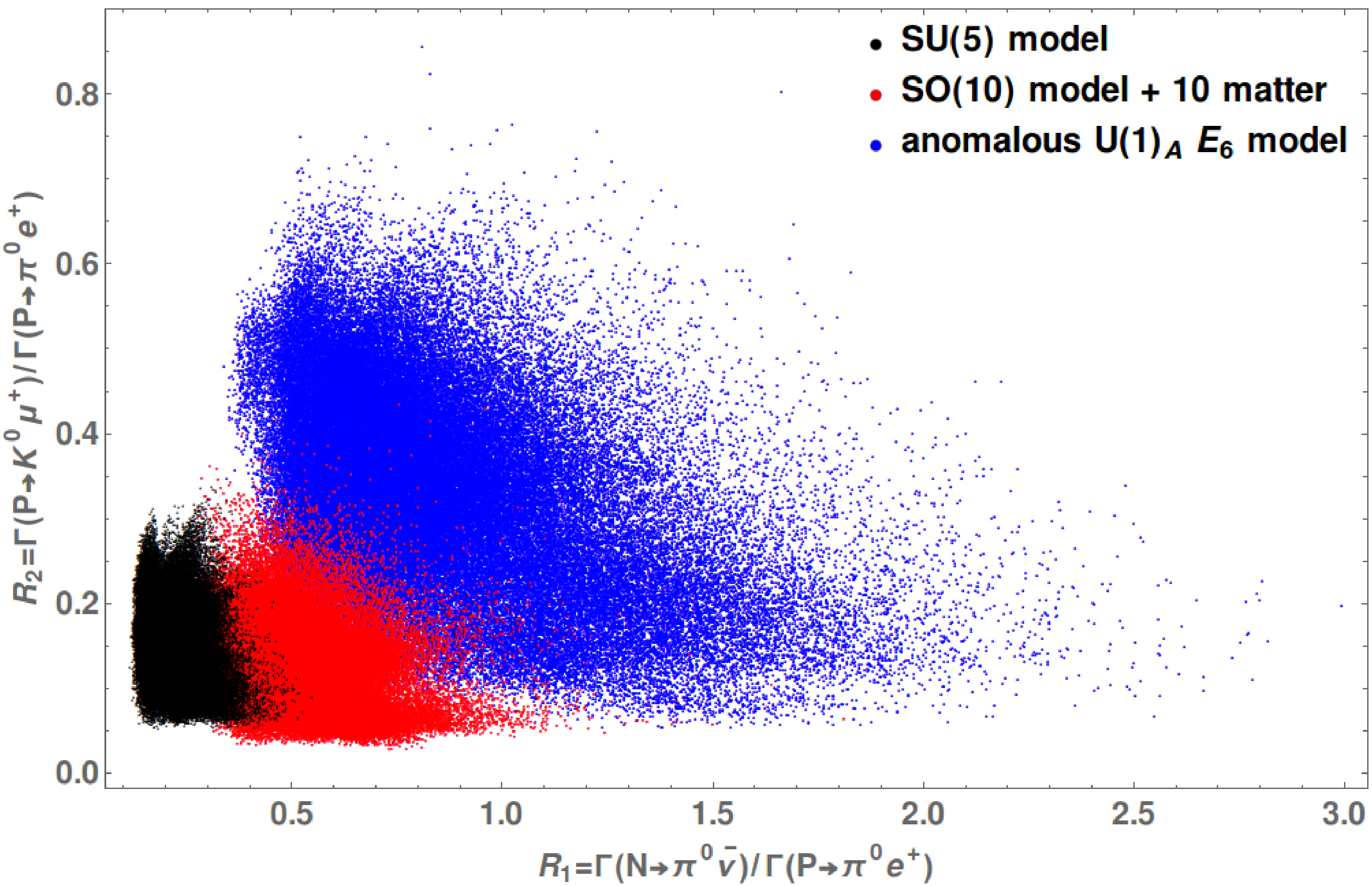
Maekawa (2002)

intermediation scales ⬅ $x > v_c$

Sometimes intermediation scales spoil gauge coupling unification.

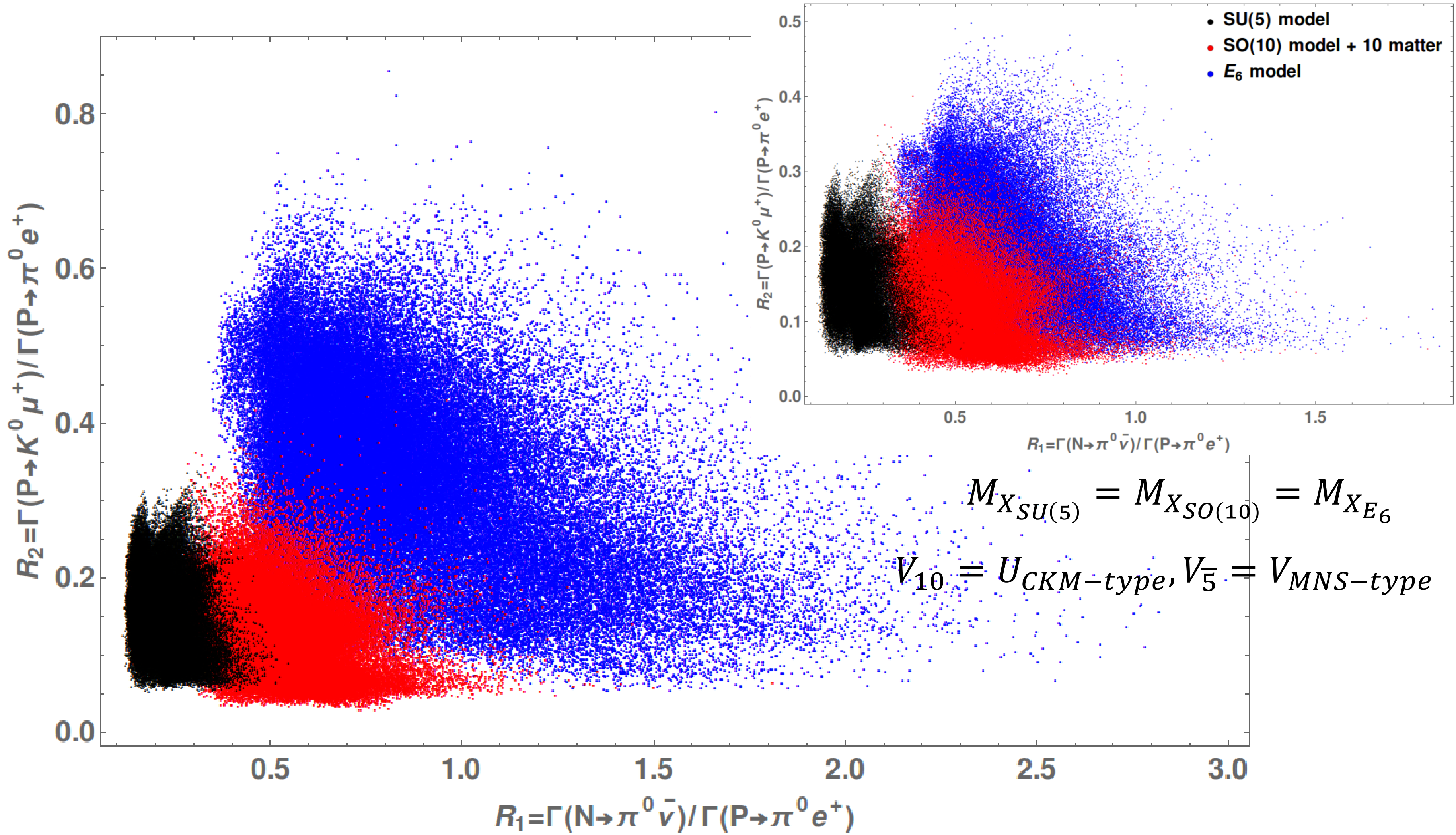
In anomalous $U(1)_A$ SUSY GUT model, gauge coupling unification can be realized even if there are intermediation scales.

R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2} M_{X_{E_6}}$$
$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

R_1 vs R_2



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

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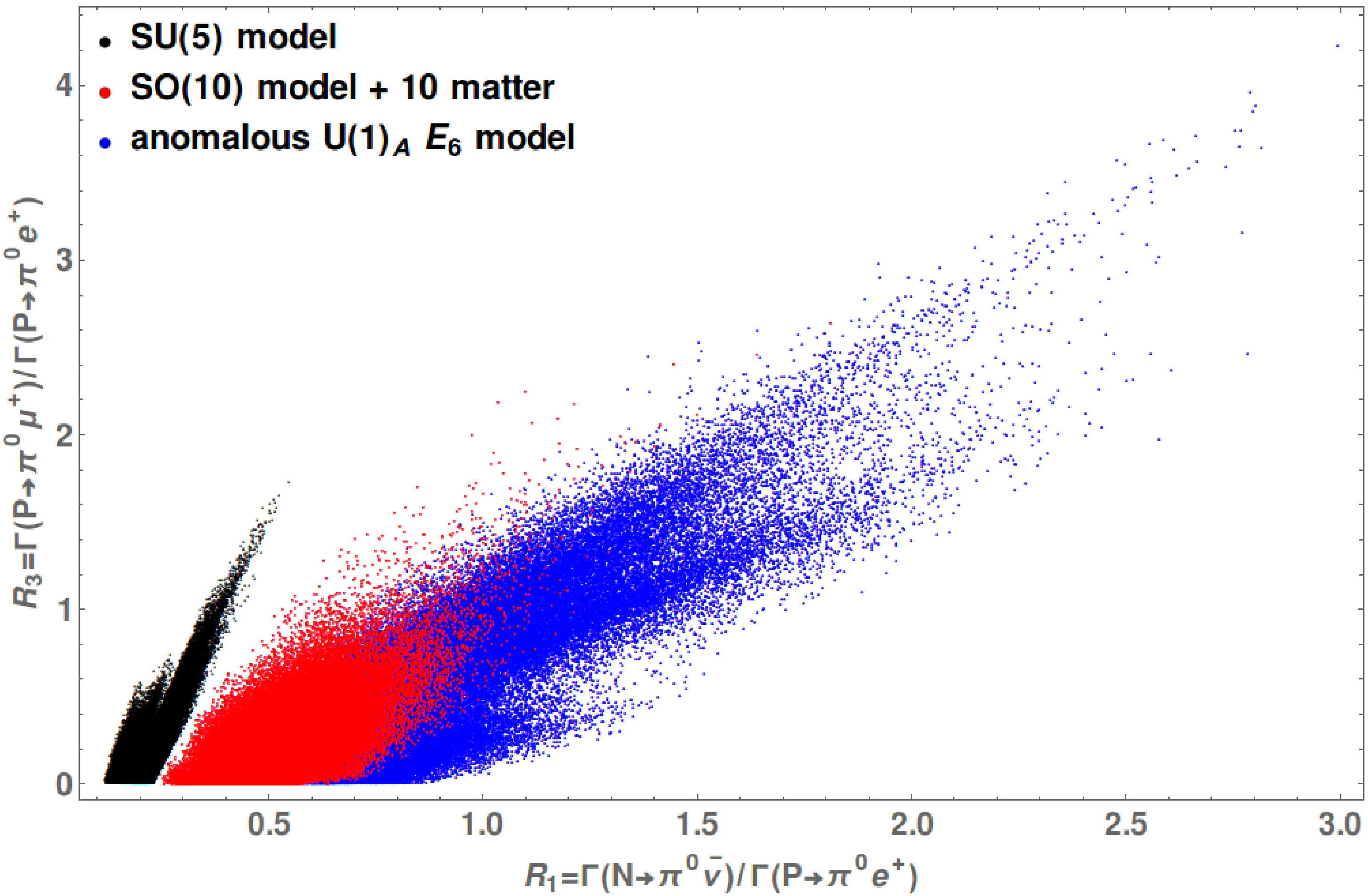
$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2} M_{X_{E_6}}$$

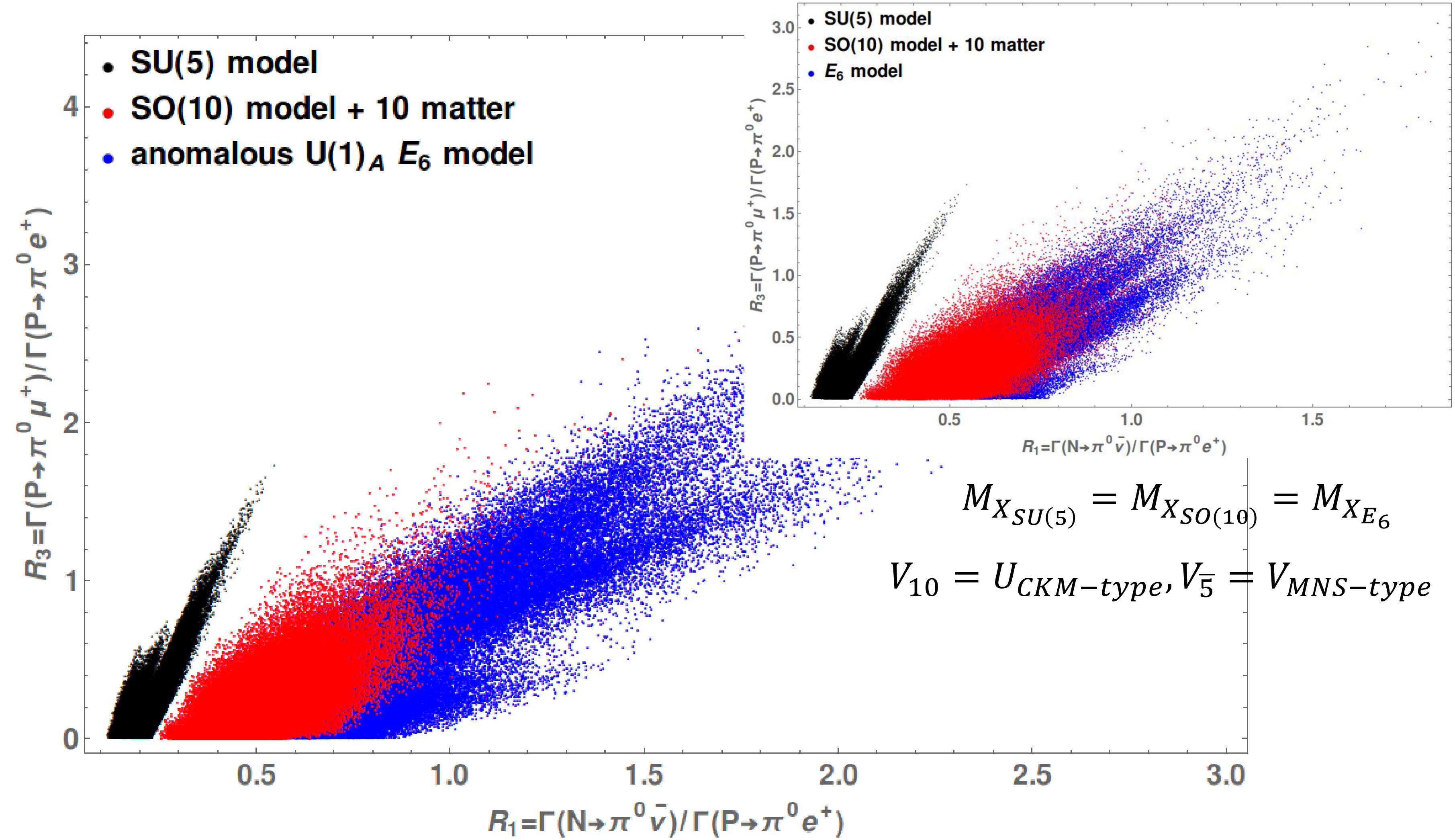
$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

R_1 vs R_3



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2}M_{X_{E_6}}$$
$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

R_1 vs R_3



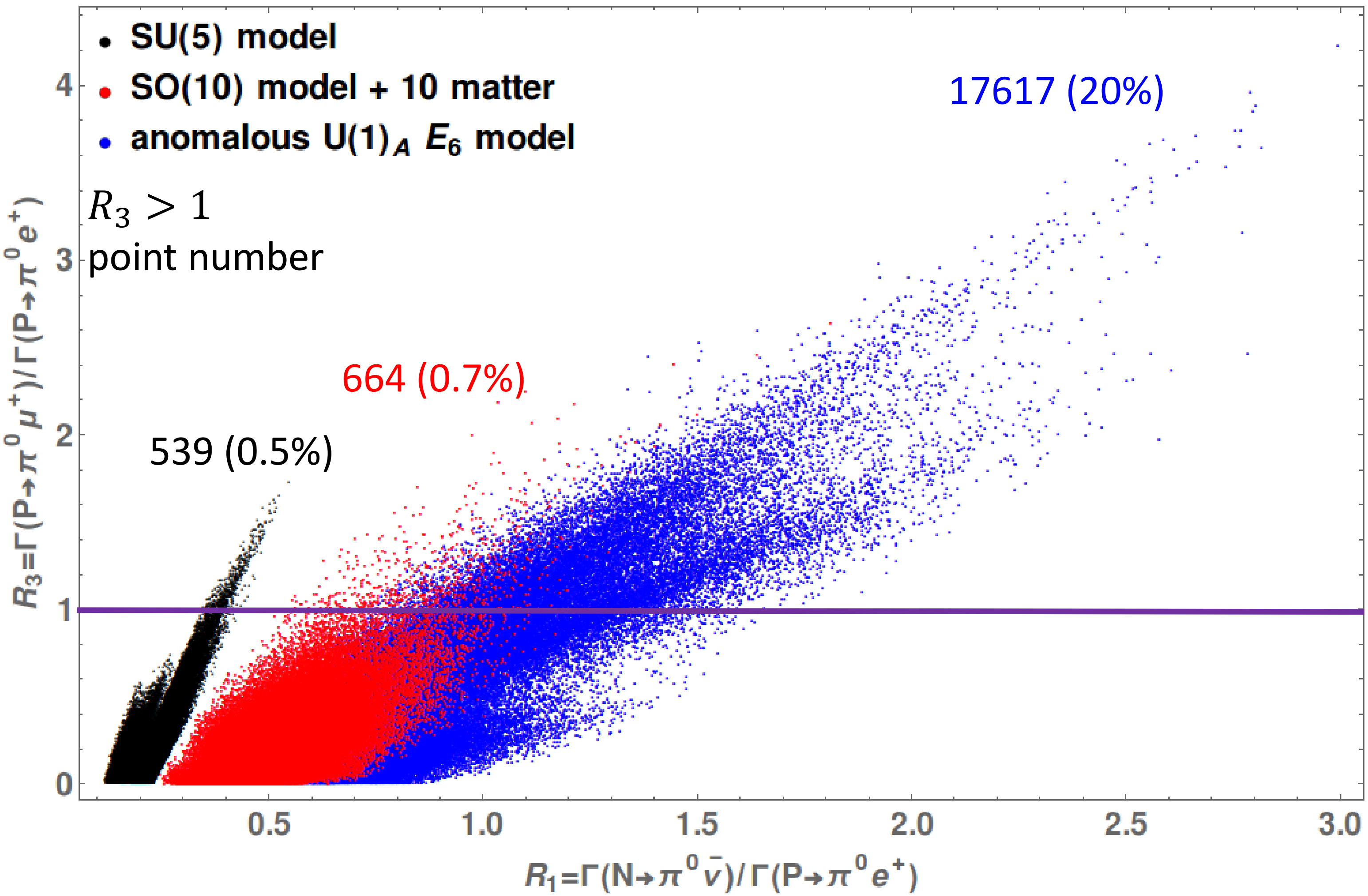
$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2} M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

R_1 vs R_3



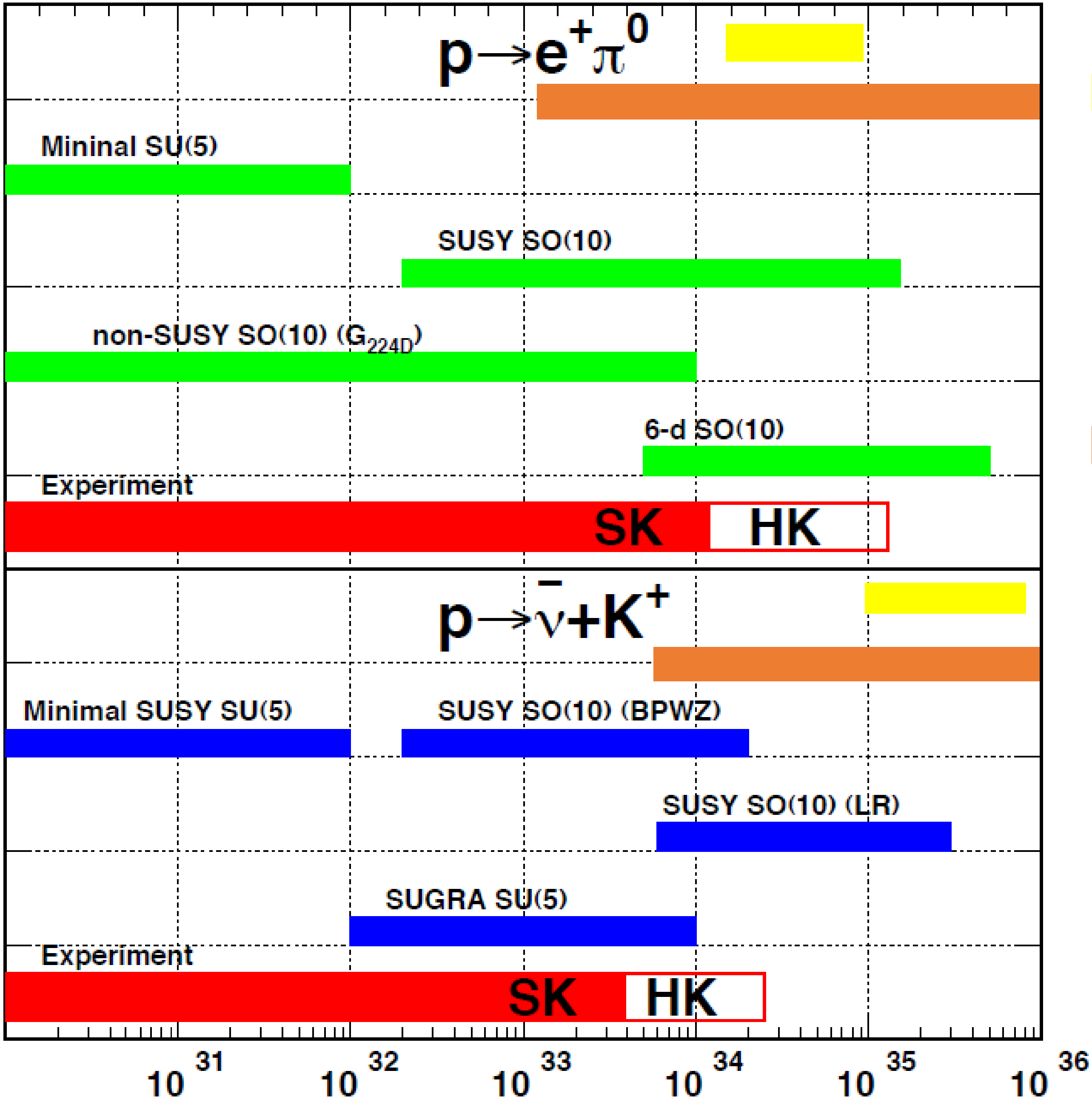
$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2}M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\bar{5}} = V_{MNS-type}$$

Summary

- Anomalous $U(1)_A$ E_6 SUSY GUT model has testable nucleon decay prediction.
- $U(1)_A$ charge determines model predictions therefore mechanisms to solve problems of GUT model induce this testable nucleon decay prediction.
- We can expect observation of nucleon decay in Hyper-Kamiokande because $M_X/g_{GUT} = 10^{16}$ GeV.

Summary



anomalous $U(1)_A$ E_6 SUSY GUT

$$\frac{M_{X_{SU(5)}}}{g_{GUT}} = \frac{M_{X_{SO(10)}}}{g_{GUT}} = \frac{\sqrt{2}M_{X_{E_6}}}{g_{GUT}} = 10^{16}\ \text{GeV}$$

anomalous $U(1)_A$ E_6 SUSY GUT

$$\frac{M_{X_{SU(5)}}}{g_{GUT}} = \frac{M_{X_{SO(10)}}}{g_{GUT}} = \frac{\sqrt{2}M_{X_{E_6}}}{g_{GUT}} = \boxed{0.5 \times 10^{16}\text{ - }2 \times 10^{16}\ \text{GeV}}$$

from $O(1)$ uncertainty

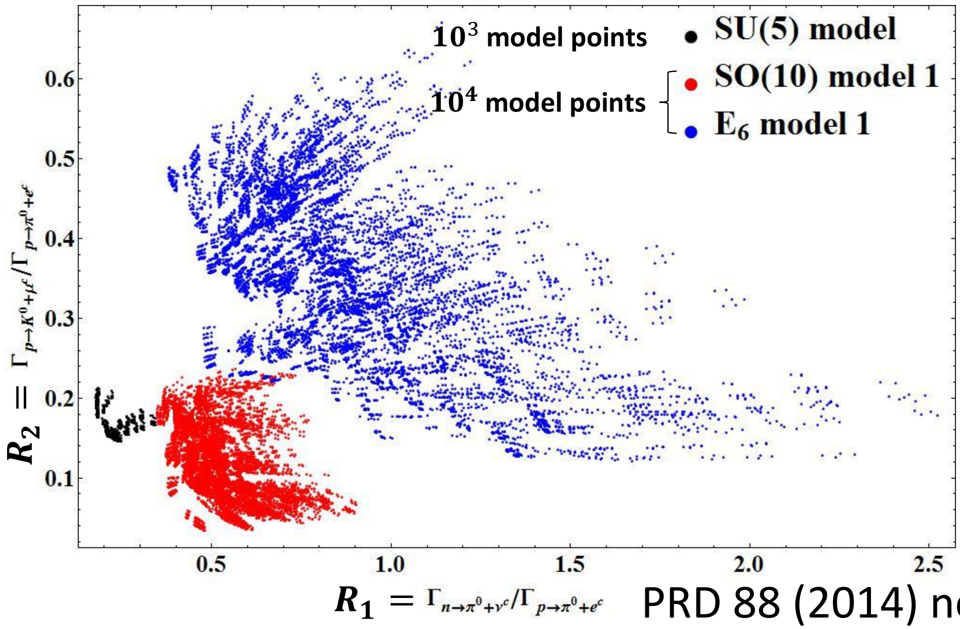
HK working group (2011)


Guan Yu(関羽)
60 m

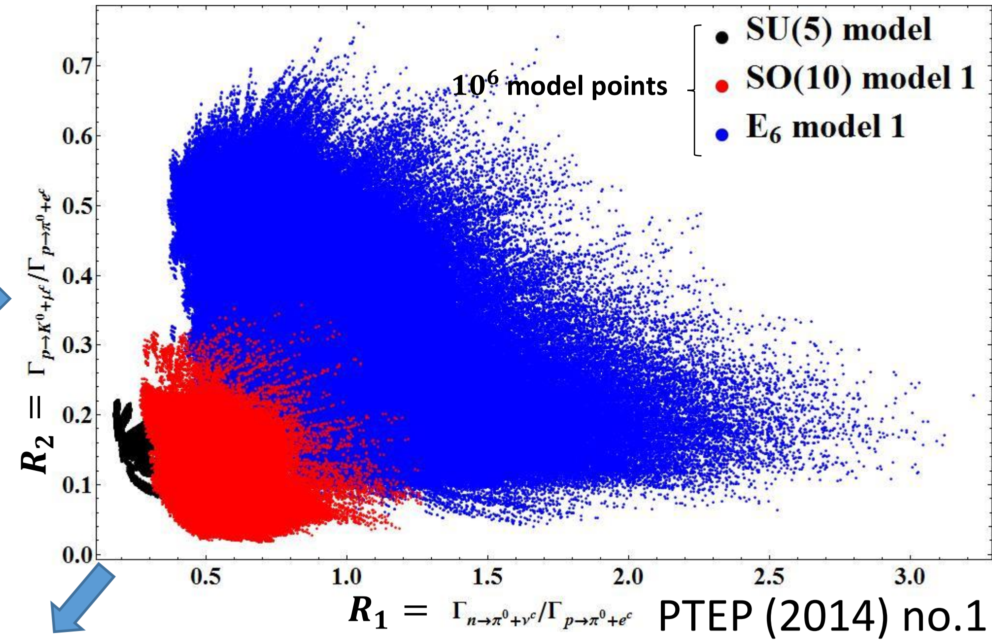
Back up

Theory

unfair assumption…



PRD 88 (2014) no.9



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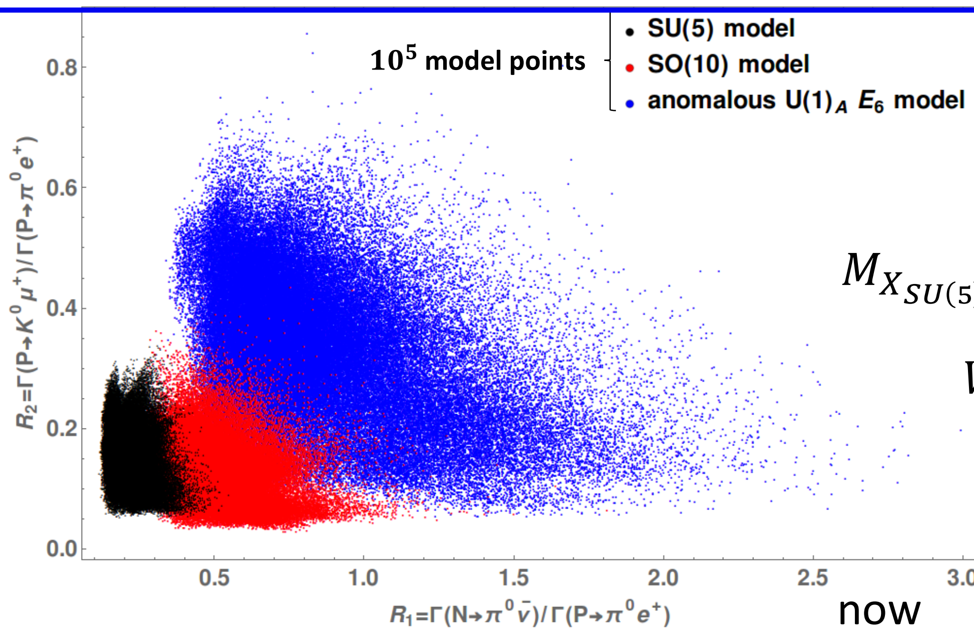
We used unfair assumption.



now

$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = \sqrt{2}M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type},$$
$$V_{\bar{5}} = V_{MNS-type}$$

Especially difference between $SU(5)$ GUT model region is large

In $SU(5)$ GUT model

Theoretically

Without loss of generality we can fix two diagonalizing matrix.

$$\overline{\psi} D_\mu \gamma^\mu \psi = \overline{\psi'} D_\mu \gamma^\mu \psi'$$

$$\psi \to \psi' = U\psi$$

In our past calculation

For $\overline{\mathbf{5}}$ matter we fix R_d.

$$d_R^c \to {d'}_R^c = R_d^\dagger d_R^c \qquad {d''}_R^c = R_d {d'}_R^c = \underline{d_R^c}$$

fix

$$e_L \to {e'}_L = L_e^\dagger e_L \qquad {e''}_L = R_d {e'}_L = \underline{R_d L_e^\dagger} e_L$$

new unitary matrix

$$\nu_L \to {\nu'}_L = L_\nu^\dagger \nu_L \qquad {\nu''}_L = R_d {\nu'}_L = \underline{R_d L_\nu^\dagger} \nu_L$$

new unitary matrix

We generator two unitary matrices and

Apply $O(1)$ test

lose generality
and
unfair assumption

In our new calculation we generate all diagonalizing matrices.

uncertainty of diagonalizing matrix is overestimated?

In minimal $SU(5)$ GUT model we can fix all diagonalizing matrix.

minimal particle contents which include SM particles
but it is hard to realize realistic quark and lepton masses and mixing... ($SU(5)$ Yukawa relation)

To realize realistic quark and lepton masses and mixing we introduce new degree of freedoms.

Especially adjoint contributions are important.

e.g.

minimal renormalizable $SU(5)$ GUT

$$Y_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H \qquad Y = Y_d = Y_e^T$$

$SU(5)$ Yukawa relation

non-renormalizable $SU(5)$ GUT

unification group breaking VEV

$$Y_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H + \frac{{Y'}_{ij}}{\Lambda}\mathbf{10}_i\mathbf{24}_H\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H + \cdots = \underline{\left(Y + Y'\frac{\langle\mathbf{24}_H\rangle}{\Lambda}\right)_{ij}}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$$

$$Y_d \neq Y_e^T$$

$\Lambda = \Lambda_{\text{Planck}}$ or smaller

$$\downarrow$$

$$\frac{\langle\mathbf{24}_H\rangle}{\Lambda} \geq 10^{-3} \Longleftrightarrow \frac{m_u}{m_t} \sim 10^{-5}$$

especially non-renormalizable terms contribute to first-second generation mixing

Hadron matrix element (form factor)

calculated by lattice - Aoki, Shintani, Soni (2013)

Matrix element	W_0^{RL}, W_0^{LR}
$\langle \pi^0 \vert (ud)u \vert p \rangle$, $\langle \pi^0 \vert (du)d \vert n \rangle$	$-0.103(23)(34)$
$\langle \pi^+ \vert (ud)d \vert p \rangle$, $-\langle \pi^- \vert (du)u \vert n \rangle$	$-0.146(33)(48)$
$\langle K^0 \vert (us)u \vert p \rangle$, $-\langle K^- \vert (ds)d \vert n \rangle$	$0.098(15)(12)$
$\langle K^+ \vert (us)d \vert p \rangle$, $-\langle K^0 \vert (ds)u \vert n \rangle$	$-0.054(11)(9)$
$\langle K^+ \vert (ud)s \vert p \rangle$, $-\langle K^0 \vert (du)s \vert n \rangle$	$-0.093(24)(18)$
$\langle K^+ \vert (ds)u \vert p \rangle$, $-\langle K^0 \vert (us)d \vert n \rangle$	$-0.044(12)(5)$
$\langle \eta \vert (ud)u \vert p \rangle$, $-\langle \eta \vert (du)d \vert n \rangle$	$0.015(14)(17)$

Statistical and systematic error is 20 and 30 percent of central value, respectively.

W_0^{RL}, W_0^{LR} (statistical error)(systematic error)

Reason why $\boldsymbol{R_1}$ is useful

R_1 (strictly speaking $\Gamma_{N\to\pi^0\bar{\nu}}$) is very useful to test unification group.

When $V_{10} = U_{CKM-type} \sim \mathbf{1}_{3\times3}, V_{\bar{5}} = V_{MNS-type}$

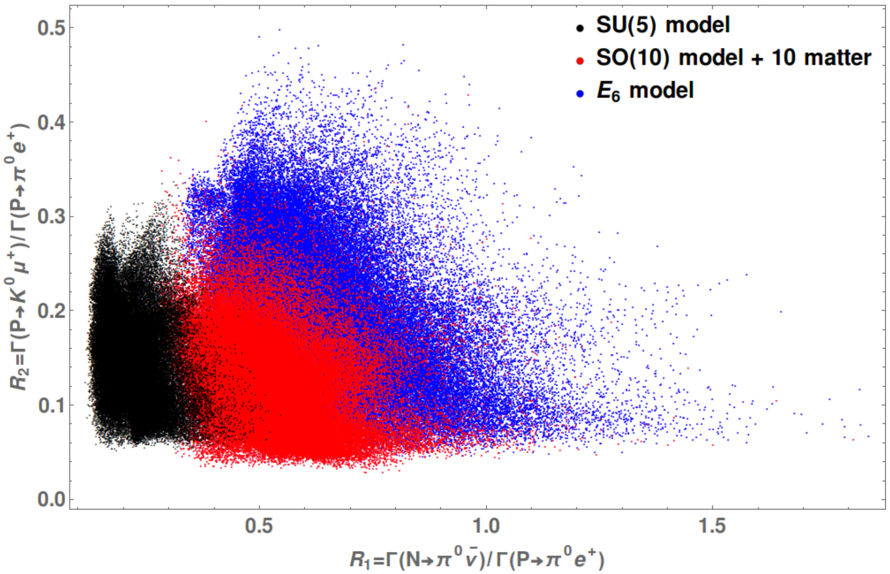
always positive contribution

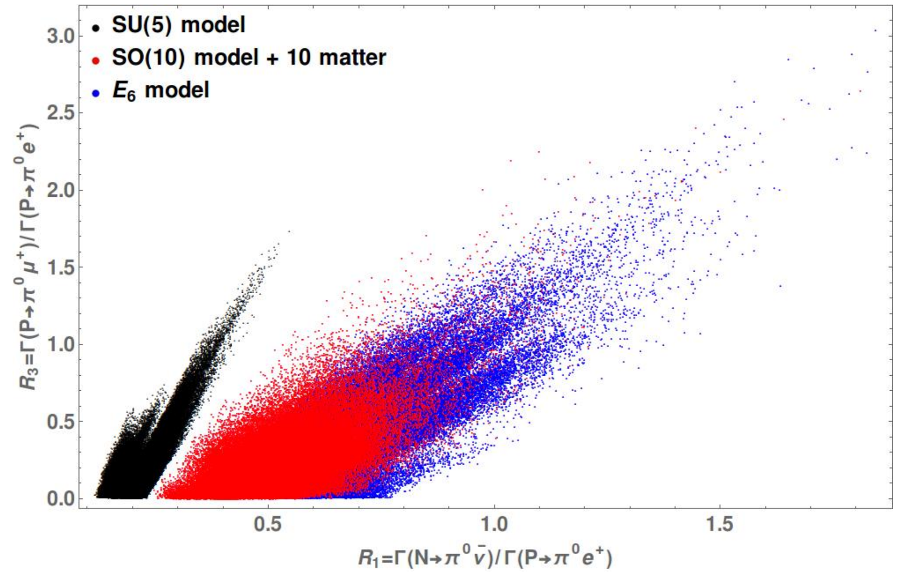
$$\frac{\Gamma_{N\to\pi^0\bar{\nu}}}{\underline{(\Gamma_{N\to\pi^0\bar{\nu}})_{SU(5)}}} = 1 + \frac{M_{X^2_{SU(5)}}}{M_{X^2_{SO(10)}}}\left(2+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{SO(10)}}}\right)|(R_d)_{11}|^2 + \frac{M_{X^2_{SU(5)}}}{M_{X^2_{E_6}}}\left(2+\frac{M_{X^2_{SU(5)}}}{M_{X^2_{E_6}}}\right)|(R_d)_{21}|^2$$

➡ $\Gamma_{N\to\pi^0\bar{\nu}}$ in $SU(5)$ GUT model

This is because in neutrino final state neutrino flavors (from electron to tau) are summed up.

Reason why $\boldsymbol{R_1}$ vs $\boldsymbol{R_3}$ plot is more useful than $\boldsymbol{R_1}$ vs $\boldsymbol{R_2}$





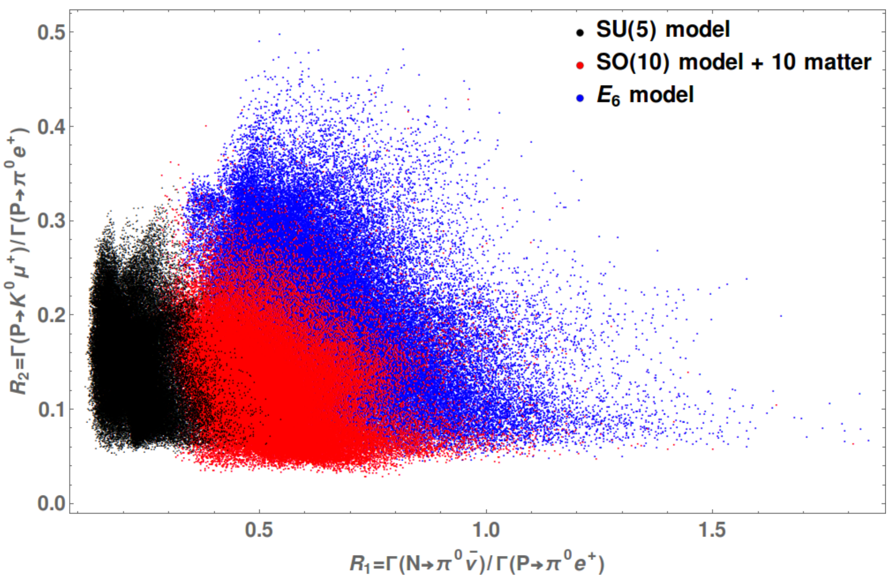
Comparison of "$SU(5)$ model" and "$SO(10)$ model + 10 matter"

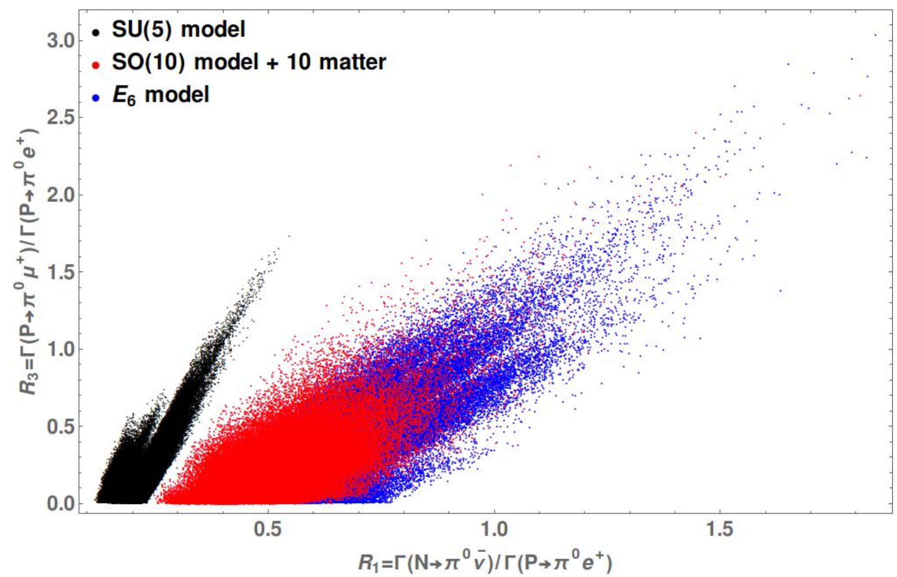
In $SO(10)$ GUT model

R_1 ↗ R_2 ↘ R_3 ↗

but these results are related to each other.
When R_1 becomes larger, then is R_2 become smaller and is R_3 becomes larger?

Reason why $\boldsymbol{R_1}$ vs $\boldsymbol{R_3}$ plot is more useful than $\boldsymbol{R_1}$ vs $\boldsymbol{R_2}$





In $SO(10)$ GUT model

$R_1 \nearrow$ $R_2 \searrow$ $R_3 \nearrow$

When $\Gamma_{P\to\pi^0e^+}$ becomes smaller

$R_1 \nearrow$ $R_2 \nearrow$ $R_3 \nearrow$

In R_1 vs R_2 plot it is not easy both R_1 and R_2 take testable value.

$\boldsymbol{P \to \pi^0 \mu^+}$ from dimension 5 operators

dimension 5 operators are induced by Yukawa interaction

second generation final state is favored

It is easy to realize $R_3 > 1$

But problem is $P \to K^0\bar{\nu}$ mode.

no observation of $P \to K^0\bar{\nu}$ mode

$R_3 > 1$

Is it possible to realize these at one time?

In $SO(10)$ (low-energy) SUSY GUT Lucas, Raby (1996)

TABLE IV. Partial mean lifetime for proton decaying into a kaon plus antineutrino and ratios of the rates of proton decay into various decay products versus rate of decay into a kaon plus antineutrino for various values of the GUT scale parameters, when the $\mathcal{O}_{13}$ operator is included.

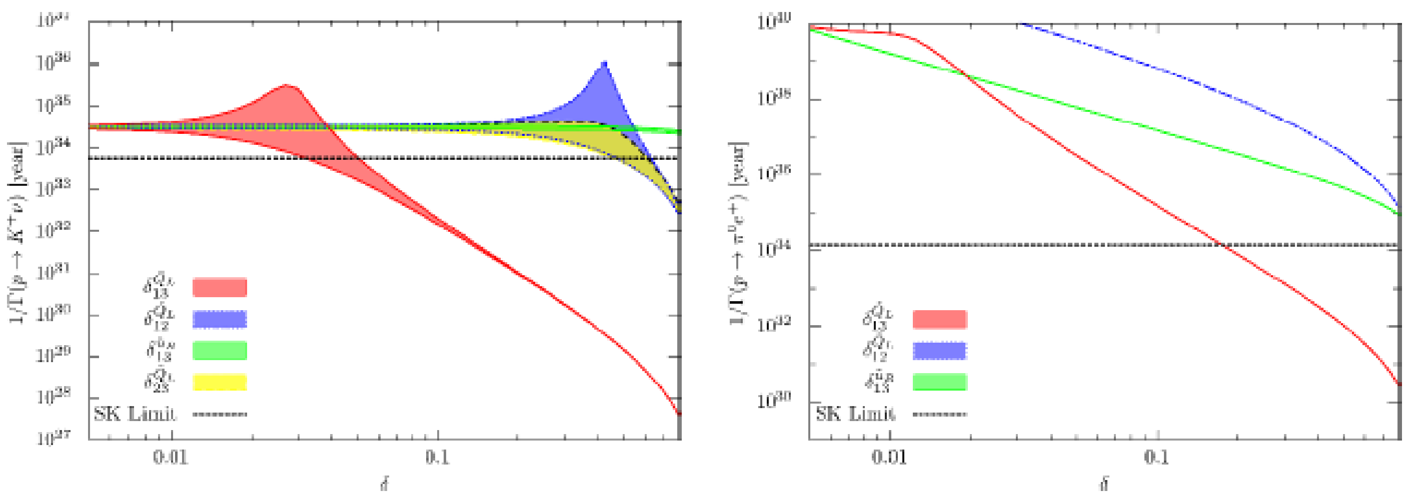
Run	$\tau(p\to K^+\overline{\nu})/(10^{32}$ yr)			$\frac{\Gamma(p\to\pi^+\overline{\nu})}{\Gamma(p\to K^+\overline{\nu})}$			$\frac{\Gamma(p\to K^0\mu^+)}{\Gamma(p\to K^+\overline{\nu})}\times 10^2$			$\frac{\Gamma(p\to \pi^0\mu^+)}{\Gamma(p\to K^+\overline{\nu})}\times 10^2$			$\frac{\Gamma(p\to \eta\mu^+)}{\Gamma(p\to K^+\overline{\nu})}\times 10^2$		
No.	max	$\beta=-\alpha$	min	max	$\beta=-\alpha$	min	max	$\beta=-\alpha$	min	max	$\beta=-\alpha$	min	max	$\beta=-\alpha$	min
I	27	15	15	0.44	0.38	0.38	0.53	0.29	0.29	0.29	0.16	0.16	0.10	0.056	0.056
II	61	38	37	0.36	0.32	0.31	0.53	0.33	0.33	0.29	0.18	0.18	0.10	0.063	0.061
III(1)	220	130	99	1.1	0.74	0.68	0.31	0.19	0.14	0.12	0.073	0.055	0.028	0.017	0.013
III(2)	150	98	75	1.5	1.0	0.93	0.37	0.23	0.18	0.11	0.071	0.054	0.017	0.011	0.0084
III(3)	110	76	59	1.5	1.1	1.0	0.34	0.23	0.18	0.092	0.063	0.049	0.011	0.0078	0.0061

very small

First signal can not be $P\to\pi^0\mu^+$.

It should be $P\to K^+\bar{\nu}$.

In $SU(5)$ split SUSY GUT Nagata, Shirai (2014)



(a) $p \to K^+\bar{\nu}$

(b) $p \to \pi^0 e^+$

(c) $p \to K^0\mu^+$

(d) $p \to \pi^0\mu^+$

possible ?
The squark and slepton observation is not possible.
Because this result comes from large flavor violation.

Dimopoulos-Wilczek (DW) mechanism

to realize DT splitting

adjoint Higgs A has DW form VEV

In $SO(10)$ GUT model VEV $\langle \mathbf{45}_A \rangle$ for 45 rep. Higgs A

$$\langle \mathbf{45}_A \rangle = i\sigma_2 \times \begin{pmatrix} x & & & & \\ & x & & & \\ & & x & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

σ_i : Pauli matrix

To realize DW form VEV we need some operators and have to forbid some other operators by SUSY zero mechanism. Therefore, $U(1)_A$ charges have following relation.

$$\Rightarrow -a < -\frac{1}{2}(c + \bar{c}) \quad (a < 0)$$

And $x \sim \lambda^{-a}\Lambda$. Therefore $x \sim \lambda^{-a}\Lambda > \lambda^{-\frac{1}{2}(c+\bar{c})}\Lambda \sim v_c$.

DT splitting

In $SO(10)$ GUT model

negative $U(1)_A$ charge to forbid HH term

$$\mathbf{10}:\underline{\mathrm{H}(h<0,+)},\mathrm{H}'(h'>0,-)$$

$$\mathbf{45}:\mathrm{A}(a<0,-)$$

Z_2 parity

$$\mathrm{W}=\lambda^{h+h'+a}\mathrm{HAH}'+\lambda^{2h'}H'H'\ (h+h'+a>0)$$

$$\mathbf{10}\to\mathbf{5}+\overline{\mathbf{5}}\,(SO(10)\to SU(5))$$

mass term for Higgs $\mathbf{10}$

$$(\mathbf{5}_H \quad \mathbf{5}_{H'})M_{10}\begin{pmatrix}\bar{\mathbf{5}}_H \\ \bar{\mathbf{5}}_{H'}\end{pmatrix}$$

M_{10} : mass matrix for Higgs $\mathbf{10}$

$$M_{10} = \begin{pmatrix} 0 & \lambda^{h+h'+a}\langle A\rangle \\ \lambda^{h+h'+a}\langle A\rangle & \lambda^{2h'}\Lambda \end{pmatrix}$$

doublet Higgs mass matrix M_D

$$M_D = \begin{pmatrix} 0 & 0 \\ 0 & \lambda^{2h'}\Lambda \end{pmatrix}$$

- one massless mode

  =SM doublet Higgs
- one massive mode

triplet Higgs mass matrix M_T

$$M_T = \begin{pmatrix} 0 & \lambda^{h+h'+a}x \\ \lambda^{h+h'+a}x & \lambda^{2h'}\Lambda \end{pmatrix}$$

- two massive modes

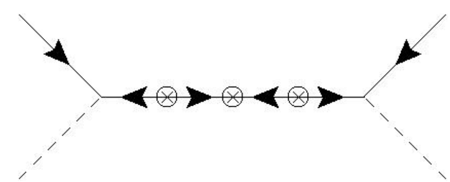
We can realize DT splitting by DW mechanism.

Nucleon decay via dimension-5 operators suppression

$$M_D = \begin{pmatrix} 0 & 0 \\ 0 & \lambda^{2h'}\Lambda_{SUSY\ GUT} \end{pmatrix} \qquad M_T = \begin{pmatrix} 0 & \lambda^{h+h'+a}\langle A\rangle \\ \lambda^{h+h'+a}\langle A\rangle & \lambda^{2h'}\Lambda_{SUSY\ GUT} \end{pmatrix}$$

$\mathbf{3}_H$ is not coupled with $\overline{\mathbf{3}}_H$ these are GUT partner for MSSM doublet doublet directly.

$$\underbrace{\begin{pmatrix}\mathbf{3}_H\\ \mathbf{2}_H\end{pmatrix}}_{\mathbf{5}_H} \overbrace{\longleftrightarrow}^{\lambda^{h+h'}} \underbrace{\begin{pmatrix}\bar{\mathbf{3}}_{H'}\\ \bar{\mathbf{2}}_{H'}\end{pmatrix}}_{\bar{\mathbf{5}}_{H'}} \overbrace{\longleftrightarrow}^{\lambda^{2h'}} \underbrace{\begin{pmatrix}\mathbf{3}_{H'}\\ \mathbf{2}_{H'}\end{pmatrix}}_{\mathbf{5}_{H'}} \overbrace{\longleftrightarrow}^{\lambda^{h+h'}} \underbrace{\begin{pmatrix}\bar{\mathbf{3}}_{H}\\ \bar{\mathbf{2}}_{H}\end{pmatrix}}_{\bar{\mathbf{5}}_{H}}$$



"Effective" triplet Higgs mass suppresses nucleon decay via dimension-5 operators.

Effective triplet Higgs mass ($m_{T\ eff}$) is

$$m_{T\ eff} \sim \frac{\lambda^{h+h'}\Lambda_{SUSY\ GUT}\cdot\lambda^{h+h'}\Lambda_{SUSY\ GUT}}{\lambda^{2h'}\Lambda_{SUSY\ GUT}} = \lambda^{2h}\Lambda_{SUSY\ GUT}$$

$$> \Lambda_{SUSY\ GUT}\ (h < 0, \lambda < 1)$$

Gauge coupling unification (GCU) in anomalous $U(1)_A$ SUSY GUT model

Assumption

i. the unification group is simple

ii. Higgs VEVs are

$$\begin{cases} \langle H^+ \rangle = 0 \ (h^+ > 0) \\ \langle H^- \rangle = \lambda^{-h^-} \Lambda \ (h^- < 0) \end{cases}$$

cut-off scale

iii. MSSM and GCU is realized without GUT particle at $\Lambda_{\mathrm{SUSY\ GUT}}$

To realize GCU in anomalous $\mathrm{U}(1)_A$ SUSY GUT model (with GUT particle contribution)

- $\Lambda \sim \Lambda_{\mathrm{SUSY\ GUT}}$

cut-off scale is around minimal $SU(5)$ SUSY GUT scale 2×10^{16} GeV

- $\tilde{h} \sim 0$

This means $m_{eff}^{H_T} \sim \Lambda_{\mathrm{SUSY\ GUT}}$

fermion masses and mixings through $\bar{\mathbf{5}}$ mixings

In $SO(10)$ GUT model add $\mathbf{10}$ rep. as SM quarks and leptons

$$\mathbf{10} \to \underbrace{D_R^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}'} + \underbrace{\overline{D_R^c}(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + \overline{L_L}(\mathbf{1},\mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}$$

$$\begin{pmatrix} \mathbf{16}_{\psi_1} \\ \mathbf{16}_{\psi_2} \\ \mathbf{16}_{\psi_3} \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix}$$

hierarchy of **16**

10 rep.

hierarchy of **10** of $SU(5)$

$$\begin{pmatrix} \mathbf{10}_{\psi_1} \\ \mathbf{10}_{\psi_2} \\ \mathbf{10}_{\psi_3} \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix}$$

realistic

hierarchy of $\bar{\mathbf{5}}$ of $SU(5)$

$\bar{\mathbf{5}}$ rep.

$$\begin{pmatrix} \bar{\mathbf{5}}_{\psi_1} \\ \bar{\mathbf{5}}_{\psi_2} \\ \bar{\mathbf{5}}_{\psi_3} \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix}$$

$(\bar{\mathbf{5}}'_T) \Rightarrow (\lambda^{2.5})$

add $\bar{\mathbf{5}}'$ from **10** of $SO(10)$

unrealistic

$$\begin{pmatrix} \bar{\mathbf{5}}_{\psi_1} \\ \bar{\mathbf{5}}'_T \\ \bar{\mathbf{5}}_{\psi_2} \\ \bar{\mathbf{5}}_{\psi_3} \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^3 \\ \lambda^{2.5} \\ \lambda^2 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \bar{\mathbf{5}}_{\psi_1} \\ \bar{\mathbf{5}}'_T \\ \bar{\mathbf{5}}_{\psi_2} \\ \bar{\mathbf{5}}_{\psi_3} \end{pmatrix} \sim \begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \\ \bar{\mathbf{5}}_M \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{\begin{matrix} \lambda^3 \\ \lambda^{2.5} \\ \lambda^2 \end{matrix}} \\ 1 \end{pmatrix}$$

massless $\bar{\mathbf{5}}$ of $SU(5)$ (SM quarks and leptons)

large Yukawa = superheavy

realistic

fermion masses and mixings through $\bar{\mathbf{5}}$ mixings

In E_6 GUT model

In $SO(10)$ GUT model, addition of $\mathbf{10}$ induces realistic quark and lepton masses and mixings.

$\mathbf{27} \to \mathbf{16} + \mathbf{10} + \mathbf{1}$

include one $\bar{\mathbf{5}}$ rep., individually $\Rightarrow$ six $\bar{\mathbf{5}}$ rep.'s

in $SO(10)$ GUT

massless $\bar{\mathbf{5}}$ rep. in $SU(5)$ group (SM quarks and leptons)

$$\begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \\ \bar{\mathbf{5}}_M \end{pmatrix} \sim \begin{pmatrix} \bar{\mathbf{5}}_{\psi_1} \\ \bar{\mathbf{5}}'_T \\ \bar{\mathbf{5}}_{\psi_2} \\ \bar{\mathbf{5}}_{\psi_3} \end{pmatrix}$$

in E_6 GUT

massless $\bar{\mathbf{5}}$ rep. in $SU(5)$ group (SM quarks and leptons)

$$\begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \\ \bar{\mathbf{5}}_{M_1} \\ \bar{\mathbf{5}}_{M_2} \\ \bar{\mathbf{5}}_{M_3} \end{pmatrix} \sim \begin{pmatrix} \bar{\mathbf{5}}_{\psi_1} \\ \bar{\mathbf{5}}'_{\psi_1} \\ \bar{\mathbf{5}}_{\psi_2} \\ \bar{\mathbf{5}}'_{\psi_2} \\ \bar{\mathbf{5}}_{\psi_3} \\ \bar{\mathbf{5}}'_{\psi_3} \end{pmatrix}$$

difference

- There are three massive $\bar{\mathbf{5}}$ rep.'s.
- $\bar{\mathbf{5}}_2$ comes from $\bar{\mathbf{5}}$ which belongs to $\mathbf{10}$ of $SO(10)$ and $\mathbf{27}_1$ of E_6.

$\boldsymbol{O(1)}$ is $\mathbf{0.5-1.5}$ for $\boldsymbol{U_{CKM-type}}$

small mixing matrix

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.22 & 0.011 \\ 0.22 & 1 & 0.048 \\ 0.011 & 0.048 & 1 \end{pmatrix} \Leftrightarrow |U_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.037 \\ 0.23 & 0.97 & 0.042 \\ 0.0087 & 0.041 & 1.0 \end{pmatrix}$$

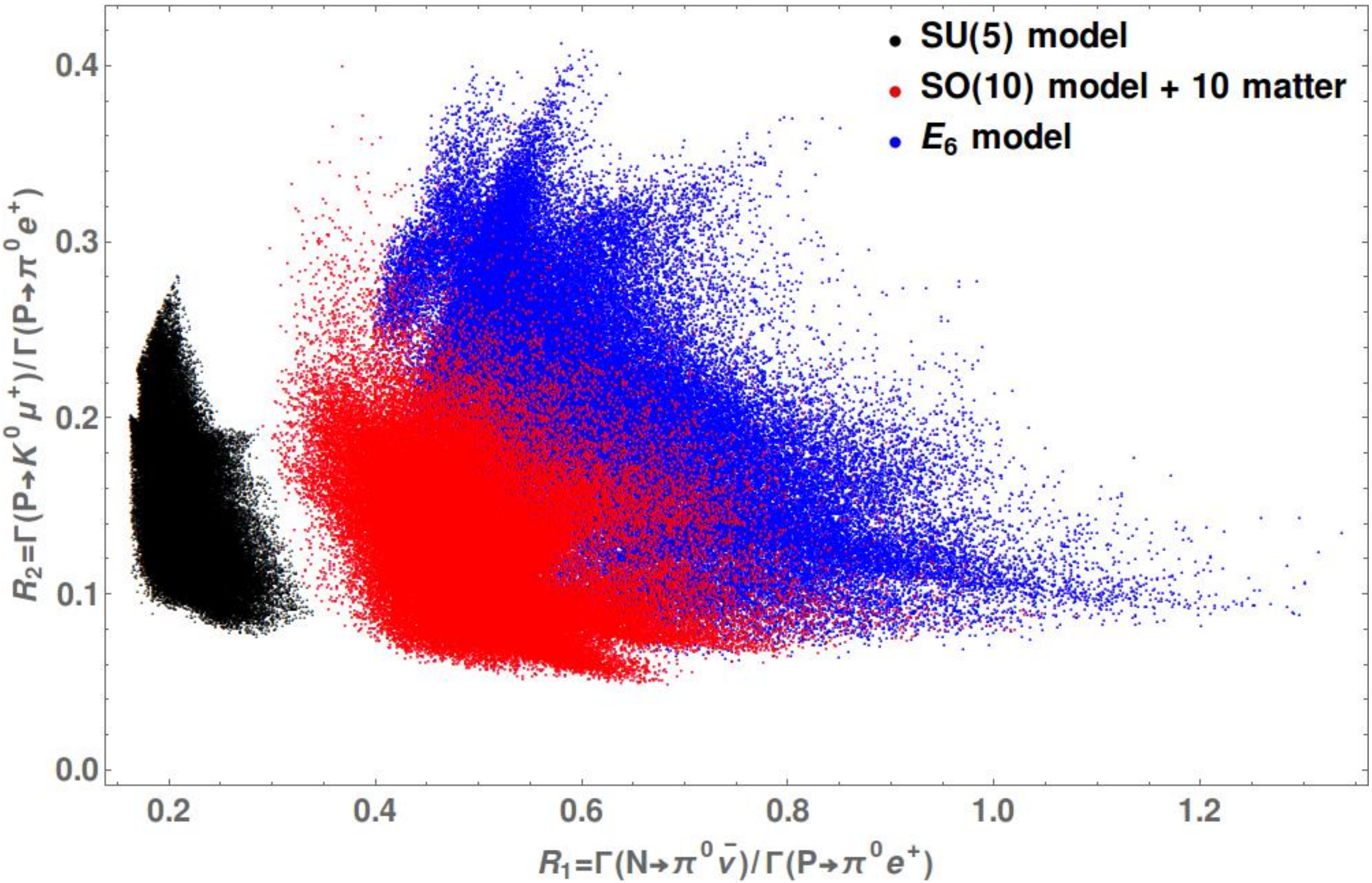
large mixing matrix

$$U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.47 & 0.22 \\ 0.47 & 1 & 0.47 \\ 0.22 & 0.47 & 1 \end{pmatrix} \Leftrightarrow |U_{MNS}| = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.47 & 0.52 & 0.71 \\ 0.31 & 0.65 & 0.69 \end{pmatrix}$$

$$\Rightarrow \quad 2 \times \left(U_{CKM-type}\right)_{12} = 0.44 \sim |(U_{MNS})_{12}| \sim |(U_{MNS})_{21}|$$

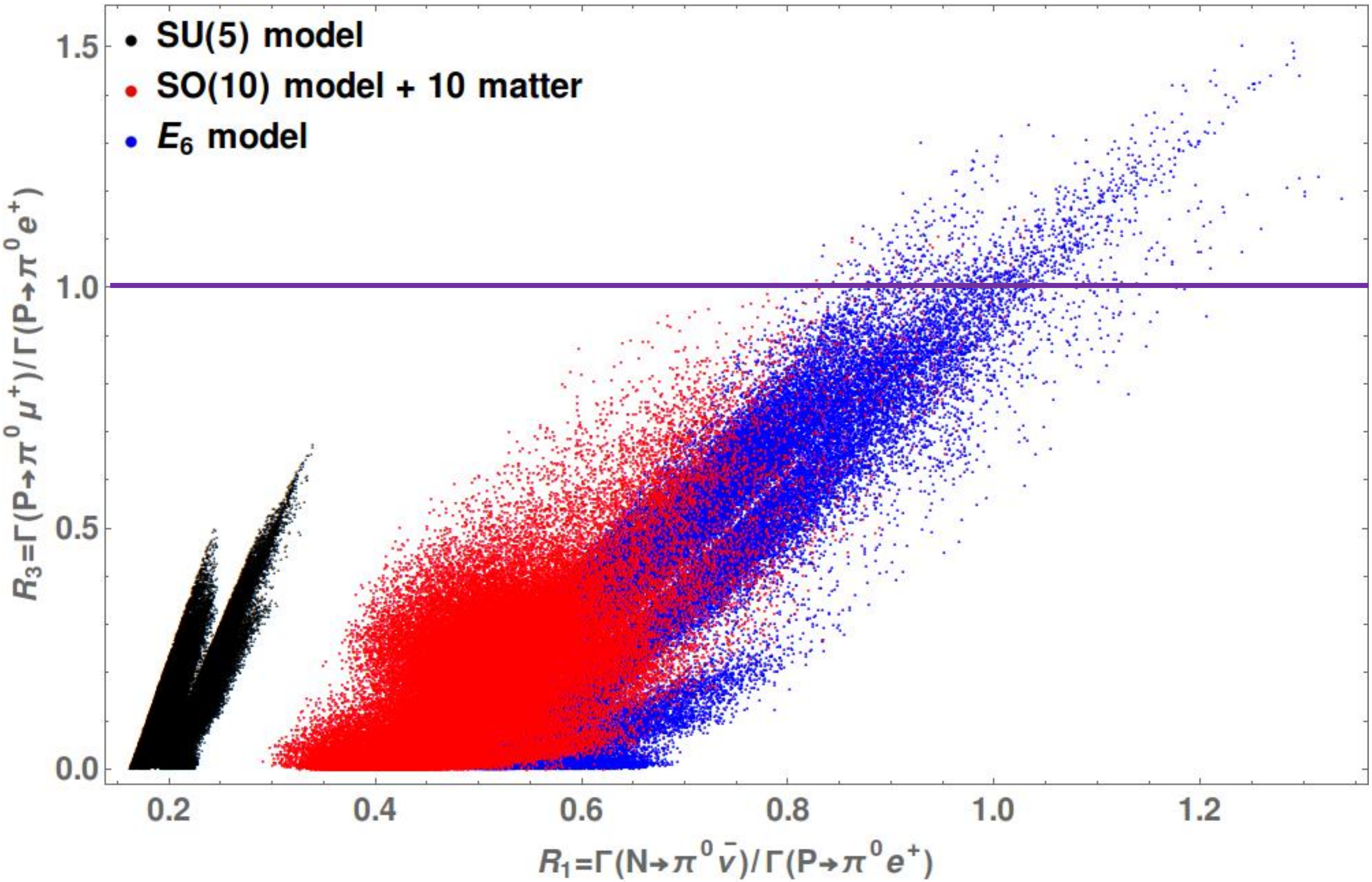
Is it small ?

Therefore we assume $O(1)$ uncertainties for small mixing matrices is 0.5-1.5.



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

$\boldsymbol{O}(\mathbf{1})$ is $\mathbf{0.5-1.5}$ for all matrices

small mixing matrix

$$U_{CKM-type} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.22 & 0.011 \\ 0.22 & 1 & 0.048 \\ 0.011 & 0.048 & 1 \end{pmatrix} \Leftrightarrow |U_{CKM}| = \begin{pmatrix} 0.97 & 0.23 & 0.037 \\ 0.23 & 0.97 & 0.042 \\ 0.0087 & 0.041 & 1.0 \end{pmatrix}$$

large mixing matrix

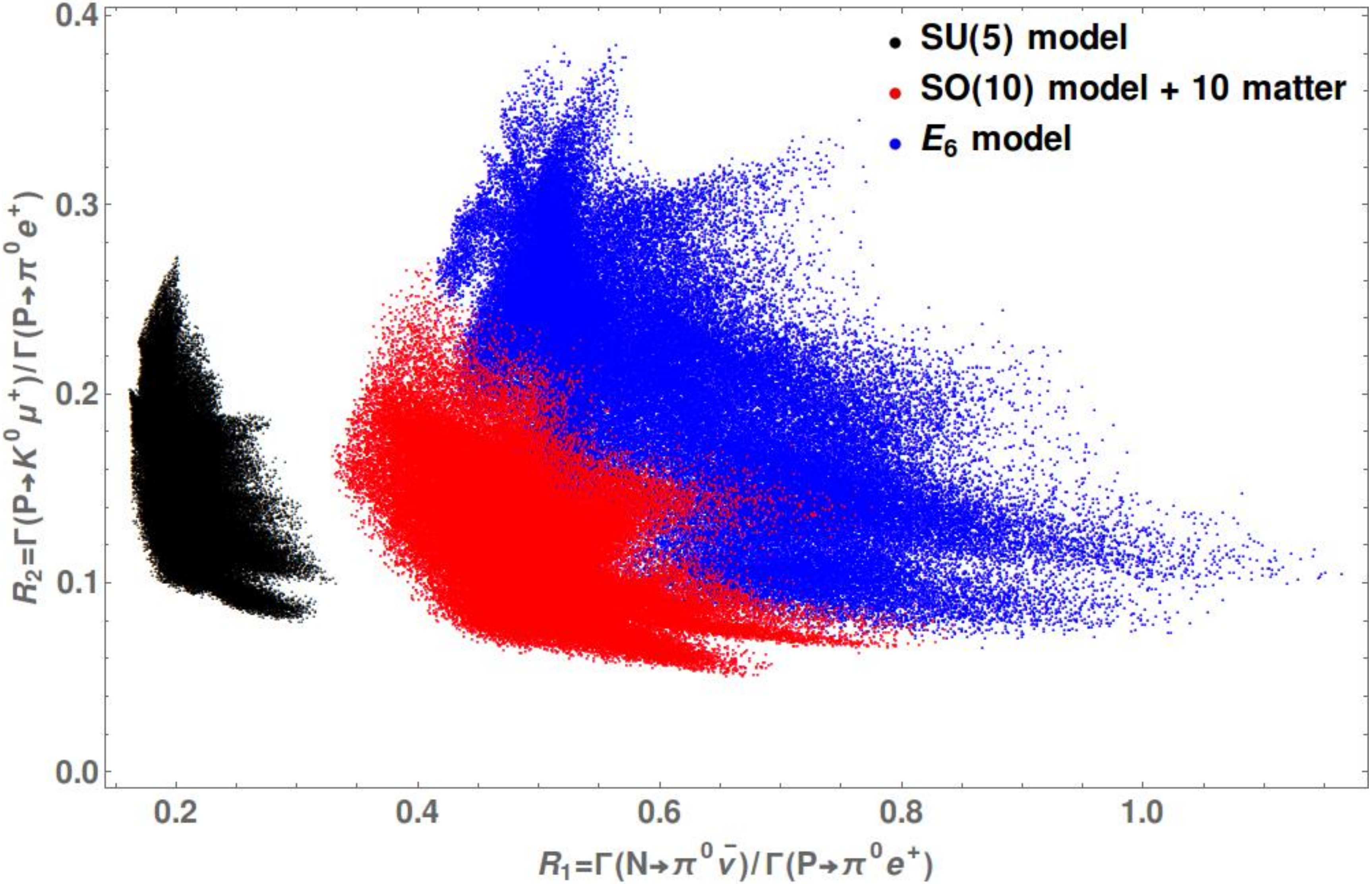
$$U_{MNS-type} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.47 & 0.22 \\ 0.47 & 1 & 0.47 \\ 0.22 & 0.47 & 1 \end{pmatrix} \Leftrightarrow |U_{MNS}| = \begin{pmatrix} 0.83 & 0.55 & 0.15 \\ 0.47 & 0.52 & 0.71 \\ 0.31 & 0.65 & 0.69 \end{pmatrix}$$

$$\Rightarrow 2 \times \left(U_{MNS-type}\right)_{12} = 0.94 > 0.7$$

larger than maximal mixing

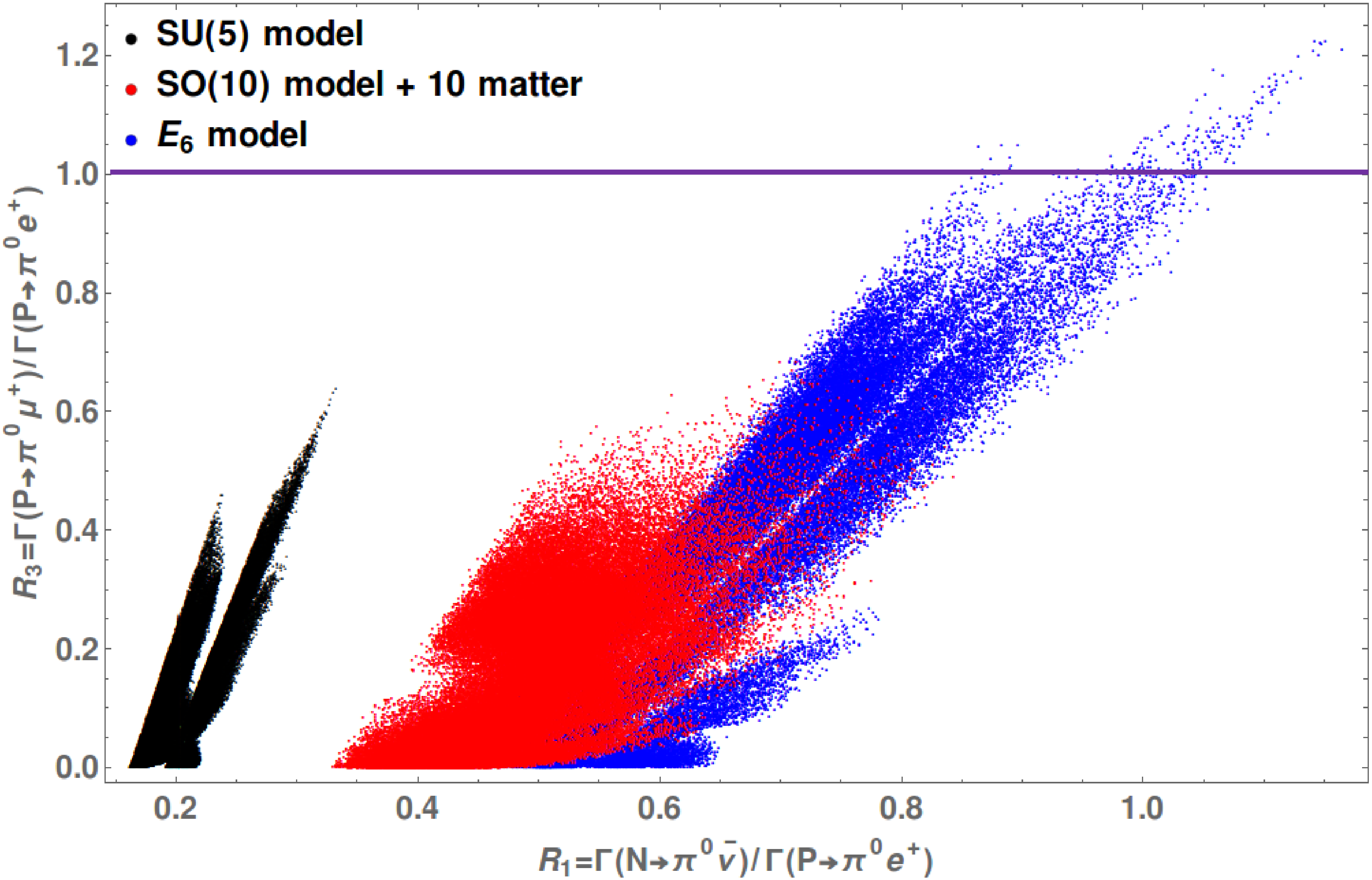
Therefore we assume $O(1)$ uncertainties for all matrices is 0.5-1.5.

$$1.5 \times \left(U_{MNS-type}\right)_{12} \sim 0.7$$



$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

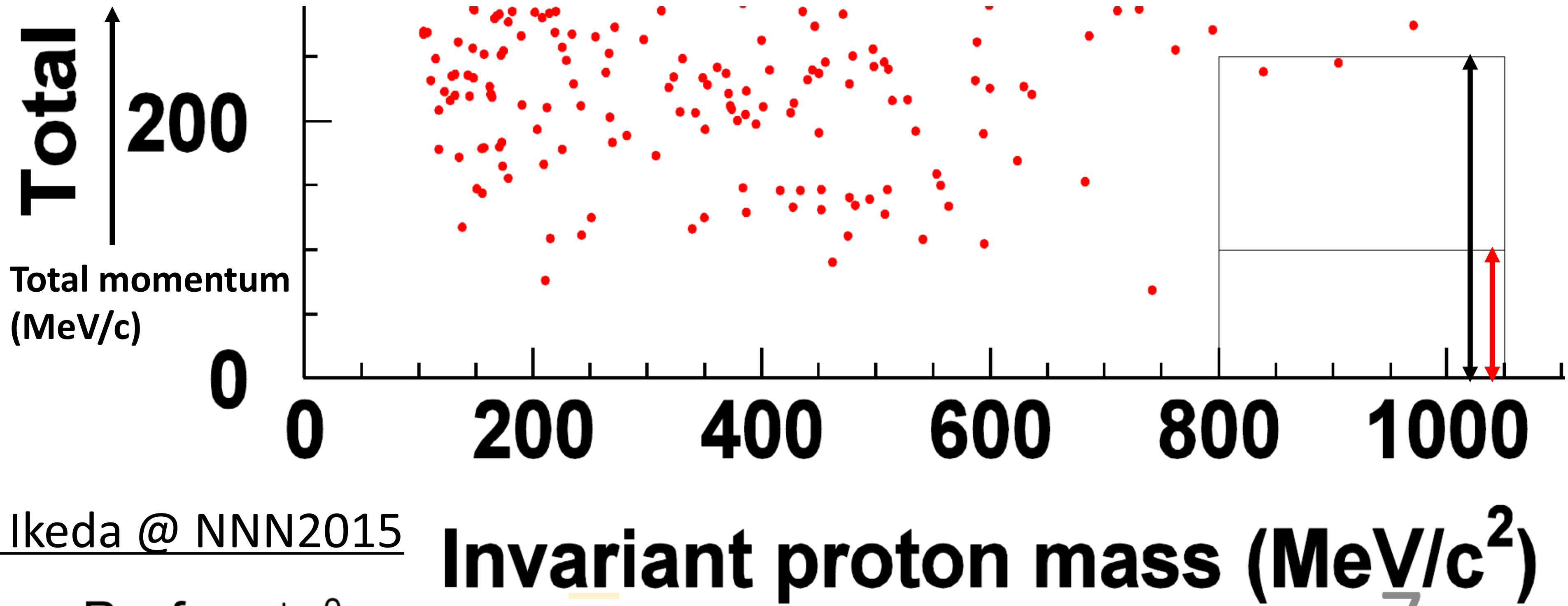


$$M_{X_{SU(5)}} = M_{X_{SO(10)}} = M_{X_{E_6}}$$

$$V_{10} = U_{CKM-type}, V_{\overline{5}} = V_{MNS-type}$$

Experiment

nucleon decay signal region



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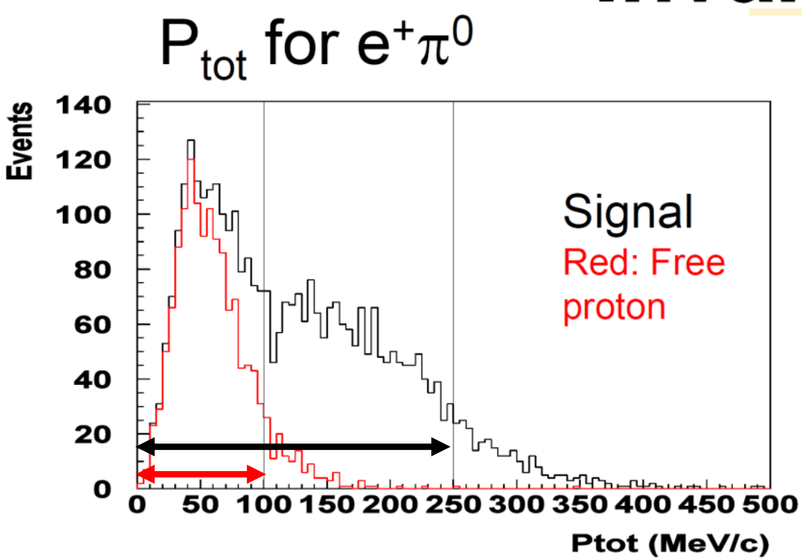


- free protons from hydrogen
- bound protons from oxygen

nuclear binding energy and fermi motion of proton

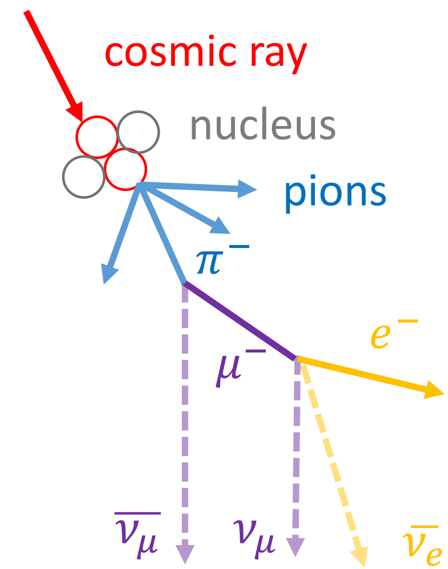
nucleon decay selection criteria

1. The number of Cherenkov rings is two or three.
2. One of the rings is e-like (μ-like) for $p \rightarrow \pi^0 e^+$ ($p \rightarrow \pi^0 \mu^+$) and all the other rings are e-like.
3. Check the meson invariant mass (if it is possible to reconstructed).
4. The number of electron from muon decay is 0 (1) for$p \rightarrow \pi^0 \mu^+$.

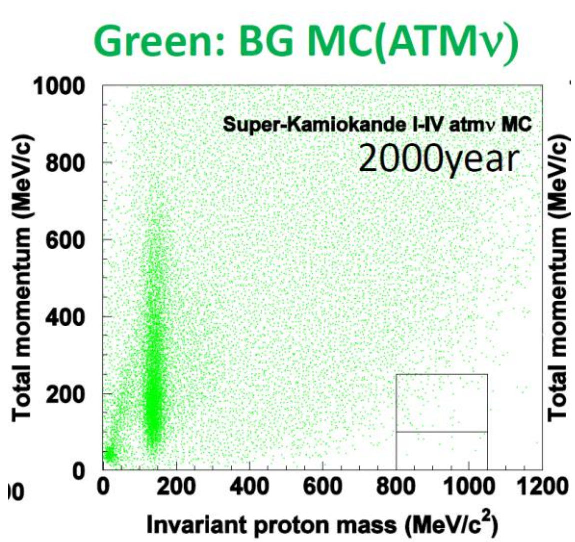
   Because of this selection criteria the efficiency for $p \rightarrow \pi^0 \mu^+$ is lower than that for $p \rightarrow \pi^0 e^+$.
5. Check the total invariant mass and the total momentum (if it is possible to reconstructed).

background to the nucleon decay search

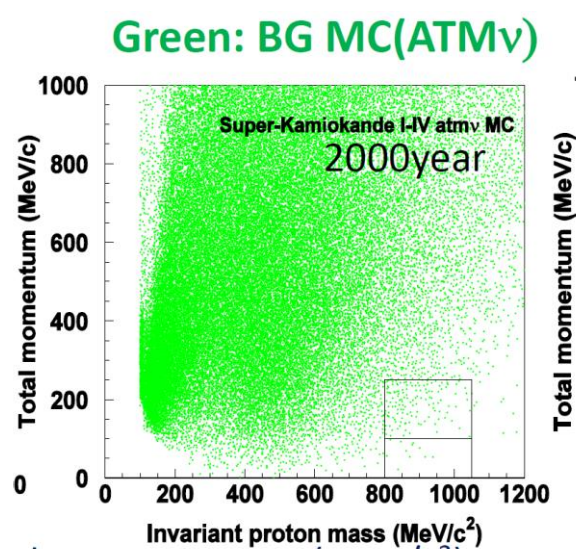
atmospheric neutrino interactions
charged current interactions : $\nu N \rightarrow lN'\pi^0$
neutral current interactions : $\nu N \rightarrow \nu N'\pi(\pi's)$



The selection criteria 4 is useful
to reduce background muon.



background for $p \rightarrow \pi^0 e^+$



background for $p \rightarrow \pi^0 \mu^+$

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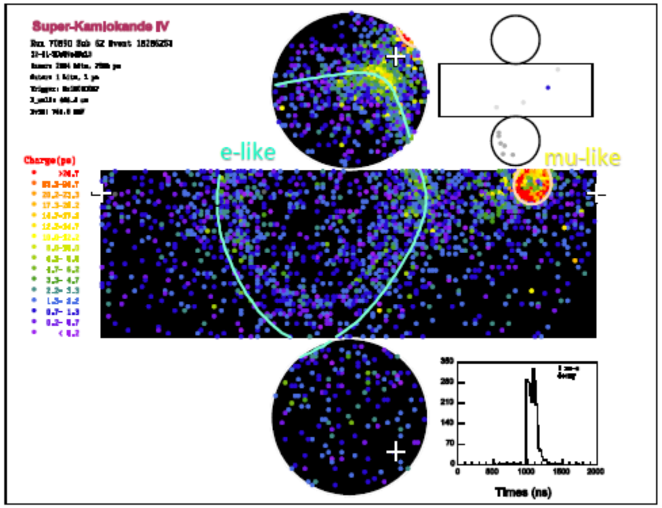
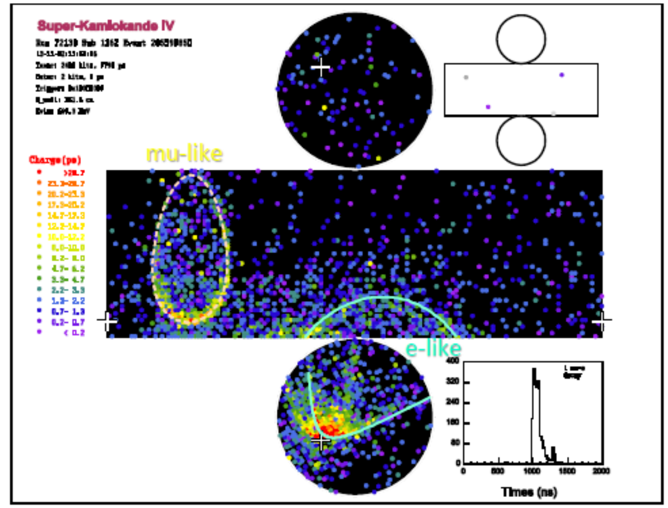
Details for the 2 candidate

Event #1	Event #2
	

Event #1

(M_p, P_{tot}) : (902.5, 248.0)MeV
Wall : 466.0cm
\# ring : 2
P_e: 374.9MeV/c
P_μ: 551.1MeV/c
$\theta_{e\text{-}mu}$: 157.9°

Event #2

(M_p, P_{tot}) : (832.4, 237.9)MeV
Wall : 351.6cm
\# ring : 2
P_e: 460.5MeV/c
P_μ: 391.3MeV/c
$\theta_{e\text{-}mu}$: 148.9°

(additional ring by manual fit → $M_{\pi 0}$: 406MeV/c^2. See supplement)

	P_{tot}<100MeV/c	100≤P_{tot}<250MeV/c
Total #BKG (SKI-IV)	~0.05	~0.82
Data(SKI-IV)	0	2

- Poisson prob. (≥2; 0.82): 19.9%

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